Naturally inflating on steep potentials through electromagnetic dissipation

Lorenzo Sorbo

UMass Amherst

IPhT

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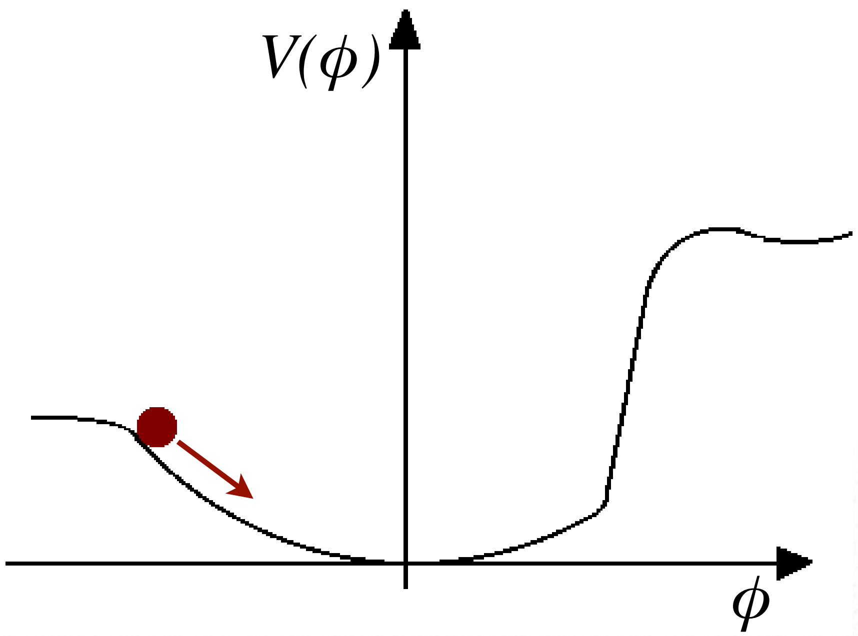
M. Anber, LS, PRD 2010, PRD 2012

INFLATION



✓ very early Universe filled by scalar field ϕ, potential $V(\phi)>0$

✓ to induce acceleration, $V(\phi)$ must be *flat*

$$\boxed{|V'(\phi)| \ll V(\phi)/M_P}$$

✓ to have long enough inflation, $V(\phi)$ must *stay* flat for long enough

$$\boxed{|V''(\phi)| \ll V(\phi)/M_P^2}$$

Simple way of realizing $|V'(\phi)| << V(\phi)/M_P$, $|V''(\phi)| << V(\phi)/M_P^2$: monomial potential, with ϕ large enough

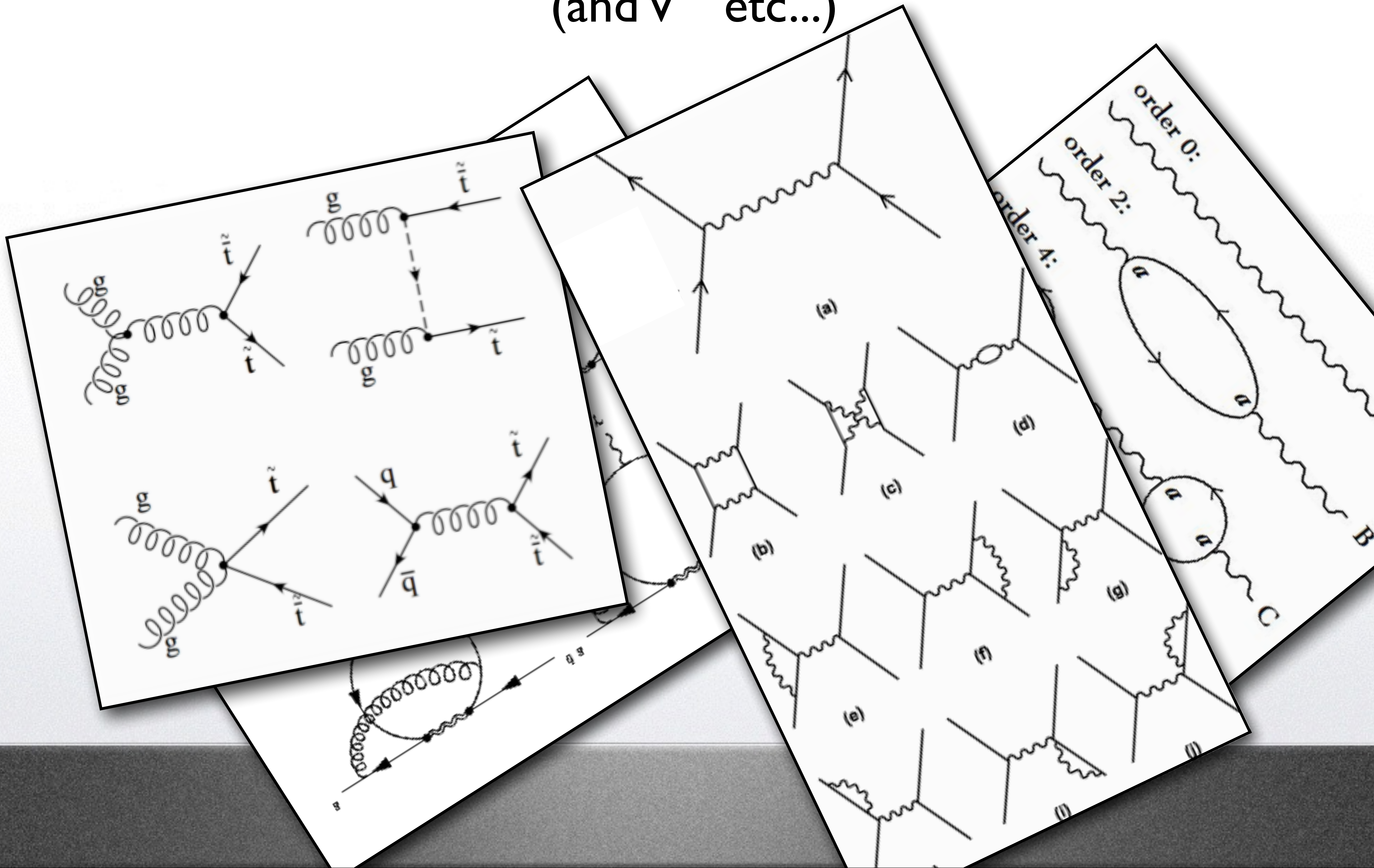
Most famous example: quadratic potential *(chaotic inflation)*

