Quarkyonic matter: theory and phenomenology

Based on PRL107:152301,2011, 1204.3272 Stefano Lottini, PRL111 012301 with Sascha Vogel and Bjoern Beauchle
Also, 1006.2471 (PRC), with Igor Mishustin, 1105.0188 (JHEP) with Piero Nicolini

G. Torrieri

Helmholtz International Center for FAIR
Synopsis

**What is** quarkyonic matter

**Bold claim n.1:** I can define and explain what it is!

**Large** $N_c$ : A short introduction

**An estimate** from a percolation Ansatz

**Towards a pheonomenolgy** of quarkyonic matter in supernova and at FAIR/NICA/SPS/RHIC@low $\sqrt{s}$

**Bold claim n.2:** I might have experimental signatures for it
What is "Quarkyonic matter"

Name introduced in L. MacLerran, R. Pisarski, NPA796 (2007) 83-100

300 Citations, 5 conferences, 1 wikipedia entry. So its a big deal! but definitions found in the literature so far include, these and more...

- Coexistence between Confinement + pQCD (Mclerran, Pisarski, 2007)
- Confinement + Chiral restoration (Fukushima, McLerran, 2008)
- Deconfinement + Chirally broken (Satz, Csernai, ...)
- Chiral spiral inhomogeneities (Kojo, Pisarski, Tsvelik, 2009)
- Generic chirally inhomogeneous regions (Buballa et al)
- Condensation of "baryons" in 2-color QCD (Hands, Skullerud, Giudice)
All relevant for “high density low temperature” matter, produced in neutron stars or “low energy” uRHICs

- RHIC low energy scan
- SPS experiment NA61
- FAIR
- NICA
What is "Quarkyonic matter"

The "minimalist answer": A name invented in a highly cited paper, *Nucl. Phys. A* 796, 83 (2007), by McLerran and Pisarski, to describe matter at \( \mu_Q \geq \Lambda_{QCD}, T < T_c \).

In "physical terms", of chemical potential of more than "one baryon per baryonic volume" but "low temperature wrt deconfinement".

By definition, this is the matter we hope to produce at FAIR/NICA/RHIC scan, and which should exist in neutron stars! So "quarkyonic matter" is simply "quark matter" dense enough that a Fermi surface forms.

Was the name a gimmick, Or is there something more to this?
The issue: QCD at $\mu_Q \geq \Lambda_{QCD}, T < T_c$ is really not understood

Hadronic or EFTs (σ,NJL,PNJL etc): assume $p_i - p_j \ll \Lambda_{\text{fundamental}}$

Only scale in QCD is $\Lambda_{\text{fundamental}} = \Lambda_{QCD}$, and $p_i - p_j \sim \mu_Q \sim \Lambda_{QCD}$

So EFT at $\mu_Q \simeq \Lambda_{QCD}$ means Taylor-expanding around 1!

For any operator $\hat{O}(x)$ (e.g. $q, P, ...$) Not guaranteed $\hat{O}^n \ll \hat{O}^{n-1}$ The critical point illustrates this painfully: different models (σ,NJL,PNJL, give wildly different predictions...

M.Stephanov

hep-lat/0701002
**Lattice QCD** has the **sign problem**, any expansion is good for $\mu_q \ll T$

Fodor et al: Reweighting (yes!)  
Karsch et al. (Taylor) maybe  
Philipsen et al. Taylor+i $\mu$ No?

By arguments in the previous slide, most Taylor-expanding methods fail at critical point. We are still not sure critical point exists!

**AdS/CFT** apart from the many unrealistic assumptions, classical Gauge dual depends on $N_c \to \infty$, on which **more later**
Any high density calculation is an essentially educated guess. Expect surprizes, dont be disappointed if your favourite model not even qualitatively correct. No reason for it to be!!!! (eg, the critical point might not exist, no matter how many models predict it. Separation of confinement and chiral symmetry, or any chiral inhomogeneous phases, also in doubt)

Chiral symmetry breaking likely 80% due to confinement (since $M_{baryon} \sim 80\% N_c/R_{baryon}$) so models incorporating chiral symmetry without confinement unreliable eg, Heinz+Giacosa, PRD85 (2012) 056005, $T^\sigma_{c-model} \sim \langle \phi_0(T = 0) \rangle \sim \sqrt{N_c} f_\pi$, $T^{QCD}_c \sim \Lambda_{QCD} \sim N_c^0$ Confinement quintessentially non-perturbative, EFTs problematic

FAIR/NICA/RHICbes is a “shot in the dark”, requiring what if phenomenology ("If in FAIR regime X happens, we should see Y")
The **only** hierarchy that seems to be **roughly correct** is the large $N_c$ limit

$$N_c \simeq 3 \gg 1, N_c^{-1} \ll 1$$

You may laugh, but it establishes a **rigorous hierarchy**: “fast” quarks (quantum degrees of freedom) vs “slow” baryons (immobile heavy classical background)

- Quasi-particle picture of mesons
- Quasi-classical structure of baryons (Skyrme model)
- OZI rule

all compatible with this hierarchy
What do we mean by "varying $N_c$"?
't Hooft, over 20 years ago, showed that provided a continuous limit exists where $N_c \to \infty$, $g_{YM} \to 0$, $g_{YM}^2 N_c \to \lambda$.

