Predicting the Dirac CP Violation Phase in the Lepton Sector

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All compelling $\nu-$Oscillation Data: $3-\nu$ Mixing (a reference scheme)

$$\nu_{l\mathrm{L}} = \sum_{j=1,2,3} U_{lj}\, \nu_{j\mathrm{L}}\,, \qquad l = e, \mu, \tau.$$

Three Neutrino Mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\,\nu_{j\mathrm{L}} \ .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6
CP-violating phases:				
• ν_j- Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• ν_j- Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P.,1980

PMNS Matrix: Standard Parametrization

$$U = V\,P\,, \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CPV phase, $\delta = [0, 2\pi]$; CP inv.: $\delta = 0, \pi, 2\pi$;
- α_{21}, α_{31} - Majorana CPV phases; CP inv.: $\alpha_{21(31)} = k(k')\pi$, $k(k') = 0, 1, 2...$

S.M. Bilenky, J. Hosek, S.T.P., 1980

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.54 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),
- $|\Delta m^2_{31(32)}| \cong 2.48\ (2.44) \times 10^{-3}$ eV^2, $\sin^2\theta_{23} \cong 0.425\ (0.437)$, NH (IH),
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} = 0.0234\ (0.0239)$, NH (IH).

F. Capozzi, E. Lisi *et al.*, arXiv:1312.2878

- $\text{sgn}(\Delta m^2_{\text{atm}}) = \text{sgn}(\Delta m^2_{31})$ not determined

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{31} > 0, \quad \text{normal mass ordering}$$

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{32} < 0, \quad \text{inverted mass ordering}$$

Convention: $m_1 < m_2 < m_3$ - NMO, $m_3 < m_1 < m_2$ - IMO

$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> \Delta m^2_{\text{atm}},\ \text{QD};\ m_j \gtrsim 0.10\ \text{eV}.$$

- **Dirac phase δ**: $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A_{CP}^{(l,l')} \propto J_{CP} \propto \sin\theta_{13} \sin\delta$:

P.I. Krastev, S.T.P., 1988

$$J_{CP} = \text{Im}\left\{U_{e1}\, U_{\mu 2}\, U_{e2}^*\, U_{\mu 1}^*\right\} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

Current data: $|J_{CP}| \lesssim 0.035|\sin\delta|$ (can be relatively large!).

- **Majorana phases α_{21}, α_{31}**:
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

    S.M. Bilenky, J. Hosek, S.T.P.,1980;
    P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

  - $|<m>|$ in $(\beta\beta)_{0\nu}$–decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \rightarrow e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\delta, \alpha_{21,31}$!

- Fogli et al., Phys. Rev. D86 (2012) 013012, global analysis, b.f.v.: $\sin^2\theta_{13} = 0.0241$ (0.0244), NH (IH).

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.54 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),
- $|\Delta m^2_{31(32)}| \cong 2.48$ (2.44) $\times 10^{-3}$ eV^2, $\sin^2\theta_{23} \cong 0.425$ (0.437), NH (IH),
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} = 0.0234$ (0.0239), NH (IH).
- $1\sigma(\Delta m^2_{21}) = 2.6\%$, $1\sigma(\sin^2\theta_{12}) = 5.4\%$;
- $1\sigma(|\Delta m^2_{31(23)}|) = 3\%$, $1\sigma(\sin^2\theta_{23}) = 14\%$;
- $1\sigma(\sin^2\theta_{13}) = 10\%$,
- $3\sigma(\Delta m^2_{21})$: $(6.99 - 8.18) \times 10^{-5}$ eV^2; $3\sigma(\sin^2\theta_{12})$: $(0.259 - 0.359)$;
- $3\sigma(|\Delta m^2_{31(23)}|)$: $2.19(2.17) - 2.62(2.61) \times 10^{-3}$ eV^2;
$3\sigma(\sin^2\theta_{23})$: $0.331(0.335) - 0.637(0.663)$;
- $3\sigma(\sin^2\theta_{13})$: $0.0169(0.0171) - 0.0313(0.0315)$.

Fogli *et al.*, arXiv:1205.5254v3

After the discovery of the $\nu-$Oscillations, the most important positive result was the high precision measurement of the "CHOOZ" angle θ_{13} in the Daya Bay and RENO experiments.

- March 8, 2012, Daya Bay: 5.2σ evidence for $\theta_{13} \neq 0$, $\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005$.

- April 4, 2012, RENO: 4.9σ evidence for $\theta_{13} \neq 0$, $\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019$.

- Nu'2012 (June 4-9, 2012), T2K, Double Chooz: 3.2σ and 2.9σ evidence for $\theta_{13} \neq 0$.

- RENO, 12/09/2013 (TAUP 2013): $\sin^2 2\theta_{13} = 0.100 \pm 0.010\ (stat.) \pm 0.012$.

- Daya Bay, 23/08/2013: $\sin^2 2\theta_{13} = 0.090 \pm 0.009$.


Sendai
Super Kamiokande
295km
JAERI
(Tokai)
Japan
KEK
Tokyo
Gifu
Nagoya
Kawasaki
Kyoto
Yokohama
Osaka
(c) 2000 ESRI
~ 420.0 mi / 675.8 km across

T2K: Search for $\nu_\mu \to \nu_e$ oscillations

T2K: first results March 2011 (2 events);

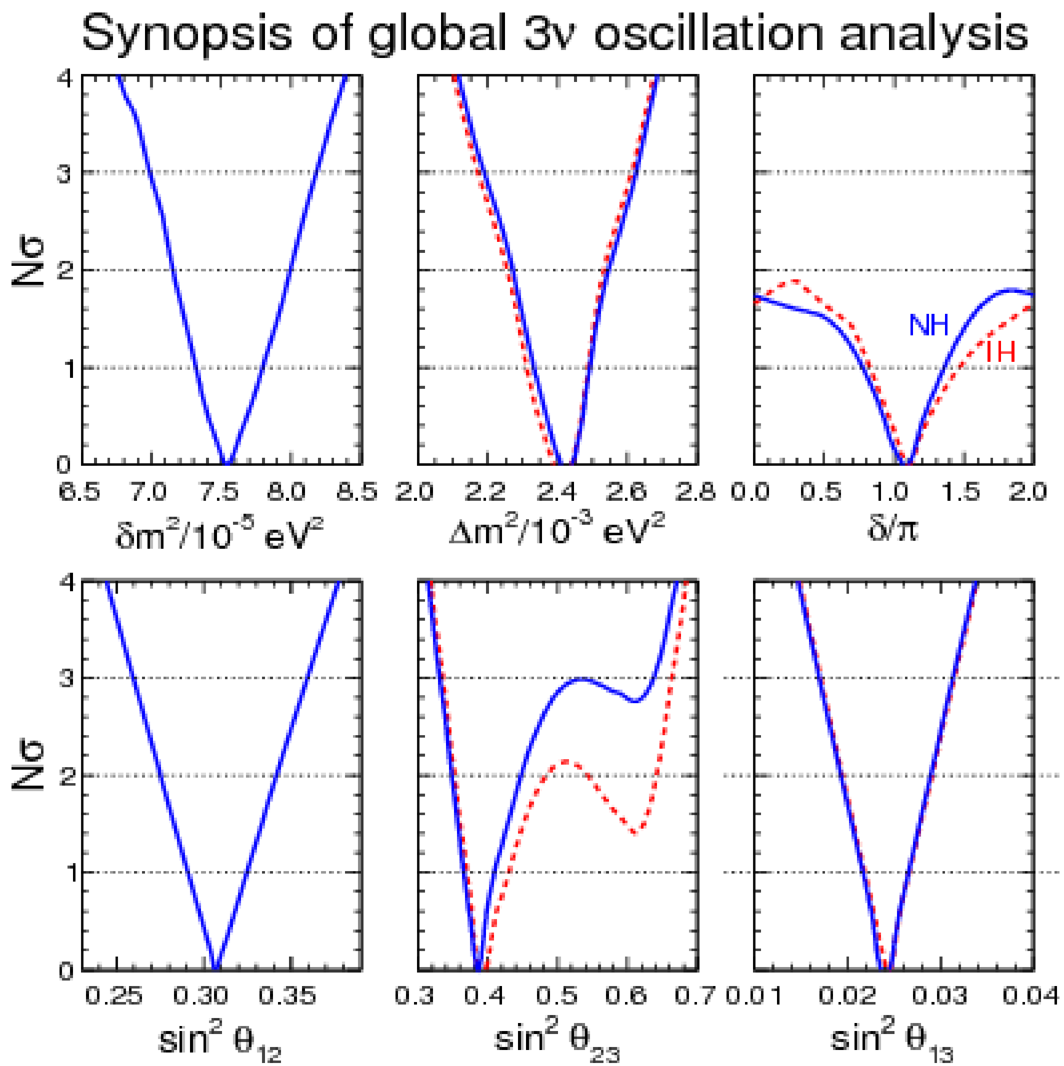
June 14, 2011 (6 events): evidence for $\theta_{13} \neq 0$ at 2.5σ;

