Higher derivatives in Type II and M-theory on Calabi-Yau threefolds

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Abstract

The four- and five-dimensional effective actions of Calabi-Yau threefold compactifications are derived with a focus on terms involving up to four space-time derivatives. The starting points for these reductions are the ten- and eleven-dimensional supergravity actions supplemented with the known eight-derivative corrections that have been inferred from Type II string amplitudes. The corrected background solutions are determined and the fluctuations of the Kähler structure of the compact space and the form-field background are discussed. It is concluded that the two-derivative effective actions for these fluctuations only takes the expected supergravity form if certain additional ten- and eleven-dimensional higher-derivative terms for the form-fields are included. The main results on the four-derivative terms include a detailed treatment of higher-derivative gravity coupled to Kähler structure deformations. This is supplemented by a derivation of the vector sector in reductions to five dimensions. While the general result is only given as an expansion in the fluctuations, a complete treatment of the one-Kähler modulus case is presented for both Type II theories and M-theory.

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1 Introduction and Summary

Compactifications of Type II string theories and M-theory on Calabi-Yau manifolds are interesting from various interrelated perspectives reaching from phenomenologically motivated model building to purely mathematical studies of quantum geometry. In particular, reductions on Calabi-Yau threefolds have been studied over several decades leading to intriguing discoveries such as mirror symmetry. Compactifications of Type IIA and Type IIB string theory on Calabi-Yau threefolds result in four-dimensional effective theories that admit $\mathcal{N} = 2$ supersymmetry [1], or sixteen preserved four-dimensional supercharges. Similarly the compactifications of M-theory on Calabi-Yau threefolds lead to five-dimensional supersymmetric theories that preserve sixteen supercharges or five-dimensional $\mathcal{N} = 2$. Four-dimensional $\mathcal{N} = 1$ supersymmetric effective theories are obtained from Calabi-Yau threefold compactifications including D-branes and orientifold planes [3,4]. So far, however, the effective actions arising in Calabi-Yau threefold compactifications are only fully understood at the two-derivative level. In this work we aim to go beyond this and systematically include terms up to four derivatives into the effective action.

Our starting point for the compactifications are the ten- or eleven-dimensional low energy effective action of of Type II string theory or M-theory. At the two derivative level these are the well-known ten-dimensional Type IIA and Type IIB supergravity actions with $\mathcal{N} = 2$ supersymmetry, and the unique eleven-dimensional supergravity theory with $\mathcal{N} = 1$ supersymmetry. The Type II supergravity actions are modified by two types of stringy. First, there are α'-corrections imprinted in higher derivative terms. Second, the worldsheet genus expansion in g_s leading to higher dimensional operators in the effective theory depending on higher orders of the dilaton. Using perturbative string theory these corrections can be computed by explicitly evaluation string scattering amplitudes. While this can be notoriously difficult, various corrections are known in the literature. For example, it is well established that the $\mathcal{R}^4$-terms in Type II theories at order α'^3 are the complete set of purely gravitational eight-derivative terms [5,33,10,8,9]. Additionally, there are higher derivative terms in ten dimensions involving the NS-NS two-form B_2 [7,14]. It was conjectured in [14] that the B_2 completion of the $\mathcal{R}^4$-terms can almost be completely captured by introducing a connection with torsion, where $H_3 = \mathrm{d}B_2$ acquires the role of the torsion. A strategy to extract corrections to the eleven-dimensional supergravity action is to up-lift the Type IIA corrections computed by string amplitudes. From the known Type IIA terms one can thus infer $\mathcal{R}^4$-terms [11,13] and terms involving the M-theory three-form C_3. It is eminent that the only expansion parameter in eleven dimensions is the eleven-dimensional Planck length.

Obtaining effective actions in four and five dimensions taking into account higher derivative corrections in ten or eleven dimensions is an important yet challenging task. Since the first corrections at order α'^3 involve terms quartic in the Riemann tensor, the derivation of the effective couplings of the geometric moduli constitutes a computationally challenging task. Furthermore, although the $\mathcal{R}^4$-couplings in ten and eleven dimensions

are well established, less is known about the completion of the action at eight derivative level for the full NS-NS and R-R sector. Important partial results can be found, for example, in [15, 16, 14, 17], but it is desirable to obtain a complete action via, for example, supersymmetric completion or comparison with scattering amplitudes. Another important challenge is the construction of general higher derivative supergravity theories in various dimensions independent of their string theory origins. While many standard formulations of supergravity at two derivatives are well established, there exist only partial results beyond the two derivative truncation. In the considered string compactifications a general higher derivative supergravity action would not only allow us to check consistency with the expected preserved supersymmetry, but also suggest a clear systematics to determine physical couplings in terms of geometric data of the compactification space.

Higher derivative corrections in string theory and M-theory have led to interesting modifications of the low energy dynamics and phenomenological models obtained in compactifications. In the following we will only give a very incomplete list of past applications for which higher derivative corrections play a prominent role. It is well known that higher derivative corrections give rise to corrections of the Kähler potential of $\mathcal{N} = 2$ compactifications involving characteristic classes of the compactification Calabi-Yau threefold [19, 18]. It was later argued in [20, 21] that some of these corrections survive the truncation to $\mathcal{N} = 1$ supergravity in compactifications including fluxes and an orientifold projection. Although obtaining a general higher derivative theory in five dimensions is not yet available, an off-shell completion of R^2-terms in five-dimensional $\mathcal{N} = 2$ supergravity and its on shell version for pure supergravity is known [22]. These results were used to study the effect of higher derivative corrections on AdS black holes as well as gauge theories using holographic anomaly matching [23]. Four derivative $\mathcal{N} = 1$ and $\mathcal{N} = 2$ supergravity theories in four spacetime dimensions are still poorly understood. Recently, progress towards classifying four derivative superspace operators in the context of $\mathcal{N} = 1$ supergravity was made in [24]. This interest arose from the observation that higher derivative terms in Type IIB orientifolds might stabilze Kähler moduli [25]. Let us emphazise out that a first complete treatment of the kinetic terms for the Kähler deformations originating from $\mathcal{R}^4$-terms in eleven dimensions was presented in [26–28], where M-theory was compactified to three dimensions on a Calabi-Yau fourfold.

Our goal in this work is to study various compactifications of M-theory and Type II string theories on Calabi-Yau threefolds taking into account the known eight derivative terms. We review the considered known and conjectured terms in the ten and eleven-dimensional action contributing at order α'^3. This will allow us to solve the equations of motion determining background solutions possessing a compact, six-dimensional manifold which is topologically a Calabi-Yau threefold. We find that the modified solution in Type IIA involves a non-trivial background of the dilaton given by the Euler density in six dimensions on the lowest order Calabi-Yau geometry. Furthermore, the metric receives a correction leading to a deviation from Ricci flatness, which, however, does not alter the cohomology class of the curvature two-form. We then briefly comment on the relation between M-theory and Type IIA at order α'^3 and explain why adding a certain term in

eleven dimensions is necessary.

The main part of this work is devoted to a detailed discussion of the dimensional reduction of M-theory and Type II on Y_3 including infinitesimal deformations of the Kähler structure. In the M-theory reduction we also present results on the five-dimensional two- and four-derivative effective action also including vectors arsing from the M-theory three-form, while in Type II theories we comment on including the dilaton and modes from the NS-NS two-from. We first derive the five-dimensional two-derivative effective action and show that eleven-dimensional eight-derivative terms only lead to a modification of the scalar field corresponding to the overall classical volume of Y_3 in accordance with [12]. Furthermore, we then derive the five-dimensional four-derivative terms originating from the eleven-dimensional eight-derivative terms. Increasing computational complexity forces us to restrict to terms quadratic in the fluctuation. The main couplings at four-derivative level are found to be divisor integrals of the second Chern class of Y_3 and two new tensorial structures $Z_{i\bar{j}k\bar{l}}$, $\mathcal{X}^{(0)}_{ai\bar{j}}$, see (3.27) and (3.40), involving one and two Riemann tensors on Y_3. However, the physical significance of the latter two remains unclear, since there might exist a field redefinition ambiguity in the description of higher derivative theories. We also preform the dimensional reduction of Type II ten-dimensional effective actions with a focus the deformations of the Kähler structure and scalars b^a arising from the zero mode expansion of the NS-NS two-form. The two-derivative four-dimensional effective action for the Kähler structure deformations contains the well known shift with the Euler characteristic of Y_3 [**?**]. In Type IIA reductions we then use the fact that the Kähler structure deformations and the scalars from the NS-NS two-form combine into the complexified Kähler moduli in $\mathcal{N} = 2$ vector multiplets. In order to obtain their kinetic terms from a $\mathcal{N} = 2$ prepotential [19], we find that a string tree-level structure for the kinetic terms of the scalars b^a is missing. This forces us to add a minimal novel set of H_3-terms in ten dimensions. In other words, we use four-dimensional supersymmetry constraints to suggest missing higher-derivative terms in the ten-dimensional action.

In the last part of this work we exploit the fact that for the case of one modulus u all computations can be performed exactly. We derive all four derivative couplings stemming from the $\mathcal{R}^4$-terms in M-theory and both Type II theories. Furthermore, we discuss the possibility of higher derivative field redefinitions. This allows us to show that there exists a field basis in which the four derivative effective actions can be chosen to only consist of the Gauss-Bonnet term and a four derivative interaction $(\partial u)^4$. We also comment on the orientifold truncation of this $\mathcal{N} = 2$ result to a minimal supersymmetric setting in four dimensions. After truncation we aim to make contact to the proposal of [25] to stabilize moduli using higher derivative terms. Our reduction clarifies the structure of the terms $(\partial u)^4$, but we point out some caveats in reading off the coefficient functions. Unfortunately, our findings cannot settle the phenomenological relevance of higher derivative terms for moduli stabilization and a direct derivation of the scalar potential remains crucial.

2 Higher derivatives and circle reduction

The purpose of this section is to collect and state the bosonic actions of $\mathcal{N} = 1$ supergravity in eleven and the $\mathcal{N} = (1, 1)$ Type IIA supergravity action in ten dimensions including known and conjectured eight derivative corrections stemming from the α' expansion of the sigma-model worldsheet action of the underlying string theory. Note that due to the lack of a microscopic formulation of M-theory, all known corrections are indirectly obtained by lifting ten-dimensional correction of Type IIA to eleven dimensions. At the classical level, i.e. at lowest order in α', this duality is simply given by a circle reduction. Once higher derivative terms are taken into account it has to be refined and corrected. This correction turns out to be crucial as we will argue in the following and is indeed necessary for compatibility with supersymmetry in lower dimensions.

The specific types of higher derivative terms we consider are the well known eight derivative terms quartic in the Riemann tensor [7–13] and a set of terms conjectured in [14, 29] quadratic in the NS-NS three-form field strength on the Type IIA side, which lift to eight derivative terms of the four-form field strength in the M-theory picture.

In this section we first collect the various pieces contributing to the actions in ten and eleven dimensions. We explain which terms are well established and which ones are conjectured. In the following we argue, that a certain term in the action has to be added in eleven dimensions to take into account the deviation of the dilaton from being constant in the backgrounds we are considering. Thus, the additional term can be seen as a necessity to account for the absence of the dilaton in M-theory.

2.1 Ten-dimensional Type IIA supergravity action at eight derivatives

In the following we will use a tilde to denote a field or operator in ten dimensions and capital letters at the beginning of the alphabet for tensor indices in ten dimensions. The bosonic field content of ten-dimensional Type IIA supergravity is given by the ten-dimensional metric $\tilde{g}_{AB}$, the dilaton in ten dimensions $\tilde{\phi}$ and the two-form $\tilde{B}_2$ all of which descend as massless modes from the NS-NS sector of the RNS superstring. Those terms are augmented by further fields from the R-R sector which however do not play a role in the following. We will therefore restrict ourselves completely to the NS-NS fields.

The part of the $\mathcal{N} = (1, 1)$ low energy effective action of the Type IIA superstring we are considering takes the schematic form

$$S_{\text{IIA}} = S_{\text{IIA}}^{\text{class}} + \alpha S_{\tilde{R}^4}^{\text{tree}} + \alpha S_{\tilde{R}^4}^{\text{loop}} + \alpha S_{\tilde{H}^2} \,, \tag{2.1}$$

where we already introduced the expansion parameter

$$\alpha = \frac{\alpha'^3}{3 \cdot 2^{11}}. \tag{2.2}$$

The classical action at lowest order in the parameter α has for the NS-NS fields the form

$$2\kappa_{10}^2 S_{\text{IIA}}^{\text{class}} = \int_{\mathcal{M}_{10}} \mathrm{e}^{-2\tilde{\phi}} \left(\tilde{R} \,\tilde{*}\, 1 + 4\mathrm{d}\tilde{\phi} \wedge \tilde{*}\, \mathrm{d}\tilde{\phi} - \frac{1}{2} \tilde{H}_3 \wedge \tilde{*}\, \tilde{H}_3 \right), \tag{2.3}$$

where $\mathcal{M}_{10}$ is the spacetime manifold, $\tilde{R}$ is the Ricci scalar in ten spacetime dimensions and $\tilde{H}_3 = \mathrm{d}\tilde{B}_2$ is the field strength of the NS-NS two-form. The gravitational coupling in ten dimensions, denoted by κ_{10} is related to the slope parameter α' by the relation

$$2\kappa_{10}^2 = (2\pi)^7 \alpha'^4. \tag{2.4}$$

At eight derivatives the action gets supplemented by additional terms quartic in the Riemann tensor at both tree-level and one-loop in the string coupling g_s. Those pieces of the action take the schematic form [30]

$$2\kappa_{10}^2 S_{\tilde{R}^4}^{\text{tree}} = \zeta(3) \int_{\mathcal{M}_{10}} \mathrm{e}^{-2\tilde{\phi}} \left(\tilde{t}_8 \tilde{t}_8 + \frac{1}{8} \epsilon_{10} \epsilon_{10} \right) \tilde{R}^4 \,\tilde{*}\, 1 \,, \tag{2.5}$$

$$2\kappa_{10}^2 S_{\tilde{R}^4}^{\text{loop}} = \frac{\pi^2}{3} \int_{\mathcal{M}_{10}} \left(\tilde{t}_8 \tilde{t}_8 - \frac{1}{8} \epsilon_{10} \epsilon_{10} \right) \tilde{R}^4 \,\tilde{*}\, 1. \tag{2.6}$$

For the detailed form of these terms and the definition of the tensor t_8 in ten and eleven dimensions we refer the reader to appendix B. Note that the relative sign flip of the one-loop contribution compared to the tree-level piece is characteristic for Type IIA and does not appear in Type IIB. Additionally, there is an an eight derivative coupling involving the NS-NS two-form $\tilde{B}_2$ and four Riemann tensors, which will however not play a role in the Type IIA discussion. Its M-theory counterpart will be taken into account when the four derivative couplings in five dimensions are computed.

It is furthermore conjectured in [14], that the completion of the eight derivative terms with respect to the NS-NS two-form is almost completely captured by introducing a connection with torsion. The claim is then, that higher derivative terms involving $\tilde{B}_2$ can be obtained from the $\tilde{R}^4$ terms by computing the latter with respect to the aforementioned torsionful connection. We will not collect all the structures emerging from this procedure but outline the strategy how to get the eight derivative terms we need for our discussion.

Both the tree-level and one-loop contributions to the $\tilde{R}^4$ action can be expressed in terms of the two 'superinvariants'

$$\mathcal{J}_0 = \left(\tilde{t}_8 \tilde{t}_8 + \frac{1}{8} \epsilon_{10} \epsilon_{10} \right) \tilde{R}^4 \,, \tag{2.7}$$

$$\mathcal{J}_1 = \tilde{t}_8 \tilde{t}_8 \tilde{R}^4 - \frac{1}{4} \tilde{t}_8 \epsilon_{10} \tilde{B}_2 \tilde{R}^4 \tag{2.8}$$

such that the $\tilde{R}^4$-terms read

$$2\kappa_{10}^2 S_{\tilde{R}^4}^{\text{tree}} = \zeta(3) \int_{\mathcal{M}_{10}} \mathrm{e}^{-2\tilde{\phi}} \mathcal{J}_0 \tilde{*} 1 \tag{2.9}$$

$$2\kappa_{10}^2 S_{\tilde{R}^4}^{\text{loop}} = \frac{\pi^2}{3} \int_{\mathcal{M}_{10}} (2\mathcal{J}_1 - \mathcal{J}_0) \,\tilde{*}\, 1. \tag{2.10}$$

The last term in (2.8) corresponds to the eight derivative coupling involving $\tilde{B}_2$ which we do not consider in the Type IIA context and is therefore also ignored in (2.10). The $\tilde{B}_2$ field completion at eight derivatives is according to [14] then given by the replacements

$$\mathcal{J}_0 \to \mathcal{J}_0(\Omega_+) + \Delta\mathcal{J}_0(\Omega_+, \tilde{H}_3) =$$

$$= \left(\tilde{t}_8\tilde{t}_8 + \frac{1}{8}\epsilon_{10}\epsilon_{10}\right)\tilde{R}^4(\Omega_+) + \frac{1}{3}\epsilon_{10}\epsilon_{10}\tilde{H}_3^2\tilde{R}^3(\Omega_+) \tag{2.11}$$

$$\mathcal{J}_1 \to \mathcal{J}_1(\Omega_+) = \tilde{t}_8\tilde{t}_8\tilde{R}^4(\Omega_+) - \frac{1}{8}\epsilon_{10}\,\tilde{t}_8\,\tilde{B}_2\left(\tilde{R}^4(\Omega_+) + \tilde{R}^4(\Omega_-)\right), \tag{2.12}$$

where the Riemann tensor with respect to the connection with torsion $\Omega_\pm$ is given in components by

$$\tilde{R}(\Omega_\pm)_{A_1 A_2}{}^{B_1 B_2} = \tilde{R}_{A_1 A_2}{}^{B_1 B_2} \pm \tilde{\nabla}_{[A_1}\tilde{H}_{3 A_2]}{}^{B_1 B_2} + \frac{1}{2}\tilde{H}_{3[A_1}{}^{B_1 B_3}\tilde{H}_{3 A_2]B_3}{}^{B_2}. \tag{2.13}$$

For the detailed structure of the coupling $\epsilon_{10}\epsilon_{10}\tilde{H}_3^2\tilde{R}^3$ we again refer the reader to appendix B. The terms generated by this replacement up to quadratic order in the NS-NS three-form field strength are denoted by $S_{\tilde{H}^2}$ in (2.1).

However, as we will explain in section 4, the replacements (2.11) and (2.12) are not consistent with the correction to the four dimensional prepotential. We therefore propose a corrected replacement given by

$$\mathcal{J}_0 \to \mathcal{J}_0(\Omega_+) + \Delta\mathcal{J}_0(\Omega_+, \tilde{H}_3) + \delta\mathcal{J}$$

$$\mathcal{J}_1 \to \mathcal{J}_1(\Omega_+) + \frac{1}{2}\delta\mathcal{J}, \qquad \text{where}$$

$$\delta\mathcal{J} = -2\int_{\mathcal{M}_{10}}\tilde{t}_8\,\tilde{t}_8\,\tilde{H}_3^2\,\tilde{R}^3\,\tilde{\ast}\,1 \tag{2.14}$$

and the explicit index expression of (2.14) is given by

$$\tilde{t}_8\,\tilde{t}_8\,\tilde{H}_3^2\,\tilde{R}^3 = \tilde{t}_8^{A_1\cdots A_2}\,\tilde{t}_{8\,B_1\cdots B_8}\,\tilde{H}_{3\,A_1 A_2 C}\,\tilde{H}_3^{B_1 B_2 C}\,\tilde{R}^{B_3 B_4}{}_{A_3 A_4}\cdots\tilde{R}^{B_7 B_8}{}_{A_7 A_8}. \tag{2.15}$$

Notice, that the structure of the modified replacement (2.14) is such that the tree-level terms get modified, whereas the one-loop terms remain untouched. Furthermore, note that the index contraction of (2.15) is such that it cannot be obtained via the metric replacement (2.13) applied to the $\mathcal{R}^4$-terms.

2.2 Corrected Y_3 background solution at order α'^3 in Type IIA

In the classical, i.e. lowest order in α' case, a solution of the equations of motion with a complex three-dimensional compact internal space preserving $\mathcal{N} = 2$ supersymmetry is simply given by a product manifold $\mathcal{M}_{10} = \mathbb{M}_{1,3} \times Y_3$, a constant dilaton and no fluxes. Here $\mathbb{M}_{1,3}$ is four-dimensional Minkowski space and Y_3 is a Calabi-Yau manifold. Due to

the eight derivative couplings at order α'^3 we need to look for corrected solutions to the corrected equations of motion which reduce to the classical Y_3 solution at lowest order in the expansion parameter α introduced in (2.2). We therefore take

$$\langle \mathrm{d}\tilde{s}^2 \rangle = \eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + \big(g_{mn}^{(0)} + \alpha g_{mn}^{(1)} \big) \mathrm{d}y^m \mathrm{d}y^n \tag{2.16}$$

$$\langle \tilde{\phi} \rangle = \phi_0 + \alpha \langle \phi^{(1)}(y) \rangle$$

$$\langle \tilde{H}_3 \rangle = 0$$

as our ansatz for solving the corrected equations of motion. In the ansatz (2.16) $\eta_{\mu\nu}$ denotes the four-dimensional Minkowski metric, $g_{mn}^{(0)}$ is the lowest order Calabi-Yau metric and ϕ_0 is a constant. The goal is now to fix the correction to the dilaton $\langle \phi^{(1)} \rangle$ as well as the correction to the metric $g_{mn}^{(1)}$. Note that in this section we work entirely with real indices $m, n = 1, ..., 6$.