Not solution to all problems: $g_{YM}$ weak, but $\lambda$ has approximately same running as QCD, hence $\Lambda_{QCD} \sim N_c^0$.

Theory still strongly coupled and confining below $\Lambda_{QCD}$

but in this limit drastic simplifications are possible, as some observables $\sim N_c^2$, some $\sim N_c^0$ etc. Plugging in $N_c = 3 \to O(10)$ hierarchy
$N_c$ scaling results...

- Planar diagrams dominate, $\Rightarrow$ Strong force $\leftrightarrow$ strings
  Tension $\sim \lambda$, breaking probability $\sim N_c^{-1}$
  AdS/CFT ultimately comes from this analogy!

- Mesons $\rightarrow$ weakly interacting quasiparticles
  Confinement "survives" in $\sim N_c^{-1}$ coupling constant

- Baryons $\rightarrow$ strongly interacting semi-classical states

- The phase diagram...
If deconfinement \( \iff \) quark-hole loops “beat” gluon antiscreening...

Deconfinement line flattens, for deconfinement \( \mu_Q \sim N_c^{1/2} N_f^{-1/2} \Lambda_{QCD} \)

NB: higher \( n \) order hierarchy \( \sim (N_c/N_f)^{n(n-1)} \), does not help!
Note: Above is a big if

Above reasoning contradicts, for example, bag model intuition, where $\mu_{Q}^{\text{crit}} \sim T_{c} \sim \Lambda_{QCD} \sim N_{c}^{0}$. The “trick is” it assumes non-perturbative contributions to $\beta$-function/confinement order parameters don’t have a different $N_{c}$ dependence, which could dominate at $N_{c} = 3$. Lets continue to assume this, but its unproven! either alternative is instersting
line separating "vacuum" from "dense nuclear matter" narrows, since baryon abundance in vacuum phase $\sim \exp(-N_c \Lambda_{QCD}/T) \to 0$
McLerran+Pisarski, arXiv:0706.2191: line at

$$\Lambda_{QCD} \leq \mu_Q \leq \sqrt{N_f/N_c} \Lambda_{QCD}$$

defines new "quarkyonic" phase!

NB: AGS,SIS $\mu_B \simeq 800$ MeV < $m_N$, so it might still be out there!
Inter-quark distance in this phase $\sim N_c^{-1/3} \to 0$, asymptotic freedom in configuration space! Confined but quasi-free quarks below fermi surface and $P \sim N_c$ (quark-hole?)

NB: If color can propagate at inter-baryonic distances, “quarkyonic matter” $\equiv$ QGP, “bag model intuition” correct ). otherwise , A new phase to look for at low energy, high density (Neutron stars, FAIR, NICA, etc.), In alternative to critical point, but...
Even if we assume our large $N_c$ limits are under control....

Can we exclude phase transitions in $N_f/N_c$?

<table>
<thead>
<tr>
<th>Quantity</th>
<th>$N_c \to \infty$</th>
<th>QCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^{binding}_{Nucleus}$</td>
<td>$N_c \Lambda_{QCD}$</td>
<td>$\ll \Lambda_{QCD}, m_\pi$</td>
</tr>
<tr>
<td>$\Delta E_{spin-flip}$</td>
<td>$\sim \Lambda_{QCD}/N_c$</td>
<td>$\sim \Lambda_{QCD}$</td>
</tr>
<tr>
<td>Ground state</td>
<td>Crystal</td>
<td>Liquid</td>
</tr>
</tbody>
</table>
When you are expanding around the right vacuum, a \( \sim 30\% \) correction is OK. When you are expanding around the wrong vacuum, any correction is catastrophic. Sometimes it’s easy to see this (tachyons!), sometimes not (confinement?)
In fact, phase transitions in $N_c$ are certain to happen!

Confined $SU(N_c)_{N_f=0}$ invariant under symmetry $Z_N$, spontaneously broken by deconfinement at high $T$.
These symmetry principles dictate that deconfinement is a phase transition, at $N_f = 0$

At $N_f/N_c \sim 1$, according to the lattice, deconfinement is a cross-over.

So, unless something weird is going on (GW point?), there is a critical point in $N_c$ for confinement.
“finding” a dual gravity description of this critical point, and measuring its critical exponents, an important test for Gauge/Gravity duality (M.Sprenger, P.Nicolini, M.Kaminski, GT, work in progress)
In fact, phase transitions in $N_c$ are certain to happen II

At $N_c \to \infty$, $\mu_B/N_c \sim \Lambda_{QCD}$, the ground state of nuclear matter is widely understood to be a Skyrme crystal. I.Klebanov, Nucl.Phys.B262:133,1985

From that paper... Of course, this treatment ignores the kinetic energy of skyrmions. It can be roughly estimated to be $1/Mc a^2 \sim 100 \text{ MeV}$. Energy of this order is enough to unbind the crystal at $N_c = 3$

Roughly speaking... baryon mass $\sim N_c$, baryon Fermi motion energy $\sim N_c^0$, so baryon Fermi motion momentum $\sim N_c^{1/2}$, inter-baryon binding energy $\sim N_c$. As we go down in $N_c$, crystal melts into a fluid; This must be a phase transition, as symmetries change!
The Landau algorithm:

a) Formulate simple picture of the problem
b) Solve it

The best physicist in the USSR is Yakov Frenkel, who uses in his papers only quadratic equations. I am slightly worse, I sometimes use differential equations.