July, 2013 (28 events).

For $|\Delta m^2_{23}| = 2.4 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} = 1$, $\delta = 0$, NO (IO) spectrum:

$\sin^2 2\theta_{13} = 0.14$ (0.17), best fit.

This value is by a factor of ~ 1.6 (1.9) bigger than the b.f. value obtained in the Daya Bay experiment.



Global data: best fit $\sin^2\theta_{23} \cong 0.39$;

$\cos\delta = -1$ favored over $\cos\delta = +1$;

best fit: $\sin\theta_{13}\cos\delta \cong -0.16$;

Fogli *et al.*, Phys. Rev. D86 (2012) 013012

Neutrino Mixing: New Symmetry?

- $\theta_{12} = \theta_{\odot} \cong \frac{\pi}{5.4}, \quad \theta_{23} = \theta_{atm} \cong \frac{\pi}{4}(?), \quad \theta_{13} \cong \frac{\pi}{20}$

$$U_{PMNS} \cong \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \epsilon \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}}(?) \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}(?) \end{pmatrix};$$

Very different from the CKM-matrix!

- $\theta_{12} \cong \pi/4 - 0.19, \quad \theta_{13} \cong 0 + \pi/20, \quad \theta_{23} \cong \pi/4 - 0.08.$
- U_{PMNS} due to new approximate symmetry?

A Natural Possibility (vast literature):

$$U = U_{\text{lep}}^{\dagger}(\theta^{\ell}_{ij}, \psi)\; Q(\phi, \varphi) U_{\text{tri,bim,LC}}\; P^0(\alpha_1, \alpha_2)\,,$$

with

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\text{BM}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

- $U_{\text{lep}}^{\dagger}(\theta^{\ell}_{ij}, \psi)$ - from diagonalization of the l^- mass matrix;
- $U_{\text{TBM,BM,...}} P^0(\alpha_{21}, \alpha_{31})$ - from diagonalization of the ν mass matrix;
- $Q(\phi, \varphi)$, - from diagonalization of both the l^- and ν mass matrices.

P. Frampton, STP, W. Rodejohann, 2003

$U_{\rm LC}$, $U_{\rm GRAM}$, $U_{\rm GRBM}$, $U_{\rm HGM}$:

$$U_{\rm LC} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{c^{\nu}_{23}}{\sqrt{2}} & \frac{c^{\nu}_{23}}{\sqrt{2}} & s^{\nu}_{23} \\ \frac{s^{\nu}_{23}}{\sqrt{2}} & -\frac{s^{\nu}_{23}}{\sqrt{2}} & c^{\nu}_{23} \end{pmatrix}; \quad \mu-\tau \text{ symmetry}: \quad \theta^{\nu}_{23} = \mp\pi/4;$$

$$U_{\rm GR} = \begin{pmatrix} c^{\nu}_{12} & s^{\nu}_{12} & 0 \\ -\frac{s^{\nu}_{12}}{\sqrt{2}} & \frac{c^{\nu}_{12}}{\sqrt{2}} & -\sqrt{\frac{1}{2}} \\ -\frac{s^{\nu}_{12}}{\sqrt{2}} & \frac{c^{\nu}_{12}}{\sqrt{2}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\rm HGM} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad \theta^{\nu}_{12} = \pi/6.$$

$U_{\rm GRAM}$: $\sin^2\theta^{\nu}_{12} = (2+r)^{-1} \cong 0.276$, $r = (1+\sqrt{5})/2$
(GR: $r/1$; $a/b = a + b/a$, $a > b$**)**

$U_{\rm GRBM}$: $\sin^2\theta^{\nu}_{12} = (3-r)/4 \cong 0.345$.

- U_{TBM}: $s^2_{12} = 1/3$, $s^2_{23} = 1/2$, $s^2_{13} = 0$; $s^2_{13} = 0$ must be corrected; if $\theta_{23} \neq \pi/4$, $s^2_{23} = 0.5$ must be corrected .

- U_{BM}: $s^2_{12} = 1/2$, $s^2_{23} = 1/2$, $s^2_{13} = 0$; $s^2_{13} = 0$, $s^2_{12} = 1/2$ and possibly $s^2_{23} = 1/2$ must be corrected.

$U_{\text{TBM(BM)}}$: Groups A_4, S_4, T', ... (vast literature)

(Reviews: G. Altarelli, F. Feruglio, arXiv:1002.0211; M. Tanimoto et al., arXiv:1003.3552; S. King and Ch. Luhn, arXiv:1301.1340)

- U_{GR}: Group A_5,...;

L. Everett, A. Stuart, arXiv:0812.1057;...

- U_{LC}: alternatively $U(1)$, $L' = L_e - L_\mu - L_\tau$

S.T.P., 1982

- U_{LC}: $s^2_{12} = 1/2$, $s^2_{13} = 0$, s^ν_{23} - free parameter; $s^2_{13} = 0$ and $s^2_{12} = 1/2$ must be corrected.

None of the symmetries leading to U_{TBM}, U_{BM}, U_{LC}, U_{GRM}, U_{HGM} can be exact.

Which is the correct approximate symmetry, i.e., approximate form of U_{PMNS} (if any)?

In all cases of U_ν given by U_{tri}, or U_{bim}, or U_{LC}, etc. the requisite corrections of some of the mixing angles are small and can be considered as perturbations to the corresponding symmetry values.

