

On four-derivative terms in IIB Calabi-Yau orientifold reductions

Matthias Weissenbacher

Kavli Institute for the Physics and Mathematics of the Universe, University of Tokyo,
Kashiwa-no-ha 5-1-5, 277-8583, Japan

ABSTRACT

We perform a Kaluza-Klein reduction of IIB supergravity including purely gravitational α'^3 -corrections on a Calabi-Yau threefold, and perform the orientifold projection accounting for the presence of $O3/O7$ -planes. We consider infinitesimal Kähler deformations of the Calabi-Yau background and derive the complete set of four-derivative couplings quadratic in these fluctuations coupled to gravity. In particular, we find four-derivative couplings of the Kähler moduli fields in the four-dimensional effective supergravity theory, which are referred to as friction couplings in the context of inflation.

Contents

1	Introduction	1
2	The 4d four-derivative Lagrangian	3
2.1	IIB higher-derivative action	3
2.2	Supersymmetric background	4
2.3	Reduction results	5
2.3.1	Weyl rescaling	7
3	The 4d, $\mathcal{N} = 1$ action	9
3.1	Orientifold projection	9
3.2	4d, $\mathcal{N} = 1$ linear multiplets	10
4	Conclusions	12
	Appendix	12
A	Conventions, definitions, and identities	12

1 Introduction

The dimensional reduction of ten-dimensional IIB supergravity on Calabi-Yau orientifolds yields four-dimensional $\mathcal{N} = 1$ supergravity theories [1], which are of particular phenomenological interest. The resulting couplings are given by topological quantities of the internal space which are computable for explicit backgrounds, and thus provide a fruitful environment for string model building [2, 3, 4, 5, 6, 7]. The compactification on a Calabi-Yau threefold preserves a quarter of the supersymmetry of ten dimensions and thus results in a $\mathcal{N} = 2$ theory in four dimensions, which is then broken to $\mathcal{N} = 1$ by the presence of orientifold planes. Incorporating gauge fields by adding D-branes in the Calabi-Yau background one is led to introduce extended objects with negative tension to cancel gravitational and electro/magnetic tadpoles, given by

the orientifold planes, which however carry no physical degrees of freedom by themselves [8]. String theory provides an infinite series in α' of higher-derivative corrections to the leading order two-derivative IIB supergravity action. However, even the next to leading order α'^3 -correction to the four-dimensional action arising in Calabi-Yau (orientifold) compactifications are only marginally understood, but have proven to be of high relevance to string phenomenology [9, 10].

In this work, we discuss a set of four-derivative couplings that arise in four-dimensional $\mathcal{N} = 2$ and $\mathcal{N} = 1$ supergravity theories resulting from purely gravitational eight-derivative α'^3 -corrections to ten-dimensional IIB supergravity [11, 12, 13], upon compactification on a Calabi-Yau threefold and orientifold, respectively. Such corrections are of conceptual as well as of phenomenological importance. Four-dimensional $\mathcal{N} = 1$ and $\mathcal{N} = 2$ supergravity theories with four-derivative interaction terms are only marginally understood [14, 15, 16, 17], and the knowledge of the relevant couplings is desirable. A recent progress is the classification of $4d$, $\mathcal{N} = 1$ four-derivative superspace operators for ungauged chiral multiplets [18]. On the other hand higher-derivative couplings have a prominent role in phenomenological models such as inflation [19, 20, 21, 22] and have been used in the context of moduli stabilization recently [23].

Dimensionally reducing ten-dimensional IIB supergravity on a supersymmetric background must yield an effective four-dimensional $\mathcal{N} = 1$, $\mathcal{N} = 2$ supergravity theory depending on how much supersymmetry is preserved by the background. However, the supersymmetric completion at order α'^3 of IIB supergravity is not known, thus an exhaustive study of the four-derivative effective action at order α'^3 in four dimensions is out of reach. Hence our strategy will be to focus on a complete subset of the ten-dimensional IIB supergravity theory at order α'^3 and argue that the resulting couplings in the four-dimensional theory cannot be altered by any other sector of the higher-dimensional theory. More concretely, the terms we analyze in ten dimensions carry four Riemann tensors, thus are schematically of the form $\mathcal{R}^4$ and are shown to be complete [24, 25, 26]. In other words all other possible $\mathcal{R}^4$ -terms are related to this sector via a higher-derivative field-redefinition of the metric. We hence restrict our analysis to a subset of four-dimensional couplings, which can only originate from the $\mathcal{R}^4$ -sector and thus must also be complete in the above sense. In particular we focus on Kähler deformations of the internal space, which give rise to a set of real scalar fields in the external space. We do not allow for background fluxes or localized sources for D-branes in this work, furthermore we neglect higher-derivative corrections arising due to D-branes and O-planes.

It is well known that the classical Einstein-Hilbert term gives rise to the kinetic terms for the Kähler moduli. The $\mathcal{R}^4$ -sector generically corrects the couplings of the kinetic terms at order α'^3 by some expression carrying six internal space derivatives [27], which was also discussed in the context of M-theory/F-theory in [28, 29, 30, 31, 32]. However, these α' -corrections will not be addressed in this work. Furthermore, note that the two-derivative kinetic terms generically receive backreaction effects at order α'^3 from the modified supersymmetric background at this order in the string length. However, the four-derivative external terms arising from $\mathcal{R}^4$ do not

receive corrections from the modified background since these would be even higher order in α' . The interaction terms of the Kählermoduli fields with the four-dimensional metric moreover can only arise from purely gravitational terms in ten dimensions given at order α'^3 solely by the $\mathcal{R}^4$ -sector. We restrict ourselves to study four-derivative couplings at most quadratic in the infinitesimal Kählermoduli deformations. However, a complete analysis would need to also take into account cubic and quartic infinitesimal Kählermoduli deformations, which will be discussed in a forthcoming work [33].

This paper is organized as follows. In section 2 we review the relevant $\mathcal{R}^4$ -terms in ten dimensions, comment on the supersymmetric background, and discuss the four-derivative couplings quadratic in the Kählermoduli deformations, arising upon dimensional reduction on a Calabi-Yau threefold. In section 3 we then perform the orientifold projection to yield the $\mathcal{N} = 1$ couplings at fourth order in derivatives.

2 The 4d four-derivative Lagrangian

This section discusses the dimensional reduction of IIB supergravity including purely gravitational eight-derivative corrections on a Calabi-Yau threefold to four dimensions. We fluctuate the background metric by Kähler deformations and focus on couplings which carry four external space derivatives and are at most quadratic in the infinitesimal Kähler deformations. We first review the relevant α'^3 R^4 -corrections to ten-dimensional IIB supergravity and the supersymmetric background.

2.1 IIB higher-derivative action

The IIB higher-derivative action at order α'^3 has various contributions [34, 35, 36, 37, 38, 39, 40]. For the discussion at hand only the $\mathcal{R}^4$ -sector containing four ten-dimensional Riemann tensors will be relevant. This subsector of the IIB supergravity action at order α'^3 in the Einstein-frame is given by

$$S_{\text{grav}} = S_{EH} + \alpha S_{\hat{R}^4} \quad , \quad \text{with} \quad \alpha = \frac{\zeta(3)\alpha'^3}{3 \cdot 2^{10}} \quad , \quad (2.1)$$

and

$$S_{EH} = \frac{1}{2\kappa_{10}^2} \int \hat{R} \hat{*} 1 \quad , \quad (2.2)$$

where $2\kappa_{10}^2 = (2\pi)^7 \alpha'^4$. The higher-derivative contribution can be schematically written as

$$S_{\hat{R}^4} = \frac{1}{2\kappa_{10}^2} \int e^{-\frac{3}{2}\hat{\phi}} \left(t_8 t_8 + \frac{1}{8} \epsilon_{10} \epsilon_{10} \right) \hat{R}^4 \hat{*} 1 \quad , \quad (2.3)$$

where the explicit tensor contractions are given by

$$\begin{aligned}\epsilon_{10}\epsilon_{10}\hat{R}^4 &= \epsilon^{R_1 R_2 M_1 \dots M_8} \epsilon_{R_1 R_2 N_1 \dots N_8} \hat{R}^{N_1 N_2}_{M_1 M_2} \hat{R}^{N_3 N_4}_{M_3 M_4} \hat{R}^{N_5 N_6}_{M_5 M_6} \hat{R}^{N_7 N_8}_{M_7 M_8} , \\ t_8 t_8 \hat{R}^4 &= t_8^{M_1 \dots M_8} t_{8 N_1 \dots N_8} \hat{R}^{N_1 N_2}_{M_1 M_2} \hat{R}^{N_3 N_4}_{M_3 M_4} \hat{R}^{N_5 N_6}_{M_5 M_6} \hat{R}^{N_7 N_8}_{M_7 M_8} .\end{aligned}\quad (2.4)$$