The Feynman algorithm

a) Write down the problem
b) Think REALLY hard
c) Write down solution

The rest of this talk: Toy models which hopefully reproduce the issues discussed until now!
Nuclei and their interactions at large $N_c$ use the Van Der Waals EoS

$$(\rho^{-1} - b) \left( P + a\rho^2 - g\rho^3 \right) = T$$

$b$ is the excluded volume

$a, g$ are the interaction. For any radial interaction $V(r)$, they came out as terms in the expansion of $\prod_{ij} \int dx_{ij} e^{-\frac{V(x_{ij})}{T}}$

Solvable analytically, universal, connected to black holes (A. Chamblin, R. Emparan, C. V. Johnson and R. C. Myers, PRD 60, 064018 (1999))
Only scale at of theory large $N_c$ is $\Lambda_{QCD}$! This is the inverse of the confinement scale (empirically $\Lambda_{QCD}^3 \sim \langle \bar{\psi} \psi \rangle$).

It is therefore natural to decompose VdW equation into dimensionless components (functions of $N_c$) and the appropriate power of $\Lambda_{QCD}$

$$(\rho^{-1} - \alpha) \left( P + \beta \rho^2 - \gamma \rho^3 \right) = T$$

$\alpha$ is in $\Lambda_{QCD}^{-3}$

$\beta$ is in $\Lambda_{QCD}^2$

$\gamma$ is in $\Lambda_{QCD}^5$

Factors of $\Lambda_{QCD}$ neglected henceforward
BUT universality has limits ... 

- No chiral symmetry *(Ask me at the end!)*

- in VdW, interactions integrated out so carry **no entropy**.
  
  inappropriate for measuring the entropy content of, say, electron gas (interaction-dominated).

  So might be inappropriate for understanding the **liquid** phase if its entropy resonance (or residual quark interaction) dominated as in the quarkyonic conjecture

  **...but** can still give phase transition line!
How does $\alpha$ depend on $N_c$?

- $\alpha$ can’t go below unity (deconfinement).

- In the large $N_c$ limit, the only scale is $\Lambda_{QCD}$. It is therefore natural that

$$\lim_{N_c \to \infty} \alpha = \Lambda_{QCD}^{-3}$$

It can not have an $N_c^{a>1}$ leading term, since Baryon size does not diverge. But in our world, $\alpha \gg \Lambda_{QCD}^3$

$$\alpha \sim 1 + \frac{A}{N_c}$$

and the $A$ term dominates!
My guess is, we don’t live in a large $N_c$ world!

The other scale of the problem is the number of neighbours in tightly packed systems, "kissing number", exact dependence on $d$ unknown $k(d) \sim 2^{\zeta d}$, $k(1,2,3,4) = 2, 6, 10, 24$, of course $\sim N_c^0$, $k(d = 3) \gg 3$.
Pauli exclusion principle in valence picture irrelevant for $N_c \gg k(d)$, but not for $N_c = 3$. Keeps nuclei further apart than $\Lambda_{QCD}^{-1}$.
\[
\alpha \sim 1 + \frac{N_N}{N_c} \sim 1 + \frac{k(d)}{N_c} \sim 1 + \frac{10}{N_c}|_{3d}
\]

- Fits nuclear VdW at \( N_c = 3 \)
- Compatible with strongly coupled nuclear matter at \( N_c \gg 3 \)
- Understandable by Pauli exclusion principle
  Spin, flavor complicates things. But in our world \( \Delta E|_{spin\,flip} \sim \Lambda_{QCD} \), flipping flavor suppressed
\[ \alpha \sim 1 + \frac{N_N}{N_c} \sim 1 + \frac{k(d)}{N_c} \sim 1 + \frac{10}{N_c|_{3d}} \]

What this means:

• confinement scale \( \gg \) nuclear separation up to \( \sim \) deconfinement potential!

• Expansion in \( \rho^n / \Lambda_{QCD}^{3n} \) \underline{progressively worse} but \underline{always converges}

Trust diagram, but not factors of \( \mathcal{O}(1) \)
• $\beta, \gamma$ Have to scale the same way, since same interaction

• Witten’s solitonic picture of the nucleon: $\beta, \gamma \sim N_c$
  Weak ($\ll$ even $m_\pi$ ) nuclear force an accidental cancellation.
  Witten says that all $(2, 3, n)$ body forces scale as $N_c$ . Weinberg’s
  hyperarchy, $n - body$ nuclear forces $\sim (k/\Lambda_{QCD})^n \sim (\rho^{1/3}/\Lambda_{QCD})^n$
  complementary: $N$ body forces all $\sim N_c$ but $2 > 3 > ...n$ Same as VdW
  expansion! .

• Y. Hidaka, T. Kojo, L. McLerran and R. D. Pisarski, 1004.2261 :
  This picture is wrong (skyrmion unstable, stabilized by large quantum
  corrections which put $N_c - 1$ quarks into diquarks).
  Nuclear force carried by remaining quark, so $\beta, \gamma \sim N_c^0$ or $\sim \log N_c$ Weak
  nuclear force natural

  Room for phenomenological playing: Try $\beta, \gamma \sim N_c^\nu, \nu = 0, 1$
Can we say anything more about a critical $N_c$?