What is the minimal U_{lep} providing the requisite corrections to $U_{\text{TBM,BM,LC,GRM,HGM}}$?

The simplest case ($SU(5) \times T'$,...)

$U_{\text{lep}} \cong O^{\ell}_{12}(\theta^{\ell}_{12})$; now $Q = \text{diag}(e^{i\varphi}, 1, 1)$;

$\sin^{\ell}\theta_{13}$, $\sin^{\ell}\theta_{23}$ - negligibly small ($SU(5) \times T'$,...).

$U_{\text{BM(LC)}}$: $\sin^2\theta_{12} = \frac{1}{2} + \sin 2\theta^{\nu}_{12}\, \sin\theta_{13}\, \cos\delta$,

δ is the Dirac CPV phase;

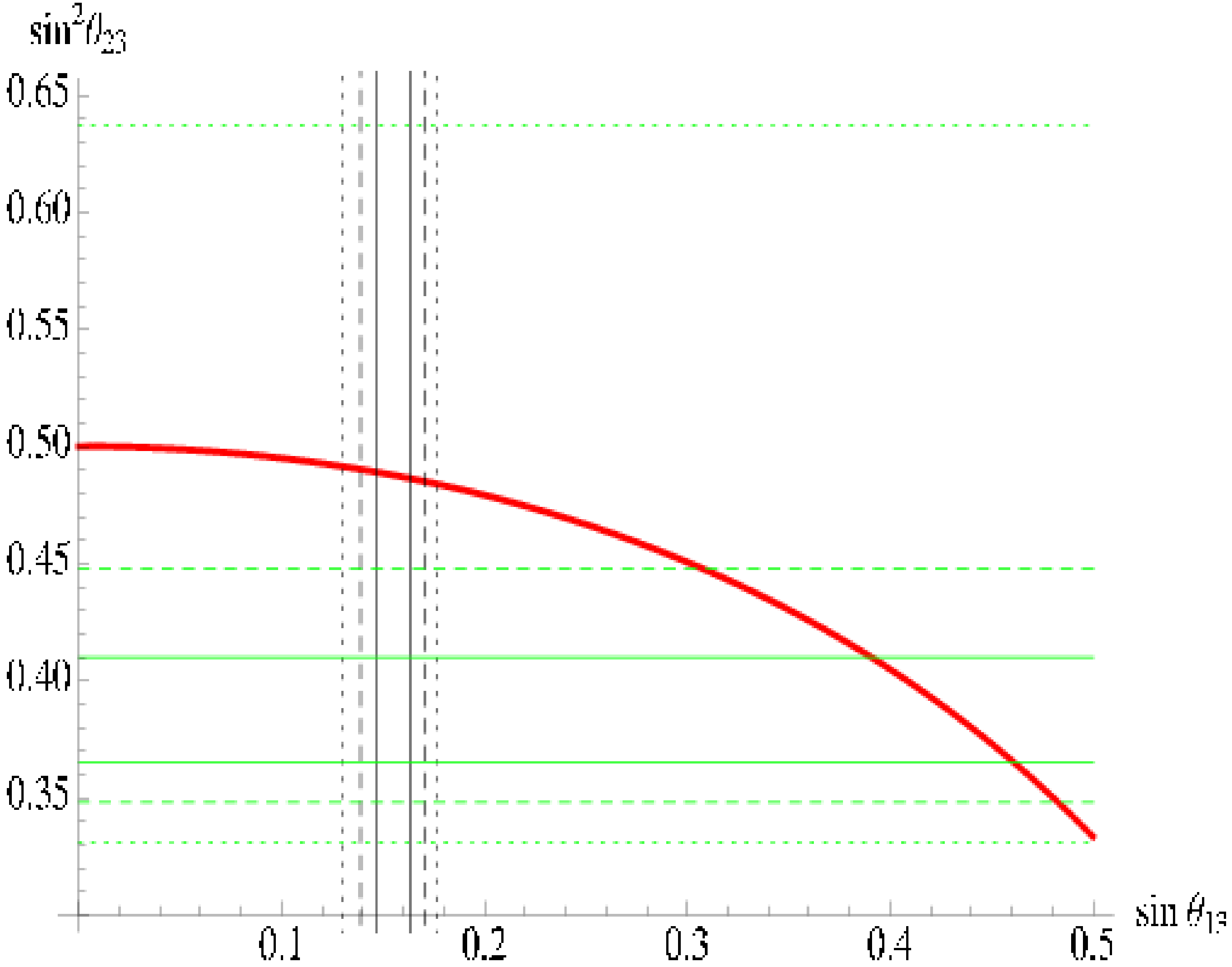
U_{BM}: requires $\cos\delta \cong \mp 1$ as $\sin 2\theta^{\nu}_{12} = \pm 1$.

U_{TBM}: $\sin^2\theta_{12} = \frac{1}{3} \mp 2\,\frac{\sqrt{2}}{3}\, \sin\theta_{13}\, \cos\delta$.

Problem for $U_{\text{TBM,BM,GRM,HGM}}$ if $\sin^2\theta_{23} \cong 0.42$:

$$\sin^2\theta_{23} = \frac{1-2\sin^2\theta_{13}}{2(1-\sin^2\theta_{13})} \cong 0.5(1 - \sin^2\theta_{13}).$$

P. Frampton, S.T.P., W. Rodejohann, 2004;
S. King, 2005; S. Antusch, S. King, 2005; I. Masina, 2006;
K. Hochmuth, S.T.P., W. Rodejohann, 2007



sin²θ23
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.1
0.2
0.3
0.4
0.5
sin θ13

Larger correction to $\sin^2\theta^\nu_{23} = 0.5$ needed.

Minimal case: $U_{\text{lep}} \cong U_{\text{lep}}(\theta^\ell_{12}, \theta^\ell_{23})$,

$Q = \text{diag}(1, e^{-i\psi}, e^{-i\omega})$.

Two possibilities.

"Standard" Ordering:

$U_{\text{lep}}(\theta^\ell_{12}, \theta^\ell_{23}) = O^T_{23}(\theta^\ell_{23}) O^T_{12}(\theta^\ell_{12})$ (GUTs typically);

in many theories - a consequence of $m^2_e \ll m^2_\mu \ll m^2_\tau$.

"Inverse" Ordering:

$U_{\text{lep}}(\theta^\ell_{12}, \theta^\ell_{23}) = O^T_{12}(\theta^\ell_{12}) O^T_{23}(\theta^\ell_{23})$

Results are drastically different in the two cases.

Standard Ordering

$$U = O_{12}(\theta^{\ell}_{12})O_{23}(\theta^{\ell}_{23})\,\mathrm{diag}(1, e^{-i\psi}, e^{-i\omega})\,O_{23}(\theta^{\nu}_{23})O_{12}(\theta^{\nu}_{12})\bar{P}\,,$$

$$\bar{P} = \mathrm{diag}(1, e^{i\kappa_1}, e^{i\kappa_2})\,.$$

Can be shown to be equivalent to:

$$U = O_{12}(\theta^{\ell}_{12})\,\mathrm{diag}(1, e^{i\phi}, 1)\,O_{23}(\hat{\theta}_{23})O_{12}(\theta^{\nu}_{12})P^0(\alpha_1, \alpha_2)$$

$\hat{\theta}_{23} = \hat{\theta}_{23}(\theta^{\ell}_{23}, \psi - \omega, \theta^{\nu}_{23})$, $\quad \phi = \phi(\theta^{\ell}_{23}, \psi, \omega, \theta^{\nu}_{23})$.