Where ϵ_{10} is the ten-dimensional Levi-Civita tensor and the explicit definition of the tensor t_8 can be found in [41]. Let us note that we do not discuss higher-derivative terms of the dilaton, since we lack completeness of the ten-dimensional action. However, the complete axio-dilaton dependence of the R^4 -terms is known to be

$$S_{\hat{R}^4}^{(2)} = \frac{1}{2\kappa_{10}^2} \int E(\tau, \bar{\tau})^{3/2} \left(t_8 t_8 + \frac{1}{8} \epsilon_{10} \epsilon_{10} \right) \hat{R}^4 \star 1 , \quad (2.5)$$

where $E(\tau, \bar{\tau})^{3/2}$ is the $SL(2, \mathbb{Z})$ -invariant Eisenstein Series given by

$$E(\tau, \bar{\tau})^{3/2} = \sum_{(m,n) \neq (0,0)} \frac{\tau_2^{3/2}}{|m + n\tau|^3} , \quad (2.6)$$

with $\tau = \hat{C}_0 + ie^{-\hat{\phi}} := \tau_1 + i\tau_2$ the axio-dilaton. In the large τ_2 limit, which corresponds to the small string coupling limit (2.6) results in

$$E(\tau, \bar{\tau})^{3/2} = 2\zeta(3) \tau_2^{3/2} + \frac{2\pi^2}{3} \tau_2^{-1/2} + \mathcal{O}(e^{-2\pi\tau_2}) . \quad (2.7)$$

We will use this approximation in (2.5) in the following discussion, and only look at the leading order contribution in g_s , the string coupling, given by (2.3).

2.2 Supersymmetric background

The supersymmetric background of ten-dimensional IIB supergravity at the two-derivative level, thus at leading order in α' is given by a Calabi-Yau threefold Y_3 . For simplicity we do not consider localized sources and background fluxes, and thus the line element is given by

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + 2g_{m\bar{n}}^{(0)} dy^m dy^{\bar{n}} , \quad (2.8)$$

with $\eta_{\mu\nu}$ the Minkowski metric, where $\mu = 0, 1, 2, 3$ is a $4d$ external space world index and $m = 1, \dots, 3$ is the index of the complex three-dimensional internal Calabi-Yau manifold with metric $g_{m\bar{n}}^{(0)}$, where $m, \bar{m}$ are holomorphic and anti-holomorphic indices, respectively. Taking into account the higher-curvature corrections (2.3) in ten dimensions, (2.8) is no longer a supersymmetric background but needs to be modified such that the internal manifold is no longer Ricci flat. It was shown that the internal space metric is modified as

$$g_{m\bar{n}}^{(0)} \longrightarrow g_{m\bar{n}}^{(0)} + \alpha'^3 g_{m\bar{n}}^{(1)} , \quad (2.9)$$

where $g_{m\bar{n}}^{(1)}$ is a solution to the modified Einstein equation $R_{m\bar{n}} = \alpha'^3 \partial_m \partial_{\bar{n}} Q$, with Q the six-dimensional Euler-density (A.12), [42]. However, we can restrict our analysis to the case of the leading order metric (2.8), since at order α'^3 the four-derivative couplings only receive corrections from the $\mathcal{R}^4$ -terms evaluated on the zeroth order Calabi-Yau background. We do not incorporate for internal flux in this work, since the considered sector decouples, and also do not allow for localized D-brane sources, which would give rise to a warpfactor in (2.8).

In the following we freeze the complex structure moduli, and allow solely for the Kähler deformations given by the harmonic $(1,1)$ -forms $\{\omega_i\}$, with $i = 1, \dots, h^{1,1}$, where $h^{(1,1)} = \dim H^{(1,1)}$ the dimension of the $(1,1)$ -cohomology group. The harmonicity is w.r.t. the zeroth order Calabi-Yau metric. These give rise to the massless Kähler moduli fields by varying the background metric by

$$g_{m\bar{n}}^{(0)} \rightarrow g_{m\bar{n}}^{(0)} - i \delta v^i \omega_{i m\bar{n}} \quad , \quad (2.10)$$

where δv^i are the real scalar infinitesimal Kähler deformations.² Let us emphasize that also (2.10) receives α'^3 -corrections [30], however, these do not affect the four-derivative couplings at the relevant order in α' . A preliminary study for allowing both the complex structure deformations and Kähler deformations simultaneously at the higher-derivative level arising from the $\mathcal{R}^4$ -sector in the context of M-theory can be found in [43]. In this work we consider four-derivative couplings which are up to quadratic order in the infinitesimal Kähler deformations δv^i .

2.3 Reduction results

Compactifying the action (2.1) on the Calabi-Yau background (2.8) we expand the result at four external derivative level up to quadratic order in the infinitesimal Kähler deformations (2.10). The reduction result may be expressed entirely in terms of the second Chern-form c_2 , see (A.10), the Kähler form (A.6) and a higher-derivative object $Z_{m\bar{m}n\bar{n}}$ [14] given by

$$Z_{m\bar{m}n\bar{n}} = \frac{1}{(2\pi)^2} \epsilon_{m\bar{m}m_1\bar{m}_1m_2\bar{m}_2} \epsilon_{n\bar{n}n_1\bar{n}_1n_2\bar{n}_2} R^{\bar{m}_1m_1\bar{n}_1n_1} R^{\bar{m}_2m_2\bar{n}_2n_2} \quad . \quad (2.11)$$

Its analog for a Calabi-Yau four-fold has been encountered in the context of M-theory/F-theory in [30]. $Z_{m\bar{m}n\bar{n}}$ in (2.11) obeys the following relations

$$\begin{aligned} Z_{m\bar{m}n\bar{n}} &= -Z_{m\bar{n}n\bar{m}} = Z_{n\bar{m}m\bar{n}} & Z_{m\bar{m}} &= Z_{m\bar{m}n}{}^n = -2i(*c_2)_{m\bar{m}} & Z_{m\bar{m}}\omega_i^{\bar{m}m} &= 2i*(c_2 \wedge \omega_i) \\ Z_{m\bar{m}}g^{\bar{m}m} &= Z_m{}^m{}_n{}^n = 2*(c_2 \wedge J) & Z_{m\bar{m}n\bar{n}}R^{m\bar{m}n\bar{n}} &= -3!2\pi*c_3 \quad . \end{aligned} \quad (2.12)$$

Note that $Z_{m\bar{m}n\bar{n}}$ has the symmetry properties of the Riemann tensor build from a Kähler metric. It is itself not topological but is related to second and third Chern form of a Calabi-

²Note that we choose the fluctuation to be $-i \delta v^i \omega_{i m\bar{n}}$. The choice of sign is such that combined with the convention $J_{m\bar{n}} = i g_{m\bar{n}}$, to give a positive sign in $\delta J = \delta v^i \omega_i$.

Yau manifold of dimension $n \geq 3$. In the following we dress objects constructed from the background Calabi-Yau metric with the symbol - $^{(0)}$ - as e.g. $Z_{m\bar{m}n\bar{n}}^{(0)}$.