$N_c << k(d)$

$k(d)$: "kissing number"
$k(d=3) \sim O(10)$

$N_c >> k(d)$

$\mathcal{O}N_c \to \infty$ baryons classical. In-medium ($\rho_B \sim \Lambda_{QCD}^3$), $N_c \to \infty$ is when Pauli principle satisfied by color rotations:

$N_c \geq N_{neighbors} \sim k(d = 3) \sim \mathcal{O}(10)$. 
such a quantum-to-classical transition might drive $E_{\text{binding}}^{NN} \sim \mathcal{O}(10) \text{GeV} \ll m_\pi, \Lambda_{\text{QCD}}$.

"quarkyonic matter" might be nuclear matter at $N_c \gg N_{\text{neighbours}}$. Or not as dependence on flavor, density not so clear. But $N_{\text{neighbors}}$ scaling motivates percolation.
Basic idea: You have a (regular or irregular) lattice of sites, which can be "on" and "off" (links "switched on", particles "in sites", etc), with probability $p$. Count adjacent sites $\langle N_{\text{sites}} \rangle$. When $p \simeq p_c$, $\langle N_{\text{sites}} \rangle \rightarrow \infty$
• second order transition ($\langle N_{sites} \rangle \equiv \text{correlation}$), with critical behavior.

• $p_c(1D) = 1, p_c(2D) \sim \mathcal{O}(0.5), p_c(3D) \sim \mathcal{O}(0.2)$ (depends on $N_{neighbors}$). So "small" $\sim N_c^{-1}$ correction could trigger it.

Some people have tried to describe deconfinement by percolation of strings/bags, but order of phase transition missed.
an EFT of $\mu_Q \sim \Lambda_{QCD}, N_c \gg 1$ matter

**Baryons** are heavy and immobile “background”

**Quarks** are delocalized, since $\rho_{baryon}^{-1/3} \leq R_{baryon}$ Such delocalization compatible with confinement

An immediate physical analogy: conductor in QED, with baryons playing the role of atoms.
Such a “conducting phase”, not predicted by any EFT, could be the “surprise” we were looking for.

But remember, conductor insulator phase transition is governed by number of electrons in the “conducting band”.

However, since Quark/baryon $\sim N_c$, conductor/insulator transition in full $T - \mu_Q - N_c$ space!
$N_c$ scaling and Percolation at $\mu_Q = \Lambda_{QCD}$

Intuitively, relevance of percolation **clear**. With $N_c$ colors, ways two baryons can interact with one another grows **fast** with $N_c$. Correlation length **diverges** at percolation, so existence of transition **independent** of microscopic details (within reason)
Calculating percolation probability at $\mu_Q = \Lambda_{QCD}$

In large $N_c$ limit, assume "perturbative" ($\sim \lambda N_c^{-1}$) interactions between "confining" quarks. Picture insensitive to further details.

**NB:** all dependence on $N_c$ only, the $N_c$ vs $N_{neighbors}$ requirement for classical baryons also depends on $N_f$. This transition different from VdW, as only scales with $N_c$!
An ansatz with confinement and correct $N_c$ scaling

\[ p = 1 - (q_{(1),ij})^{(N_c)^\alpha} \quad , \quad q_{(1),ij} = \int f_A(x_i) \, dx_i \int f_B(x_j) \, dx_j \, (1 - F(|x_i - x_j|)) \]

Mathematically very similar to Glauber model, dont need to get $\sigma$ exactly right to get $N_{part}$ dependence. In same way, we put in sample propagators to get $N_c$ dependence.
We assume a density distribution with a range of $\rho$ of the form

$$f_{A,B}(x) = \rho \left( \Lambda_{QCD}^{-1} - |x - x_{A,B}^{\text{center}}| \right)$$

A range of $\rho$ considered.
...and a range of probability amplitudes for the exchange $i \leftrightarrow j$ which respect

- Confinement (rapid fall-off at distances $\Lambda_{QCD}^{-1}$)
- $N_c$ scaling ($\sim \lambda/N_c$)

$$F(y) = \frac{\lambda}{N_c} \mathcal{N} \left\{ \begin{array}{l} \theta(1 - \frac{y}{r_T}) \\ \exp \left(- \frac{3 y^2}{4 r_T^2}\right) \\ \frac{2 r_T^2}{\pi y^2} \sin^2 \left(\frac{y}{r_T}\right) \end{array} \right. $$

(Theta-function and Gribov-Zwanziger propagators)
Varying $r_T/\Lambda_{\text{QCD}}$

($=0.7,1.0,1.5,1.9,2.3,2.7,3.5$)

Rapid growth with $N_c$ at $p = p_c$ independently of details of propagator.
Transition seems universal at $N_c \sim \mathcal{O}(10)$
Critical $N_c$ for $\Theta$-function $P_{i \leftrightarrow j}$ in position and momentum

“typical” Parameters of order unity give a critical number of colors for percolation well above 3. These are lower limits, since we assume hexagonal lattice (Skyrme cubic and disordered $p_c$ higher). So $N_c^{crit} = 3$ disfavored but not excluded at $\mu_Q = \Lambda_{QCD}, T = 0$. 
But let's vary $\mu_Q$: Percolation and deconfinement