Linde 1983

$$V(\phi)=m^2\,\phi^2\,/2$$

Amplitude of perturbations produced during inflation $\Rightarrow$ $m \sim 10^{13}$ GeV

...but, in general, quantum loops will contribute to V’ and V’’ (and V’’’ etc...)



Radiative corrections can disrupt the inflationary potential in two ways

1- affect the functional form of $V(\phi)$

2- affect value of the parameters that appear in $V(\phi)$

Chaotic inflation example

1- adds terms $\propto \phi^n$, $n=4, 6, \ldots$

2- push m to larger values (e.g. M_P - cf EW hierarchy pbm)

How to make sure that radiative effects are under control?

The situation is actually not so horrible...

Smolin 80
Linde 88

<u>If</u> we have a theory where ϕ interacts only with gravity

<u>then</u> quantum corrections are not a problem!

Indeed: for potential $V(\varphi)$, quantum gravity effects are

$\mathcal{O}(1)\ V(\varphi)^2/M_P^4$ and $\mathcal{O}(1)\ V''(\varphi)\ V(\varphi)/M_P^2$

negligible during inflation

---

however, in general there will be couplings to other fields

reheating

How to make sure that radiative effects are under control?

A very well-known system that contains "controllably small" quantities is the **Standard Model**: "small" quantities are protected against radiative effects by ***symmetries***

If a model has a symmetry, quantum effects cannot violate it (unless the symmetry is anomalous...)

If the symmetry is broken, quantum effects cannot make the breaking much larger
(ie the breaking parameter is controllably small)

A field ϕ has a *shift symmetry* if the theory that describes it is invariant under the transformation

$$\boxed{\varphi \longrightarrow \varphi + c}$$

(c=arbitrary constant)

If this symmetry is exact, the only possible potential for ϕ is $V(\phi)$=constant

(i.e. a cosmological constant)

an exact shift symmetry is an overkill...
...but we can break the symmetry a bit and generate a potential

An (important) example

If ϕ is a phase, then shift symmetry ⇔ global *U(1)*

- Theory with a spontaneously broken global *U(1)*

$$\mathcal{L} = \partial_\mu H^* \, \partial^\mu H - \lambda \left(|H|^2 - v^2 \right)^2$$

