Prepared for submission to JHEP IPMU-

F-theory Vacua and α' – Corrections

Matthias Weissenbacher

Kavli Institute for the Physics and Mathematics of the Universe, University of Tokyo, Kashiwa-no-ha 5-1-5, 277-8583, Japan

E-mail: matthias.weissenbacher@ipmu.jp

Abstract: In this work we analyze F-theory and Type IIB orientifold compactifications to study α'-corrections to the four-dimensional, $\mathcal{N}=1$ effective actions. We discuss the role of novel α'-corrections in moduli stabilization and the possibility of generating (meta)-stable vacua.

In particular, we obtain corrections to the Kählermoduli space metric and its complex structure for generic dimension originating from eight-derivative corrections to eleven-dimensional supergravity. We propose a completion of the G^2R^3 and $(\nabla G)^2R^2$-sector in eleven-dimensions relevant in Calabi–Yau fourfold reductions. We suggest that the three-dimensional, $\mathcal{N}=2$ Kähler coordinates may be expressed as topological integrals depending on the first, second, and third Chern-forms of the divisors of the internal Calabi–Yau fourfold.

The divisor integral Ansatz for the Kähler potential and Kähler coordinates may be lifted to four-dimensional, $\mathcal{N}=1$ F-theory vacua. We identify a novel correction to the Kähler potential and coordinates at order α'^2, which is leading compared to other known corrections in the literature. At weak string coupling the correction arises from the intersection of $D7$-branes and $O7$-planes with base divisors and the volume of self-intersection curves of divisors in the base. In the presence of the novel α'-correction resulting from the divisor interpretation the no-scale structure may be broken. Furthermore, we propose a model independent scenario to achieve non-supersymmetric AdS vacua for Calabi-Yau orientifold backgrounds with negative Euler-characteristic.

Contents

1 Introduction

Four-dimensional minimal super-gravity theories are of particular phenomenological interest. The effective actions are commonly derived by dimensionally reducing ten-dimensional supergravity actions arising in string theory with localized brane sources. The stringy imprint arises in the form of α'-corrections[1] to the Kähler potential and coordinates of the leading two-derivative action or in form of high-derivative couplings in four dimensions. Such corrections have been shown to be crucial in determining the vacua of the effective theory in the process of moduli stabilization. However, to compute α'-corrections in a truly minimal supersymmetric i.e. $\mathcal{N} = 1$ set-up has been a challenging endeavor. A promising approach is to utilize F-theory which is a formulation of Type IIB string theory with space-time filling seven-branes at varying string coupling [1]. It captures the string coupling dependence in the geometry of an elliptically fibered higher-dimensional manifold. The general effective actions of F-theory compactifications have been studied using the duality with M-theory [2, 3]. A wide range of phenomenologically promising geometric F-theory backgrounds are known to giving rise non-Abelian gauge groups [2, 4, 5].

The starting point of the M/F-theory duality is the long wave length limit of M-theory, i.e. eleven-dimensional supergravity. Higher-derivative or higher-order l_{M}-corrections can then be followed through the duality to give rise to α'-corrections in the resulting four-dimensional $\mathcal{N} = 1$ theory. We first compactify eleven-dimensional supergravity including the next to leading order eight-derivative or l_{M}^6-couplings to three dimensions on a supersymmetry preserving 8-dimensional background. More precisely, we preform a classical Kaluza-Klein reduction of the purely gravitational M-theory R^4-terms [6–11] on elliptically fibered Calabi–Yau fourfolds. Furthermore, one needs to consider the G^2R^3 and $(\nabla G)^2R^2$-sector, where G is the M-theory four-form field strength. One easily verifies that all those couplings carry eight derivatives. We then implement the F-theory limit by decompactifying the thee-dimensional theory to four space-time dimensions and interpret the resulting α'-corrections to the two-derivative effective theory. In particular, we study l_{M}^6-corrections to the three-dimensional Kähler potential and Kähler coordinates of the $\mathcal{N} = 2$ theory, which then modify the four-dimensional Kähler potential and Kähler coordinates in the F-theory limit. In particular, we identify a new leading order α'^2-correction to the Kähler potential and coordinates which may break the no-scale structure. It is then of interest to study its effects in moduli stabilization scenarios.

We start the discussion in section 2 by reviewing the G^2R^3 and $(\nabla G)^2R^2$-sector. No super-symmetric completion of those sectors is known. In this work we propose a completion of the bosonic terms relevant for Calabi–Yau fourfold reductions. We

[1] Which is given by $\alpha' = l_S^2$ with string length l_S. The canonical convention for the definition of α' is w.r.t, the string tension T as $T^{-1} = 2\pi\alpha'$.

start from a general basis and fix the coefficients via comparison to controlled theories upon dimensional reduction. In particular, we compactify on Calabi–Yau threefolds and verify compatibility with $5d, \mathcal{N} = 2$ supergravity. Furthermore, upon reduction on $S^1 \times K3$ we make use of the Heterotic/IIA-theory duality.

This then allows us to fix the parameters such that we can perform a controlled dimensional reduction on Calabi–Yau fourfolds with a generic number of Kähler deformations in section 3. Also in this section we review our previous results for the one-modulus case for which the integration in a three-dimensional Kähler potential and coordinates can be performed exactly.

In section 4 we suggest a proposal for the three-dimensional Kähler potential and coordinates for a generic number of Kähler moduli of the Calabi–Yau fourfold background. The key new approach in contrast to our previous attempts [12, 13] is the formulation of the higher-derivative contributions as divisor integrals, analogous to the discussion of the warp-factor in [12]. We argue in 4.1 that the new formulation can indeed give rise to all relevant higher-derivative couplings in the reduction result obtained in 3.2. However, to match the reduction result is beyond the aim of this work and we suggest that non-trivial identities relating the higher-derivative objects are needed to perform this tasks. Let us stress that obtaining the correct building blocks from a Kähler potential and Kähler coordinates is a big leap forward as this steps meets heavy obstacles as pointed out in [12]. We then proceed in 4.2 by showing that the divisor integral Kähler coordinates can be re-expressed as topological integrals. This is very intriguing as it will allow for a F-theory interpretation. Lastly in section 4.3 we show compatibility with to the one-modulus case where the Kähler potential and coordinates could be fixed exactly [14].

In section 5 we discuss the F-theory uplift of the three-dimensional l_{M}^6-corrected Kähler potential and coordinates to four dimensions. The classical uplift of the topological integrals is well understood and can be performed rigorously. It is expected that the F-theory lift receives loop-corrections which result from integrating out Kaluza-Klein states on the $4d/3d$ circle at one-loop. As we encounter a l_{M}^6-correction to the Kähler coordinates with [14] logarithmic dependence on the Calabi–Yau fourfold volume reminiscent of such a loop correction we comment on a one-loop modification of the F-theory uplift. However, to present a complete analysis of the F-theory uplift at one-loop is beyond the scope of this work. Due to this the resulting α'^2-corrected four-dimensional Kähler potential and coordinates thus carry free parameters we are not able to fix in this work. We conclude, however, that the divisor integral interpretation of the three-dimensional Kähler coordinates generically leads to a breaking of the no-scale structure which remains present in four-dimensions. This breaking of the no-scale structure is also consistent with the one-modulus case [14].

To give an independent interpretation of the novel α'^2-correction we take the Type IIB weak string coupling limit [15]. The correction is proportional to the volume of the intersection curve of $D7$-branes and the $O7$-plane with divisors in the Kähler base of the elliptically fibered Calabi–Yau fourfold. Moreover, it depends on the volume of the self-intersection curves of those divisors in the base. We also identify a second correction which survives the F-theory limit. However it vanishes due to conspiration of pre-factors. The latter correction is proportional to the self-intersection of divisors in the base intersecting the $D7$-branes and the $O7$-plane. Both are expected to arise from tree-level string amplitudes involving oriented open strings with the topology of a disk a and non-orientable closed strings with the topology of a projective plane analogous to the α'^2-correction encountered in [16, 17]. We also discuss the latter in this work.

In section 6 we discuss the implications of the α'^2-corrections on moduli stabilization. In 6.1 we analyze the structure of the novel leading order α'^2-correction to the scalar potential. We argue that for specific values of the topological quantities of the geometric background the correction takes a form in which all but one two-cycle volumes are fixed in a Minkowski vacuum with a flat direction, i.e. a flat direction for the overall volume. In more realistic scenarios this feature can be exhibited in particular for backgrounds with $\chi(B_3) = 0$, i.e. Calabi–Yau fourfolds with vanishing Euler-characteristic of the base B_3, or for large enough vacuum values of the string coupling. Lastly in section 6.2 we propose a model independent scenario to achieve non-supersymmetric AdS vacua for geometric backgrounds with negative Euler-characteristic $\chi(B_3) < 0$, where B_3 is the base of the elliptically fibered Calabi-Yau fourfold in F-theory. In the IIB picture thus for Calabi–Yau oreintifold backgrounds with negative Euler-characteristic. The vacua are obtained due to an interplay of the Euler-Characteristic correction [18] and the α'^2-corrections to the scalar potential.[2] We close by emphasizing that the discussion can be performed analogously for Calabi–Yau fourfolds with $\chi(B_3) > 0$ which leads to de Sitter extrema. We thus suggest that the scenarios may suffice to construct an explicit counter example to the recent conjecture by [21]. Let us emphasize that we do not study explicit geometric backgrounds in this work but derive constraints on the topological quantities such that vacua may be obtained.

2 Towards a completion of the G^2R^3 and $(\nabla G)^2R^2$ sectors

In section 2.1 we review the known eleven-dimensional supergravity action at eight-derivatives. We consider the possibility of having additional $G^2\mathcal{R}^3$ and $(\nabla G)^2R^2$-terms in the eleven-dimensional action in section 2.2, where G denotes the M-theory four-form field strength, and R is an abbreviation for the Riemann tensor. We propose a completion of these two sectors relevant for Calabi–Yau fourfold reductions.

[2] The form of the scalar potential due to the α'^2-correction obtained in [16, 17] is similar to the one obtained at order α'^3 in [19, 20].

Due to these potential novel terms one encounters an additional parameter freedom in the reduction result in section 3. However, as we make not use of this parameter freedom in the remaining work let us stress that this section stands independent. The reader more interested in the three and four-dimensional effective actions can thus safely skip the technical section 2.2 and carry on with section 3.

2.1 Higher-derivative corrections in M-theory

In this section we review the eleven-dimensional supergravity action including the relevant eight-derivative terms. Note that we comment on a completion of the $G^2\mathcal{R}^3$ and $(\nabla G)^2 R^2$-sector relevant for a Calabi–Yau fourfold CY_4 reductions in the next section 2.2. The bosonic part of the classical two-derivative $\mathcal{N}=1$ action in eleven dimensions is given by

$$2\kappa_{11}^2 \, S_{11} = \int_{M_{11}} R \ast 1 - \frac{1}{2} G \wedge \ast G - \frac{1}{6} C \wedge G \wedge G \, . \tag{2.1}$$

The purely gravitational sector is corrected at eight-derivatives by R^4-terms given by

$$2\kappa_{11}^2 \, S_{R^4} = \int_{M_{11}} \big(t_8 t_8 - \tfrac{1}{24} \epsilon_{11} \epsilon_{11} \big) \mathcal{R}^4 \ast 1 - 3^2 2^{13} \, C \wedge X_8 \quad . \tag{2.2}$$

First derived in [22] these terms can be shown to be re related to the R-symmetry and conformal anomaly of the world-volume theory of a stack of N M5-branes [11]. Secondly the known contributions [23] to $G^2\mathcal{R}^3$ and $(\nabla G)^2 R^2$-sector of the four-form field strength are given by

$$2\kappa_{11}^2 \, S_{\mathcal{G}} = \int_{M_{11}} - \big(t_8 t_8 + \tfrac{1}{96} \epsilon_{11} \epsilon_{11} \big) G^2 \, R^3 \ast 1 + s_{18} \left(\nabla G \right)^2 R^2 \ast 1 + 256 \, Z G \wedge \ast G \, . \tag{2.3}$$

The last term in (2.3) was argued to be necessary to ensure Type IIA/M-theory duality when considering Calabi–Yau threefold compactifications [20]. The precise definition of the higher-derivative terms in (2.2) and (2.3) can be found in the appendix in B.3. The detailed index structure of the terms $\left(\nabla G \right)^2 R^2$ in (2.3) can be found in B.3.

2.2 Checks on the $G^2 R^3$ and $(\nabla G)^2 R^2$-sector

It is well known that no supersymmetric completion of the eleven-dimensional $G^2\mathcal{R}^3$-sector and $(\nabla G)^2 R^2$-sector is known. The eleven-dimensional eight-derivative terms involving two powers of the four-form field strength are lifted from the corresponding terms in the Type IIA effective action. Those arise at the level of the five point-functions in the Type IIA superstring and partial indirect conclusions can be drawn at the level of the six-point function [23]. However, let us stress that a conclusive study at the level of the six-point function and especially at higher order n-point functions remains absent. In particular a supersymmetric completion of the $G^2\mathcal{R}^3$-sector and $(\nabla G)^2 R^2$-sector employing the Noether coupling method would be of great interest. It is thus desirable to discuss possible extensions of the $G^2\mathcal{R}^3$ and $(\nabla G)^2 R^2$-sector beyond the known terms. In this section we accomplish this task and provide a complete maximal extension of the eleven-dimensional $G^2\mathcal{R}^3$ and $(\nabla G)^2 R^2$-sector relevant for Calabi–Yau fourfold reductions.[3]

Instead of computing string amplitudes or employing the Noether coupling method we take a more pragmatic way here. In [14] a complete basis of eight-derivative terms of the schematic form $G^2\mathcal{R}^3$ was constructed. We then compliment this with a basis for the $(\nabla G)^2 R^2$-sector given in appendix (B.3), both of which then upon dimensional reduction contribute to the kinetic terms of the three-dimensional vectors. To constrain the free parameters we follow the same logic as in our previous work [14, 20], namely deriving constraints on the parameter of the eleven-dimensional Ansatz by verifying compatibility upon dimensional reduction with lower-dimensional supersymmetry. For example, as the R^4-sector is known to be complete one can fix certain lower-dimensional supersymmetry variables solely by deriving its dimensional reduction, which then can be compared to the ones derived from the $G^2\mathcal{R}^3$ and $(\nabla G)^2 R^2$-sector.

Let us next discuss the general form of the relevant terms in the basis of $G^2\mathcal{R}^3$ and $(\nabla G)^2\mathcal{R}^2$. The terms contributing to the three-dimensional effective action are those, which do not contain any Ricci tensors or scalars as these vanish trivially on a Calabi–Yau manifold. Taking into account the first Bianchi identity for the Riemann tensor a minimal basis of these terms is given in appendix B.3. The general expansion of terms which may contribute in addition to (2.3) to the three-dimensional action is then

$$2\kappa_{11}^2\, S^{\text{extra, gen}} = \alpha^2 \int_{M_{11}} \sum_{i=1}^{17} C_i\, \mathcal{B}_i\, *1 + \sum_{i=1}^{24} C_{i+17}\, B_i\, *1 \tag{2.4}$$

for some coefficients $C_i \in \mathbb{R}$. To restrict the parameters in the Ansatz (2.4) we first take a detour to Calabi–Yau threefold compactifications and furthermore discuss the dimensional reduction on $K3 \times S^1$. Thus in particular, we provide the maximal complete extensions of the eleven-dimensional $G^2\mathcal{R}^3$ and $(\nabla G)^2 R^2$-sector (2.4), which is compatible upon dimensional reduction with five-dimensional, $\mathcal{N} = 2$ supersymmetry, i.e. by dimensional reduction on Calabi–Yau threefolds to five dimension for a generic number of Kähler moduli. Moreover, we perform the dimensional reduction on $K3 \times S^1$ to six dimensions and employ the Heterotic - IIA duality to compare the resulting four-derivative couplings to the well known terms on the Heterotic side

[3]In other words due to the Calabi-Yau condition certain terms in the Ansatz yield zero upon reduction. Those coefficients can not be fixed by our arguments but constitute a complete description relevant for Calabi–Yau fourfold reductions.

of the duality. It turns out that these arguments are very restrictive and allow us to parametrize the $G^2\mathcal{R}^3$ basis with only five parameters [14]. However when allowing for an interplay with the $(\nabla G)^2R^2$-sector the number of independent parameters reduces from forty-one to thirteen.

Moreover the above analysis allows us to infer that the $G^2\mathcal{R}^3$ and $(\nabla G)^2R^2$-terms are consistent with the partially known six-point function results [23]. Let us stress that it would be of great interest to study additional constrains on this eleven-dimensional sector by circular reduction to type IIA effective supergravity. Any combination of novel terms need to be vanishing at the level of the five-point one-loop string scattering amplitude with two NS-NS two-form field and three graviton vertex operator insertions. We suggest that such a study will lead to fix the remaining parameter freedom in the eleven-dimensional action.

By dimensionally reducing the extension (2.4) one modifies the kinetic couplings of the three-dimensional vectors and introduces an additional parameter freedom. One may use to this to rewrite the reduction result in terms of $3d$, $\mathcal{N}=2$ variables. In section 3.2 we perform the dimensional reduction of the $G^2\mathcal{R}^3$ and $(\nabla G)^2R^2$-extensions to three space-time dimensions on Calabi–Yau fourfolds with arbitrary number of Kähler moduli.

Calabi–Yau threefold checks to $5d,\mathcal{N}=2$. In the following we derive constraints on the coefficients C_i in (2.4) by demanding compatibility with $\mathcal{N}=2$ supersymmetry in five dimensions upon compactification on a Calabi–Yau threefold. The $l_{\rm M}^6$-corrections give contributions to the five-dimensional vector multiplets of the $\mathcal{N}=2$ supergravity which is expressed in terms of a real pre-potential $\mathcal{F}(X^I)$ and real special coordinates X^I. Note that physical scalars in the vector multiplets obey

$$\mathcal{F}(X^I) = \tfrac{1}{3!}C_{IJK}\, X^I X^J X^K = 1 \quad . \tag{2.5}$$

The totally symmetric and constant tensor C_{IJK} is entirely determined by the $U(1)$ Chern-Simons terms $\sim C_{IJK}\, A^I F^J F^K$, which however do not receive $l_{\rm M}^6$–corrections. One concludes that also the physical scalars X^I remain uncorrected.

We dimensionally reduce the action (2.4) with general coefficients C_i on a Calabi–Yau threefold Y_3 to five dimensions. As our focus is on the kinetic terms for the vectors we note that in order to dimensionally reduce one expands

$$G = F^i_{\rm 5D} \wedge \omega_i^{CY_3}\,, \tag{2.6}$$

with the field strength of the five-dimensional vectors $F^i_{\rm 5D}$ and the harmonic (1, 1)-forms on the Calabi–Yau threefold $\omega_i^{CY_3}$, $i=1,\ldots,h^{1,1}(CY_3)$. The constraints imposed by supersymmetry are then inferred by making use of Shouten and total derivative identities on the internal space CY_3. The condition one encounters is that novel terms (2.4) may no contribute to the five-dimensional couplings, which is equivalent to the non-renormalisation of (2.5). The computation is in principal straightforward (but tedious) and leads us to impose the relations among the coefficients $C_1 \dots, C_{41}$. Details can be found in the appendix (B.21).

Heterotic and type IIA duality. In this section we compactify (2.4) on $K3 \times S^1$. We first circular reduce the basis of forty-one $G^2\mathcal{R}^3$ and $(\nabla G)^2 R^2$-terms to ten dimensions on $\mathbb{R}^{1,9} \times S^1$ to obtain a l_M^6-modified IIA supergravity theory. The only terms relevant for us are the ones which arise from

$$G_{11MNO} = e^{\frac{\phi}{3}} H_{MNO} \quad , \quad M,N,O = 1,\dots,10 \quad , \tag{2.7}$$

where 11 denotes the direction along S^1 and with H the field strength of the type IIA Kalb-Ramond tensor field. We then check compatibility of the novel induced H^2R^3-terms making use of the IIA - Heterotic duality by dimensional reduction on $K3$. Compactifying type IIA on $K3$ is dual to the Heterotic string on $\mathbb{T}^4$. For our purpose it is enough to show that when compactifying the novel H^2R^3-terms on $K3$ those do not induce any l_M^6-correction to the six-dimensional action. In particular. the absence of four-derivative terms is imposed, which results in one further constraint on the parameters. The additional constraints on the C's arises from imposing the vanishing of the four-derivative terms such as e.g.

$$\sim \chi(K3)\; H^{6D\mu\nu\rho}\, H^{6D}{}_{\mu}{}^{\nu_1\rho_1}\, R^{6D}{}_{\mu\mu_1\nu\nu_1} \quad , \tag{2.8}$$

with $\mu,\nu = 1,\dots,6$. One then infers the additional constraints on the parameters in (2.4) to be

$$C_2 = 0 \quad , \quad C_1 = -\tfrac{1}{6}\big(8C_3 + 2C_{31} + C_{35} + 36C_4 + 3C_6\big) \quad . \tag{2.9}$$

This concludes that by fixing the parameter (2.9) the proposed maximal extension of $G^2\mathcal{R}^3$ and $(\nabla G)^2R^2$-terms in the M-theory effective action is fully consistent with the indirect six-point functions results discussed in [23].

3 Three-dimensional effective actions revisited

F-theory can be viewed as a map of dualities which allows one to derive controlled IIB orientifold backgrounds at weak string coupling which incorporate for back-reacted $D7$ branes and $O7$-planes on the axio-dilaton [1–3]. The starting point of this journey is eleven-dimensional supergravity, which compactified on an appropriate eight-dimensional internal space gives a $3d$, $\mathcal{N}=2$ supergravity theory which can then be related via the F-theory lift to a $4d$, $\mathcal{N}=1$ supergravity theory. The main objective of this section is the dimensional reduction of eleven-dimensional supergravity including the novel eight-derivative couplings (2.4) on Calabi–Yau fourfolds for a generic number of Kähler moduli in section 3.2. We start our discussion with a review of the generic properties of $3d$, $\mathcal{N}=2$ supergravity theories in section 3.1. Finally, we conclude this section with a review of the one-modulus case in which the warp-factor as well as the higher-derivative couplings can be matched to the $3d$, $\mathcal{N}=2$ variables [14].

Background solution. Let us set the stage by reviewing the fourfold solutions including eight-derivative terms studied in [24–26]. The background solution is taken to be an expansion in terms of the dimensionful parameter [4]

$$\alpha^2 = \frac{(4\pi\,\kappa_{11}^2)^{\frac{2}{3}}}{(2\pi)^4\,3^2\cdot 2^{13}}\,, \qquad 2\kappa_{11}^2 = (2\pi)^5\, l_{\mathrm{M}}^9\,, \tag{3.1}$$

which reduces to the ordinary direct product solution $\mathbb{R}^{1,2}\times CY_4$ without fluxes and warping to lowest order in α. At order α^2 a warp-factor $W^{(2)} = W^{(2)}(z,\bar z)$ and fluxes are induced. The background solution is known [25, 26] to then take the form

$$\langle ds^2\rangle = \epsilon^{\alpha^2\,\Phi^{(2)}}\Big(\epsilon^{-2\alpha^2\,W^{(2)}}\eta_{\mu\nu}\,dx^\mu dx^\nu + 2\epsilon^{\alpha^2\,W^{(2)}}\,g_{m\bar m}\,dz^m d\bar z^{\bar m}\Big)\,, \tag{3.2}$$

$$\langle G\rangle = \alpha\, G^{(1)} + \mathrm{dvol}_{\mathbb{R}^{1,2}}\wedge d\big(\epsilon^{-3\alpha^2\,W^{(2)}}\big)\,. \tag{3.3}$$

By solving the eleven-dimensional E.O.M.'s for the metric $g_{m\bar m}$ of the internal space one encounters that it seizes to be Ricci flat i.e. Calabi–Yau [16]. It receives a correction at order α^2 as

$$g_{m\bar m} = g^{(0)}_{m\bar m} + \alpha^2\, g^{(2)}_{m\bar m}\,, \qquad g^{(2)}_{m\bar m} \sim \partial_m\bar\partial_{\bar m}\;*^{(0)}\left(J^{(0)}\wedge J^{(0)}\wedge F_4\right), \tag{3.4}$$

where $g^{(0)}$ is the lowest order, Ricci-flat Calabi–Yau metric and $J^{(0)}$ is its associated Kähler form and where F_4 the non-harmonic part of the third Chern form. Latter is however irrelevant for the following discussion, as it only contributes couplings to the effective action which are total derivatives [17]. Furthermore, (3.4) includes an overall Weyl factor $\Phi^{(2)} = -\frac{512}{3}*^{(0)}\left(c_3^{(0)}\wedge J^{(0)}\right)$, which was first discussed in [26] and a warp-factor $W^{(2)}(z,\bar z)$ satisfying the warp-factor equation

$$\Delta^{(0)}\,\epsilon^{3\alpha^2 W^{(2)}}\,\mathrm{dvol}^{(0)}_{Y_4} + \frac{1}{2}\alpha^2\, G^{(1)}\wedge G^{(1)} - 3^2 2^{13}\,\alpha^2 X_8^{(0)} = 0\,. \tag{3.5}$$

The background value of the four-form field strength (3.3) is given by the sum of the internal flux $G^{(1)}\in H^4(CY_4)$ and a warp-factor contribution. Due to lowest order supersymmetry constraints the flux is to be self-dual with respect to the lowest order Calabi–Yau metric. Note that we do not discuss the corrections to the gravitino variations at order l_{M}^6 here but refer the reader to [26] for a detailed discussion. Let us emphasize that the l_{M}^6-gravitino variations are not known as a supersymmetric completion of eleven-dimensional supergravity at higher l_{M}-order remains elusive. However, it is widely believed that (3.2)-(3.5) constitutes a supersymmetric background.

[4] We follow the conventions of [11].