Percolation: $\rho - N_c$ anti correlated.
Deconfinement: $\rho - N_c$ correlated $\mu_B^{\text{dec}} \sim N_c^{1/2} N_f^{-1/2} m_B \sim N_c^{3/2} N_f^{-1/2} \mu_q$

(\text{Since for deconfinement:})

$\rho_B^{\text{deconfinement}} \sim N_c^{1/2} N_f^{-1/2}$

\[ \sum_\text{color} \]

\[ \sim \sum_\text{hole} \]

\[ \sum_\text{quark} \]
Remember 1 percolating quark negligible for wavefunction of hadron. Need $\mathcal{O} \left( N_c^{1/2} N_f^{-1/2} \right)$ or higher quarks to break hadron apart. But $N_c = 3$ !!!
Deconfinement happens below percolation, i.e., percolation transition does not exist separately from deconfinement.

Percolation, deconfinement separate (Quarkyonic phase?)
What is this critical $N_c$? Percolation in a "glass": Conceptually similar, technically more involved

- "Nearest neighbor" not uniquely defined: Baryons overlap
- Interactions to arbitrary distance $\rightarrow$ percolation for arbitrarily low thresholds?
Solution: MC renormalization

Decimate glass to a cubic grid, over many “glass events”. Do percolation over cubic grid

Since percolation at critical point, critical probability should be fixed point of renormalization step, independent of $b$
Gimel,Nicolai,Durand, J Phys A Math Gen 32 L515 (1999)

\[ p^* (b, \Theta(x_T, \lambda, N_c)) = \Pi_{physical} (\Theta(x_T, \lambda, N_c)) + \beta b^{-y}, \quad y = 0.81 \]
Density and $N_c$ tightly correlated. Percolation at $N_c = 3$ excluded at $\rho_B \sim \Lambda_{QCD}^3$. But could there be percolating region at $\Lambda_{QCD}^3 < \rho_B < \rho_B^{\text{deconfinement}}$?
Equations for confinement: \textbf{Ideal} gas of non-relativistic baryons, mesons

\[ n_{\text{conf}}^{\Lambda^3_{\text{QCD}}} = G \sum_{n=1}^{\infty} (-1)^n \frac{n \gamma^2}{\beta} \sinh \left( (\sqrt{N_c \beta})^n \right) K_2 (n \gamma \beta) \]

\[ e_{\text{conf}}^{\Lambda^3_{\text{QCD}}} = G \sum_{n=1}^{\infty} 3(-1)^n \frac{n \gamma^3}{\beta} \cosh \left( (\sqrt{N_c \beta})^n \right) \left( \frac{3}{\gamma \beta} K_2(n \gamma \beta) + K_1(n \gamma \beta) \right) \]

Where \( G = \frac{4\pi g_f^2 g_s(N_c)}{(2\pi)^3 \sqrt{N_f}} N_c^{5/2} (T - T_c) \) and

\[ \frac{T}{\mu_B} = \frac{1}{\beta N_c^{1/2}} \quad , \quad \frac{m}{\mu_B} = \frac{\gamma}{N_c^{1/2}} \quad , \quad \frac{p}{\mu_B} = \frac{\alpha}{N_c^{1/2}} = 1 \]

\( * \ T \simeq 0 : \) All energy carried by baryons. \( T \simeq T_c : \) deconfinement happens at all \( \mu_B : \) Parametrize confinement line by \( T^2 + N_c^2 \mu_q^2 = \mathcal{O} (1) \Lambda_{\text{QCD}}^2 \)
Quarkyonic phase might exist at $\Lambda_{QCD} \leq \mu Q \leq N_c N_f^{-1} \Lambda_{QCD}$.

In PRL we neglected Density-$N_c$ curvature and fixed density to $\mu_B \sim \Lambda_{QCD}$.

\[ N_c \rho / \Lambda^3 \]

A sliver of $n - \rho - N_c = 3$ space which is percolating but confined seems to be there, but...
Width depends a lot on whether $N_f = 2$ or $N_f = 3$. “Systematic error too big. Need phenomenology!”
What does a percolating phase look like?
How do confinement and free quarks coexist? McLerran, Pisarski, Kojo: quark Fermi surface and baryonic excitations. But..

\[
\frac{dS}{dV} = \frac{dP}{dT} = \frac{P + \rho - \mu n}{T}
\]

And any diagrams of this type will give \( T\mu_B \) contributions to pressure, and hence \( dS/dV \). So need theory with confinement but free quarks! Physical example: Electrons in a metal
Confinement and quasi-free quarks: spin-color-flavor separation?

Confinement remains, so regions above $\sim 1 \text{fm}$ can-not be color charged. (Same problem at $T \geq T_c$, but correlations required to maintain confinement can be $\left( N_c^0 \Lambda_{QCD}^{-1} \right) \ll s (T \geq T_c) \sim N_c^2 T^3$

Spin-color-flavor separation can achieve this and maintain $N_c, N_f$ scaling! 

Pisarski, McLerran, Kojo, NPA843 (2010) 37-58 and subsequent works: implement this by 1D WZW model.

$$S = S_{2N_f}^{WZW} [h_{\text{color}}] + S_{N_c}^{WZW} [h_{\text{flavor}}]$$

which generalized spin-charge separation to $SU(N_f), SU(N_c)$.

