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Universal upper limit on inflation energy scale from cosmic magnetic field

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Abstract. Recently observational lower bounds on the strength of cosmic magnetic fields were reported, based on γ-ray flux from distant blazars. If inflation is responsible for the generation of such magnetic fields then the inflation energy scale is bounded from above as $\rho_{\rm inf}^{1/4} < 2.5 \times 10^{-7} M_{\rm Pl} \times (B_{\rm obs}/10^{-15}{\rm G})^{-2}$ in a wide class of inflationary magnetogenesis models, where $B_{\rm obs}$ is the observed strength of cosmic magnetic fields. The tensor-to-scalar ratio is correspondingly constrained as $r < 10^{-19} \times (B_{\rm obs}/10^{-15}{\rm G})^{-8}$. Therefore, if the reported strength $B_{\rm obs} \geq 10^{-15}$G is confirmed and if any signatures of gravitational waves from inflation are detected in the near future, then our result indicates some tensions between inflationary magnetogenesis and observations.

Keywords: inflation, primordial magnetic fields

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Contents

1 Introduction

In 2010, the first detection of cosmic magnetic fields was reported [1] (see also [2–9]). Although High Energy Stereoscopic System (HESS) γ-ray telescopes observed TeV scale γ-rays from several blazars, Fermi space telescope did not observe GeV scale γ-rays from same blazars. Without cosmic magnetic fields, these two observations would contradict each other because some parts of TeV scale γ-rays traveling through the inter-galactic medium are converted into GeV scale γ-rays by electromagnetic cascade reaction; TeV scale γ-rays emit electron/positron pairs by scattering with extragalactic background lights and then created electrons and positrons emit GeV scale γ-rays by inverse Compton scattering with CMB photons until traveling $\mathcal{O}(1)$Mpc typically [4]. In the presence of cosmic magnetic fields, on the other hand, they can bend the trajectory of charged particles and consequently decrease the flux of secondary GeV scale γ-rays. From the observational lower limit on the bending angle the lower limit on the cosmic magnetic field strength was obtained. Several works were devoted to this subject [1–9]. Some of them took account of possible time variance of intrinsic blazar fluxes. The reported lower limit ranges $\mathcal{O}(10^{-14}\text{–}10^{-20})$G .

The problem is what the origin of cosmic magnetic fields is. No astrophysical process or early universe phenomenology is known to explain sufficient amount of magnetogenesis [10]. As for the inflationary magnetogenesis, many scenarios were proposed, aiming to explain the origin of magnetic fields of galaxies or galaxy clusters as well as cosmic magnetic fields [11–21]. The major obstacle in those models is the so-called "back reaction problem" [18, 22, 23]. Generation of magnetic fields during inflation inevitably increases the energy density of the electromagnetic field. If it becomes comparable with the energy density of inflaton then the dynamics of inflation is significantly altered. Consequently, the inflationary epoch may end or generation of magnetic fields is drastically suppressed. Demozzi, et al. [23] pointed out that in some specific models this problem is crucial and prevents generation of sufficient magnetic fields.

In the present paper we conduct a model independent analysis of inflationary magnetogenesis. Specifically, we derive an upper limit on the inflation energy scale by assuming that all observed cosmic magnetic fields are generated during inflation. Our constraint depends on neither details of the model lagrangian, the behavior of photon mode functions nor the shape of magnetic field spectrum. If the strength of cosmic magnetic fields is stronger than 10^{-15}G, the upper limit on the inflation energy density is $2.5 \times 10^{-7} M_{\rm Pl}$. As a consequence, tensor-to-scalar ratio r is severely constraint as $r < 10^{-19}$. Therefore in this case, if all observed cosmic magnetic fields are generated during inflation, it is almost impossible to detect gravitational waves from inflation in near future observations. Conversely, if any signatures of inflationary gravitational waves are detected, then the cosmic magnetic fields should have another origin.

The rest of the paper is organized as follows. In section 2, we introduce the observational lower limit on the strength of the magnetic field. In section 3, we discuss the assumptions which are needed to derive the main result. In section 4, the derivation of the upper limit on the inflation energy scale is presented. In section 5, we investigate the validity of the third assumption and explore the possibility to evade the constraint. Section 6 is devoted to a summary of this paper. In appendix, we derive the observational lower bound of cosmic magnetic fields in terms of the magnetic power spectrum.

