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The effects of anisotropy and non-adiabaticity on the evolution of the curvature perturbation

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We derive a general equation for the evolution of the curvature perturbation on comoving slices $\mathcal{R}_c$ in the presence of anisotropic and non-adiabatic pressure terms in the energy-momentum tensor of matter field. The equation is obtained by manipulating the perturbed Einstein equations in the comoving slicing. It could be used to study the evolution of perturbations for a system with an anisotropic energy-momentum tensor, such as in the presence of a vector field or in the presence of non-adiabaticity, such as in a multi-field system.

I. INTRODUCTION

The theory of cosmological perturbations is very useful to study the early stages of the Universe, especially during inflation, that is, an exponential expansion phase which the standard cosmological model hypothesizes to explain observations such as anisotropies in the cosmic microwave background radiation (CMB). One quantity which is particularly important in this context is the curvature perturbation on comoving slices, $\mathcal{R}_c$. In slow-roll single field inflationary models this quantity is conserved on super-horizon scales[1, 2], which has important implications on the relation between primordial perturbations and late-time observables such as CMB anisotropies. For a globally adiabatic system in a single field model this quantity may not be conserved [3]. Other possible causes of super-horizon evolution could be anisotropic or non-adiabatic pressure components of the energy-momentum tensor. In this short note we derive the equations for the curvature perturbation on comoving slices, $\mathcal{R}_c$, including these two terms showing that they act, as expected, as source terms which are relevant also on super-horizon scales. Our approach is quite generic and can be applied to any system which can be described by an energy-momentum tensor of the form we use, not only to a multi-scalar scalar system.

The derivation is based on manipulating the Einstein equations in order to obtain an equation involving only $\mathcal{R}_c$, the anisotropy and non-adiabatic pressure terms and background quantities. The equation can be used to study phenomenologically the effects of anisotropy and non-adiabaticity without assuming any specific model.

One useful application could be to study models which violate the non-Gaussianity consistency relation [4] that was derived in fact based on the assumption of the conservation of the comoving curvatue perturbation on superhorizon scales.

II. EVOLUTION OF COMOVING CURVATURE PERTURBATIONS

The Einstein equations in a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) background are

$$3\mathcal{H}^2 = a^2(\eta)\,\rho\,, \tag{1}$$

$$2(\mathcal{H}' - \mathcal{H}^2) = -a^2(\eta)\,(\rho + P)\,. \tag{2}$$

Here a prime denotes a derivative with respect to the conformal time η and $\mathcal{H}$ stands for the conformal Hubble parameter defined by $\mathcal{H} = a'/a$. ρ and P represent the background energy density and pressure of the matter field respectively. We use the units in which $8\pi G = c = 1$.

Scalar perturbations on a spatially flat FLRW metric can be written as

$$ds^2 = a^2(\eta)\Big[-(1+2A)\mathrm{d}\eta^2 + 2\partial_i B \mathrm{d}x^i \mathrm{d}\eta + \\ + \big\{(1+2\mathcal{R})\delta_{ij} + 2\partial_i\partial_j E\big\}\mathrm{d}x^i \mathrm{d}x^j\Big]\,, \tag{3}$$

where the Latin indices run from 1 to 3. The corresponding energy-momentum tensor takes the form :

$$T^0{}_0 = -(\rho+\delta\rho)\,, \quad T^0{}_i = \frac{\rho+P}{a(\eta)}u_i\,,$$
$$T^i{}_j = (P+\delta P)\delta^i{}_j + \Pi^i{}_j\,; \tag{4}$$

where

$$u_i = a(\eta)\,\partial_i(v+B)\,, \tag{5}$$

$$\Pi^i{}_j = \delta^{ik}\partial_k\partial_j\Pi - \frac{1}{3}\overset{(3)}{\Delta}\Pi\delta^i{}_j\,, \quad \Pi^i{}_i = 0\,. \tag{6}$$

In the above equations $\Pi^i{}_j$ is the anisotropic pressure component of the energy-momentum tensor, v is the velocity potential, Π is the anisotropy potential and we have defined $\overset{(3)}{\Delta} \equiv \delta^{ij}\partial_i\partial_j$.