We have now set the stage to discuss the reduction results. By fluctuating the Calabi-Yau metric with the Kähler deformations, the higher-derivative α'^3 -terms (2.3) at two-derivative level give rise to a α'^3 -modified four-dimensional Einstein-Hilbert term [44] and α'^3 -corrections to the kinetic terms for the Kähler moduli fields [27]. The explicit form of these corrections has been also worked out in the context of M-theory on Calabi-Yau fourfolds in [28, 29, 30, 32]. The four-dimensional dilaton ϕ arises as $\hat{\phi} \rightarrow \phi$. Its internal component is constant at leading order but is given by $\phi \propto \alpha'^3 Q$ at the order of consideration. However, for the discussion at hand only the leading order constant part is relevant. The focus of this work is to derive the four-derivative corrections to the leading order two-derivative 4d Lagrangian, as discussed next. The reduction of the classical Einstein-Hilbert term gives

$$\frac{1}{2\kappa_{10}^2} \int \hat{R} \hat{*} 1 \longrightarrow \frac{1}{2\kappa_{10}} \int_{\mathcal{M}_4} \left[\Omega R + \nabla_\mu \delta v^i \nabla^\mu \delta v^j \int_{Y_3} \left(\frac{1}{2} \omega_{im\bar{n}} \omega_j^{\bar{n}m} - \omega_{im}^m \omega_{jn}^n \right) \right] *_4 1 + \mathcal{O}(\alpha), \quad (2.13)$$

with

$$\Omega = \int_{Y_3} \left[1 - i \delta v^i \omega_{im}^m + \frac{1}{2} \delta v^i \delta v^j (\omega_{im\bar{n}} \omega_j^{\bar{n}m} - \omega_{im}^m \omega_{jn}^n) \right] *_6 1, \quad (2.14)$$

where the $\mathcal{O}(\alpha)$ corrections in (2.13) arise due to the mentioned α'^3 -modification of the background. However, these terms do not interfere with our analysis. It is necessary to consider the Weyl rescaling factor (2.14) up to order $(\delta v)^2$. The four-derivative corrections arising from the ten-dimensional $\mathcal{R}^4$ -terms result in

$$\begin{aligned} & \frac{1}{2\kappa_{10}^2} \int e^{-\frac{3}{2}\hat{\phi}} \left(t_8 t_8 + \frac{1}{8} \epsilon_{10} \epsilon_{10} \right) \hat{R}^4 \hat{*} 1 \longrightarrow \\ & \frac{192(2\pi)^2}{2\kappa_{10}^2} \int_{M_4} e^{-\frac{3}{2}\phi} \left[\left[4 R_{\mu\nu} R^{\mu\nu} - R^2 \right] \left(\int_{Y_3} c_2^{(0)} \wedge J^{(0)} + \delta v^i \int_{Y_3} c_2^{(0)} \wedge \omega_i + \delta v^i \delta v^j \int_{Y_3} \delta_j (c_2^{(0)} \wedge \omega_i) \right) \right. \\ & \quad + \left[\left(-2R_{\mu\nu} + \frac{1}{2} R g_{\mu\nu} \right) \nabla^\mu \delta v^i \nabla^\nu \delta v^j + \nabla_\mu \nabla^\mu \delta v^i \nabla_\nu \nabla^\nu \delta v^j \right] \int_{Y_3} Z_{m\bar{m}n\bar{n}}^{(0)} \omega_i^{\bar{m}m} \omega_j^{\bar{n}n} *_4 1 \\ & \quad \left. - 2 \nabla_\mu \nabla_\nu \delta v^i \nabla^\mu \nabla^\nu \delta v^j \int_{Y_3} Z_{m\bar{m}n\bar{n}}^{(0)} \omega_i^{\bar{m}m} \omega_j^{\bar{n}n} *_4 1 \right] *_4 1, \end{aligned} \quad (2.15)$$

where δ_i denotes the variation resulting from the metric shift (2.10). Note that

$$\int_{Y_3} \delta_j (c_2^{(0)} \wedge \omega_i) = 0, \quad (2.16)$$

since $c_2 \wedge \omega_i$ is a topological quantity and hence its variation results in a total derivative. Furthermore, let us note that the four-dimensional Euler-density is given by

$$e(\nabla) = R^2 - 4R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}, \quad \text{with} \quad \int_{M_4} e(\nabla) *_4 1 = \chi(M_4), \quad (2.17)$$

where $\chi(M_4)$ is the Euler-characteristic of the external space M_4 . Comparing (2.17) to (2.15) one infers that one may express the reduction result at zeroth order in δv^i in terms of $R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$,

plus the topological term dependent on $\chi(M_4)$. However, we will not perform this substitution since there is a more intuitive way of expressing the result as we will discuss in the next section. Let us stress that (2.15) is not the complete reduction result at the four-derivative level arising from the $\mathcal{R}^4$ -sector, but we have neglected terms cubic and quartic in the fluctuations δv^i . Their derivation is crucial for a complete understanding, and we refer the reader to future work.

2.3.1 Weyl rescaling

In this section we perform the Weyl rescaling of the four-dimensional action composed of (2.13) and (2.15) to the canonical Einstein-frame. Furthermore, we discuss the extension of the infinitesimal Kähler deformations to finite fields. The Weyl rescaling of the classical Einstein-Hilbert term gives

$$\frac{1}{2\kappa_{10}^2} \int_{\mathcal{M}_4} \Omega R * 1 \xrightarrow{\text{Weyl}} \frac{1}{(2\pi)^4 \alpha'} \int_{\mathcal{M}_4} R * 1 - \frac{3}{2} \nabla_\mu \delta v^i \nabla^\mu \delta v^j \frac{1}{\mathcal{V}^{(0)2}} \mathcal{K}_i^{(0)} \mathcal{K}_j^{(0)} * 1 . \quad (2.18)$$

Where we have used identities (A.15) for the intersection numbers $\mathcal{K}_i^{(0)}, \mathcal{K}_{ij}^{(0)}, \mathcal{K}_{ijk}^{(0)}$, whose definitions are given in (A.14). Moreover, note that from the definition (A.14) it is manifest that the volume $\mathcal{V}^{(0)}$ and the intersection numbers $\mathcal{K}_i^{(0)}, \mathcal{K}_{ij}^{(0)}, \mathcal{K}_{ijk}^{(0)}$ are dimensionless and are expressed in terms of the length scale α' . In this conventions also the fields δv^i are dimensionless.

Due to the appearance of the four-derivative term the Weyl rescaling of the action is more involved. One may show that by using (A.16) and (A.17) up to total derivative contributions at order α'^3 one finds

$$\begin{aligned} \int_{M_4} e^{-\frac{3}{2}\phi} \left[\begin{aligned} & \left[4 R_{\mu\nu} R^{\mu\nu} - R^2 \right] \left(\int_{Y_3} c_2^{(0)} \wedge J^{(0)} + \delta v^i \int_{Y_3} c_2^{(0)} \wedge \omega_i \right) \end{aligned} \right] * 1 \\ \xrightarrow{\text{Weyl}} \\ \int_{M_4} e^{-\frac{3}{2}\phi} \left[\begin{aligned} & + \left[4 R_{\mu\nu} R^{\mu\nu} - R^2 \right] \left(\int_{Y_3} c_2^{(0)} \wedge J^{(0)} + \delta v^i \int_{Y_3} c_2^{(0)} \wedge \omega_i \right) \\ & - \left[4 R_{\mu\nu} - R g_{\mu\nu} \right] \nabla^\mu \nabla^\nu \delta v^i \frac{2}{\mathcal{V}^{(0)}} \mathcal{K}_i^{(0)} \left(\int_{Y_3} c_2^{(0)} \wedge J^{(0)} + \delta v^i \int_{Y_3} c_2^{(0)} \wedge \omega_i \right) \\ & - \left[4 R_{\mu\nu} - R g_{\mu\nu} \right] \nabla^\mu \delta v^i \nabla^\nu \delta v^j \frac{1}{\mathcal{V}^{(0)2}} \mathcal{K}_j^{(0)} \mathcal{K}_i^{(0)} \int_{Y_3} c_2^{(0)} \wedge J^{(0)} \\ & + \left[4 \nabla_\mu \nabla_\nu \delta v^i \nabla^\nu \nabla^\mu \delta v^j - \nabla_\mu \nabla^\mu \delta v^i \nabla_\nu \nabla^\nu \delta v^j \right] \frac{1}{\mathcal{V}^{(0)2}} \mathcal{K}_i^{(0)} \mathcal{K}_j^{(0)} \int_{Y_3} c_2^{(0)} \wedge J^{(0)} \end{aligned} \right] *_4 1 + \dots \end{aligned} \quad (2.19)$$

The ellipses denote terms where more than two fields δv^i carry derivatives and furthermore terms, which have derivatives acting on the dilaton. An exhaustive derivation of the four-derivative dilaton action would require the knowledge of the ten-dimensional higher-derivative dilaton action [36, 38, 39, 40], which lacks completeness and is hence beyond the scope of our study.