3.1 Three-dimensional gauged $\mathcal{N} = 2$ supergravity

In this section we briefly review $\mathcal{N} = 2$ gauged supergravity in three dimensions where all shift symmetries are gauged. Shift symmetries corresponds to an isometry of the geometry of the scalar field space. Three-dimensional maximal and non-maximal supergravities are discussed in [27]. For our purpose it is sufficient to consider three-dimensional $\mathcal{N} = 2$ supergravity coupled to chiral multiplets with complex scalars N^a, which are gauged along the isometries I^{ab} and subject to the constant embedding tensor Θ_{ab}. One then infers the simply form of the $\mathcal{N} = 2$ action to be

$$S_{N=2} = \int_{M_3} \tfrac{1}{2} R * 1 - K_{a\bar{b}} \nabla N^a \wedge * \nabla \bar{N}^{\bar{b}} - \tfrac{1}{2} \Theta_{ab} A^a \wedge F^b - \big(V_D + V_F\big) * 1 \,, \qquad (3.6)$$

where $K_{a\bar{b}} = \partial_{N^a} \partial_{\bar{N}^{\bar{b}}} K$ is a Kähler metric with Kähler potential K. The gauge covariant derivative ∇N^a is defined by $\nabla N^a = dN^a + \Theta_{bc} I^{ab} A^c$. The F-term scalar potential in (3.6) is given by

$$V_F = \epsilon^K \big(K^{a\bar{b}} D_i W \overline{D_b W} - 4|W|^2\big) \,, \qquad (3.7)$$

with $K^{a\bar{b}} = (K^{-1})^{a\bar{b}}$ the inverse of the Kähler metric given by a hermitian matrix and W a holomorphic super potential. Furthermore, one finds that $V_D = K^{a\bar{b}} \partial_a D \partial_{\bar{b}} D - D^2$ where D is a real function of the chiral fields N^i. Lastly, note that the vectors in the Chern-Simons term (3.6) are non-dynamical.

Dualization of the action. One may now split the chiral fields as $N^a = (M^I, T_i)$ and dualizes the chiral multiplets in (3.6) with bosonic component T_i into vector multiplets [28]. Note that dualization is in general not possible but requires $\mathrm{Im}T_i$ to admit a shift symmetry. Upon Legendre dualization the theory depends on the kinematic potential $\tilde{K}$ which is expressed in terms of the quantities of the dual theory as

$$K(M,T) = \tilde{K}(M,L) - \mathrm{Re}T_i \, L^i \quad , \quad L^i = -\frac{\partial K}{\partial \mathrm{Re}T_i} \quad . \qquad (3.8)$$

One then derives the dual action to take the form[5]

$$\begin{aligned} S_{\mathcal{N}=2,\,\mathrm{dual}} = &\int_{M_3} \tfrac{1}{2} R * 1 - \tilde{K}_{M^I \bar{M}^J} \mathcal{D} M^I \wedge * \mathcal{D} \bar{M}^{\bar{J}} + \tfrac{1}{4} \tilde{K}_{L^i L^j} dL^i \wedge * dL^j \\ &+ \int_{M_3} \tfrac{1}{4} \tilde{K}_{L^i L^j} F^i \wedge * F^j + \tfrac{1}{2} \Theta_{ij} A^i \wedge F^j + F^i \wedge \mathrm{Im}\big[\tilde{K}_{L^i M^I} \nabla M^I\big] \\ &- \int_{M_3} \big(V_D + V_F\big) * 1 \,, \end{aligned} \qquad (3.9)$$

[5]One may choose a constant embedding tensor such that

$$I^{ij} = -2i \, dx^{ij} \,, \qquad I^{IJ} = \tilde{I}^{i\bar{J}} = 0 \,, \qquad I^{iJ} = 0 \,, \qquad \Theta_{IJ} = 0 \,.$$

with kinematic couplings given by

$$\tilde{K}_{L^iL^j} = \partial_{L^i}\partial_{L^j}\tilde{K} \quad . \tag{3.10}$$

Note that the scalars L^i belong to vector multiplets. One may furthermore infer from (3.8) that

$$K_{T_i\bar{T}_j} = -\tfrac{1}{4}\tilde{K}^{L^iL^j} \ , \qquad \mathrm{Re}T_i = \tilde{K}_{L^i} \ , \qquad \frac{\partial L^i}{\partial T_j} = \tfrac{1}{2}\tilde{K}^{L^iL^j} \ . \tag{3.11}$$

Left to discuss is the dualization of the scalar potential.[6] The F-term scalar potential in the vector multiplet language is then given by

$$V_F = e^K \Big[\tilde{K}^{M^I\bar{M}^{\bar{J}}} D_{M^I}W\,\overline{D_{M^J}W} - \big(4 + L^i\,\tilde{K}_{L^iL_j}\,L^j\big)\big|W\big|^2\Big] . \tag{3.13}$$

where we have assumed that the superpotential does not depend on the scalars L^i in the vector multiplet. This case is relevant when matching to the string theory reduction result in which the superpotential does not depend on the Kähler moduli, i.e. non-perturbative effects such as $M5$-brane instantons are absent. For the discussion in this work this will be sufficient but one may choose to generalize (3.13) easily.

3.2 Calabi–Yau fourfold reduction for generic $h^{1,1}$

In this section we discuss the reduction result of M-theory involving the eight-derivative action (2.1)-(2.3) and (2.4) on the warped background (3.2)-(3.5) and allow for an arbitrary number of Kähler moduli of the internal manifold. Latter is achieved by deforming the background metric as

$$g_{m\bar{n}} \to g_{m\bar{n}} + i\delta v^i \omega^{(0)}_{im\bar{n}} \quad , \tag{3.14}$$

where $\delta v^i = \delta v^i(x)$ are infinitesimal scalar deformations and $\{\omega^{(0)}_i\}$ are harmonic $(1,1)$-forms w.r.t the background Calabi–Yau metric $g^{(0)}$, with $i = 1,\ldots,h^{1,1}(CY_4)$. The non-vanishing contribution for the dynamical three-dimensional vectors A^i_μ is derived by[7]

$$G_{\mu\nu m\bar{n}} = F^i_{\mu\nu}\omega^{(0)}_{im\bar{n}} \quad , \quad F^i = dA^i \quad . \tag{3.15}$$

To enhance the readability of the main text in the following we shift the more technical steps to the appendix. To express the reduction result we need to introduce several

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[6] The D-term results in

$$V_D = \tilde{K}^{M^I\bar{M}^{\bar{J}}}\partial_{M^I}\mathcal{T}\,\partial_{\bar{M}^{\bar{J}}}\mathcal{T} - \tilde{K}^{L^iL^j}\partial_{L^i}\mathcal{T}\,\partial_{L^j}D - D \quad , \quad D = -\tfrac{1}{2}L^i\,\Theta_{ij}\,L^j\,. \tag{3.12}$$

[7] Note that in the presence of l_{M}^6-correction the deformations (3.14) and (3.15) may receive higher-order corrections as discussed in [12, 13], none of which alter the dynamics of the resulting theory. We thus omit them from the present discussion.

higher-derivative building blocks. Among them the familiar second and third Chern-forms c_2 and c_3, respectively, and $Z, Z_{m\bar{m}}, Z_{m\bar{m}n\bar{n}}$ and $\mathcal{Y}_{ij}, \Omega_{ij}$. All higher-derivative objects are w.r.t. the zeroth α-order Calabi–Yau metric. Their precise definition can be found in appendix A, in particular (A.19)-(A.26). Here let us schematically note that

$$Z,\, Z_{m\bar{m}},\, Z_{m\bar{m}n\bar{n}} \sim \big(R\big)^3 \quad , \quad \mathcal{Y}_{ij} \sim (\nabla\omega_i)(\nabla\omega_j)\big(R\big)^2 \quad , \quad \Omega_{ij} \sim (\omega_i)(\omega_j)\big(R\big) \ . \tag{3.16}$$

where R denotes the Riemann tensor on the internal manifold and ∇ is the covariant derivative w.r.t. the Calabi–Yau metric. The warp-factor dependence can be elegantly captured by introducing the warped volume and warped metric

$$\mathcal{V}_{\mathscr{W}} = \mathcal{V} + 3\mathscr{W} \quad , \quad \mathscr{W} = \int_{Y_4} W^{(2)} *^{(0)} 1 \ , \qquad G^{\mathscr{W}}_{ij} = \frac{1}{2\mathcal{V}_{\mathscr{W}}} \int_{Y_4} e^{3\alpha^2 W^{(2)}} \omega^{(0)}_i \wedge *^{(0)} \omega^{(0)}_j \ , \tag{3.17}$$

which at zeroth order in α reduce to $\mathcal{V}$ and $G_{ij} = \frac{1}{2\mathcal{V}} \int_{Y_4} \omega^{(0)}_i \wedge *^{(0)} \omega^{(0)}_j$. We also introduce

$$\mathcal{K}^{\mathscr{W}}_i = i\mathcal{V}_{\mathscr{W}}\, \omega^{(0)\,m}_{im} + \frac{9}{2}\alpha^2 \int_{Y_4} \partial_i W^{(2)}| *^{(0)} 1 \ , \tag{3.18}$$

which at lowest order simply reduces to $\mathcal{K}^{(0)}_i = i\mathcal{V}\, \omega^{(0)\,m}_{im} = \frac{1}{3!} \int_{Y_4} \omega^{(0)}_i \wedge J^{(0)} \wedge J^{(0)} \wedge J^{(0)}$. Note that we use the notation $\mathcal{K}^{(0)}_i$ to abbreviate the intersection number evaluated in the background, in contrast to the analogue quantities $\mathcal{K}_i$ which may vary over the Kähler moduli space. With these definitions we state that the action including the l^6_{M}-corrections to the kinetic terms [12, 13] is given by

$$\begin{aligned} S_{\mathrm{kin}} = \frac{1}{2\kappa_{11}} \int_{\mathcal{M}_3} \Big[& R * 1 - (G^{\mathscr{W}}_{ij} + \mathcal{V}^{-2}_{\mathscr{W}} K^{\mathscr{W}}_i K^{\mathscr{W}}_j) d\delta v^i \wedge * d\delta v^j - \mathcal{V}^2_{\mathscr{W}} G^{\mathscr{W}}_{ij} F^i \wedge * F^j \\ & - d\delta v^i \wedge * d\delta v^j \frac{\alpha^2}{\mathcal{V}_0} \int_{CY_4} \Big(768 Z \omega^{(0)\,m}_{im} \omega^{(0)\,n}_{jn} - 3072 i Z_{m\bar{n}} \omega^{(0)\,\bar{n}m}_i \omega^{(0)\,s}_{js} \Big) *^{(0)} 1 \\ & + d\delta v^i \wedge * d\delta v^j \frac{\alpha^2}{\mathcal{V}_0} \int_{CY_4} 3072 Z_{m\bar{n}r\bar{s}} \omega^{(0)\bar{n}m}_i \omega^{(0)\bar{s}r}_j *^{(0)} 1 \\ & - F^i \wedge * F^j \alpha^2 \mathcal{V}_0 \int_{CY_4} \Big(-256 Z \omega^{(0)}_{im\bar{n}} \omega^{(0)\bar{n}m}_j + 192(7 - a_1) i Z_{m\bar{n}} \omega^{(0)\bar{r}m}_i \omega^{(0)\bar{n}}_{j\ \ \bar{r}} \Big) *^{(0)} 1 \\ & + F^i \wedge * F^j \alpha^2 \mathcal{V}_0 \int_{CY_4} 384(1 + a_1) Z_{m\bar{n}r\bar{s}} \omega^{(0)\bar{n}m}_i \omega^{(0)\bar{s}r}_j *^{(0)} 1 \quad + \Theta_{ij} A^i \wedge F^i \Big] \ . \end{aligned} \tag{3.19}$$

The one parameter freedom a_1 arises from the uncertainty inherent in the $(\nabla G)^2 R^2$-sector. From the novel sector [20] we find

$$\delta S_1 = 256\, F^i \wedge *F^j \alpha^2 \mathcal{V} \int_{Y_4} Z \omega^{(0)}_{im\bar{n}} \omega^{(0)\bar{n}m}_j *^{(0)} 1 \quad . \tag{3.20}$$

Note that novel eleven-dimensional terms (3.20) is precisely cancelled by the same structure in (3.19). Lastly, one performs the dimensional reduction of (2.4) to yield

$$\begin{aligned}\delta S_2 = \quad & F^i \wedge *F^j \alpha^2 \mathcal{V} \int_{Y_4} \Big(8i(a_3 + a_4) Z_{m\bar{n}} \omega^{(0)\bar{n}m}_i \omega^{(0)s}_{js} * 1 - 8a_3 Z_{m\bar{n}r\bar{s}} \omega^{(0)\bar{n}m}_i \omega^{(0)\bar{s}r}_j \Big) *^{(0)} 1 \\ & + F^i \wedge *F^j \alpha^2 \mathcal{V} \int_{Y_4} a_2\, c_2 \wedge \Omega_{ij} \quad ,\end{aligned} \tag{3.21}$$

with the coefficients a_3, a_4 result from the unfixed eleven dimensional parameters, $a_3 = -C_{22} + 4C_3$ and $a_4 = 18C_4$. Let us close this section with some remarks. Note that in (3.21) one obtains a term proportional to the second Chern-form. In the limit $h^{1,1} \to 1$, i.e. the one-modulus case we see that

$$\delta S_2 \to a_4 \mathcal{Z} \quad , \tag{3.22}$$

as the term Ω_{ij} vanishes. For the physical arguments provided in [14] where the one-modulus case is discussed we infer that $\delta S_2 \to 0$ as it would change the physical interpretation else-wise. Hence in the remainder of this work we assume $C_{14} = 0$ and thus $a_4 = 0$.[8] Furthermore, note that the action (3.19) depends on the infinitesimal deformation δv^i. To establish the connection to the full field space v^i, i.e. the coordinates on the Kähler moduli space we replace $\delta v^i \to v^i$ in the following.[9] This will become relevant for the discussion in section 4.

3.3 Review one-modulus Kähler potential and coordinates

The dimensional reduction of the eleven-dimensional supergravity action including higher-derivative terms on a warped Calabi–Yau fourfold background with one Kähler modulus, i.e. $h^{1,1} = 1$ case was discussed rigorously in [14]. We devote this section to reviewing this discussion, in particular the derivation of the Kähler potential and coordinates of the $3d$, $\mathcal{N} = 2$ theory. As a starting point we may take the limit $h^{1,1} \to 1$ of the generic Calabi–Yau fourfold reduction result presented in (3.19)-(3.21). One then infers that the one-modulus l_{M}^6-corrected action r takes the standard form

$$S^{3d} = \int_{M_3} \frac{1}{2} R * 1 + \frac{1}{4} \tilde{G}_{LL}(L)\, dL \wedge * dL + \frac{1}{4} \tilde{G}_{LL}(L)\ F \wedge * F \,, \tag{3.23}$$

with

$$\tilde{G}_{LL}(L) = -\frac{4}{L^2} \Big(1 - 384\, \alpha^2\, \tilde{\mathcal{Z}}\, L \Big) = -\frac{4}{L^2} + 1536\, \alpha^2\, \tilde{\mathcal{Z}}\, \frac{1}{L} \,. \tag{3.24}$$

[8] Comparison to five point-scattering and six-point amplitudes can in principle fix the 11-dimensional coefficient of the basis, thus also C_{14}.

[9] Possible obstructions and subtleties to this step for higher-derivative couplings of non-topological nature were discussed in [29].

and with the topological coupling depending on the third Chern-form given by

$$\mathcal{Z} = (2\pi^3) \int_{CY_4} c_3 \wedge J \quad , \qquad \mathcal{Z} = \mathcal{V}^{\frac{1}{4}} \tilde{\mathcal{Z}} \quad , \tag{3.25}$$

where we have used that $J = \omega_0 \mathcal{V}^{\frac{1}{4}}$. We can integrate the metric $\tilde{G}_{LL}$ to obtain the kinetic potential $\tilde{K}(L)$ and coordinate

$$\tilde{K} = 4 \log L + 1536\, \alpha^2\, \tilde{\mathcal{Z}}\, L \left(\log(L) - 1 \right) + 4\,, \tag{3.26}$$

$$L = \mathcal{V}^{-\frac{3}{4}} - 3\, \alpha^2\, \mathcal{W}\, \mathcal{V}^{-\frac{7}{4}}\,, \tag{3.27}$$

where we have chosen the integration constants in a convenient way.

Determining the Kähler potential. One may next dualize the vector multiplet to a chiral multiplet, whose metric derives from a Kähler potential. As outlined in section 3.1 this is achieved by a Legendre transformation of the kinetic potential

$$K = \tilde{K} - L\, \mathrm{Re}\, T\,, \qquad \mathrm{Re}\, T = \partial_L \tilde{K}\,. \tag{3.28}$$

One thus derives the Kähler potential $K(T + \bar{T})$ to be

$$K = 4 \log L - 1536\, \alpha^2\, \tilde{\mathcal{Z}}\, L = -3 \log \Big(\mathcal{V} + \alpha^2 \big(4\mathcal{W} + 512\, \mathcal{Z} \big) \Big)\,, \tag{3.29}$$

with corresponding coordinate

$$\mathrm{Re} T = \frac{4}{L} + 1536 \alpha^2\, \tilde{\mathcal{Z}}\, \log L = 4 \mathcal{V}^{\frac{3}{4}} + 12 \alpha^2\, \mathcal{V}^{-\frac{1}{4}} \mathcal{W} - 1152\, \alpha^2\, \tilde{\mathcal{Z}}\, \log \mathcal{V}\,. \tag{3.30}$$

Note that all quantities in the Kähler potential (3.29) depend on the one-modulus $\mathcal{V}$, i.e. the overall volume.

The no-scale condition and the scalar potential. We next argue that the ℓ_{M}^6-suppressed corrections to the Kähler potential in (3.29) generically lead to a breaking of the no-scale condition and thus generate a F-term scalar potential. One straightforwardly computes that

$$K_T\, K^{T\bar{T}}\, K_{\bar{T}} = \frac{K_T^2}{K_{T\bar{T}}} = 4 - 1536\, \frac{\alpha^2}{\mathcal{V}} \mathcal{Z}\,. \tag{3.31}$$

One may next infer the scalar potential originating from the breaking of the no-scale condition. It enters the effective action via the F-term scalar potential[10]

$$V_F = \epsilon^K \big(K^{T\bar{T}}\, D_T W \overline{D_T W} - 4 |W|^2 \big) = -1536\, \alpha^2\, \frac{|W_0|^2}{\mathcal{V}^4} \mathcal{Z} \quad . \tag{3.32}$$

[10] Note that superpotential can not be renormalized perturbatively but may be subject to e.g. $M5$-instanton corrections which correspond to $D3$-instantons in the F-theory limit [30].

Note that it exhibits a runaway direction for $\mathcal{V} \to \infty$ if $\int_{Y_4} c_3 \wedge J < 0$ [11]. In (3.32) we assumed that the complex structure moduli are stabilized by the GVW superpotential [31] given by

$$W = \frac{1}{\ell_{\rm M}^3} \int_{Y_4} G^{(1)} \wedge \Omega \,, \qquad \Omega \in H^{4,0}(Y_4)\,. \tag{3.33}$$

which in the vacuum then takes the constant value W_0. A critical assessment of this two step procedure is discussed in [32–34]. The runaway behavior of (3.32) for large volume $\mathcal{V}$ signals an instability of the solution for the case of a non-vanishing W_0 as recently examined in [35].

Let us conclude this section by emphasizing the importance of the one-modulus results in particular the integration into a Kähler potential and coordinates. In a following section we will show compatibility with the generic moduli case which is exceedingly more complicated due to the appearance of non-topological higher-derivative contributions to the Kähler metric.

4 Three-dimensional Kähler potential and coordinates

The eleven-dimensional higher-derivative corrections manifest themselves in terms of $l_{\rm M}^6$-modifications of the kinematic couplings of the two-derivative three-dimensional supergravity theory as discussed in the previous section 3.2. The objective is to express these $l_{\rm M}^6$-modifications to the kinematic couplings in the language of three-dimensional, $\mathcal{N} = 2$ supergravity. Namely these must result from a $l_{\rm M}^6$-correction to the Kähler potential and Kähler coordinates, i.e. fixing the complex structure on the Kähler moduli space. We reviewed this procedure for the one-modulus case, i.e. $h^{1,1} = 1$ in 3.3. In this section we propose a novel description of the Kähler coordinates in terms of divisor integrals. Due to these specific divisor integrals of the Calabi–Yau fourfold one manages to reproduce all high-derivative structures appearing in the reduction result of the Kähler metric (3.19)-(3.21) which we discuss in section 4.1. To motivate our Ansatz note that the Kähler coordinates are expected to linearise the action of M5-brane instantons on divisors D_i.[12] This implies that the T_i's are expected to be integrals over divisors D_i. In particular the Ansatz depends on the first, second and third Chern-form of the Divisors $\tilde{c}_{1,2,3} = \tilde{c}_{1,2,3}(D_i)$. Let us first recall further definitions

$$\mathcal{Z}_i = (2\pi)^3 \int_{CY_4} c_3 \wedge \omega_i = (2\pi)^3 \int_{D_i} c_3 \;\; , \;\; \mathcal{W}_i = \int_{D_i} W^{(2)} * 1 \;\; , \;\; \mathcal{F}_i = 1536 \int_{D_i} F_6 * 1 \;\; . \tag{4.1}$$

[11] An example with this property and $h^{1,1} = 1$ is the sextic fourfold. For the sextic one finds $\int_{Y_4} c_3 \wedge \omega = -420$.

[12] In fact, as discussed in [36] a holomorphic super-potential of the schematic form $W \propto e^{-T_i}$ can be induced by such instanton effects.

The Ansatz for the Kähler potential and coordinates depends on the real parameters $\alpha_1, \ldots, \alpha_9$ and $\kappa_1, \ldots, \kappa_6$. We assert the Kähler potential to take the form

$$K = -3\log\Big(\mathcal{V} + \alpha^2\big(4\mathscr{W}_i v^i + \kappa_1 \mathcal{Z}_i v^i + \kappa_2 \mathcal{T}_i v^i\big)\Big) \quad , \tag{4.2}$$

and for the Kähler coordinates to be[13]

$$\mathrm{Re}T_i = \mathcal{K}_i + \alpha^2\Big(\mathscr{F}_i + 3\mathscr{W}_i + \kappa_3 \frac{\mathcal{K}_i}{\mathcal{V}}\mathcal{Z}_j v^j + \kappa_4 \mathcal{Z}_i \log\mathcal{V} + \kappa_5 \frac{\mathcal{K}_i}{\mathcal{V}}\mathcal{T}_j v^j + \kappa_6 \mathcal{T}_i\Big) \quad . \tag{4.3}$$

Note that the warp-factor part of this Ansatz was fixed in [13, 37].[14] In (4.3) we introduce a novel divisor integral higher-order correction

$$\begin{aligned} \mathcal{T}_i \; = \;\; & \alpha_1 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{c}_1 + \alpha_2 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_2 \; + \alpha_3 \int_{D_i} \tilde{c}_3 + \alpha_4 \int_{D_i} \mathcal{C}_1 \, \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{J} \\ + \; & \alpha_5 \int_{D_i} \mathcal{C}_1^2 \, \tilde{c}_1 \wedge \tilde{J} \wedge \tilde{J} + \alpha_6 \int_{D_i} \mathcal{C}_1 \, \tilde{c}_2 \wedge \tilde{J} + \alpha_7 \int_{D_i} *_6(\tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_2 \\ + \; & \alpha_8 \int_{D_i} *_6(\tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_1 \wedge \tilde{c}_1 + \alpha_9 \int_{D_i} \mathcal{C}_1 \, \tilde{c}_1 \wedge *_6 \, \tilde{c}_1 \quad , \end{aligned} \tag{4.4}$$

with $\mathcal{C}_1 = *_6(\tilde{c}_1 \wedge \tilde{J}^2) = 2R_m{}^m{}_n{}^n$ and $i = 1, ..., h^{1,1}$ and where $D_i = PD(\omega_i)$ are the Poincare-dual divisors to the harmonic forms ω_i the Calabi–Yau fourfold. Furthermore, $\tilde{c}_1, \tilde{c}_2, \tilde{c}_3$ are the corresponding Chern-forms of the Divisor and $\tilde{J} = i^* J$ the pull-back of the Kähler form $i : D_j \to CY_4$. In the following c_3 is the third Chern-form of CY_4. Note that although $c_1(CY_4) = 0$ the divisors i.e. sub-manifolds of complex co-dimension one generically have $c_1(D_i) := \tilde{c}_1 \neq 0$. Let us use the notation

$$\mathcal{Z} = \mathcal{Z}_i v^i \quad , \tag{4.5}$$

in the following. Furthermore, we choose the normalization

$$\alpha_3 = 1 \quad , \tag{4.6}$$

which is argued for in section 4.1. Note that as in the Ansatz (4.3) we allow for additional pre-factors (4.6) can be imposed without loss of generality.

Let us next briefly outline the logic of this section. In 4.1 we compute the variation of the Ansatz (4.4) w.r.t. Kähler deformations of the Calabi–Yau fourfold and show the correlation with the higher-derivative structures encountered in the reduction result. We will argue in section 4.2 that the Ansatz (4.2) and (4.3) can be rewritten solely in terms of topological quantities of the divisors. All the higher-derivative structures of the reduction result (3.19)-(3.21) can be matched. This steps

[13]We omit constants shifts such as $\mathcal{Z}_i$ in the definition of the Kähler coordinates.

[14]Comparison of the warp-factor contribution of the one modulus Kähler coordinates (3.30) and (4.3) suggest that $\mathscr{F}_i \to 9\mathscr{W}\mathcal{V}^{-1/4}$ in the one-modulus case.

fixes the relative factors $\alpha_1, \ldots, \alpha_9$ with one remaining free parameter α_2. In section 4.3 we discuss the compatibility of this Ansatz with the one-modulus case which can be integrated exactly into a Kähler potential [14] which induces certain relations among the κ's in the Ansatz. However, a precise determination of the reminiang κ-parameters is beyond the aim of this work and we suggest that the matching of the reduction result is possible with the Ansatz (4.4), (4.2) and (4.3) which then may fix all the parameters uniquely. Lastly, we provide further indirect evidence for this claim by comparison to the newly discovered structures (3.21) proportional to the second Chern form of the Calabi–Yau fourfold which may also be reproduced by the novel Ansatz. This insight however is not used in the direct line of arguments which precede through the following sections.