- Decompose $H = (v + \delta H) \; e^{i\phi/v}$

  where δH is massive and φ is a massless Goldstone boson *(pseudoscalar)*

- The global *U(1)* is broken e.g. by some strong dynamics

$$\delta \mathcal{L} = \Lambda^3 \; (H + H^*) + \ldots$$

- A potential is generated: $\delta V \sim \Lambda^3 \, v \, \cos(\phi/v)$

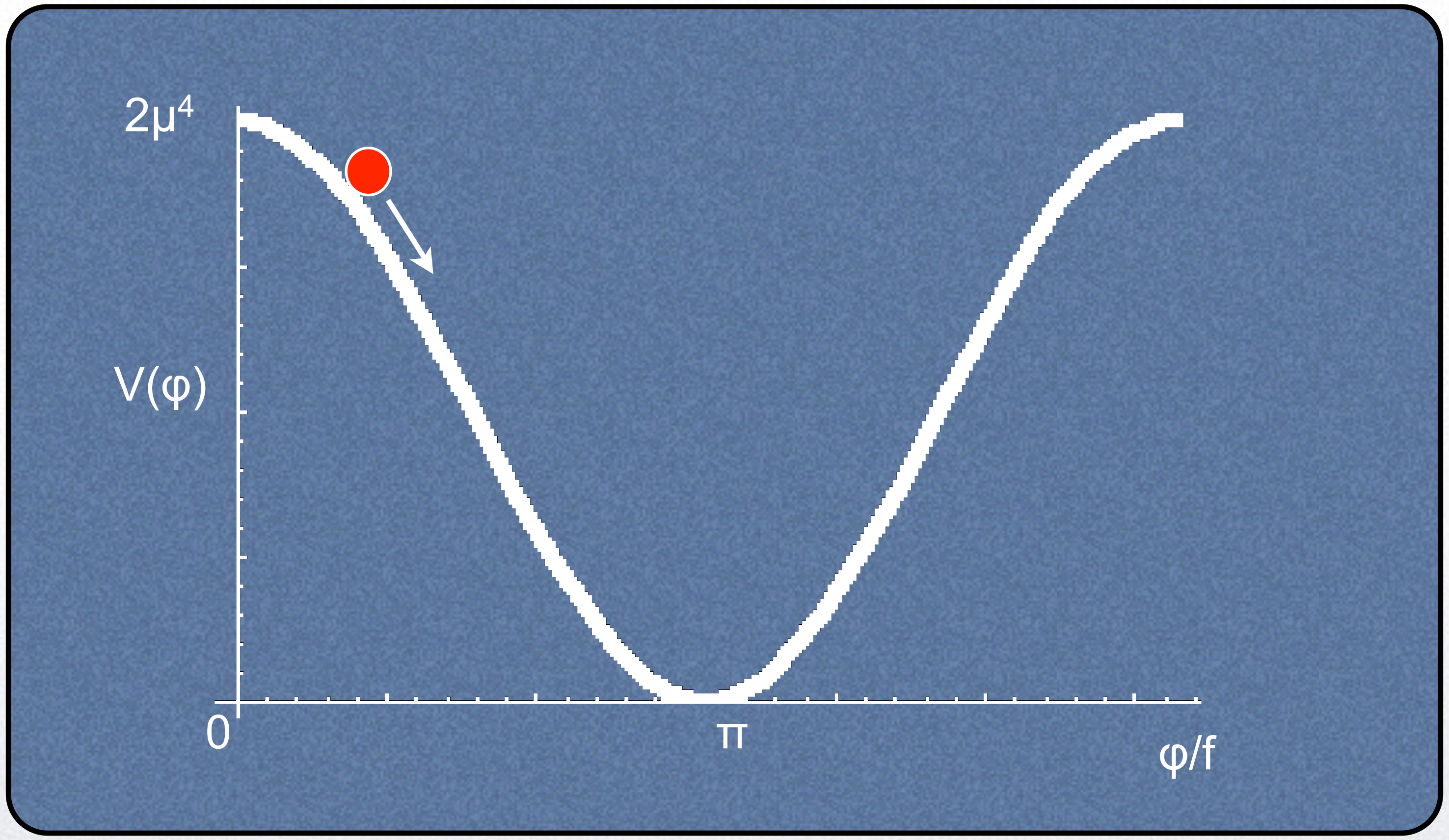
Pseudo-Nambu-Goldstone boson
PNGb

...using a pNGB as an inflaton...

Natural inflation

Freese et al 1990

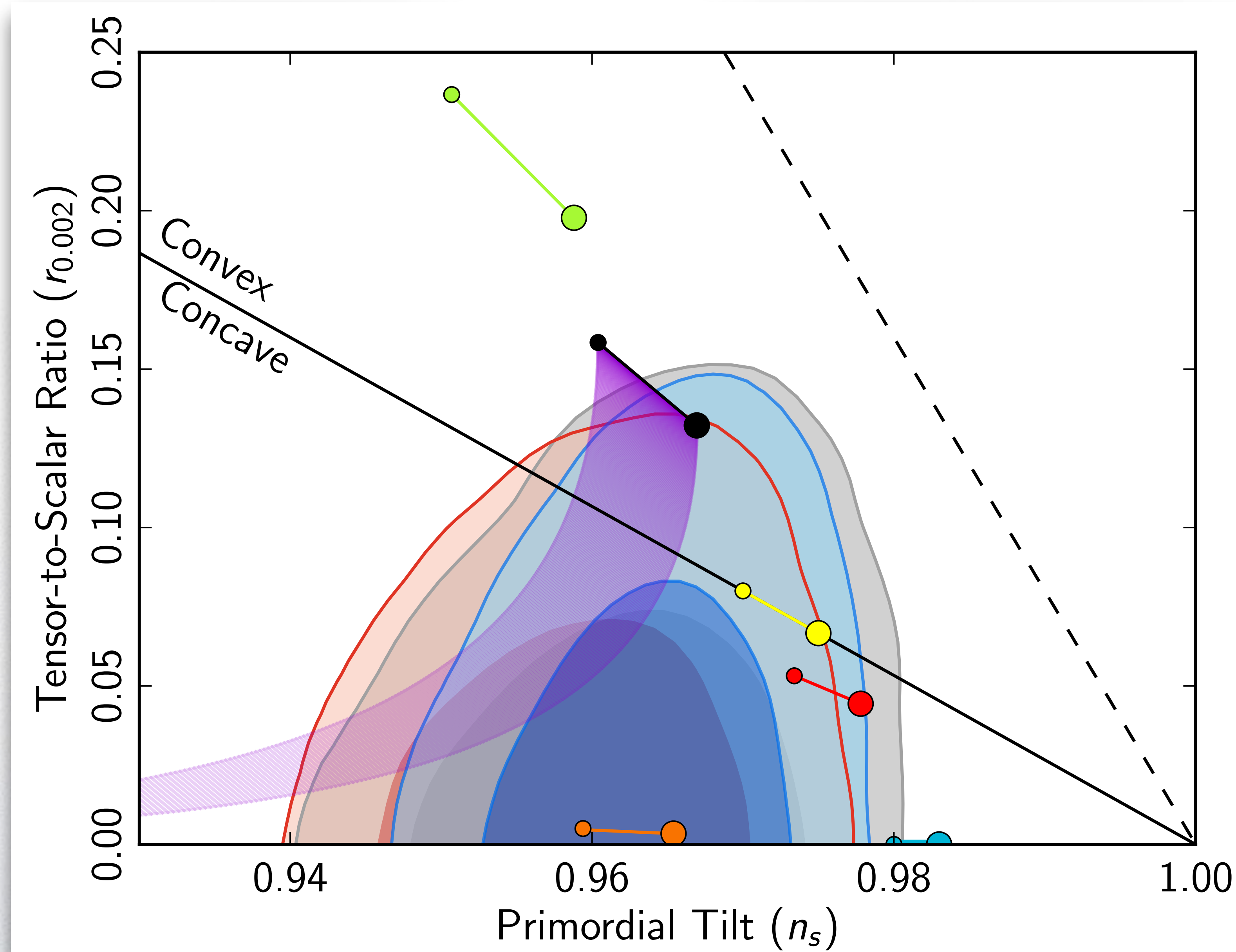
$$V(\varphi)=\mu^4 \, [\cos(\varphi/f)+1]$$