Before collecting the contributions arising due to the Weyl rescaling (A.16), (A.17) and combining it with the reduction results (2.13) and (2.15) let us first lift the infinitesimal Kähler fluctuations around the background metric to full fields. We proceed by making the naive replacement $v^i = v^{(0)i} + \delta v^i$, where $J^{(0)} = v^{(0)i} \omega_i$ is the background Kähler form. This substitution is straightforward when the couplings are given by topological quantities as in the case of them being intersection numbers, where one simply infers e.g. $\mathcal{K}_i^{(0)} \rightarrow \mathcal{K}_i$. Analogously, one infers in the case of the topological higher-derivative coupling that

$$\int_{Y_3} c_2^{(0)} \wedge J^{(0)} + \delta v^i \int_{Y_3} c_2^{(0)} \wedge \omega_i \longrightarrow \int_{Y_3} c_2 \wedge J , \quad (2.20)$$

where $J = v^i \omega_i$, and c_2 is constructed from the metric $g_{m\bar{n}} = -i v^i \omega_i$. However, the uplift of the coupling $\int_{Y_3} Z_{m\bar{m}n\bar{n}}^{(0)} \omega_i^{\bar{m}m} \omega_j^{\bar{n}n}$ is less trivial since it does not represent a topological quantity of the internal Calabi-Yau threefold. We will write the uplift of this coupling in the action by naively replacing the background metric by $g_{m\bar{n}}$, thus one yields $\int_{Y_3} Z_{m\bar{m}n\bar{n}} \omega_i^{\bar{m}m} \omega_j^{\bar{n}n}$. However, a more refined analysis would be required to fully justify this choice.

Combining the uplift of the reduction result (2.13), (2.15) and the terms, which arose due to Weyl rescaling (A.16) and (A.17), and by using the definition $\mathcal{G}_{\mu\nu} := R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R$, which is defined in close analogy to the Einstein tensor³ one finds

$$\begin{aligned} S_{\text{kin}} = \frac{1}{(2\pi)^4 \alpha'} \int_{M_4} \left[R + \nabla_\mu v^i \nabla^\mu v^j \frac{1}{v} \left(\frac{1}{2} \mathcal{K}_{ij} - \frac{1}{v} \mathcal{K}_i \mathcal{K}_j \right) + \frac{\zeta(3) \alpha'}{4} e^{-\frac{3}{2} \phi} \left(\mathcal{G}_{\mu\nu} \mathcal{G}^{\mu\nu} \mathcal{Z} - \mathcal{G}_{\mu\nu} \nabla^\nu \nabla^\mu v^i \left(\frac{2}{v} \mathcal{K}_i \mathcal{Z} \right) \right. \right. \\ \left. \left. + \mathcal{G}_{\mu\nu} \nabla^\mu v^i \nabla^\nu v^j \left(\mathcal{Z}_{ij} - \frac{1}{v^2} \mathcal{K}_i \mathcal{K}_j \mathcal{Z} \right) - \frac{1}{2} \nabla_\mu \nabla^\mu v^i \nabla_\nu \nabla^\nu v^j \left(\mathcal{Z}_{ij} + \frac{1}{2v^2} \mathcal{K}_i \mathcal{K}_j \mathcal{Z} \right) \right. \right. \\ \left. \left. + \nabla_\mu \nabla_\nu v^i \nabla^\mu \nabla^\nu v^j \left(\mathcal{Z}_{ij} + \frac{1}{v^2} \mathcal{K}_i \mathcal{K}_j \mathcal{Z} \right) \right] \star_4 1 . \quad (2.21) \end{aligned}$$

Where we have used the dimensionless quantities

$$\mathcal{Z} = \frac{1}{2\pi\alpha'} \int_{Y_3} c_2 \wedge J , \quad \mathcal{Z}_i = \frac{1}{2\pi\alpha'} \int_{Y_3} c_2 \wedge \omega_i , \quad \mathcal{Z}_{ij} = -\frac{1}{4\pi\alpha'} \int_{Y_3} Z_{m\bar{m}n\bar{n}} \omega_i^{\bar{m}m} \omega_j^{\bar{n}n} \star 1 , \quad (2.22)$$

obeying the relations

$$\mathcal{Z}_i = \mathcal{Z}_{ij} v^j = \mathcal{Z}_{ji} v^j \quad \text{and} \quad \mathcal{Z} = \mathcal{Z}_i v^i , \quad (2.23)$$

which can be seen by using (2.12). Note that as expected $\frac{\delta}{\delta v^i} \mathcal{Z} = \mathcal{Z}_i$ but $\frac{\delta}{\delta v^j} \mathcal{Z}_i = 0$, thus $\mathcal{Z}_{ij}$ cannot be obtained easily by taking derivatives w.r.t. δv^i . Let us stress that we have neglected α' -corrections to the two-derivative part of this action [27], since those will not interfere with the four-derivative couplings. Furthermore, note that due to the uplift to finite fields v^i , terms in (2.21) may have a higher power in the fields v^i , in contrast to the quadratic dependence of the infinitesimal Kähler deformations. Let us close this section by remarking that the higher-derivative effective action (2.21) can be rewritten using field redefinitions involving higher-derivative pieces themselves. Thus the given presentation is a particular choice, which results

³ The Einstein tensor is given by $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$.

naturally after dimensional reduction. However, one may perform field redefinitions as e.g.

$$g_{\mu\nu} \rightarrow g_{\mu\nu} + aR_{\mu\nu} + bRg_{\mu\nu} \quad a, b \in \mathbb{R} . \quad (2.24)$$

One concludes that the higher-derivative couplings in (2.21) are presented in one particular frame of the fields $g_{\mu\nu}$ and v^i . A more sophisticated analysis of the supersymmetric completion at the four-derivative level would be required to select a canonical frame.

3 The 4d, $\mathcal{N} = 1$ action

In this section we perform the orientifold projection on the effective action (2.21), which amounts to adding $O3/O7$ planes to the Calabi-Yau background [8, 45, 46, 47, 48]. For consistency we are required to also consider $D3/D7$ branes in this setup. However, we will not discuss any α'^3 -corrections arising from these sources, but let us emphasize that a complete treatment would require such a refined analysis. Already at the classical level these would source a warp-factor and background fluxes, which we chose not to account for.

In 3.1 we review the well known properties of the orientifold projection on Calabi-Yau threefolds [1, 49], and apply it to the four-derivative effective action derived in the previous section. We then proceed in 3.2 by expressing the truncated spectrum in terms of the real scalar fields of the linear multiplet of 4d, $\mathcal{N} = 1$ supergravity.

3.1 Orientifold projection

In the following we consider $O3/O7$ planes in the Calabi-Yau threefold background, known as Calabi-Yau orientifold, denoted in the following as X . The presence of orientifold planes truncates the effective theory from $\mathcal{N} = 2$ to $\mathcal{N} = 1$ supersymmetry. Orientifold planes manifest themselves as an isometric, holomorphic involution $\sigma : X \rightarrow X$, thus $\sigma^2 = id$ and $\sigma^*g = g$ on the internal Calabi-Yau space with metric g , such that

$$\sigma^*J = J . \quad (3.1)$$

Moreover, the presence of $O3/O7$ planes results in $\sigma^*\Omega = -\Omega$, where Ω is the holomorphic $(3,0)$ -form. Furthermore, considering the action of $\Omega_p(-1)^{F_L}$ on the space-time fields, where Ω_p is the world-sheet parity and F_L the space-time fermion number of the left moving sector, one finds that

$$\Omega_p(-1)^{F_L}\phi = \phi \quad \text{and} \quad \Omega_p(-1)^{F_L}g = g . \quad (3.2)$$

The cohomology groups $H^{p,q}$ naturally decompose in odd and even eigenspaces under the action of σ^* as $H^{p,q} = H_+^{p,q} \oplus H_-^{p,q}$. Since the Kähler form is invariant under the orientifold projection

(3.1), only the Kähler deformations related to the even eigenspace $H_+^{1,1}$ remain in the spectrum, such that $J = v^a \omega_a$, $a = 1, \dots, h_+^{1,1}$.