$\theta^{\nu}_{12} = \pi/4$ (BIM,LC) or $\sin^{-1}(1/\sqrt{3})$ (TBM)

Thus, $\theta_{12}, \theta_{23}, \theta_{13}, \delta$ - functions of $\theta^{\ell}_{12}, \phi, \hat{\theta}_{23}$.

Expect $\delta = \delta(\theta_{12}, \theta_{23}, \theta_{13})$(!)

Standard Ordering

$$U = O_{12}(\theta^{\ell}_{12}) O_{23}(\theta^{\ell}_{23}) \text{diag}(1, e^{-i\psi}, e^{-i\omega}) O_{23}(\theta^{\nu}_{23}) O_{12}(\theta^{\nu}_{12}) \bar{P}$$

Can be shown to be equivalent to:

$$U = O_{12}(\theta^{\ell}_{12}) \text{diag}(1, e^{i\phi}, 1) O_{23}(\hat{\theta}_{23}) O_{12}(\theta^{\nu}_{12}) P^0(\alpha_1, \alpha_2)$$

Indeed,

$$O_{12}(\theta^{\ell}_{12}) O_{23}(\theta^{\ell}_{23}) \text{diag}(1, e^{-i\psi}, e^{-i\omega}) O_{23}(\theta^{\nu}_{23}) O_{12}(\theta^{\nu}_{12}) \bar{P}$$

$$= O_{12}(\theta^{\ell}_{12}) Q \,\text{diag}(1, e^{i\phi}, 1) O_{23}(\hat{\theta}_{23}) \,\tilde{P} O_{12}(\theta^{\nu}_{12}) \bar{P}$$

$Q = \text{diag}(1, 1, e^{-i\alpha})$, $\tilde{P} = \text{diag}(1, 1, e^{i\beta})$, $P^0 = \tilde{P}\bar{P}$,

$\alpha = (\gamma + \psi + \omega)$ - unphysical,

$\beta = (\gamma - \phi)$ - adds to the Majorana phases,

$$
\begin{aligned}
\sin^2\widehat{\theta}_{23} &= \frac{1}{2}\left(1-2\sin\theta^{\ell}_{23}\cos\theta^{\ell}_{23}\cos(\omega-\psi)\right), \\
\phi &= \arg\left(e^{-i\psi}\cos\theta^{\ell}_{23}+e^{-i\omega}\sin\theta^{\ell}_{23}\right), \\
\gamma &= \arg\left(-e^{-i\psi}\cos\theta^{\ell}_{23}+e^{-i\omega}\sin\theta^{\ell}_{23}\right), \\
P^0 &= \mathrm{diag}(1, e^{i\kappa_1}, e^{i(\kappa_2+\beta)}), \\
U &= O_{12}(\theta^{\ell}_{12})\,\mathrm{diag}(1, e^{i\phi}, 1)\,O_{23}(\widehat{\theta}_{23})\,O_{12}(\theta^{\nu}_{12})\,P^0 .
\end{aligned}
$$

$$\sin\theta_{13} = |U_{e3}| = \sin\theta^{\ell}_{12}\sin\hat{\theta}_{23}\,,$$

$$\sin^2\theta_{23} = \frac{|U_{\mu3}|^2}{1-|U_{e3}|^2} = \sin^2\hat{\theta}_{23}\frac{\cos^2\theta^{\ell}_{12}}{1-\sin^2\theta^{\ell}_{12}\sin^2\hat{\theta}_{23}}\,,$$

$$\sin^2\theta_{12} = \frac{|U_{e2}|^2}{1-|U_{e3}|^2} = \frac{\left|\sin\theta^{\nu}_{12}\cos\theta^{\ell}_{12}+e^{i\phi}\cos\theta^{\nu}_{12}\cos\hat{\theta}_{23}\sin\theta^{\ell}_{12}\right|^2}{1-\sin^2\theta^{\ell}_{12}\sin^2\hat{\theta}_{23}}\,,$$

J_{CP}: $\quad \delta \cong -\phi\,.$

From first two eqs.:

$\theta^{\ell}_{12} = \theta^{\ell}_{12}(\theta_{13},\theta_{23})$, $\hat{\theta}_{23} = \hat{\theta}_{23}(\theta_{13},\theta_{23})$;

substitute in the third.

$$\sin^2\theta_{23} = \frac{\sin^2\hat{\theta}_{23} - \sin^2\theta_{13}}{1 - \sin^2\theta_{13}}, \quad \hat{\theta}_{23} \cong \theta_{23}\,.$$

BM, LC: $\sin^2\theta_{12} = \frac{1}{2} + \frac{1}{2}\frac{\sin 2\theta_{23}\sin\theta_{13}\cos\phi}{1-\cos^2\theta_{23}\cos^2\theta_{13}}$

TBM: $\sin^2\theta_{12} = \frac{1}{3}\left(2 + \frac{\sqrt{2}\sin 2\theta_{23}\sin\theta_{13}\cos\phi - \sin^2\theta_{23}}{1-\cos^2\theta_{23}\cos^2\theta_{13}}\right)$

$\cos\phi = \cos\phi(\theta_{12}, \theta_{13}, \theta_{23})$, both for BM, LC and TBM.

Similar results - for GRM and HGM.

Comparing the imaginary and real parts of $U^*_{e1}U^*_{\mu 3}U_{e3}U_{\mu 1}$ in the two parametrisations:

- BM, LC cases

$$\sin\delta = -\frac{\sin\phi(\theta_{12},\theta_{13},\theta_{23})}{\sin 2\theta_{12}}\,,$$

$$\cos\delta = \frac{\cos\phi}{\sin 2\theta_{12}}\left(\frac{2\sin^2\theta_{23}}{\sin^2\theta_{23}\cos^2\theta_{13}+\sin^2\theta_{13}} - 1\right)\,;$$

$$\cos\delta = -\frac{1}{2\sin\theta_{13}}\cot 2\theta_{12}\tan\theta_{23}\left(1-\cot^2\theta_{23}\sin^2\theta_{13}\right)\,.$$

For $\sin^2\theta_{12} = 0.31$, $\sin^2\theta_{23} = 0.39$, $\sin\theta_{13} = 0.16$

$\sin\delta \cong \pm 0.170$, $\cos\delta \cong -0.985$, i.e., $\delta \cong \pi$.