2 Observational Constraint on Magnetic Power Spectrum

From the observations of blazars, current strength of cosmic magnetic fields is constrained [1–9]. While the constraints in those literatures are given in terms of the correlation length of magnetic fields, it is straightforward to rewrite it for the power spectrum of magnetic fields as

$$B_{\rm eff}^2(\eta_{\rm now}) \equiv \int \frac{{\rm d}k}{k} F(kL)\,\mathcal{P}_B(\eta_{\rm now},k) \;\geq\; B_{\rm obs}^2, \tag{2.1}$$

where $\mathcal{P}_B(\eta_{\rm now},k)$ is the power spectrum of the magnetic field at the present,

$$F(z) \equiv \frac{3}{2} z^{-2}\left[\cos(z) - \frac{\sin(z)}{z} + z{\rm Si}(z)\right], \tag{2.2}$$

and ${\rm Si}(z)$ denotes the sine integral function. (See Appendix for derivation.) Here, η is the conformal time, the subscript "now" denotes the present value, $L \equiv 1$Mpc stands for the characteristic length scale for energy losses of charged particles due to inverse Compton scattering [4], $B_{\rm obs}$ is the observational lower limit on the strength of cosmic magnetic fields. For $z \geq 0$, $F(z)$ satisfies

$$0 < F(z) \leq 1, \quad 0 \leq zF(z) \leq \alpha, \quad \alpha \equiv {\rm Max}[zF(z)] \simeq 2.48. \tag{2.3}$$

Although $B_{\rm obs}$ still has a few orders of uncertainty, in the present paper we adopt the value reported by ref.[7] in which the latest data were analyzed. According to ref.[7], $B_{\rm obs} \simeq 10^{-15}$G unless the time variance of intrinsic blazar fluxes is significant.

The derivation of the formula (2.1) is given in Appendix. Here, instead of showing the detailed derivation, we provide intuitive understanding of it. For this purpose, let us replace $F(kL)$ by its asymptotic forms,

$$F(z) \sim \begin{cases} 1 + \mathcal{O}(z^2) & (z \ll 1) \\ \frac{3\pi}{4z} + \mathcal{O}(z^{-2}) & (z \gg 1) \end{cases}, \tag{2.4}$$

and drop $\mathcal{O}(1)$ numerical factors to obtain the approximate formula as

$$B_{\rm eff}^2(\eta_{\rm now}) \sim \int_0^{1/L} \frac{{\rm d}k}{k} \Big[\mathcal{P}_B(\eta_{\rm now}, k) \Big] + \int_{1/L}^{\infty} \frac{{\rm d}k}{k} \left[\frac{1}{kL} \mathcal{P}_B(\eta_{\rm now}, k) \right]. \tag{2.5}$$

Let us now think of a Fourier mode of the magnetic field. For $kL \ll 1$, the corresponding magnetic field can be treated as a homogeneous field, as far as the particle's trajectory (with the total length L) is concerned. Thus modes with $kL \ll 1$ contribute to the bending angle as if they are homogeneous fields. This explains the first term in the right hand side of (2.5). On the other hand, for $kL \gg 1$, the direction of the corresponding magnetic field randomly changes $N \sim kL$ times while the charged particle travels the total length L. If we were interested in the trajectory of the charged particle within one of short segments of the length $\sim k^{-1}$ then the magnetic field could be treated as a homogeneous field. Actually, we are interested in the total bending angle due to N segments. Because of the randomness of the direction, the total bending angle from N segments adds up to only $\sqrt{N}$ times the contribution from each segment. Therefore the contribution of modes with $kL \gg 1$ to the variance of the bending angle should acquire the weight of order $1/N \sim 1/(kL)$. This explains the second term in the right hand side of (2.5).

3 Four Assumptions

To derive the upper limit on the inflation energy scale, we need four assumptions.

3.1 Assumption 1: the form of kinetic term

First, we assume that the kinetic term of the photon field A_μ is of the form

$$\mathcal{L}_{\rm kin} = -\frac{1}{4} I^2(\eta) F_{\mu\nu} F^{\mu\nu}, \tag{3.1}$$

where it is understood that the time-dependence of $I(\eta)$ is due to its dependence on homogeneous, time-dependent fields present in the theory. Thus, $\mathcal{L}_{\rm kin}$ includes various interactions between the photon field and other fields [12, 13, 18, 23]. This form of coupling does not have to break either gauge or local Lorentz symmetry. In general the photon field can have additional interactions $\mathcal{L}_{\rm int}$:

$$\mathcal{L}_A = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm int}. \tag{3.2}$$

However we let $\mathcal{L}_{\rm int}$ unspecified. Even so, under the four assumptions introduced in this section, we can derive the upper limit on the inflation energy scale in a model independent way. Note that when $I = 1$ and $\mathcal{L}_{\rm int} = 0$, the usual Maxwell theory is restored.