Subjected to the orientifold projection the reduction result (2.21) has to be modified accordingly and one straightforwardly arrives at

$$S_{\text{kin}} = \frac{1}{(2\pi)^4 \alpha'} \int_{M_4} \left[R + \nabla_\mu v^i \nabla^\mu v^b \frac{1}{\mathcal{V}} \left(\frac{1}{2} \mathcal{K}_{ab} - \frac{1}{\mathcal{V}} \mathcal{K}_a \mathcal{K}_b \right) + \frac{\zeta(3) \alpha'}{4} e^{-\frac{3}{2} \phi} \left(\mathcal{G}_{\mu\nu} \mathcal{G}^{\mu\nu} \mathcal{Z} - \mathcal{G}_{\mu\nu} \nabla^\nu \nabla^\mu v^a \left(\frac{2}{\mathcal{V}} \mathcal{K}_a \mathcal{Z} \right) \right. \right. \\ \left. \left. + \mathcal{G}_{\mu\nu} \nabla^\mu v^a \nabla^\nu v^b \left(\mathcal{Z}_{ab} - \frac{1}{\mathcal{V}^2} \mathcal{K}_a \mathcal{K}_b \mathcal{Z} \right) - \frac{1}{2} \nabla_\mu \nabla^\mu v^a \nabla_\nu \nabla^\nu v^b \left(\mathcal{Z}_{ab} + \frac{1}{2\mathcal{V}^2} \mathcal{K}_a \mathcal{K}_b \mathcal{Z} \right) \right. \right. \\ \left. \left. + \nabla_\mu \nabla_\nu v^a \nabla^\mu \nabla^\nu v^b \left(\mathcal{Z}_{ab} + \frac{1}{\mathcal{V}^2} \mathcal{K}_a \mathcal{K}_b \mathcal{Z} \right) \right) \right] \star_4 1 . \quad (3.3)$$

Where we have used the properties of the orientifold projection to conclude that

$$\mathcal{Z} = \frac{1}{2\pi\alpha'} \int_{Y_3} c_2 \wedge J = \frac{1}{2\pi\alpha'} \int_X c_2 \wedge J , \quad \mathcal{Z}_a = \frac{1}{2\pi\alpha'} \int_{Y_3} c_2 \wedge \omega_a = \frac{1}{2\pi\alpha'} \int_X c_2 \wedge \omega_a \quad (3.4) \\ \mathcal{Z}_{ab} = -\frac{1}{4\pi\alpha'} \int_{Y_3} Z_{m\bar{m}n\bar{n}} \omega_a^{\bar{m}m} \omega_b^{\bar{n}n} \star 1 = -\frac{1}{4\pi\alpha'} \int_X Z_{m\bar{m}n\bar{n}} \omega_a^{\bar{m}m} \omega_b^{\bar{n}n} \star 1 ,$$

obeying the analogous relations to (2.23) given by

$$\mathcal{Z}_a = \mathcal{Z}_{ab} v^b = \mathcal{Z}_{ba} v^b \quad \text{and} \quad \mathcal{Z} = \mathcal{Z}_a v^a . \quad (3.5)$$

3.2 4d, $\mathcal{N} = 1$ linear multiplets

The canonical form of the 4d, $\mathcal{N} = 1$ action for the real scalars L^a in the linear multiplets takes the form

$$S = \frac{1}{(2\pi)^4 \alpha'} \int_{M_4} R \star 1 + \frac{1}{2} G_{ab} \nabla_\mu L^a \nabla^\mu L^b \star 1 , \quad (3.6)$$

with the couplings G_{ab} , which can be inferred from a kinematic potential $\tilde{K}$ as $G_{ab} = \frac{\delta}{\delta L^a} \frac{\delta}{\delta L^b} \tilde{K}$. The identification of the Kähler moduli fields v^a with the real scalars in the linear multiplet of the 4d, $\mathcal{N} = 1$ supergravity theory at leading order in α' is given by

$$L^a = \frac{v^a}{\mathcal{V}} . \quad (3.7)$$

Eventual α' -modifications of (3.7) due to the two-derivative analysis at this order in α' [27] do not alter the four-derivative couplings at the relevant order in α' , thus it suffices to express the action in terms of (3.7). To determine all the relevant four-derivative couplings of L^i one requires knowledge of the couplings cubic and quartic in the infinitesimal fluctuations δv^i arising from the $\mathcal{R}^4$ -sector. This is however, beyond the study of this work and we have thus omitted such terms also arising due to the Weyl rescaling in (2.19). However, one may show that one can express the couplings $T_{\mu\nu} \nabla^\nu \nabla^\mu v^a$ and $T_{\mu\nu} \nabla^\mu v^a \nabla^\nu v^b$ in terms of the fields L^a in the linear multiplets without making use of information of the neglected sector. This does not apply to the

$\nabla_\mu \nabla^\mu v^a$ $\nabla_\nu \nabla^\nu v^b$ and $\nabla_\mu \nabla^\nu v^a$ $\nabla_\nu \nabla^\mu v^b$ terms where the knowledge of the other four-derivative couplings is crucial. Hence we will not consider the latter in the following. Expressing (3.3) in terms of the linear multiplets one finds

$$S_{\text{kin}} = \frac{1}{(2\pi)^4 \alpha'} \int_{M_4} \left[R + \frac{1}{2} \nabla_\mu L^a \nabla^\mu L^b \mathcal{V} \left(\mathcal{K}_{ab} - \frac{1}{\mathcal{V}} \mathcal{K}_a \mathcal{K}_b \right) + \frac{\zeta(3) \alpha'}{4} e^{-\frac{3}{2} \phi} \left(\mathcal{G}_{\mu\nu} \mathcal{G}^{\mu\nu} \mathcal{Z} + \mathcal{G}_{\mu\nu} \nabla^\nu \nabla^\mu L^a \mathcal{K}_a \mathcal{Z} \right. \right. \\ \left. \left. + \mathcal{G}_{\mu\nu} \nabla^\mu L^a \nabla^\nu L^b \left(\mathcal{V}^2 \mathcal{Z}_{ab} + \frac{5}{2} \mathcal{K}_a \mathcal{K}_b \mathcal{Z} - 3 \mathcal{V} \mathcal{K}_{ab} \mathcal{Z} - \mathcal{V} \mathcal{K}_a \mathcal{Z}_b \right) \right) \right] * 1. \quad (3.8)$$

Classically one then encounters the Kähler metric on the moduli space to be given by

$$G_{ab} = \mathcal{V} \int_X \omega_a \wedge * \omega_b = \mathcal{V} \left(\mathcal{K}_{ab} - \frac{1}{\mathcal{V}} \mathcal{K}_a \mathcal{K}_b \right), \quad (3.9)$$

arising from the kinematic potential $\tilde{K} = -2 \log \mathcal{V} = \log \mathcal{K}_{ijk} L^i L^j L^k$. The resulting novel couplings at order α'^3 , couple derivatives of the real scalars L^a to the tensor $\mathcal{G}_{\mu\nu}$, which is composed out of the Ricci tensor and the Ricci scalar. The higher-derivative coupling $\mathcal{G}_{\mu\nu} \mathcal{G}^{\mu\nu} \mathcal{Z}$ has been analyzed in [17], and leads to a propagating massive spin 2 ghost mode. However, let us note that the appearance of ghost modes in effective field theories is not an immediate issue since it is related to the truncation of the ghost-free infinite series resulting from string theory.