Varying the corrected action (2.1) with respect to the fields and evaluating the resulting equations of motion on the ansatz (2.16) yields the following equations:

- **external Einstein equation.**

$$0 = g^{(0)mn} R^{(1)}{}_{mn} + 4 \nabla_n^{(0)} \nabla^{(0)\,n} \langle \phi^{(1)}(y) \rangle \tag{2.17}$$

- **internal Einstein equations.**

$$0 = R_{mn}^{(1)} - \frac{1}{2} g_{mn}^{(0)} g^{(0)kl} R^{(1)}{}_{kl} - 2 g_{mn}^{(0)} \nabla_k^{(0)} \nabla^{(0)\,k} \langle \phi^{(1)}(y) \rangle + 2 \nabla_m^{(0)} \nabla_n^{(0)} \langle \phi^{(1)}(y) \rangle \tag{2.18}$$

$$+ 768 \, (2\pi)^3 \Big(\zeta(3) + \frac{\pi^2}{3} \mathrm{e}^{2\phi_0} \Big) J^{(0)\,r}_{m} J^{(0)\,s}_{n} \nabla_r^{(0)} \nabla_s^{(0)} \big(*_6^{(0)} c_3^{(0)} \big) .$$

In (2.17) and (2.18) we have employed the definition $R_{mn}(g^{(0)} + \alpha g^{(1)}) \equiv R_{mn}^{(0)} + \alpha R_{mn}^{(1)}$ and furthermore used the complex structure $J^{(0)\,n}_{m}$, the Hodge star $*_6^{(0)}$ and the third Chern class $c_3^{(0)}$ all of which are evaluated on the zeroth order Calabi-Yau background. The equation of motion for the NS-NS two-form is trivially satisfied and the equation of motion of the dilaton coincides with the external Einstein equation in the string frame. Compatibility of the external and internal Einstein equation is achieved by taking the trace of the internal Einstein equation and comparing it to the external Einstein equation, which in turn fixes the correction of the dilaton to

$$\langle \phi^{(1)}(y) \rangle = 384 \, (2\pi)^3 \Big(\zeta(3) + \frac{\pi^2}{3} \mathrm{e}^{2\phi_0} \Big) *_6^{(0)} c_3^{(0)} . \tag{2.19}$$

Plugging (2.19) again into the internal Einstein equation results in

$$R_{mn}^{(1)} = -768 \, (2\pi)^3 \Big(\zeta(3) + \frac{\pi^2}{3} \mathrm{e}^{2\phi_0} \Big) \big(\nabla_m^{(0)} \nabla_n^{(0)} + J^{(0)\,r}_{m} J^{(0)\,s}_{n} \nabla_r^{(0)} \nabla_s^{(0)} \big) *_6^{(0)} c_3^{(0)} . \tag{2.20}$$

Going to complex indices in (2.20) shows that $R_{i\bar{j}}^{(1)} \sim \partial_i \bar{\partial}_{\bar{j}} (*_6^{(0)} c_3^{(0)})$ which in turn has a solution $g_{i\bar{j}}^{(1)} \sim \partial_i \bar{\partial}_{\bar{j}} f(y)$ for some specific function f which depends on the compact

manifold and serves of as a Kähler potential. The holomorphic and antiholomorphic indices can take the values $i = 1, 2, 3$ and $\bar{j} = \bar{1}, \bar{2}, \bar{3}$. The precise form of f can be computed explicitly, which we will only do for the case of M-theory in section 2.4 since the procedure is the same and the precise form of f is of no physical importance, as the metric correction turns out to completely decouple from low energy dynamics.

2.3 Eleven-dimensional supergravity action at eight derivatives

The higher derivative corrections obtained for the Type IIA superstring can then be lifted to eleven dimensions by moving to the strong coupling limit which corresponds to M-theory. Regarding notation we will use hats to indicate that a certain object is defined in eleven dimensions and eleven-dimensional indices $M, N, ...$ from the middle of the alphabet. The lowest order action we are considering is the action of eleven-dimensional $\mathcal{N} = 1$ supergravity which was first constructed in [31]. Its bosonic part is given by

$$S_{\mathrm{M}}^{\mathrm{class}} = \frac{1}{2\kappa_{11}^2} \int_{\mathcal{M}_{11}} \hat{R} \hat{*} 1 - \frac{1}{2} \hat{G}_4 \wedge \hat{*} \hat{G}_4 - \frac{1}{6} \hat{C}_3 \wedge \hat{G}_4 \wedge \hat{G}_4 \,. \tag{2.21}$$

The dynamical degrees of freedom are the eleven-dimensional metric $\hat{g}_{MN}$ and the M-theory three-form $\hat{C}_3$ with its field strength $\hat{G}_4 = \mathrm{d}\hat{C}_3$. We introduce similarly to the Type IIA action the expansion parameter

$$\hat{\alpha} = \frac{(4\pi\kappa_{11}^2)^{2/3}}{(2\pi)^4 3^2 2^{13}} \,, \tag{2.22}$$

where κ_{11} is related to the eleven-dimensional Planck length ℓ_M as $\kappa_{11}^2 = \frac{1}{2}(2\pi)^8 \ell_M^9$ such that $\hat{\alpha} \propto \ell_M^6$. The eight derivative action of M-theory up to terms quadratic in $\hat{G}_4$ takes the following schematic form [14]

$$S_{\mathrm{M}} = S_{\mathrm{M}}^{\mathrm{class}} + \hat{\alpha} S_{\hat{R}^4} + \hat{\alpha} S_{\hat{C}\hat{X}_8} + \hat{\alpha} S_{\hat{G}^2\hat{R}^3} + \hat{\alpha} S_{(\hat{\nabla}\hat{G})^2\hat{R}^2} \,. \tag{2.23}$$

Let us proceed by stating the pieces contributing to the eight derivative action (2.23). The most prominent part is the well known $\hat{R}^4$ combination which is

$$S_{\hat{R}^4} = \frac{1}{2\kappa_{11}^2} \int_{\mathcal{M}_{11}} \left(\hat{t}_8 \hat{t}_8 - \frac{1}{24} \epsilon_{11} \epsilon_{11} \right) \hat{R}^4 \hat{*} 1 \tag{2.24}$$

and gets supplemented by an $\hat{R}^4$ coupling to the M-theory three-form $\hat{C}_3$ via an eight-form curvature polynomial $\hat{X}_8$, namely

$$S_{\hat{C}\hat{X}_8} = -\frac{3^2 2^{13}}{2\kappa_{11}^2} \int_{\mathcal{M}_{11}} \hat{C}_3 \wedge \hat{X}_8. \tag{2.25}$$

The sector involving the four-form field strength $\hat{G}_4$ is obtained by lifting the conjectured terms, which we recalled in section 2.1, to eleven dimensions [14]. The terms we are

considering are the ones quadratic in $\hat{G}_4$ and their corresponding actions in (2.23) are

$$S_{\hat{G}^2\hat{R}^3} = -\frac{1}{2\kappa_{11}^2} \int_{\mathcal{M}_{11}} \left(\hat{t}_8\hat{t}_8 + \frac{1}{96}\epsilon_{11}\epsilon_{11}\right)\hat{G}_4^2\hat{R}^3 \hat{*} 1, \tag{2.26}$$

$$S_{(\hat{\nabla}\hat{G})^2\hat{R}^2} = \frac{1}{2\kappa_{11}^2} \int_{\mathcal{M}_{11}} \hat{s}_{18}(\hat{\nabla}\hat{G}_4)^2\hat{R}^2 \hat{*} 1 \,. \tag{2.27}$$

For the detailed structure of all the terms in (2.23) we once again refer the reader to appendix B. Note however that the new tensorial structure $\hat{s}_{18}$ is not fully known but contains six unfixed coefficients a_i [32, 33]. The reason for this ambiguity is that the analysis carried out in [32] is sensitive only to terms which do not vanish at the level of the four point function. There are however six independent combinations of contractions which require a five point function analysis to fix their corresponding coefficient in the effective action.

We will however argue in section 2.5 that we need to add another term to the effective action in eleven dimensions to ensure M-theory - Type IIA duality for the backgrounds we are considering.

2.4 Corrected Y_3 background solution at order ℓ_M^6 in M-theory

We now apply a similar strategy as in section 2.2 for the case of Type IIA and determine a background solution of M-theory at eight derivatives. This solution should again have the property that it reduces to the classical direct product solution of five-dimensional Minkowski spacetime and a compact Calabi-Yau threefold $\mathcal{M}_{11}^{\text{class}} = \mathbb{M}_{1,4} \times Y_3$ as $\hat{\alpha} \to 0$ considered in [2]. This problem was already solved in the case of a three-dimensional Minkowski spacetime and a complex four-dimensional compact internal space, which is at lowest order a Calabi-Yau fourfold [26, 34], a Spin(7) holonomy manifold [35] or an internal manifold with G_2 holonomy [36].

The ansatz for the fourfold solution in [26] involves a warp factor, fluxes and an overall Weyl rescaling. The necessity for warping and fluxes can be traced back to the fact, that the eight-form curvature polynomial $\hat{X}_8$ does not vanish on the internal Calabi-Yau manifold and one therefore has to take into account fluxes in order to ensure, that the $\hat{C}_3$ equation of motion is satisfied. In our case the situation is different, since the eight-form $\hat{X}_8$ trivially vanishes on the Calabi-Yau threefold Y_3. Thus the modified ansatz for the background solution can be taken to have the following form

$$\langle d\hat{s}^2 \rangle = e^{\hat{\alpha}\langle \Phi^{(1)}(y^m) \rangle} \left(\eta_{\mu\nu}dx^\mu dx^\nu + (g_{mn}^{(0)} + \hat{\alpha}g_{mn}^{(1)}) dy^m dy^n\right), \tag{2.28}$$

$$\langle \hat{G}_4 \rangle = 0.$$

The variation of the quantum corrected action (2.23) then gives rise to the following conditions on the corrections $\langle \Phi^{(1)}(y^m) \rangle$ and $g_{mn}^{(1)}(y^m)$:

- **external Einstein equation.**

$$g^{(0)mn} R^{(1)}_{mn} - 9 g^{(0)mn} \nabla^{(0)}_m \nabla^{(0)}_n \langle \Phi^{(1)} \rangle = 0 \tag{2.29}$$

- **internal Einstein equation.**

$$0 = R^{(1)}_{mn} - \frac{1}{2} g^{(0)}_{mn} g^{(0)kl} R^{(1)}_{kl} - \frac{9}{2} \nabla_m \nabla_n \langle \Phi^{(1)} \rangle + \frac{9}{2} g^{(0)}_{mn} \nabla_k \nabla^k \langle \Phi^{(1)} \rangle \tag{2.30}$$
$$+ 768(2\pi)^3 J^{(0)l}_m J^{(0)k}_n \nabla_l \nabla_k \big(*^{(0)}_6 c^{(0)}_3 \big) \,.$$

In order to fix the global Weyl factor $\langle \Phi^{(1)} \rangle$ one again takes the trace over the internal Einstein equation and eliminates all expressions involving $R^{(1))}_{mn}$ by making use of the external Einstein equation resulting in

$$\langle \Phi^{(1)} \rangle = -\frac{512}{3} (2\pi)^3 \, *^{(0)}_6 \, c^{(0)}_3 , \tag{2.31}$$

$$R^{(1)}_{mn} = -768(2\pi)^3 \left(\nabla^{(0)}_m \nabla^{(0)}_n + J^{(0)l}_m J^{(0)k}_n \nabla^{(0)}_l \nabla^{(0)}_k \right) *^{(0)}_6 \, c^{(0)}_3 . \tag{2.32}$$

It is important to notice that the correction to the internal Ricci tensor is governed by an expression which is twice a covariant derivative of the Hodge dual of the third Chern class. The strategy to solve this equation is to split $*^{(0)}_6 c^{(0)}_3$ into a part which is constant on the internal space and therefore drops out of the equation of motion and a part which varies non trivially over Y_3. To make this separation one uses the fact that the third Chern class satisfies $\mathrm{d} c^{(0)}_3 = 0$ but $\mathrm{d}^\dagger c^{(0)}_3 \neq 0$ and can therefore be expanded as

$$c^{(0)}_3 = \Pi_H c^{(0)}_3 + i \partial \bar{\partial} \xi \tag{2.33}$$

by applying the $\partial \bar{\partial}$-Lemma, where $\Pi_H c^{(0)}$ is the harmonic part of $c^{(0)}_3$ which is unique by virtue of the Hodge decomposition theorem and ξ is a $(2,2)$-form satisfying $\partial^\dagger \xi = \bar{\partial}^\dagger \xi = 0$. The unspecified $(2,2)$- form clearly parametrizes the non-harmonicity of the third Chern class. Since $\Pi_H c^{(0)}_3$ is harmonic by definition and $[\Delta_\mathrm{d}, *_6] = 0$ it follows immediately that the scalar function $h(y^m) \equiv *^{(0)}_6 \Pi_H c^{(0)}_3$ is constant on the compact Y_3 and can therefore be ignored in the equation of motion. Furthermore, using that on a Kähler manifold the Laplacian satisfies

$$\Delta_\mathrm{d} = 2 \Delta_\partial = 2 \Delta_{\bar{\partial}} \tag{2.34}$$

one shows that

$$i *^{(0)}_6 \partial \bar{\partial} \xi = -\frac{1}{2} \Delta^{(0)} *^{(0)}_6 \left(J^{(0)} \wedge \xi \right) , \tag{2.35}$$

where $J^{(0)}$ is the Kähler form of Y_3 and $\Delta^{(0)} = \nabla^{(0)}_k \nabla^{(0)k}$ is the Laplace - Beltrami operator. Having determined the non-trivial part of the correction to the internal Ricci tensor the equation determining $g^{(1)}$ reads

$$R^{(1)}_{mn} = 384(2\pi)^3 \left(\nabla^{(0)}_m \nabla^{(0)}_n \nabla^{(0)}_k \nabla^{(0)k} + J^{(0)r}_m J^{(0)s}_n \nabla^{(0)}_r \nabla^{(0)}_s \nabla^{(0)}_k \nabla^{(0)k} \right) *^{(0)}_6 \left(J^{(0)} \wedge \xi \right) \tag{2.36}$$

whose solution can be checked to be

$$g^{(1)}_{mn} = -768(2\pi)^3 \left(J^{(0)k}_m J^{(0)l}_n \nabla_k \nabla_l + \nabla_m \nabla_n \right) *^{(0)}_6 \left(J^{(0)} \wedge \xi \right).$$ (2.37)

We again observe in (2.37), that the correction to the metric is twice the derivative of a scalar function. We once more want to stress, that one of the key ingredients for the derivation of consistent two-derivative effective actions from dimensional reduction is the interplay between higher derivative corrections and the fully backreacted background solution, as we will see in section 3.

2.5 Comments on M-theory - Type IIA duality on Calabi-Yau backgrounds at eight derivative level

In the following we will argue that we need to include another piece in the eleven-dimensional action in order to obtain after compactification an action in five dimensions which is consistent with $\mathcal{N} = 2$ supergravity. The reason for the necessity of such a term is the fact, that for the background solutions we are considering a non-trivial correction of the ten-dimensional dilaton occurs, see (2.19). The additional part of the action then compensates for the absence of the dilaton in eleven dimensions. For the discussion in this section we will closely follow [36] and extend it by including the M-theory three-form. The additional piece of the eight derivative action at order $\hat{\alpha}$ we need is

$$\Delta S_{\mathrm{M}} = 256 \int_{\mathcal{M}_{11}} \hat{Z} \, \hat{G}_4 \wedge \hat{*} \, \hat{G}_4 \,,$$ (2.38)

where $\hat{Z}$ is the generalization of the six-dimensional Euler density to eleven dimensions

$$\hat{Z} = \frac{1}{12} \left(\hat{R}_{M_1 M_2}{}^{M_3 M_4} \hat{R}_{M_3 M_4}{}^{M_5 M_6} \hat{R}_{M_5 M_6}{}^{M_1 M_2} - 2 \hat{R}_{M_1}{}^{M_2}{}_{M_3}{}^{M_4} \hat{R}_{M_2}{}^{M_5}{}_{M_4}{}^{M_6} \hat{R}_{M_5}{}^{M_1}{}_{M_6}{}^{M_2} \right).$$ (2.39)

We will now argue that the corrected dilaton in ten dimensions can not be identified with the overall Weyl factor (2.31) but requires the inclusion of (2.38). This indicates that the M-theory - Type IIA duality at eight derivatives has to be modified and thus deviates from the simple S^1 reduction of the classical case. As we showed in section 2.4 the corrected background solution in M-theory requires a Weyl factor (2.31) whereas the Type IIA equations of motion can be solved without the latter. We therefore move to a frame in eleven dimensions which leads to the same Einstein equations for the backgrounds we are considering. This is achieved by the redefinition

$$\hat{g}_{MN} \to \mathrm{e}^{-\frac{512}{3} \hat{\alpha} \hat{Z}} \hat{g}_{MN} \,,$$ (2.40)

such that the classical action picks up two additional terms from this field redefinition

$$2\kappa_{11}^2 S_{\mathrm{M}}^{\mathrm{class}} \to 2\kappa_{11}^2 S_{\mathrm{M}}^{\mathrm{class}} - \int_{\mathcal{M}_{11}} 768 \, \hat{\alpha} \hat{Z} \, \hat{R} \, \hat{*} \, 1 - 128 \hat{\alpha} \, \hat{Z} \, \hat{G}_4 \wedge \hat{*} \, \hat{G}_4 \,.$$ (2.41)

It is now easy to check, that that the equations of motion for an ansatz involving five-dimensional Minkowski spacetime and a compact complex dimension three internal space in this frame can be solved by a direct product

$$\langle \mathrm{d}\hat{s}^2 \rangle = \eta_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu} + 2\big(g^{(0)}_{i\bar{j}} + \hat{\alpha}g^{(1)}_{i\bar{j}}\big)\mathrm{d}z\mathrm{d}\bar{z}^{\bar{j}}, \qquad (2.42)$$

$$\langle \hat{G}_4 \rangle = 0, \qquad (2.43)$$

where $g^{(1)}_{i\bar{j}}$ is simply (2.37) in complex indices. The additional term in (2.41) represent terms which become backreaction effects in the frame, where the Weyl factor has to be included. The expectation is therefore, that with the choice $\mathcal{M}_{10} = \mathbb{M}_{1,3} \times Y_3$ the circle reduction of (2.41) is equivalent to the classical Type IIA action up to a possible field redefinition of the dilaton. This redefinition of the dilaton is expected, since the background solution of Type IIA includes a non-constant dilaton (2.19). Concretely, we will consider an eleven-dimensional ansatz of the form

$$\mathrm{d}\hat{s}^2 = \mathrm{e}^{-\frac{2}{3}\tilde{\phi}}\tilde{g}_{AB}\mathrm{d}x^A\mathrm{d}x^B + \mathrm{e}^{\frac{4}{3}\tilde{\phi}}\mathrm{d}y^2, \qquad (2.44)$$

$$\hat{G}_4 = \tilde{H}_3 \wedge \mathrm{d}y, \qquad (2.45)$$

with the circle coordinate $y \sim y + 1$ and the ten-dimensional metric $\tilde{g}_{AB}$ given by (2.42) with three-dimensional Minkowski spacetime. We will furthermore keep terms linear in the dilaton, since those are not expected to occur at higher derivative level in the string frame and should therefore be captured by a field redefinition. The reduced action (2.41) including linear dilaton terms and the correction ΔS_{M} then reads

$$2\kappa_{11}^2 S^{(10)} = \int_{\mathcal{M}_{10}} \mathrm{e}^{-2\tilde{\phi}}\Big(\tilde{R} \tilde{*} 1 + 4\mathrm{d}\tilde{\phi} \wedge \tilde{*}\mathrm{d}\tilde{\phi} - \frac{1}{2}\tilde{H}_3 \wedge \tilde{*}\tilde{H}_3\Big) \qquad (2.46)$$

$$- \int_{\mathcal{M}_{10}} 768\hat{\alpha}\,\tilde{Z}\,\tilde{R}\tilde{*}1 + 3072\hat{\alpha}\,\tilde{Z}\,\tilde{\Box}\tilde{\phi}\tilde{*}1 - 128\hat{\alpha}\,\tilde{Z}\,\tilde{H}_3 \wedge \tilde{*}\tilde{H}_3 + \Delta S_{\mathrm{M}}. \qquad (2.47)$$

We then notice that we can recover the classical Type IIA action if we perform a redefinition of the dilaton according to

$$\phi \to \phi - 384\hat{\alpha}\,\tilde{Z} \qquad (2.48)$$

and if we identify ΔS_{M} with the expression (2.38). Consequently, the additional part of the eleven-dimensional action (2.38) captures a part of the effect which the corrected dilaton has on the low energy effective action. We stress that this analysis relies on the background geometry stated in this section explicitly. It is therefore expected, that the general analysis requires more complicated structures. However, the analysis of [36] for the case, where the internal space is a G_2 manifold, shows the same properties with the quantity $\hat{Z}$ playing a major role. From now on we will consider the action (2.38) as a part of the full eleven-dimensional action (2.23).

3 M-theory on Calabi-Yau threefolds

We study the dimensional reduction of M-theory on the background solution found in section 2.4 preserving $\mathcal{N} = 2$ supersymmetry in five dimensions. We perturb the background solution and derive the two derivative effective action as well as four derivative operators quadratic in the lower dimensional fields.