- TBM case

$$\sin\delta = -\frac{2\sqrt{2}}{3}\frac{\sin\phi}{\sin 2\theta_{12}},$$

$$\cos\delta = \frac{2\sqrt{2}}{3\sin 2\theta_{12}}\cos\phi\left(-1+\frac{2\sin^2\theta_{23}}{\sin^2\theta_{23}\cos^2\theta_{13}+\sin^2\theta_{13}}\right) + \frac{1}{3\sin 2\theta_{12}}\frac{\sin 2\theta_{23}\,\sin\theta_{13}}{\sin^2\theta_{23}\cos^2\theta_{13}+\sin^2\theta_{13}}.$$

$$\cos\delta = \frac{\tan\theta_{23}}{3\sin 2\theta_{12}\sin\theta_{13}}\times \left[1+\left(3\sin^2\theta_{12}-2\right)\left(1-\cot^2\theta_{23}\,\sin^2\theta_{13}\right)\right].$$

For $\sin^2\theta_{12}=0.31$, $\sin^2\theta_{23}=0.39$, $\sin\theta_{13}=0.16$

$\sin\delta \cong \pm 0.999$, $\cos\delta \cong -0.0490$, i.e., $\delta \cong \pi/2$ or $3\pi/2$.

In all three cases BM, LC, TBM:

- New sum rules relating θ_{12},θ_{13}, θ_{23} and δ;
- $J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13})$.
- BM, LC cases: $\delta \cong \pi$
- TBM case: $\delta \cong 3\pi/2$ or $\pi/2$;

b.f.v. of θ_{ij}: $\delta \cong 265^\circ$ or 95°.

- GRAM case, b.f.v. of θ_{ij}: $\delta \cong 286^\circ$ or 74°.

- GRBM case, b.f.v. of θ_{ij}: $\delta \cong 261^\circ$ or 99°.

- HGM case, b.f.v. of θ_{ij}: $\delta \cong 296^\circ$ or 64°.

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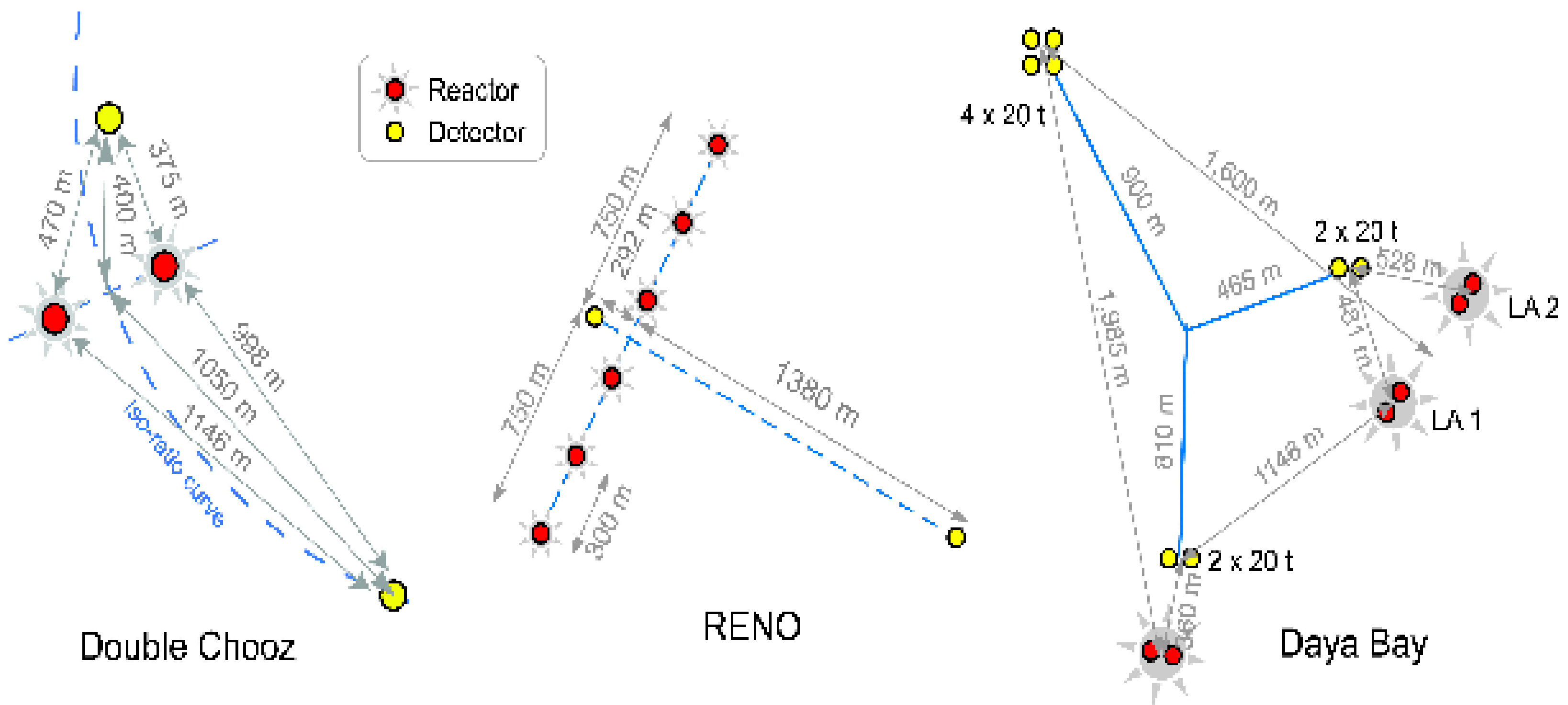
June 14, 2011 (6 events): evidence for $\theta_{13} \neq 0$ at 2.5σ;

July, 2013 (28 events).
For $|\Delta m^2_{23}| = 2.4 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} = 1$, $\delta = 0$, NO (IO) spectrum:

$\sin^2 2\theta_{13} = 0.14$ (0.17), best fit.

This value is by a factor of ~ 1.6 (1.9) bigger than the b.f. value obtained in the Daya Bay experiment.



M. Mezzetto, T. Schwetz, arXiv:1003.5800[hep-ph]

$$P(\bar{\nu}_e \to \bar{\nu}_e) = P_{ee}(\theta_{12}, \theta_{13}, \Delta m^2_{31}, \Delta m^2_{21})$$

$$P(\nu_\mu \to \nu_e) = P_{\mu e}(\theta_{12}, \theta_{13}, \Delta m^2_{31}, \Delta m^2_{21}, \theta_{23}, \delta)$$

Up to 2nd order in the two small parameters $|\alpha| \equiv |\Delta m^2_{21}|/|\Delta m^2_{31}| \ll 1$ and $\sin^2\theta_{13} \ll 1$:

$$P_m^{3\nu\ man}(\nu_\mu \to \nu_e) \cong P_0 + P_{\sin\delta} + P_{\cos\delta} + P_3\,,$$

$$P_0 = \sin^2\theta_{23}\,\frac{\sin^2 2\theta_{13}}{(A-1)^2}\,\sin^2[(A-1)\Delta],$$