This assumption on the form of the kinetic term is necessary to quantize the photon field and to define the kinetic energy density. The photon field A_μ can be separated into scalar and vector modes as

$$A_\mu(\eta, \mathbf{x}) = (A_0, V_i + \partial_i S), \qquad \partial_i V_i = 0, \tag{3.3}$$

where A_0 and S are the scalar modes and V_i are the vector modes. Let us quantize the vector modes. After Fourier transformation with respect to the spatial coordinates, expansion by polarization vectors and mode expansion, we impose the standard commutation relation on the creation and annihilation operators.

$$V_i(\eta, \mathbf{x}) = \sum_{p=1}^{2} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \epsilon_i^{(p)}(\hat{\mathbf{k}}) \left[a_{\mathbf{k}}^{(p)} u_k^{(p)}(\eta) + a_{-\mathbf{k}}^{\dagger(p)} u_k^{(p)*}(\eta) \right] \tag{3.4}$$

$$\left[a_{\mathbf{k}}^{(p)}, a_{\mathbf{k}'}^{\dagger(p')} \right] = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}') \delta^{pp'}, \tag{3.5}$$

where $u_k(\eta)$ is the mode function of the photon vector mode, p $(= 1, 2)$ is the polarization label and $\epsilon_i^{(p)}(\hat{\mathbf{k}})$ is the polarization vector satisfying

$$k_i \epsilon_i^{(p)}(\hat{\mathbf{k}}) = 0 \quad (p = 1, 2), \qquad \sum_{p=1}^{2} \epsilon_i^{(p)}(\hat{\mathbf{k}}) \epsilon_j^{(p)}(-\hat{\mathbf{k}}) = \delta_{ij} - \frac{k_i k_j}{k^2}. \tag{3.6}$$

Then the canonical commutation relation for V_i requires the normalization condition of mode function $u_k(\eta)$ as

$$I^2 \left(u_k^{(p)} \partial_\eta u_k^{(p)*} - u_k^{(p)*} \partial_\eta u_k^{(p)} \right) = i \qquad (p = 1, 2). \tag{3.7}$$

Now let us define the kinetic energy density of the electromagnetic field as

$$\rho_{\mathrm{kin}}(\eta) = \frac{I^2}{2} \int \frac{\mathrm{d}k}{k} \left[\mathcal{P}_E(\eta, k) + \mathcal{P}_B(\eta, k) \right], \tag{3.8}$$

where we have defined power spectra of electric and magnetic fields as

$$\mathcal{P}_E(\eta, k) = \frac{k^3 |u_k'(\eta)|^2}{\pi^2 a^4(\eta)}, \quad \mathcal{P}_B(\eta, k) = \frac{k^5 |u_k(\eta)|^2}{\pi^2 a^4(\eta)}. \tag{3.9}$$

3.2 Assumption 2: Avoidance of strong coupling

The second assumption is that

$$I(\eta) \geq 1 \quad \text{for } \eta \geq \eta_i, \tag{3.10}$$

where η_i is the conformal time at the beginning of inflation.

This assumption essentially states that the effective coupling constants of the photon field to other fields should be always smaller than present values. For example, let us consider the interaction between the photon and a charged fermion as

$$\mathcal{L}_{\mathrm{int}} \ni -e\bar{\psi}\gamma^\mu \psi A_\mu. \tag{3.11}$$

In order to evaluate the effective coupling constant, we should canonically normalize the fields. Let us suppose that the fermion ψ is already canonically normalized. The canonically normalized photon field is $A_\mu^c \equiv I A_\mu$. Then the interaction term is rewritten as

$$\mathcal{L}_{\mathrm{int}} \ni -\frac{e}{I} \bar{\psi}\gamma^\mu \psi A_\mu^c. \tag{3.12}$$

It is now clear that e/I is the effective coupling constant. Therefore if $I \ll 1$, the effective coupling constant becomes large and the tree level analysis would be invalidated. In order to justify the tree level analysis, we need to assume that I is bounded from below by a positive constant I_0. For simplicity we set I_0 to be the present value of I, i.e. $I_0 = 1$.

3.3 Assumption 3: Small back reaction

The third assumption is that the kinetic energy density of electromagnetic field is smaller than that of inflaton,

$$\rho_{\rm kin}(\eta) < \rho_{\rm inf} \quad \text{for } \eta_i \le \eta \le \eta_f, \tag{3.13}$$

where η_i and η_f are the conformal time at the beginning and the end of inflation, and hereafter we ignore the time-dependence of the inflaton energy density $\rho_{\rm inf}$.

