



Sommerfeld enhancement from unparticle exchange for dark matter annihilation

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ABSTRACT

We investigate the implication of unparticle exchange for the possible Sommerfeld enhancement in dark matter annihilation process. Assuming the unparticle exchange during WIMP collision, we solve the Schrödinger equation for the effective potential, and find that the Sommerfeld enhancement factor is dictated by the scale dimension of unparticle as $1/v^{3-2d_U}$. Numerically the Sommerfeld enhancement could be $\mathcal{O}(10-10^3)$.

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1. Introduction

It has been known that our universe is made of not only the stuff of the standard model (SM) with the occupancy of mere 4%, but also dark matter and dark energy with the abundance of 22% and 74%, respectively [1]. Therefore, it must be one of the most important issue to understand what the dark matter is and how to explore it by various observations. Hopefully, through high energy colliders such as the Large Hadron Collider (LHC) at CERN, we may directly observe dark matter soon. In the mean time we may also have the chance to probe dark matter indirectly by the study of the high energy cosmic-ray.

Recently, the collaborations of PAMELA [2], ATIC [3], FERMI-LAT [4], HESS [5], etc. have published quite astonished events in cosmic-ray measurements, in which PAMELA observes the excess in the positron flux ratio over 10 GeV till PAMELA's observational limit of about 100 GeV, whereas others measure consistent anomalies in the electron + positron flux in the 300–1000 GeV range. Inspired by the new finds at the satellite, balloon and ground-based experiments, the excess could be readily ascribed to dark matter annihilation, even though there still are the possibilities of

existing new young pulsars [6]. Although dark matter decays could be the origin of such anomaly, however, for making the lifetime as long as $\mathcal{O}(10^{25})$ seconds, the extreme fine-tuning on the coupling of interaction [7,8] cannot be avoided. For escaping the fine-tuning problem, hereafter, we will focus on the mechanism of dark matter annihilation only. In addition, the candidate of dark matter in our following analysis is regarded as weakly interacting massive particle (WIMP).

Although the WIMP annihilation could be the source for the excess of cosmic-ray, however, due to low reacting rate in the annihilating process, an enhanced boost factor of a few orders of magnitude, e.g. Sommerfeld enhancement [9–16], has to appear during the annihilation of WIMP. Therefore, a new force carrier in the dark matter annihilation to dictate the enhancement is required. As an example, an interesting mechanism for the Sommerfeld enhancement is arisen from the light boson exchange between dark matter [13,17], where the resulted interaction is Yukawa potential and the force carrier has the significant influence in the range of Compton wavelength, denoted by αM_χ with α being the fine structure constant of the interaction and of order 10^{-2} .

In this Letter, we study another kind of new force that may be alive in an invisible sector and dictated by scale invariant. As known that an exact scale invariant stuff cannot have a definite mass unless it is zero, therefore for distinguishing from the conventional particles, Georgi named the stuff as unparticle [18,19]. Interestingly, it is found that the unparticle with the scaling dimension d_U behaves like a non-integral number d_U of invisible

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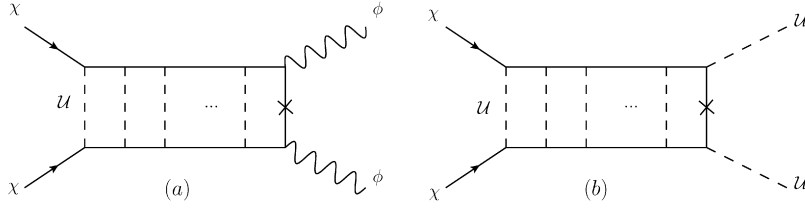


Fig. 1. Dark matter annihilation with unparticle-mediated Sommerfeld enhancement.

particles [18]. Further implications of the unparticle to colliders and low energy physics could be referred to Refs. [20–22]. In order to concentrate on the Sommerfeld effect, here we don't study the general effective interactions with unparticle, e.g. in our analysis, we have suppressed the interactions between unparticle and Higgs [23]. Although WIMP and unparticle both weakly couple to the SM particles, however, there is no any reason to limit that the interactions between them should be weak. Therefore, when WIMPs collide each other with small speed, we speculate that the Sommerfeld enhancement could be arisen from the unparticle exchange during the collision, sketched in Fig. 1, where the χ and \mathcal{U} denotes the WIMP and unparticle, respectively. And ϕ represents the (generic light) particle that might weakly decay to SM particles and the constraints on the couplings will be controlled by current observed fluxes of cosmic rays such as electrons, positrons, antiproton, etc. Here, its appearance is responsible for the possible connection between dark and visible sectors, but not for the Sommerfeld enhancement. Hence, we don't further discuss the detailed couplings to the SM stuff and the related issue for this decay. Our motivation is to understand that if there exists unparticle in the invisible sector, what are its unique character on the Sommerfeld factor and the differences from Coulomb and Yukawa interactions?