4.1 Kähler coordinates as integrals on CY_4

To write the integrals (4.4) defined over Divisors $D_i = PD(\omega_i)$ as integrals over the Calabi–Yau fourfold we note that e.g.

$$\int_{D_i} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{c}_1 = \int_{CY_4} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{c}_1 \wedge \omega_i . \tag{4.7}$$

Note that it is crucial to maintain $\tilde{c}_1$ instead of c_1 as latter would vanish due to the Calabi–Yau condition. The induced metric on D_i inherited from the ambient space is itself Kähler [38, 39] but generically not Calabi–Yau. Let us note that in previous work we considered the correction written in terms of topological quantity namely the third Chern-form of the Calabi–Yau fourfold. One may write the Kähler coordinates (4.4) in terms of a basis of well defined CY_4-integrals in terms the Calabi–Yau metric and covariant quantities thereof such as the Riemann tensors if the parameters in (4.4) obey the following relations

$$\begin{aligned} \alpha_5 &= -\tfrac{1}{8}\alpha_1 + \tfrac{1}{24} + \tfrac{1}{4}\alpha_4 \ , \\ \alpha_6 &= \tfrac{1}{2}\alpha_2 + \tfrac{1}{2} \ , \\ \alpha_7 &= \alpha_2 + 1 \ , \\ \alpha_8 &= \tfrac{1}{2}\alpha_1 - \tfrac{1}{3} - \alpha_4 \ , \\ \alpha_9 &= -\alpha_1 + \tfrac{1}{6} \ . \end{aligned} \tag{4.8}$$

Thus in other words by imposing (4.8) we can rewrite the Kähler coordinates in terms of a higher-derivative density on the Calabi–Yau fourfold, which as we argue in appendix A.1 may take the form

$$\mathcal{T}_i = \int_{CY_4} \omega_i \wedge \mathcal{X} \quad , \quad \mathcal{X} \sim R^3 \tag{4.9}$$

with the higher-derivative $(3,3)$-form $\mathcal{X}$ defined in the appendix (A.31). One can easily verify the property

$$\mathcal{T}_i v^i = \mathcal{Z} \ . \tag{4.10}$$

To compute the Kähler metric we need to take derivatives of the Kähler potential w.r.t. to the Kähler coordinates as

$$K_{ij} = \frac{\partial^2 K}{\partial \mathrm{Re}T_i \partial \mathrm{Re}T_j} = \frac{\partial^2 v^k}{\partial \mathrm{Re}T_i \partial \mathrm{Re}T_j}\frac{\partial K}{\partial v^k} + \frac{\partial v^k}{\partial \mathrm{Re}T_i}\frac{\partial v^l}{\partial \mathrm{Re}T_j}\frac{\partial^2 K}{\partial v^k \partial v^l} \ , \tag{4.11}$$

with

$$\frac{\partial v^i}{\partial \mathrm{Re}T_j} = \Big(\frac{\partial \mathrm{Re}T_j}{\partial v^i}\Big)^{-1} = \mathcal{K}^{ij} - \alpha^2 \kappa_5 \mathcal{K}^{ik}\Big(\frac{\partial}{\partial v^k}\mathcal{T}_l + \dots\Big)\mathcal{K}^{lj} \ , \tag{4.12}$$

where $\mathcal{K}^{ij}$ is the inverse intersection number of the Calabi–Yau fourfold defined in the appendix (A.11). The variation of $\mathcal{T}_i$ w.r.t. to the Kähler moduli fields of the Calabi–Yau fourfold constitutes the crucial new ingredient to generate and match the higher-derivative structures in the reduction result (3.19)-(3.21) of the Kähler metric. Let us next discuss it in more detail.

Variational derivative of Kähler coordinates. The aim of this section is to argue that the Ansatz for the Kähler potential (4.2) and Kähler coordinates (4.3) may reproduce the Kähler metric in the Legendre dual variables which are in agreement with the reduction results. In other words we are able to encounter all relevant higher-derivative structures found in the reduction result (3.19)- (3.21). However, let us stress that to precisely match the factors in the reduction result is beyond the aim of this work. It is expected that additional non-trivial identities relating the higher-derivative building blocks (4.14) and (3.19) -(3.21), and (4.16) are required to perform this task.

Let us proceed with the main argument. It is straight forward to compute derivatives of the previously encountered topological objects [17] w.r.t. to the Kähler moduli fields as

$$\frac{\partial}{\partial v^i}\mathcal{Z} = \mathcal{Z}_i \quad , \quad \frac{\partial}{\partial v^j}\mathcal{Z}_j = 0 \ . \tag{4.13}$$

Let us note that due to (4.13) no terms proportional to the logarithm of the volume - $\log \mathcal{V}$ - appear in the Kähler metric nor in the Legendre dual variables and thus (4.3) and (4.2) are in agreement with the reduction result in this regard.

Let us next compute the variation of $\mathcal{T}_i$ in (4.9) w.r.t. to the Kähler moduli fields which gives

$$\frac{\partial}{\partial v^j}\mathcal{T}_i = \frac{1}{\mathcal{V}}\mathcal{K}_i\mathcal{T}_j - \frac{2}{\mathcal{V}}\mathcal{K}_j\mathcal{T}_i + 4\,\mathcal{T}_{ij} + 4\frac{1}{\mathcal{V}}\mathcal{Z}_i\mathcal{K}_j + 2i\int_{CY_4} Z_{m\bar{n}}\omega_i{}^{\bar{n}s}\omega_{js}{}^m * 1 \ , \tag{4.14}$$

where

$$\mathcal{T}_{ij} = \int_{CY_4} *_8\big(\omega_i \wedge \omega_j \wedge J\big) \wedge \mathcal{X} \ . \tag{4.15}$$

To compute (4.14) we make extensive use of the compute Algebra package xTensor [40]. We provide some more technical details in appendix A.3. There we also discuss couplings of the Kähler metric proportional to the second Chern form of the Calabi–Yau fourfold. By using the relation

$$\mathcal{Y}_{ij} = -\frac{1}{6}\int_{Y_4}(iZ_{m\bar{n}}\omega_i{}^{\bar{r}m}\omega_j{}^{\bar{n}}{}_{\bar{r}} + 2Z_{m\bar{n}r\bar{s}}\omega_i{}^{\bar{n}m}\omega_j{}^{\bar{s}r}) * 1\,, \tag{4.16}$$

one infers that (4.14) can be put in relation to $Z_{m\bar{n}r\bar{s}}\omega_i^{\bar{n}m}\omega_i^{\bar{r}s}$ and $\mathcal{Y}_{ij}$. Let us emphasize that establishing the relation of topological Kähler coordinates and the building blocks of the Kähler metric obtained by dimensional reduction $\sim Z_{m\bar{n}r\bar{s}}\omega_i^{\bar{n}m}\omega_i^{\bar{r}s}$ as well as $\sim Z_{m\bar{n}}\omega_i{}^{\bar{n}s}\omega_{js}{}^m$ has been a long standing problem posed in our previous work [12, 13].

Let us close this section by providing further arguments for the completeness of higher-derivative building blocks in (4.14). By evaluating (4.11) one obtains that the Kähler metric K_{ij} contains $\mathcal{V}\mathcal{K}_{kl}\mathcal{K}_{ijk}\mathcal{T}_j$ and $\mathcal{K}_{(i}\mathcal{T}_{j)}$.[15] Those structures arise naturally from the variation of the Kähler coordinates (4.14), in particular $\mathcal{T}_{ij} \sim \mathcal{K}^{kl}\mathcal{K}_{ijk}\mathcal{T}_i + \ldots$. It has been argued for analogous relations in [41, 42]. Concludingly, the divisor integral Ansatz (4.3) manages to reproduce all relevant higher-derivative building blocks which appear in the reduction result (3.19)-(3.21). However, we also find that we have one abundant object namely $\mathcal{Y}_{ij}$ which does not appear in the reduction result but would be generated by our Ansatz. In [12] we had argued for a relation in between the $\mathscr{F}$ and higher-derivative objects which in the light of this work most certainly needs a revision. Let us close this section with remarks on the warp-factor in the Kähler potential and coordinates and its potential connection to the higher-derivative structures. In appendix A.3 we review the integration of the warp-factor into a Kähler potential in particular in (A.51) - (A.62) . From the definition (A.20) one immediately infers that $\mathcal{Y}_{ij}v^j = \mathcal{Y}_{ji}v^j = 0$ and thus it takes special simplified role in the process of matching the reduction result. We thus suggest that a relation $\mathcal{Y}_{ij} \sim \mathscr{F}_{ij}$ might be established to proof the conjectured integration into a Kähler potential which revises the claims of [12].

4.2 Topological divisor integrals as Kähler coordinates

In this section we argue that the Ansatz for the Kähler coordinates (4.4) may be rewritten in terms of "topological quantities" by fixing the coefficients in the Ansatz. The quotation marks refer to an abuse of the word as the integrands can be reduced to topological integrands by factorizing out Kähler moduli deformations, e.g. the intersection number of the Calabi–Yau fourfold $\mathcal{K}_{ijkl}$ is a topological quantity, in contrast to the volume of a complex curve $\mathcal{K}_{ijk}$ which is not as it depends on the position in moduli space. However one may write it in terms of the topological intersection numbers by factorizing out the Kähler moduli fields as $\mathcal{K}_{ijk} = \mathcal{K}_{ijkl}v^l$.

[15] The precise form of the Kähler metric results from (4.11) by inserting our Ansatz (4.3),(4.2) and by using the properties of the intersection numbers listed in equation (A.11). Furthermore, one may use the relations on the higher-derivative building blocks (A.33) and (A.34)

To set the stage note that any closed form such as $\tilde{c}_1$ may be written in terms of its harmonic part plus a double exact contribution

$$\tilde{c}_1 = H\tilde{c}_1 + \partial\bar{\partial}\lambda \ , \tag{4.17}$$

where λ is a function on the divisor. From the closure of $\tilde{c}_1$ and by using inferred relation thereof in appendix A.2 one may show that the Ansatz for the Kähler coordinates (4.4) can be rewritten as

$$\begin{aligned}
\mathcal{T}_i = \ & \alpha_1 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{c}_1 + \alpha_2 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_2 \ + \alpha_3 \int_{D_i} \tilde{c}_3 + \frac{\alpha_4}{\mathcal{K}_i} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{J} \\
& + \frac{\alpha_5}{\mathcal{K}_i^2} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 + \frac{\alpha_6}{\mathcal{K}_i} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_2 \wedge \tilde{J} \\
& + 2\alpha_6 \int_{D_i} \tilde{*}_6 (H\tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_2 - \big(2\alpha_4 + 8\alpha_5\big) \int_{D_i} \tilde{*}_6 (H\tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_1 \wedge \tilde{c}_1 \\
& - \frac{4\alpha_5}{\mathcal{K}_i} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_1 \wedge *_6 H\tilde{c}_1 \ ,
\end{aligned} \tag{4.18}$$

where $\mathcal{K}_i$ denotes the volume of the divisor D_i. Note that in order to obtain (4.18) one fixes the coefficients such that

$$\alpha_7 = 2\alpha_6 \ , \quad \alpha_8 = 2\alpha_4 - 8\alpha_5 \ , \quad \alpha_9 = -4\alpha_5 \ . \tag{4.19}$$

Additionally requiring that we can write $\mathcal{T}_i$ as integrals on the Calabi–Yau fourfold one is led to additional constraints which in combination with (4.8) then impose

$$\begin{aligned}
&\alpha_1 = \tfrac{1}{6} \ , && \alpha_3 = 1 \ , && \alpha_4 = -\tfrac{1}{12} \ , && \alpha_5 = 0 \ , \\
&\alpha_6 = \tfrac{1}{2} + \tfrac{1}{2}\alpha_2 \ , && \alpha_7 = 1 + \alpha_2 \ , && \alpha_8 = -\tfrac{1}{6} \ , && \alpha_9 = 0 \ .
\end{aligned} \tag{4.20}$$

One thus infers from (4.20) the final form of the higher-derivative Kähler coordinate divisor integral to be

$$\begin{aligned}
\mathcal{T}_i = \ & \tfrac{1}{6} \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{c}_1 + \alpha_2 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_2 \ + \int_{D_i} \tilde{c}_3 - \frac{1}{12\mathcal{K}_i} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{J} \\
& + \tfrac{1}{2}\big(1 + \alpha_2\big) \frac{1}{\mathcal{K}_i} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_2 \wedge \tilde{J} + \big(1 + \alpha_2\big) \int_{D_i} \tilde{*}_6 (H\tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_2 \\
& - \tfrac{1}{6} \int_{D_i} \tilde{*}_6 (H\tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_1 \wedge \tilde{c}_1 \ .
\end{aligned} \tag{4.21}$$

Let us note that (4.21) is in indeed a sum of "topological integrals". In this sense after factorizing out Kähler moduli deformations one may vary the integrands of (4.21) w.r.t. the induced metric on the divisors D_i and find that the resulting variation constitutes a total derivative. This follows straightforwardly from the properties of $\tilde{c}_1, \tilde{c}_2, \tilde{c}_3$ and $\tilde{J}$. The integrands involving the hodge star $\tilde{*}_6$ crucially have it act on only the harmonic part of the first Chern-form $H\tilde{c}_1$.

4.3 One-modulus compatibility

The one-modulus case can be integrated exactly into a Kähler potential as discussed in section 3.3. Thus in this section we examine the limit $h^{1,1} \to 1$ of the generic moduli case 3.2 to impose constraints on the κ-parameters in the Ansatz. We focus on the higher-derivative components and do not discuss the warp-factor contributions $\mathscr{W}$ and $\mathscr{F}$ here. Recall that

$$\mathcal{Z}_i = (2\pi)^3 \int_{CY_4} c_3 \wedge \omega_i \quad , \qquad \mathcal{Z} = \mathcal{Z}_i v^i \quad , \tag{4.22}$$

We made the Ansatz for the Kähler potential

$$K = -3 \log \Big(\mathcal{V} + \alpha^2 \big(\kappa_1 \mathcal{Z}_i v^i + \kappa_2 \mathcal{T}_i v^i \big) \Big) \quad , \tag{4.23}$$

and for the Kähler coordinates

$$\mathrm{Re} T_i = \mathcal{K}_i + \alpha^2 \Big(\mathscr{F}_i + 3 \mathscr{W}_i + \kappa_3 \frac{\mathcal{K}_i}{\mathcal{V}} \mathcal{Z}_j v^j + \kappa_4 \mathcal{Z}_i \log \mathcal{V} + \kappa_5 \frac{\mathcal{K}_i}{\mathcal{V}} \mathcal{T}_j v^j + \kappa_6 \mathcal{T}_i \Big) \quad . \tag{4.24}$$

Let us next analyse these expressions (4.23) and (4.24) in the case $h^{1,1} = 1$. One finds that

$$\mathcal{K}_i \to 4 \mathcal{V}^{\frac{3}{4}} \quad , \quad \mathcal{K}_{ij} \to 12 \mathcal{V}^{\frac{1}{2}} \quad , \quad \mathcal{K}_{ijk} \to 24 \mathcal{V}^{\frac{1}{4}} \quad , \quad \mathcal{K}_{ijkl} \to 1 \quad , \quad \mathcal{K}^{ij} \to \tfrac{1}{12} \mathcal{V}^{-\frac{1}{2}} \tag{4.25}$$

and from the expression (4.9) and (4.10) that in the one-modulus case

$$\mathcal{T}_i \to \tilde{\mathcal{Z}} \qquad \text{with} \quad \tilde{\mathcal{Z}} = (2\pi)^3 \int_{CY_4} c_3 \wedge \omega^0 \quad . \tag{4.26}$$

The relation (4.26) follows from (4.4) and (4.18) due to the Calabi–Yau condition which leads to a vanishing of terms proportional to $\tilde{c}_1$. One furthermore notes that $J = \omega_0 \mathcal{V}^{\frac{1}{4}}$ and thus

$$\mathcal{Z} = \mathcal{V}^{\frac{1}{4}} \tilde{\mathcal{Z}} = (2\pi)^3 \int_{CY_4} c_3 \wedge J \quad . \tag{4.27}$$

Ones concludes that (4.23) and (4.24) for $h^{1,1} \to 1$ by using relations (4.25)-(4.27) become

$$K \to -3 \log \Big(\mathcal{V} + \alpha^2 \, (\kappa_1 + \kappa_2) \mathcal{V}^{\frac{1}{4}} \tilde{\mathcal{Z}} \Big) \quad . \tag{4.28}$$

and

$$\mathrm{Re} T_i \to 4 \mathcal{V}^{\frac{3}{4}} + \alpha^2 \Big((4\kappa_3 + 4\kappa_5 + \kappa_6) \tilde{\mathcal{Z}} + \kappa_4 \tilde{\mathcal{Z}} \log \mathcal{V} \Big) \quad , \tag{4.29}$$

Thus one infers by comparison to the one-modulus case (3.29) (3.30) that

$$\kappa_1 + \kappa_2 = 512 \quad , \quad 4\kappa_3 + 4\kappa_5 + \kappa_6 = 0 \quad , \quad \kappa_4 = -1152 \quad . \tag{4.30}$$

Additionally one aims to match the Legendre dual coordinates to the one modulus case. To proceed one needs to specify the precise form of the Kähler coordinates in terms of Calabi–Yau fourfold integrals. In section 4.1 we emphasized that the match with the divisor integral form remains ambiguous. Let us proceed with a simple version given in (4.15) for the remainder of this section. One can then use

$$L^i = -\frac{\partial K}{\partial T_i} = -\frac{\partial K}{\partial v^j i}\frac{\partial v^j}{\partial T_i} \ , \tag{4.31}$$

to find

$$L^i = \frac{v^i}{\mathcal{V}} + \frac{\alpha^2}{\mathcal{V}}\Big(\kappa_6 \mathcal{K}^{ij}\mathcal{T}_j + (3\kappa_1 - 4\kappa_3 - 4\kappa_5 - \kappa_6)\mathcal{K}^{ij}\mathcal{Z}_j - \frac{v^i}{3\mathcal{V}}(3\kappa_1 - \kappa_3 - \kappa_5 + \kappa_4)\mathcal{Z}\Big) \ . \tag{4.32}$$

To compute (4.32) we only used the fact that $\big(\frac{\partial}{\partial v^j}\mathcal{T}_i\big)v^i = -\mathcal{T}_j + \mathcal{Z}_j$ which follows from $\mathcal{T}_i v^i = \mathcal{Z}$. Lastly by imposing (4.30) one infers a match of (3.27) with comparison of the one-modulus limit of (4.32), i.e. the order α-contributions vanishes in the limit. One can furthermore compute the scalar potential by evaluating (4.11) which can be performed by using (A.34) and (A.11) contracted with (4.32). One finds for a non vanishing flux-superpotential W_0 that

$$V_F = \frac{|W_0|^2}{\mathcal{V}^4}\frac{4}{3}\big(9\kappa_1 + 5\kappa_4 + 9\kappa_6\big)\mathcal{Z} \tag{4.33}$$

which by imposing (4.30) matches the one modulus case given in (3.32). Moreover, note that from (4.33) one infers that for the Ansatz (4.2) and (4.3) the no-scale structure is broken due to the imposed compatibility with the one-modulus case.

Let us close this section with a critical remark. In section 4.1 and 4.2 we pointed out that the lift of the divisor integral expressions to integrals on the Calabi–Yau fourfold leaves certain parameters unfixed. In order to compute other quantities such as (4.33) in full generality we suggest that a better understanding of the $\mathcal{T}_i$ contribution is to be developed.

5 F-theory uplift to 4d, $\mathcal{N} = 1$

In this section we utilize the duality between M-theory and F- theory to lift the l_{M}-corrections in the three-dimensional theory obtained in the previous section to α'-corrections to the four-dimensional effective theory arising from F-theory compactified on CY_4. This requires the Calabi–Yau manifold to be elliptically fibered over a three-dimensional Kähler base B_3.

In the following we consider the classical result of the F-theory uplift [3]. One may parametrize the shrinking of the torus fiber by the parameter $\epsilon \to 0$. One then infers the scaling of the fields $v^0 \sim \epsilon$ and $v^\alpha \sim \epsilon^{-1/2}$. This leads to an identification of the $3d, \mathcal{N} = 2$ multiplet field $L^0 = \frac{v^0}{\mathcal{V}} = \frac{1}{r^2}$ with r the radius of the $4d/3d$ circular reduction. To keep the base volume finite in the limit one finds

$$2\pi v_b^\alpha = \sqrt{v^0}v^\alpha \ . \tag{5.1}$$

For simplicity, let us restrict to a smooth Weierstrass model, i.e. a geometry without non-Abelian singularities, that can be embedded in an ambient fibration with typical fibers being the weighted projective space $W\mathbb{P}_{231}$. This implies having just two types of divisors D_i, $i = 1, ..., h^{1,1}(CY_4)$. There is the horizontal divisor corresponding to the zero-section D_0, and the vertical divisors D_α, $\alpha = 1, ..., h^{1,1}(B_3)$, corresponding to elliptic fibrations over base divisors D^b_α. Denoting the Poincare-dual two-forms to the divisors by $\omega_i = (\omega_0, \omega_\alpha)$, one expands the Kähler form as

$$J = v^0\omega_0 + v^\alpha\omega_\alpha \quad , \tag{5.2}$$

where v^0 is the volume of the elliptic fiber, and we choose the harmonic representatives of the class. We are now in a position to discuss the F-theory uplift of the individual terms in

$$\begin{aligned}\mathcal{T}_i = &\tfrac{1}{6}\int_{D_i} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{c}_1 + \alpha_2 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_2 \; + \int_{D_i} \tilde{c}_3 - \frac{1}{12\mathcal{K}_i}\int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{J} \\ &+ \tfrac{1}{2}\big(1+\alpha_2\big)\frac{1}{\mathcal{K}_i}\int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_2 \wedge \tilde{J} + \big(1+\alpha_2\big)\int_{D_i} \tilde{*}_6(H\tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_2 \\ &- \tfrac{1}{6}\int_{D_i} \tilde{*}_6(H\tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_1 \wedge \tilde{c}_1 \quad .\end{aligned} \tag{5.3}$$

and the correction

$$\mathcal{Z}_\alpha = (2\pi)^3 \int_{CY_4} c_3 \wedge \omega_\alpha \quad , \tag{5.4}$$

where $\mathcal{K}_i$ is the volume of the divisor D_i. Latter was discussed already in [16, 17] however, we review these results in section 5.2. Note that the relation between the eleven-dimensional Planck length l_{M} and the string length l_s by the M/F-theory duality is obtained as

$$2\pi l_s = \mathcal{V}^{\frac{1}{2}} l_{\mathrm{M}} \quad . \tag{5.5}$$

As in the F-theory limit one sends $v^0 \to 0$ decompactifying the fourth dimension by sending to infinity the radius of the $4d/3d$ circle $r \sim \mathcal{V}^{3/2} \to \infty$. Thus after the limit all volumes of the base B_3 are expressed in terms of the string units l_s. In the following we omit the warp-factor $\mathscr{W}$ and thus $\mathscr{F}$ from the discussion.

In section 5.1 we shortly comment on the uplift of F-theory involving one-loop corrections resulting from integrating out massive KK-modes at one-loop in the circular reduction from four to three dimensions. As those results are not well studied in the literature we present an superficial discussion. Let us stress however, that as we are not able to fix all parameters in the $3d$, $\mathcal{N} = 2$ coordinates the ambiguity of the "one-loop" up-lift can be hidden in the following section in the uncertainty of the parameters and the generic conclusions of this work are expected to be unchanged. In section 5.2 we then analyse the terms in the Kähler potential (4.2) and Kähler metric (4.3) surviving the F-theory uplift. Finally, in section (5.3) we then combine the conclusions of sections 5.1 and 5.2 to discuss the $4d, \mathcal{N} = 1$ Kähler potential and Kähler metric. In particular we give a string theory interpretation of the novel corrections and discuss the breaking of the no-scale structure and the α'^2-modified scalar potential.