Let us next comment on the term $\mathcal{G}_{\mu\nu} \nabla^\mu L^a \nabla^\nu L^b$. Firstly, note that this higher-derivative coupling does not correct the propagator of L^a , since it vanishes in the Minkowski background. Thus it does not give rise to any ghost modes for L^a . The analogous case of the Einstein-tensor coupled to a scalar field is well studied and relevant in the context of inflation. It was observed that such a coupling of a scalar field to curvature terms favors slow roll inflation, in other words rather steep potentials can exhibit the feature of slow roll. It is expected that this coupling (3.8) could be used to implement these scenarios in the context of Kähler moduli inflation. It is an old approach in the context of string theory to drive slow roll inflation by a Kähler modulus [50, 51]. It would be interesting to analyze the consequences of the derived novel couplings to such inflationary models and their relevance due to their α'^3 -suppression. Finally, let us discuss the coupling $\mathcal{G}_{\mu\nu} \nabla^\nu \nabla^\mu L^a$. As in the above case it does not correct the propagator of L^a . In contrast to the previous case these couplings are poorly studied in inflation literature and hence their embedding in string inflation models is desirable. In both cases coefficients dependent on topological quantities $\mathcal{Z}, \mathcal{Z}_a$, see (3.4), of the internal Calabi-Yau orientifold and are trivially related to the analog quantities (2.22) of the Calabi-Yau threefold, and are thus computable in the context of algebraic geometry. However, the semi-topological coupling $\mathcal{Z}_{ab}$ requires the knowledge of the Calabi-Yau metric and although derivable in principle it is beyond the capability of current available techniques.

4 Conclusions

Considering purely gravitational $\mathcal{R}^4$ -corrections at order α'^3 to the leading order IIB supergravity action in ten dimensions, we performed a dimensional reduction to four dimensions on a Calabi-Yau threefold. Analyzing the reduction result at four-derivative level and quadratic in the infinitesimal Kähler deformations we derived novel couplings of the Kähler moduli fields and gravity. We argued that these are complete in a sense that the couplings cannot be altered by other sectors of the IIB action at order α'^3 , or by modifications of the background. We then performed the orientifold projection to derive a minimal supergravity theory in four dimensions. Let us stress that for a complete analysis one needs to derive the reduction result up to quartic order in the infinitesimal Kähler deformations. Only then one is able to draw definite conclusions for all of the resulting four-derivative couplings involving the Kähler moduli fields and gravity. This is an interesting question to be answered and the obvious next step in this research program. Let us conclude by emphasizing that a detailed analysis of the novel couplings in the context of Kähler moduli inflation in IIB orientifold setups is desirable.

Acknowledgments

I would like to thank Federico Bonetti, Thomas Grimm, Dieter Luest, Taizan Watari and Itamar Yakov for helpful discussions and comments. In particular, I would like to express my thankfulness to the theoretical high-energy physics groups of the Walter Burke institute at Caltech, the center for the fundamental laws of nature in Harvard, and the Max-Planck institute for physics in Munich, for their hospitality during my visits. This work was supported by the Grant-in-Aid for Scientific Research on Innovative Areas 2303, MEXT, and the WPI program of Japan.

Appendix

A Conventions, definitions, and identities

In this work we denote the ten-dimensional space indices by capital Latin letters $M, N = 0, \dots, 9$, the external ones by $\mu, \nu = 0, 1, 2, 3$, and the internal complex ones by $m, n, p = 1, 2, 3$ and $\bar{m}, \bar{n}, \bar{p} = 1, 2, 3$. The metric signature of the ten-dimensional space is $(-, +, \dots, +)$. Furthermore, the convention for the totally anti-symmetric tensor in Lorentzian space in an orthonormal

frame is $\epsilon_{012\dots 9} = \epsilon_{012} = +1$. The epsilon tensor in d dimensions then satisfies

$$\epsilon^{R_1 \dots R_p N_1 \dots N_{d-p}} \epsilon_{R_1 \dots R_p M_1 \dots M_{d-p}} = (-1)^s (d-p)! p! \delta^{N_1}_{[M_1} \dots \delta^{N_{d-p}}_{M_{d-p}]}, \quad (\text{A.1})$$

where $s = 0$ if the metric has Riemannian signature and $s = 1$ for a Lorentzian metric. We adopt the following conventions for the Christoffel symbols and Riemann tensor

$$\begin{aligned} \Gamma^R_{MN} &= \frac{1}{2} g^{RS} (\partial_M g_{NS} + \partial_N g_{MS} - \partial_S g_{MN}), & R_{MN} &= R^R_{MRN}, \\ R^M_{NRS} &= \partial_R \Gamma^M_{SN} - \partial_S \Gamma^M_{RN} + \Gamma^M_{RT} \Gamma^T_{SN} - \Gamma^M_{ST} \Gamma^T_{RN}, & R &= R_{MNG}^{MN}, \end{aligned} \quad (\text{A.2})$$

with equivalent definitions on the internal and external spaces. Written in components, the first and second Bianchi identity are

$$\begin{aligned} R^O_{PMN} + R^O_{MNP} + R^O_{NPM} &= 0 \\ (\nabla_L R)^O_{PMN} + (\nabla_M R)^O_{PNL} + (\nabla_N R)^O_{PLM} &= 0. \end{aligned} \quad (\text{A.3})$$

Let us specify in more detail our conventions regarding complex coordinates in the internal space. For a complex Hermitian manifold M with complex dimension n the complex coordinates $z^1, \dots, z^n$ and the underlying real coordinates $\xi^1, \dots, \xi^{2n}$ are related by

$$(z^1, \dots, z^n) = \left(\frac{1}{\sqrt{2}} (\xi^1 + i\xi^2), \dots, \frac{1}{\sqrt{2}} (\xi^{2n-1} + i\xi^{2n}) \right). \quad (\text{A.4})$$

Using these conventions one finds

$$\sqrt{g} d\xi^1 \wedge \dots \wedge d\xi^{2n} = \sqrt{g} (-1)^{\frac{(n-1)n}{2}} i^n dz^1 \wedge \dots \wedge dz^n \wedge d\bar{z}^1 \wedge \dots \wedge d\bar{z}^n = \frac{1}{n!} J^n, \quad (\text{A.5})$$

with g the determinant of the metric in real coordinates and $\sqrt{\det g_{mn}} = \det g_{m\bar{n}}$. The Kähler form is given by

$$J = i g_{m\bar{n}} dz^m \wedge d\bar{z}^{\bar{n}}. \quad (\text{A.6})$$

Let $\omega_{p,q}$ be a (p, q) -form, then its Hodge dual is the $(n-q, n-p)$ form

$$\begin{aligned} \star \omega_{p,q} &= \frac{(-1)^{\frac{n(n-1)}{2}} i^n (-1)^{pn}}{p! q! (n-p)! (n-q)!} \omega_{m_1 \dots m_p \bar{n}_1 \dots \bar{n}_q} \epsilon^{m_1 \dots m_p}_{\bar{r}_1 \dots \bar{r}_{n-p}} \\ &\quad \times \epsilon^{\bar{n}_1 \dots \bar{n}_q}_{s_1 \dots s_{n-q}} dz^{s_1} \wedge \dots \wedge dz^{s_{n-q}} \wedge d\bar{z}^{\bar{r}_1} \wedge \dots \wedge d\bar{z}^{\bar{r}_{n-p}}. \end{aligned} \quad (\text{A.7})$$

Finally, let us record our conventions regarding Chern forms. To begin with, we define the curvature two-form for Hermitian manifolds to be

$$\mathcal{R}^m_n = R^m_{n r \bar{s}} dz^r \wedge d\bar{z}^{\bar{s}}, \quad (\text{A.8})$$

and we set

$$\begin{aligned}
\text{Tr}\mathcal{R} &= R^m_{mr\bar{s}} dz^r \wedge d\bar{z}^{\bar{s}} , \\
\text{Tr}\mathcal{R}^2 &= R^m_{nr\bar{s}} R^n_{mr_1\bar{s}_1} dz^r \wedge d\bar{z}^{\bar{s}} \wedge dz^{r_1} \wedge d\bar{z}^{\bar{s}_1} , \\
\text{Tr}\mathcal{R}^3 &= R^m_{nr\bar{s}} R^n_{n_1r_1\bar{s}_1} R^{n_1}_{mr_2\bar{s}_2} dz^r \wedge d\bar{z}^{\bar{s}} \wedge dz^{r_1} \wedge d\bar{z}^{\bar{s}_1} \wedge dz^{r_2} \wedge d\bar{z}^{\bar{s}_2} .
\end{aligned} \tag{A.9}$$