This assumption is closely related to the condition for avoidance of the back reaction problem,

$$|\rho_{\rm kin}(\eta) + \rho_{\rm int}(\eta)| < \rho_{\rm inf} \quad \text{for } \eta_i \le \eta \le \eta_f. \tag{3.14}$$

Note that eq.(3.13) and eq.(3.14) are different. In general, the total energy density of the photon field includes not only the kinetic energy density $\rho_{\rm kin}$ but also the interaction energy density $\rho_{\rm int}$ due to the additional interaction terms $\mathcal{L}_{\rm int}$. If the interaction energy density is non-negative ($\rho_{\rm int} \ge 0$) then eq.(3.14) requires eq.(3.13). Even if the interaction energy is negative ($\rho_{\rm int} < 0$), unless the two contributions $\rho_{\rm kin}$ and $\rho_{\rm int}$ cancel each other with a sufficiently good precision, eq.(3.14) generically requires eq.(3.13). Therefore the third assumption eq.(3.13) is mandatory unless negative $\rho_{\rm int}$ precisely cancels out positive $\rho_{\rm kin}$.

In section 5, we confirm the necessity of the third assumption in the case of gauge and local Lorentz invariant quadratic interactions and explore the possibility of the precise cancellation between $\rho_{\rm kin}$ and $\rho_{\rm int}$.

3.4 Assumption 4: Magnetogenesis during inflation

The fourth assumption is that all observed magnetic fields are generated during inflation. In particular, the conformal symmetry of the photon field action is broken appreciably only in the inflationary era. Since the electric conductivity of the universe increases after the end of inflation [11], we have

$$B_{\rm eff}^2(\eta_{\rm now}) \le a_f^4 B_{\rm eff}^2(\eta_f), \tag{3.15}$$

where we have set $a(\eta_{\rm now}) = 1$.

By using eq.(3.10), (3.13) and the fact that $B_{\rm eff}$ is smaller than the usual definition of magnetic field strength ($0 < F(kL) \le 1$), we obtain

$$B_{\rm eff}^2(\eta_i) < 2\rho_{\rm inf}. \tag{3.16}$$

Assuming the instantaneous reheating, we find the scale factor at the end of inflation is given by

$$a_f^4 = \frac{\rho_\gamma}{\rho_{\rm inf}} \tag{3.17}$$

where $\rho_\gamma \simeq 5.7 \times 10^{-125} M_{\rm Pl}^4 \simeq 5.2 \times 10^{-12}{\rm G}^2$ is the present energy density of radiation. Eq.(3.15), (3.16) and (3.17) lead to the following inequality.

$$\frac{a_i^4 B_{\rm eff}^2(\eta_i)}{a_f^4 B_{\rm eff}^2(\eta_f)} < 10^{-68} \times \exp[-4(N_{\rm tot} - 50)] \left(\frac{B_{\rm low}}{10^{-15}G}\right)^{-2}, \tag{3.18}$$

where $N_{\rm tot} = \ln(a_f/a_i)$ is the total e-folding number of inflation. This inequality implies that

$$a_i^4 B_{\rm eff}^2(\eta_i) \ll a_f^4 B_{\rm eff}^2(\eta_f), \tag{3.19}$$

and thus states that the magnetic fields have to be significantly amplified during inflation to explain the observational lower limit eq.(2.1).

4 Upper Limit on Inflation Energy Scale

With the four assumptions stated in the previous section, we are now ready to derive the upper limit on the inflation energy scale. The derivation is independent of details of inflationary magnetogenesis models, the behavior of photon mode functions or the spectrum of the electromagnetic fields.

Independently from the specific functional form of $u_k(\eta)$, it can be shown that

$$\begin{aligned}|u_k(\eta_f)|^2 - |u_k(\eta_i)|^2 &= \int_{\eta_i}^{\eta_f} \mathrm{d}\eta\ 2|u_k(\eta)|\,|u_k(\eta)|' \\ &\le \int_{\eta_i}^{\eta_f} \frac{\mathrm{d}\eta}{k}\ 2k|u_k(\eta)|\,|u_k'(\eta)| \\ &\le \int_{\eta_i}^{\eta_f} \frac{\mathrm{d}\eta}{k}\left(k^2|u_k(\eta)|^2 + |u_k'(\eta)|^2\right),\end{aligned} \tag{4.1}$$

where we have used the inequality $2xy \le x^2 + y^2$ for real numbers x and y, in order to obtain the last inequality. Multiplying the both ends of eq.(4.1) by $F(kL)k^4/\pi^2$ and integrating it over k, we obtain