The Letter is organized as follows: First, we derive the static unparticle potential, solve the associated radial Schrödinger equation with suitable boundary condition, and find the resultant formula for s -wave Sommerfeld enhancement. Then, we do the numerical analysis on the resulted Sommerfeld factor and present it as a function of involved parameters in two-dimensional contour plots. Finally we give the conclusion.

2. Unparticle potential and its Sommerfeld factor

Although the nonperturbative Sommerfeld effect can be calculated by the combined contributions of a set of ladder diagrams shown as in Fig. 1, however in the non-relativistic limit, the effect could be equivalent to solving the Schrödinger equation with an effective potential, which is arisen from the single particle exchange. In the considered mechanism, here the exchanged particle is the unparticle. Following the scheme proposed in Ref. [18], the interaction of WIMP to unparticle can be written as

$$\frac{\lambda}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{\chi}^{(c)} \chi \mathcal{O}_{\mathcal{U}} \quad (1)$$

where we have assumed that the WIMP is a Dirac (or Majorana) fermion, λ is dimensionless parameter and $\Lambda_{\mathcal{U}}$ denotes the living scale of unparticle. For displaying the character of scale invariant stuff, we concentrate only on the scalar unparticle. Please note that when unparticle is realized in the framework of conformal field theories, the propagators for vector and tensor unparticles should be modified appropriately to preserve the conformal symmetry [24]. To obtain the unparticle potential in non-relativistic limit, we use the propagator of the scalar unparticle operator given by [18,19]

$$\int e^{iqx} \langle 0 | T \mathcal{O}_{\mathcal{U}}(x) \mathcal{O}_{\mathcal{U}}(0) | 0 \rangle = i \frac{A_{d_{\mathcal{U}}}}{2 \sin d_{\mathcal{U}} \pi} \frac{1}{(-q^2)^{2-d_{\mathcal{U}}}}, \quad \text{where} \\ A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}} + 1/2)}{\Gamma(d_{\mathcal{U}} - 1)\Gamma(2d_{\mathcal{U}})}. \quad (2)$$

Combining Eqs. (1) and (2), the four-fermion effective interacting term in momentum space could be expressed by

$$\bar{\chi}^{(c)} \chi \left[\left(\frac{\lambda}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \right)^2 \frac{A_{d_{\mathcal{U}}}}{2 \sin d_{\mathcal{U}} \pi} \frac{1}{(-q^2)^{2-d_{\mathcal{U}}}} \right] \bar{\chi}^{(c)} \chi. \quad (3)$$

By Fourier transformation and with $q^0 = 0$, the static unparticle potential in WIMP interaction resulted by Eq. (3) is found by

$$V(r) = -\frac{\alpha}{r^t} \quad (4)$$

with $t = 2d_{\mathcal{U}} - 1$,

$$\alpha = \frac{\xi_{\Gamma}}{2\pi^{2d_{\mathcal{U}}}} \left(\frac{\lambda}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \right)^2, \\ \xi_{\Gamma} = \frac{\Gamma(d_{\mathcal{U}} + 1/2)\Gamma(d_{\mathcal{U}} - 1/2)}{\Gamma(2d_{\mathcal{U}})}. \quad (5)$$

Interestingly, we see that the power of unparticle potential is associated with the scaling dimension $d_{\mathcal{U}}$ and not an integer.

Since the unparticle potential is independent of the polar and azimuth angles in spherical coordinate system, the relevant piece for the Sommerfeld factor is the radial Schrödinger equation, read by

$$\left[\frac{d^2}{dr^2} + \left(-\frac{\ell(\ell+1)}{r^2} + \frac{2\mu\alpha}{r^t} + k^2 \right) \right] u_{k\ell}(r) = 0, \quad (6)$$

where we have used the nature unit with $\hbar = c = 1$, $E = k^2/2\mu$ with μ being the reduced mass of system and $R_{k\ell}(r) = u_{k\ell}(r)/r$. Once we solve the differential equation with the proper boundary condition, the Sommerfeld effect associated with each angular momentum ℓ is obtained by [15]

$$S_{\ell} = \left| \frac{(2\ell+1)!!}{|k|^{\ell} \ell!} \frac{\partial^{\ell} R_{k\ell}(r)}{\partial r^{\ell}} \right|_{r=0}^2. \quad (7)$$

Due to the power t of unparticle potential in r being not an integer, there is no hope to find a general close form for the solution. Nevertheless, since the required Sommerfeld factor is estimated at $r = 0$, our strategy for finding the solution is to look for a good approximation to extract the behavior of radial wave function at $r \sim 0$.