5.1 The F-theory uplift

In this section we review the supergravity perspective of the F-theory lift identifying the connection in-between the four and three-dimensional fields and their kinematic couplings [3]. Note that by compactifying a general four-dimensional, $\mathcal{N} = 1$ supergravity theory on a circle one matches the original four-dimensional Kähler potential with the three-dimensional Kähler potential K or kinetic potential $\tilde{K}$. The resulting kinetic potential arising in the $4d/3d$ circular dimensional reduction takes the form

$$\tilde{K}(r, T_\alpha) = -\log(r^2) + K^F(T_\alpha) \, . \tag{5.6}$$

To match (5.6) with the natural three-dimensional multiplets one may split L^i and T_i such that

$$L^i = \left(L^0 \equiv R \, , \;\; L^\alpha \right) \, , \qquad T_i = \left(T_0 \, , \, T_\alpha \right) \, . \tag{5.7}$$

One is then led to identify that R is given by $R = r^{-2}$, where r is the radius of the $4d/3d$ circle [3]. Furthermore, the fields T_α remain complex scalars in four dimensions whilst T_0 should be dualized already in three dimensions into vector multiplets with (R, A^0) and then uplifted to four dimensions as it arises from the four-dimensional metric. Note that one computes the dualized kinetic potential $\tilde{K}(R, \mathrm{Re}\, T_\alpha)$ by Legendre dualization as discussed in 3.1. In the F-theory limit one then identifies

$$L^\alpha_b = L^\alpha|_{\epsilon=0} \, , \qquad T^b_\alpha = T_\alpha|_{\epsilon=0} \, , \tag{5.8}$$

where we denote the four-dimensional fields L^α_b and T^b_α due to the fact that they correspond to fields with couplings related to the base B_3 representing the Calabi–Yau orientifold in the IIB picture, i.e. in the F-theory limit. Let us next review the classical analysis to determine $K^F(T^b_\alpha)$. Evaluating the intersection numbers $\mathcal{K}_{ijkl}$ for an elliptic fibration the non-vanishing coupling is

$$\mathcal{K}_{0\alpha\beta\gamma} = \mathcal{K}^b_{\alpha\beta\gamma} \quad , \quad \mathcal{K}^b_{\alpha\beta\gamma} = \int_{B_3} \omega_\alpha \wedge \omega_\beta \wedge \omega_\gamma \, . \tag{5.9}$$

The kinetic potential and coordinates take the following form for an elliptic fibration

$$\tilde{K}(L^i) = \log(R) - 2\log\Big(\mathcal{V}^b + \mathcal{O}(R)\Big) + 4 \, , \tag{5.10}$$

$$\mathrm{Re}\, T_\alpha = \mathcal{K}^b_\alpha + \mathcal{O}(R) \, , \qquad \mathcal{V} = \frac{1}{3!}\mathcal{K}_{\alpha\beta\gamma} v^\alpha_b v^\beta_b v^\gamma_b \, , \tag{5.11}$$

or equivalently

$$\tilde{K}(L^i) = \log(R) + \log\Big(\frac{1}{3!}\mathcal{K}^b_{\alpha\beta\gamma}L^\alpha_b L^\beta_b L^\gamma_b + \mathcal{O}(R)\Big) + 4\ , \tag{5.12}$$

$$\mathrm{Re}\, T_\alpha = \frac{1}{2!}\frac{\mathcal{K}^b_{\alpha\beta\gamma}L^\beta_b L^\gamma_b}{\hat{\mathcal{V}}^b(L_b)} + \mathcal{O}(R)\ , \qquad \hat{\mathcal{V}}^b(L^b) = \frac{1}{3!}\mathcal{K}_{\alpha\beta\gamma}L^\alpha_b L^\beta_b L^\gamma_b\ , \tag{5.13}$$

where we have replaced the L^α with L^α_b by means of (5.8) and made use of the relation $\hat{\mathcal{V}}^b(L_b) = (\mathcal{V}^b)^{-2}$. Performing the Legendre transform in order to express everything in terms of T^b_α and comparing the result with (5.6) by setting $R = r^{-2}$ in the limit $r \to \infty$ one encounters

$$K^F(T^b_\alpha) = -2\log\big(\mathcal{V}^b\big) = \log\big(\hat{\mathcal{V}}^b(L_b)\big)\ , \qquad \mathrm{Re}\, T^b_\alpha = \mathcal{K}^b_\alpha = \frac{1}{2!}\frac{\mathcal{K}^b_{\alpha\beta\gamma}L^\beta_b L^\gamma_b}{\hat{\mathcal{V}}^b(L_b)}\ , \tag{5.14}$$

where one has to solve T^b_α for $L^\alpha_b(T^b_\alpha)$ and insert the result into K^F.

Let us next comment on the case present in this work namely where one encounters higher-order l_{M}-corrections to the three-dimensional fields. As suggested by the generic $4d/3d$ circular reduction result and one infers for the corrected the Kähler coordinates that

$$\mathrm{Re}T_\alpha \quad \to \quad \mathrm{Re}T^b_\alpha\ , \tag{5.15}$$

where we analyse $\mathrm{Re}T^b_\alpha$ in the next section 5.2. The corrected Kähler potential (4.2) can be re-written as

$$K = -\log\big(R\big) - 2\log\Big(\mathcal{V}^b\Big(1 + \alpha^2\tfrac{3}{2\mathcal{V}^b}\big((512-\kappa_2)\,\mathcal{Z}^b_\alpha v^\alpha_b + \kappa_2\mathcal{T}^b_\alpha v^\alpha_b\big)\Big) + \mathcal{O}(R)\Big)\ , \tag{5.16}$$

by making use of the sub-leading order of α^2. Thus in the limit $r \to \infty$ one encounters

$$K^F(T^b_\alpha) = -2\log\Big(\mathcal{V}^b + \alpha^2\big((768-\tilde{\kappa}_2)\mathcal{Z}^b_\alpha v^\alpha_b + \tilde{\kappa}_2\mathcal{T}^b_\alpha v^\alpha_b\Big)\ , \tag{5.17}$$

where is $\mathcal{Z}^b_\alpha$ the F-theory limit of $\mathcal{Z}_\alpha$ derived in in the following section 5.2 and $\tilde{\kappa}_2 = \frac{3}{2}\kappa_2$. The identification of the dependence T^b_α is implicit.

Let us close this section with remarks on one-loop corrections to the F-theory limit resulting from integrating out massive KK-modes which is expected to modify the relation (5.15). The $\log\mathcal{V}$-correction to the Kähler coordinates (4.3) is reminiscent of such a one loop correction. To see this one is to preform a dimensional reduction of a general $4d$, $\mathcal{N} = 1$ supergravity theory on the circle to three dimensions where massive KK-modes are integrated out at one-loop. The case for pure supergravity is discussed in [43] which yields the three-dimensional Kähler coordinates

$$\mathrm{Re}T_0^{1-loop} = \frac{2\pi^2}{R} - \frac{7}{48}\log(R)\ . \tag{5.18}$$

However, we are interested in a theory with additional chiral multiplets and vector multiplets which will lead to a modification of the purely gravitational result (5.18).

We are not aware of such a discussion in the literature and thus have no rigorous tool to argue for the up-lift of the $\mathcal{Z}_\alpha \log \mathcal{V}$ correction in F-theory except the comments made in [14]. Note that the main result of this work is obtained from the novel divisor integral modification of the Kähler potential and coordinates in (4.2) and (4.3) thus the one-loop discussion is not expected to change these conclusions. Let us assume in the following that the $\log \mathcal{V}$-correction in the Kähler coordinates (4.3) is absorbed entirely by the F-theory uplift. This leads us to write

$$\mathrm{Re}T_i = \mathcal{K}_i + \alpha^2 \kappa_3 \mathcal{Z}_i \log \mathcal{V} \ , \tag{5.19}$$

where for simplicity we only write the logarithmic correction to the Kähler coordinates. Considering (5.19) on the elliptically fibered Calabi–Yau fourfold one finds

$$\begin{aligned} \mathrm{Re}T_0 &= \frac{1}{R} - \tfrac{\alpha^2\kappa_3}{3} \mathcal{Z}_0^b \log R - \tfrac{\alpha^2\kappa_3}{3} \mathcal{Z}_0^b \log\left((\mathcal{V}^b)^{-3} + \mathcal{O}(R) \right) , \\ \mathrm{Re}T_\alpha &= \mathcal{K}_\alpha^b - \tfrac{\alpha^2\kappa_3}{3} \mathcal{Z}_\alpha^b \log(R) - \tfrac{\alpha^2\kappa_3}{3} \mathcal{Z}_\alpha^b \log\left((\mathcal{V}^b)^{-3} + \mathcal{O}(R) \right) \ . \end{aligned} \tag{5.20}$$

The assumption that it is absorbed in the uplift immediately leads us to a revision of (5.15) to

$$\begin{aligned} \mathrm{Re}T_0^{1-loop} &\to \frac{1}{R} - q_0 \log R + \frac{1}{2} q_0 K^{4d} \ , \\ \mathrm{Re}T_\alpha^{1-loop} &\to \mathrm{Re}T_\alpha^{b\,tree} - q_\alpha \log R + \frac{1}{2} q_\alpha K^{4d} \ . \end{aligned} \tag{5.21}$$

where K^{4d} is the four-dimensional classical Kähler potential. By matching (5.20) and (5.21) one fixes the charges to $q_i = -\frac{\alpha^2\kappa_3}{3} \mathcal{Z}_i^b + \mathcal{O}(R)$. Let us stress that an assumption leads us to find (5.21) but a honest one-loop computation needs to be performed to check its validity. Note that (5.21) implies that by integrating out massive KK-modes only the three-dimensional Kähler coordinates receive modifications whilst the Kähler potential remains uncorrected.

5.2 Topological integrals on elliptic Calabi–Yau fourfolds

In this section we discuss the F-theory uplift of the higher-order l_M-corrections appearing in (5.3) and (5.4) resulting in α'-corrections. For topological integrals we can use adjunction formulae to express Chern-classes of CY_4 and the divisors D_α in terms of Chern-classes of the base B_3. For details of the derivation of the adjunction formulae see appendix B.2. One infers that

$$\begin{aligned} \tilde{c}_3(D_\alpha) &= c_3(B_3) - c_1(B_3) \wedge c_2(B_3) - 60 c_1^3(B_3) - 60 c_1^2(B_3) \wedge \omega_0 - \tilde{c}_2(D_\alpha) \wedge \omega_\alpha \ , \\ \tilde{c}_2(D_\alpha) &= c_2(B_3) + 11 c_1^2(B_3) + 12 c_1(B_3) \wedge \omega_0 + \omega_\alpha^2 \ , \\ \tilde{c}_1(D_\alpha) &= -\,\omega_\alpha \ , \end{aligned} \tag{5.22}$$

where the $c_{i=1,2,3}(B_3)$ on the r.h.s. of these expressions denote the Chern classes of B_3 pulled-back to CY_4 restricted to D_α. Note that the Poincare duals of the harmonic $(1,1)$-forms in (5.22) are given by $PD(\omega_0) = B_3$, and $PD(\omega_\alpha) = D_\alpha$. We choose to omit the pull-back map in expressions in this section for notational simplicity. One furthermore finds that

$$\omega_0^2 = -c_1(B_3) \wedge \omega_0 \quad . \tag{5.23}$$

Note that the new contribution to the Kähler coordinates $\mathcal{T}_i$ is expressed as integrals on the divisors D_i where the Kählerform is inherited from the ambient CY_4 and one thus may use the decomposition (5.2) as well for $\tilde{J}$. In the F-theory limit one finds the scalings discussed at the beginning of this section to imply

$$v^\alpha \sim \epsilon^{-\frac{1}{2}} \,, \qquad v^0 \sim \epsilon \quad \Rightarrow \quad \mathcal{V}_\alpha = \mathcal{K}_\alpha \sim \epsilon^0 \quad . \tag{5.24}$$

Using (5.22) and (5.24) one infers the contributions in (4.3) and (4.2) which survive the F-theory limit. For the object defined as the third Chern-form of the Calabi–Yau fourfold (5.4) one finds in the limit

$$\mathcal{Z}_\alpha \quad \longrightarrow \quad \mathcal{Z}^b_\alpha = -60\,(2\pi)^2 \int_{D^b_\alpha} c_1(B_3) \wedge c_1(B_3) \quad . \tag{5.25}$$

The leading order contributions which are non vanishing in the limit must scale as $\mathcal{T}_\alpha \sim \mathcal{O}(\epsilon^0)$. The integrals in (5.4) which thus contribute are

$$\begin{aligned}
&\int_{D_\alpha} \tilde{c}_1 \wedge \tilde{c}_2 && \longrightarrow && -12 \int_{D^b_\alpha} c_1(B_3) \wedge \omega^b_\alpha \;, \\
&\int_{D_\alpha} \tilde{c}_3 && \longrightarrow && -60 \int_{D^b_\alpha} c_1(B_3) \wedge c_1(B_3) - 12 \int_{D^b_\alpha} c_1(B_3) \wedge \omega^b_\alpha \;, \\
&\int_{D_\alpha} \tilde{*}_6 (H \tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_2 && \longrightarrow && -\frac{12}{\mathcal{K}^b_\alpha} \int_{D^b_\alpha} \omega^b_\alpha \wedge J^b \int_{D^b_\alpha} c_1(B_3) \wedge J^b + 12 \int_{D^b_\alpha} c_1(B_3) \wedge \omega^b_\alpha \;, \\
&\frac{1}{\mathcal{K}_\alpha} \int_{D_\alpha} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_\alpha} \tilde{c}_2 \wedge \tilde{J} && \longrightarrow && -\frac{12}{\mathcal{K}^b_\alpha} \int_{D^b_\alpha} \omega^b_\alpha \wedge J^b \int_{D^b_\alpha} c_1(B_3) \wedge J^b \;,
\end{aligned} \tag{5.26}$$

where we used (A.14) and where D^b_α are the divisors of the base such that their pre-image w.r.t. the projection $\pi : CY_4 \to B_3$ gives the vertical divisors of the Calabi–Yau fourfold as $D_\alpha = \pi^{-1}(D^b_\alpha)$.[16] One thus infers the divisor integral contribution of the Kähler coordinates in the limit to take the form

[16] Note that in order to rewrite the integrals we note that e.g.

$$\int_{B_3} c_1(B_3) \wedge \omega^b_\alpha \wedge \omega^b_\alpha = \int_{D^b_\alpha} c_1(B_3) \wedge \omega^b_\alpha \;, \tag{5.27}$$

where we again omit the pull-back map on $c_1(B_3)$ in the r.h.s. of the equality.

$$\mathcal{T}_\alpha \quad \longrightarrow \quad \mathcal{T}^b_\alpha = \mathcal{Z}^b_\alpha - 18(1+\alpha_2)\,(2\pi)^2 \frac{1}{\mathcal{K}^b_\alpha} \int_{D^b_\alpha} \omega^b_\alpha \wedge J^b \int_{D^b_\alpha} c_1(B_3) \wedge J^b \tag{5.28}$$

with $\mathcal{K}^b_\alpha = \frac{1}{2!}\int_{B_3} \omega^b_\alpha \wedge J^{b2}$ the volume of the divisor D^b_α and the Kähler form $J^b = \omega^b_\alpha v^\alpha_b$. For further use let us define

$$\mathcal{W}^b_\alpha \;:=\; \frac{(2\pi)^2}{\mathcal{K}^b_\alpha} \int_{D^b_\alpha} \omega^b_\alpha \wedge J^b \int_{D^b_\alpha} c_1(B_3) \wedge J^b \;\; , \tag{5.29}$$

$$\mathcal{U}^b_\alpha \;:=\; (2\pi)^2 \int_{D^b_\alpha} c_1(B_3) \wedge \omega^b_\alpha \;\; . \tag{5.30}$$

The contribution (5.29) takes a special role as it depends on the Kähler form of the divisor and thus is non-vanishing upon taking derivatives w.r.t. Kähler moduli fields. This will be of particular interest in the following sections. Note that the F-theory uplift absorbs two-derivatives along the fiber thus the resulting corrections are of order α'^2. The $\mathcal{U}^b_\alpha$-correction (5.30) vanishes from (5.28) due to a vanishing pre-factor. As one may find that our constraints imposed are too restrictive this correction may survive if an additional parameter freedom is somehow introduced in the present discussion of divisor integrals. In the following we thus as well comment on its potential origin and interpretation.

Let us next comment on some special cases before providing a Type IIB string interpretation of the α'-corrections in (5.25) and (5.28). Firstly, for a trivial elliptic fibration, i.e. $CY_4 = CY_3 \times T^2$ with CY_3 a Calabi–Yau threefold, one infers that $c_i(CY_4) = c_i(CY_3)$, $i = 1,2,3$, in particular $c_1(CY_3) = 0$. Furthermore, the divisors relevant in the Kähler coordinates (4.3) are a direct product and obey $c_1(D^b_\alpha \times T^2) = c_1(D^b_\alpha)$, $c_2(D^b_\alpha \times T^2) = c_2(D^b_\alpha)$ and $c_3(D^b_\alpha \times T^2) = 0$, see appendix B.2 for details. One infers that in this case all corrections in (5.26) and (5.25) go to zero due to their scaling behavior in the limit $v^0 \to 0$ and thus the α'-corrections in the resulting $4d$, $\mathcal{N} = 2$ theory are absent.

Secondly, one may study other $\mathcal{N} = 2$ F-theory vacua by taking $Y_4 = \mathrm{K3} \times \mathrm{K3}$, a configuration discussed in [44] with a focus on α'-corrections. In this case $c_3(Y_4) = 0$ and thus the $\mathcal{Z}^b$-correction (5.25) vanishes identically. The corrections resulting from the divisors (5.26) vanish due to analogous arguments as in the above case. Concludingly, the α'-corrections discussed in this work vanish in these $\mathcal{N} = 2$ set-ups.

Finally, let us stress that there are several additional l_{M}-corrections to the fourfold volume surviving the F-theory limit. Let us again go back to the example of the product geometry $Y_4 = X_3 \times T^2$, without $D7$-branes. The α'-corrections involving the Type IIB axio-dilaton τ have been computed by integrating out the whole tower of T^2 Kaluza-Klein modes of the 11d supergravity multiplet [7], which results in $v^{0-\frac{1}{2}}\,\chi(CY_3)\,E_{3/2}(\tau,\bar\tau)$ with $E_{3/2}$ the non-holomorphic Eisenstein series. Note that it obeys the correct scaling behavior to survive the F-theory limit. One expects that the proper treatment of the KK-modes in a generic elliptic fibration is crucial to encounter the Euler-characteristic α'^3-correction [18] to the $4d, \mathcal{N}=1$ Kähler potential inside the F-theory framework.[17]

5.3 4d, $\mathcal{N}=1$ Kähler potential and coordinates

The discussion of the uplift of the α'-corrections in the previous sections 5.1 and 5.2 enables us to infer the resulting $4d, \mathcal{N}=1$ Kähler potential and coordinates. Let us use the dimensionless coefficients from now one, where all dimensionful quantities, e.g. α'-corrections are expressed in terms of the string length l_s, we thus write

$$\alpha^2 \to \frac{1}{3^2 \cdot 2^{13}} \quad . \tag{5.31}$$

One infers that

$$\mathcal{K}^{4d,\mathcal{N}=1} = -2\log\Big(\mathcal{V}_b + \alpha^2\big((768-\tilde{\kappa}_2)\,\mathcal{Z}^b{}_\alpha v_b^\alpha + \tilde{\kappa}_2 \mathcal{T}^b{}_\alpha v_b^\alpha\big)\Big) \;, \tag{5.32}$$

and

$$\mathrm{Re}T^b_\alpha = \mathcal{K}^b_\alpha + \alpha^2\left(\kappa_3 \frac{\mathcal{K}^b_\alpha}{\mathcal{V}_b}\mathcal{Z}^v_\alpha v^\alpha_b + \kappa_5 \frac{\mathcal{K}^b_\alpha}{\mathcal{V}_b}\mathcal{T}^v_\alpha v^\alpha_b - 4(\kappa_3+\kappa_5)\,\mathcal{T}^b_\alpha\right) \quad . \tag{5.33}$$

Note that we have argued in 5.1 that the term $\kappa_3 \mathcal{Z}^{\mathbf{b}}_\alpha \log \mathcal{V}_b$ can be absorbed in the F-theory uplift as a one-loop correction and is thus not present in (5.33). To verify this assumption is of great interest. One expects, however, that this remains independent of the discussion of the contribution proportional to $\mathcal{T}^b_\alpha$ which constitutes the main protagonist of this work. Note that (5.32) and (5.33) depend on four unfixed parameters, due to the additional freedom in (5.28). Further studies are required need to proof the existence of the α'-corrections in (5.32) and (5.33). Nevertheless, let us proceed by giving a string theory interpretation of the novel corrections to the four-dimensional Kähler potential and coordinates.

String theory interpretation and weak string-coupling limit. We follow the weak string-coupling limit by Sen [15] which is is performed in the complex structure moduli space of CY_4 to give a weakly coupled description of F-theory in terms of Type IIB string theory on a Calabi–Yau threefold with an $O7$-plane and $D7$-branes. Where CY_3 is a double cover of the base B_3 branched along the $O7$-plane. Let us stress that the class of this branching locus is the pull-back of $c_1(B_3)$ to CY_3. In section 5.2 we considered the topological divisor integrals on the geometries described by the smooth Weierstrass model i.e. non-Abelian singularities are absent. In this case Sen's limit contains a single recombined $D7$-brane wrapping a divisor

[17] An alternative approach was taken in [45].

of class $8c_1(B_3)$. This follows from the seven-brane tadpole cancellation condition. As was noted in [46, 47] this $D7$-brane is of the characteristic Whitney-umbrella shape. It would be interesting to extend the study to geometries with non-Abelian singularities analogously to [17].

The $\mathcal{Z}^b_\alpha$-correction (5.25) was discussed extensively in [16, 17] and we refer the reader to this work for details. Let us mention here however, that in more generic geometries it morally counts the number of self-intersections of stacks of $D7$-branes and the $O7$-plane. It should arise at tree-level in string theory and is of order α'^2. In the geometry studied in this work this can be checked by identifying

$$\mathcal{V}_{\mathrm{D7}\cap\mathrm{O7}} = 8\int_{CY_3} c_1^2(B_3)\wedge J_b\ , \tag{5.34}$$

where we omitted the pull-back map from B_3 to its double cover CY_3 in the integrand. To give the string theory interpretation one identifies the string amplitude capturing it by considering the Einstein-Hilbert term of the four-dimensional action in the string frame[18]

$$S_{(4)} \supset \frac{1}{(2\pi)^7 l_s^2 g_{IIB}^2}\int \Big(\mathcal{V}_b^s - \tfrac{5\pi^2}{2} g_{IIB}\mathcal{V}^s_{\mathrm{D7}\cap\mathrm{O7}}\Big) R^s_{\mathrm{sc}} *^s_4 1\ \ . \tag{5.35}$$

Let us recall the general formula for the Euler number of Riemann surfaces, possibly non-orientable and with boundaries, is

$$\chi(\Sigma) = 2 - 2g - b - c\,, \tag{5.36}$$

where g, b, c denote the genus, the number of boundaries, and the number of cross caps, respectively. One thus infers that the correction in (5.35) arises from a string amplitude that involves the sum over two topologies, namely the disk $g = c = 0$, $b = 1$ and the projective plane $g = b = 0$, $c = 1$. These are tree-level amplitudes of the orientable open strings and non-orientable closed strings which is in agreement with the property that the correction is intrinsically $\mathcal{N} = 1$, i.e. its presence is constrained by having $D7$-branes intersecting with an $O7$-plane.

Let us next give a string theory interpretation of (5.28). In fact, at weak string coupling one infers that

$$\int_{D^b_\alpha} c_1(B_3)\wedge J^b \sim \big(\mathcal{V}_{\mathrm{D7}\cap\mathrm{D}^\mathrm{b}_\alpha} + 4\mathcal{V}_{\mathrm{O7}\cap\mathrm{D}^\mathrm{b}_\alpha}\big)\,, \tag{5.37}$$

where $\mathcal{V}_{\mathrm{D7}}$ and $\mathcal{V}_{\mathrm{O7}}$ are the volumes of the $D7$-brane and the $O7$-plane in CY_3, respectively. Both volumes are in the Einstein frame and in units of l_s. By tadpole cancellation one infers $\mathcal{V}_{\mathrm{D7}} = 8\mathcal{V}_{\mathrm{O7}}$. It follows that

$$\int_{D^b_\alpha} c_1(B_3)\wedge \omega^b_\alpha \quad\sim\quad \mathrm{D7}\cap\mathrm{D}^\mathrm{b}_\alpha\cap\mathrm{D}^\mathrm{b}_\alpha + 4\,\mathrm{O7}\cap\mathrm{D}^\mathrm{b}_\alpha\cap\mathrm{D}^\mathrm{b}_\alpha, \tag{5.38}$$

[18] The superscript ^{s} denotes quantities computed using the string frame metric.

are the self-intersection curves of the base Divisors D^b_α intersected with the $D7$-brane and the $O7$-plane in CY_3, respectively. Lastly, the $\mathcal{W}^b_\alpha$-correction in (5.29) which is of order α'^2 and depends on the volume the self intersection curve of D^b_α

$$\int_{D^b_\alpha} \omega_\alpha \wedge J^b \;\sim\; \mathcal{V}_{D^b_\alpha \cap D^b_\alpha} \;, \tag{5.39}$$

and furthermore

$$\int_{D^b_\alpha} c_1(B_3) \wedge J^b \;\sim\; \mathcal{V}_{\mathrm{D7}\cap \mathrm{D}^b_\alpha} + 4\,\mathcal{V}_{\mathrm{O7}\cap \mathrm{D}^b_\alpha} \;, \tag{5.40}$$

the volume to the intersection curves of the $D7$-branes and $O7$-planes with the base divisors D^b_α. One concludes that (5.29) is a product of the curve volumes (5.39) and (5.40) weighted over the volume of the divisor D^b_α. It is of same order in the string coupling as (5.35) and is thus expected to arise equivalently from a tree-level amplitude of the orientable open string and non-orientable closed string amplitude in an orientifold background.