$$a_f^4 B_{\rm eff}^2(\eta_f) - a_i^4 B_{\rm eff}^2(\eta_i) < \frac{\alpha}{L}\int_{\eta_i}^{\eta_f} \mathrm{d}\eta\ a^4(\eta)\int\frac{\mathrm{d}k}{k}\left[\mathcal{P}_E(\eta,k) + \mathcal{P}_B(\eta,k)\right], \tag{4.2}$$

where we have used the second inequality listed in (2.3). Using the second, third and fourth assumptions as well as eq.(3.19), we obtain

$$B_{\rm eff}^2(\eta_{\rm now}) < \frac{2\alpha}{L}\rho_{\rm inf}\int_{\eta_i}^{\eta_f} d\eta\, a^4(\eta) \simeq \frac{2\alpha}{3H_{\rm inf}L}a_f^3\rho_{\rm inf} \tag{4.3}$$

where $H_{\rm inf}$ ($\simeq$ const.) is the Hubble expansion rate during inflation.

Substituting eq.(2.1) and eq.(3.17) into eq.(4.3), we finally obtain the upper limit on the inflation energy scale,

$$\rho_{\rm inf}^{1/4} < \frac{2\alpha}{\sqrt{3}L}\rho_\gamma^{3/4} M_{\rm Pl} B_{\rm obs}^{-2} \approx 2.5\times 10^{-7} M_{\rm Pl}\times\left(\frac{B_{\rm obs}}{10^{-15}G}\right)^{-2}. \tag{4.4}$$

Note this upper limit can become even stronger if details of reheating is taken into consideration instead of eq.(3.17). Provided that the dominant energy density behaves like matter ($\propto a^{-3}$) during reheating, the right-hand side of eq.(4.4) is multiplied by an additional factor $(\rho_{\rm reh}/\rho_{\rm inf})^{1/4} < 1$, where $\rho_{\rm reh}$ is the energy density at the end of reheating era.

Eq.(4.4) can be converted into the upper bound on the tensor-to-scalar ratio r under the slow-roll approximation,

$$r < 10^{-19}\times\left(\frac{B_{\rm obs}}{10^{-15}G}\right)^{-8}. \tag{4.5}$$

Therefore, if all observed cosmic magnetic fields are generated during inflation, it is extremely difficult to detect any signatures of primordial gravitational waves, for example direct detections or CMB B mode polarization. Conversely, if some observations reveal that r is larger than the upper bound (4.5), it implies that inflation cannot explain the origin of cosmic magnetic fields under the four assumptions.

Now let us discuss the intuitive understanding of the reason why we obtain the upper limit on the inflation energy scale. Eq.(3.13) can be rewritten as

$$\rho_{\rm kin} \;=\; \left(a^4\rho_{\rm kin}\right)\exp\left[4\sqrt{\frac{\rho_{\rm inf}}{3M_{\rm Pl}^2}}(t_f-t)\right]\frac{\rho_{\rm inf}}{\rho_\gamma} \;<\; \rho_{\rm inf}, \tag{4.6}$$

where t is the cosmic time. Here, we have used eq.(3.17) and the equation of the exponential expansion, $a_f/a(t) = \exp\left[H(t_f-t)\right]$. From eq.(4.6) we acquire several observations. First, at the end of inflation ($t = t_f$) eq.(4.6) does not constrain $\rho_{\rm inf}$. It is because if $\rho_{\rm inf}$ increases, $\rho_{\rm kin} \propto a^{-4}$ also increases at the same rate. Hence we have to consider the condition (4.6) before the end of inflation to obtain the upper bound of $\rho_{\rm inf}$. Second, during inflation ($t < t_f$) only left-hand side of eq.(4.6) depends on $\rho_{\rm inf}$. It is because $\rho_{\rm kin}$ decreases rapidly due to the cosmic expansion while $\rho_{\rm inf}$ is nearly constant. The further past from the end of inflation it is, the more difficult to satisfy eq.(4.6) it becomes. The only way to weaken the dilution of $\rho_{\rm kin}$ is decreasing $\rho_{\rm inf}$. Therefore the lower $\rho_{\rm inf}$ is favored and the upper limit exists. Third, the upper limit on $\rho_{\rm inf}$ is apparently relaxed if $a^4\rho_{\rm kin}$ is very small until right before inflation ends. However, since $B \propto u_k(\eta)$ and $E \propto u_k'(\eta)$, a rapid amplification of magnetic fields causes a huge increase of electric fields energy. Such a rapid amplification is restricted by the avoidance of the back reaction problem.