Before discussing the solution to the differential equation, first we analyze the limit on $t = 2d_{\mathcal{U}} - 1$. To avoid the crossing of the branching cut at $1/\sin d_{\mathcal{U}} \pi$ in Eq. (2), we require $1 < d_{\mathcal{U}} < 2$ ($1 < t < 3$). In order to further understand whether the upper limit of t can be bounded by the boundary condition when solving differential equation, we examine the case with $t = 2$, in which the potential could be exactly solved at $r \rightarrow 0$. Hence, by taking $t = 2$ and $u_{k\ell} \sim r^{\sigma}$ and considering $r \rightarrow 0$, from Eq. (6) we get

$$\sigma = \frac{1}{2} + \sqrt{\left(\ell + \frac{1}{2}\right)^2 - 2\mu\alpha}. \quad (8)$$

Thus, the corresponding solution for $u_{k\ell}(r)$ can be expressed by $u_{k\ell}(r) = r^\sigma e^{ikr} f(r)$, where the function of $f(r)$ is controlled by

$$r \frac{d^2 f}{dr^2} + (2\sigma + 2ikr) \frac{df}{dr} + 2ik\sigma f(r) = 0. \quad (9)$$

Compare to the confluent hypergeometric differential equation, $xy'' + (c-x)y' - ay = 0$, in which the solution is confluent hypergeometric function of the first kind $y(x) = {}_1F_1(a, c; x)$, we see that the solution to Eq. (9) can be found by taking $a = \sigma$, $c = 2\sigma$ and $x = -2ikr$. For $\ell = 0$, from Eq. (8) it is easy to find $\sigma < 1$. Since ${}_1F_1(\sigma, 2\sigma, -2ikr) \rightarrow 1$ when $r \rightarrow 0$, as a result, the wave function $R_{k\ell}(r)$ is singular at origin. In other words, $t = 2$ leads to an ill-defined solution and should be taken as the new upper limit. Therefore, we find the new bound on the scale dimension of unparticle as $1 < d_U < 3/2$ ($1 < t < 2$).

After obtaining the allowed power of unparticle potential, next we are looking for a good approximation to get the proper solution for near origin. As used before, if we regard $u_{k\ell}(r) = r^{\ell+1} e^{ikr} f(r)$ as a general solution to the radial Schrödinger equation, according to Eq. (6), the function $f(r)$ should satisfy the differential equation

$$r \frac{d^2 f}{dr^2} + (2(\ell+1) + 2ikr) \frac{df}{dr} + \left(2ik(\ell+1) + \frac{2\mu\alpha}{r^{t-1}}\right) f = 0. \quad (10)$$

Although this equation cannot be solved for arbitrary value of t , we find that if we concentrate on small r , the terms associated with $2ikr$ and $2ik(\ell+1)$ can be safely neglected and the solution to the simplified differential equation can be found by

$$f_t(r) = d^{-(2\ell+1)/2(2-t)} (2-t)^{(2\ell+1)/(2-t)} \frac{1}{r^{(2\ell+1)/2}} \times \Gamma\left(\frac{2\ell-t+3}{2-t}\right) J\left(\frac{2\ell+1}{2-t}, \frac{2\sqrt{d}r^{(2-t)/2}}{2-t}\right),$$

where $J_\nu(x) = J(\nu, x)$ is the Bessel function and the asymptotic form for $0 < x \ll \sqrt{\nu+1}$ is $J_\nu \rightarrow (x/2)^\nu / \Gamma(\nu+1)$. Hence, the solution to radial Schrödinger equation in the region of small r can be expressed by

$$R_{k\ell}(r) = \frac{u_{k\ell}(r)}{r} = r^\ell e^{ikr} f_t(r), \quad (11)$$

where a suitable normalization for the wave function has to be further dealt with.