3.1 $\mathcal{N} = 2$ supergravity in five dimensions

For later reference the basic ingredients of five-dimensional $\mathcal{N} = 2$ ungauged supergravity are collected. In the dimensional reduction of M-theory we focus entirely on the massless sector. For this reason, the relevant massless multiplets are given in Table 3.1. Note that

multiplet	bosonic field content	# of multiplets
gravity multiplet	metric $g_{\mu\nu}$, graviphoton A^0_μ	1
tensor multiplet	tensor $B_{\mu\nu}$, real scalar ϕ	n_T
vector multiplet	vector A^a_μ, real scalar Φ^a	n_V
hypermultiplet	four real scalars $q^{u=1,\cdots,4}$	n_H

Table 3.1: multiplets of five-dimensional $\mathcal{N} = 2$ supergravity and their field content

the entire tensor multiplet can be dualized into a vector multiplet since in five dimensions a two form B is dual to a vector. Let us now turn to the geometry of the scalar field space in the various multiplets. Since the main focus will lie on the vector- and gravity multiplet, we will only briefly discuss the hypermultiplet sector. The scalar field space $\mathcal{M}_{\text{scalar}}$ is locally given as the direct product [37, 38]

$$\mathcal{M}_{\text{scalar}} = \mathcal{M}_{\text{real sp.}} \times \mathcal{M}_{\text{quat. Käh.}}, \qquad (3.1)$$

where $\mathcal{M}_{\text{quat. Käh.}}$ is a quaternionic Kähler manifold parametrized by the hypermultiplet scalars. The vector multiplet scalar geometry is encoded in a real very special manifold $\mathcal{M}_{\text{real sp.}}$ whose whole information is captured by a cubic potential. This sector is highly restrictive and allows for precise tests of higher derivative couplings based on supersymmetry. The vectormultiplet scalar geometry is describe by the $(n_V + 1)$ very special coordinates L^a, where $a = 0, ..., n_V$ exceeds the number counting the actual vector multiplet scalars by one. However in the end, the scalars L^a parametrize only n_V degrees of freedom. This can be understood in a geometric way as follows. The scalar sector of the vectormultiplet can be interpreted as a n_V dimensional submanifold embedded in an ambient $(n_V + 1)-$dimensional manifold with coordinates L^a. The hypersurface spanned

by the vectormultiplet scalars is defined by a cubic polynomial, which in general takes the form

$$\mathcal{N}(L) = \frac{1}{3!}C_{abc}L^aL^bL^c,\tag{3.2}$$

where C_{abc} is a constant and symmetric tensor. The hypersurface constraint which has to be satisfied by the very special coordinates L^a is then simply given by

$$\mathcal{N}(L) = \frac{1}{3!}C_{abc}L^aL^bL^c = 1.\tag{3.3}$$

The canonical $\mathcal{N} = 2$ supergravity action in the bosonic sector can then be written as [39, 38]

$$S_{\text{can.}}^{(5)} = \int_{\mathcal{M}_5} \frac{1}{2}R\star 1 - \frac{1}{2}G_{ab}\,\mathrm{d}L^a \wedge \star\,\mathrm{d}L^b - h_{uv}\,\mathrm{d}q^u \wedge \star\,\mathrm{d}q^v\tag{3.4}$$

$$- \frac{1}{2}G_{ab}F^a \wedge \star F^b - \frac{1}{6}C_{abc}A^a \wedge F^b \wedge F^c.$$

The notation indicates that the vector in the gravity multiplet A^0 was included in a collective notation such that the index $a = 0, ..., n_V$. The hypermultiplet metric h_{uv} does not play a role in the following and will therefore not be treated. The restrictive nature of $\mathcal{N} = 2$ supergravity in five dimensions does not only follow from the constraint (3.3) which has to be satisfied by the physical scalar fields. The metric for the vector multiplet scalars furthermore follows from the cubic polynomial $\mathcal{N}$ by taking derivatives

$$G_{ab} = -\frac{1}{2}\partial_{L^a}\partial_{L^b}\log\mathcal{N}\bigg|_{\mathcal{N}=1} = -\frac{1}{2}\mathcal{N}_{ab} + \frac{1}{2}\mathcal{N}_a\mathcal{N}_b\bigg|_{\mathcal{N}=1},\tag{3.5}$$

where the notation $\mathcal{N}_a \equiv \partial_{L^a}\mathcal{N}$ was introduced. Thus, the geometry of the vectormultiplet space is fully determined by the cubic potential $\mathcal{N}$.

3.2   The two derivative effective action

The first step in our analysis will be the derivation of the two derivative effective action for the gravity- and vectormultiplet fields as well as a hypermultiplet scalar, which in the classical case is the volume modulus of Y_3. To perform the dimensional reduction of M-theory we perturb the background solution found in section 2.4. A crucial observation is the fact, that the correction to the Calabi-Yau metric $g_{i\bar{j}}^{(1)}$ drops out of the final expression, since it can be written as twice the derivative of a scalar function and therefore only contributes as a total derivative. So effectively the dimensional reduction of (2.23) is performed on the metric background

$$\mathrm{d}\hat{s}^2 = \mathrm{e}^{\hat{\alpha}\Phi^{(1)}}\Big(g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + \big(g_{i\bar{j}}^{(0)} - i\delta v^a\omega_{a\,i\bar{j}}\big)\mathrm{d}z^i\mathrm{d}\bar{z}^{\bar{j}}\Big),\tag{3.6}$$

$$\Phi^{(1)} = -\frac{512}{3}(2\pi)^3\,*_6\,c_3 = \langle\Phi^{(1)}\rangle + \langle\partial_a\Phi^{(2)}\rangle\delta v^a + \frac{1}{2}\langle\partial_a\partial_b\Phi^{(1)}\rangle\delta v^a\delta v^b + \mathcal{O}(\delta v^3),$$

where the deformations of the Kähler class of Y_3 parametrized by δv^a are expanded in a real basis $\omega_a \in H_{\bar{\partial}}^{1,1}(Y_3)$, $a = 1, ..., h^{1,1}$ as

$$\delta g_{i\bar{j}} = -i \delta v^a \omega_{a i\bar{j}}, \tag{3.7}$$

and we have introduced the notation $\partial_a \equiv \partial_{\delta v^a}$. In the M-theory three-form zero-mode expansion we only keep terms contributing to the vector and gravity multiplet in five dimensions. More precisely, we only take into account modes giving rise to vectors A^a in five dimensions. The expansion is thus

$$\hat{C}_3 = A^a \wedge \omega_a, \tag{3.8}$$

$$\hat{G}_4 = \mathrm{d}\hat{C}_3 = F^a \wedge \omega_a,$$

i.e. along the $H_{\bar{\partial}}^{1,1}(Y_3)$ cohomology. In principle, the massless modes in the effective theory do not have to coincide with the ones from the classical reduction. The reason for this is the fact, that the linearized equations of motion, which are solved by the massless modes, can receive non-trivial corrections. Along the lines of [27], using the fact that the massless deformations of the corrected background in the compactification ansatz should preserve the Kähler condition as well as the Bianchi identity for the four-form field strength in the absence of M5-branes, it is possible to show on general grounds that the possible corrections to the massless fields at most contribute as total derivatives to the effective action and therefore decouple. Thus, we will ignore those corrections in the following and treat the perturbations as the ones of the classical M-theory reduction on a Calabi-Yau threefold. We will proceed by recording the results of the contributions of the classical and the eight derivative action to the kinetic terms separately. Finally we will consider all contributions at quadratic order without any five-dimensional derivative, i.e. terms contributing to a scalar potential.

Classical action.

First let us perform the dimensional reduction of the classical Einstein-Hilbert term on the perturbed and α'-corrected background (3.6). Focusing on terms carrying two derivatives in five dimensions up to second order in the fluctuations we obtain

$$\int_{\mathcal{M}_{11}} \hat{R} \,\hat{*}\, 1 \Big|_{\mathrm{kin.}} = \int_{\mathcal{M}_5} \left(\mathcal{V}_{\mathrm{M}} - 768 \, (2\pi)^3 \, \hat{\alpha} \, \chi(Y_3) \right) R \star 1 \tag{3.9}$$

$$+ \int_{\mathcal{M}_5} \mathrm{d}\delta v^a \wedge \star \, \mathrm{d}\delta v^b \int_{Y_3} \left(\frac{1}{2} \omega_{a\,i\bar{j}} \, \omega_b^{\,\bar{j}i} - \omega_{a\,i}{}^{i} \, \omega_{b\,j}{}^{j} \right) \star_6^{(0)} 1$$

$$- \int_{\mathcal{M}_5} 768 \, \hat{\alpha} \left(\frac{1}{2} \mathscr{R}_{ab} + \mathscr{T}_{ab} \right) \mathrm{d}\delta v^a \wedge \star \, \mathrm{d}\delta v^b,$$

where we made use of the shorthand notation

$$\mathcal{V}_{\mathrm{M}} = \int_{Y_3} \left[1 - i\delta v^a \omega_{a\,i}{}^i + \frac{1}{2}\left(\omega_{a\,i\bar{j}}\,\omega_b{}^{\bar{j}i} - \omega_{a\,i}{}^i\omega_{b\,j}{}^j \right)\delta v^a \delta v^b \right] *_6^{(0)} 1, \tag{3.10}$$

$$\mathscr{R}_{ab} = (2\pi)^3 \int_{Y_3} \omega_{a\,i\bar{j}}\,\omega_b{}^{\bar{j}i}\, c_3^{(0)}, \tag{3.11}$$

$$\mathscr{T}_{ab} = (2\pi)^3 \int_{Y_3} \omega_{a\,i}{}^i\omega_{b\,j}{}^j\, c_3^{(0)}, \tag{3.12}$$

which we will use extensively throughout this work. From the classical action we further-more pick up a correction to the kinetic terms of the vectors and a Chern-Simons term in five dimensions

$$\int_{\mathcal{M}_{11}} -\frac{1}{2}\hat{G}\wedge\hat{*}\hat{G}_4 - \frac{1}{6}\hat{C}_3\wedge\hat{G}_4\wedge\hat{G}_4\Big|_{\mathrm{kin.+C.S.}} = \int_{\mathcal{M}_5} \frac{1}{2}F^a\wedge\star F^b \int_{Y_3} \omega_{a\,i\bar{j}}\,\omega_b{}^{\bar{j}i} *_6^{(0)} 1$$

$$- \int_{\mathcal{M}_5} 128\,\hat{\alpha}\,\mathscr{R}_{ab}\,F^a\wedge\star F^b$$

$$+ \int_{\mathcal{M}_5} \frac{1}{6}\mathcal{K}_{abc}\,A^a\wedge F^b\wedge F^c, \tag{3.13}$$

where we introduced the triple intersection numbers $\mathcal{K}_{abc} = \int_{Y_3} \omega_a\wedge\omega_b\wedge\omega_c$ on Y_3, which appear as the coefficients of the Chern-Simons term. We furthermore record the reduction of (2.38) yielding

$$\Delta S_{\mathrm{M}}\big|_{\mathrm{kin}} = - \int_{\mathcal{M}_5} 256\,\hat{\alpha}\,\mathscr{R}_{ab}\,F^a\wedge\star F^b. \tag{3.14}$$

Eight derivative action.

We obtain further contributions to the kinetic terms of the five-dimensional theory by reducing the eight derivative terms in the action (2.23) on the lowest order Calabi-Yau background. The $\mathcal{R}^4$-terms (2.24) lead to a correction to the kinetic term of the Kähler class deformations and a correction to the Ricci scalar

$$2\kappa_{11}^2 S_{\hat{R}^4}\big|_{\mathrm{kin.}} = \int_{\mathcal{M}_5} 768\,(2\pi)^3\,\chi(Y_3)\,R\star 1 + 384\,\mathscr{R}_{ab}\,\mathrm{d}\delta v^a\wedge\star\mathrm{d}\delta v^b, \tag{3.15}$$

and from (2.26) and (2.27) we obtain the corrections to the kinetic terms of the vectors

$$2\kappa_{11}^2\big(S_{\hat{G}^2\hat{R}^3} + S_{(\hat{\nabla}\hat{G})^2 R^2}\big)\big|_{\mathrm{kin.}} = \int_{\mathcal{M}_5} 384\,\mathscr{R}_{ab}\,F^a\wedge\star F^b. \tag{3.16}$$

Note that in order to obtain the result (3.16) we had to fix $a_1 = a_2$ in $\hat{s}_{18}$. This is nec-essary to arrive at an expression, which is solely built of internal space Riemann tensors and harmonic (1,1)-forms without explicit derivatives after applying internal space total derivative identities. Then the final result can be shown to be independent of the unfixed coefficients a_n by applying Schouten identites. Before putting the results obtained in this

section together to obtain the five-dimensional two derivative effective action we briefly comment on the scalar potential.

Scalar potential.

Let us consider the higher curvature terms proportional to $\hat{R}^4$ once again. The dimensional reduction of those, focusing on the zero external derivative contributions gives

$$2\kappa_{11}^2\, S_{\hat{R}^4}\big|_{\text{sc. p.}} = -768(2\pi)^3 \int_{\mathcal{M}_5} \delta v^a \delta v^b \star 1 \int_{Y_3} \nabla_k \nabla^k (*_6^{(0)} c_3^{(0)})\, \omega_{a\,i\bar{j}}\, \omega_b{}^{\bar{j}i} \,*_6^{(0)} 1\,. \qquad (3.17)$$

This indeed looks like a mass term arising for the fluctuations δv^a. Another potential source for a scalar potential is the classical Einstein-Hilbert action, since it is possible to pick up a mass term if one performs the dimensional reduction on the $\hat{\alpha}$-corrected background solution (3.6). The reduced Einstein-Hilbert term then reads

$$\int_{\mathcal{M}_{11}} \hat{R}\,\hat{*}\,1\bigg|_{\text{sc. p.}} = 768(2\pi)^3\,\alpha^2 \int_{\mathcal{M}_5} \delta v^a \delta v^b \star 1 \int_{Y_3} \nabla_k \nabla^k (*_6^{(0)} c_3^{(0)})\, \omega_{a\,i\bar{j}}\, \omega_b{}^{\bar{j}i} \,*_6^{(0)} 1\,, \qquad (3.18)$$

which is exactly the contribution needed to cancel the one coming from the higher curvature terms (3.17). From the reduction results (3.17) and (3.18) it is also possible to see that their contribution entirely stems from the non-harmonicity of the third Chern class given by $i\partial\bar{\partial}\xi$. Moreover this cancellation shows that taking into account the backreaction and expanding the perturbations around a consistent background solution is crucial for the five-dimensional effective action. Note that this cancellation is already expected from a previous analysis on a fourfold in [28].

We are now in a position to collect the various pieces (3.9), (3.13), (3.14), (3.15), (3.16) and merge them into the five-dimensional two derivative effective action

$$2\kappa_{11}^2 S^{(5)} = \int_{\mathcal{M}_5} \mathcal{V}_{\mathrm{M}}\, R \star 1 + \int_{\mathcal{M}_5} \mathrm{d}\delta v^a \wedge \star \mathrm{d}\delta v^b \int_{Y_3} \left(\frac{1}{2}\omega_{a i\bar{j}}\omega_b{}^{\bar{j}i} - \omega_{ai}{}^i\omega_{bj}{}^j\right) *_6^{(0)} 1 \qquad (3.19)$$

$$- \int_{\mathcal{M}_5} 768\,\hat{\alpha}\,\mathscr{T}_{ab}\, \mathrm{d}\delta v^a \wedge \star \mathrm{d}\delta v^b - \frac{1}{2}\int_{\mathcal{M}_5} F^a \wedge \star F^b \int_{Y_3} \omega_a \wedge \star \omega_b$$

$$- \int_{\mathcal{M}_5} \frac{1}{6}\mathcal{K}_{abc}\, A^a \wedge F^b \wedge F^c\,.$$

Note that all terms involving the coupling $\mathscr{R}_{ab}$ in (3.19) canceled. Those cancellations are in fact crucial for compatibility with $\mathcal{N} = 2$ supergravity in five dimensions, as the coupling $\mathscr{R}_{ab}$ is not proportional to the Euler characteristic $\chi(Y_3)$, since the non-harmonic part of the third Chern class prevents us from performing an integral split. The surviving coupling $\mathscr{T}_{ab}$ however satisfies

$$\mathscr{T}_{ab} = -(2\pi)^3 \frac{\chi(Y_3)}{\mathcal{V}^{(0)}} \mathcal{K}_a^{(0)} \mathcal{K}_b^{(0)}\,, \qquad (3.20)$$

since the traces of the harmonic (1,1)-forms are constant. The quantity $\mathcal{V}^{(0)}$ in (3.20) denotes the volume of the zeroth order Calabi-Yau manifold and $\mathcal{K}_a^{(0)}$, $\mathcal{K}_{ab}^{(0)}$ are contractions

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of the intersection numbers with the Kähler moduli evaluated in the background, whose precise form can be found in appendix A. In the following we will denote quantities evaluated in the background with a zero superscript. We now perform a Weyl rescaling of (3.19) according to $g_{\mu\nu} \to \mathcal{V}_{\mathrm{M}}^{-2/3} g_{\mu\nu}$ and the uplift from infinitesimal Kähler class deformations to finite fields v^a leading to the action in Einstein frame

$$2\kappa_{11}^2 S^{(5)} = \int_{\mathcal{M}_5} \left[R \star 1 + \frac{1}{2\mathcal{V}} \left(\mathcal{K}_{ab} - \frac{5}{3\mathcal{V}} \mathcal{K}_a \mathcal{K}_b \right) \mathrm{d}v^a \wedge \star \, \mathrm{d}v^b \right. \tag{3.21}$$

$$+ \frac{1}{2\mathcal{V}^{\frac{1}{3}}} \left(\mathcal{K}_{ab} - \frac{1}{2\mathcal{V}} \mathcal{K}_a \mathcal{K}_b \right) F^a \wedge \star F^b - \frac{1}{6} \mathcal{K}_{abc} A^a \wedge F^b \wedge F^c$$

$$\left. + 768 (2\pi)^3 \, \hat{\alpha} \, \frac{\chi(Y_3)}{\mathcal{V}^3} \mathcal{K}_a \mathcal{K}_b \mathrm{d}v^a \wedge \star \, \mathrm{d}v^b \right] .$$

In (3.21) we used the definition of the Calabi-Yau volume $\mathcal{V} = \frac{1}{3!} \mathcal{K}_{abc} v^a v^b v^c$. We will now make contact with $\mathcal{N} = 2$ supergravity outlined in section 3.1. It is already clear from (3.4) that we make the identification $C_{abc} = \mathcal{K}_{abc}$, since there is no correction to the Chern-Simons term in five dimensions, leading to the cubic constraint

$$\mathcal{N}(L) = \frac{1}{3!} \mathcal{K}_{abc} L^a L^b L^c , \tag{3.22}$$

which the physical scalars in the vectormultiplet L^a have to obey. This constraint is obviously solved by $L^a = \mathcal{V}^{-\frac{1}{3}} v^a$ which is equivalent to the classical case. Due to the relation (3.5) this data is enough to completely fix the geometry on the vectormultiplet and shows, that there are no quantum corrections present in this sector. The correction $\sim \chi(Y_3)$ in (3.21) must therefore be the one of a hypermultiplet. Using the explicit form of the physical scalars L^a one can show that (3.21) is equivalent to

$$\kappa_{11}^2 S^{(5)} = \int_{\mathcal{M}_5} \frac{1}{2} R \star 1 - \frac{1}{2} G_{ab}(L) \, \mathrm{d}L^a \wedge \star \, \mathrm{d}L^b - \frac{1}{2} G_{ab}(L) \, F^a \wedge \star F^b - \frac{1}{4} \mathrm{d} \log \mathcal{V} \wedge \star \, \mathrm{d} \log \mathcal{V}$$

$$+ \int_{\mathcal{M}_5} 384 \, \hat{\alpha} \, \frac{\chi(Y_3)}{\mathcal{V}} \, \mathrm{d} \log \mathcal{V} \wedge \star \, \mathrm{d} \log \mathcal{V} - \frac{1}{6} \mathcal{K}_{abc} A^a \wedge F^b \wedge F^c , \tag{3.23}$$

where we used the metric G_{ab} derived from the cubic potential (3.22) given by

$$G_{ab} = -\frac{1}{2} \partial_a \partial_b \log \mathcal{N}(L) \Big|_{\mathcal{N}=1} = -\frac{1}{2} \mathcal{K}_{abc} L^c + \frac{1}{8} \mathcal{K}_{acd} \mathcal{K}_{bef} L^c L^d L^e L^f . \tag{3.24}$$

Classically one identifies one of the hypermultiplet scalars D with $D = -\frac{1}{2} \log \mathcal{V}$ [38], whereas when taking quantum corrections into account we find the corrected hypermultiplet scalar

$$D = -\frac{1}{2} \log \left(\mathcal{V} + 768 \, \hat{\alpha} \, \chi(Y_3) \right) , \tag{3.25}$$

such that the final action is

$$\kappa_{11}^2 S^{(5)} = \int_{\mathcal{M}_5} \frac{1}{2} R \star 1 - \frac{1}{2} G_{ab}(L) \, \mathrm{d}L^a \wedge \star \, \mathrm{d}L^b - \frac{1}{2} \mathscr{G}_{ab}(L) \, F^a \wedge \star F^b - \mathrm{d}D \wedge \star \, \mathrm{d}D$$

$$- \int_{\mathcal{M}_5} \frac{1}{6} \mathcal{K}_{abc} A^a \wedge F^b \wedge F^c . \tag{3.26}$$

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We have thus shown that our dimensional reduction of M-theory at two derivative level is compatible with $\mathcal{N} = 2$ supergravity. The metric of the vectormultiplet coincides with the one of the classical reduction, such that the only net effect at two derivatives is a corrected field identification of one hypermultiplet scalar. This can in turn be interpreted as a renormalization of the volume of Y_3 at order $\hat{\alpha}$, see (3.25).