The curvature perturbation on comoving slices $\mathcal{R}_c$ is a gauge-invariant quantity defined as the curvature perturbation $\mathcal{R}$ evaluated on the hypersurfaces in which $v+B$

vanish. The spatial Fourier expansion of the linearly perturbed Einstein equations on comoving slices [5] takes the form :

$$2k^2(\mathcal{R}_c - \mathcal{H}\sigma_c) = a^2\delta\rho_c\,, \tag{7}$$

$$\mathcal{R}_c' - \mathcal{H}A_c = 0\,, \tag{8}$$

$$2(\mathcal{H}' - \mathcal{H}^2)A_c = a^2\Big[\delta P_c - (2k^2/3)\Pi_c\Big]\,, \tag{9}$$

$$\sigma_c' + 2\mathcal{H}\sigma_c - A_c - \mathcal{R}_c = a^2\Pi_c\,, \tag{10}$$

where $\sigma = E' - B$ is the scalar shear.

In general we can decompose the pressure perturbation as

$$\delta P_c = c_s^2(\eta)\delta\rho_c + \Gamma_c\,, \tag{11}$$

where we can interpret c_s and Γ_c as the adiabatic sound speed and the non-adiabatic part of the pressure respectively. For a minimally coupled scalar field model $c_s = 1$ and Γ_c is zero, but in general one would expect that Γ_c could be non-vanishing. Our goal is to derive an equation for $\mathcal{R}_c$ in the presence of both anisotropic stress ${\Pi^i}_j$ and non-adiabatic pressure Γ_c.

First we use Eq. (8) to express A_c in terms of $\mathcal{R}_c$,

$$A_c = \frac{\mathcal{R}_c'}{\mathcal{H}}\,. \tag{12}$$

We substitute this A_c and δP_c given in Eq. (11) into Eq. (9), and solve it for $\delta\rho_c$:

$$\delta\rho_c = \frac{\mathcal{H}\left(2k^2\Pi_c - 3\Gamma_c\right) - 3(\rho+P)\mathcal{R}_c'}{3\mathcal{H}c_s^2}\,. \tag{13}$$

We then insert this into eq. (7) to get an expression for σ_c :

$$\sigma_c = \frac{\mathcal{R}_c}{\mathcal{H}} - \frac{a^2\Big[\mathcal{H}(2k^2\Pi_c - 3\Gamma_c) - 3(\rho+P)\mathcal{R}_c'\Big]}{6k^2\mathcal{H}^2c_s^2}\,. \tag{14}$$

Finally we substitute A_c and σ_c given by Eqs. (12) and (14), respectively, into Eq. (10) to obtain

$$\mathcal{R}_c'' + \frac{(z^2)'}{z^2}\mathcal{R}_c' - c_s^2 \overset{(3)}{\Delta} \mathcal{R}_c + \frac{\mathcal{H}}{\rho+P}Y_c = 0\,, \tag{15}$$

where we have defined

$$z^2 \equiv \frac{a^4(\rho+P)}{c_s^2\mathcal{H}^2}\,, \tag{16}$$

$$Y_c \equiv \left[\log\left(\frac{a^4}{\mathcal{H}{c_s}^2}\right)\right]'\left(\frac{2}{3}\overset{(3)}{\Delta}\Pi_c + \Gamma_c\right) + 2\mathcal{H}c_s^2\overset{(3)}{\Delta}\Pi_c + \frac{2}{3}\overset{(3)}{\Delta}\Pi_c' + \Gamma_c'\,. \tag{17}$$

This is the main result of this note. As expected, for adiabatic ($\Gamma_c = 0$) and isotropic perturbations ($\Pi_c = 0$) the above equation takes the well-known form :

$$\mathcal{R}_c'' + \frac{(z^2)'}{z^2}\mathcal{R}_c' - c_s^2\overset{(3)}{\Delta}\mathcal{R}_c = 0\,, \tag{18}$$

III. CONCLUSIONS

We have derived a general equation for the evolution of the comoving curvature perturbation by taking into account the effects of anisotropic and non-adiabatic stress components. The equation can be applied to study the generic evolutionary behavior of the curvature perturbation in a system where anisotropic and/or non-adiabatic stress perturbations play important roles.

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