The Chern forms can then be expressed in terms of the curvature two-form as

$$\begin{aligned}
c_0 &= 1 , \\
c_1 &= \frac{1}{2\pi} i \text{Tr}\mathcal{R} , \\
c_2 &= \frac{1}{(2\pi)^2} \frac{1}{2} (\text{Tr}\mathcal{R}^2 - (\text{Tr}\mathcal{R})^2) , \\
c_3 &= \frac{1}{3} c_1 c_2 + \frac{1}{(2\pi)^2} \frac{1}{3} c_1 \wedge \text{Tr}\mathcal{R}^2 - \frac{1}{(2\pi)^3} \frac{i}{3} \text{Tr}\mathcal{R}^3
\end{aligned} \tag{A.10}$$

The Chern forms of an n -dimensional Calabi-Yau manifold Y_n reduce to

$$c_2(Y_{n \geq 2}) = \frac{1}{(2\pi)^2} \frac{1}{2} \text{Tr}\mathcal{R}^2 \quad \text{and} \quad c_3(Y_{n \geq 3}) = -\frac{1}{(2\pi)^3} \frac{i}{3} \text{Tr}\mathcal{R}^3 \tag{A.11}$$

The six dimensional Euler-density is given by

$$Q = -\frac{1}{3} \left(R^{(0)}_{m_1}{}^{m_2}{}_{n_1}{}^{n_2} R^{(0)}_{m_2}{}^{m_1}{}_{n_2}{}^{n_3} R^{(0)}_{n_2}{}^{n_1}{}_{n_3}{}^{n_2} + R^{(0)}_{m_1}{}^{m_2}{}_{n_1}{}^{n_2} R^{(0)}_{m_2}{}^{m_3}{}_{n_2}{}^{n_3} R^{(0)}_{m_3}{}^{m_1}{}_{n_3}{}^{n_1} \right) . \tag{A.12}$$

It satisfies

$$Q = (2\pi)^3 *_6 c_3 , \quad \int_{Y_3} Q *_6 1 = (2\pi)^3 \chi , \tag{A.13}$$

where χ is the Euler-Characteristic of the internal Calabi-Yau manifold. Let us next define the intersection numbers

$$\begin{aligned}
\mathcal{K}_{ijk} &= \frac{1}{(2\pi\alpha')^3} \int_{Y_3} \omega_i \wedge \omega_j \wedge \omega_k , & \mathcal{K}_{ij} &= \frac{1}{(2\pi\alpha')^3} \int_{Y_3} \omega_i \wedge \omega_j \wedge J = \mathcal{K}_{ijk} v^k , \\
\mathcal{K}_i &= \frac{1}{2(2\pi\alpha')^3} \int_{Y_3} \omega_i \wedge J \wedge J = \frac{1}{2} \mathcal{K}_{ijk} v^j v^k , & \mathcal{V} &= \frac{1}{3!(2\pi\alpha')^3} \int_{Y_3} J \wedge J \wedge J = \frac{1}{3!} \mathcal{K}_{ijk} v^i v^j v^k ,
\end{aligned} \tag{A.14}$$

where $\{\omega_i\}$ are harmonic (1,1) -forms w.r.t. to the Calabi- Yau metric $g_{m\bar{n}}$. Let us state the useful identities

$$\omega_{im}{}^m = i \frac{\mathcal{K}_i}{\mathcal{V}} , \quad \omega_{im\bar{n}} \omega_j{}^{\bar{n}m} *_6 1 = \omega_i \wedge \omega_j \wedge J - \frac{1}{\mathcal{V}^2} \mathcal{K}_i \mathcal{K}_j *_6 1 . \tag{A.15}$$

We present the formulae for a Weyl rescaling $g_{\mu\nu} \rightarrow \Omega g_{\mu\nu}$ of the four-derivative terms, $R_{\mu\nu} R^{\mu\nu}$, R^2 . These expressions can be derived straight forwardly, and are given by

$$\begin{aligned}
R^2 \xrightarrow{\text{Weyl}} & \frac{1}{\Omega^2} R^2 - 6 R \frac{1}{\Omega^3} (\nabla_\mu \nabla^\mu \Omega) + 3 \frac{1}{\Omega^4} R (\nabla_\mu \Omega) (\nabla^\mu \Omega) + 9 \frac{1}{\Omega^5} (\nabla_\mu \nabla^\mu \Omega) (\nabla_\nu \nabla^\nu \Omega) \\
& - 9 \frac{1}{\Omega^5} (\nabla_\mu \Omega) (\nabla^\mu \Omega) (\nabla_\nu \nabla^\nu \Omega) + 9 \frac{1}{4\Omega^6} (\nabla_\mu \Omega) (\nabla^\mu \Omega) (\nabla_\nu \Omega) (\nabla^\nu \Omega) ,
\end{aligned} \tag{A.16}$$

and

$$\begin{aligned}
R_{\mu\nu}R^{\mu\nu} \xrightarrow{\text{Weyl}} & \frac{1}{\Omega^2}R_{\mu\nu}R^{\mu\nu} - R\frac{1}{\Omega^3}(\nabla_\mu\nabla^\mu\Omega) + 3\frac{1}{\Omega^4}R_{\mu\nu}\nabla^\mu\Omega\nabla^\nu\Omega - 2\frac{1}{\Omega^4}R_{\mu\nu}\nabla^\mu\nabla^\nu\Omega \\
& + 2\frac{1}{\Omega^5}(\nabla_\mu\nabla^\mu\Omega)(\nabla_\nu\nabla^\nu\Omega) + \frac{1}{\Omega^5}(\nabla^\mu\nabla^\nu\Omega)(\nabla_\mu\nabla_\nu\Omega) - \frac{3}{2}\frac{1}{\Omega^5}(\nabla_\mu\nabla^\mu\Omega)(\nabla_\nu\Omega)(\nabla^\nu\Omega) \\
& - 3\frac{1}{\Omega^5}(\nabla^\mu\nabla^\nu\Omega)(\nabla_\nu\Omega)(\nabla_\mu\Omega) + 9\frac{1}{4\Omega^6}(\nabla_\mu\Omega)(\nabla^\mu\Omega)(\nabla_\nu\Omega)(\nabla^\nu\Omega) . \quad (\text{A.17})
\end{aligned}$$

References

- [1] T. W. Grimm and J. Louis, “The Effective action of $N = 1$ Calabi-Yau orientifolds,” *Nucl. Phys.* **B699** (2004) 387–426, [hep-th/0403067](#).
- [2] R. Blumenhagen, M. Cvetič, P. Langacker, and G. Shiu, “Toward realistic intersecting D-brane models,” *Ann. Rev. Nucl. Part. Sci.* **55** (2005) 71–139, [hep-th/0502005](#).
- [3] M. Grana, “Flux compactifications in string theory: A Comprehensive review,” *Phys. Rept.* **423** (2006) 91–158, [hep-th/0509003](#).
- [4] M. R. Douglas and S. Kachru, “Flux compactification,” *Rev. Mod. Phys.* **79** (2007) 733–796, [hep-th/0610102](#).
- [5] R. Blumenhagen, B. Kors, D. Lust, and S. Stieberger, “Four-dimensional String Compactifications with D-Branes, Orientifolds and Fluxes,” *Phys. Rept.* **445** (2007) 1–193, [hep-th/0610327](#).
- [6] N. Akerblom, R. Blumenhagen, D. Lust, and M. Schmidt-Sommerfeld, “Thresholds for Intersecting D-branes Revisited,” *Phys. Lett.* **B652** (2007) 53–59, [0705.2150](#).
- [7] L. McAllister, E. Silverstein, and A. Westphal, “Gravity Waves and Linear Inflation from Axion Monodromy,” *Phys. Rev.* **D82** (2010) 046003, [0808.0706](#).
- [8] S. B. Giddings, S. Kachru, and J. Polchinski, “Hierarchies from fluxes in string compactifications,” *Phys. Rev.* **D66** (2002) 106006, [hep-th/0105097](#).
- [9] F. Denef and M. R. Douglas, “Distributions of flux vacua,” *JHEP* **05** (2004) 072, [hep-th/0404116](#).
- [10] V. Balasubramanian, P. Berglund, J. P. Conlon, and F. Quevedo, “Systematics of moduli stabilisation in Calabi-Yau flux compactifications,” *JHEP* **03** (2005) 007, [hep-th/0502058](#).
- [11] E. Kiritsis and B. Pioline, “On R^{*4} threshold corrections in IIB string theory and (p, q) string instantons,” *Nucl. Phys.* **B508** (1997) 509–534, [hep-th/9707018](#).