Unlike conventional approach that the normalization of wave function for scattering process is chosen at $r \rightarrow \infty$, in order to get a suitable normalization for the wave function at near origin, we have to consider an alternative method which could provide correct value of wave function at $r = 0$. We find that the purpose can be achieved by using integral form instead of directly solving Eq. (6). In other words, the solution for $u_{k\ell}(r)$ could be expressed by

$$u_{k\ell}(r) \propto j_\ell(kr) + \int_0^\infty dr' G(r, r') U(r') u_{k\ell}(r') \quad (12)$$

with $U(r) = 2\mu V(r)$ and

$$G(r, r') = \begin{cases} j_\ell(kr) n_\ell(kr')/k & \text{for } r < r', \\ n_\ell(kr) j_\ell(kr')/k & \text{for } r > r', \end{cases} \quad (13)$$

where $G(r, r')$ is the Green's function of the differential equation. After compared to the partial wave expansion given by $\exp(ikz) =$

$\sum_{\ell=0}^\infty i^\ell (2\ell+1) j_\ell(kr) P_\ell(\cos\theta)/kr$, we find that the solution for $r \rightarrow 0$ is

$$u_{k\ell} = \frac{j_\ell(kr)}{k} + \frac{j_\ell(kr)}{k^2} \int_0^\infty dr' n_\ell(kr') j_\ell(kr') U(r'). \quad (14)$$

Now we match both solutions at $r \approx 0$ by solving differential equation and the integration with Green function. It has been known that the Sommerfeld factor of Coulomb potential (*i.e.* $t = 1$) for s -wave can be obtained exactly as

$$S_0 = \frac{2\pi\alpha/v}{1 - \exp(-2\pi\alpha/v)}. \quad (15)$$

For the case of $v \ll 1$, we have $S_0 \approx 2\pi\alpha/v$. Based on the known result, we find that in order to get correct approximation for Coulomb potential, matching condition for Eqs. (11) and (14) has to be adopted as

$$N_\ell^2 r^{\ell+1} f(r) = \frac{4}{k^2} j_\ell(kr) \int_0^\infty dr' n_\ell(kr') j_\ell(kr') U(r').$$

Using the asymptotic results $f(r \rightarrow 0) \rightarrow 1$ and $j_\ell(kr \rightarrow 0) \rightarrow \frac{2^\ell \ell!}{(2\ell+1)!} (kr)^{\ell+1}$, the normalization constant is found by

$$N_\ell^2 = 8\mu\alpha \frac{2^\ell \ell!}{(2\ell+1)!} k^{\ell+t-2} X_\ell \quad (16)$$

with $X_\ell = \int_0^\infty dz j_\ell(z) n_\ell(z)/z^t$. According to Eq. (7), the Sommerfeld factor for s -wave is obtained by $S_0 = 8\alpha |X_0| \mu^{t-1}/v^{2-t}$, where we have used $k = \mu v$. One can see that for $t = 1$, $X_0 = -\pi/4$ and $S_0 = 2\pi\alpha/v$. Hence, the s -wave Sommerfeld factor induced by unparticle exchange is given by

$$S_0 = \left(\frac{2\lambda}{\pi d_U}\right)^2 \xi_\Gamma \left(\frac{\mu}{\Lambda_U}\right)^{2(d_U-1)} \frac{|X_0|}{v^{3-2d_U}}. \quad (17)$$

3. Numerical analysis and discussions

We now analyze the formula for S_0 numerically. From Eq. (17), we see that the involved free parameters are λ , d_U , Λ_U and v . Unlike the case of Coulomb or Yukawa potential, the velocity-dependent factor appears by $1/v^{3-2d_U}$ with $1 < d_U < 3/2$. The Sommerfeld factor is increasing when d_U approaches to unity. Consequently, if we focus on the maximum of S_0 , then we find that S_0 is insensitive to the values of μ and Λ_U , where they show up by $(\mu/\Lambda_U)^{2(d_U-1)}$. Note that to explain the excess of cosmic rays from both PAMELA and ATIC/Fermi-LAT, it has been known that the mass of dark matter is of order of TeV. On the other hand, in order to produce the unusual stuff at LHC, the interesting scale to form unparticle should be also at TeV scale. Therefore, without lose of generality we set $\mu = \Lambda_U$ in our numerical estimates.

In order to see the influence of remaining parameters, we will fix one parameter in turn when we make two-dimensional contours as a function of remaining two parameters. First, we analyze the contour as a function of λ and d_U when the speed of WIMP is fixed. Although dark stuff weakly couples to visible particle, however, the interaction between invisible particles may not be small. Accordingly, we set the allowed range for λ be within one order of magnitude, *i.e.* $1 < \lambda < 10$. As a result, the contour for Sommerfeld factor S_0 induced by unparticle contributions as a function of coupling λ and scale dimension d_U with $v = 10^{-3}$ is shown in Fig. 2, where the numbers in the plot stand for the values of S_0 . By the figure, we see that S_0 is increasing while d_U is decreasing.