Because of its radiative stability,

A pNGB gives an extremely well motivated model of inflation from the point of view of effective field theory

What about data?



f>5 M_P

from Planck, Inflation

Stringy models of natural inflation?

YES, *in principle*
(string theory contains a plethora of pNGBs)

However

Banks, Dine, Fox and Gorbatov 03

String Theory appears to require $f<M_P$

n-instanton actions contribute $\propto e^{-(n M_P/f)} \cos(n\,\phi/f)$ to pNGB potential

$$\Downarrow$$

first f/M_P harmonics in $V(\phi)$ matter

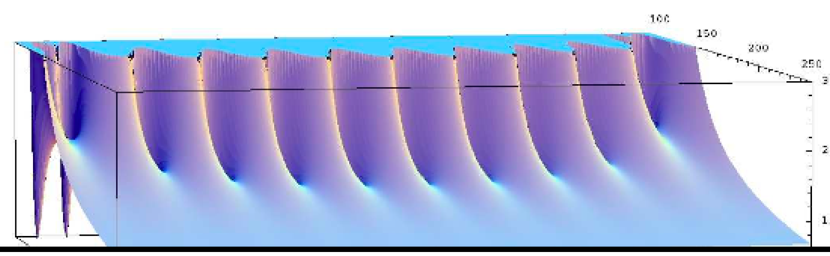
Ways out?

Kim, Nilles and Peloso 2004

- *Two* pNGBs $$V = \Lambda_1^4\left[1-\cos\left(\frac{\theta}{f_1}+\frac{\rho}{g_1}\right)\right] + \Lambda_2^4\left[1-\cos\left(\frac{\theta}{f_2}+\frac{\rho}{g_2}\right)\right]$$

Blanco-Pillado et al 2004

- PNGBs and moduli

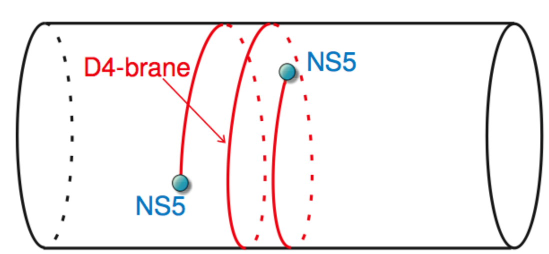


Dimopoulos et al 2005

- *Many* pNGBs $$\mathcal{L} = -\sqrt{-g}\sum_{i=1}^{N}\left\{\frac{1}{2}(\partial\phi_i)^2 + \Lambda_i^4\left[1+\cos(\phi_i/f_i)\right]\right\}$$

Silverstein and Westphal, 2008

- Monodromy



Kaloper and LS, 2008
Kaloper, Lawrence and LS.

- Mixing with 4-form $$L= \frac{M_{Pl}^2}{2}R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{48}F_{\mu\nu\lambda\sigma}^2 + \frac{\mu\phi}{24}\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}}F_{\mu\nu\lambda\sigma} + \ldots)$$

A different way of approaching the problem...

The inflaton can be slowed down
(even on a steep potential!)
if it ***dissipates*** its kinetic energy

e.g. particle production associated to motion of ϕ
rate depends on $\dot{\phi}$

...back to the origins...

In the early '70s (pre-inflation), try to explain isotropy from initial anisotropy by particle production

Today, chaotic inflation paradigm allows to ignore primordial anisotropy problem-but still need flat potential

Particle production
can help mitigate
the requirement of flat potential

Trapped inflation (I)

Green et al 2009

Idea: field χ with mass $m_\chi(\phi(t))$

At some time t_0, $m_\chi(t_0)=0$, with $\dot{m}_\chi(t_0)\neq 0$.

$\Rightarrow$ Heisenberg inequality $\hbar \lesssim \Delta E\, \Delta t \sim m_\chi\,(m_\chi/\dot{m}_\chi)$ violated

Concept of number of quanta of χ not well defined

Quanta of χ are produced

Trapped inflation (II)

Particles created at expenses of inflaton kinetic energy

(the only useful energy available)

Chung et al 1999

Inflaton rolling is slowed down for ~ 1 efold

To get 60 efolds, need many production events

Green et al 2009

$$\frac{1}{2} g^2 \sum_i (\phi - \phi_i)^2 \chi_i^2 \ .$$

depending on parameters, 1 to 10^{12} events per efold are needed

this structure can be present in some stringy constructions

A mechanism analogous to trapping is built in in natural inflation

Anber and LS 09

Idea: pNGB driving natural inflation is “naturally” coupled to gauge fields

$$\mathcal{L}_{\phi F_{\mu\nu}} = \alpha \frac{\phi}{4f} \epsilon_{\mu\nu\rho\lambda} F^{\mu\nu} F^{\rho\lambda}$$