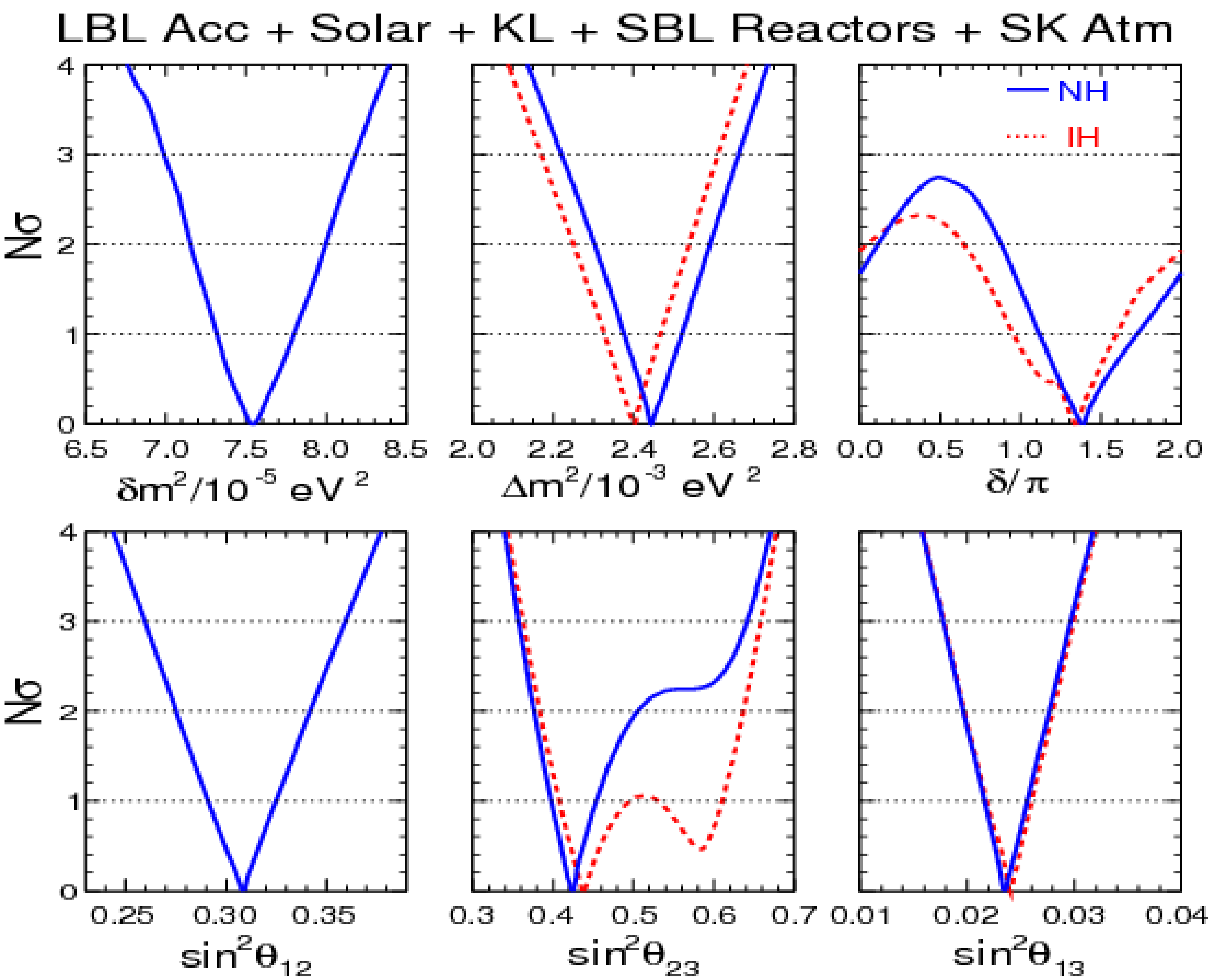
$$P_3 = \alpha^2\,\cos^2\theta_{23}\,\frac{\sin^2 2\theta_{12}}{A^2}\,\sin^2(A\Delta),$$

$$P_{\sin\delta} = -\,\alpha\,\frac{8\,J_{CP}}{A(1-A)}(\sin\Delta)(\sin A\Delta)\,(\sin[(1-A)\Delta]),$$

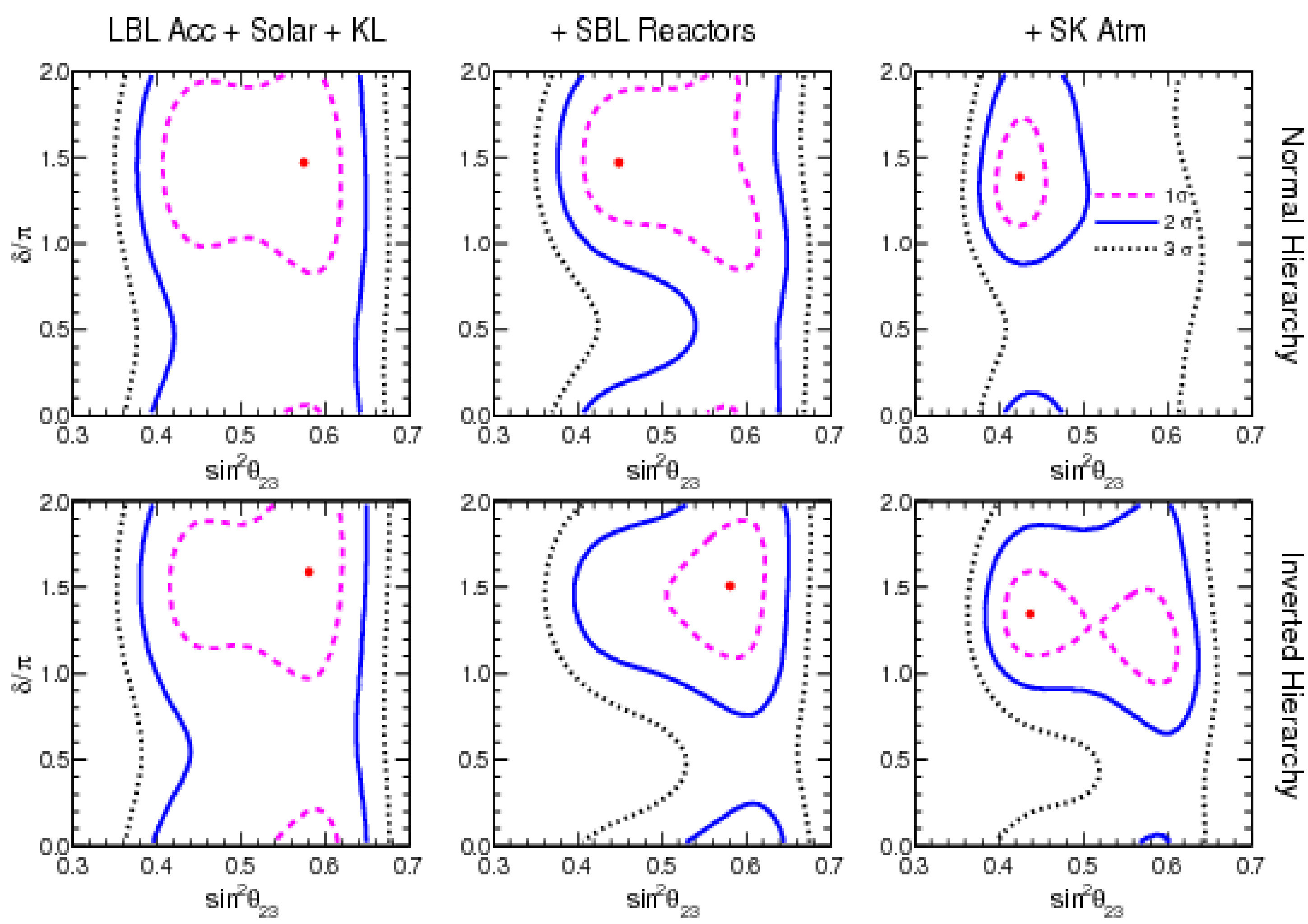
$$P_{\cos\delta} = \alpha\,\frac{8\,J_{CP}\cot\delta}{A(1-A)}(\cos\Delta)(\sin A\Delta)\,(\sin[(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m^2_{31}\,L}{4E},\;\; A = \sqrt{2}G_{\mathrm{F}}N_e^{man}\frac{2E}{\Delta m^2_{31}}.$$