3.3 Four derivative terms of the Kähler moduli

We now aim to include four derivative terms in the effective action including at most two fluctuations δv^a. Obviously this truncation misses e.g. the four derivative interaction of the form $(\partial v^a)^4$ since this would require at least an analysis up to order δv^4, which is however technically very involved. In this section we therefore consider the background perturbations (3.7) up to second order. We now introduce the coupling capturing the four derivative terms we find for the Kähler deformations as well as most of the four derivative terms of the vectors in five dimensions derived in section 3.4. It is a non-topological, co-closed (2,2)-form Z whose components are given by

$$Z_{i\bar{j}k\bar{l}} = \varepsilon_{i\bar{j}\bar{i}_1\bar{j}_1 i_2\bar{j}_2}\, \varepsilon_{k\bar{l}k_1\bar{l}_1 k_2\bar{l}_2}\, R^{\bar{j}_1 i_1 \bar{l}_1 k_1}\, R^{\bar{j}_2 i_2 \bar{l}_2 k_2} \tag{3.27}$$

satisfying the relations

$$Z_{i\bar{j}k}{}^k = -2i(2\pi)^2\, (*_6\, c_2)_{i\bar{j}}\,, \tag{3.28}$$

$$Z_l{}^l{}_k{}^k = 2(2\pi)^2\, *_6\, (c_2 \wedge J)\,, \tag{3.29}$$

$$Z_{i\bar{j}k}{}^k \omega_a{}^{\bar{j}i} = 2i(2\pi)^2\, *_6\, (c_2 \wedge \omega_a)\,. \tag{3.30}$$

This object was already recognized to play a role in the context of $\mathcal{N} = 2$ four derivative couplings arising from string compactifications in [29, 40]. The four derivative couplings from the $\hat{R}^4$ terms we find by reducing them in a straightforward way to five dimensions are then

$$2\kappa_{11}^2 S_{\hat{R}^4}\big|_{\text{four der.}} = \int_{\mathcal{M}_5} 192(2\pi)^2 \left[R^2 \star 1 - 4R_{\mu\nu}R^{\mu\nu} \star 1 - 16\,\text{Tr}\,\mathcal{R} \wedge \star\mathcal{R}\right] \int_{Y_3} c_2^{(0)} \wedge J$$

$$- \int_{\mathcal{M}_5} 96\, \mathcal{Z}_{ab}^{(0)} \left[R\, \mathrm{d}\delta v^a \wedge \star\, \mathrm{d}\delta v^b - 4R^{\mu\nu}\partial_\mu\delta v^a \partial_\nu\delta v^b \star 1 + 2(\Box\delta v^a)(\Box\delta v^b) \star 1\right], \tag{3.31}$$

where we performed external spacetime integrations by parts and defined $\Box \equiv \nabla_\mu\nabla^\mu$. Additionally, we introduced the five-dimensional curvature two-form $\mathcal{R}^\mu{}_\nu$ satisfying

$R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \star 1 = -8 \operatorname{Tr} \mathcal{R} \wedge \star \mathcal{R}$ and the shorthand notation

$$\mathcal{Z}_{ab} = \int_{Y_3} Z_{i\bar{j}k\bar{l}} \, \omega_a^{\bar{j}i} \omega_b^{\bar{l}k} *_6 1 \,, \tag{3.32}$$

$$\mathcal{Z}_a = (2\pi)^2 \int_{Y_3} c_2 \wedge \omega_a \,,$$

$$\mathcal{Z} = (2\pi)^2 \int_{Y_3} c_2 \wedge J \,,$$

which we will use frequently in the following[2]. Note that the Kähler form in (3.31) coupling to the second Chern class of Y_3 is with respect to the fluctuated metric $J = J^{(0)} + \delta v^a \omega_a$ and the second order fluctuations of the terms built of two Riemann tensors in five dimensions cancel in a non-trivial way.

In the following we will discuss the Riemann squared terms in (3.31) when moving to the five-dimensional Einstein frame. Note that the second line in (3.31) is already second order in the fluctuations δv^a such that the Weyl rescaling simply leads to an overall factor proportional to the zeroth order Calabi-Yau volume $\mathcal{V}^{(0)}$. The terms involving two five-dimensional Riemann tensors however come with at most linear terms in δv^a and get therefore a less trivial modification from the Weyl rescaling considering terms up to $\mathcal{O}(\delta v^2)$. The explicit form of the action after the Weyl rescaling in terms of the fluctuations is very involved and we will not display it here. Since we are in the end interested in the corresponding action of the finite fields we will give an action, which precisely reproduces the Weyl rescaled action when expanding it in infinitesimal fluctuations. We therefore introduce the scalar

$$u = \log \mathcal{V} \quad \text{with} \quad \mathcal{V} = \frac{1}{3!} \mathcal{K}_{abc} v^a v^b v^c \,. \tag{3.33}$$

The Riemann squared action in Einstein frame then reads

$$2\kappa_{11}^2 S_{R^2}^{\text{Einst.}} = \int_{\mathcal{M}_5} 192 \, \hat{\alpha} \, \mathcal{Z} e^{-u/3} \Big[R^2 \star 1 - 4 \, R_{\mu\nu} R^{\mu\nu} \star 1 - 16 \operatorname{Tr} \mathcal{R} \wedge \star \mathcal{R} \Big] \tag{3.34}$$

$$+ \int_{\mathcal{M}_5} \frac{1}{3} \hat{\alpha} \, \mathcal{Z} \, e^{-u/3} \Big[1536 \, R \, (\Box u) \star 1 - 1536 \, R^{\mu\nu} \, \nabla_\mu \nabla_\nu u \star 1 - 512 \, R \, \mathrm{d}u \wedge \star \, \mathrm{d}u$$

$$+ 256 \, R^{\mu\nu} \, \partial_\mu u \, \partial_\nu u \star 1 + 1024 \, (\Box u)^2 \star 1 \Big] \,,$$

where we now have the moduli dependent coupling

$$\mathcal{Z} \equiv \mathcal{Z}(v^a) = (2\pi)^2 \int_{Y_3} c_2 \wedge J \,, \quad \text{with} \quad J = v^a \omega_a \,. \tag{3.35}$$

The uplift of the terms in the second line in (3.31) is however not obvious, due to the non-topological nature of the coupling $\mathcal{Z}_{ab}^{(0)}$. The naive guess would be to to promote $\mathcal{Z}_{ab}^{(0)} \to \mathcal{Z}_{ab} \equiv \mathcal{Z}_{ab}(v^a)$ to its moduli dependent counterpart and to lift $\delta v^a \to v^a$. A higher order analysis in the fluctuations δv^a might provide further evidence for this claim.

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[2]Note that we always denote evaluation on the background with a zero superscript.

3.4   Four derivative terms of the five-dimensional vectors

Before we continue with the results of the four derivative terms involving five-dimensional vectors, or rather two powers of their field strength to be more precise, let is note that it is well known that there is also a five-dimensional gauge-gravitational Chern-Simons term present [12, 41, 38]. This contribution to the five-dimensional effective action arises upon dimensional reduction of (2.25) and is in our conventions

$$S_{\hat{C}\hat{X}_8} = 768 \int_{\mathcal{M}_5} \mathcal{Z}_a^{(0)} A^a \wedge \text{Tr } \mathcal{R} \wedge \mathcal{R}. \tag{3.36}$$

It was worked out in [22] that the coefficients of this gauge-gravitational Chern-Simons term and the coefficient of the $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ term in (3.31) are related by supersymmetry and matches our computation. This known fact serves as a crosscheck at this point.

We proceed by dimensionally reducing the action

$$S_{\hat{G}_4} = S_{\hat{G}^2\hat{R}^2} + S_{(\hat{\nabla}\hat{G})^2\hat{R}^2} \tag{3.37}$$

keeping terms containing four derivatives in $\mathcal{M}_5$ and up two two five-dimensional field strengths $F^a = \mathrm{d}A^a$. We will keep the coefficients a_n completely generic, however keeping in mind that the discussion of the two derivative action forced us to impose $a_1 = a_2$.

The part of the five-dimensional effective action containing the Ricci scalar and two field strengths of the vectors is

$$2\kappa_{11}^2 S_{\hat{G}_4}\big|_{R\,F^a\wedge\star F^b} = -96 \int_{\mathcal{M}_5} \mathcal{Z}_{ab}^{(0)} R\, F^a \wedge \star F^b \tag{3.38}$$

and the terms involving the five-dimensional Ricci tensor reduce to

$$2\kappa_{11}^2 S_{\hat{G}_4}\big|_{R^{\mu\nu}\,F^a F^b} = 192 \int_{\mathcal{M}_5} \mathcal{Z}_{ab}^{(0)} R^{\mu\nu} F^a_{\ \rho\mu} F^{b\,\rho}_{\ \ \nu} \star 1. \tag{3.39}$$

The treatment of the terms containing one five-dimensional Riemann tensor fully contracted on two field strength tensors F^a is however more complicated. In addition to internal space total derivative identities we made use of the first Bianchi identity for the five-dimensional Riemann tensor and introduced a new object $\mathcal{X}_a^{(0)} = \mathcal{X}_{ai\bar{j}}^{(0)} \mathrm{d}z^i \wedge \mathrm{d}\bar{z}^{\bar{j}}$ whose components are given by

$$\mathcal{X}_{ai\bar{j}}^{(0)} = R_{i\bar{j}k\bar{l}}^{(0)} \omega_a^{\bar{l}k}. \tag{3.40}$$

It is interesting to note, that with this new building block the reduction can be performed without having to fix any of the parameters a_n. We will therefore give the general result, however keeping in mind that $a_1 = a_2$. We then obtain the following additional coupling of the vectors to the five-dimensional Riemann tensor

$$2\kappa_{11}^2 S_{\hat{G}_4}\big|_{R^{\mu\nu\rho\sigma}F^a F^b} = \int_{\mathcal{M}_5} R^{\mu\nu\rho\sigma} F^a_{\ \mu\nu} F^b_{\ \rho\sigma} \star 1 \tag{3.41}$$

$$\times \int_{Y_3} \Big[f_1^{(\alpha)} \mathcal{X}_{a\,i\bar{j}}^{(0)} \mathcal{X}_b^{(0)\,\bar{j}i} + f_2^{(\alpha)} Z_{i\bar{j}k}^{(0)\,k} \omega_a^{\bar{j}n} \omega_{b\,n}^{\ \ i} + f_3^{(\alpha)} Z^{(0)\,k\ l}_{\ \ k\ l} \omega_{a\,i\bar{j}} \omega_b^{\bar{j}i}$$

$$+ f_4^{(\alpha)} Z_{i\bar{j}k}^{(0)\,k} \omega_a^{\bar{j}i} \omega_{b\,l}^{\ \ l} + f_5^{(\alpha)} Z^{(0)\,k\ l}_{\ \ k\ l} \omega_{a\,i}^{\ \ i} \omega_{b\,j}^{\ \ j} + f_6^{(\alpha)} Z_{i\bar{j}k\bar{l}}^{(0)} \omega_a^{\bar{j}i} \omega_b^{\bar{l}k} \Big] \star_6^{(0)} 1.$$

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The coefficients $f_i^{(\alpha)}$, $i = 1, ..., 6$ depend on the unfixed a_n from the definition of the tensor $\hat{s}_{18}$ and are given by the linear relations

$$f_1^{(\alpha)} = -96a_1 + 24a_2 - 36a_3 - 24a_4 + 4a_5 - 2a_6 \,, \tag{3.42}$$

$$f_2^{(\alpha)} = -192a_1 + 72a_2 - 66a_3 - 48a_4 + 6a_5 - 4a_6 \,,$$

$$f_3^{(\alpha)} = 48 + 96a_1 - 24a_2 + 36a_3 + 24a_4 - 4a_5 + 2a_6 \,,$$

$$f_4^{(\alpha)} = 96a_1 - 24a_2 + 36a_3 + 24a_4 - 4a_5 + 2a_6 \,,$$

$$f_5^{(\alpha)} = -48a_1 + 12a_2 - 18a_3 - 12a_4 + 2a_5 - a_6 \,,$$

$$f_6^{(\alpha)} = -48 + 48a_1 - 36a_2 + 12a_3 + 12a_4 + a_6 \,.$$

Observe that we have $f_1^{(\alpha)} = -f_4^{(\alpha)} = 2f_5^{(\alpha)}$. The last contribution we are lacking is the structure with the schematic form $\nabla F^a \nabla F^b$. The reduction reveals, that there are two different structures present, which are however related o each other by exploiting the Bianchi identity of the vectors A^a given by $\mathrm{d}F^a = 0$. In components this means $3\nabla_{[\rho}F^a_{\mu\nu]} = 0$ giving us the identity

$$3\nabla^\mu F^{a\nu\rho}\nabla_{[\mu}F^b_{\nu\rho]} = 0 \quad \Rightarrow \quad \nabla_\nu F^a_{\ \mu\rho}\nabla^\rho F^{b\mu\nu} = \frac{1}{2}\nabla_\mu F^a_{\ \nu\rho}\nabla^\mu F^{b\nu\rho} \,, \tag{3.43}$$

which allows us to to eliminate one of the two structures. The resulting piece in the five-dimensional Lagrangian is then

$$2\kappa_{11}^2 S_{G_4}\big|_{\nabla F^a \nabla F^b} = \int_{\mathcal{M}_5} \nabla_\mu F^a_{\ \rho\sigma}\nabla^\mu F^{b\rho\sigma} \star 1 \tag{3.44}$$

$$\times \int_{Y_3} \left[f_1^{(\beta)} \mathcal{X}^{(0)}_{a\,i\bar{j}} \mathcal{X}^{(0)\,\bar{j}i}_b + f_2^{(\beta)} Z^{(0)\,k}_{i\bar{j}k} \omega_a^{\ \bar{j}l}\omega_{b\,l}^{\ \ i} + f_3^{(\beta)} Z^{(0)\,k\ l}_{\ \ k\ l} \omega_{a\,i\bar{j}}\omega_b^{\ \bar{j}i} \right.$$

$$\left. + f_4^{(\beta)} Z^{(0)\,k}_{i\bar{j}k} \omega_a^{\ \bar{j}i}\omega_{b\,l}^{\ \ l} + f_5^{(\beta)} Z^{(0)\,k\ l}_{\ \ k\ l} \omega_{a\,i}^{\ \ i} \omega_{b\,j}^{\ \ j} + f_6^{(\beta)} Z^{(0)}_{i\bar{j}k\bar{l}} \omega_a^{\ \bar{j}i}\omega_b^{\ \bar{l}k} \right] *_6^{(0)} 1$$

and its corresponding coefficients in terms of the a_n are

$$f_1^{(\beta)} = -48a_1 - 24a_3 - 12a_4 + 4a_5 \,, \tag{3.45}$$

$$f_2^{(\beta)} = -72a_1 + 24a_2 - 24a_3 - 12a_4 \,,$$

$$f_3^{(\beta)} = 24a_1 + 12a_3 + 6a_4 \,,$$

$$f_4^{(\beta)} = 24a_1 + 12a_3 \,,$$

$$f_5^{(\beta)} = 12a_1 + 6a_3 \,,$$

$$f_6^{(\beta)} = -12 + 24a_1 - 24a_2 + 6a_3 + 6a_4 \,,$$

where we again find a linear relation among the coefficients $f_4^{(\beta)} = 2f_5^{(\beta)}$. We can now put the results together and use the identities (3.28)-(3.30) to obtain the four derivative

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acion of the vectors quadratic in F^a

$$2\kappa_{11}^2 S_F^{(5)} = \int_{\mathcal{M}_5} \zeta_{ab} \left[192\, R^{\mu\nu}\, F^a_{\ \rho\mu} F^{b\rho}_{\ \ \nu} \star 1 - 96\, R\, F^a \wedge \star F^b \right] \tag{3.46}$$

$$+ \int_{\mathcal{M}_5} R^{\mu\nu\rho\sigma} F^a_{\ \mu\nu} F^b_{\ \rho\sigma} \star 1 \int_{Y_3} (2\pi)^2 \Big[-\tilde{f}_1^{(\alpha)} \mathcal{X}_a^{(0)} \wedge *_6^{(0)} \mathcal{X}_b^{(0)} - 2i f_2^{(\alpha)} c_2^{(0)}{}_{i\bar{j}} \omega_a^{\ \bar{j}k} \omega_{b\,k}^{\ \ i} *_6^{(0)} 1$$

$$+ 2 f_3^{(\alpha)} \omega_{b\,i\bar{j}} \omega_b^{\ i\bar{j}} c_2^{(0)} \wedge J^{(0)} + 2i f_4^{(\alpha)} \omega_{b\,l}^{\ \ l} c_2^{(0)} \wedge \omega_a$$

$$+ 2 f_5^{(\alpha)} \omega_{b\,i}^{\ \ i} \omega_{b\,j}^{\ \ j} c_2^{(0)} \wedge J^{(0)} + \tilde{f}_6^{(\alpha)} Z_{i\bar{j}k\bar{l}}^{(0)} \omega_a^{\ \bar{j}i} \omega_b^{\ \bar{l}k} *_6^{(0)} 1 \Big]$$

$$+ \int_{\mathcal{M}_5} \nabla_\mu F^a_{\ \rho\sigma} \nabla^\mu F^{b\,\rho\sigma} \star 1 \int_{Y_3} (2\pi)^2 \Big[-\tilde{f}_1^{(\beta)} \mathcal{X}_a^{(0)} \wedge *_6^{(0)} \mathcal{X}_b^{(0)} - 2i f_2^{(\beta)} c_2^{(0)}{}_{i\bar{j}} \omega_a^{\ \bar{j}k} \omega_{b\,k}^{\ \ i} *_6^{(0)} 1$$

$$+ 2 f_3^{(\beta)} \omega_{a\,i\bar{j}} \omega_b^{\ \bar{j}i} c_2^{(0)} \wedge J^{(0)} + 2i f_4^{(\beta)} \omega_{b\,l}^{\ \ l} c_2^{(0)} \wedge \omega_a$$

$$+ 2 f_5^{(\beta)} \omega_{b\,i}^{\ \ i} \omega_{b\,j}^{\ \ j} c_2^{(0)} \wedge J^{(0)} + \tilde{f}_6^{(\beta)} Z_{i\bar{j}k\bar{l}}^{(0)} \omega_a^{\ \bar{j}i} \omega_b^{\ \bar{l}k} *_6^{(0)} 1 \Big].$$

We furthermore introduced $\tilde{f}_1^{(\alpha,\beta)} = (2\pi)^{-2} f_1^{(\alpha,\beta)}$. A general four derivative $\mathcal{N} = 2$ supergravity theory combined with taking into account the possibility to perform higher derivative field redefinitions could in principle, if available, be used to constrain or even completely fix the coefficients a_n.

4 Type IIA supergravity on Calabi-Yau threefolds

In this section we derive the four dimensional two derivative effective action of Type IIA supergravity including both tree-level and one-loop eight derivative terms in ten dimensions. We stress that higher derivative terms of the ten-dimensional dilaton and the RR-fields are not known and therefore not taken into account. We therefore focus on the $\mathcal{R}^4$ terms in ten dimensions and and the conjectured $\tilde{H}_3$ completion of the one-loop Type IIA terms from [14]. Since those conjectured terms are based on introducing a connection with torsion, upon which the superinvariants $\mathcal{J}_{0,1}$ are evaluated, we assume for this section that the tree-level terms are completed in the same fashion. We already saw in the context of the M-theory Y_3 reduction, that the one-loop terms are compatible with $\mathcal{N} = 2$ supegravity in five dimensions. One consistency check of our computations will be, that we obtain the correction to the prepotential, from which the metric of the complexified Kähler moduli $t^a = b^a + iv^a$ derives, which takes the form

$$f(t) = f_{\text{class}}(t) - i \frac{\zeta(3)}{2(2\pi)^3} \chi(Y_3) = \frac{1}{3!} \mathcal{K}_{abc} t^a t^b t^c - i \frac{\zeta(3)}{2(2\pi)^3} \chi(Y_3) + ..., \tag{4.1}$$

where the ellipses denote terms $\sim k_{ab} t^a t^b + c_a t^a$, which however do not contribute to the Kähler potential, as well as non-perturbative contributions stemming from worldsheet instantons. This correction is well known [12, 19, 42], we however show a first derivation which is solely based on a dimensional reduction of Type IIA supergravity.

4.1 The two derivative effective action

The purpose of this section is to dimensionally reduce the ten-dimensional Type IIA supergravity action including the eight derivative corrections introduced in section 2.1 to four dimensions. We are considering the modified Y_3 solution (2.16) including deformations of the Kähler class parametrized by the fluctuations δv^a as before and in addition scalars b^a from the zero-mode expansion of the NS-NS two-form

$$\tilde{B}_2 = b^a \omega_a \tag{4.2}$$

in harmonic $(1,1)$-forms. Our focus will lie on the kinetic terms of those scalar modes, since they combine into the complexified Kähler moduli $t^a = b^a + iv^a$ and the geometry on the space of those is specified by a prepotential $f(t^a)$. We will split again the reduction into two separate parts: the reduction of the classical Type IIA action on the corrected background (2.16) and the reduction of the eight derivative terms in ten dimensions on the lowest order Calabi-Yau background.

Classical action.