$4d$ **Scalar Potential and no-scale condition.** We next comment on the scalar potential resulting from (5.32) and (5.33). We assume that the complex structure moduli have been fixed and thus the superpotential remains to be a function of the Kähler moduli. The F-term scalar potential of a $4d$, $\mathcal{N}=1$ theory is well known and adjusted to our case results in

$$V_F^{4d} = e^K \Big(K_{\alpha\beta} D^\alpha W \overline{D^\beta W} - 3\big|W\big|^2 \Big) \;, \tag{5.41}$$

with the superpotential $W = W(\mathrm{Re}T_\alpha)$ and the Kähler covariant derivative given by

$$D^\alpha W = \frac{\partial K}{\partial \mathrm{Re}T_\alpha} W + \frac{\partial W}{\partial \mathrm{Re}T_\alpha} \;. \tag{5.42}$$

Let us next discuss the special case in which the superpotential is given generated by fluxes in F-theory (3.33) and non-perturbative effects are absent. We denote W_0 as the vacuum expectation value of the superpotential resulting after stabilizing the complex structure moduli. One then infers that for the Kähler potential (5.32) and Kähler coordinates (5.33) the F-term scalar potential (5.41) results in

$$\begin{aligned} V_F \;&=\; \alpha^2 \frac{3|W_0|^2}{\mathcal{V}_b^3} \Big(\tilde{\kappa}_2\, \mathcal{T}^b_\alpha\, v^\alpha_b \;+\; (768 - \tilde{\kappa}_2)\, \mathcal{Z}^b_\alpha\, v^\alpha_b \Big) \\ &=\; \frac{3|W_0|^2}{\mathcal{V}_b^3} \Big(\hat{\kappa}_2 \mathcal{W}^b_\alpha\, v^\alpha_b \;+\; 768\,\alpha^2\, \mathcal{Z}^b_\alpha\, v^\alpha_b \Big) \;, \quad \hat{\kappa}_2 = -3^3\alpha^2 \cdot (1+\alpha_2)\,\kappa_2 \;. \end{aligned} \tag{5.43}$$

Note that we cannot fix $\hat{\kappa}_2$ in this work. Furthermore, we have argued for a vanishing of the term proportional to $\kappa_3 \mathcal{Z}^b_\alpha \log \mathcal{V}^b$ in (5.33) due to an assumption on the F-theory uplift. As the uplift of this one-loop term remains elusive note that the pre-factor of the $\mathcal{Z}^b_\alpha$-correction in (6.34) might be subject to change. In the context of this work however it is suggested that the α'^2-correction breaks the no-scale structure as seen from (5.43). The presence of $\mathcal{T}_\alpha^{\mathbf{b}}$ in the $4d, \mathcal{N} = 1$ Kähler potential always lead to a breaking of the no-scale condition as can be inferred from (5.43). Let us emphasize that all corrections $\mathcal{Z}_\alpha^b$ and $\mathcal{W}_\alpha^b$ are of order α'^2 and are thus leading with respect to the well known Euler-characteristic correction [18]. Finally note that both contributions in (5.43) result from the α'-correction to the Kähler potential (5.32). The terms in the Kähler coordinates (5.33) admit a functional structure which remarkably never breaks the no-scale condition, i.e. thus (5.43) is independent of the additional parameters.

Let us close this section with two critical remarks. Firstly, the F-theory lift is performed by shrinking the fiber i.e. making the geometry singular and thus other higher-order corrections may become relevant. However, let us emphasize that all the corrections discussed in this work are of topological nature and thus are expected to be protected in the F-theory limit. Secondly, let us stress that we did not aim to prove the integration into $3d, \mathcal{N} = 2$ variables of the reduction result. However, we suggest an Ansatz for the Kähler coordinates and Kähler potential which allow to obtain all the higher-derivative couplings in the Kähler metric obtained by dimensional reduction from the l_{M}^6 eight-derivative couplings to eleven-dimensional supergravity. This is a necessary but not sufficient step, and it thus remains to ultimately decide the faith of the α'-corrections $\mathcal{W}_\alpha^b$ and $\mathcal{Z}_\alpha^b$.

6 Moduli Stabilisation

In this section we comment on the vacuum structure of the potential generated by the novel α'-correction (5.43). Furthermore, we study the interplay with the well known Euler-characteristic α'^3-correction to the Kähler potential

$$\xi = -g_s^{-\frac{3}{2}} (2\pi)^3 \frac{\zeta(3)}{4} \chi(CY_3) \quad , \tag{6.1}$$

with $\chi(CY_3)$ Euler-characteristic of CY_3. Note that it is of order $\mathcal{O}(\alpha'^3)$ and it depends on the Type IIB string coupling.[19] It is obtained from the parent $\mathcal{N} = 2$ theory arising from compactification of Type IIB on Calabi–Yau orientifolds [18, 48].[20] Note that Calabi-Yau threefold in IIB is the double cover of the base B_3 branched along the $O7$-plane. Thus in particular we find that $\chi(CY_3) = \chi(B_3)$. As we discuss intrinsic $\mathcal{N} = 1$ vacua in this work we continue in the terminology of F-theory. We comment on the potential F-theoretic origin of the correction (6.1) in

[19] The correction is known to depend on the dilaton $e^{-\Phi}$. We assume that the dilaton is stabilized by the flux background and we thus encounter the string coupling constant $g_s = \langle e^{\Phi} \rangle$.

[20] To compute the correction to the scalar potential resulting from (6.1) we use the Kähler potential and coordinates obtained in [48].

section 5.2. Note that the string coupling dependence of (6.1) makes it parametrically relevant although being sub leading in α' compared to (5.43).

Lastly let us close with a remark on the stability of the following scenarios in regard to higher-order corrections in α' and g_s in the light of [49]. The classical correction to the scalar potential vanishes due to the no-scale condition and thus the leading order g_s and α'-correction determine the vacuum. Higher-order α'-corrections are parametrically under control as one stabilizes the internal space at large volumes. Moreover the string coupling constant g_s may be achieved to be parametrically small thus higher-order string loop corrections can be safely neglected.

6.1 α'-stabilisation scenario for Minkowski vacua?

In this section we analyse minima of the four-dimensional scalar potential generated solely by the novel α'^2-correction $\mathcal{W}^b_\alpha$ given in (5.29). We study the simplified scenario in which neither the Euler-characteristic α'^3-correction (6.1) nor the α'^2-correction in (5.43) contributes, i.e. for the cases $\chi(B_3) = 0$ and $\mathcal{Z}^b_\alpha = 0$, respectively.[21] The correction takes the form

$$V_F^{new} \sim \frac{1}{\mathcal{V}_b^3} \mathcal{W}^b_\alpha v^\alpha_b \quad , \tag{6.2}$$

where we cannot fix the pre-factor in (6.2) and thus in particular its sign in the context of this work. The following scenario however remains applicable in either case of the pre-factor in (6.2) determined by $\hat{\kappa}_2$, as it leads to different conditions on the topological quantities. Let us next introduce some useful abbreviations. One may write

$$\mathcal{W}^b_\alpha = \frac{1}{\mathcal{K}^b_\alpha} \mathcal{S}_\alpha \mathcal{C}_\alpha \quad , \tag{6.3}$$

with the volume of the self-intersection curve $\mathcal{S}_\alpha$ and the volume of the intersection curve $\mathcal{C}_\alpha$ in between of $D7$-branes and $O7$-planes with the divisors

$$\mathcal{S}_\alpha = \int_{B_3} \omega^b_\alpha \wedge \omega^b_\alpha \wedge J^b \quad , \qquad \mathcal{C}_\alpha = 2\pi \int_{B_3} c_1(B_3) \wedge \omega^b_\alpha \wedge J^b \quad . \tag{6.4}$$

and furthermore $\mathcal{S}_\alpha = \mathcal{S}_{\alpha\beta} v^\beta_b$ and $\mathcal{C}_\alpha = \mathcal{C}_{\alpha\beta} v^\beta_b$, where one simply uses $J^b = v^\alpha_b\, \omega^b_\alpha$. Note that $\mathcal{C}_{\alpha\beta}$ is symmetric in its components whilst $\mathcal{S}_{\alpha\beta}$ is not.

We will argue that for a generic number of Kähler moduli there exist values for $\mathcal{S}_\alpha$ and $\mathcal{C}_\alpha$ such that all but one modulus, i.e. the overall volume is fixed in a Minkowski minimum with a remaining flat direction for $\mathcal{V}^b$. One may want to extend this results for $\chi(CY_3) \neq 0$ which then will fix all cycles up to scaling by the volume $\mathcal{V}^b$ and induce a runaway direction $\mathcal{V}^b \to \infty$ for $\chi(B_3) < 0$ and $\mathcal{V}^b \to 0$ for $\chi(B_3) > 0$. Thus one may furthermore use additional contributions to the scalar potential such as

[21] Note that alternatively one may stabilize the potential in a regime where the α^2-correction is subleasing due to its α'-suppression. In a weakly coupled string regime the Euler-characteristic correction is always expected to be relevant.

non-perturbative effects to the superpotential e.g. $M5$-brane instanton in F-theory, i.e. $D3$-brane instantons in IIB [30, 50] to stabilize the overall volume such that all moduli are fixed. We do not explicitly study the latter scenario in this work but rather analyse the intriguing structure of the potential generated by $\mathcal{W}^b_\alpha$. Let us emphasize that we won't study explicit geometries in this section where $\mathcal{S}_\alpha$ and $\mathcal{C}_\alpha$ are computed, but we instead provide generic conditions on those quantities such that the Minkowski vacua with one remaining flat direction is generated. It would be interesting in future work to construct explicit examples, i.e. elliptically fibered Calabi–Yau fourfold geometries over the base B_3 which exhibit this features.

The case $\mathbf{h^{1,1} = 3}$. Let us first show the claim for the case of three Kähler moduli of the base. The scalar potential (6.2) in particular $\mathcal{S}_\alpha \in \mathbb{Z}$ and $\mathcal{C}_\alpha \in \mathbb{Z}$ may be chosen such that the Minkowski minimum is located at

$$v_b^2 = \gamma_1 \, v_b^1 \quad , \quad v_b^3 = \gamma_2 \, v_b^1 \quad , \quad \text{with} \quad \gamma_1, \gamma_2 \in \mathbb{Q}^+_{>0} \quad , \tag{6.5}$$

where we consider the region $v_b^1 \geq 0$.[22] The self-intersection numbers are such that

$$\mathcal{S}_{\alpha\beta}\gamma^\beta = 0 \quad , \quad \forall \, \alpha = 1, 2, 3 \quad , \tag{6.6}$$

with $\gamma^\beta = (1, \gamma_1, \gamma_2)$ is a vector with three components, which may always be chosen such that (6.6) is satisfied as $\mathcal{S}_{\alpha\beta}$ admits $3 \times 3 = 9$ independent components.[23] Note that from (6.6) it follows that the potential (6.2) vanishes at the chosen coordinates (6.5) if the denominator depending on $\mathcal{K}^b_\alpha$ and $\mathcal{V}^b$ is not singular.[24] To actually obtain an extremum one fixes

$$\mathcal{C}_{\alpha\beta}\gamma^\beta = 0 \quad \forall \, \alpha \quad , \quad \text{and} \quad \mathcal{C}_{1\beta} \neq 0 \quad \forall \, \beta \quad , \tag{6.7}$$

and furthermore

$$\mathcal{C}_{22} = 0 \quad , \qquad \mathcal{C}_{33} = 0 \quad , \tag{6.8}$$

which implies that

$$\gamma_1 = -\frac{\mathcal{C}_{13}}{\mathcal{C}_{23}} \quad , \quad \gamma_2 = -\frac{\mathcal{C}_{12}}{\mathcal{C}_{23}} \quad , \quad \mathcal{C}_{11} = 2\frac{\mathcal{C}_{12}\mathcal{C}_{13}}{\mathcal{C}_{23}} \quad . \tag{6.9}$$

where we have used the symmetry of the tensor $\mathcal{C}_{\alpha\beta}$. Note that as we demanded the γ's to be strictly positive this implies

$$\mathcal{C}_{13}, \mathcal{C}_{12} > 0 \quad \& \quad \mathcal{C}_{23} < 0 \qquad \text{or} \qquad \mathcal{C}_{13}, \mathcal{C}_{12} < 0 \quad \& \quad \mathcal{C}_{23} > 0 \quad . \tag{6.10}$$

[22] Note that $\gamma_1, \gamma_2 \neq 0$.

[23] Let us stress again that in principle the geometric background fixes the components of $\mathcal{S}_{\alpha\beta}$ thus those cannot be chosen freely.

[24] Note that the constraints in this do not specify all intersection numbers of B_3 thus one is to check that the remaining intersection numbers are such that the $\mathcal{K}^b_\alpha$ and $\mathcal{V}^b$ are non-singular in the minimum defined by (6.5).

These conditions lead to an extremum of (6.2), which one can easily verify. To show that we have a minimum one analyses the determinant of the Hessian matrix which however computes to zero due to the remaining flat direction. To compute the eigenvalues of the Hessian matrix is straightforward and one can verify that the minimum will be obtained for many possible combinations of the remaining parameters. Additionally for a satisfactory vacuum we want the volume to be positive and large such that other higher-order α'-corrections are under control. Latter is trivially satisfied as due to the flat direction for the volume $\mathcal{V}^b$. The positivity needs to be verified in the explicit geometry by computing the topological quantities.

Generic moduli case. In this paragraph we generalize the argument for the three moduli case to the generic case. As before we only impose restrictions on $\mathcal{C}_{\alpha\beta}$ and $\mathcal{S}_{\alpha\beta}$ to obtain an extremum. The positivity of all eigenvalues of the Hessian matrix remains to be verified for concrete examples. We proceed analogously as before.

The potential in particular $\mathcal{S}_\alpha \in \mathbb{Z}$ and $\mathcal{C}_\alpha \in \mathbb{Z}$ may be chosen such that the Minkowski minimum is located at

$$v_b^\alpha = \gamma^\alpha\, v_b^1 \quad , \quad \gamma^\alpha \in (\mathbb{Q}^+_{>0})^{h^{1,1}(B_3)} \text{ with } \gamma^1 = 1 \ , \tag{6.11}$$

where we consider the region $v_b^1 \geq 0$. The self-intersection numbers are such that

$$\mathcal{S}_{\alpha\beta}\gamma^\beta = 0 \ , \ \forall\, \alpha = 1, \ldots, h^{1,1}(B_3) \ , \tag{6.12}$$

$\mathcal{S}_{\alpha\beta}$ admits $h^{1,1}(B_3) \times h^{1,1}(B_3)$ independent components.[25] Note that from (6.12) it follows that the potential (6.2) vanishes at the chosen coordinates (6.11), if the denominator depending on $\mathcal{K}_\alpha^b$ and $\mathcal{V}^b$ is not singular.[26] Furthermore, one constraints

$$\mathcal{C}_{1\beta} \neq 0 \quad \beta = 1, \ldots, h^{1,1}(B_3) \ , \tag{6.13}$$

which implies that for $h^{1,1}$ odd

$$\gamma^k = -\frac{\mathcal{C}_{1k}}{\mathcal{C}_{k\,k+1}} \quad , \quad \mathcal{C}_{k\,k+1}, \mathcal{C}_{1k} \neq 0 \quad , \quad k = 2, \ldots, h^{1,1}(B_3) \ , \tag{6.14}$$

and for $h^{1,1}$ even

$$\gamma^k = -\frac{\mathcal{C}_{1k}}{\mathcal{C}_{k\,k+1}} \quad , \ \mathcal{C}_{h^{1,1}(B_3)\,h^{1,1}(B_3)}, \mathcal{C}_{k\,k+1}, \mathcal{C}_{1k} \neq 0 \ , \ k = 2, \ldots, h^{1,1}(B_3) - 1 \ . \tag{6.15}$$

Moreover, all other $\mathcal{C}$'s are vanishing. Note that this in particular leads to

$$\mathcal{C}_{\alpha\beta}\gamma^\beta = 0 \ , \quad \forall\, \alpha \ . \tag{6.16}$$

[25]The geometric background fixes the components of $\mathcal{S}_{\alpha\beta}$.

[26]The totally symmetric tensor $\mathcal{K}^b_{\alpha\beta\gamma}$ admits $h^{1,1}(B_3)(h^{1,1}(B_3)+1)(h^{1,1}(B_3)+2)\,/\,3!$ independent components. The condition (6.12) only determines $h^{1,1}(B_3)^2$ components and thus the remaining intersection numbers can be found such that the $\mathcal{K}^b_\alpha$ and $\mathcal{V}^b$ are non-singular in the minimum defined by (6.11).

The conditions (6.13) -(6.16) lead to an extremum of (6.2), which one can easily verify. To see this note that now the scalar potential is a sum of fractions with simply functional dependence of the numerator being a product of to components both vanishing separately for (6.11). Thus upon taking partial derivatives and applying the chain rule one of the components always remains and assures vanishing of each term of the sum, respectively. The condition (6.13) assures that indeed all moduli except for one are fixed. Let us close this section by noting that other values of $\mathcal{S}_{\alpha\beta}$, $\mathcal{C}_{\alpha\beta}$ can exhibit the same behavior leading to a Minkowski minimum with a flat remaining direction of the overall volume $\mathcal{V}^b$. It would be interesting to see if the suggested conditions on the topological quantities can be obtained in explicit geometric backgrounds of elliptically fibered Calabi-Yau fourfolds with base B_3.

6.2 Extrema in the generic moduli case

In this section we discuss a scenario in which all Kähler moduli might be stabilized in a non-supersymmetric anti-de Sitter minimum for manifolds with $\chi(B_3) < 0$. We argue for a model independent extremum and provide a sufficient condition for the existence of a local minimum in generic geometric backgrounds such as e.g.

$$\langle \mathcal{K}^b_\alpha \rangle \langle \mathcal{K}^b_\alpha \rangle \quad > \quad -\tfrac{1}{2} \langle \mathcal{K}^b_{\alpha\alpha} \rangle \langle \mathcal{V}^b \rangle \quad , \quad \forall\, \alpha = 1, \ldots, h^{1,1}(B_3) \ . \tag{6.17}$$

However, to show that is a true local minimum further studies in explicit geometries are required. The stabilization is achieved by an interplay of the correction proportional to $\mathcal{Z}^b_\alpha$ and $\mathcal{W}^b_\alpha$ in (5.43), respectively, with the α'^3 Euler-characteristic correction (6.1) to the Kähler potential [18, 48]. We discuss the two cases in which only the $\mathcal{Z}^b_\alpha$ and $\mathcal{W}^b_\alpha$-correction is present, respectively. To achieve positivity of the four-cycle volumes in the vacuum the α'-corrections additionally need to obey strict positivity and negativity conditions, i.e. the geometric background must be suitable. The combined case is a straightforward generalization and relaxes the positivity and negativity constraints. Note that due to a similar potential all Kähler moduli may be stabilized for $\chi(B_3) > 0$ as discussed in [19]. Let us emphasize that the we do not require non-perturbative effects which are generically exponentially suppressed by the volume of the cycles. In future work [51] we study a modified scenario by additionally considering the $\alpha'^3 g_s^{-3/2}$-correction to the scalar potential discussd in [19, 20].

Stabilisation with the $\mathcal{Z}^b_\alpha$-correction. We henceforth assume that we consider geometries in which all self-intersection numbers are vanishing and thus the correction $\mathcal{W}^b_\alpha$ is identically zero. The resulting potential in the large volume limit then takes the form[27]

$$V_F = \frac{3|W_0|^2}{\mathcal{V}_b^3} \left(3\,\xi \; + 768\,\alpha^2\, \mathcal{Z}^b_\alpha\, v^\alpha_b \right) \ . \tag{6.18}$$

[27] We refer to the large volume limit to the regime at large volumes $\mathcal{V}^b$ and weak string coupling such that higher order α' and g_s-corrections can be neglected.

We note that the functional structure is similar to the α'-correction discussed in [19, 20]. One finds the AdS vacua where all four-cycle volumes $\mathcal{K}^b_\alpha$ are stabilized at

$$\langle \mathcal{K}^b_\alpha \rangle = -\,\Lambda^2\,\mathcal{Z}^b_\alpha \quad , \quad \text{with} \quad \Lambda = -4\,\xi \cdot \frac{1}{\mathcal{Z}^b_\alpha \langle v_0^\alpha \rangle} \quad , \tag{6.19}$$

where $\langle v_0^\alpha \rangle = \frac{1}{\Lambda} \langle v_b^\alpha \rangle$ is the expectation value of the fields such that

$$\mathcal{K}^b_{\alpha\beta\gamma} \langle v_0^\beta \rangle \langle v_0^\gamma \rangle = -\,\mathcal{Z}^b_\alpha \quad . \tag{6.20}$$

This scaling is required to ensure that $\mathcal{Z}^b_\alpha \langle v_b^\alpha \rangle = -4\,\xi$. In other words this additional condition can always be satisfied as one concludes from (6.20) which fixes $\langle v_0^\alpha \rangle$ uniquely, and thus implies (6.19). One infers the volume in the extremum to be

$$\langle \mathcal{V}^b \rangle = \frac{4\,\xi\,\Lambda^2}{3} \sim g_s^{-\frac{9}{2}} \quad , \tag{6.21}$$

and moreover that the value of the potential in the extremum takes the form

$$\langle V^F \rangle = -\frac{9\,\xi}{8} \cdot \frac{|W_0|^2}{\langle \mathcal{V}^b \rangle^3} \sim g_s^{12}\,|W_0|^2 \quad < 0 \quad . \tag{6.22}$$

Note that since $\chi(B_3) < 0$ one infers that $\xi > 0$. In the weakly coupled string regime $g_s < 1$ one generically achieves a large positive overall volume $\mathcal{V}^b > 0$ in (6.21). Moreover, positivity of all four-cycles volumes $\mathcal{K}^b_\alpha > 0$ for $\mathcal{Z}^b_\alpha < 0$ for all $\alpha = 1, \ldots, h^{1,1}(B_3)$ in (6.19) and (6.20).[28] From (6.22) one finds that one may achieve small values of $\langle V^F \rangle$ also for a moderately large $|W_0|$ due to the strong string coupling suppression. By analyzing the matrix of second derivatives in the extremum one infers

$$\left\langle \frac{\partial^2\, V_F}{\partial v_b^\alpha \partial v_b^\beta} \right\rangle = \frac{3|W_0|^2 \Lambda^2}{\langle \mathcal{V}_b \rangle^5} \Big(\gamma_1 \mathcal{K}^b_{\alpha\beta} + \gamma_2 \mathcal{Z}^b_\alpha \mathcal{Z}^b_\beta \Big) \quad , \quad \gamma_1 = \tfrac{3}{2}\xi^2 \quad , \quad \gamma_2 = \tfrac{9}{4}\xi\Lambda^2 \quad , \tag{6.23}$$

where one concludes that $\gamma_1 > 0$ and $\gamma_2 > 0$. The matrix $\gamma_2 \mathcal{Z}^b_\alpha \mathcal{Z}^b_\beta$ is positive semi-define, however it was argued in [52] that $\mathcal{K}^b_{\alpha\beta}$ is of signature $(1, h^{1,1}(B_3))$, i.e. it exhibits one positive eigenvalue in the direction of the vector $\langle v_b^\alpha \rangle$. Thus to argue for a local minimum one needs to analyse (6.23) in explicit models. One may rewrite (6.23) to be in the form

$$\left\langle \frac{\partial^2\, V_F}{\partial v_b^\alpha \partial v_b^\beta} \right\rangle = \frac{3^3\,|W_0|^2 \xi}{2^3 \langle \mathcal{V}^b \rangle^4} \Big(\tfrac{1}{\langle \mathcal{V}^b \rangle} \langle \mathcal{K}^b_\alpha \rangle \langle \mathcal{K}^b_\beta \rangle + \tfrac{1}{2} \langle \mathcal{K}^b_{\alpha\beta} \rangle \Big) \quad , \tag{6.24}$$

from which one infers a sufficient condition on the geometry for positive semi-definiteness of (6.24) and thus for the existence of a local minimum to be

$$\langle \mathcal{K}^b_\alpha \rangle \langle \mathcal{K}^b_\alpha \rangle \quad > \quad -\tfrac{1}{2} \langle \mathcal{K}^b_{\alpha\alpha} \rangle \langle \mathcal{V}^b \rangle \quad , \quad \forall\, \alpha = 1, \ldots, h^{1,1}(B_3) \quad . \tag{6.25}$$

[28] Note that the mechanism could also be applied for different sign of the pre-factor of the $\mathcal{Z}^b_\alpha$-correction in (6.34) and would then lead to $\mathcal{Z}^b_\alpha > 0$ for all $\alpha = 1, \ldots, h^{1,1}(B_3)$ with opposite overall sign in (6.19) and (6.20).

Note that in this paragraph we have assumed that the self-intersection numbers are vanishing to argue for the vanishing of the $\mathcal{W}^b_\alpha$ in the scalar potential (6.34). Thus (6.25) is automatically satisfied by the non-vanishing of all four-cycle volumes in the vacuum. Thus one encounters a local minimum for those geometries. Let us next compare the gravitino mass with the string and Kaluza-Klein scale [53] for which one finds that

$$m_S \sim \langle \mathcal{V}^b \rangle^{-\frac{1}{2}} \quad , \quad m_{KK} \sim \langle \mathcal{V}^b \rangle^{-\frac{2}{3}} \quad , \quad m_{3/2} \sim \frac{|W_0|}{\langle \mathcal{V}^b \rangle} \ . \tag{6.26}$$

Thus one infers by using (6.21) that

$$\frac{m_{3/2}}{m_S} \sim \frac{|W_0|}{2|\Lambda|}\sqrt{\frac{3}{\xi}} g_s^{\frac{9}{4}} \sim |W_0| \ , \quad \frac{m_{3/2}}{m_{KK}} \sim g_s^{\frac{3}{2}} |W_0| \ , \quad \frac{m_{KK}}{m_S} \sim g_s^{\frac{3}{4}} \ . \tag{6.27}$$

Thus for a weakly coupled string regime the hierarchies in (6.27) can be satisfied accordingly. Let us conclude that this mechanism might lead to a stabilization for all four-cycles for geometric backgrounds with $\chi(B_3) < 0$. This is achieved solely by an interplay of the Euler-Characteristic α'^3-correction [18] with the α^2-correction [16, 17]. As the volume can be stabilized at sufficiently large values higher-order α'-corrections are under control i.e. the vacuum may not be shifted.

Let us close this section with some remarks concerning the recent conjecture by [21] which in particular implies the absence of local de Sitter extrema in any controlled string theory set-up. Note that the discussion in this section can be performed analogously for $\chi(B_3) > 0$ which then leads to a de Sitter extremum as seen by equation (6.22) with $\xi < 0$. To achieve a positive overall volume $\mathcal{V}^b > 0$ in (6.21) and positivity of all four-cycles volumes $\mathcal{K}^b_\alpha > 0$ one infers that $\mathcal{Z}^b_\alpha > 0$ for all $\alpha = 1, \ldots, h^{1,1}(B_3)$ and the opposite overall sign choice in (6.19) and (6.20) which as well constitutes a solution. It would be interesting to study explicit geometries where $\mathcal{Z}^b_\alpha$ takes values such that de Sitter are obtained. Let us close this section by emphasizing that the scenario in this section might suffice as the starting point for a concrete counter example to the conjecture [21].