$\bar{\nu}_\mu \to \bar{\nu}_e$: $\delta,\;\; A \to (-\delta),\;\; (-A)$



F. Capozzi, E. Lisi *et al.*, arXiv:1312.2878



F. Capozzi, E. Lisi *et al.*, arXiv:1312.2878

Large $\sin\theta_{13} \cong 0.16$ (Daya Bay, RENO) - far-reaching implications for the program of reserach in neutrino physics:

- For the determination of the type of $\nu-$ mass spectrum (or of $\text{sgn}(\Delta m^2_{\text{atm}})$) in neutrino oscillation experiments.

- For understanding the patern of the neutrino mixing and its origins (symmetry, etc.?).

- For the predictions for the $(\beta\beta)_{0\nu}$-decay effective Majorana mass in the case of NH light ν mass spectrum (possibility of a strong suppression).

Large $\sin\theta_{13} \cong 0.16$ (Daya Bay, RENO) + $\delta = 3\pi/2$ - far-reaching implications:

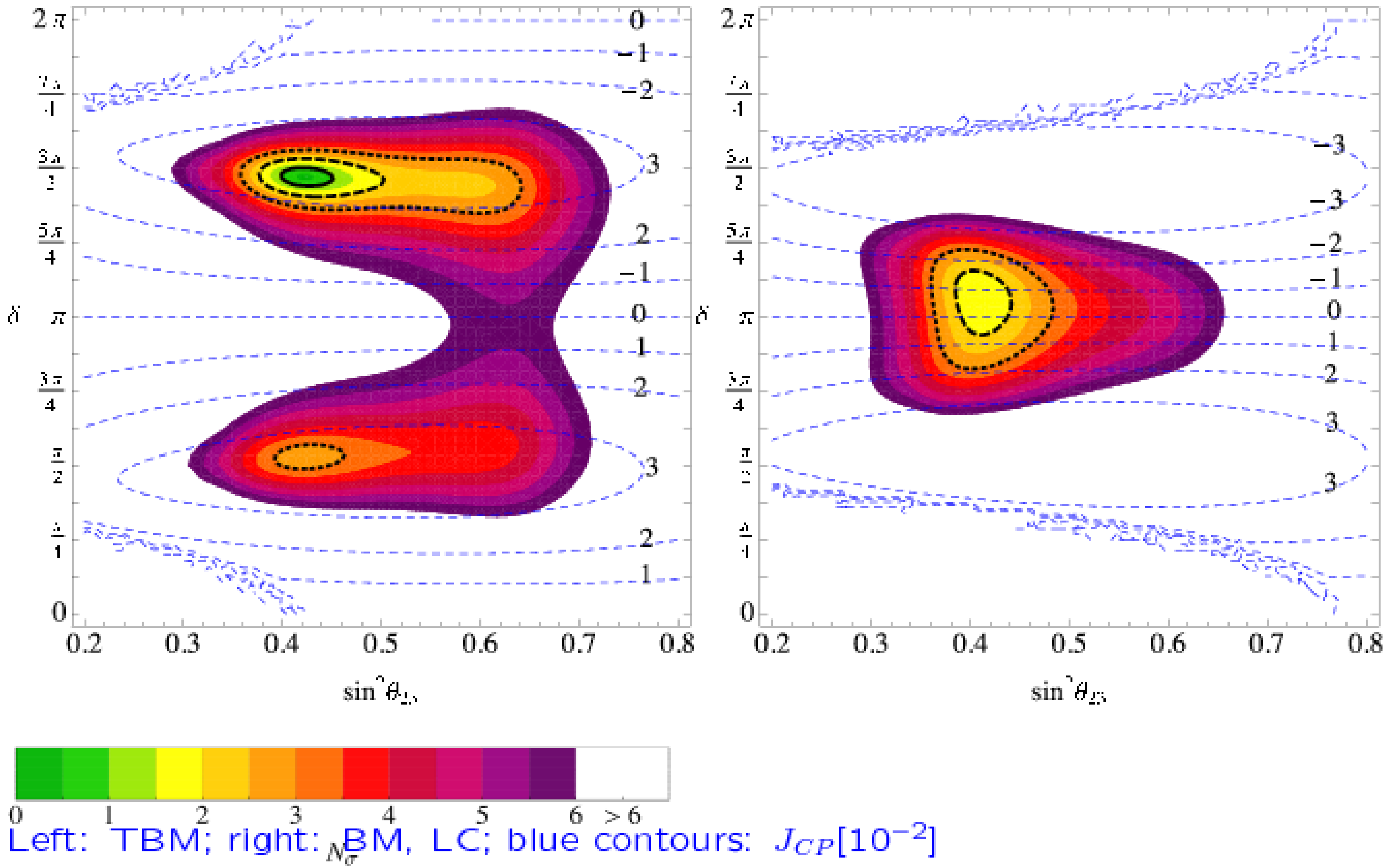
- For the searches for CP violation in ν-oscillations; for the b.f.v. one has $J_{CP} \cong -0.035$;

- Important implications also for the "flavoured" leptogenesis scenario of generation of the baryon asymmetry of the Universe (BAU).