α=dimensionless constant

we will consider *N* copies of $U(1)$ gauge fields

Equation for the *U(1)* field in the presence of $\phi(t)$:

$$A''_{\pm} + \left(k^2 \pm \alpha \frac{\Phi'}{f} k\right) A_{\pm} = 0$$

$A_{\pm}$= >ve and <ve helicity comoving modes
of the vector potential

One of the two modes has
a ***negative***, ***time dependent*** "mass term"

Exponential amplification
of one helicity mode

Equation for $A_{\pm}$ can be solved by assuming $\dot{\phi}$, H=constant

Modes with $k/a < \alpha\dot{\phi}/f$
feel tachyonic mass until $k=aH$

$\Downarrow$

amplification by

$\sim exp[\alpha\dot{\phi}/fH]$

more precisely...

$$A_+(\tau, \vec{k}) \simeq \frac{1}{\sqrt{2|\vec{k}|}} \left(\frac{|\vec{k}|}{2\,\xi\, a\, H} \right)^{1/4} e^{-2\sqrt{2\xi\,|\vec{k}|/aH} + \pi\,\xi}$$

Exponential amplification term!

$$\xi \equiv \frac{\alpha\,\dot{\phi}}{2\,f\,H}$$

Slowing down the inflaton

backreaction equation

$$\ddot{\phi} + 3\,H\,\dot{\phi} + V'(\phi) = -\mathcal{N}\,\frac{\alpha}{f}\,\langle \vec{E}\cdot\vec{B}\rangle$$

with

$$\langle \vec{E}\cdot\vec{B}\rangle \propto exp\{\pi\alpha\dot{\phi}/fH\}$$

As φ starts increasing under the effect of the steep potential, the backreaction term gets important, slowing it down.

Slow roll equation⇒ $\boxed{V'(\phi) = -\mathcal{N}\,\frac{\alpha}{f}\,\langle \vec{E}\cdot\vec{B}\rangle}$

the slow roll solution...

$$\dot{\phi} \simeq \frac{f\,H}{\alpha\,\pi}\,\log\left\{\frac{10^4}{\mathcal{N}\,\alpha}\,\xi^4\,\frac{M_P^4}{V(\phi)}\,\frac{f\,V'(\phi)}{V(\phi)}\right\}$$

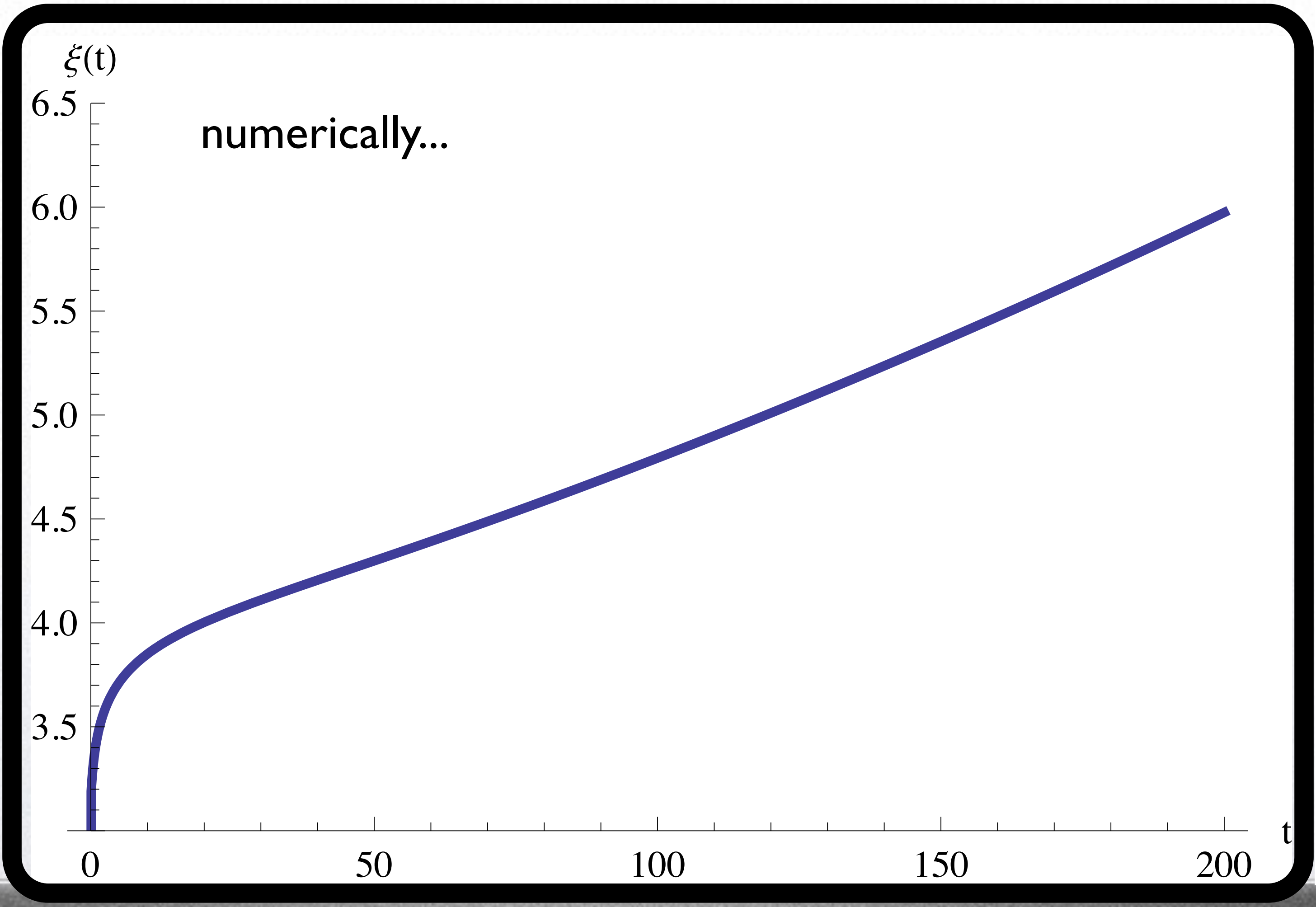
O(1)