The classical action gives rise to the following contribution to the kinetic terms in four dimensions

$$2\kappa_{10}^2 S_{\text{IIA}}^{\text{class}}\big|_{\text{kin.}} = \int_{\mathcal{M}_4} \left[\mathcal{V}_{\text{M}} - 1536 \, (2\pi)^3 \, \alpha \, \chi(Y_3) \, (\ell_0 + \ell_1) \right] R \star 1 \tag{4.3}$$

$$+ \int_{\mathcal{M}_4} \mathrm{e}^{-2\phi_0} \, \mathrm{d}\delta v^a \wedge \star \mathrm{d}\delta v^b \int_{Y_3} \left(\frac{1}{2} \omega_{a\,i\bar{\jmath}} \, \omega_b{}^{\bar{\jmath}i} - \omega_{a\,i}{}^i \omega_{b\,j}{}^j \right) \ast_6^{(0)} 1$$

$$- \int_{\mathcal{M}_4} 768 \, \alpha \, (\ell_0 + \ell_1) \left(\mathcal{T}_{ab} + \frac{1}{2} \mathcal{R}_{ab} \right) \mathrm{d}\delta v^a \wedge \star \mathrm{d}\delta v^b$$

$$+ \int_{\mathcal{M}_4} \frac{1}{2} \mathrm{e}^{-2\phi_0} \mathrm{d}b^a \wedge \star \mathrm{d}b^b \int_{Y_3} \omega_{a\,i\bar{\jmath}} \, \omega_b{}^{\bar{\jmath}i} \ast_6^{(0)} 1 - 384 \, \alpha \, (\ell_0 + \ell_1) \mathcal{R}_{ab} \, \mathrm{d}b^a \wedge \star \mathrm{d}b^b \,,$$

where we defined the constants

$$\ell_0 = \zeta(3)\mathrm{e}^{-2\phi_0} \,, \qquad \ell_1 = \frac{\pi^2}{3} \tag{4.4}$$

and made use of the definition (3.10).

Eight derivative action.

The reduction of the eight derivative terms in ten dimensions yields the action

$$2\kappa_{10}^2 \big(S_{\tilde{R}^4}^{\text{tree}} + S_{\tilde{R}^4}^{\text{loop}} + S_{\tilde{H}^2} \big)\big|_{\text{kin.}} = \int_{\mathcal{M}_4} -768 \, (2\pi)^2 \, (\ell_0 - \ell_1) \, \chi(Y_3) \, R \star 1 \tag{4.5}$$

$$+ \int_{\mathcal{M}_4} 384 \, (\ell_0 + \ell_1) \mathcal{R}_{ab} \, \mathrm{d}b^a \wedge \star \mathrm{d}b^b$$

$$+ \int_{\mathcal{M}_4} 384 \, (\ell_0 + \ell_1) \mathcal{R}_{ab} \, \mathrm{d}\delta v^a \wedge \star \mathrm{d}\delta v^b \,,$$

which again has the property that, combining it with the reduction from the classical action (4.3), all couplings involving $\mathscr{R}_{ab}$ cancel, such that the four-dimensional action is independent of it. Summing up the two pieces (4.3) and (4.5) of the action in four dimensions leads to

$$2\kappa_{10}^2 S^{(4)} = \int_{\mathcal{M}_4} \mathcal{V}_{\mathrm{IIA}} R \star 1 + \frac{1}{2} \mathrm{e}^{-2\phi_0} \Big(\mathcal{K}_{ab}^{(0)} - \frac{1}{\mathcal{V}^{(0)}}\mathcal{K}_a^{(0)}\mathcal{K}_b^{(0)}\Big) \mathrm{d}b^a \wedge \star \mathrm{d}b^b \tag{4.6}$$

$$+ \int_{\mathcal{M}_4} \frac{1}{2} \mathrm{e}^{-2\phi_0} \Big(\mathcal{K}_{ab}^{(0)} + \frac{1}{\mathcal{V}^{(0)}}\mathcal{K}_a^{(0)}\mathcal{K}_b^{(0)}\Big) \mathrm{d}\delta v^a \wedge \star \mathrm{d}\delta v^b$$

$$+ \int_{\mathcal{M}_4} 768 \,(2\pi)^3 \,(\ell_0 + \ell_1) \,\frac{\chi(Y_3)}{\mathcal{V}^{(0)\,2}} \,\mathcal{K}_a^{(0)}\mathcal{K}_b^{(0)} \mathrm{d}\delta v^a \wedge \star \mathrm{d}\delta v^b \,.$$

We furthermore introduced the notation for the prefactor of the four-dimensional Ricci scalar

$$\mathcal{V}_{\mathrm{IIA}} = \mathcal{V}_{\mathrm{M}} \mathrm{e}^{-2\phi_0} - 1536 \,(2\pi)^3 \,\alpha \,\ell_0 \,\chi(Y_3) \,, \tag{4.7}$$

which can be removed by a Weyl rescaling of the metric $g_{\mu\nu} \to \mathcal{V}_{\mathrm{IIA}}^{-1} \,g_{\mu\nu}$. Performing this rescaling as well as the uplift from infinitesimal to finite fields v^a leads to the effective action in Einstein frame

$$S^{(4)} = S_{\alpha'0}^{(4)} + \alpha'^3 \,S_{\alpha'3}^{(4)}, \tag{4.8}$$

where we have restored the explicit α'-dependence by using (2.2). We split the action (4.8) into a classical and quantum (tree-level and one-loop) corrected part which are explicitly given by

$$2\kappa_{10}^2 S_{\alpha'0}^{(4)} = \int_{\mathcal{M}_4} R \star 1 + \frac{1}{2\mathcal{V}} \Big(\mathcal{K}_{ab} - \frac{1}{\mathcal{V}}\mathcal{K}_a\mathcal{K}_b\Big) \mathrm{d}b^a \wedge \star \mathrm{d}b^b \tag{4.9}$$

$$+ \int_{\mathcal{M}_4} \frac{1}{\mathcal{V}} \Big(\frac{1}{2}\mathcal{K}_{ab} - \frac{1}{\mathcal{V}}\mathcal{K}_a\mathcal{K}_b\Big) \mathrm{d}v^a \wedge \star \mathrm{d}v^b$$

and

$$2\kappa_{10}^2 S_{\alpha'3}^{(4)} = \int_{\mathcal{M}_4} \frac{\chi(Y_3)\,\zeta(3)\,\pi^3}{\mathcal{V}^2} \Big(\mathcal{K}_{ab} - \frac{1}{\mathcal{V}}\mathcal{K}_a\mathcal{K}_b\Big) \mathrm{d}b^a \wedge \star \mathrm{d}b^b \tag{4.10}$$

$$+ \int_{\mathcal{M}_4} \frac{\chi(Y_3)\,\zeta(3)\,\pi^3}{\mathcal{V}^2} \Big(\mathcal{K}_{ab} - \frac{4}{\mathcal{V}}\mathcal{K}_a\mathcal{K}_b\Big) \mathrm{d}v^a \wedge \star \mathrm{d}v^b$$

$$+ \int_{\mathcal{M}_4} \frac{\chi(Y_3)\,\pi^5}{3\mathcal{V}^3} \,\mathrm{e}^{2\phi_0} \,\mathcal{K}_a\mathcal{K}_b \,\mathrm{d}v^a \wedge \star \mathrm{d}v^b.$$

Let us now comment on the origin of the various corrections. It is evident in (4.6) that the corrections to the kinetic term of the scalars b^a cancel and can thus only come from the Weyl rescaling. This in turn implies, that the correction is at tree-level in g_s, since the contributions to the Ricci scalar for the one-loop terms precisely cancel. This in turn

can be traced back to the fact, that the $\mathcal{R}^4$ terms in Type IIA suffer from a relative sign flip when comparing tree-level to one-loop. The Kähler moduli receive both tree-level and one-loop corrections. The one-loop corrections are marked in (4.10) with a factor proportional to the dilaton vev.

Now let us discuss the possible corrections of the vectormultiplets containing the complexified Kähler moduli t^a. The vectormultiplet metric can not depend on a scalar in a hypermultiplet, in particular the dilaton, which identifies the term $\sim \mathrm{e}^{2\phi_0}$ in (4.10) as a correction to a hypermultiplet. It is furthermore well known, that the prepotential in Type IIA obtains a tree-level (in g_s) correction of the prepotential according to

$$f(t) = \frac{1}{3!}\mathcal{K}_{abc}\, t^a t^b t^c - i\, a\, \chi(Y_3)\,, \qquad a \in \mathbb{R}\,. \tag{4.11}$$

The constant a is already completely fixed from the terms $\sim \mathcal{K}_{ab}$ in (4.10), while terms $\sim \mathcal{K}_a \mathcal{K}_b\, \mathrm{d}v^a \wedge \star \mathrm{d}v^b$ can in principle be absorbed in the definition of a hypermultiplet scalar. One can then compute the metric following from the prepotential (4.11) and compare the coefficient with the one of the $\sim \mathcal{K}_{ab}$ terms in (4.10) which leads to

$$a = 4\,\alpha'^3\,\pi^3\,\zeta(3)\,. \tag{4.12}$$

Choosing units as $\ell_s = 2\pi\sqrt{\alpha'} = 1$ one obtains the desired result for the prepotential correction

$$f(t) = \frac{1}{3!}\,\mathcal{K}_{abc}t^a t^b t^c - i\frac{\zeta(3)}{2(2\pi)^3}\,\chi(Y_3)\,. \tag{4.13}$$

Note however, that a correction $\sim \mathcal{K}_a\,\mathcal{K}_b\,\mathrm{d}b^a \wedge \star\, \mathrm{d}b^b$ cannot be absorbed in the definition of a hypermultiplet scalar ϕ_4 with e.g. a contribution $\phi_4 = \tilde{\phi} + \mathcal{K}_a b^a + ...$, thus spoiling the shift symmetry of b^a. In order to match also the b^a sector with the metric derived from (4.13) we therefore need the additional contribution

$$\Delta S_b^{(4)} = -\int_{\mathcal{M}_4} \frac{\zeta(3)}{8(2\pi)^3}\chi(Y_3)\,\mathcal{V}^{-3}\,\mathcal{K}_a\mathcal{K}_b\,\mathrm{d}b^a \wedge \star\, \mathrm{d}b^b \tag{4.14}$$

from the reduction of the tree-level eight derivative terms including $\tilde{H}_3$. This missing structure strongly indicates, that the logic of obtaining the $\tilde{B}_2$ field completion of the eight derivative terms employed in [14] cannot be applied to the tree-level terms. However, if one considers the modified replacement (2.14) the new structure $\tilde{t}_8\tilde{t}_8\tilde{H}_3^2\tilde{R}^3$ gives precisely the lacking contribution (4.14).

4.2 Four derivative terms of the Kähler moduli

We proceed by determining the four derivative terms of Type IIA supergravity in a similar way to section 3.3. Due to the lack of a complete eight derivative action of the dilaton in ten dimensions we do not take into account the latter. We thus consider the action

$$S_{\tilde{R}^4}^{\mathrm{IIA}} = S_{\tilde{R}^4}^{\mathrm{tree}} + S_{\tilde{R}^4}^{\mathrm{loop}}\,, \tag{4.15}$$

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since the classical action does not lead to any four derivative coupling in five dimensions. However, let us add two comments on the impact of the classical action on the four derivative couplings. After the reduction, the four derivative terms at $\mathcal{O}(\alpha'^3)$ are not in Einstein frame. Performing a Weyl rescaling of the Einstein-Hilbert term after the reduction given by $\sim \mathrm{e}^{-2\phi}\mathcal{V}R$ one obtains the Einstein frame four derivative couplings. Notice that any $\mathcal{O}(\alpha'^3)$ contribution to the Ricci scalar does not alter the form of the four derivative terms at $\mathcal{O}(\alpha'^3)$.

Tree-level terms.

The tree-level terms give rise to the four derivative couplings

$$
\begin{aligned}
2\kappa_{10}^2 \, \ell_0^{-1} \, S_{\tilde{R}^4}^{\text{tree}}\big|_{\text{four der.}} &= \int_{\mathcal{M}_4} 768 \, \mathcal{Z}_a^{(0)} \left[R\left(\Box \delta v^a\right) - 2R^{\mu\nu} \, \nabla_\mu \nabla_\nu \delta v^a \right] \star 1 \\
&\quad + \int_{\mathcal{M}_4} 768 \, (2\pi)^2 \left[R_{\mu\nu} R^{\mu\nu} - 4R^2 \right] \star 1 \int_{Y_3} c_2^{(0)} \wedge J \qquad (4.16) \\
&\quad + \int_{\mathcal{M}_4} 96 \, \mathcal{Z}_{ab}^{(0)} \left[R \, \mathrm{d}\delta v^a \wedge \star \, \mathrm{d}\delta v^b - 4 \, R^{\mu\nu} \, \partial_\mu \delta v^a \partial_\nu \delta v^b \star 1 \right] \\
&\quad + \int_{\mathcal{M}_4} 192 \, \mathcal{Z}_{ab}^{(0)} \left[(\Box \delta v^a)(\Box \delta v^b) \star 1 - 2\nabla_\mu \nabla_\nu \delta v^a \nabla^\mu \nabla^\nu \delta v^b \right] \star 1 \, ,
\end{aligned}
$$

where $J = J^{(0)} + \delta v^a \omega_a$. Notice that the terms in the first line in (4.16) coupling to divisor integrals of the second Chern class vanish up to derivative terms of the dilaton by virtue of the contracted second Bianchi identity. Performing the Weyl rescaling $g_{\mu\nu} \to \mathrm{e}^{2\phi}\mathcal{V}^{-1}g_{\mu\nu}$ one obtains the effective action in Einstein frame. Due to the appearance of the R^2 terms in four dimensions the Weyl rescaling is rather involved. We therefore refrain from spelling out the effective action in terms of fluctuations but rather give an action in terms of the fields after the uplift, which precisely reproduces the action obtained in terms of the fluctuations δv^a . By defining $\mathcal{G}_{\mu\nu} = R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu}$, in close analogy to the Einstein tensor[3] one finds

$$
\begin{aligned}
2\kappa_{10}^2 \, \ell_0^{-1} \, S_{\tilde{R}^4}^{\text{tree}}\big|_{\text{four der.}} &= \int_{\mathcal{M}_4} 768\,\alpha \left[\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu}\mathcal{Z} + \tfrac{2}{\mathcal{V}}\mathcal{K}_a \mathcal{Z}\, \mathcal{G}_{\mu\nu}\, \nabla^\nu\nabla^\mu v^a \right] \star 1 \\
&\quad - \int_{\mathcal{M}_4} 768\,\alpha \left(\tfrac{1}{2}\mathcal{Z}_{ab} - \tfrac{2}{\mathcal{V}}\mathcal{K}_{ab}\mathcal{Z} + \tfrac{1}{\mathcal{V}^2}\mathcal{K}_a\mathcal{K}_b\mathcal{Z} \right)\mathcal{G}_{\mu\nu}\, \nabla^\mu v^a \nabla^\nu v^b \\
&\quad + \int_{\mathcal{M}_4} 192\,\alpha \left(\mathcal{Z}_{ab} - \tfrac{1}{\mathcal{V}^2}\mathcal{K}_a\mathcal{K}_b\mathcal{Z} \right)(\Box v^a)\,(\Box v^b) \star 1 \\
&\quad - \int_{\mathcal{M}_4} 384\,\alpha \left(\mathcal{Z}_{ab} - \tfrac{2}{\mathcal{V}^2}\,\mathcal{K}_a\mathcal{K}_b\mathcal{Z} \right)\nabla_\mu\nabla_\nu v^a \; \nabla^\mu\nabla^\nu v^b \star 1 \\
&\quad + \int_{\mathcal{M}_4} 768\,\alpha\, \mathcal{Z}_a \left[R\left(\Box v^a\right) - 2R^{\mu\nu}\,\nabla_\mu\nabla_\nu v^a \right] \star 1 \, . \qquad (4.17)
\end{aligned}
$$

---

[3] The Einstein tensor is given by $\mathcal{G}_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$.

The last line in (4.17) involving the coupling to the divisor integrals of the second Chern class again vanishes up to dilaton terms upon integrating by parts. We once more stress, that the trivial uplift $\mathcal{Z}_{ab}^{(0)} \to \mathcal{Z}_{ab}$ is not necessarily the complete answer and the role the tensor $Z_{i\bar{j}k\bar{l}}^{(0)}$ plays both from a mathematical and a physical point of view is not clear.

One-loop terms.

The one-loop terms in ten dimensions give rise to the four derivative terms

$$2\kappa_{10}^2 \, \ell_1^{-1} \, S_{\tilde{R}^4}^{\text{loop}}\big|_{\text{four der.}} = \int_{\mathcal{M}_4} 768 \, \mathcal{Z}_a^{(0)} \big[2R^{\mu\nu} \nabla_\mu \nabla_\nu \delta v^a - R \, (\Box \delta v^a) \big] \star 1$$

$$+ \int_{\mathcal{M}_4} 192 \, (2\pi)^2 \big[2R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2 \big] \star 1 \int_{Y_3} c_2^{(0)} \wedge J$$

$$- \int_{\mathcal{M}_4} 96 \, \mathcal{Z}_{ab}^{(0)} \big[R \, \mathrm{d}\delta v^a \wedge \star \, \mathrm{d}\delta v^b - 4 \, R^{\mu\nu} \, \partial_\mu \delta v^a \partial_\nu \delta v^b \star 1$$

$$+ 2(\Box \delta v^a)(\Box \delta v^b) \big] \, . \qquad (4.18)$$

In the following we will again consider the Weyl rescaling of the squared Riemann terms in (4.18). To do this we redefine the metric as $g_{\mu\nu} \to \mathrm{e}^{2\phi} \mathcal{V}^{-1} g_{\mu\nu}$ leading to a canonically normalized Einstein-Hilbert term. Performing the rescaling and the uplift to finite fields one obtains

$$2\kappa_{10}^2 \, \ell_1^{-1} \, S_{\tilde{R}^4}^{\text{loop}}\big|_{\text{four der.}} = \int_{\mathcal{M}_4} 384 \, \mathcal{Z} \left[R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \tfrac{1}{\mathcal{V}} R \, \mathcal{K}_a \, (\Box v^a) - 2 \, \mathcal{G}_{\mu\nu} \mathcal{G}^{\mu\nu} \right] \star 1$$

$$+ \int_{\mathcal{M}_4} 384 \left[\mathcal{Z}_{ab} \, \mathcal{G}_{\mu\nu} + \tfrac{1}{\mathcal{V}} \mathcal{Z} \, R \, \mathcal{K}_{ab} \, g_{\mu\nu} - \tfrac{3}{2\mathcal{V}^2} \mathcal{Z} \, R \, \mathcal{K}_a \, \mathcal{K}_b \, g_{\mu\nu} \right] \nabla^\mu v^a \nabla^\nu v^b \star 1$$

$$- \int_{\mathcal{M}_4} 192 \left[\mathcal{Z}_{ab} - \tfrac{3}{\mathcal{V}^2} \mathcal{Z} \, \mathcal{K}_a \, \mathcal{K}_b \right] (\Box v^a) \, (\Box v^b) \star 1 \, . \qquad (4.19)$$

4.3 Type IIB supergravity on Calabi-Yau threefolds at four-derivatives

In this section we summarize the four derivative couplings involving Kähler moduli derived in [40] via the same logic as in the previous section but starting from type IIB supergravity in ten dimensions. The intention is to present a complete discussion in this work. The two derivative discussion of type IIB supergravity at order α'^3 was recently revisited in [21] for generic $h^{1,1}$, and reproduces the well known Euler-characteristic correction to the Kähler potential [20]. This discussion involves the parametrisation of the higher-derivative dilaton action and is thus beyond the treatment in this work. However, let us note that the complete axio-dilaton dependence of the $\mathcal{R}^4$-terms in type IIB is known to be

$$S_{\tilde{R}^4}^{\text{IIB}} = \frac{1}{4\kappa_{10}^2} \int E(\tau, \bar{\tau})^{3/2} \Big(\tilde{t}_8 \tilde{t}_8 + \tfrac{1}{8} \epsilon_{10} \epsilon_{10} \Big) \tilde{R}^4 \tilde{\star} 1 \, , \qquad (4.20)$$

where $E(\tau, \bar\tau)^{3/2}$ is the $SL(2, \mathbb{Z})$-invariant Eisenstein series given by

$$E(\tau, \bar\tau)^{3/2} = \sum_{(m,n)\neq(0,0)} \frac{\tau_2^{3/2}}{|m + n\,\tau|^3} \ , \tag{4.21}$$

with $\tau = \tilde{C}_0 + ie^{-\tilde\phi} := \tau_1 + i\tau_2$ the axio-dilaton. When performing the large τ_2 limit, corresponding to the small string coupling limit (4.21) becomes

$$E(\tau, \bar\tau)^{3/2} = 2\zeta(3)\,\tau_2^{3/2} + \tfrac{2\pi^2}{3}\tau_2^{-1/2} + \mathcal{O}(e^{-2\pi\tau_2}) \ . \tag{4.22}$$

In the following discussion, we will use this approximation in (4.20) and only consider the leading order g_s contribution, given by

$$S_{\tilde{R}^4}^{\text{IIB}} = \frac{1}{2\kappa_{10}^2} \int e^{-\frac{3}{2}\tilde\phi} \big(\tilde{t}_8\tilde{t}_8 + \tfrac{1}{8}\epsilon_{10}\epsilon_{10}\big) \tilde{R}^4 \tilde\ast 1 \ . \tag{4.23}$$