Stabilisation with the $\mathcal{W}^b_\alpha$-correction. In this paragraph we henceforth consider a regime in which the $\mathcal{Z}^b_\alpha$-correction in (5.43) may be neglected, e.g. geometries in which it results in zero. The resulting potential in the large volume limit then takes the form[29]

$$V_F = \frac{3|W_0|^2}{\mathcal{V}_b^3}\Big(3\,\xi \,+\, \hat{\kappa}_2 \mathcal{W}^{\mathbf{b}}_\alpha\, v^\alpha_b \Big) \ . \tag{6.28}$$

One finds the AdS vacua where all four-cycle volumes $\mathcal{K}^b_\alpha$ are stabilized at

$$\langle \mathcal{K}^b_\alpha \rangle = \pm\,\Lambda^2 \left\langle \tfrac{\partial}{\partial v^\alpha} \mathcal{W}^b \right\rangle \ , \quad \text{with} \quad \Lambda = -\frac{3^3\,\xi}{2^3\,\hat{\kappa}_2} \cdot \frac{1}{\langle \mathcal{W}^b_\alpha \rangle \langle v^\alpha_0 \rangle} \ , \tag{6.29}$$

[29] In a large volume regime higher order α'-corrections and non-perturbative effects can be safely neglected.

where we have defined $\mathcal{W}^b = \mathcal{W}^{\mathbf{b}}_\alpha v_b^\alpha$ and $\langle v_0^\alpha\rangle = \frac{1}{\Lambda}\langle v_b^\alpha\rangle$ is the expectation value of the fields such that

$$\mathcal{K}^b_{\alpha\beta\gamma}\langle v_0^\beta\rangle\langle v_0^\gamma\rangle = \pm\big\langle \tfrac{\partial}{\partial v^\alpha}\mathcal{W}^b\big\rangle \ . \tag{6.30}$$

Note that (6.30) is independent of Λ and thus constitutes an implicit definition for $\langle v_0^\alpha\rangle$.[30] Furthermore, (6.29) ensures that $\langle\mathcal{W}^b\rangle = -3^2\xi/(2^3\hat{\kappa}_2)$. The volume and potential in the extremum are given by

$$\langle\mathcal{V}^b\rangle = \mp\frac{9\,\xi\,\Lambda^2}{8\,\hat{\kappa}_2} \quad , \quad \langle V^F\rangle = -\frac{9}{8}\,\xi\,\frac{|W_0|^2}{\langle\mathcal{V}^b\rangle^3} \ . \tag{6.31}$$

For $\xi > 0$ one infers from (6.31) the "upper" sign choice in (6.29) and (6.30) if $\hat{\kappa}_2 < 0$ and the "lower" one if $\hat{\kappa}_2 > 0$. The positivity of the four-cycle volumes then imposes that $\big\langle\frac{\partial}{\partial v^\alpha}\mathcal{W}^b\big\rangle > 0$ and $\big\langle\frac{\partial}{\partial v^\alpha}\mathcal{W}^b\big\rangle < 0$, respectively. From computing the second partial derivative of the potential in the extremum one finds the sufficient condition for a minimum to be

$$\langle\mathcal{K}^b_\alpha\rangle\langle\mathcal{K}^b_\alpha\rangle \ \mp \tfrac{\Lambda^2}{2}\big\langle\tfrac{\partial}{\partial v^\alpha}\tfrac{\partial}{\partial v^\beta}\mathcal{W}^b\big\rangle\Big|_{\beta=\alpha} \ > \ -\tfrac{1}{2}\langle\mathcal{K}^b_{\alpha\alpha}\rangle\langle\mathcal{V}^b\rangle \ , \tag{6.32}$$

for all $\alpha = 1,\ldots,h^{1,1}(B_3)$.[31] Let us close by commenting on the case in which $\chi(B_3) > 0$. From (6.31) it is clear that in this case one finds a de Sitter extremum, which is likely to be a local maximum or a saddle point.

Stabilisation with the $\mathcal{Z}^b_\alpha$ and $\mathcal{W}^b_\alpha$-correction. This constitutes the generic case. The scalar potential in a large volume regime then takes the form

$$V_F = \frac{3|W_0|^2}{\mathcal{V}_b^3}\Big(3\,\xi \,+\, \hat{\kappa}_2\mathcal{W}^b_\alpha\, v_b^\alpha \ + 768\,\alpha^2\,\mathcal{Z}^b_\alpha\, v_b^\alpha\Big) \ . \tag{6.34}$$

In this case one finds the AdS vacua where all four-cycle volumes $\mathcal{K}^b_\alpha$ are stabilized at

$$\langle\mathcal{K}^b_\alpha\rangle = \pm\,\Lambda^2\Big(\big\langle\tfrac{\partial}{\partial v^\alpha}\mathcal{W}^b\big\rangle + \gamma\mathcal{Z}^b_\alpha\Big) \quad , \quad \text{with} \quad \Lambda = -\frac{3^3\,\xi}{8\ \hat{\kappa}_2}\cdot\frac{1}{\langle(\mathcal{W}^b_\alpha\rangle + \gamma\mathcal{Z}^b_\alpha)\langle v_0^\alpha\rangle} \ , \tag{6.35}$$

with $\gamma = \frac{3^3}{2^5\hat{\kappa}_2}$ and where yet again $\langle v_0^\alpha\rangle = \frac{1}{\Lambda}\langle v_b^\alpha\rangle$ is the expectation value of the fields such that

$$\mathcal{K}^b_{\alpha\beta\gamma}\langle v_0^\beta\rangle\langle v_0^\gamma\rangle = \pm\big\langle\tfrac{\partial}{\partial v^\alpha}\mathcal{W}^b\big\rangle \ \pm\gamma\mathcal{Z}^b_\alpha \ . \tag{6.36}$$

Note that (6.36) is independent of Λ and thus constitutes an implicit definition for $\langle v_0^\alpha\rangle$. Intriguingly the remainder of the discussion is exactly equivalent to previous

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[30] Due to the functional form of $\mathcal{W}^b_\alpha$ see (6.3) one expects that a solution for $\langle v_0^\alpha\rangle$ exists generically.

[31] One infers that $v_b^\alpha\frac{\partial}{\partial v^\alpha}\mathcal{W}^b_\beta = 0$ and thus one finds that

$$\langle v_b^\alpha\rangle\big\langle\tfrac{\partial}{\partial v^\alpha}\tfrac{\partial}{\partial v^\beta}\mathcal{W}^b\big\rangle\langle v_b^\beta\rangle \ = \ 0 \ . \tag{6.33}$$

case and one infers the volume and potential in the extremum are given by (6.31). From computing the second partial derivative of the potential in the extremum one finds the sufficient condition for a minimum to be (6.32). Note that the undetermined parameter $\hat{\kappa}_2$ in (6.35) and (6.36) leads to two different cases in which positive four-cycles volumes are achieved. Noteworthy, due to the interplay of the $\mathcal{Z}^b_\alpha$ and $\mathcal{W}^b_\alpha$-correction in (6.35) the strict positivity or negativity of the α'-correction can be relaxed in contrast to the previous cases (6.19) and (6.29). Furthermore, one concludes that in particular for $\chi(B_3) > 0$ from (6.31) one infers a de Sitter extremum. Lastly let us emphasize once more that it would be of great interest to construct explicit geometries in which the local anti-de Sitter minimum or de Sitter extremum is realized, respectively, due to the presence of the α'-corrections.

7 Conclusions

In this work we established a connection in between eight-derivative l_{M}^6-couplings in eleven-dimensional supergravity i.e. the low wave length limit of M-theory, and α'-corrections to the Kähler potential and Kähler coordinates of four-dimensional $\mathcal{N} = 1$ supergravity. The derivation relies on the M/F-theory duality. In particular we argue for two novel corrections to the Kähler coordinates and potential at order α'^2. Noteworthy one of them breaks the no-scale structure. However, we are not able to ultimately determine the faith of the proposed correction as a more complete analysis of the $3d, \mathcal{N} = 2$ variables needs to be performed. This work constitutes the foundation for such a future study. We provide the completion of the eleven-dimensional G^2R^3 and $(\nabla G)^2R^2$-sectors relevant four Calabi–Yau fourfold reductions. We suggest that it would be of great interest to match our proposal against 5-point and 6-point scattering amplitudes. Furthermore, we provide the reduction result of the G^2R^3 and $(\nabla G)^2R^2$-sectors for Calabi–Yau fourfolds with an arbitrary number of Kähler moduli.

One of the main achievements presented is the establishment of a divisor integral basis for the three-dimensional Kähler coordinates at higher-order in l_{M}. This allows us to derive the non-topological higher-derivative couplings obtained in the dimensional reduction from the novel Ansatz for the Kähler potential and Kähler coordinates. We suggest that in order to prove the integration into the proposed $3d, \mathcal{N} = 2$ variables additional non-trivial identifies relating the higher-derivative building blocks are required. Then this amounts to fixing the remaining parameters in our Ansatz. We are able to fix several parameters by ensuring compatibility with the one-modulus case in which the Kähler potential and Kähler coordinates can be determined exactly as no non-trivial higher-derivative couplings appear in the Kähler metric.

To connect the l_{M}-corrections in the three-dimensional Kähler coordinates and Kähler potential to the α'^2-corrections in the $4d, \mathcal{N} = 1$ theory we employ the classical well understood F-theory uplift. Although it is expected that a one-loop modification of the F-theory lift is needed we argue that in particular the novel $\mathcal{T}^b_\alpha$- contribution to the Kähler potential and coordinates is expected to remain untouched by such an extension. It would be interesting to perform a dimensional reduction of a generic $4d, \mathcal{N} = 1$ supergravity theory in particular with vector and chiral multiplets where the Kaluza Klein-modes are integrated out at one-loop extending the work of [43]. The novel divisor integral contribution in four-dimensions is of order α'^2. The α'^2-contribution to the Kähler potential moreover generically break the no-scale structure. Let us stress that the ultimate faith of the novel α'-corrections to the scalar potential shall be decided in a forthcoming work as the present result admits one free parameter. Let us continue with a critical remark. The F-theory lift is performed by shrinking the fiber of the Calabi–Yau fourfold, i.e. the geometry becomes singular in this process. In this limit other higher-order UV-corrections may become relevant and modify the uplift. However, the corrections discussed in this work are of topological nature and are thus expected to be protected in the F-theory limit.

Although not all parameters in the resulting α'-corrected scalar potential are fixed it is of interest to study possible scenarios to obtain stable vacua. We discuss two scenarios. One in which only the $\mathcal{W}^b_\alpha$-correction is present, in which all two-cycle volumes are fixed relative to each other in a Minkowski minima however a flat direction for the overall volume remains. We do not study explicit geometric Calabi–Yau fourfold backgrounds, but put generic constraints on the topological quantities of the base B_3. Secondly, we propose scenarios in which the $\mathcal{W}^b_\alpha$ and $\mathcal{Z}^b_\alpha$-correction at order α'^2 interplay with the α'^3 Euler-characteristic correction to achieve a model-independent non-supersymmetric an anti-de Sitter minimum for geometric backgrounds with $\chi(B_3) < 0$. Moreover constraints on the topological quantities of the geometric backgrounds are derived such that a minimum may be obtained. It would be of great interest to realize our constraints in explicit examples of elliptically fibered Calabi-Yau fourfolds. Furthermore, we note that the scenarios provide a model independent de Sitter extremum for geometric backgrounds with $\chi(B_3) > 0$. One may extend the present analysis [51] by additionally considering the α'^3-correction to the scalar potential discussd in [19, 20]. Lastly let us point the reader to an obvious extension of the present work. Our analysis of geometries does not allow for no-Abelian singularities, i.e. no non-Abelian four-dimensional gauge fields are present. It would be highly desirable to analyse the uplift of the Kähler potential and Kähler coordinates for such backgrounds.

Acknowledgements.
In particular, I am grateful to Andreas Braun, Thomas Grimm and Raffaele Savelli for enlightening discussions and comments on the draft. Moreover, I would like to

take this opportunity to express my gratitude to Sharon Law for being a beacon of light and for her constant kind support. Lastly, many thanks to the string theory group at the Seoul National University for its hospitality during my stay, where parts of this work were initiated. This work was supported by the WPI program of Japan.

A Conventions, definitions, and identities

In this work we denote the eleven-dimensional space indices by capital Latin letters $M, N = 0, \ldots, 10$ and the external ones by $\mu, \nu = 0, 1, 2$, and the internal complex ones by $m, n, p = 1, ..., 4$ and $\bar{m}, \bar{n}, \bar{p} = 1, \ldots, 4$. The metric signature of the eleven-dimensional space is $(-, +, \ldots, +)$. Furthermore, the convention for the totally anti-symmetric tensor in Lorentzian space in an orthonormal frame is $\epsilon_{012\ldots10} = \epsilon_{012} = +1$. The epsilon tensor in d dimensions then satisfies

$$\epsilon^{R_1 \cdots R_p N_1 \ldots N_{d-p}} \epsilon_{R_1 \ldots R_p M_1 \ldots M_{d-p}} = (-1)^s (d-p)! p! \delta^{N_1}{}_{[M_1} \ldots \delta^{N_{d-p}}{}_{M_{d-p}]} \, , \tag{A.1}$$

where $s = 0$ if the metric has Riemannian signature and $s = 1$ for a Lorentzian metric. We adopt the following conventions for the Christoffel symbols and Riemann tensor

$$\begin{aligned} \Gamma^R{}_{MN} &= \frac{1}{2} g^{RS} (\partial_M g_{NS} + \partial_N g_{MS} - \partial_S g_{MN}) \, , & R_{MN} &= R^R{}_{MRN} \, , \\ R^M{}_{NRS} &= \partial_R \Gamma^M{}_{SN} - \partial_S \Gamma^M{}_{RN} + \Gamma^M{}_{RT} \Gamma^T{}_{SN} - \Gamma^M{}_{ST} \Gamma^T{}_{RN} \, , & R &= R_{MN} g^{MN} \, , \end{aligned} \tag{A.2}$$

with equivalent definitions on the internal and external spaces. Written in components, the first and second Bianchi identity are

$$\begin{aligned} R^O{}_{PMN} + R^O{}_{MNP} + R^O{}_{NPM} &= 0 \\ (\nabla_L R)^O{}_{PMN} + (\nabla_M R)^O{}_{PNL} + (\nabla_N R)^O{}_{PLM} &= 0 \quad . \end{aligned} \tag{A.3}$$

Differential p-forms are expanded in a basis of differential one-forms as

$$\Lambda = \frac{1}{p!} \Lambda_{M_1 \ldots M_p} dx^{M_1} \wedge \ldots \wedge dx^{M_p} \quad . \tag{A.4}$$

The wedge product between a p-form $\Lambda^{(p)}$ and a q-form $\Lambda^{(q)}$ is given by

$$(\Lambda^{(p)} \wedge \Lambda^{(q)})_{M_1 \ldots M_{p+q}} = \frac{(p+q)!}{p! q!} \Lambda^{(p)}_{[M_1 \ldots M_p} \Lambda^{(q)}_{M_1 \ldots M_q]} \quad . \tag{A.5}$$

Furthermore, the exterior derivative on a p-form Λ results in

$$(d\Lambda)_{N M_1 \ldots M_p} = (p+1) \partial_{[N} \Lambda_{M_1 \ldots M_p]} \quad , \tag{A.6}$$

while the Hodge star of p-form Λ in d real coordinates is given by

$$(*_d\Lambda)_{N_1\ldots N_{d-p}} = \frac{1}{p!}\Lambda^{M_1\ldots M_p}\epsilon_{M_1\ldots M_p N_1\ldots N_{d-p}} \quad . \tag{A.7}$$

Moreover,

$$\Lambda^{(1)} \wedge *\Lambda^{(2)} = \frac{1}{p!}\Lambda^{(1)}_{M_1\ldots M_p}\Lambda^{(2)M_1\ldots M_p}*_1 \quad , \tag{A.8}$$

which holds for two arbitrary p-forms $\Lambda^{(1)}$ and $\Lambda^{(2)}$.

Let us next define the intersection numbers, where $\{\omega_i\}$ are harmonic w.r.t. to the Calabi- Yau metric $g_{m\bar{n}}$

$$\begin{aligned} &\mathcal{K}_{ijkl} = \int_X \omega_i \wedge \omega_j \wedge \omega_k \wedge \omega_l \,, \qquad \mathcal{K}_{ij} = \mathcal{K}_{ijkl}v^l \,, \qquad\qquad \mathcal{K}_{ij} = \frac{1}{2}\mathcal{K}_{ijkl}v^k v^l, \\ &\quad \mathcal{K}_i = \frac{1}{3!}\mathcal{K}_{ijkl}v^j v^k v^l \,, \qquad\qquad \mathcal{V} = \frac{1}{4!}\mathcal{K}_{ijkl}v^i v^j v^k v^l \,. \end{aligned} \tag{A.9}$$

Let us review well known identities such as

$$\int \omega_i \wedge *_8\omega_j = -\mathcal{K}_{ij} + \frac{1}{\mathcal{V}}\mathcal{K}_i\mathcal{K}_j \quad . \tag{A.10}$$

Let us note that the intersection numbers obey the properties

$$\begin{aligned} \mathcal{K}_i v^i = 4\mathcal{V} \quad &, \quad \mathcal{K}_{ij}v^j = 3\mathcal{K}_j \quad , \quad \mathcal{K}_{ijk}v^k = 2\mathcal{K}_{ij} \\ \mathcal{K}_{ijkl}v^l = \mathcal{K}_{ijk} \quad &, \quad \mathcal{K}^{ik}\mathcal{K}_{jk} = \delta^i_j \quad , \quad \mathcal{K}^{ik}\mathcal{K}_k = \tfrac{1}{3}v^i \\ \Big(\frac{\partial}{\partial v^k}\mathcal{K}^{ij}\Big)\mathcal{K}_j = -\tfrac{2}{3}\delta^i_k \quad &, \quad \Big(\frac{\partial}{\partial v^k}\mathcal{K}^{ij}\Big)\mathcal{K}_{jl} = -\mathcal{K}^{ij}\mathcal{K}_{kjl} \quad , \end{aligned} \tag{A.11}$$

with the inverse intersection matrix $\mathcal{K}^{ij}$. The intersection numbers for the Kähler base are given by

$$\begin{aligned} &\mathcal{K}^b_{\alpha\beta\gamma} = \int_{B_3} \omega_\alpha \wedge \omega_\beta \wedge \omega_\gamma \,, \qquad \mathcal{K}_{\alpha^b\beta} = \mathcal{K}^b_{\alpha\beta\gamma}v^\gamma_b \,, \qquad \mathcal{K}^b_\alpha = \frac{1}{2}\mathcal{K}^b_{\alpha\beta\gamma}v^\beta_b v^\gamma_b, \\ &\quad \mathcal{V}^b = \frac{1}{3!}\mathcal{K}^b_{\alpha\beta\gamma}v^\alpha_b v^\beta_b v^\gamma_b \,. \end{aligned} \tag{A.12}$$

One may show that for a six-dimensional Kähler manifold

$$*_6\,(\omega^b_\alpha \wedge J^b) = \frac{\mathcal{K}^b_\alpha}{\mathcal{V}^b}J^b - \omega^b_\alpha \quad . \tag{A.13}$$

with intersection numbers defined analogously to (A.9). In particular, this implies the analogous relation

$$\int_{D_\alpha} \tilde{*}_6(H\tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_2 \;= \frac{1}{\mathcal{K}_\alpha}\int_{D_\alpha} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_\alpha} \tilde{c}_2 \wedge \tilde{J} - \int_{D_\alpha} \tilde{c}_1 \wedge \tilde{c}_2 \quad , \tag{A.14}$$

which holds due to the harmonicity of $H\tilde{c}_1(D_\alpha)$.

We define the curvature two-form for Hermitian manifolds to be

$$\mathcal{R}^m{}_n = R^m{}_{nr\bar{s}} dz^r \wedge d\bar{z}^{\bar{s}} \quad , \tag{A.15}$$

and

$$\begin{aligned} \text{Tr}\mathcal{R} \;&= R^m{}_{mr\bar{s}} dz^r \wedge d\bar{z}^{\bar{s}} \,, \\ \text{Tr}\mathcal{R}^2 &= R^m{}_{nr\bar{s}} R^n{}_{mr_1\bar{s}_1} dz^r \wedge d\bar{z}^{\bar{s}} \wedge dz^{r_1} \wedge d\bar{z}^{\bar{s}_1} \,, \\ \text{Tr}\mathcal{R}^3 &= R^m{}_{nr\bar{s}} R^n{}_{n_1r_1\bar{s}_1} R^{n_1}{}_{mr_2\bar{s}_2} dz^r \wedge d\bar{z}^{\bar{s}} \wedge dz^{r_1} \wedge d\bar{z}^{\bar{s}_1} \wedge dz^{r_2} \wedge d\bar{z}^{\bar{s}_2} \,. \end{aligned} \tag{A.16}$$

The Chern forms can be expressed in terms of the curvature two-form as

$$\begin{aligned} c_1 &= i\text{Tr}\mathcal{R} \,, \\ c_2 &= \frac{1}{2}\left(\text{Tr}\mathcal{R}^2 - (\text{Tr}\mathcal{R})^2\right) \,, \\ c_3 &= \frac{1}{3}c_1c_2 + \frac{1}{3}c_1 \wedge \text{Tr}\mathcal{R}^2 - \frac{i}{3}\text{Tr}\mathcal{R}^3 \,, \\ c_4 &= \frac{1}{24}\left(c_1^4 - 6c_1^2\text{Tr}\mathcal{R}^2 - 8ic_1\text{Tr}\mathcal{R}^3\right) + \frac{1}{8}((\text{Tr}\mathcal{R}^2)^2 - 2\text{Tr}\mathcal{R}^4) \,. \end{aligned} \tag{A.17}$$

The Chern classes of a n complex-dimensional Calabi-Yau manifold CY_n reduce to

$$c_3(CY_{n\geq 3}) = -\frac{i}{3}\text{Tr}\mathcal{R}^3 \quad \text{and} \quad c_4(CY_{n\geq 4}) = \frac{1}{8}((\text{Tr}\mathcal{R}^2)^2 - 2\text{Tr}\mathcal{R}^4) \,, \tag{A.18}$$

with $\text{Tr}\mathcal{R}^4$ defined analogous as in (A.16). Let us next define a set of higher-derivative building blocks identified in [13] as

$$Z_{m\bar{m}n\bar{n}} = \frac{1}{4!}\epsilon^{(0)}_{m\bar{m}m_1\bar{m}_1m_2\bar{m}_2m_3\bar{m}_3}\epsilon^{(0)}_{n\bar{n}n_1\bar{n}_1n_2\bar{n}_2n_3\bar{n}_3} R^{(0)\bar{m}_1m_1\bar{n}_1n_1} R^{(0)\bar{m}_2m_2\bar{n}_2n_2} R^{(0)\bar{m}_3m_3\bar{n}_3n_3} \,, \tag{A.19}$$

and

$$\begin{aligned} Y_{ijm\bar{n}} = \frac{1}{4!}\epsilon^{(0)}_{m\bar{m}m_1\bar{m}_1m_2\bar{m}_2m_3\bar{m}_3}\epsilon^{(0)}_{n\bar{n}n_1\bar{n}_1n_2\bar{n}_2n_3\bar{n}_3} \nabla^{(0)n}\omega_i^{(0)\bar{m}_1m_1}\nabla^{(0)\bar{m}}\omega_j^{(0)\bar{n}_1n_1} \\ \times R^{(0)\bar{m}_2m_2\bar{n}_2n_2} R^{(0)\bar{m}_3m_3\bar{n}_3n_3} \,. \end{aligned} \tag{A.20}$$

It turns out that the tensor $Z_{m\bar{m}n\bar{n}}$ given in (A.19) plays a central role in the following and is related to the key topological quantities on Y_4. It satisfies the identities

$$Z_{m\bar{m}n\bar{n}} = Z_{n\bar{m}m\bar{n}} = Z_{m\bar{n}n\bar{m}} \,, \qquad \nabla^{(0)m}Z_{m\bar{m}n\bar{n}} = \nabla^{(0)\bar{m}}Z_{m\bar{m}n\bar{n}} = 0 \,. \tag{A.21}$$

It is related to the third Chern-form $c_3^{(0)}$ via

$$Z_{m\bar{m}} = i2Z_{m\bar{m}n}{}^n = (2\pi)^3\frac{1}{2}(*^{(0)}c_3^{(0)})_{m\bar{m}} \,,$$

$$Z = i2Z_m{}^m = (2\pi)^3 *^{(0)} (J^{(0)} \wedge c_3^{(0)}) \,, \quad (2\pi)^3 *^{(0)} (c_3^{(0)} \wedge \omega_i^{(0)}) = -2 Z_{m\bar{n}} \omega_i^{(0)\bar{n}m} \,, \tag{A.22}$$

and yields the fourth Chern-form $c_4^{(0)}$ by contraction with the Riemann tensor as

$$Z_{m\bar{m}n\bar{n}} R^{(0)\bar{m}m\bar{n}n} = (2\pi)^4 *^{(0)} c_4^{(0)} \,. \tag{A.23}$$

We note that $Y_{ijm\bar{n}}$ is also related to $Z_{m\bar{m}n\bar{n}}$ upon integration as

$$\int_{Y_4} Y_{ijm}{}^m *^{(0)} 1 = -\frac{1}{6} \int_{Y_4} (i Z_{m\bar{n}} \omega_i^{(0)\bar{r}m} \omega_j^{(0)\bar{n}}{}_{\bar{r}} + 2 Z_{m\bar{n}r\bar{s}} \omega_i^{(0)\bar{n}m} \omega_j^{(0)\bar{s}r}) *^{(0)} 1 \,, \tag{A.24}$$

where the right hand side represents the same linear combination that will be relevant in 4.1. Let us for further use define

$$\mathcal{Y}_{ij} := \int_{Y_4} Y_{ijm}{}^m *^{(0)} 1 \;\; . \tag{A.25}$$

Lastly in this work we encounter a new (2,2)-form object

$$\Omega_{ij} = R^{(0)}_{m\bar{n}r\bar{s}} \omega_i^{(0)r}{}_t \omega_j^{(0)\bar{s}}{}_{\bar{u}} \;\; dz^m \wedge dz^t \wedge d\bar{z}^{\bar{n}} \wedge d\bar{z}^{\bar{u}} \;\; . \tag{A.26}$$