$V \sim \Lambda^4$, $N \sim 10^5$, $\alpha \sim 1000$

$$\xi \sim \frac{2}{\pi}\,\log\frac{M_P}{\Lambda} \sim 3 \div 20$$

the slow roll solution...

$$\dot{\phi} \simeq \frac{f\,H}{\alpha\,\pi}\,\log\left\{\frac{10^4}{\mathcal{N}\,\alpha}\,\xi^4\,\frac{M_P^4}{V(\phi)}\,\frac{f\,V'(\phi)}{V(\phi)}\right\}$$



$\Lambda = 10^{-3}\,M_P,\ f = 0.1\,M_P,\ \alpha = 300,\ \mathcal{N} = 10^5$

...and a (first) constraint on the model:

$\Delta\phi = \pi f$ from top to bottom of potential

$\Downarrow$

total # efoldings $\sim H\,\Delta\phi/\dot{\phi} \sim \alpha \log \left\{ \frac{1}{\mathcal{N}\,\alpha} \frac{M_P^4}{V(\phi)} \frac{f\,V'(\phi)}{V(\phi)} \right\}^{-1}$

$\Downarrow$

$$\boldsymbol{\alpha \sim O(10^3)}$$

Consistency

We have assumed that the contribution to the electromagnetic modes to the Hubble parameter is negligible.

True?

YES!

$$\frac{\rho_{EM}}{V(\phi)} \sim \mathcal{N}\,\frac{\vec{E}^2+\vec{B}^2}{V(\phi)} \sim \frac{\xi}{\alpha}\,\frac{f\,V'}{V}$$

negligible, for $\alpha >> \xi$, unless at the bottom of $V(\phi)$

Reheating!

...but... is it really inflation?

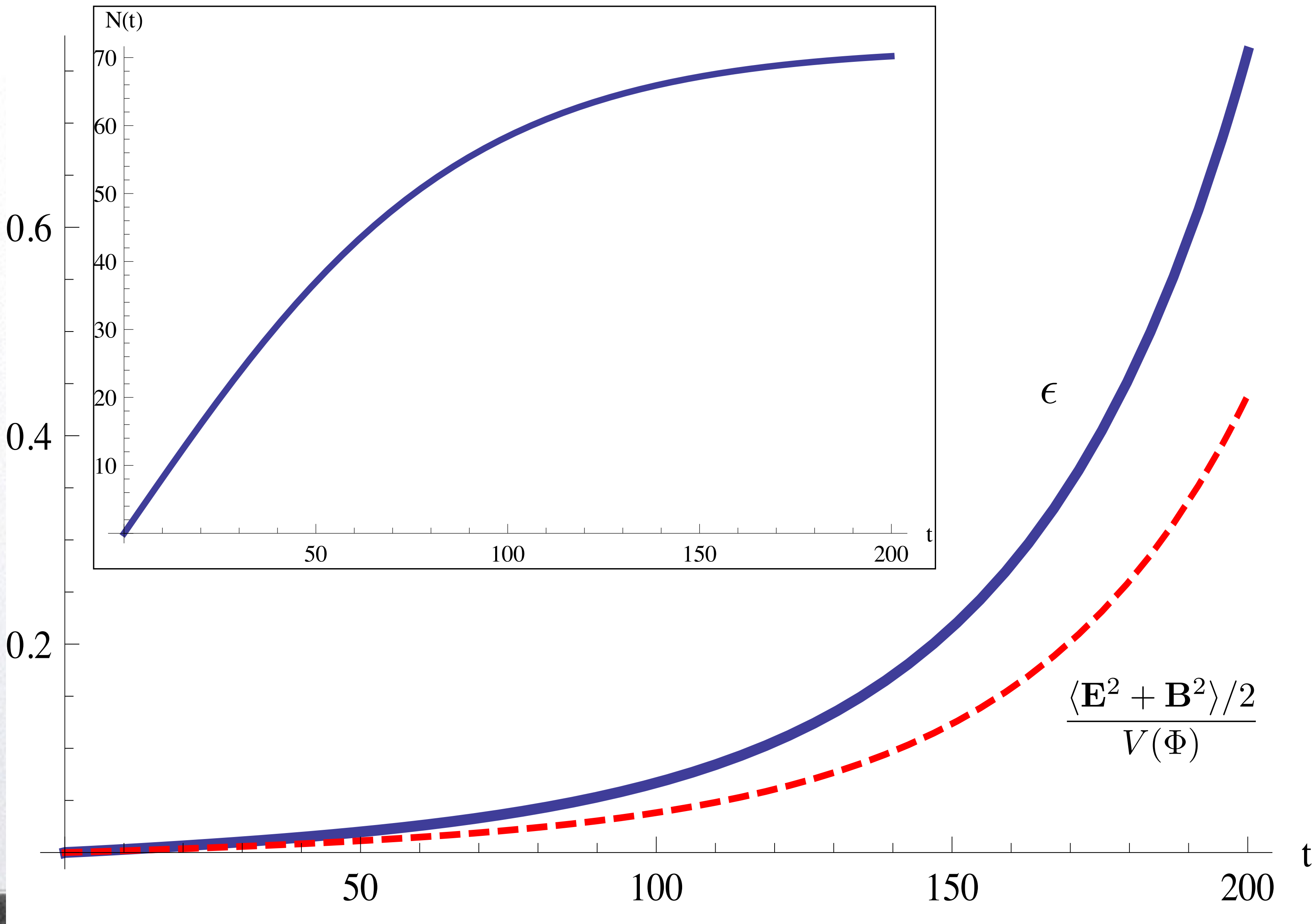
Need the slow roll parameters ε and η <<*1*:

$$\begin{aligned}\epsilon &= \frac{\dot{\Phi}^2}{2\,H^2\,M_P^2} \simeq \frac{2\,f^2}{\alpha^2\,M_P^2}\\ \eta &= 2\,\epsilon - \frac{f}{\pi\alpha}\left(\frac{V''(\Phi)}{V'(\Phi)} - 2\,\frac{V'(\Phi)}{V(\Phi)}\right)\end{aligned}$$

Bottom line - background evolution

For $\alpha \sim O(1000)$, possible to get *~60* efolds of inflationary expansion

Numerical example



$$\Lambda = 10^{-3}\, M_P,\ f = 0.1\, M_P,\ \alpha = 300,\ \mathcal{N} = 10^5$$

How to get such a large $\alpha \sim 10^3$?

Choi and Kim 85

One example

Two axions in $E_8 \times E_8$

$$
\begin{aligned}
\mathcal{L}_{\text{axions}} &= \frac{1}{2}(\partial_\mu a_1)^2 + \frac{1}{2}(\partial_\mu a_2)^2 \\
&- \frac{1}{2}\left(\frac{a_1}{M_1} + \frac{a_2}{M_2}\right) F^i_{\mu\nu}\tilde{F}^i_{\mu\nu} \\
&- \frac{1}{2}\left(\frac{a_1}{M_1} - \frac{a_2}{M_2}\right) F'^i_{\mu\nu}\tilde{F}'^i_{\mu\nu} \\
&= \frac{1}{2}(\partial_\mu a)^2 + \frac{1}{2}(\partial_\mu a')^2 - \frac{a}{2M} F\tilde{F} \\
&- \frac{a'}{2M'}\left(F\tilde{F} + \frac{M_2^2+M_1^2}{M_1^2-M_2^2} F'\tilde{F}'\right)
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
a &= (M_1 a_1 + M_2 a_2)/(M_1^2 + M_2^2)^{\frac{1}{2}} \\
a' &= (M_2 a_1 - M_1 a_2)/(M_1^2 + M_2^2)^{\frac{1}{2}}
\end{aligned}
}
$$

$$
\begin{aligned}
M &= \frac{1}{2}(M_1^2 + M_2^2)^{\frac{1}{2}} \\
M' &= M_1 M_2 (M_1^2 + M_2^2)^{\frac{1}{2}}/(M_1^2 - M_2^2)
\end{aligned}
$$

Perturbations

Equation for perturbations (only one family of gauge fields for now)

$$\delta\ddot{\phi} + 3\,H\,\delta\dot{\phi} + \left(-\nabla^2 + V''(\phi)\right)\,\delta\phi = -\frac{\alpha}{f}\,\delta\left[\vec{E}\cdot\vec{B}\right]$$

Rigorously: perturb φ and E and B

System can be reduced to one integro-differential eqn

(very, very complicated!)

Perturbations

Equation for perturbations (only one family of gauge fields for now)

$$\delta\ddot{\phi} + 3\,H\,\delta\dot{\phi} + \left(-\nabla^2 + V''(\phi)\right)\,\delta\phi = -\frac{\alpha}{f}\,\delta\left[\vec{E}\cdot\vec{B}\right]$$

Trick

Two contributions to $\delta\left[\vec{E}\cdot\vec{B}\right]$:

$$\delta\left[\vec{E}\cdot\vec{B}\right] \simeq \left[\vec{E}\cdot\vec{B} - \langle\vec{E}\cdot\vec{B}\rangle\right]_{\delta\phi=0} + \frac{\partial\langle\vec{E}\cdot\vec{B}\rangle}{\partial\dot{\phi}}\,\delta\dot{\phi}$$

intrinsic fluctuations $\delta_{\vec{E}\cdot\vec{B}}$

dominate at small scales

fluctuations induced by fluctuations in ϕ

dominate at large scales

Perturbations

Fourier-transformed effective equation for perturbations

$$\delta\ddot{\phi}_p + H\left(3 + \frac{\pi\,\alpha\,V'(\Phi_0)}{3\,f\,H^2}\right)\delta\dot{\phi}_p + \left(\frac{p^2}{a^2} + V''(\Phi_0)\right)\phi_p = -\frac{\alpha}{f}\,\delta_{\vec{E}\cdot\vec{B}}(p)$$