The four-derivative corrections arising from the ten-dimensional $\mathcal{R}^4$-terms result in

$$2\kappa_{10}^2\ell_0^{-1}S_{\tilde{R}^4}^{\text{IIB}}\big|_{\text{four der.}} = 192\int_{\mathcal{M}_4}(2\pi)^2e^{-\frac{3}{2}\tilde\phi}\big[4\,R_{\mu\nu}R^{\mu\nu} - R^2\big]\star 1\int_{Y_3}c_2^{(0)} \wedge J \tag{4.24}$$

$$+ 192\int_{\mathcal{M}_4}e^{-\frac{3}{2}\tilde\phi}\mathcal{Z}_{ab}^{(0)}\Big[\big(-2R_{\mu\nu} + \tfrac{1}{2}Rg_{\mu\nu}\big)\nabla^\mu\delta v^a\nabla^\nu\delta v^b\Big]\star 1$$

$$+ 192\int_{\mathcal{M}_4}e^{-\frac{3}{2}\tilde\phi}\mathcal{Z}_{ab}^{(0)}\Big[(\Box\delta v^a)\,(\Box\delta v^b) - 2\nabla_\mu\nabla_\nu\delta v^a\,\nabla^\mu\nabla^\nu\delta v^b\Big]\star 1 \, ,$$

where we once again have $J = J^{(0)} + \delta v^a\omega_a$ and we have dropped the tilde on $\tilde\phi$ to indicate that it is now a four-dimensional field. The final result is derived by combing the Weyl rescaling of the reduction of tyhe classical Einstein-Hilbert action, with the uplift of the reduction result (4.24). The action one obtains is then

$$2\kappa_{10}^2\ell_0^{-1}S_{\text{kin}}^{(4)} = \int_{\mathcal{M}_4} R + \tfrac{1}{\mathcal{V}}\Big(\tfrac{1}{2}\mathcal{K}_{ab} - \tfrac{1}{\mathcal{V}}\mathcal{K}_a\mathcal{K}_b\Big)\mathrm{d}v^a \wedge \star\,\mathrm{d}v^b$$

$$+ \int_{\mathcal{M}_4} 768\,\alpha\,e^{-\frac{3}{2}\tilde\phi}\Big[\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu}\mathcal{Z} + \tfrac{2}{\mathcal{V}}\mathcal{K}_a\mathcal{Z}\,\mathcal{G}_{\mu\nu}\,\nabla^\nu\nabla^\mu v^a\Big]\star 1$$

$$- \int_{\mathcal{M}_4} 768\,\alpha\,e^{-\frac{3}{2}\tilde\phi}\Big(\tfrac{1}{2}\mathcal{Z}_{ab} - \tfrac{2}{\mathcal{V}}\mathcal{K}_{ab}\mathcal{Z} + \tfrac{1}{\mathcal{V}^2}\mathcal{K}_a\mathcal{K}_b\mathcal{Z}\Big)\mathcal{G}_{\mu\nu}\,\nabla^\mu v^a\nabla^\nu v^b$$

$$+ \int_{\mathcal{M}_4} 192\,\alpha\,e^{-\frac{3}{2}\tilde\phi}\Big(\mathcal{Z}_{ab} - \tfrac{1}{\mathcal{V}^2}\mathcal{K}_a\mathcal{K}_b\mathcal{Z}\Big)(\Box v^a)\,(\Box v^b)\star 1$$

$$- \int_{\mathcal{M}_4} 384\,\alpha\,e^{-\frac{3}{2}\tilde\phi}\Big(\mathcal{Z}_{ab} - \tfrac{2}{\mathcal{V}^2}\,\mathcal{K}_a\mathcal{K}_b\mathcal{Z}\Big)\nabla_\mu\nabla_\nu v^a\,\nabla^\mu\nabla^\nu v^b\star 1 \, . \tag{4.25}$$

As expected, since the tree-level $\mathcal{R}^4$-terms of both Type IIA and Type IIB have the same structure, the results of the dimensional reduction coincide, see (4.17) and (4.25). On the

other hand, compared to the one-loop $\mathcal{R}^4$-terms in Type IIB, the corresponding one-loop $\mathcal{R}^4$-terms in Type IIA suffer from a relative sign flip between the two basic structures given by $\tilde{t}_8\tilde{t}_8\tilde{R}^4$ and $\epsilon_{10}\epsilon_{10}\tilde{R}^4$. The two main differences between one-loop contribution to the four dimensional action at four derivatives are: the appearance of a $R^2_{\mu\nu\rho\sigma}$ term in Type IIA, whereas this term is not present in the case of Type IIB and the lack of a term with the structure $\nabla_\mu\nabla_\nu v^a \nabla^\mu\nabla^\nu v^b$ in the case of Type IIA compared to Type IIB. Both cases can be traced back to the different sign structures of the one-loop terms in ten dimensions leading to non-trivial cancellations.

5 Threefold reduction up to four derivatives - the one modulus case

In the following we simplify the discussion and assume that the background geometries under consideration have only one modulus. The advantage of the simplified discussion is, that all computations can be done exactly yet leading to non-trivial results. We consider the $\mathcal{R}^4$-terms in M-theory and both Type II theories, where for the latter we consider tree-level and one-loop terms.

5.1 M-theory one modulus reduction

In this section we comment on the M-theory compactification on a Calabi-Yau threefold including higher derivative terms for the case of a single (volume) modulus. We only consider the gravitational terms in eleven dimensions in the following discussion. The compactification ansatz for the metric is then given by

$$\mathrm{d}\hat{s}^2_{11} = \mathrm{e}^{\hat{\alpha}\Phi^{(1)}} \left(g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + 2\mathrm{e}^{u/3}g^{(0)}_{i\bar{j}}\mathrm{d}z^i\mathrm{d}\bar{z}^{\bar{j}} \right), \tag{5.1}$$

$$\Phi^{(1)} = -\frac{512}{3}(2\pi)^3 *^{(0)}_6 c^{(0)}_3, \tag{5.2}$$

where in (5.1) the Weyl factor $\Phi^{(1)}$ is computed using the metric $g_{i\bar{j}} = \mathrm{e}^{u/3}g^{(0)}_{i\bar{j}}$. The metric $g_{i\bar{j}}$ is furthermore normalized to unit volume, such that the relation

$$\int_{Y_3} *_6 1 = \int_{Y_3} \mathrm{d}^6y\sqrt{g} = \mathrm{e}^u \int_{Y_3} \mathrm{d}^6y\sqrt{g^{(0)}} = \mathrm{e}^u \tag{5.3}$$

holds. Note that the scalar field $u(x)$ is related to the volume modulus by $u(x) = \log \mathcal{V}(x)$. As a first step we dimensionally reduce the action on the background (5.1) including the $\mathcal{R}^4$-terms in eleven dimensions up to four external spacetime derivatives. One obtains in

five dimensions the following action

$$2\kappa_{11}^2 S^{(5)} = \int_{\mathcal{M}_5} e^u R \star 1 + \frac{5}{6} e^u \mathrm{d}u \wedge \star \mathrm{d}u + 768(2\pi)^3 \hat{\alpha}\chi(Y_3)\mathrm{d}u \wedge \star \mathrm{d}u \tag{5.4}$$

$$+ \int_{\mathcal{M}_5} \hat{\alpha}\,\mathcal{Z}^{(0)}\, e^{u/3} \left[384\,R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 768\,R_{\mu\nu}R^{\mu\nu} + 192R^2\right] \star 1$$

$$+ \int_{\mathcal{M}_5} \hat{\alpha}\,\mathcal{Z}^{(0)}\, e^{u/3} \left[\frac{256}{3}R^{\mu\nu}\,\partial_\mu u\,\partial_\nu u \star 1 + 512R^{\mu\nu}\nabla_\mu\nabla_\nu u \star 1\right.$$

$$\left. - 256R\left(\Box u\right)\star 1 - 64R\,\mathrm{d}u \wedge \star\mathrm{d}u\right]$$

$$+ \int_{\mathcal{M}_5} \hat{\alpha}\,\mathcal{Z}^{(0)}\, e^{u/3} \left[\frac{128}{3}(\Box u)^2 \star 1 + \frac{128}{9}(\Box u)\,\mathrm{d}u \wedge \star\mathrm{d}u + \frac{80}{9}(\partial u)^4 \star 1\right].$$

Note that the quantity $\mathcal{Z}^{(0)}$ is computed using the metric $g_{i\bar{j}}^{(0)}$ which is normalized to unit volume. We furthermore made use of the schematic notation $(\partial u)^4 \equiv \partial_\mu u\,\partial^\mu u\,\partial_\nu u\,\partial^\nu u$. For a canonical normalization of the five-dimensional Einstein-Hilbert term we perform a Weyl rescaling

$$g_{\mu\nu} \to e^\sigma g_{\mu\nu}, \tag{5.5}$$

$$\sigma = -\frac{2}{3}u,$$

which results in the action

$$2\kappa_{11}^2 S^{(5)} = \int_{\mathcal{M}_5} R \star 1 - \frac{1}{2}\mathrm{d}u \wedge \star\mathrm{d}u + 768(2\pi)^3 \hat{\alpha}\chi(Y_3)e^{-u}\mathrm{d}u \wedge \star\mathrm{d}u \tag{5.6}$$

$$+ \int_{\mathcal{M}_5} \hat{\alpha}\,\mathcal{Z}^{(0)} \left[384\,R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 768\,R_{\mu\nu}R^{\mu\nu} + 192R^2\right] \star 1$$

$$+ \int_{\mathcal{M}_5} \hat{\alpha}\,\mathcal{Z}^{(0)} \left[384\,(\Box u)^2 \star 1 - \frac{1280}{3}(\Box u)\,\mathrm{d}u \wedge \star\mathrm{d}u + \frac{368}{3}(\partial u)^4 \star 1\right]$$

$$+ \int_{\mathcal{M}_5} \hat{\alpha}\,\mathcal{Z}^{(0)} \left[256R\left(\Box u\right)\star 1 - \frac{448}{3}R\,\mathrm{d}u \wedge \star\mathrm{d}u\right]$$

in Einstein frame, where we have furthermore performed integrations by parts. It is clear, that higher derivative actions of the form (5.6) suffer from ambiguities due to the possibility of performing higher derivative field redefinitions and integrations by parts. Nevertheless, we will propose new field variables such that the action (5.6) takes a rather simple form. If one redefines the five-dimensional metric $g_{\mu\nu}$ and the scalars u as

$$g_{\mu\nu} \to g_{\mu\nu} + a_1^{(\mathrm{M})}\,\alpha\,\mathcal{Z}^{(0)}\,(\Box u)\,g_{\mu\nu} + a_2^{(\mathrm{M})}\,\hat{\alpha}\,\mathcal{Z}^{(0)}\,(\partial u)^2\,g_{\mu\nu} + a_3^{(\mathrm{M})}\,\hat{\alpha}\,\mathcal{Z}^{(0)}\,R\,g_{\mu\nu} \tag{5.7}$$

$$+ a_4^{(\mathrm{M})}\,\hat{\alpha}\,\mathcal{Z}^{(0)}\,R_{\mu\nu} + a_5^{(\mathrm{M})}\,\hat{\alpha}\,\mathcal{Z}^{(0)}\,\partial_\mu u\,\partial_\nu u + a_6^{(\mathrm{M})}\,\alpha\,\mathcal{Z}^{(0)}\,\nabla_\mu\nabla_\nu u\,,$$

$$u \to u + a_7^{(\mathrm{M})}\,\hat{\alpha}\,\mathcal{Z}^{(0)}\,(\Box u) + a_8^{(\mathrm{M})}\,\hat{\alpha}\,\mathcal{Z}^{(0)}\,(\partial u)^2 + a_9^{(\mathrm{M})}\,\hat{\alpha}\,\mathcal{Z}^{(0)}\,R\,, \tag{5.8}$$

where the nine coefficients $a_i^{(M)}$ take the values

$$a_1^{(M)} = -\frac{512}{3} \qquad a_2^{(M)} = \frac{320}{9} \qquad a_3^{(M)} = -128$$

$$a_4^{(M)} = 768 \qquad a_5^{(M)} = 384 \qquad a_6^{(M)} = 0 \qquad (5.9)$$

$$a_7^{(M)} = -384 \qquad a_8^{(M)} = \frac{896}{3} \qquad a_9^{(M)} = 0 \,,$$

the five-dimensional action boils down to

$$2\kappa_{11}^2 S^{(5)} = \int_{\mathcal{M}_5} R \star 1 - \frac{1}{2} \mathrm{d}u \wedge \star \, \mathrm{d}u + 768(2\pi)^3 \, \hat{\alpha}\chi(Y_3)\mathrm{e}^{-u} \mathrm{d}u \wedge \star \, \mathrm{d}u \qquad (5.10)$$

$$+ \int_{\mathcal{M}_5} 384 \, \hat{\alpha} \, \mathcal{Z}^{(0)} \left[R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 \, R_{\mu\nu} R^{\mu\nu} + R^2 \right] \star 1$$

$$+ \int_{\mathcal{M}_5} 192 \, \hat{\alpha} \, \mathcal{Z}^{(0)} \, (\partial u)^4 \star 1.$$

The only four derivative couplings in (5.10) surviving this field redefinition are the squared Riemann terms in the Gauss-Bonnet combination and the $(\partial u)^4$ interaction which leads to a particularly simple form of the effective action. Effectively, one obtains a massless scalar field u coupled to Gauss-Bonnet (super-)gravity[4] and a $(\partial u)^4$ like interaction term. The four derivative terms couple to the theory by means of the divisor integral

$$\mathcal{Z}^{(0)} = \int_{Y_3} c_2^{(0)} \wedge J^{(0)} = \int_D c_2^{(0)} \,, \qquad (5.11)$$

where $D \equiv [J]_{\mathrm{PD}}$ is the divisor Poincare dual to J. At this stage the coupling $\mathcal{Z}^{(0)}$ is independent of the modulus $u = \log \mathcal{V}$, since it is computed using the metric $g_{i\bar{\jmath}}$. The modulus dependent coupling $\mathcal{Z}$

$$\mathcal{Z} = \int_{Y_3} c_2 \wedge J = \mathrm{e}^{u/3} \int_{Y_3} c_2^{(0)} \wedge J^{(0)} = \mathrm{e}^{u/3} \, \mathcal{Z}^{(0)} \,, \qquad (5.12)$$

where the quantities c_2 and J are now associated to the metric $g_{i\bar{\jmath}} = \mathrm{e}^{u/3} g_{i\bar{\jmath}}^{(0)}$, can now be implemented in (5.10) in a straightforward way using (5.12).

5.2 Type IIA one modulus reduction

Now we turn to the discussion of the one-modulus case in Type IIA. We therefore consider in a similar fashion as in the M-theory context shown above the ansatz for the metric

$$\mathrm{d}\tilde{s}_{10}^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + 2\mathrm{e}^{u/3} g_{i\bar{\jmath}}^{(0)} \mathrm{d}z^i \mathrm{d}\bar{z}^{\bar{\jmath}} \,, \qquad (5.13)$$

$$\tilde{\phi} = \phi_0 + \alpha \, \langle \phi^{(1)} \rangle \,, \qquad (5.14)$$

---

[4]once its supersymmetric completion is taken into account

where we again compute $\langle\phi^{(1)}\rangle$ given in (2.16) using the metric $g_{i\bar{j}} = \mathrm{e}^{u/3}g_{i\bar{j}}^{(0)}$. Performing the dimensional reduction of the Type IIA action including tree-level and one-loop corrections results in a four-dimensional effective action of the form

$$S^{(4)} = S^{(4)}_{2\text{ der.}} + \alpha\, S^{(4)}_{R^2} + \alpha\, S^{(4)}_{Ru} + \alpha\, S^{(4)}_{u}\,. \tag{5.15}$$

The various contributions to (5.15) are the two derivative action $S^{(4)}_{2\text{ der.}}$ [5], the piece containing the quadratic Riemann tensor terms $S^{(4)}_{R^2}$, the part of the action containing the mixed Riemann tensor and u terms $S^{(4)}_{Ru}$ and finally the contribution where all four derivatives act on the u scalars, denoted by $S^{(4)}_{u}$. The various pieces are

$$2\kappa_{10}^2 S^{(4)}_{2\text{ der.}} = \int_{\mathcal{M}_4} \left[\mathrm{e}^{-2\phi_0}\mathrm{e}^u - 1536\,\alpha\,(2\pi)^3\,\chi(Y_3)\,\ell_0\right]R\star 1 + \frac{5}{6}\,\mathrm{e}^{-2\phi_0}\,\mathrm{e}^u\,\mathrm{d}u\wedge\star\mathrm{d}u$$
$$+ 768\,\alpha\,(\ell_0 + \ell_1)\,\chi(Y_3)\,(2\pi)^3\,\mathrm{d}u\wedge\star\mathrm{d}u\Big]\,, \tag{5.16}$$

$$2\kappa_{10}^2 S^{(4)}_{R^2} = \int_{\mathcal{M}_4}\mathcal{Z}\left[384\,\ell_1\,R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + (\ell_0-\ell_1)\,R_{\mu\nu}R^{\mu\nu} - 192\,(\ell_0-\ell_1)\,R^2\right]\star 1\,, \tag{5.17}$$

$$2\kappa_{10}^2 S^{(4)}_{Ru} = \int_{\mathcal{M}_4}\mathcal{Z}\left[-\frac{256}{3}\,(\ell_0-\ell_1)\,R^{\mu\nu}\partial_\mu u\,\partial_\nu u\star 1 - 512\,(\ell_0-\ell_1)\,R^{\mu\nu}\,\nabla_\mu\nabla_\nu u\right.$$
$$\left.+ 256\,(\ell_0-\ell_1)\,R\,(\Box u)\star 1 + 64\,(\ell_0-\ell_1)\,R\,\mathrm{d}u\wedge\star\mathrm{d}u\right]\,, \tag{5.18}$$

$$2\kappa_{10}^2 S^{(4)}_{u} = \int_{\mathcal{M}_4}\mathcal{Z}\left[-\frac{128}{3}\,(\ell_0-\ell_1)\,(\Box u)^2\star 1 - \frac{128}{9}\,(\ell_0-\ell_1)\,(\Box u)\,\mathrm{d}u\wedge\star\mathrm{d}u\right.$$
$$+ \frac{80}{9}\,(\ell_0+\ell_1)\,(\partial u)^4\star 1 + \frac{256}{9}\,\ell_0\,\partial^\mu u\,\nabla_\mu\nabla_\nu u\,\partial^\nu u\star 1$$
$$\left.+ \frac{256}{3}\,\ell_0\,\nabla_\mu\nabla_\nu u\,\nabla^\mu\nabla^\nu u\star 1\right]\,, \tag{5.19}$$

where we made use of the shorthand notation (4.4) and (5.12). A Weyl rescaling according to

$$g_{\mu\nu} \to \left[\mathrm{e}^{-2\phi_0}\mathrm{e}^u - 1536\,\alpha\,(2\pi)^3\,\chi(Y_3)\ell_0\right]^{-1}g_{\mu\nu} \tag{5.20}$$

transforms the action (5.15) into the four dimesnional Einstein frame action

$$S^{(4)}_{\text{IIA}} = S^{(4)}_{\text{class}} + \alpha\,\ell_0\,S^{(4)}_{\text{tree}} + \alpha\,\ell_1\,S^{(4)}_{\text{loop}}\,, \tag{5.21}$$

where we split the action (5.21) in a classical piece $S^{(4)}_{\text{class}}$, a tree-level (in g_s) part $S^{(4)}_{\text{tree}}$ and a one-loop correction $S^{(4)}_{\text{loop}}$. Using external space total derivative identities on can show that the relations

$$\int_{\mathcal{M}_4}\mathrm{d}^4 x\sqrt{-g}\,\mathrm{e}^{u/3}\,\nabla^\mu u\,\nabla_\mu\nabla_\nu u\,\nabla^\nu u = -\frac{1}{2}\int_{\mathcal{M}_4}\mathrm{d}^4 x\sqrt{-g}\,\mathrm{e}^{u/3}\left[\frac{1}{3}(\partial u)^4 + (\Box u)\,(\partial u)^2\right] \tag{5.22}$$

---

[5]including the corrections at $\mathcal{O}(\alpha'^3)$

as well as

$$\int_{\mathcal{M}_4} \mathrm{d}^4 x \sqrt{-g}\, \mathrm{e}^{u/3}\, \nabla_\mu \nabla_\nu u\, \nabla^\mu \nabla^\nu u = \int_{\mathcal{M}_4} \mathrm{d}^4 x \sqrt{-g}\, \mathrm{e}^{u/3} \Big[(\Box u)^2 + \frac{1}{2}\, (\Box u)\, (\partial u)^2 \qquad (5.23)$$

$$+ \frac{1}{18}\, (\partial u)^4 - R^{\mu\nu}\, \partial_\mu u\, \partial_\nu u \Big]$$

hold. The constituents of (5.21) can then be shown to take the form

$$2\kappa_{10}^2 S_{\mathrm{class}}^{(4)} = \int_{\mathcal{M}_4} R \star 1 - \frac{2}{3} \mathrm{d}u \wedge \star\, \mathrm{d}u\,, \qquad (5.24)$$

$$2\kappa_{10}^2 S_{\mathrm{tree}}^{(4)} = \int_{\mathcal{M}_4} \mathcal{Z} \left[768\, R_{\mu\nu} R^{\mu\nu} - 192\, R^2 \right] \star 1 - 2560\, (2\pi)^3\, \chi(Y_3)\, \mathrm{e}^{2\phi_0}\, \mathrm{e}^{-u}\, \mathrm{d}u \wedge \star\, \mathrm{d}u$$

$$+ \int_{\mathcal{M}_4} \mathcal{Z} \left[\frac{6592}{9}\, (\Box u)^2 \star 1 - 128\, (\Box u)\, \mathrm{d}u \wedge \star\, \mathrm{d}u + \frac{2048}{81}\, (\partial u)^4 \star 1 \right]$$

$$+ \int_{\mathcal{M}_4} \mathcal{Z} \left[\frac{2048}{9}\, R^{\mu\nu}\, \partial_\mu u\, \partial_\nu u \star 1 + 1024\, R^{\mu\nu} \nabla_\mu \nabla_\nu u \star 1 \right.$$

$$\left. - 128\, R\, (\Box u) \star 1 - 128\, R\, \mathrm{d}u \wedge \star\, \mathrm{d}u \right]\,, \qquad (5.25)$$

$$2\kappa_{10}^2 S_{\mathrm{loop}}^{(4)} = \int_{\mathcal{M}_4} \mathcal{Z} \left[384\, R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 768\, R_{\mu\nu} R^{\mu\nu} + 192\, R^2 \right] \star 1$$

$$+ \int_{\mathcal{M}_4} 768\, (2\pi)^3\, \mathrm{e}^{2\phi_0}\, \chi(Y_3)\, \mathrm{e}^{-u}\, \mathrm{d}u \wedge \star\, \mathrm{d}u$$

$$+ \int_{\mathcal{M}_4} \mathcal{Z} \left[448 (\Box u)^2 \star 1 - \frac{6016}{9}\, (\Box u)\, \mathrm{d}u \wedge \star\, \mathrm{d}u + \frac{4928}{27}\, (\partial u^4) \star 1 \right]$$

$$+ \int_{\mathcal{M}_4} \mathcal{Z} \left[- 256\, R^{\mu\nu}\, \partial_\mu u \partial_\nu u \star 1 + 512\, R^{\mu\nu} \nabla_\mu \nabla_\nu u \star 1 \right.$$

$$\left. + 128\, R\, (\Box u) \star 1 - 256\, R\, \mathrm{d}u \wedge \star\, \mathrm{d}u \right]\,. \qquad (5.26)$$