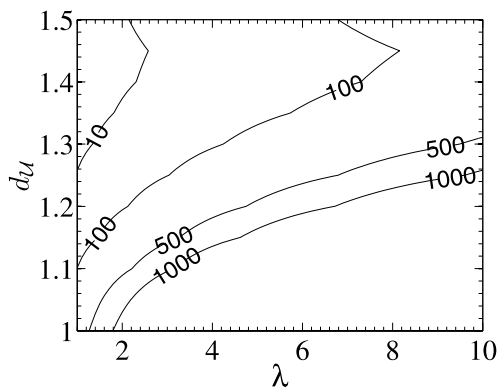


Fig. 2. Contour for Sommerfeld factor induced by unparticle-mediated as a function of λ and d_U with $v = 10^{-3}$. The numbers in the plot denote the values of S_0 .

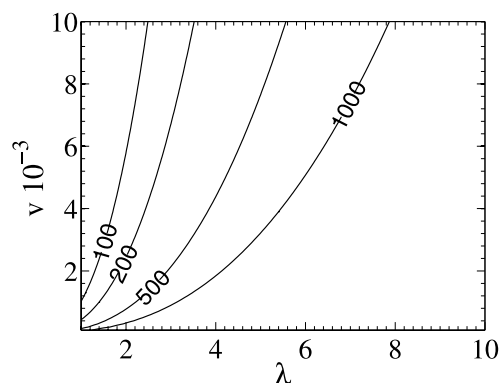


Fig. 3. Legend is similar to Fig. 2, where $d_U = 1.1$ and the variables for the contour are λ and v (in units of 10^{-3}).

Furthermore, the value of d_U for $S_0 \sim \mathcal{O}(10^3)$ can be somewhat larger, when λ is away from unity. It is clear that Sommerfeld factor could be as large as $\mathcal{O}(10^3)$ by the new force mediated by unparticle.

Secondly, we also study the behavior of S_0 in λ and v (in units of 10^{-3}) by fixing the value of d_U . We present the resultant contour with $d_U = 1.1$ in Fig. 3. Clearly, with the value of d_U that is close to unity, S_0 can easily reach $\mathcal{O}(10^3)$ even with $\lambda \sim 1$ and $v \sim 10^{-3}$. On the other hand, S_0 of $\mathcal{O}(10^3)$ can be preserved when v and λ both are increasing simultaneously.

Finally, for understanding the dependence of speed of WIMP and scale dimension of unparticle on Sommerfeld factor, we checked the contour for S_0 as a function of v (in units of 10^{-3}) and d_U with $\lambda = 1$. And we found that Sommerfeld enhancement of $\mathcal{O}(10^2)$ for explaining the excess of observed cosmic-ray through WIMP annihilation can still be achieved with $v \sim \mathcal{O}(10^{-3})$ and $\lambda = 1$ while $1 < d_U < 1.1$.

As summary, inspired by the excess of electrons and/or positrons observed at PAMELA, ATIC, Fermi-LAT, etc., the issue of dark matter annihilation for the solution is revived and studied broadly. Besides a new mechanism is needed for the production of excessive cosmic-ray, usually we also need a new force that interacts between dark matter for overcoming the low cross section when dark matter collides. For studying the possible new force, we have

investigated the impact of unparticle which is ruled by scale invariant.

We find that the power of the static unparticle potential in r , resulted from the exchange of scalar unparticle, is non-integral number and it depends on the scale dimension of unparticle, expressed by $1/r^{2d_U-1}$. By the boundary condition for wave function at $r = 0$, the upper bound of scale dimension is found by $d_U|_{\max} < 3/2$. By looking for the suitable boundary condition and matching condition for the radial wave function at $r \rightarrow 0$, we are led to the Sommerfeld factor for s -wave collision, in which the result could return to that of Coulomb potential. However, unlike Coulomb or Yukawa potential, the speed dependence is related to scale dimension of unparticle and dictated by $1/v^{3-2d_U}$. Although the Sommerfeld factor is associated several parameters, however, our results are only sensitive to the parameters λ , d_U and v . According to our numerical calculations, we conclude that with the allowed range of free parameters, the Sommerfeld enhancement induced by unparticle-mediated effects could be $\mathcal{O}(10-10^3)$ and the factor could provide the necessity for enhancing the cross section of dark matter annihilation.

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