Modifications to $S_{2N_f}$ could localize color, maintain $\sim N_c$ degeneracy.

“Naively” WZW incompatible with percolation (1D), but could work as EFT in percolation regime. Work in progress.
All of this is very nice, but let us recap the tower of assumptions!

**Fundamentally** the linking of deconfinement with the perturbative $\beta$ function scaling with $N_{f,c}$ might be incorrect. Bag model intuition might hold!

$N_c$ might be too low and $N_f$ might be too high
(Counting on “2/3 being high because $m_s \gg \Lambda_{QCD}$ shaky!

**Color-flavor-spin** separation not yet worked out. Interplay of confinement and asymptotic freedom not clear

I do not see a way of investigating the **theoretical** validity of each of these in a model independent way. **We need a quarkyonic phenomenology!**
Quarkyonic phenomenology on the lattice
Quenched lattice very close to $N_c$ invariant (Panero et al.), but need at least 1 flavor for the effects described here. One would need to vary $N_{f,c}$ at finite $\mu_Q$, possibly $\mu_Q \sim \Lambda_{QCD}$

Lattice at finite $N_f,N_c$ and finite density?

I can already see you making such a poster!

Sounds simple!

But hear me out!
**Strong coupling expansion**  Binding energy and EoS should drastically change with $N_c, N_f$ (NB: Percolation sensitive to $N_c$, “kissing transition” to $N_c N_f$ so different)

Strong coupling expansion has no sign problem and relatively cheap!

**“Baryon molecules”**  $T = 0$ wavefunction should drastically change shape with $N_c$

**Hopping approximation and Reweighting**  found jump in baryon density at $N_c = 3, \mu_Q \simeq \Lambda_{QCD}$ .

But this is “trivial”, due to high baryon mass!

Need to check pressure behavior with $N_c$ . difficult but possible!
Astrophysical implications
If quarkyonic phase realized in proto-neutron star, pressure, entropy $\sim O(3)$ corresponding nuclear matter. EoS similar to pQCD (stiffer than nuclear matter), but no mixed phase/latent heat: Stiffness gradually turns on!

Such an EoS might make it easier for supernovae to explode?
pQCD but not quite: the role of baryons

Unlike pQCD, quarkyonic matter’s “vacuum” is a classical dense baryon state. Treating baryons as mean fields will give a momentum-dependent form factor $F(k)$

$F(k)$ gives the F.T. of the baryonic gluon content. For the equation of state, it should just be a $\mathcal{O}(1)$ normalization factor, but for scattering processes it is a qualitative difference from naive QCD. Spin-color-flavor separation can ensure color neutrality with quark-like degrees of freedom. Baryons motion doesn't influence quarks up to $N_c^{-1}$ corrections.
NB: Quarks delocalized by tunneling, not confinement

Gluons, antiquarks still confined, only processes with outgoing quarks allowed!
This description coincides with Larry and Rob’s

In momentum space lower-lying modes are quark-like. Since $\lambda \sim 1/p$ these are the most long-wavelength modes, which are exactly the modes feeling the mean field
From EoS to dynamics: An EFT of percolating matter

In percolation regime, asymptotically free quark wavefunctions of different baryons can superimpose across large distances. Thus, even if $E_{\text{state}} \sim 1/L_{\text{baryon}} \sim N_c^0 \ll N_c^{1/2} \Lambda_{\text{QCD}}$|deconfinement degrees of freedom quark-like, so $P \sim N_c, s \sim N_c$ (In the same way electrons in a metal have a much lower energy than ionization). Periodic wavefunctions ⇒ leading component always $p \geq \Lambda_{\text{QCD}}$^{-1}
Modeling quarkyonic matter for RHIC/NICA/FAIR

Quark dynamics: QED, QCD + form factor
F(k) given by BARYON distribution

$R_{qq \to X} = \Psi(k) \Psi^*(k') M_{qq \to X}^2$ Where $M_{qq \to X}$ is the pQCD matrix element

$\Psi(k) \sim \exp \sum_i [ikx_{0i}] F(k) \sim \exp \left[ ikx_{0i} - \frac{k^2}{\Lambda_{QCD}} \right]$ $F(k)$ is the quark function inside a “classical” proton potential well (\sim Gaussian) and $x_{0i}$ are the baryon locations. The latter is given by uRQMD.
Photon production in this approach

As antiquarks, gluons suppressed leading channel is quark Brehmsstrahlung.

\[
\mathcal{M}^2 = L^2(k_1, k_2 \rightarrow k_3, k_4, p) + L^2(k_1 \leftrightarrow k_2, k_3 \leftrightarrow k_4)
\]

\[
L^2 = -\frac{1}{4}e^2 \lambda^2 N_c^{-2}(k_2 - k_4)^{-4}Tr \left[ k_4 \gamma^\sigma k_2 \gamma_\rho \right] Tr \left[ k_3 Z_\sigma^\mu k_1 Z_\mu^\rho \right]
\]

\[
Z_\beta^\alpha = \gamma_\alpha (k_1 - p)^{-1} \gamma_\beta + \gamma_\beta (k_3 + p)^{-1} \gamma_\alpha
\]
\[
\frac{dN_\gamma}{d^3 p} = \int \frac{d^4 k_1}{k_1^0} \frac{d^4 k_2}{k_2^0} \frac{d^4 k_3}{k_3^0} \frac{d^4 k_4}{k_4^0} (\mathcal{M} (k_1, k_2 \rightarrow k_3, k_4, p) \Psi(k_1) \Psi(k_2))^2
\]