Similar to the M-theory discussion in section 5.1 we can again consider higher derivative field redefinitions. The general ansatz for those redefinitions reads

$$g_{\mu\nu} \to g_{\mu\nu} + a_1^{(\mathrm{A})}\, \alpha\, \mathcal{Z}^{(0)}\, \mathrm{e}^{u/3}\, (\Box u)\, g_{\mu\nu} + a_2^{(\mathrm{A})}\, \hat{\alpha}\, \mathcal{Z}^{(0)}\, \mathrm{e}^{u/3}\, (\partial u)^2\, g_{\mu\nu} + a_3^{(\mathrm{M})}\, \hat{\alpha}\, \mathcal{Z}^{(0)}\, \mathrm{e}^{u/3}\, R\, g_{\mu\nu}$$
$$(5.27)$$

$$+ a_4^{(\mathrm{A})}\, \hat{\alpha}\, \mathcal{Z}^{(0)}\, \mathrm{e}^{u/3}\, R_{\mu\nu} + a_5^{(\mathrm{A})}\, \hat{\alpha}\, \mathcal{Z}^{(0)}\, \mathrm{e}^{u/3}\, \partial_\mu u\, \partial_\nu u + a_6^{(\mathrm{A})}\, \alpha\, \mathcal{Z}^{(0)}\, \mathrm{e}^{u/3}\, \nabla_\mu \nabla_\nu u\,,$$

$$u \to u + a_7^{(\mathrm{A})}\, \hat{\alpha}\, \mathcal{Z}^{(0)}\, \mathrm{e}^{u/3}\, (\Box u) + a_8^{(\mathrm{A})}\, \hat{\alpha}\, \mathcal{Z}^{(0)}\, \mathrm{e}^{u/3}\, (\partial u)^2 + a_9^{(\mathrm{A})}\, \hat{\alpha}\, \mathcal{Z}^{(0)}\, \mathrm{e}^{u/3}\, R\,. \qquad (5.28)$$

The coefficients $a_i^{(\mathrm{A})}$ can then have tree-level and one-loop contributions proportional to $\zeta(3)\mathrm{e}^{-2\phi_0}$ and $\pi^2/3$ respectively which we again choose in a way, such that the action

takes a particularly simple form. The factors multiplying these tree-level and one-loop coefficients in the various parameters $a_i^{(A)} \equiv \alpha_i\,\ell_0 + \beta_i\,\ell_1$ are listed in Table 5.1.

	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$
α_i	-384	$\frac{2048}{9}$	-192	768	$\frac{512}{9}$	1024	$-\frac{3184}{3}$	-96	0
β_i	-384	$\frac{1280}{3}$	-192	768	$-\frac{256}{3}$	512	-592	$\frac{928}{3}$	0

Table 5.1: Our choice of the coefficients in the field redefinitions.

The various pieces in the four-dimensional action we obtain after those redefinitions are then

$$2\kappa_{10}^2 S_{\text{class}}^{(4)} = \int_{\mathcal{M}_4} R \star 1 - \frac{2}{3}\mathrm{d}u \wedge \star\,\mathrm{d}u\,, \tag{5.29}$$

$$2\kappa_{10}^2 S_{\text{tree}}^{(4)} = -\int_{\mathcal{M}_4} 2560\,(2\pi)^3\,\chi(Y_3)\,\mathrm{e}^{2\phi_0}\,\mathrm{e}^{-u}\,\mathrm{d}u \wedge \star\,\mathrm{d}u\; +\; \frac{5632}{81}\,\mathcal{Z}\,(\partial u)^4 \star 1, \tag{5.30}$$

$$2\kappa_{10}^2 S_{\text{loop}}^{(4)} = \int_{\mathcal{M}_4} 384\,\mathcal{Z}\left[R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4\,R_{\mu\nu}R^{\mu\nu} + R^2\right] \star 1 \tag{5.31}$$

$$+ \int_{\mathcal{M}_4} 768\,(2\pi)^3\,\mathrm{e}^{2\phi_0}\,\chi(Y_3)\,\mathrm{e}^{-u}\,\mathrm{d}u \wedge \star\,\mathrm{d}u - \frac{3008}{27}\,\mathcal{Z}\,(\partial u)^4 \star 1\,.$$

5.3   Type IIB one modulus reduction

We finally aim to include the eight derivative $\mathcal{R}^4$-terms for the case of one modulus in Type IIB. We will take into account the classical Einstein-Hilbert action, which is included in Type IIB supergravity, as well as the tree-level and one-loop $\mathcal{R}^4$-corrections in ten dimensions. Since the higher derivative terms of the dilaton and the NS-NS two-form are not known in Type IIB, we do not include them as dynamical fields. The action we are considering thus takes the form

$$S^{\text{IIB}} = S_{\text{class}}^{\text{IIB}} + \alpha\,S_{\tilde{\mathcal{R}}^4}^{\text{IIB}}\,, \tag{5.32}$$

with the classical action

$$S_{\text{class}}^{\text{IIB}} = \int_{\mathcal{M}_{10}} \tilde{R}\,\tilde{\star}\,1 - \frac{1}{2\tau_2^2}\mathrm{d}\tau \wedge \tilde{\star}\,\mathrm{d}\bar{\tau} + \dots\,. \tag{5.33}$$

and the leading order $\mathcal{R}^4$-action defined in (4.20). The ellipses in (5.33) stand for the NS-NS two-form and the R-R fields which we do not display here. The ansatz for the ten-dimensional fields is the background solution found in [21]

$$\mathrm{d}\tilde{s}^2 = \mathrm{e}^{\alpha\Phi^{(1)}}\left(g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + 2\mathrm{e}^{u/3}g_{i\bar{j}}^{(0)}\mathrm{d}z^i \mathrm{d}\bar{z}^{\bar{j}}\right) \tag{5.34}$$

$$\Phi^{(1)} = -192\,(2\pi)^3\,\mathscr{Y}\left(*_6^{(0)}\,c_3^{(0)}\right)$$

$$\tilde{\phi} = \phi_0 + \mathcal{O}(\alpha)\,,$$

where we have defined

$$\mathscr{Y} \equiv \zeta(3) \, e^{-3\phi_0/2} + \frac{\pi^2}{3} \, e^{\phi_0/2} \, . \tag{5.35}$$

As in the M-theory and Type IIA reduction, we introduced the modulus u, such that the internal Calabi-Yau metric $g_{i\bar{j}}^{(0)}$ is normalized to unit volume and the overall Weyl factor $\Phi^{(1)}$ is again computed using $g_{i\bar{j}} = e^{u/3} g_{i\bar{j}}^{(0)}$. There is furthermore a correction to the dilaton $\tilde{\phi}^{(1)} \sim *_6 \, c_3$, which however only contributes at $\mathcal{O}(\alpha^2)$ to the action and can therefore be ignored.

The results of the dimensional reduction are very similar to the tree-level terms of Type IIA presented in section 5.2. We therefore refrain from giving the results before the Weyl rescaling and simply state the result in Einstein frame. The four dimension effective action is then

$$2\kappa_{10}^2 S_{\text{IIB}}^{(4)} = \int_{\mathcal{M}_4} R \star 1 - \frac{2}{3} du \wedge \star du - 2560 \, (2\pi)^3 \, \alpha \, \chi(Y_3) \, \mathscr{Y} \, e^{-u} \, du \wedge \star du \tag{5.36}$$

$$+ \int_{\mathcal{M}_4} \alpha \, \mathscr{Z} \, \mathscr{Y} \left[768 \, R_{\mu\nu} R^{\mu\nu} - 192 \, R^2 \right] \star 1$$

$$+ \int_{\mathcal{M}_4} \alpha \, \mathscr{Z} \, \mathscr{Y} \left[\frac{2048}{9} R^{\mu\nu} \partial_\mu u \, \partial_\nu u \star 1 + 1024 \, R^{\mu\nu} \nabla_\mu \nabla_\nu u \star 1 \right.$$

$$\left. - 128 \, R \, (\Box u) \star 1 - 128 \, R \, du \wedge \star du \right]$$

$$+ \int_{\mathcal{M}_4} \alpha \, \mathscr{Z} \, \mathscr{Y} \left[\frac{6592}{9} (\Box u)^2 \star 1 - 128 \, (\Box u) \, du \wedge \star du + \frac{2048}{81} (\partial u)^4 \star 1 \right] .$$

We can furthermore use the field redefinitions (5.27) and (5.28) and the coefficients in the first column of Table 5.1 to simplify (5.36). Note that we also have to replace $\zeta(3) \, e^{-2\phi_0} \to \mathscr{Y}$. This finally leads to the action

$$2\kappa_{10}^2 S_{\text{IIB}}^{(4)} = \int_{\mathcal{M}_4} R \star 1 - \frac{2}{3} du \wedge \star du - 2560 \, (2\pi)^3 \, \alpha \, \chi(Y_3) \, \mathscr{Y} \, e^{-u} \, du \wedge \star du$$

$$- \int_{\mathcal{M}_4} \alpha \, \frac{5632}{81} \, \mathscr{Z} \, \mathscr{Y} \, (\partial u)^4 \star 1 \, . \tag{5.37}$$

Note that compared to (5.15) the couplings in the action (5.36) involve different powers of the dilaton vev ϕ_0. This is due to the fact, that we started on the one hand in the ten-dimensional string frame for Type IIA, whereas on the other hand our starting point for Type IIB was already the Einstein frame in ten dimensions. We can obtain the same powers of the dilaton as in (5.15) if we perform the shift $u \to u - \frac{3}{2}\phi_0$.

5.4   Comments on the $\mathcal{N} = 1$ orientifold truncation

In this final subsection we will discuss the orientifold truncation of the $\mathcal{N} = 2$ compactification performed in the previous subsection. The resulting theory is expected to be a

$\mathcal{N} = 1$ supergravity theory and we will focus on the subsector of the theory involving the Kähler structure deformations. In addition to this restricted focus we will not try to complete the compactification to a fully consistent $\mathcal{N} = 1$ setup. In fact, the presence of orientifold planes would require a more complete treatment involving D-branes, which themselves eventually contribute higher-derivative terms to the four-dimensional theory. With these caveats in mind we can nevertheless try to push our $\mathcal{N} = 2$ results and comment on the recent proposal of [25] to stabilize moduli. This procedure of direct truncation from $\mathcal{N} = 2$ to $\mathcal{N} = 1$ has been used before in the determination of corrections to the $\mathcal{N} = 1$ Kähler potential in [20, 21].

In the $\mathcal{N} = 1$ settings one faces similar difficulties as for in the discussion of the $\mathcal{N} = 2$ Calabi-Yau threefold compactifications of the previous sections. In particular, a general $\mathcal{N} = 1$ four derivative on-shell action to which one can compare the effective action after performing a reduction is currently not available. There are, however, partial results extracted by expanding certain four derivative terms in superspace [43]. Therefore, it is tempting to compare the $\mathcal{N} = 1$ truncated one modulus reduction of section 5.3 to the action of [43] as suggested in [25]. We will therefore briefly review the required results. The main idea is to take into account a four derivative $\mathcal{N} = 1$ Lagrangian and infer from the four derivative coupling a corresponding scalar potential. The relevant four derivative Lagrangian is [43, 25]

$$\frac{\mathcal{L}_{\text{bos.}}}{\sqrt{-g}} = \tfrac{1}{2}R - G_{i\bar{j}}(A, \bar{A})\, \partial_\mu A^i \partial^\mu \bar{A}^{\bar{j}} - 2\mathrm{e}^K\, T^{\bar{l}\ \ k}_{\ i\ \bar{j}}\, D_k W\, \overline{D_{\bar{l}}W}\, \partial_\mu A^i \partial^\mu \bar{A}^{\bar{j}} \tag{5.38}$$

$$+ T_{ij\bar{k}\bar{l}}\, \partial_\mu A^i \partial^\mu A^j\, \partial_\nu \bar{A}^{\bar{k}} \partial^\nu \bar{A}^{\bar{l}} - V(A, \bar{A})\,.$$

In (5.38) A^i are complex scalars in chiral multiplets, W is the holomorphic superpotential, $T_{ij\bar{k}\bar{l}}$ is the coupling tensor for the four derivative interaction and D_j is the Kähler covariant derivative. Additionally, the Kähler metric $G_{i\bar{j}}$ is given in terms of a Kähler potential $G_{i\bar{j}} = \partial_i \partial_{\bar{j}} K(A, \bar{A})$. The scalar potential in (5.38) consists of two terms $V(A, \bar{A}) = V_{(0)} + V_{(1)}$, where

$$V_{(0)} = \mathrm{e}^K \left(G^{i\bar{j}} D_i W\, \overline{D_{\bar{j}}W} - 3|W|^2 \right),$$

$$V_{(1)} = -\mathrm{e}^{2K} T^{\bar{i}\bar{j}kl} \overline{D_{\bar{i}}W}\, \overline{D_{\bar{j}}W}\, D_k W\, D_l W\,. \tag{5.39}$$

In a simplified setup with only a single Kähler modulus sitting in a chiral multiplet after the $\mathcal{N} = 1$ truncation we have to compare (5.38) with the reduction result (5.37).

In order to perform the suggested comparison we first have to determine the correct $\mathcal{N} = 1$ coordinates. It is well-known [44, 20, 45] that at leading order one has to introduce complex fields $A \equiv \rho + i\,\mathrm{e}^{\frac{2}{3}u}$, where ρ is the appropriately re-scaled scalar arising from the R-R four-form and we recall $u = \log \mathcal{V}$ is the logarithm of the Einstein-frame volume of Y_3 as seen in (5.3). Taking into account the $\mathcal{O}(\alpha'^3)$ corrections at the two-derivative level they potentially correct the $\mathcal{N} = 1$ coordinates. However, it was argued in [20, 21] that the above A and the complex dilaton-axion $\tau = C_0 + ie^{-\phi}$ are still the correct $\mathcal{N} = 1$

coordinates. In the following we will freeze τ and only consider the dynamics of the field A.

In order to match the action (5.38) with the reduction result (5.37) we now have to assume, that the complexified coordinates A, especially the scalars ρ, arrange themselves in way, such that only the contribution $(\partial A)^2(\partial \bar{A})^2$ enters the four dimensional action. Comparing the action in the correct field variables with (5.38) leads to [6]

$$T_{AA\bar{A}\bar{A}} = -\frac{11}{192} \frac{(2\pi)^2}{\mathcal{V}^4} \left(\mathrm{Im}\tau\right)^{3/2} \int_{Y_3} c_2 \wedge J .$$

(5.40)

This fixes, at least under the stated assumptions, the numerical factor discussed in [25]. Let us stress two points. First, we have used a non-trivial coordinate redefinition to obtain (5.37) and it would be desirable to study its significance in this $\mathcal{N} = 1$ truncated scenario. Second, it would be desirable to compute the terms involving ρ in order to justify the crucial assumption about the dependence on the complex moduli A. In this computation one would have to use the same non-trivial coordinate redefinition, which would provide a non-trivial check.

Let us close this section by some further comments. So far we have failed to generalize the derivation of $T_{ij\bar{k}\bar{l}}$ or other couplings for the kinetic terms of v^i for more than one modulus, since the computational complexity increases significantly. Also the derivation of the actual scalar potential seems currently difficult, since many of the required higher-derivative terms in ten dimensions are not known. While (5.40) seems to suggest that a potential $V_{(1)}$ given in (5.38) is induced if a non-trivial superpotential is included, we are not able to give any further evidence for that. It has been shown, for example, in [46] that it can happen in string reductions that the potential vanishes if one finds other structures for the kinetic terms not appearing in (5.38). It would be desirable to explore this further.

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[6]In this expression we have set $2\pi\sqrt{\alpha'} = 1$.

A Definitions and conventions

The metric signature of the ten and eleven-dimensional spacetime is $(-, +, \ldots, +)$. Our conventions for the totally antisymmetric tensor in Lorentzian signature in an orthonormal frame are $\epsilon_{012\ldots 9(10)} = \epsilon_{0123(4)} = +1$. The epsilon tensor in d dimensions then satisfies

$$\epsilon^{R_1 \cdots R_p N_1 \ldots N_{d-p}} \epsilon_{R_1 \ldots R_p M_1 \ldots M_{d-p}} = (-1)^s (d-p)! p! \delta^{N_1}{}_{[M_1} \ldots \delta^{N_{d-p}}{}_{M_{d-p}]} , \tag{A.1}$$

where $s = 0$ if the metric has Euclidean signature and $s = 1$ for a Lorentzian metric.

We adopt the following conventions for the Christoffel symbols and Riemann tensor

$$\Gamma^R{}_{MN} = \frac{1}{2} g^{RS} (\partial_M g_{NS} + \partial_N g_{MS} - \partial_S g_{MN}) , \qquad\qquad R_{MN} = R^R{}_{MRN} ,$$

$$R^M{}_{NRS} = \partial_R \Gamma^M{}_{SN} - \partial_S \Gamma^M{}_{RN} + \Gamma^M{}_{RT} \Gamma^T{}_{SN} - \Gamma^M{}_{ST} \Gamma^T{}_{RN} , \qquad R = R_{MN} g^{MN} , \tag{A.2}$$

with equivalent definitions on the internal and external spaces. Written in components, the first and second Bianchi identity are

$$R^O{}_{PMN} + R^O{}_{MNP} + R^O{}_{NPM} = 0$$

$$\nabla_L R^O{}_{PMN} + \nabla_M R^O{}_{PNL} + \nabla_N R^O{}_{PLM} = 0 \quad . \tag{A.3}$$

Differential p-forms are expanded in a basis of differential one-forms as

$$\Lambda = \frac{1}{p!} \Lambda_{M_1 \ldots M_p} dx^{M_1} \wedge \ldots \wedge dx^{M_p} \quad . \tag{A.4}$$

The wedge product between a p-form $\Lambda^{(p)}$ and a q-form $\Lambda^{(q)}$ is given by

$$(\Lambda^{(p)} \wedge \Lambda^{(q)})_{M_1 \ldots M_{p+q}} = \frac{(p+q)!}{p! q!} \Lambda^{(p)}_{[M_1 \ldots M_p} \Lambda^{(q)}_{M_1 \ldots M_q]} \quad . \tag{A.5}$$

Furthermore, the exterior derivative on a p-form Λ reads

$$(d\Lambda)_{N M_1 \ldots M_p} = (p+1) \partial_{[N} \Lambda_{M_1 \ldots M_p]} \quad , \tag{A.6}$$

while the Hodge star of p-form Λ in d real coordinates is given by

$$(*_d \Lambda)_{N_1 \ldots N_{d-p}} = \frac{1}{p!} \Lambda^{M_1 \ldots M_p} \epsilon_{M_1 \ldots M_p N_1 \ldots N_{d-p}} \quad . \tag{A.7}$$

Moreover, the identity

$$\Lambda^{(1)} \wedge *\Lambda^{(2)} = \frac{1}{p!} \Lambda^{(1)}_{M_1 \ldots M_p} \Lambda^{(2) M_1 \ldots M_p} * 1 \tag{A.8}$$

holds for two arbitrary p-forms $\Lambda^{(1)}$ and $\Lambda^{(2)}$.