5 Additional Interaction Terms

The action for the photon field consists of not only the kinetic term $\mathcal{L}_{\rm kin}$ but also the additional interaction terms $\mathcal{L}_{\rm int}$. As already mentioned after (3.14), the third assumption eq.(3.13) is mandatory unless negative $\rho_{\rm int}$ precisely cancels out positive $\rho_{\rm kin}$. Therefore whether such a precise cancellation is possible is a significant question. The answer we shall draw in the following discussion is that it is rather difficult to achieve such a cancellation.

Here, it is perhaps worthwhile stressing that, as long as the four assumptions (including the third one) are satisfied, our main result eq.(4.4) holds even if $\rho_{\rm int}$ and $\rho_{\rm kin}$ precisely cancel out.

5.1 Gauge and Lorentz invariant quadratic term

In the quadratic level, the most general renormalizable interaction term which preserves gauge and local Lorentz symmetry is given by

$$\mathcal{L}_{\rm int} = \frac{1}{8}f(\eta)\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + \frac{1}{2}m^2(\eta)A_\mu A^\mu, \tag{5.1}$$

where $\epsilon^{\mu\nu\rho\sigma}$ is the totally anti-symmetric tensor with $\epsilon^{0123} = 1/\sqrt{-g}$, $f(\eta)$ is a function of homogeneous scalars. The first term is called axial coupling term. The second term is the effective mass term of the photon induced by expectation values of charged scalars. It stems from the kinetic term of the charged scalars, and the positivity of the time kinetic term implies the positivity of the mass squared m^2. This term spontaneously breaks the $U(1)$ gauge symmetry, and the longitudinal mode of photon field becomes a physical degree of freedom.

Actually, the axial coupling term does not contribute to the energy density of the photon field. Since the axial coupling term does not depend on the metric, its contribution to the energy momentum tensor is exactly zero. The effective mass term does contribute to $\rho_{\rm int}$ but the contribution is always positive because of the positivity of the mass squared. Therefore the cancellation between $\rho_{\rm int}$ and $\rho_{\rm kin}$ cannot occur.

5.2 Model with negative interaction energy

There is an existing model which gives a *negative* energy contribution from an additional interaction term. Turner and Widrow [11] proposed a model with non-minimal coupling, $\mathcal{L}_{\rm int} \propto RA_\mu A^\mu$, where R is the Ricchi scalar. This coupling can become an effective mass term of photon with negative mass squared. However this model has three critical problems. First, the longitudinal mode of photon becomes ghost [23]. Second, the negative energy contribution from $\mathcal{L}_{\rm int}$ exceeds $\rho_{\rm inf}$ and the back reaction spoils inflation when we require generated magnetic field is sufficient [23]. Third, this coupling explicitly breaks the gauge symmetry.

5.3 Energy conserving term

From purely phenomenological viewpoints, let us investigate the additional interaction term of the form

$$\sqrt{-g}\mathcal{L}_{\rm int} = \frac{1}{2}a^2 J^2(\eta) V_i^2 \qquad (J^2 > 0) \tag{5.2}$$

where $J(\eta)$ is a function of homogeneous scalar fields and V_i is the photon vector mode defined in eq.(3.3). This term is effective mass term of photon vector mode with *negative* mass squared. Note that this term has neither gauge invariance nor Lorentz invariance. It does not yield ghost field because it contains only vector modes by breaking Lorentz symmetry and we still assume that the kinetic term of photon is given by eq.(3.1). Although it may be hard to embed such a term in a viable elementary particle theory, it is worth investigating it since we can find an interesting way to realize the cancellation between $\rho_{\rm int}$ and $\rho_{\rm kin}$.

From eq.(5.2), the equation of motion is given by

$$u_k'' + \left(k^2 - a^2 J^2\right) u_k = 0. \tag{5.3}$$

Here we have assumed $I(\eta) = 1$ for simplicity, since otherwise the weak coupling effect due to $I \gg 1$ would make the interaction term irrelevant. At the same time we require the cancellation between $\rho_{\rm int}$ and $\rho_{\rm kin}$ for each mode,

$$|u_k'|^2 + (k^2 - a^2 J^2)|u_k|^2 = 0. \tag{5.4}$$

It is easy to show that eq.(5.3), eq.(5.4) and eq.(3.7) imply that

$$a^2 J^2(\eta) = \text{const}. \tag{5.5}$$

In other words, the coefficient of the quadratic term (5.2) should be constant.