- **Quarkyonic quark wavefunctions**
  \[
  \Psi(k) \sim \exp \sum_i [ikx_{0i}] F(k) \sim \exp \left[ ikx_{0i} - \frac{k^2}{\Lambda_{QCD}} \right], \text{uRQMD } \Rightarrow x_{0i}
  \]

- **Can we go beyond \( N_c \rightarrow \infty \) and incorporate baryon flow?**
  “Boosted quarkyonic” : Same wavefunction as above boosted to flow of a “random” baryon: An upper limit to \( N_c^{-1} \) backreaction (effect of baryon flow on quark wavefunction)
Calculate

$$\frac{dN}{d^3p} = \frac{dN}{dp_Tdy} \left[ 1 + 2 \sum_{n=1}^{\infty} v_n \cos(n(\phi - \Psi_{reaction})) \right]$$

for

**Quarkyonic and Boosted quarkyonic matter** described above

**thermalized QGP** cross-sections described above and quark wavefunctions

$$\Psi(k)\Psi(k') = \delta(k' - k) \exp[-k_\mu u^\mu / T]$$

**Hadron gas** calculated with uRQMD molecular dynamics model (same as the one used for quarkyonic wavefunctions!)
Very little difference. NB “static baryon” approximation breaks down away from mid-rapidity.
Quarkyonic wavefunction similar to cold quark gluon plasma, unrealistic temperatures. NB: “boosted quarkyonic” increases flow, but still cold!
Random distribution of quark wavefunctions quenches total $v_2$ but produces big fluctuation in event and $p_T$: oscillation frequency $\sim p_T \rho_B^{-1/3}$
“pure” quarkyonic effect, it is due to sensitivity of quark wavefunctions to baryon location. signature?
dileptons potentially more direct probe but more complicated

Both quarks and holes needed  Sensitivity to equilibration

\[ \tilde{F}(M^2) \] connects baryon distribution to \( M^2 \) dilepton spectrum

\[
\langle \hat{\Psi} \rangle = \text{Tr} \left\{ \exp \left[ \frac{\hat{H} - \mu_q \hat{N}}{T} \right] \left[ \frac{1}{3N} \left( \sum_{i,j,k} \hat{a}_i(k_i) \hat{a}_j(k_j) \hat{a}_k(k_k) \right) \right] \right\}
\]

where \( a_i \) solutions of confining potential wells centered around baryons,

\[
\hat{H} = \sum \hat{k}_i^2 + \sum_{i}^{\text{baryons}} V \left( \hat{x}_i^{\text{baryon}} - v_i^{\text{baryon}} t \right)
\]
Hadronic resonance peaks, $M > 0.5$ GeV
($\eta, \omega, \rho, \phi, \ldots$)

QGP Continuum

$\rho_q(\text{M})$

$M \sim \rho_{B}^{1/3}$

$\sim 0.2 - 0.4$ GeV gap

Flavor excitations

Color excitations

If baryons were regular (pasta phase?) one could observe bloch waves!
("upside down resonance"?)
Event by event fireball structure not regular, but Collective structures exist in events flow profile (radial, longitudinal flow) and baryons have repulsive potential, so structures in 3D dilepton spectral function $Q_{z,r,\phi}$ bound to exist!
Is there a Gauge/Gravity angle to all this?

• Since phase transition happens at critical $N_c$, it can only be realized at subleading $g_s$. Asymptotic freedom limit for quark-quark interactions at large $N_c$ also requires $\alpha'$ corrections!

• In string world flavor $\leftrightarrow$ D7,8 branes. So $N_c \sim N_f$ means so many overlapping branes string loops among them can not be neglected.

• This might explain why, despite compelling argument for $s \sim N_c@\mu_q \geq \Lambda_{QCD}$, all AdS/CFT setups so far have $s \sim N_c^0$ in that regime. $P \sim s \sim N_c$ argument explicitly based on asymptotic freedom. Not implementable in supergravity.
Gauge/Gravity at subleading $g$ , $\alpha$?

Sounds even simpler!

I cannot see a sure road into percolation, but some qualitative insights could be obtained back at $\mu_Q \to 0$ . remember the order of confinement!
Here is how to make arguments in previous slides compatible with AdS/CFT:

Above leading order in $g_s$. Leading order misses auxiliary minimum where $s \sim N_c$ so only minimum at $s \sim N_c^0$. Van Der Waals example shows correction can be small (but not infinitesimal) for this to happen!
Confinement and black holes

In normal space, black hole decays and has a negative heat capacity → Thermodynamically unstable state!

Let’s put the black hole in a reflecting box (One “physical way” of doing it: A negative cosmological constant, AdS!