Let us specify in more detail our conventions regarding complex coordinates in the internal space. For a complex Hermitian manifold $\mathcal{M}$ with complex dimension n the complex coordinates $z^1, \ldots, z^n$ and the underlying real coordinates $\xi^1, \ldots, \xi^{2n}$ are related by

$$(z^1, ..., z^n) = \left(\frac{1}{\sqrt{2}}(\xi^1 + i\xi^2), \ldots, \frac{1}{\sqrt{2}}(\xi^{2n-1} + i\xi^{2n}) \right) . \tag{A.9}$$

Using these conventions one finds

$$\sqrt{g}\mathrm{d}\xi^1 \wedge \ldots \wedge \mathrm{d}\xi^{2n} = \sqrt{g}(-1)^{\frac{(n-1)n}{2}} i^n \mathrm{d}z^1 \wedge \ldots \wedge \mathrm{d}z^n \wedge \mathrm{d}\bar{z}^1 \wedge \ldots \wedge \mathrm{d}\bar{z}^n = \frac{1}{n!}J^n , \tag{A.10}$$

with g the determinant of the metric in real coordinates and $\sqrt{\det g_{mn}} = \det g_{i\bar{j}}$. The Kähler form is given by

$$J = i g_{i\bar{j}} \mathrm{d}z^i \wedge \mathrm{d}\bar{z}^{\bar{j}} . \tag{A.11}$$

Let $\omega_{p,q}$ be a (p,q)-form, then its Hodge dual is the $(n-q, n-p)$ form

$$*\omega_{p,q} = \frac{(-1)^{\frac{n(n-1)}{2}} i^n (-1)^{pn}}{p! q! (n-p)! (n-q)!} \omega_{m_1 \ldots m_p \bar{n}_1 \ldots \bar{n}_q} \epsilon^{m_1 \ldots m_p}{}_{\bar{r}_1 \ldots \bar{r}_{n-p}}$$

$$\times \epsilon^{\bar{n}_1 \ldots \bar{n}_q}{}_{s_1 \ldots s_{n-q}} \mathrm{d}z^{s_1} \wedge \cdots \wedge \mathrm{d}z^{s_{n-q}} \wedge \mathrm{d}\bar{z}^{\bar{r}_1} \wedge \cdots \wedge \mathrm{d}\bar{z}^{\bar{r}_{n-p}} . \tag{A.12}$$

Finally, let us record our conventions regarding Chern forms. To begin with, we define the curvature two-form for Hermitian manifolds to be

$$\mathcal{R}^i{}_j = R^i{}_{jk\bar{l}} \mathrm{d}z^k \wedge \mathrm{d}\bar{z}^{\bar{l}} \tag{A.13}$$

and we set

$$\mathrm{Tr}\,\mathcal{R} = R^m{}_{mr\bar{s}} \mathrm{d}z^r \wedge \mathrm{d}\bar{z}^{\bar{s}} ,$$

$$\mathrm{Tr}\,\mathcal{R}^2 = R^m{}_{nr\bar{s}} R^n{}_{mr_1\bar{s}_1} \mathrm{d}z^r \wedge \mathrm{d}\bar{z}^{\bar{s}} \wedge \mathrm{d}z^{r_1} \wedge \mathrm{d}\bar{z}^{\bar{s}_1} ,$$

$$\mathrm{Tr}\,\mathcal{R}^3 = R^m{}_{nr\bar{s}} R^n{}_{n_1 r_1 \bar{s}_1} R^{n_1}{}_{mr_2 \bar{s}_2} \mathrm{d}z^r \wedge \mathrm{d}\bar{z}^{\bar{s}} \wedge \mathrm{d}z^{r_1} \wedge \mathrm{d}\bar{z}^{\bar{s}_1} \wedge \mathrm{d}z^{r_2} \wedge \mathrm{d}\bar{z}^{\bar{s}_2} . \tag{A.14}$$

The Chern forms can then be expressed in terms of the curvature two-form as

$$c_0 = 1 ,$$

$$c_1 = \frac{i}{2\pi} \mathrm{Tr}\,\mathcal{R} ,$$

$$c_2 = \frac{1}{(2\pi)^2} \frac{1}{2} \left(\mathrm{Tr}\,\mathcal{R}^2 - (\mathrm{Tr}\,\mathcal{R})^2 \right) , \tag{A.15}$$

$$c_3 = \frac{1}{3} c_1 \wedge c_2 + \frac{1}{(2\pi)^2} \frac{1}{3} c_1 \wedge \mathrm{Tr}\,\mathcal{R}^2 - \frac{1}{(2\pi)^3} \frac{i}{3} \mathrm{Tr}\,\mathcal{R}^3 ,$$

$$c_4 = \frac{1}{24} \left(c_1^4 - \frac{6}{(2\pi)^2} c_1^2 \wedge \mathrm{Tr}\,\mathcal{R}^2 - \frac{8i}{(2\pi)^3} c_1 \wedge \mathrm{Tr}\,\mathcal{R}^3 \right) + \frac{1}{(2\pi)^4} \frac{1}{8} \left((\mathrm{Tr}\,\mathcal{R}^2)^2 - 2\mathrm{Tr}\,\mathcal{R}^4 \right) .$$

The Chern forms of an n-dimensional Calabi-Yau manifold Y_n reduce to

$$c_3(Y_{n\geq3}) = -\frac{1}{(2\pi)^3}\frac{i}{3}\text{Tr }\mathcal{R}^3 \text{ and } c_4(Y_{n\geq4}) = \frac{1}{(2\pi)^4}\frac{1}{8}\left((\text{Tr }\mathcal{R}^2)^2 - 2\text{Tr }\mathcal{R}^4\right). \quad \text{(A.16)}$$

We furthermore introduce the intersection numbers $\mathcal{K}_{abc}$ as

$$\mathcal{K}_{abc} = D_a \cdot D_b \cdot D_c = \int_{Y_3} \omega_a \wedge \omega_b \wedge \omega_c, \quad \text{(A.17)}$$

where $D_a = [\omega_a]_{\text{PD}}$ are divisors Poincare dual to the harmonic (1,1)-forms ω_a. Contractions of the intersection numbers with Kähler moduli v^a, such as the four-cycle volumes $\mathcal{K}_a$ are defined as

$$\mathcal{K}_a = \frac{1}{2}\mathcal{K}_{abc}v^b v^c \quad \text{(A.18)}$$

$$\mathcal{K}_{ab} = \mathcal{K}_{abc}v^c.$$

The corresponding quantities on the backgroud Calabi-Yau are denoted by $\mathcal{K}_{ab}^{(0)}$, $\mathcal{K}_a^{(0)}$ are defined in an analogous way.

B   Eight derivative terms in ten and eleven dimensions

B.1   Eight derivative terms in ten dimensions

Here we collect the explicit forms of the eight derivative terms relevant for the discussion in the main part. The relevant structures containing four Riemann tensors are the familiar $\tilde{t}_8\tilde{t}_8\tilde{R}^4$ and $\epsilon_{10}\epsilon_{10}\tilde{R}^4$ combinations. Their explicit form in terms of ten-dimensional indices is

$$\tilde{t}_8\tilde{t}_8\tilde{R}^4 = \tilde{t}^{A_1\cdots A_8}\,\tilde{t}_{B_1\cdots B_8}\tilde{R}^{B_1 B_2}{}_{A_1 A_2}\cdots\tilde{R}^{B_7 B_8}{}_{A_7 A_8} \quad \text{(B.1)}$$

$$\epsilon_{10}\epsilon_{10}\tilde{R}^4 = \epsilon_{10}^{C_1 C_2 A_1\cdots A_8}\epsilon_{10\,C_1 C_2 B_1\cdots B_8}\tilde{R}^{B_1 B_2}{}_{A_1 A_2}\cdots\tilde{R}^{B_7 B_8}{}_{A_7 A_8}, \quad \text{(B.2)}$$

where ϵ_{10} is the ten-dimensional curved spacetime Levi-Civita tensor and the tensor $\tilde{t}_8$ has the explicit representation in terms of the metric tensor

$$\tilde{t}_8^{A_1\cdots A_8} = \frac{1}{16}\Big[- 2\big(\tilde{g}^{A_1 A_3}\tilde{g}^{A_2 A_4}\tilde{g}^{A_5 A_7}\tilde{g}^{A_6 A_8} + \tilde{g}^{A_1 A_5}\tilde{g}^{A_2 A_6}\tilde{g}^{A_3 A_7}\tilde{g}^{A_4 A_8} + \tilde{g}^{A_1 A_7}\tilde{g}^{A_2 A_8}\tilde{g}^{A_3 A_5}\tilde{g}^{A_4 A_6}\big)$$

$$+ 8\big(\tilde{g}^{A_2 A_3}\tilde{g}^{A_4 A_5}\tilde{g}^{A_6 A_7}\tilde{g}^{A_8 A_1} + \tilde{g}^{A_2 A_5}\tilde{g}^{A_6 A_3}\tilde{g}^{A_4 A_7}\tilde{g}^{A_8 A_1} + \tilde{g}^{A_2 A_5}\tilde{g}^{A_6 A_7}\tilde{g}^{A_8 A_3}\tilde{g}^{A_4 A_1}\big)$$

$$- (A_1 \leftrightarrow A_2) - (A_3 \leftrightarrow A_4) - (A_5 \leftrightarrow A_6) - (A_7 \leftrightarrow A_8)\Big]. \quad \text{(B.3)}$$

The $\tilde{B}_2$ completion is then obtained by introducing the connection with torsion as outlined in section 2.1

$$\Omega_{\pm A}{}^{\alpha\beta} = \Omega_A{}^{\alpha\beta} \pm \frac{1}{2}\tilde{H}_{3A}{}^{\alpha\beta}, \quad \text{(B.4)}$$

where $\Omega_A{}^{\alpha\beta}$ are the components of the $\mathfrak{so}(1,9)$ - valued connection one-form corresponding to the Levi-Civita connection. In this notation α, β are flat tangent space indices of $\mathcal{M}_{10}$. The structure $\epsilon_{10}\epsilon_{10}\tilde{H}_3^2\tilde{R}^3$ which enters the replacement in section 2.1 has the component form

$$\epsilon_{10}\epsilon_{10}\tilde{H}_3^2\tilde{R}^3(\Omega_\pm) = \epsilon_{10\,C_1A_0\cdots A_8}\,\epsilon_{10}^{C1B_0\cdots B_8}\,\tilde{H}_3^{A_1A_2}{}_{B_0}\tilde{H}_{3B_1B_2}{}^{A_0} \tag{B.5}$$

$$\times\,\tilde{R}(\Omega_+)^{A_3A_4}{}_{B_3B_4}\cdots\tilde{R}(\Omega_+)^{A_7A_8}{}_{B_7B_8}\,.$$

B.2 Eight derivative terms in eleven dimensions

The classical eleven-dimensional supergravity action gets corrected by different contributions which should be discussed in the following. The most prominent higher curvature term in M-theory is the sector containing four Riemann tensors. Those involve two different structures namely

$$S_{R^4} = \frac{1}{2\kappa_{11}^2}\int_{\mathcal{M}_{11}}\left(\hat{t}_8\hat{t}_8 - \frac{1}{24}\epsilon_{11}\epsilon_{11}\right)\hat{R}^4\bar{*}1. \tag{B.6}$$

In (B.6) the two quantities $\hat{t}_8\,\hat{t}_8\hat{R}^4$ and $\epsilon_{11}\,\epsilon_{11}\hat{R}^4$ have the index representation

$$\hat{t}_8\,\hat{t}_8\hat{R}^4 = t_8{}^{M_1\cdots M_8}t_{8N_1\cdots N_8}\hat{R}^{N_1N_2}{}_{M_1M_2}\cdots\hat{R}^{N_7N_8}{}_{M_7M_8} \tag{B.7}$$

$$\epsilon_{11}\epsilon_{11}\hat{R}^4 = \epsilon_{11}{}^{R_1R_2R_3M_1\cdots M_8}\epsilon_{11R_1R_2R_3N_1\cdots N_8}\hat{R}^{N_1N_2}{}_{M_1M_2}\cdots\hat{R}^{N_7N_8}{}_{M_7M_8}. \tag{B.8}$$

The tensor $\hat{t}_8$ is defined in a completely analogous way as in (B.3). Those $\mathcal{R}^4$ - terms are furthermore supplemented by another term quartic in the Riemann tensor. This term however also comprises a three form $\hat{C}_3$. This piece of the higher derivative action then has the form

$$S_{C_3X_8} = -\frac{3^2 2^{13}}{2\kappa_{11}^2}\int_{\mathcal{M}_{11}}\hat{C}_3\wedge\hat{X}_8 \tag{B.9}$$

where eight form $\hat{X}_8$ is defined as

$$\hat{X}_8 = \frac{1}{192}\left[\mathrm{Tr}\,\hat{\mathcal{R}}^4 - \frac{1}{4}\left(\mathrm{Tr}\,\hat{\mathcal{R}}^2\right)^2\right] \tag{B.10}$$

which is in terms of the (real) curvature two form

$$\hat{\mathcal{R}}^M{}_N = \frac{1}{2}\hat{R}^M{}_{NN_1N_2}\mathrm{d}x^{N_1}\wedge\mathrm{d}x^{N_2}. \tag{B.11}$$

In addition to those quartic Riemann tensor terms it was conjectured in [14] that the complete $\hat{G}_4$ dependence at $\mathcal{O}(\hat{G}_4^2)$ is captured by introducing

$$\hat{t}_8\,\hat{t}_8\,\hat{G}_4^2\,\hat{R}^3 = \hat{t}_8^{M_1\cdots M_8}\,\hat{t}_{8N_1\cdots N_8}\hat{G}_4{}^{N_1}{}_{M_1R_1R_2}\hat{G}_4{}^{N_2}{}_{M_2}{}^{R_1R_2}\hat{R}^{N_3N_4}{}_{M_3M_4}\cdots\hat{R}^{N_7N_8}{}_{M_7M_8} \tag{B.12}$$

$$\epsilon_{11}\epsilon_{11}\hat{G}_4^2\,R^3 = \epsilon_{11}{}^{RM_1\cdots M_{10}}\epsilon_{11RN_1\cdots N_{10}}\hat{G}_4{}^{N_1N_2}{}_{M_1M_2}\hat{G}_4{}^{N_3N_4}{}_{M_3M_4}\hat{R}^{N_5N_6}{}_{M_5M_6}\cdots\hat{R}^{N_9N_{10}}{}_{M_9M_{10}}. \tag{B.13}$$

The last eleven-dimensional eight derivative contribution involves the tensor $\hat{s}_{18}$ parametrized by six unknown coefficients $a_n \in \mathbb{R}$. We then have the additional coupling of the form

$$\hat{s}_{18}(\hat{\nabla}\hat{G}_4)^2\hat{R}^2 = \hat{s}_{18}{}^{N_1\cdots N_{18}}\hat{R}_{N_1\cdots N_4}\hat{R}_{N_5\cdots N_8}\hat{\nabla}_{N_9}\hat{G}_{4N_{10}\cdots N_{13}}\hat{\nabla}_{N_{14}}\hat{G}_{4N_{15}\cdots N_{18}}$$

$$= \mathcal{A} + \sum_{n=1}^{6} a_n\,\mathcal{Z}_n\,, \tag{B.14}$$

where the quantities $\mathcal{A}, \mathcal{Z}_n$ are defined by

$$\mathcal{A} = -24B_5 - 48B_8 - 24B_{10} - 6B_{12} - 12B_{13} + 12B_{14} + 8B_{16} - 4B_{20} + B_{22} + 4B_{23} + B_{24}$$

$$\mathcal{Z}_1 = 48B_1 + 48B_2 - 48B_3 + 36B_4 + 96B_6 + 48B_7 - 48B_8 + 96B_{10} + 12B_{12} + 24B_{13}$$

$$\mathcal{Z}_2 = -48B_1 - 48B_2 - 24B_4 - 24B_5 + 48B_6 - 48B_8 - 24B_9 - 72B_{10} - 24B_{13} + 24B_{14}$$
$$- B_{22} + 4B_{23}$$

$$\mathcal{Z}_3 = 12B_1 + 12B_2 - 24B_3 + 9B_4 + 48B_6 + 24B_7 - 24B_8 + 24B_{10} + 6B_{12} + 6B_{13} + 4B_{15}$$
$$- 4B_{17} + 3B_{19} + 2B_{21}$$

$$\mathcal{Z}_4 = 12B_1 + 12B_2 - 12B_3 + 9B_4 + 24B_6 + 12B_7 - 12B_8 + 24B_{10} + 3B_{12} + 6B_{13}$$
$$+ 4B_{15} - 4B_{17} + 2B_{20}$$

$$\mathcal{Z}_5 = 4B_3 - 8B_6 - 4B_7 + 4B_8 - B_{12} - 2B_{14} + 4B_{18}$$

$$\mathcal{Z}_6 = B_4 + 2B_{11}. \tag{B.15}$$

The elements B_i form a basis of the terms with the structure $(\hat{\nabla}\hat{G}_4)^2\hat{R}^2$ given in (B.16).

$B_1 = R_{N_1N_2N_3N_4}R_{N_5N_6N_7N_8}\nabla^{N_5}G^{N_1N_7N_8}{}_{N_9}\nabla^{N_3}G^{N_2N_4N_6N_9}$
$\quad B_2 = R_{N_1N_2N_3N_4}R_{N_5N_6N_7N_8}\nabla^{N_5}G^{N_1N_3N_7}{}_{N_9}\nabla^{N_8}G^{N_2N_4N_6N_9}$

$B_3 = R_{N_1N_2N_3N_4}R_{N_5N_6N_7N_8}\nabla^{N_5}G^{N_1N_3N_7}{}_{N_9}\nabla^{N_6}G^{N_2N_4N_8N_9}$
$\quad B_4 = R_{N_1N_2N_3N_4}R_{N_5N_6N_7N_8}\nabla_{N_9}G^{N_3N_4N_7N_8}\nabla^{N_6}G^{N_9N_1N_2N_5}$

$B_5 = R_{N_1N_2N_3N_4}R_{N_5N_6N_7}{}^{N_4}\nabla^{N_1}G^{N_2N_3}{}_{N_8N_9}\nabla^{N_5}G^{N_6N_7N_8N_9}$
$\quad B_6 = R_{N_1N_2N_3N_4}R_{N_5N_6N_7}{}^{N_4}\nabla^{N_1}G^{N_2N_5}{}_{N_8N_9}\nabla^{N_3}G^{N_6N_7N_8N_9}$

$B_7 = R_{N_1N_2N_3N_4}R_{N_5N_6N_7}{}^{N_4}\nabla^{N_1}G^{N_2N_5}{}_{N_8N_9}\nabla^{N_7}G^{N_3N_6N_8N_9}$
$\quad B_8 = R_{N_1N_2N_3N_4}R_{N_5N_6N_7}{}^{N_4}\nabla^{N_1}G^{N_3N_5}{}_{N_8N_9}\nabla^{N_2}G^{N_6N_7N_8N_9}$

$B_9 = R_{N_1N_2N_3N_4}R_{N_5N_6N_7}{}^{N_4}\nabla^{N_1}G^{N_3N_5}{}_{N_8N_9}\nabla^{N_6}G^{N_2N_7N_8N_9}$
$\quad B_{10} = R_{N_1N_2N_3N_4}R_{N_5N_6N_7}{}^{N_4}\nabla_{N_9}G^{N_3N_5N_7}{}_{N_8}\nabla^{N_9}G^{N_1N_2N_6N_8}$

$B_{11} = R_{N_1N_2N_3N_4}R_{N_5N_6N_7}{}^{N_4}\nabla_{N_8}G^{N_1N_2N_6}{}_{N_9}\nabla^{N_9}G^{N_3N_5N_7N_8}$
$\quad B_{12} = R_{N_1N_2N_3N_4}R_{N_5N_6N_7}{}^{N_4}\nabla^{N_3}G^{N_5N_6}{}_{N_8N_9}\nabla^{N_7}G^{N_2N_1N_8N_9}$

$B_{13} = R_{N_1N_2N_3N_4}R_{N_5}{}^{N_1}{}_{N_6}{}^{N_3}\nabla_{N_9}G^{N_2N_6}{}_{N_7N_8}\nabla^{N_9}G^{N_4N_5N_7N_8}$
$\quad B_{14} = R_{N_1N_2N_3N_4}R_{N_5}{}^{N_1}{}_{N_6}{}^{N_3}\nabla_{N_9}G^{N_2N_4}{}_{N_7N_8}\nabla^{N_5}G^{N_4N_7N_8N_9}$

$B_{15} = R_{N_1N_2N_3N_4}R_{N_5}{}^{N_1}{}_{N_6}{}^{N_3}\nabla^{N_2}G^{N_6}{}_{N_7N_8N_9}\nabla^{N_5}G^{N_4N_7N_8N_9}$
$\quad B_{16} = R_{N_1N_2N_3N_4}R_{N_5}{}^{N_1}{}_{N_6}{}^{N_3}\nabla^{N_2}G^{N_4}{}_{N_7N_8N_9}\nabla^{N_5}G^{N_6N_7N_8N_9}$

$B_{17} = R_{N_1N_2N_3N_4}R_{N_5}{}^{N_1}{}_{N_6}{}^{N_3}\nabla^{N_2}G^{N_5}{}_{N_7N_8N_9}\nabla^{N_4}G^{N_6N_7N_8N_9}$
$\quad B_{18} = R_{N_1N_2N_3N_4}R_{N_5}{}^{N_1}{}_{N_6}{}^{N_3}\nabla_{N_9}G^{N_5N_6}{}_{N_7N_8}\nabla^{N_4}G^{N_2N_7N_8N_9}$

$B_{19} = R_{N_1N_2N_3N_4}R_{N_5N_6}{}^{N_3N_4}\nabla_{N_9}G^{N_1N_5}{}_{N_7N_8}\nabla^{N_9}G^{N_2N_6N_7N_8}$
$\quad B_{20} = R_{N_1N_2N_3N_4}R_{N_5N_6}{}^{N_3N_4}\nabla^{N_1}G^{N_5}{}_{N_7N_8N_9}\nabla^{N_2}G^{N_6N_7N_8N_9}$

$B_{21} = R_{N_1N_2N_3N_4}R_{N_5N_6}{}^{N_3N_4}\nabla^{N_1}G^{N_5}{}_{N_7N_8N_9}\nabla^{N_6}G^{N_2N_7N_8N_9}$
$\quad B_{22} = R_{N_1N_2N_3N_4}R_{N_5}{}^{N_1N_3N_4}\nabla^{N_2}G_{N_6N_7N_8N_9}\nabla^{N_5}G^{N_6N_7N_8N_9}$

$B_{23} = R_{N_1N_2N_3N_4}R_{N_5}{}^{N_1N_3N_4}\nabla_{N_9}G^{N_2}{}_{N_6N_7N_8}\nabla^{N_9}G^{N_5N_6N_7N_8}$
$\quad B_{24} = R_{N_1N_2N_3N_4}R^{N_1N_2N_3N_4}\nabla_{N_5}G_{N_6N_7N_8}\nabla^{N_6}G^{N_5N_7N_8N_9}$

$$\tag{B.16}$$

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