The reason why only the interaction term with constant coefficient leads the cancellation is simple. It is the energy conservation. If there is no explicit dependence on time in the action (for example, if the time-dependence due to the scale factor $a(\eta)$ is canceled by time-evolving scalars), then the energy of the system is conserved by virtue of Noether's theorem. In the case of eq.(5.2), if $J(\eta)$ cancels the time dependence of $a(\eta)$, the photon energy (with respect to the conformal time η) is conserved. Note that the kinetic term of the photon field is originally free from $a(\eta)$. Therefore the energy density of photon does not increase even if the electromagnetic field strength increases. It is notable that, for this mechanism to work, the dynamics of the scalar fields included in $J(\eta)$ has to restore the time translation symmetry accidentally.

The above analysis implies that the magnetogenesis from inflation whose energy is larger than the constraint of eq.(4.4) may not be impossible in principle. However, in practice it is not easy to realize a model which exploits the energy conserving mechanism because the accidental symmetry restoration by the scalar field dynamics can be easily spoiled by various effects such as the back reaction of the photon field. Therefore it is fair to say that all the four assumptions (including the third one) are likely to be mandatory in a rather broad class of models and the derived upper limit on the inflation energy scale is considerably general.

6 Conclusion

In this paper we have derived a universal upper limit on the inflation energy scale under the following four assumptions. (i) The kinetic term of the photon field is of the canonical form up to a time-dependent overall factor. (ii) The effective coupling constants do not exceed present values and thus do not exhibit strong coupling. (iii) The kinetic energy of the photon field is always lower than the inflaton energy density during inflation. (iv) All observed cosmic magnetic fields are generated during inflation.

The derived constraint is eq.(4.4), $\rho_{\rm inf}^{1/4} < 2.5 \times 10^{-7} M_{\rm Pl} \times (B_{\rm obs}/10^{-15}G)^{-2}$. As a consequence, the tensor-to-scalar ratio r is bounded from above as eq.(4.5), $r < 10^{-19} \times (B_{\rm obs}/10^{-15}G)^{-8}$. We hardly expect that inflation is the origin of both cosmic magnetic fields and detectable gravitational waves if $B_{\rm obs} > 10^{-15}G$. Therefore the future detection of signatures of inflationary gravitational waves, if any, would imply tension between inflationary magnetogenesis and observations.

Although our constraint is valid in fairly broad class of inflationary magnetogenesis scenarios, we have investigated the possibility to evade it. In order to evade the constraint, at least one of the assumptions should be violated. The third assumption can be violated only if the energy density due to additional interaction terms and the kinetic energy density precisely cancel out. We have considered a possible mechanism which exploits a energy conservation law to realize the cancellation. However, it seems a challenge to build a realistic model equipped with such a mechanism.

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A Derivation of the Constraint on Magnetic Power Spectrum

In this appendix, we derive eq.(2.1). By Fermi and HESS observations, there is a lower limit on the bending angle of GeV scale cascade electrons and positrons in the inter-galactic medium. However, in the literatures the constraint on the cosmic magnetic field is given only in terms of the correlation length of magnetic fields [1–9] while theorists need the constraint in terms of the magnetic power spectrum. In this appendix we shall generalize the constraint on the cosmic magnetic field to more general spectra. Such a generalization makes the connection between the cosmic magnetic power spectrum $\mathcal{P}_B$ and the bending angle θ. For simplicity, we neglect effects of special relativity in this appendix.

Provided that a charged particle travels distance L in the background of a weak magnetic field $\boldsymbol{B}(\boldsymbol{r})$ from t_1 till t_2. Then the bending angle is given by

$$\boldsymbol{\theta} \simeq \frac{\boldsymbol{v}(t_1) - \boldsymbol{v}(t_2)}{v}, \tag{A.1}$$

where $\boldsymbol{v}(t)$ is the velocity vector of the particle. Note the absolute value of the velocity vector is constant. By using the equation of motion with Lorentz force, the difference of the velocity vectors is written as

$$\boldsymbol{v}(t_2) - \boldsymbol{v}(t_1) = \int_{t_1}^{t_2} \mathrm{d}\tilde{t}\ \dot{\boldsymbol{v}}(\tilde{t}) = \frac{e}{m}\int_{t_1}^{t_2} \mathrm{d}\tilde{t}\ \boldsymbol{v}(\tilde{t}) \times \boldsymbol{B}(\tilde{t}) = \frac{e}{m}\int_0^L \mathrm{d}\boldsymbol{x} \times \boldsymbol{B}(\boldsymbol{x}), \tag{A.2}$$