A.1 Divisor integrals in terms of CY_4 integrals

We define an arbitrary basis of higher-derivative $(1,1)$ -forms convenient for the computations in this work

$$\begin{aligned}
X_1 &= R_m{}^{m_2}{}_{m_5}{}^{n_2} R_{m_2}{}^{n_3}{}_{n_2}{}^{n_4} R_{n_3\bar{m}n_4}{}^{m_5} \;\; dz^m \wedge d\bar{z}^{\bar{m}} \\
X_2 &= R_m{}^{m_2}{}_{m_5}{}^{n_2} R_{m_2\bar{m}n_3}{}^{n_4} R_{n_2}{}^{m_5}{}_{n_4}{}^{n_3} \;\; dz^m \wedge d\bar{z}^{\bar{m}} \\
X_3 &= R_{m\bar{m}m_2}{}^{m_5} R_{m_5}{}^{n_2}{}_{n_3}{}^{n_4} R_{n_2}{}^{m_2}{}_{n_4}{}^{n_3} \;\; dz^m \wedge d\bar{z}^{\bar{m}} \\
X_4 &= g_{m\bar{m}} R_{m_1}{}^{m_2}{}_{m_5}{}^{n_2} R_{m_2}{}^{n_3}{}_{n_2}{}^{n_4} R_{n_3}{}^{m_1}{}_{n_4}{}^{m_5} \;\; dz^m \wedge d\bar{z}^{\bar{m}} \\
X_5 &= g_{m\bar{m}} R_{m_1}{}^{m_2}{}_{m_5}{}^{n_2} R_{m_2}{}^{m_1}{}_{n_3}{}^{n_4} R_{n_2}{}^{m_5}{}_{n_4}{}^{n_3} \;\; dz^m \wedge d\bar{z}^{\bar{m}}
\end{aligned} \tag{A.27}$$

These $(1,1)$-forms can be expressed as integrals on Calabi–Yau fourfolds which admit an interpretation as integrals on divisors D_i of a Calabi–Yau fourfold as

$$\int_{CY_4} \big(*_8 \, X_{k=1,..,5} \big) \wedge \omega_i = \int_{D_i} *_8 X_{k=1,..,5} \;\; , \tag{A.28}$$

where the r.h.s. is to be seen as pulled back to the divisor. Let us now recall the fact [38] that any complex sub-manifold of a Kähler manifold M is itself Kähler with induced metric and Kähler form g, J of M. Thus in particular we find for the Divisors $i : D_i \hookrightarrow CY_4$ the Kähler metric and form $^*ig$ and $^*iJ$, respectively, which are pulled back from the Calabi–Yau fourfold. One may thus as well restrict Riemann tensors on the Calabi–Yau fourfold to divisors D_i expressed by the induced metric which generically obeys $c_1(D_i) \neq 0$. In particular contractions of the Riemann tensors which do not vanish on the Calabi–Yau manifold due to the Calabi–Yau conditions may be pulled back to the divisors and expressed in terms of Riemann tensors in terms of the induced metric on D_i. Note that the $(1,1)$-forms in (A.27) expressed as integrals on divisors (A.28) in are of this form. By

We may write the Kähler coordinates as (4.4) in terms as the new basis on CY_4 in the following way if the coefficients obey the following relations

$$\begin{aligned} \alpha_5 &= -\tfrac{1}{8}\alpha_1 + \tfrac{1}{24}\alpha_3 + \tfrac{1}{4}\alpha_4 \ , \\ \alpha_6 &= \tfrac{1}{2}\alpha_2 + \tfrac{1}{2}\alpha_3 \ , \\ \alpha_7 &= \alpha_2 + \alpha_3 \ , \\ \alpha_8 &= \tfrac{1}{2}\alpha_1 - \tfrac{1}{3}\alpha_3 - \alpha_4 \ , \\ \alpha_9 &= -\alpha_1 + \tfrac{1}{6}\alpha_3 \ . \end{aligned} \tag{A.29}$$

one then infers that

$$\begin{aligned} \mathcal{T}_i = -\frac{i}{3}\int_{CY_4} \omega_i \wedge *_8\Big(&(\alpha_3 + 3\gamma_2 + 6\gamma_3)X_1 + (\alpha_3 + 3\gamma_1 - 12\gamma_3)X_2 + 3(\gamma_3 - \alpha_3)X_3 \\ &+ 3(-\gamma_2 + \gamma_4 + \gamma_5)X_4 - 3(\gamma_1 + 2\gamma_4 + 2\gamma_5)X_5\Big) \ , \end{aligned} \tag{A.30}$$

where $X_{i=1,2,3,4,5}$ are defined in (A.27), and where the freedom in the real parameters $\gamma_1, \ldots, \gamma_5$ results due to total derivatives which take different form on the divisors integrals and Calabi–Yau fourfold integrals, respectively. The simplest choice in this work for coefficients $\gamma_1, \gamma_2, \gamma_3$ defines the higher-derivative $(3,3)$-form to be

$$\mathcal{X} = -\frac{i}{3} *_8 \big(X_1 + X_2 - X_3\big) \ , \tag{A.31}$$

and thus the Kähler coordinate modification is

$$\mathcal{T}_i = -\frac{i}{3}\int_{CY_4} \omega_i \wedge *_8\big(X_1 + X_2 - X_3\big) \ . \tag{A.32}$$

Note that $\alpha_3 = 1$ which is in agreement with the divisor integral one-modulus limit.[32] Note that the choice of fixing α_1 does not limit the Ansatz for the Kähler coordinates as it amounts only to an overall coefficients which is anyway taken into account for in (4.3). One may easily show that thus

$$\mathcal{T}_i v^i = \int_{CY_4} J \wedge \mathcal{X} = \mathcal{Z} \ . \tag{A.33}$$

and from this property (A.33) that

$$\begin{aligned} \Big(\frac{\partial}{\partial v^j}\mathcal{T}_i\Big)v^i &= -\mathcal{T}_j + \mathcal{Z}_j \\ \Big(\frac{\partial^2}{\partial v^k \partial v^j}\mathcal{T}_i\Big)v^i &= -\Big(\frac{\partial}{\partial v^j}\mathcal{T}_k\Big) - \Big(\frac{\partial}{\partial v^k}\mathcal{T}_j\Big) \ . \end{aligned} \tag{A.34}$$

[32] The coefficients in (A.30) are chosen as $\gamma_1 = 8/3$, $\gamma_2 = -4/3$, $\gamma_3 = 2/3$, $\gamma_5 = -4/3 - \gamma_4$.

Let us comment on (A.31). the combination of basis elements $X_{i=1,2,3,4,5}$ is a choice compatible with the match to six-dimensional divisor integrals. In section A.3 we discuss the variation of $\mathcal{T}_i$ w.r.t. to the Kähler deformations.

As the matching of the correction to the Kähler coordinates in terms of CY_4 integrals to the divisor integral expression is not unique, let us close this section on remarks other possible choices of $\gamma_1, \gamma_2, \gamma_3$. Due to (A.33) the Ansatz (4.2) and (4.3) cannot depend separately on $\mathcal{T}_i v^i$. It is interesting to study the possible where (4.2) is modified by this expression as well and (4.3) by $\frac{1}{\mathcal{V}}\mathcal{K}_i\mathcal{T}_j v^j$. Let us close this section by discussing a caveat to the Ansatz in this work namely that our choice for $\mathcal{T}_i$ (A.31) may be rewritten by splitting integrals using the harmonicity of ω_i

$$\mathcal{T}_i = \tfrac{1}{3}\big(- \mathcal{Z}_i + \tfrac{1}{\mathcal{V}}\mathcal{K}_i\mathcal{Z}\big) \ . \tag{A.35}$$

Let us emphasize that the insights of this work is that the higher-derivative structures derived in dimensional Calabi–Yau fourfold reductions for $h^{1,1} > 1$ can be obtained by variation of $\mathcal{T}_i$ before applying the integral split (A.35) which suggests an interpretation in terms of divisor integrals. One infers that by imposing (A.35) first the Ansatz for the Kähler coordinates (4.3) does not carry any new information, i.e. those to steps seem not to commute. However, by choosing a more involved combination for the correction to the Kähler coordinate in terms of Calabi–Yau fourfold quantities in (A.30) this caveat can be prevented as then no analogous relation for (A.35) holds. Generically we expect the form $\mathcal{T}_i + \mathcal{T}_i^0$ where in the one-modulus limit $\mathcal{T}_i \to \tilde{\mathcal{Z}}$ and $\mathcal{T}_i^0 \to 0$. This suggests that one might need to extend the basis (A.27) to also contain terms with explicit covariant derivatives such as e.g. $\sim (\nabla R)^2$. Moreover, one may not expect to capture the information of topological quantities of divisors entirely by local covariant integral densities on the entire space but may need to include additional global obstructions to succeed in the matching.

A.2 3d Kähler coordinates as topological divisor integrals

In this section we argue that the Ansatz for the Kähler coordinates (4.4) may be rewritten in terms of topological integrals by fixing the coefficients in the Ansatz. Any closed form on such as $\tilde{c}_1$ may be written in terms of its harmonic part plus a double exact contribution

$$\tilde{c}_1 = H\tilde{c}_1 + \partial\bar{\partial}\lambda \ , \tag{A.36}$$

where λ is a function on the divisor. From the closure of $\tilde{c}_1$ we infer that

$$\nabla_{[m}\tilde{R}_{n]\bar{n}r}{}^{r} = 0 \ . \tag{A.37}$$

But equivalently one may use that

$$\begin{aligned}\nabla_m \tilde{R}_n{}^n{}_r{}^r &= \nabla_m \nabla_n \nabla^n \lambda \ , \\ \nabla^m \tilde{R}_n{}^n{}_r{}^r &= \nabla^m \nabla_n \nabla^n \lambda \ , \\ \nabla_m \tilde{R}_n{}^m{}_r{}^r &= \nabla_m \nabla_n \nabla^m \lambda \ , \\ \nabla^m \tilde{R}_m{}^n{}_r{}^r &= \nabla^m \nabla_n \nabla^m \lambda \ . \end{aligned} \tag{A.38}$$

Using the above set of equations one may show that the Ansatz for the Kähler coordinates (4.4) can be written as

$$\begin{aligned}\mathcal{T}_i = \ & \alpha_1 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{c}_1 + \alpha_2 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_2 \ + \alpha_3 \int_{D_i} \tilde{c}_3 + \frac{\alpha_4}{\mathcal{K}_i} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{J} \\ & + \frac{\alpha_5}{\mathcal{K}_i^2} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 + \frac{\alpha_6}{\mathcal{K}_i} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_2 \wedge \tilde{J} \\ & + 2\alpha_6 \int_{D_i} \tilde{*}_6 (H \tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_2 - \big(2\alpha_4 + 8\alpha_5\big) \int_{D_i} \tilde{*}_6 (H \tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_1 \wedge \tilde{c}_1 \\ & - \frac{4\alpha_5}{\mathcal{K}_i} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_1 \wedge *_6 H \tilde{c}_1 \ , \end{aligned} \tag{A.39}$$

where $\mathcal{K}_i$ denotes the volume of the divisor D_i. Note that in order to obtain (4.18) one fixes the coefficients such that

$$\alpha_7 = 2\alpha_6 \ , \quad \alpha_8 = 2\alpha_4 + 8\alpha_5 \ , \quad \alpha_9 = -4\alpha_5 \ . \tag{A.40}$$

Additionally requiring that we can write $\mathcal{T}_i$ as integrals on the Calabi–Yau fourfold i.e. the constraints (4.8) then imposes

$$\begin{aligned} &\alpha_1 = \tfrac{1}{6} \ , && \alpha_3 = 1 \ , && \alpha_4 = -\tfrac{1}{12} \ , && \alpha_5 = 0 \ , \\ &\alpha_6 = \tfrac{1}{2} + \tfrac{1}{2}\alpha_2 \ , && \alpha_7 = 1 + \alpha_2 \ , && \alpha_8 = -\tfrac{1}{6} \ , && \alpha_9 = 0 \ . \end{aligned} \tag{A.41}$$

Note that this coordinate (A.41) depends on the free parameter α_2. It would be interesting to determine it by imposing some other constraint.

A.3 Variation w.r.t. Kähler moduli fields

To compute the variation of covariant integral densities such as (3.12) w.r.t. Kähler moduli fields we deform the Calabi–Yau fourfold metric $g_{m\bar{n}}$ in complex coordinates by

$$g_{m\bar{n}} \to g_{m\bar{n}} + i\delta v^i \omega_{im\bar{n}} \quad \text{and} \quad g^{\bar{n}m} \to g^{\bar{n}m} - i\delta v^i \omega_i^{\bar{n}m} \ . \tag{A.42}$$

The determinant of the metric subject to (A.42) derives to

$$\sqrt{-g} \to \sqrt{-g} + i\sqrt{-g}\, v^i \omega_{im}{}^m \ . \tag{A.43}$$

Note that we are only interested in linear deformations here thus we need to expand the expression to $\mathcal{O}(\delta v^i)$. The Riemann tensors variation compute to

$$R_{m\bar{m}n\bar{n}} \to R_{m\bar{m}n\bar{n}} + i\delta v^i \nabla_m \nabla_{\bar{m}} \omega_{i\bar{n}n} + \tfrac{i}{2}\delta v^i R_{m\bar{n}n}{}^r \omega_{ir\bar{m}} + \tfrac{i}{2}\delta v^i R_{m\bar{m}n}{}^r \omega_{ir\bar{n}} \quad . \tag{A.44}$$

To evaluate the variation of higher-derivative object a computer algebra package such as xTensor [40] is highly desirable. One may employ its power to generate a complete set of Shouten identities, Bianchi identities and total derivatives to show that the variation of (A.32) can be written as

$$\frac{\partial}{\partial v^j}\mathcal{T}_i = \frac{1}{\mathcal{V}}\mathcal{K}_i\mathcal{T}_j - \frac{2}{\mathcal{V}}\mathcal{K}_j\mathcal{T}_i + 4\,\mathcal{T}_{ij} + \Lambda_{ij} \quad , \tag{A.45}$$

where

$$\mathcal{T}_{ij} = \int_{CY_4} *_8\big(\omega_i \wedge \omega_j \wedge J\big) \wedge \mathcal{X} \quad , \tag{A.46}$$

and

$$\begin{aligned} \Lambda_{ij} &= 4i \int_{CY_4} Z_{m\bar{n}} \omega_{(i}{}^{\bar{n}s} \omega_{j)s}{}^m * 1 + 8i \int_{CY_4} Z_{m\bar{n}} \omega_i{}^{\bar{m}n} \omega_{js}{}^s * 1 \\ &= 2i \int_{CY_4} Z_{m\bar{n}} \omega_i{}^{\bar{n}s} \omega_{js}{}^m * 1 + 4\frac{1}{\mathcal{V}}\mathcal{Z}_i\mathcal{K}_j \quad . \end{aligned} \tag{A.47}$$

Let us stress that in order to compute (4.14) we make extensive use of the computer algebra package [40], and a non-publicly self-developed extension for complex manifolds and tools to perform the above computation. By using the relation

$$\mathcal{Y}_{ij} = -\frac{1}{6}\int_{Y_4} (iZ_{m\bar{n}}\omega_i{}^{\bar{r}m}\omega_j{}^{\bar{n}}{}_{\bar{r}} + 2Z_{m\bar{n}r\bar{s}}\omega_i{}^{\bar{n}m}\omega_j{}^{\bar{s}r}) * 1\,, \tag{A.48}$$

We note in section (A.1) that the we are not able to fix $\mathcal{T}_i$ precisely in this work. Thus let us present here the variation of a different possible choice of the parameter freedom in (A.30) which one may show then leads to analogous expression as (A.45). It is intriguing to note that one can obtain also the novel higher-derivative structure in (3.21) by variation of the alternative Kähler coordinates

$$\frac{\partial}{\partial v^j}\mathcal{T}_i^{alt} \supset \int_{CY_4} c_2 \wedge \Omega_{ij} + \int_{CY_4} c_2 \wedge J \wedge \Omega^1_{ij} + \int_{CY_4} c_2 \wedge J \wedge \Omega^2_{ij} \tag{A.49}$$

with Ω_{ij} defined in (A.26) and with the $(1,1)$-forms

$$\Omega^1_{ij\,m\bar{n}} := \big(\nabla_m\nabla_{\bar{n}}\omega_{ir\bar{s}}\big)\omega_j{}^{\bar{s}r} \qquad \Omega^2_{ij\,m\bar{n}} := \big(\nabla_r\nabla^r\omega_{im\bar{s}}\big)\omega_j{}^{\bar{s}}{}_{\bar{n}} \quad . \tag{A.50}$$

Note that the second Chern-form c_2 appears in this case (A.49) in particular in the combination as in (3.21). Note that (A.49) is of schematic form and we do not specify the factors in this work.

Warp-factor and the Kähler potential. Let us next review the integration of the warp factor into a Kähler potential following [12]. To begin with, let us reduce our Ansatz (4.3) and (4.2) to the warp factor related quantities which gives

$$K = -3\log\Big(\mathcal{V} + 4\alpha^2 \mathscr{W}_i v^i\Big) \tag{A.51}$$

We therefore suggest that they take the form

$$\mathrm{Re}T_i = \mathcal{K}_i + \alpha^2\Big(\mathscr{F}_i + 3\mathscr{W}_i\Big) \tag{A.52}$$

where D_i are $h^{1,1}(Y_4)$ divisors of Y_4 that span the homology $H_2(Y_4,\mathbb{R})$. The six-form F_6 in this expression is a function of degrees of freedom associated with the internal space metric. It is constrained by a relation to the fourth Chern form c_4 such that F_6 determines the non harmonic part of c_4 as

$$c_4 = Hc_4 + i\partial\bar{\partial}F_6 \ . \tag{A.53}$$

Note that (A.53) leaves the harmonic and exact part of F_6 unfixed and we will discuss constraints on these pieces in more detail below. The justification of the first term in $\mathrm{Re}T_i$ is simpler. Remarkably, this definition of the Kähler coordinates as D_i integrals will help us to obtain the couplings $\int e^{3\alpha^2 W^{(2)}} J\wedge J\wedge\omega_i\wedge\omega_j$, which, as we stressed in our previous work [13], cannot be obtained as v^i-derivatives of the considered CY_4-integrals. In order to evaluate the derivatives of T_i with respect to v^i and to make contact with the Kähler metric found in the reduction result (3.19), we have to rewrite the integrals over D_i into integrals over CY_4. Due to the appearance of the warp-factor and the non-closed form F_6 in (A.52) this is not straightforward. In particular, one cannot simply use Poincaré duality and write T_i as an integral over CY_4 with inserted ω_i. Of course, it is always possible to write T_i as a CY_4 integral when inserting a delta-current localized on D_i, i.e.

$$\mathrm{Re}T_i = \int_{CY_4}\Big(\frac{1}{3!}e^{3\alpha^2 W^{(2)}} J\wedge J\wedge J + 1536\alpha^2 F_6\Big)\wedge\delta_i \ , \tag{A.54}$$

where δ_i is the (1,1)-form delta-current that restricts to the divisor D_i. Appropriately extending the notion of cohomology to include currents [54], we can now ask how much δ_i differs from the harmonic form ω_i in the same class. In fact, any current δ_i is related to the harmonic element of the same class ω_i by a doubly exact piece as

$$\delta_i = \omega_i + i\partial\bar{\partial}\lambda_i \ . \tag{A.55}$$

This equation should be viewed as relating currents. Importantly, as we assume D_i and hence δ_i to be v^i-independent, the v^i dependence of the harmonic form ω_i and the current λ_i has to cancel such that $\partial_j\omega_i = -i\partial\bar{\partial}\partial_j\lambda_i$. Importantly, once we determine $\partial_j \mathrm{Re} T_j$ we can express the result as Y_4-integrals without invoking currents. We therefore need to understand how each part of T_i varies under a change of moduli. This will also fix the numerical factor in front of F_6 in (A.52).

In order to take derivatives of T_i we first use the fact that D_i and hence δ_i are independent of the moduli v^i, which implies

$$\partial_j \mathrm{Re} T_i = \int_{Y_4} \Big(\frac{1}{2} e^{3\alpha^2 W^{(2)}} \omega_j \wedge J \wedge J + \frac{1}{2}\alpha^2 \partial_j W^{(2)} J \wedge J \wedge J + 1536\alpha^2 \partial_j F_6\Big) \wedge \delta_i \; . \quad \text{(A.56)}$$

We next claim that we can replace δ_i with ω_i such that finally

$$\partial_j \mathrm{Re} T_i = \frac{1}{2}\int_{Y_4} e^{3\alpha^2 W^{(2)}} \omega_i \wedge \omega_j \wedge J \wedge J + \frac{1}{2}\alpha^2 \int_{Y_4} \partial_j W^{(2)} \omega_i \wedge J \wedge J \wedge J + 1536\alpha^2 \int_{Y_4} \omega_i \wedge \partial_j F_6 \; . \quad \text{(A.57)}$$

Note that by using (A.55) the two expressions (A.56) and (A.57) only differ by a term involving $\partial\bar{\partial}\lambda_i$. By partial integration this term is proportional to

$$\begin{aligned} &\int_{Y_4} \lambda_i \partial\bar{\partial}\Big(\frac{1}{2} e^{3\alpha^2 W^{(2)}} \omega_j \wedge J \wedge J + \frac{1}{2}\alpha^2 \partial_j W^{(2)} J \wedge J \wedge J + 1536\alpha^2 \partial_j F_6\Big) \qquad \text{(A.58)} \\ &= \int_{Y_4} \lambda_i \Big(\frac{1}{2}\partial\bar{\partial}(e^{3\alpha^2 W^{(2)}}) \omega_j \wedge J \wedge J + \frac{1}{2}\alpha^2 \partial\bar{\partial}(\partial_j W^{(2)}) J \wedge J \wedge J + 1536\alpha^2 \partial\bar{\partial}\partial_j F_6\Big) \; . \end{aligned}$$

It is now straightforward to see that the terms multiplying λ_i are simply the ∂_j derivative of the warp-factor equation (3.5). One first writes (3.5) as

$$d^\dagger d e^{3\alpha^2 W^{(2)}} *_8 1 - \alpha^2 Q_8 = -\frac{1}{3} i \partial\bar{\partial}(e^{3\alpha^2 W^{(2)}}) \wedge J \wedge J \wedge J - \alpha^2 Q_8 \; . \quad \text{(A.59)}$$

Then one takes the v^j-derivative of (A.59) by using the fact that Q_8 is given via

$$Q_8 = -\tfrac{1}{2}\, G^{(1)} \wedge G^{(1)} - 3^2 2^{13}\, X_8^{(0)} \quad , \quad \text{(A.60)}$$

which can easily be inferred by comparison to (3.5) and (A.53). The moduli dependence of Q_8 only arises from the term involving F_6, i.e. one has $\partial_i Q_8 = 3072\, i\, \partial\bar{\partial}\partial_i F_6$. Hence one finds exactly the terms in (A.58) such that this λ_i dependent part of the T_i variation vanishes due to the warp-factor equation (3.5). The final expression (A.57) is then written as

$$\partial_j \mathrm{Re} T_i = \frac{1}{2}\int_{Y_4} e^{3\alpha^2 W^{(2)}} \omega_i \wedge \omega_j \wedge J \wedge J + 3\alpha^2 \mathcal{K}_i \mathscr{W}_j + 1536\alpha^2 \int_{Y_4} \omega_i \wedge \partial_j F_6 \; . \quad \text{(A.61)}$$

Evaluating (4.11) effective action will depend on the quantities

$$\int_{Y_4} \omega_i \wedge \partial_j F_6 | \quad \text{and} \quad \int_{Y_4} J \wedge \partial_i \partial_j F_6 | \; . \quad \text{(A.62)}$$

in order for the results to match the reduction result those terms need to interact with the higher-derivative building blocks. One may use the freedom in the definition (A.53) to accomplish this task. A concise match with the reduction result is beyond the scope of this work.