**Box large wrt black hole** system (hole+gas) heat capacity still negative, black hole decays

**Box small wrt black hole** Hole and photons in box in thermal equilibrium, heat capacity positive, black hole stays

The two regimes connected by **Hawking-Page** phase transition (1st order). According to Witten, confinement in d-flat or spherical space is dual to the Hawking-Page phase transition of a black hole in d+1 AdS space
The phase transition in $N_c$ and gravity

**In Gauge world**, confinement critical point is understood in terms of broken symmetries ($Z_N$).

**In Gravity world**, Hawking-Page is most likely a transition because of naked singularity conjecture. You either have a black hole, with a singularity, or you don’t! (This is why I don’t believe ”bottom-up” models where confinement is a cross-over!)

Hence, making confinement into a cross-over is equivalent to smoothening black hole singularity
We have no idea how to do this, so let’s use a Quantum-Gravity ansatz: Gravity in non-commutative geometry-inspired ansatz.

- Some people think it could come out of string theory or any generic model of quantum gravity.
  Impossible to probe scales $\leq l_p$ as this is the “size of the $g_s$ string loop” / You’ll create a black hole trying.

- Ansatz can be shown to be well-behaved (does not break unitarity and locality at distances long wrt $l_p$).

- Critical behaviour $\leftrightarrow$ universality! Insensitive to microscopic details of our model.
Non-commutative geometry-inspired Schwarzschild ansatz
The basic idea: Maintain “gravity” part classical but smear out energy momentum tensor. Black hole problem reduces to solving Einstein’s equations for infinitely rigid Gaussian energy distribution

\[ T^0_0 = \frac{1}{(2\pi l_p)^{3/2}} \exp \left[ -\frac{x^2}{2l_p^2} \right] \Rightarrow \delta(x) \]

Einsteins equations, spherical symmetry and \( T^\mu_\nu = 0 \) specify the problem uniquely. Entropy calculated via horizon area. Impossible to expand in \( l_p \)
Hawking entropy calculated the usual way. But...

**Flat space**  Black hole heat capacity becomes positive after critical radius $x_{+}^{planck} \sim l_p \rightarrow$ Ansatz used to study remnants

**AdS space**  Van Der Waals-type phase diagram
   If box small enough that $x_{+}^{planck} \sim L_{AdS}$, we reach critical point
At critical $q = l_p\Lambda_{AdS}$ Hawking-Page transition becomes a cross-over, similar to Van der Waals gas. Critical $q^* = 0.18243 \approx 1/6$ if $\mathcal{O}(1) \frac{N_f}{N_c}$ surprisingly close, for 1 flavor, to $N_c = 6 = N_{N}^{d=2+1}$.
\[ F = \underbrace{F_{\alpha=0}(T)}_{\text{kink}} + \underbrace{\Delta F(\alpha \geq 0)(T)}_{\text{smoothens}} \]

In gravity, \( \alpha = l_p \alpha \sim N_f/N_c \) in QCD? (NB: Can not Taylor-expand!)
Work in progress... a model of this type in AdS/CFT
Does the Hawking page transition become a cross-over in Witten’s original set-up, a Black hole on a sphere? \((AdS \times S_n)\)?

Witten ( hep-th/9803131v2 )

Hawking–Page in \(AdS_3 \neq AdS_1 \times S_n\) but is obvious that a similar critical point will happen in all setups with a Hawking-Page transition, although of course \(T_c\) and \(l_{pc}\) will change!
Is this the same as percolation? Not sure, but I think so!

**Critical point behaviour** identical to second order phase transition, and percolation is a 2nd order phase transition

Hawking-Page transition coincides with transition of a gas of black holes in AdS collapsing into a large black hole. It happens because of the interplay of black hole distance and the horizon. **Non-commutativity fuzzes this over**, so black holes can interact over super-horizon distances via quantum tunnelling. **Very similar to percolation!** Connection between Polyakov loops and percolation **not** trivial in Gauge picture, but understandable in gravity.
Can we make this *ansatz* testable?
The main effect of correction is to introduce a *critical point* of the $Z_2$ type (Shouldn't exist in a top-down system, and indeed doesn't seem to!).

<table>
<thead>
<tr>
<th>$d$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Gravity</th>
<th>Gauge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0</td>
<td>0.110(1)</td>
<td>0</td>
<td>$R$</td>
<td>$\langle L \rangle$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1/8</td>
<td>0.3265(3)</td>
<td>1/2</td>
<td>$TdS/dT$</td>
<td>$C_V$</td>
</tr>
</tbody>
</table>

In QCD can, ideally, be read from the lattice, either in $T - N_f/N_c$ plane *(hard)* or $T - m$ plane *(doable)* In gravity, we can have a black hole in a Box or a brane setup. Universality can mean details of the theory secondary... *critical exponents*. And both sides are in $Z_2$ class!

If exponents match and remain critical, it would be very non-trivial: Stat Mech 101 says critical exponents set by universality class and number of dimensions. Holography is a counter-example!, as number of dimensions changes. In this setup we can measure critical exponents on *both sides*
Conclusions

- “naive” hadronic EFT unreliable for regime at $\mu_Q \sim \Lambda_{QCD}$
- Large $N_c$ expansion tells us quark degrees of freedom could appear even at confinement!
- On the other hand, not at all clear $\sim \infty$
- Phenomenology of quarkyonic matter needed.

The best physicist in the USSR is Yakov Frenkel, who uses in his papers only quadratic equations. I am slightly worse, I sometimes use differential equations.

L.D. Landau, quoted in
BULLETIN OF THE American Mathematical Society
Volume 43, Number 4, October 2006, Pages 563–565