where e and m are the charge and the mass of the particle, respectively, $\boldsymbol{x}(t)$ denotes the orbit of the particle and its initial value is set to $\boldsymbol{x}(t_1) = 0$. Then, we assume θ is so small that the orbit can be approximated as a straight line, $\boldsymbol{x}(t) \simeq x_1(t)\hat{\boldsymbol{e}}_1$ where $\hat{\boldsymbol{e}}_1$ is the unit vector in the direction of the axis 1. By Fourier transforming $\boldsymbol{B}(\boldsymbol{x})$, we can perform the line integral

$$\int_0^L \mathrm{d}x_1\ \hat{\boldsymbol{e}}_1 \times \boldsymbol{B}(x_1\hat{\boldsymbol{e}}_1) = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\,\frac{e^{ik_1L}-1}{ik_1}\ \hat{\boldsymbol{e}}_1 \times \tilde{\boldsymbol{B}}(\boldsymbol{k}), \tag{A.3}$$

By using these equations, we find that the variance of $\boldsymbol{\theta}$ is given by

$$\langle \boldsymbol{\theta}^2 \rangle = \left(\frac{e}{mv}\right)^2 \int \frac{\mathrm{d}^3k\mathrm{d}^3k'}{(2\pi)^6}\,\frac{\left(e^{ik_1L}-1\right)\left(e^{ik'_1L}-1\right)}{-k_1k'_1}\,(\delta_{ij} - \delta_{i1}\delta_{j1})\langle \tilde{B}_i(\boldsymbol{k})\tilde{B}_j(\boldsymbol{k}')\rangle, \tag{A.4}$$

Since the divergence of magnetic field vanishes ($k_i\tilde{B}_i(\boldsymbol{k}) = 0$) and the cosmic magnetic fields are statistically isotropic and homogeneous, the square bracket in eq.(A.4) can be written as

$$\langle \tilde{B}_i(\boldsymbol{k})\tilde{B}_j(\boldsymbol{k}')\rangle = \frac{1}{2}(2\pi)^3\delta^{(3)}\left(\boldsymbol{k}+\boldsymbol{k}'\right)\left[\left(\delta_{ij} - \frac{k_ik_j}{k^2}\right)\frac{2\pi^2}{k^3}\mathcal{P}_B(k) + i\epsilon_{ijl}k_lH(k)\right], \tag{A.5}$$

where $\mathcal{P}_B(k)$ is the magnetic power spectrum and $H(k)$ stands for the helicity component of magnetic fields [24]. By substituting eq.(A.5) into (A.4), we obtain

$$\langle \boldsymbol{\theta}^2 \rangle = \frac{2}{3}\left(\frac{eL}{mv}\right)^2 \int \frac{\mathrm{d}k}{k}\ \mathcal{P}_B(k)\ F(kL), \tag{A.6}$$

$$F(z) \equiv \frac{3}{2}z^{-2}\left[\cos(z) - \frac{\sin(z)}{z} + z\mathrm{Si}(z)\right] \sim \begin{cases} 1 + \mathcal{O}(z^2) & (z \ll 1) \\ \frac{3\pi}{4z} + \mathcal{O}(z^{-2}) & (z \gg 1) \end{cases}, \tag{A.7}$$

where $\mathrm{Si}(z)$ denotes the sine integral function. For $z \geq 0$, $F(z)$ satisfies

$$0 < F(z) \leq 1, \quad 0 \leq zF(z) \leq \alpha, \quad \alpha \equiv \mathrm{Max}[zF(z)] \simeq 2.48. \tag{A.8}$$

In order to find a proper definition of the effective strength of the magnetic field (including its normalization) for a given spectrum $\mathcal{P}_B(k)$, as a fiducial configuration let us consider a homogeneous magnetic field whose direction is perpendicular to the particle's trajectory. Denoting the strength of the fiducial magnetic field as $B_\perp$, the bending angle is $\theta = L/R_\mathrm{L} = (eB_\perp L)/(mv)$, where R_L is the Larmor radius. On the other hand, for a statistically isotropic spectrum, the variance of the magnetic field in three-dimensions is three halves of the variance of the magnetic field projected onto the two-dimensional subspace perpendicular to the particle's trajectory. Thus, it is natural to define the effective strength of the magnetic field as

$$B_{\text{eff}}^2 \equiv \frac{3}{2}\left(\frac{mv}{eL}\right)^2 \langle \boldsymbol{\theta}^2 \rangle. \tag{A.9}$$

Combining this with the formula (A.6), we obtain

$$B_{\text{eff}}^2 = \int \frac{\mathrm{d}k}{k} F(kL)\, \mathcal{P}_B(k). \tag{A.10}$$

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