B Higher-derivatives and F-theory

B.1 11d higher-derivative Terms

The terms $t_8 t_8 R^4$ and $t_8 t_8 G^2 R^3$ in require the definition

$$
\begin{aligned}
& t_8^{N_1 \dots N_8} = \\
& \frac{1}{16}\Big(-2\left(g^{N_1N_3}g^{N_2N_4}g^{N_5N_7}g^{N_6N_8} + g^{N_1N_5}g^{N_2N_6}g^{N_3N_7}g^{N_4N_8} + g^{N_1N_7}g^{N_2N_8}g^{N_3N_5}g^{N_4N_6}\right) \\
& \quad + \quad 8\left(g^{N_2N_3}g^{N_4N_5}g^{N_6N_7}g^{N_8N_1} + g^{N_2N_5}g^{N_6N_3}g^{N_4N_7}g^{N_8N_1} + g^{N_2N_5}g^{N_6N_7}g^{N_8N_3}g^{N_4N_1}\right) \\
& \quad - \quad (N_1 \leftrightarrow N_2) - (N_3 \leftrightarrow N_4) - (N_5 \leftrightarrow N_6) - (N_7 \leftrightarrow N_8)\Big) \, .
\end{aligned} \tag{B.1}
$$

Let us now discuss the various eight-derivative couplings in in more detail. We recall the definition

$$
X_8 = \frac{1}{192}\Big(\mathrm{Tr}\mathcal{R}^4 - \frac{1}{4}(\mathrm{Tr}\mathcal{R}^2)^2\Big) \, , \tag{B.2}
$$

where $\mathcal{R}$ is the eleven-dimensional curvature two-from $\mathcal{R}^M{}_N = \frac{1}{2} R^M{}_{NPQ} dx^P \wedge dx^Q$, and

$$
\begin{aligned}
\epsilon_{11}\epsilon_{11} R^4 &= \epsilon^{R_1R_2R_3M_1\dots M_8}\epsilon_{R_1R_2R_3N_1\dots N_8} R^{N_1N_2}{}_{M_1M_2} R^{N_3N_4}{}_{M_3M_4} R^{N_5N_6}{}_{M_5M_6} R^{N_7N_8}{}_{M_7M_8} \, , \\
t_8 t_8 R^4 &= t_8^{M_1\dots M_8} t_{8\,N_1\dots N_8} R^{N_1N_2}{}_{M_1M_2} R^{N_3N_4}{}_{M_3M_4} R^{N_5N_6}{}_{M_5M_6} R^{N_7N_8}{}_{M_7M_8} \, ,
\end{aligned} \tag{B.3}
$$

where ϵ_{11} is the eleven-dimensional totally anti-symmetric epsilon tensor and t_8 is given explicitly in (B.1). Using ϵ_{11} and t_8 the explicit form for the terms in section 2.1 are precisely given by

$$
\begin{aligned}
\epsilon_{11}\epsilon_{11} G^2 R^3 &= \epsilon^{RM_1\dots M_{10}}\epsilon_{RN_1\dots N_{10}} G^{N_1N_2}{}_{M_1M_2} G^{N_3N_4}{}_{M_3M_4} \\
&\qquad \times R^{N_5N_6}{}_{M_5M_6} R^{N_7N_8}{}_{M_7M_8} R^{N_9N_{10}}{}_{M_9M_{10}} \, , \\
t_8 t_8 G^2 R^3 &= t_8^{M_1\dots M_8} t_{8N_1\dots N_8} G^{N_1}{}_{M_1R_1R_2} G^{N_2}{}_{M_2}{}^{R_1R_2} R^{N_3N_4}{}_{M_3M_4} R^{N_5N_6}{}_{M_5M_6} R^{N_7N_8}{}_{M_7M_8} \, .
\end{aligned} \tag{B.4}
$$

Finally, we need to introduce the tensor $s_{18}^{N_1\dots N_{18}}$, however its precise form not known. Significant parts of it may be fixed following [55]. We argue for an extension in 2.1 of this work. In order to express the kwon parts we use the basis B_i, $i = 1, \dots, 24$ of [55], that labels all unrelated index contractions in $s_{18}(\nabla G)^2 R^2$. The basis $\{B_i\}$ is explicitly given in section B.3. The result can then be expressed in terms of a four-point amplitude contribution $\mathcal{A}$ and a linear combination of six contributions $\mathcal{S}_{i=1,\dots,6}$ which do not affect the 4-point amplitude as

$$
s_{18}(\nabla G)^2 R^2 = s_{18}^{N_1\dots N_{18}} R_{N_1\dots N_4} R_{N_5\dots N_8} \nabla_{N_9} G_{N_{10}\dots N_{13}} \nabla_{N_{14}} G_{N_{15}\dots N_{18}} = \mathcal{A} + \sum_n a_n \mathcal{S}_n \, . \tag{B.5}
$$

The combinations $\mathcal{A}$ and $\mathcal{S}_n$ are then given in terms of the basis elements as

$$\begin{aligned}
\mathcal{A} &= -24B_5 - 48B_8 - 24B_{10} - 6B_{12} - 12B_{13} + 12B_{14} + 8B_{16} - 4B_{20} \\
&\quad + B_{22} + 4B_{23} + B_{24}\,, \\
\mathcal{S}_1 &= 48B_1 + 48B_2 - 48B_3 + 36B_4 + 96B_6 + 48B_7 - 48B_8 + 96B_{10} \\
&\quad + 12B_{12} + 24B_{13} - 12B_{14} + 8B_{15} + 8B_{16} - 16B_{17} + 6B_{19} + 2B_{22} + B_{24}\,, \\
\mathcal{S}_2 &= -48B_1 - 48B_2 - 24B_4 - 24B_5 + 48B_6 - 48B_8 - 24B_9 - 72B_{10} \\
&\quad - 24B_{13} + 24B_{14} - B_{22} + 4B_{23}\,, \\
\mathcal{S}_3 &= 12B_1 + 12B_2 - 24B_3 + 9B_4 + 48B_6 + 24B_7 - 24B_8 + 24B_{10} \\
&\quad + 6B_{12} + 6B_{13} + 4B_{15} - 4B_{17} + 3B_{19} + 2B_{21}\,, \\
\mathcal{S}_4 &= 12B_1 + 12B_2 - 12B_3 + 9B_4 + 24B_6 + 12B_7 - 12B_8 + 24B_{10} + 3B_{12} \\
&\quad + 6B_{13} + 4B_{15} - 4B_{17} + 2B_{20}\,, \\
\mathcal{S}_5 &= 4B_3 - 8B_6 - 4B_7 + 4B_8 - B_{12} - 2B_{14} + 4B_{18}\,, \\
\mathcal{S}_6 &= B_4 + 2B_{11}\,.
\end{aligned} \tag{B.6}$$

Note that $\mathcal{S}_3$ to $\mathcal{S}_6$ vanish both on the considered Calabi-Yau fourfold background solution.

B.2 Adjunction of Chern-classes

Let us next discuss the adjunction of Chern-classes of divisors on an elliptically fibered Calabi–Yau fourfold CY_4 which is a hyper-surface in a $\mathbb{P}_{321}$ bundle of the Kähler base B_3 denoted by $\mathbb{P}_{321}(\mathcal{L})$given by the vanishing locus of the Weierstrass equation

$$y^2(x^3 + fxz^4 + gz^6) = 0 \quad , \tag{B.7}$$

with f, g holomorphic sections of $\mathcal{L}^4$ and $\mathcal{L}^6$, respectively. The $SL(2,\mathbb{Z})$ line bundle $\mathcal{L}$ over B together with the choice of f, g defines the elliptic fibration. One may show that the first Chern class is given by

$$c_1(X) = c_1(B) - c_1(\mathcal{L}) \tag{B.8}$$

Then the total Chern class is given by

$$c(\mathbb{P}_{321}(\mathcal{L})) = c(B_3)(1 + 2\omega_0 + 2c_1(B_3)))(1 + 3\omega_0 + 3c_1(B_3)))(1 + \omega_0) \tag{B.9}$$

were ω_0 is the harmonic $(1,1)$-form such that $PD(\omega_0) = B$.[33] Using adjunction formulae for the

$$c(CY_4) = \frac{c(\mathbb{P}_{321}(\mathcal{L}))}{(1 + \mathcal{L})} \tag{B.10}$$

[33] We are using abuse of notation in the following using ω_0 and $c_{1,2,3}$ in the context of a concrete representative of the class as well as the class itself.

with

$$\mathcal{L} = 6\omega_0 + 6c_1(B_3) \tag{B.11}$$

one then derives

$$\begin{aligned} c_3(CY_4) &= c_3(B_3) - c_1(B_3) \wedge c_2(B_3) - 60c_1^3(B_3) - 60c_1^2(B_3) \wedge \omega_0 \\ c_2(CY_4) &= c_2(B_3) + 11c_1^2(B_3) + 12c_1(B_3) \wedge \omega_0 \\ c_1(CY_4) &= 0 \end{aligned} \tag{B.12}$$

and furthermore

$$\omega_0^2 = -c_1(B_3) \wedge \omega_0 \quad . \tag{B.13}$$

where the $c_{i=1,2,3}(B_3)$ on the r.h.s. of these expressions denote the Chern classes of B pulled-back to CY_4.

One may next iterate the adjunction formulae to find The Chern-forms of the vertical divisors D_α of the Calabi–Yau fourfold which are pullbacks of divisors of the base D^b_α. Thus we denote the class of such divisors via its representatives of harmonic $(1,1)$-forms ω_α, $\alpha = 1, ..., h^{1,1}$. Thus one may use adjunction to write

$$c(D_\alpha) = \frac{c(\mathbb{P}_{321}(\mathcal{L}))}{(1+\mathcal{L})(1+\omega_\alpha)} \quad , \tag{B.14}$$

with which one then derives

$$\begin{aligned} c_3(D_\alpha) &= c_3(B_3) - c_1(B_3) \wedge c_2(B_3) - 60c_1^3(B_3) - 60c_1^2(B_3) \wedge \omega_0 - c_2(D_\alpha) \wedge \omega_\alpha \\ c_2(D_\alpha) &= c_2(B_3) + 11c_1^2(B_3) + 12c_1(B_3) \wedge \omega_0 + \omega_\alpha^2 \\ c_1(D_\alpha) &= -\,\omega_\alpha \quad . \end{aligned} \tag{B.15}$$

where $c_{i=1,2,3}(B_3)$ on the r.h.s of the above equality are pulled back to the divisor D_α, which amounts to a simply restriction to the subspace $D_\alpha \subset CY_4$. In particular we find that the self intersection of divisors $[D_\alpha] \cdot [D_\alpha]$ is generically non-vanishing.

Let us close this section by analyzing the case where the Calabi–Yau fourfold is a direct product manifold e.g. $CY_4 = CY_3 \times T^2$ or $CY_4 = K3 \times K3$. The Chern-character on product spaces $X = Y \times Z$ obeys $c(X) = c(Y)c(Z)$. Thus we find for the Chern-forms

$$\begin{aligned} c_3(X) &= c_1(Y) \wedge c_2(Z) + c_2(Y) \wedge c_1(Z) + c_3(Y) + c_3(Z) \, , \\ c_2(X) &= c_1(Y) \wedge c_1(Z) + c_2(Y) + c_2(Z) \, , \\ c_1(X) &= c_1(Y) + c_1(Z) \quad . \end{aligned} \tag{B.16}$$

Furthermore, on may apply adjunction to compute the Chern-forms of CY_3 in therms of Chern-forms Divisors D^b_α pulled back to CY_3 which results in

$$c_1(D^b_\alpha) = \omega^b_\alpha \ , \quad c_2(D^b_\alpha) = c_2(CY_3) \quad , \tag{B.17}$$

where we have used the Calabi–Yau condition $c_1(CY_3) = 0$. Divisors inside $CY_4 = CY_3 \times T^2$ wrapping the torus are as well a direct product of $D^b_\alpha \times T^2$. Thus by combing (B.16) and (B.17) one can straightforwardly infer their Chern-forms.

B.3 Basis of the G^2R^3 and $(\nabla G)^2R^2$-sector

Basis of the G^2R^3-sector. The complete eleven-dimensional G^2R^3 terms may be written in terms of the basis [20] The basis for the potentially relevant eight-derivative terms involving the four-form field strength is

$$\begin{aligned}
\mathcal{B}_1 &= G_{M_5}{}^{M_7M_8M_9}\, G_{M_6M_7M_8M_9}\, R_{MM_2}{}^{M_4M_5}\, R^{MM_1M_2M_3}\, R_{M_1M_3M_4}{}^{M_6} \\
\mathcal{B}_2 &= G_{M_4M_6}{}^{M_8M_9}\, G_{M_5M_7M_8M_9}\, R_{MM_2}{}^{M_4M_5}\, R^{MM_1M_2M_3}\, R_{M_1M_3}{}^{M_6M_7} \\
\mathcal{B}_3 &= G_{M_4M_5}{}^{M_8M_9}\, G_{M_6M_7M_8M_9}\, R_{MM_2}{}^{M_4M_5}\, R^{MM_1M_2M_3}\, R_{M_1M_3}{}^{M_6M_7} \\
\mathcal{B}_4 &= G_{M_6M_7M_8M_9}\, G^{M_6M_7M_8M_9}\, R_{MM_2}{}^{M_4M_5}\, R^{MM_1M_2M_3}\, R_{M_1M_4M_3M_5} \\
\mathcal{B}_5 &= G_{M_6M_7M_8M_9}\, G_4^{M_6M_7M_8M_9}\, R_M{}^{M_4}{}_{M_2}{}^{M_5}\, R^{MM_1M_2M_3}\, R_{M_1M_4M_3M_5} \\
\mathcal{B}_6 &= G_{M_5}{}^{M_7M_8M_9}\, G_{M_6M_7M_8M_9}\, R_{MM_2}{}^{M_4M_5}\, R^{MM_1M_2M_3}\, R_{M_1M_4M_3}{}^{M_6} \\
\mathcal{B}_7 &= G_{M_5}{}^{M_7M_8M_9}\, G_{M_6M_7M_8M_9}\, R_M{}^{M_4}{}_{M_2}{}^{M_5}\, R^{MM_1M_2M_3}\, R_{M_1M_4M_3}{}^{M_6} \\
\mathcal{B}_8 &= G_{M_3M_6}{}^{M_8M_9}\, G_{M_5M_7M_8M_9}\, R_{MM_2}{}^{M_4M_5}\, R^{MM_1M_2M_3}\, R_{M_1M_4}{}^{M_6M_7} \\
\mathcal{B}_9 &= G_{M_3M_5}{}^{M_8M_9}\, G_{M_6M_7M_8M_9}\, R_{MM_2}{}^{M_4M_5}\, R^{MM_1M_2M_3}\, R_{M_1M_4}{}^{M_6M_7} \\
\mathcal{B}_{10} &= G_{M_3M_6}{}^{M_8M_9}\, G_{M_5M_7M_8M_9}\, R_M{}^{M_4}{}_{M_2}{}^{M_5}\, R^{MM_1M_2M_3}\, R_{M_1M_4}{}^{M_6M_7} \\
\mathcal{B}_{11} &= G_{M_3M_5}{}^{M_8M_9}\, G_{M_6M_7M_8M_9}\, R_M{}^{M_4}{}_{M_2}{}^{M_5}\, R^{MM_1M_2M_3}\, R_{M_1M_4}{}^{M_6M_7} \\
\mathcal{B}_{12} &= G_{M_4M_7}{}^{M_8M_9}\, G_{M_5M_6M_8M_9}\, R_M{}^{M_4}{}_{M_2}{}^{M_5}\, R^{MM_1M_2M_3}\, R_{M_1}{}^{M_6}{}_{M_3}{}^{M_7} \\
\mathcal{B}_{13} &= G_{M_3M_7}{}^{M_8M_9}\, G_{M_5M_6M_8M_9}\, R_{MM_2}{}^{M_4M_5}\, R^{MM_1M_2M_3}\, R_{M_1}{}^{M_6}{}_{M_4}{}^{M_7} \\
\mathcal{B}_{14} &= G_{M_3M_7}{}^{M_8M_9}\, G_{M_5M_6M_8M_9}\, R_M{}^{M_4}{}_{M_2}{}^{M_5}\, R^{MM_1M_2M_3}\, R_{M_1}{}^{M_6}{}_{M_4}{}^{M_7} \\
\mathcal{B}_{15} &= G_{M_5}{}^{M_7M_8M_9}\, G_{M_6M_7M_8M_9}\, R_{MM_1M_2}{}^{M_4}\, R^{MM_1M_2M_3}\, R_{M_3}{}^{M_5}{}_{M_4}{}^{M_6} \\
\mathcal{B}_{16} &= G_{M_4M_6}{}^{M_8M_9}\, G_{M_5M_7M_8M_9}\, R_{MM_1M_2}{}^{M_4}\, R^{MM_1M_2M_3}\, R_{M_3}{}^{M_5M_6M_7} \\
\mathcal{B}_{17} &= G_{M_4M_6}{}^{M_8M_9}\, G_{M_5M_7M_8M_9}\, R_{MM_1M_2M_3}\, R^{MM_1M_2M_3}\, R^{M_4M_5M_6M_7}.
\end{aligned} \tag{B.18}$$

Basis of the $(\nabla G)^2R^2$-sector. The complete eleven-dimensional $(\nabla G)^2R^2$ terms may be written in terms of the basis [55]. In order to discuss the term s_{18} appearing in (2.4) and (B.5) we introduce the basis

$$
\begin{aligned}
B_1 &= R_{M_1M_2M_3M_4}R_{M_5M_6M_7M_8}\nabla^{M_5}G^{M_1M_7M_8}{}_{M_9}\nabla^{M_3}G^{M_2M_4M_6M_9}\,,\\
B_2 &= R_{M_1M_2M_3M_4}R_{M_5M_6M_7M_8}\nabla^{M_5}G^{M_1M_3M_7}{}_{M_9}\nabla^{M_8}G^{M_2M_4M_6M_9}\,,\\
B_3 &= R_{M_1M_2M_3M_4}R_{M_5M_6M_7M_8}\nabla^{M_5}G^{M_1M_3M_7}{}_{M_9}\nabla^{M_6}G^{M_2M_4M_8M_9}\\
B_4 &= R_{M_1M_2M_3M_4}R_{M_5M_6M_7M_8}\nabla_{M_9}G^{M_3M_4M_7M_8}\nabla^{M_6}G^{M_9M_1M_2M_5}\\
B_5 &= R_{M_1M_2M_3M_4}R_{M_5M_6M_7}{}^{M_4}\nabla^{M_1}G^{M_2M_3}{}_{M_8M_9}\nabla^{M_5}G^{M_6M_7M_8M_9}\,,\\
B_6 &= R_{M_1M_2M_3M_4}R_{M_5M_6M_7}{}^{M_4}\nabla^{M_1}G^{M_2M_5}{}_{M_8M_9}\nabla^{M_3}G^{M_6M_7M_8M_9}\,,\\
B_7 &= R_{M_1M_2M_3M_4}R_{M_5M_6M_7}{}^{M_4}\nabla^{M_1}G^{M_2M_5}{}_{M_8M_9}\nabla^{M_7}G^{M_3M_6M_8M_9}\\
B_8 &= R_{M_1M_2M_3M_4}R_{M_5M_6M_7}{}^{M_4}\nabla^{M_1}G^{M_3M_5}{}_{M_8M_9}\nabla^{M_2}G^{M_6M_7M_8M_9}\,,\\
B_9 &= R_{M_1M_2M_3M_4}R_{M_5M_6M_7}{}^{M_4}\nabla^{M_1}G^{M_3M_5}{}_{M_8M_9}\nabla^{M_6}G^{M_2M_7M_8M_9}\,,\\
B_{10} &= R_{M_1M_2M_3M_4}R_{M_5M_6M_7}{}^{M_4}\nabla_{M_9}G^{M_3M_5M_7M_8}\nabla^{M_9}G^{M_1M_2M_6M_8}\,,\\
B_{11} &= R_{M_1M_2M_3M_4}R_{M_5M_6M_7}{}^{M_4}\nabla_{M_8}G^{M_1M_2M_6}{}_{M_9}\nabla^{M_9}G^{M_3M_5M_7M_8}\,,\\
B_{12} &= R_{M_1M_2M_3M_4}R_{M_5M_6M_7}{}^{M_4}\nabla^{M_3}G^{M_5M_6}{}_{M_8M_9}\nabla^{M_7}G^{M_2M_1M_8M_9}\,,\\
B_{13} &= R_{M_1M_2M_3M_4}R_{M_5}{}^{M_1}{}_{M_6}{}^{M_3}\nabla_{M_9}G^{M_2M_6}{}_{M_7M_8}\nabla^{M_9}G^{M_4M_5M_7M_8}\,,\\
B_{14} &= R_{M_1M_2M_3M_4}R_{M_5}{}^{M_1}{}_{M_6}{}^{M_3}\nabla_{M_9}G^{M_2M_4}{}_{M_7M_8}\nabla^{M_9}G^{M_5M_6M_7M_8}\,,\\
B_{15} &= R_{M_1M_2M_3M_4}R_{M_5}{}^{M_1}{}_{M_6}{}^{M_3}\nabla^{M_2}G^{M_6}{}_{M_7M_8M_9}\nabla^{M_5}G^{M_4M_7M_8M_9}\,,\\
B_{16} &= R_{M_1M_2M_3M_4}R_{M_5}{}^{M_1}{}_{M_6}{}^{M_3}\nabla^{M_2}G^{M_4}{}_{M_7M_8M_9}\nabla^{M_5}G^{M_6M_7M_8M_9}\,,\,,\\
B_{17} &= R_{M_1M_2M_3M_4}R_{M_5}{}^{M_1}{}_{M_6}{}^{M_3}\nabla^{M_2}G^{M_5}{}_{M_7M_8M_9}\nabla^{M_4}G^{M_6M_7M_8M_9}\,,\\
B_{18} &= R_{M_1M_2M_3M_4}R_{M_5}{}^{M_1}{}_{M_6}{}^{M_3}\nabla_{M_9}G^{M_5M_6}{}_{M_7M_8}\nabla^{M_4}G^{M_2M_7M_8M_9}\,,\\
B_{19} &= R_{M_1M_2M_3M_4}R_{M_5M_6}{}^{M_3M_4}\nabla_{M_9}G^{M_1M_5}{}_{M_7M_8}\nabla^{M_9}G^{M_2M_6M_7M_8}\,,\\
B_{20} &= R_{M_1M_2M_3M_4}R_{M_5M_6}{}^{M_3M_4}\nabla^{M_1}G^{M_5}{}_{M_7M_8M_9}\nabla^{M_2}G^{M_6M_7M_8M_9}\,,\\
B_{21} &= R_{M_1M_2M_3M_4}R_{M_5M_6}{}^{M_3M_4}\nabla^{M_1}G^{M_5}{}_{M_7M_8M_9}\nabla^{M_6}G^{M_2M_7M_8M_9}\,,\\
B_{22} &= R_{M_1M_2M_3M_4}R_{M_5}{}^{M_1M_3M_4}\nabla^{M_2}G_{M_6M_7M_8M_9}\nabla^{M_5}G^{M_6M_7M_8M_9}\,,\\
B_{23} &= R_{M_1M_2M_3M_4}R_{M_5}{}^{M_1M_3M_4}\nabla_{M_9}G^{M_2}{}_{M_6M_7M_8}\nabla^{M_9}G^{M_5M_6M_7M_8}\,,\\
B_{24} &= R_{M_1M_2M_3M_4}R^{M_1M_2M_3M_4}\nabla_{M_5}G_{M_6M_7M_8M_9}\nabla^{M_6}G^{M_5M_7M_8M_9}\,.
\end{aligned}
\tag{B.19}
$$

The contributions to $s_{18}(\nabla G)^2 R^2$ are then formed from the linear combinations described in (B.5). We write the eleven-dimensional action as

$$2\kappa_{11}^2 \, S^{\text{extra, gen}} = \alpha^2 \int_{M_{11}} \sum_{i=1}^{17} C_i \, \mathcal{B}_i \, *1 + \sum_{i=1}^{24} C_{i+17} \, B_i \, *1 \tag{B.20}$$

with real parameters $C_1, \ldots, C_{41}$ which are fixed by the reduction on a Calabi–Yau threefold and compatibility with $5d$, $\mathcal{N}_2$ super symmetry to

$$\begin{aligned}
C_5 &= -\tfrac{1}{2} C_4 \,, & C_7 &= -C_1 - \tfrac{1}{2} C_6 \,, \\
C_9 &= 4C_3 \,, & C_{10} &= -3C_1 - 2C_2 - 8C_3 - 18C_4 - \tfrac{3}{2} C_6 - C_8 \,, \\
C_{11} &= -4C_3 \,, & C_{12} &= 4C_3 \,, \\
C_{13} &= -8C_3 \,, & C_{14} &= -6C_1 - 2C_2 - 12C_3 - 36C_4 - 3C_6 - C_8 \,, \\
C_{15} &= \tfrac{1}{3} C_2 + 3C_4 \,, & C_{16} &= -2C_2 - 4C_3 \,, \\
C_{17} &= \tfrac{1}{4} C_2 \,, & C_{25} &= 2C_{22} - C_{24} \,, \\
C_{29} &= \tfrac{1}{4} C_{22} + \tfrac{1}{4} C_{23} - \tfrac{1}{4} C_{24} + \tfrac{1}{4} C_{26} \,, \\
C_{32} &= -C_1 - \tfrac{1}{3} C_{22} - \tfrac{4}{3} C_3 + \tfrac{2}{3} C_{30} - 6C_4 - \tfrac{1}{2} C_6 \,, \\
C_{33} &= C_1 - \tfrac{1}{3} C_{22} + \tfrac{4}{3} C_3 + 6C_4 + \tfrac{1}{2} C_6 \,, \\
C_{37} &= -C_1 - \tfrac{4}{3} C_3 - \tfrac{1}{3} C_{31} - \tfrac{1}{2} C_{34} - \tfrac{1}{6} C_{35} - \tfrac{2}{3} C_{36} - 6C_4 - \tfrac{1}{2} C_6 \,, \\
C_{38} &= \tfrac{1}{3} C_{30} + \tfrac{1}{3} C_{31} + \tfrac{1}{2} C_{34} + \tfrac{1}{6} C_{35} + \tfrac{2}{3} C_{36} \,, \\
C_{39} &= \tfrac{1}{4} C_1 - \tfrac{1}{24} C_{22} + \tfrac{1}{3} C_3 + \tfrac{3}{2} C_4 + \tfrac{1}{8} C_6 \,, \\
C_{40} &= \tfrac{1}{2} C_1 + \tfrac{2}{3} C_3 + \tfrac{1}{3} C_{31} + \tfrac{1}{6} C_{35} + 3C_4 + \tfrac{1}{4} C_6 \,, \\
C_{41} &= \tfrac{1}{4} C_1 + \tfrac{1}{3} C_3 + \tfrac{1}{12} C_{31} + \tfrac{1}{24} C_{35} + \tfrac{3}{2} C_4 + \tfrac{1}{8} C_6 \,,
\end{aligned} \tag{B.21}$$

We then check compatibility of the novel induces H^2R^3 terms making use of the IIA - Heterotic duality. Compactifying type IIA on $K3$ is dual to the Heterotic string on $\mathbb{T}^4$. One finds that additionally

$$C_2 = 0 \quad , \quad C_1 = -\tfrac{1}{6}\big(8C_3 + 2C_{31} + C_{35} + 36C_4 + 3C_6\big) \quad , \tag{B.22}$$

where more details can be found in section 2.2.

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