- [12] I. Antoniadis, S. Ferrara, R. Minasian, and K. Narain, “ R^2 couplings in M and type II theories on Calabi-Yau spaces,” *Nucl.Phys.* **B507** (1997) 571–588, [hep-th/9707013](#).
- [13] J. T. Liu and R. Minasian, “Higher-derivative couplings in string theory: dualities and the B-field,” [1304.3137](#).
- [14] S. Katmadas and R. Minasian, “ $\mathcal{N} = 2$ higher-derivative couplings from strings,” *JHEP* **02** (2014) 093, [1311.4797](#).
- [15] M. Koehn, J.-L. Lehnert, and B. A. Ovrut, “Higher-Derivative Chiral Superfield Actions Coupled to $N=1$ Supergravity,” *Phys. Rev.* **D86** (2012) 085019, [1207.3798](#).
- [16] G. W. Horndeski, “Second-order scalar-tensor field equations in a four-dimensional space,” *Int. J. Theor. Phys.* **10** (1974) 363–384.
- [17] L. Alvarez-Gaume, A. Kehagias, C. Kounnas, D. Lüster, and A. Riotto, “Aspects of Quadratic Gravity,” *Fortsch. Phys.* **64** (2016), no. 2-3, 176–189, [1505.07657](#).
- [18] D. Ciupke, “Scalar Potential from Higher Derivative $\mathcal{N} = 1$ Superspace,” [1605.00651](#).
- [19] L. Amendola, “Cosmology with nonminimal derivative couplings,” *Phys. Lett.* **B301** (1993) 175–182, [gr-qc/9302010](#).
- [20] S. Bielleman, L. E. Ibanez, F. G. Pedro, I. Valenzuela, and C. Wieck, “The DBI Action, Higher-derivative Supergravity, and Flattening Inflaton Potentials,” [1602.00699](#).
- [21] S. Aoki and Y. Yamada, “Impacts of supersymmetric higher derivative terms on inflation models in supergravity,” *JCAP* **1507** (2015), no. 07, 020, [1504.07023](#).
- [22] I. Dalianis and F. Farakos, “Higher Derivative D-term Inflation in New-minimal Supergravity,” *Phys. Lett.* **B736** (2014) 299–304, [1403.3053](#).
- [23] D. Ciupke, J. Louis, and A. Westphal, “Higher-Derivative Supergravity and Moduli Stabilization,” [1505.03092](#).
- [24] D. J. Gross and E. Witten, “Superstring Modifications of Einstein’s Equations,” *Nucl. Phys.* **B277** (1986) 1.
- [25] A. Rajaraman, “On a supersymmetric completion of the R^2 term in type IIB supergravity,” *Phys. Rev.* **D72** (2005) 125008, [hep-th/0505155](#).
- [26] Y. Hyakutake and S. Ogushi, “Higher derivative corrections to eleven dimensional supergravity via local supersymmetry,” *JHEP* **02** (2006) 068, [hep-th/0601092](#).
- [27] F. Bonetti and M. Weissenbacher, “The Euler characteristic correction to the Kähler potential—revisited,” *to appear* (1606.xxxx).

- [28] T. W. Grimm, J. Keitel, R. Savelli, and M. Weissenbacher, “From M-theory higher curvature terms to α' corrections in F-theory,” 1312.1376.
- [29] T. W. Grimm, R. Savelli, and M. Weissenbacher, “On α' corrections in N=1 F-theory compactifications,” *Phys.Lett.* **B725** (2013) 431–436, 1303.3317.
- [30] T. W. Grimm, T. G. Pugh, and M. Weissenbacher, “The effective action of warped M-theory reductions with higher derivative terms - Part I,” 1412.5073.
- [31] T. W. Grimm, T. G. Pugh, and M. Weissenbacher, “On M-theory fourfold vacua with higher curvature terms,” *Phys. Lett.* **B743** (2015) 284–289, 1408.5136.
- [32] T. W. Grimm, T. G. Pugh, and M. Weissenbacher, “The effective action of warped M-theory reductions with higher-derivative terms - Part II,” 1507.00343.
- [33] T. W. Grimm, K. Mayer, and M. Weissenbacher, “M-theory on Calabi-Yau threefolds, $4d, \mathcal{N} = 2$ and $\mathcal{N} = 1$ higher-derivative supergravities,” *to appear* (16xx.xxxx).
- [34] J. H. Schwarz, “Superstring Theory,” *Phys. Rept.* **89** (1982) 223–322.
- [35] M. T. Grisaru, A. E. M. van de Ven, and D. Zanon, “Four Loop Divergences for the N=1 Supersymmetric Nonlinear Sigma Model in Two-Dimensions,” *Nucl. Phys.* **B277** (1986) 409–428.
- [36] D. J. Gross and J. H. Sloan, “The Quartic Effective Action for the Heterotic String,” *Nucl. Phys.* **B291** (1987) 41–89.
- [37] M. Abe, H. Kubota, and N. Sakai, “Loop Corrections to the Heterotic String Effective Lagrangian,” *Phys. Lett.* **B200** (1988) 461–465. [Addendum: *Phys. Lett.* B203,474(1988)].
- [38] A. Kehagias and H. Partouche, “D instanton corrections as (p,q) string effects and nonrenormalization theorems,” *Int. J. Mod. Phys.* **A13** (1998) 5075–5092, hep-th/9712164.
- [39] A. Kehagias and H. Partouche, “On the exact quartic effective action for the type IIB superstring,” *Phys. Lett.* **B422** (1998) 109–116, hep-th/9710023.
- [40] R. Minasian, T. G. Pugh, and R. Savelli, “F-theory at order α'^3 ,” *JHEP* **10** (2015) 050, 1506.06756.
- [41] M. D. Freeman, C. N. Pope, M. F. Sohnius, and K. S. Stelle, “Higher Order σ Model Counterterms and the Effective Action for Superstrings,” *Phys. Lett.* **B178** (1986) 199–204.
- [42] M. D. Freeman and C. N. Pope, “Beta Functions and Superstring Compactifications,” *Phys. Lett.* **B174** (1986) 48–50.

- [43] M. Weissenbacher, “On Geometric Corrections to Effective Actions of String Theory,” <https://edoc.ub.uni-muenchen.de> (2015).
- [44] K. Becker, M. Becker, M. Haack, and J. Louis, “Supersymmetry breaking and alpha-prime corrections to flux induced potentials,” *JHEP* **06** (2002) 060, [hep-th/0204254](#).
- [45] B. S. Acharya, M. Aganagic, K. Hori, and C. Vafa, “Orientifolds, mirror symmetry and superpotentials,” [hep-th/0202208](#).
- [46] I. Brunner and K. Hori, “Orientifolds and mirror symmetry,” *JHEP* **11** (2004) 005, [hep-th/0303135](#).
- [47] I. Brunner, K. Hori, K. Hosomichi, and J. Walcher, “Orientifolds of Gepner models,” *JHEP* **02** (2007) 001, [hep-th/0401137](#).
- [48] A. Dabholkar and J. Park, “Strings on orientifolds,” *Nucl. Phys.* **B477** (1996) 701–714, [hep-th/9604178](#).
- [49] L. Andrianopoli, R. D’Auria, and S. Ferrara, “Supersymmetry reduction of N extended supergravities in four-dimensions,” *JHEP* **03** (2002) 025, [hep-th/0110277](#).
- [50] P. Binétruy and M. K. Gaillard, “Candidates for the inflaton field in superstring models,” *Phys. Rev. D* **34** (Nov, 1986) 3069–3083.
- [51] T. Banks, M. Berkooz, S. H. Shenker, G. Moore, and P. J. Steinhardt, “Modular cosmology,” *Phys. Rev. D* **52** (Sep, 1995) 3548–3562.