$O(\alpha\, M_P{}^2/f^2)$

Solution (using Green function)

$$\delta\phi_{\vec{p}}(t) = -\frac{\alpha}{f}\int dt'\,G(t,\,t')\,\delta_{\vec{E}\cdot\vec{B}}(t',\,\vec{p})$$

two point function of inflaton perturbations

$$\boxed{\langle 0|\delta\phi_{\vec{p}}\,\delta\phi_{\vec{p}'}|0\rangle = \frac{\alpha^2}{f^2}\int dt'\,G(t,\,t')\int dt''\,G(t,\,t'')\;\langle 0|\delta_{\vec{E}\cdot\vec{B}}(t',\,\vec{p})\,\delta_{\vec{E}\cdot\vec{B}}(t'',\,\vec{p}')\,|0\rangle}$$

Perturbations

Spectrum of metric perturbations

$$\mathcal{P}_\zeta \simeq \frac{5 \times 10^{-2}}{\mathcal{N}\,\xi^2} \left(\frac{k}{a\,H}\right)^{2fV''/\pi\alpha V'}$$

for N gauge fields

quasi scale invariant for large α

ξ<20 or so, so need N~10^5

$N \sim 10^5$ gauge fields can originate from $SU(\sqrt{N})$ gauge group

(e.g., from a stack of 200 branes)

need gauge coupling to be tiny to prevent gauge-field self-interactions

Perturbations

Anber and LS 12

Nongaussianities (equilateral)

$$\langle\zeta(\mathbf{k}_1)\,\zeta(\mathbf{k}_2)\,\zeta(\mathbf{k}_3)\rangle = -0.3\,\frac{1}{\mathcal{N}^2\,\xi^3}\,\frac{\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)}{k^6}$$

$f_{NL}{}^{equil} \sim -1.3\;\xi \sim -4 \div -30$

Tensors

Two sources of tensors:
(i) quasi de Sitter expansion
(ii) nonzero gauge fields

LS 11

$$\mathcal{P}^{t,L} = \frac{H^2}{\pi^2 M_p^2}\left(1 + 8.6 \times 10^{-7}\mathcal{N}\frac{H^2}{M_P^2}\frac{e^{4\pi\xi}}{\xi^6}\right),$$
$$\mathcal{P}^{t,R} = \frac{H^2}{\pi^2 M_p^2}\left(1 + 1.8 \times 10^{-9}\mathcal{N}\frac{H^2}{M_P^2}\frac{e^{4\pi\xi}}{\xi^6}\right).$$

parity violation

Tensors

$$r = \frac{1}{\mathcal{P}_\zeta}\,\frac{2\,V}{3\,\pi^2\,M_P^4} + 2.7\times 10^2\,\frac{\xi^4}{\alpha^2}\,\left(\frac{V'\,f}{V}\right)^2$$

"standard" tensor modes

chiral tensors
induced by chiral photons

new constraints

r>0.1 $\longrightarrow$ $\begin{cases} \Lambda \sim 10^{16}\,GeV, \quad \xi \sim 4 \rightarrow \alpha \gtrsim 500 \\ \Lambda \sim 10^{3}\,GeV, \quad \xi \sim 20 \rightarrow \alpha \gtrsim 10^4 \end{cases}$

Conclusions

- A “natural” system can give inflation on a steep potential
- Unfortunately large perturbations, but there is a way out
- Specific signatures possible

Other ways to suppress perts?

Suppressing the large perturbations

Why are perturbations large?

Amplitude of inhomogeneities generating perts ~ Energy in gauge field

(only one scale in the problem)

Possible solution?

Anber, LS, in progress

Make the gauge field massive!

(same energy, less fluctuations)

Suppressing the large perturbations

Equation for a massive photon

$$A''_{\pm} + \left(k^2 \pm \alpha \frac{\Phi'}{f}\, k + m^2\, a^2\right) A_{\pm} = 0$$

For $\xi > \mu \doteq m/H$:

$$A_{-} \simeq \exp\left\{\pi\left(\xi - \mu\right)\right\}$$

Suppressing the large perturbations

Analysis as before, but now

$$\xi - \mu \sim \frac{2}{\pi} \log \frac{M_P}{\Lambda} \sim 3 \div 20$$

still with

$$\mathcal{P}_\zeta \propto \frac{10^{-2}}{\xi^2}$$

Can choose $\mu \sim 10^4$ and get COBE normalization with a single photon family!

Suppressing the large perturbations

But ***how about nongaussianities***?

Work still in progress, but for time being result

$$f_{NL}^{\rm equil} \propto \xi$$

would rule the model out…

One final note…

$$A''_{\pm} + \left(k^2 \pm \alpha \frac{\Phi'}{f} k + m^2\, a^2\right) A_{\pm} = 0$$

existence proof of fact that you can get cosmologically relevant fields even if $m \gg H$

another example…

One final note…

Peloso, LS, Tasinato
one week ago

$$\mathcal{L} = e^{2\,\phi/M}\left[-\frac{1}{2}\partial_\mu\chi\,\partial^\mu\chi - \frac{m^2}{2}\,\chi^2\right]$$

the canonically normalized field $\psi = e^{\phi/M}\chi$ **obeys**

$$\psi'' + \left[k^2 - \frac{1}{\tau^2}\left(-\frac{m^2}{H^2} + 2 + 3\,\frac{\dot\phi}{M\,H} + \frac{\dot\phi^2}{M^2\,H^2}\right)\right]\psi = 0$$

scale invariant spectrum even for m≫H provided

$$3\,\frac{\dot\phi}{M\,H} + \frac{\dot\phi^2}{M^2\,H^2} = \frac{m^2}{H^2}$$