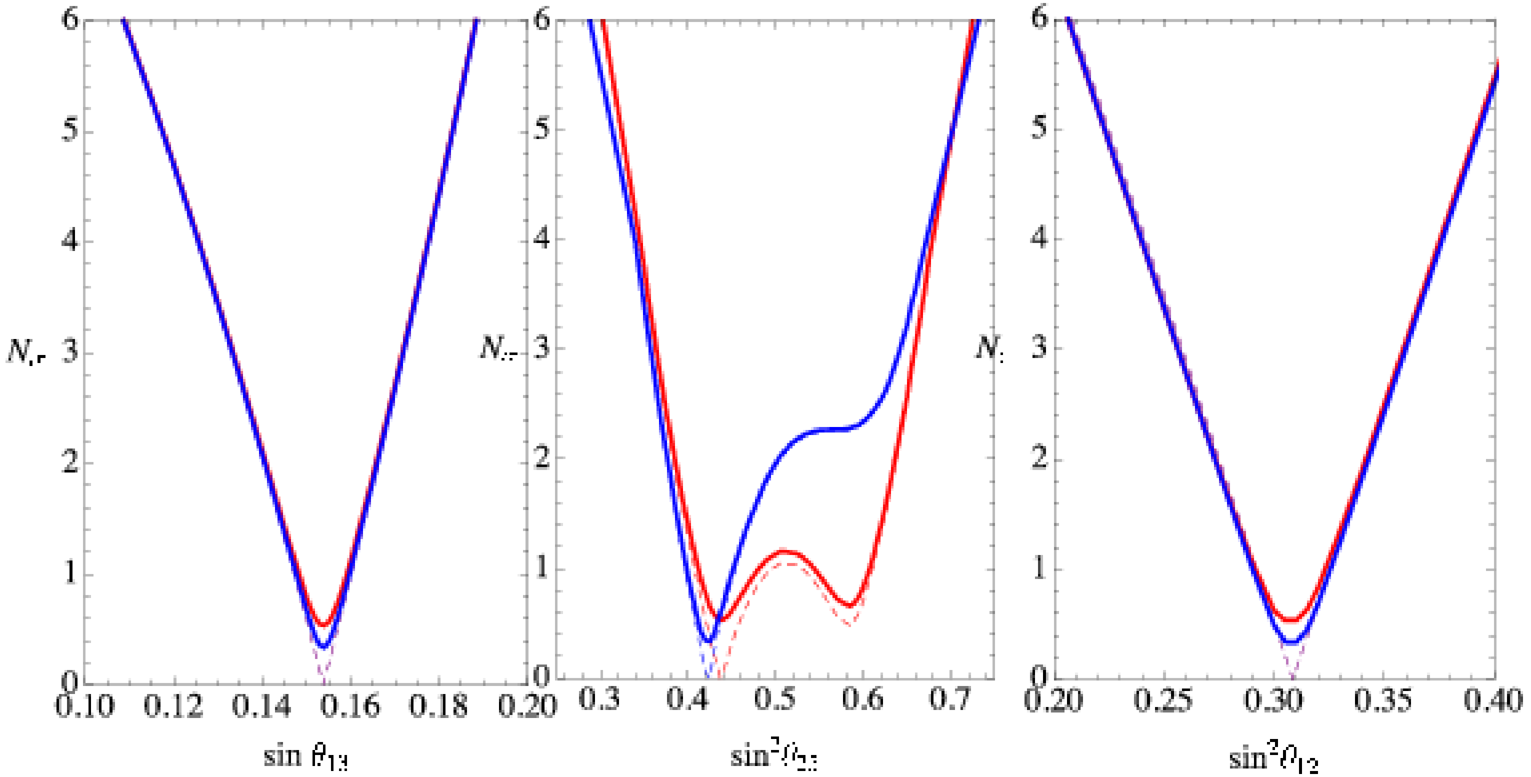
If all CPV, necessary for the generation of BAU is due to δ, a necessary condition for reproducing the observed BAU is

$$|\sin\theta_{13}\,\sin\delta| \gtrsim 0.09$$

S. Pascoli, S.T.P., A. Riotto, 2006.


2 π
7π/4
3π/2
5π/4
δ
π
3π/4
π/2
π/4
0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
sin²θ23
0
−1
−2
3
2
−1
0
1
2
3
2
1
−3
−3
−2
−1
0
1
2
3
3
0
1
2
3
4
5
6
> 6
Nσ
Left: TBM; right: BM, LC; blue contours: JCP[10−2]

Standard Ordering - TBM

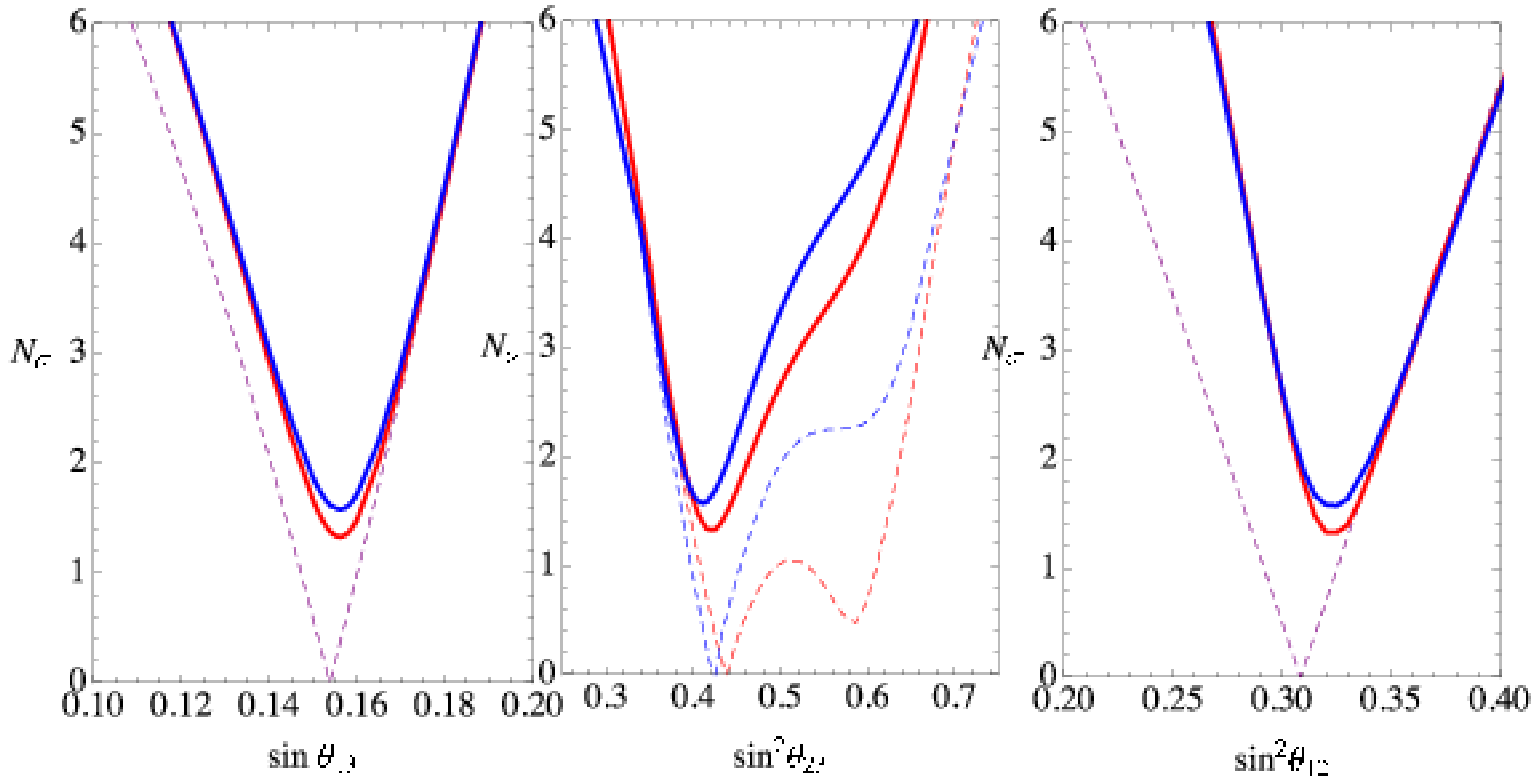


N_σ **as a function of** $\sin\theta_{13}$, $\sin^2\theta_{12}$, $\sin^2\theta_{23}$.

dashed lines - Fogli et al., solid lines - our analysis.

Blue lines - NH, red lines - IH; NH: $\sin^2\theta_{23} \leq 0.5$ **at** $\sim 2\sigma$.

Standard Ordering - BM, LC

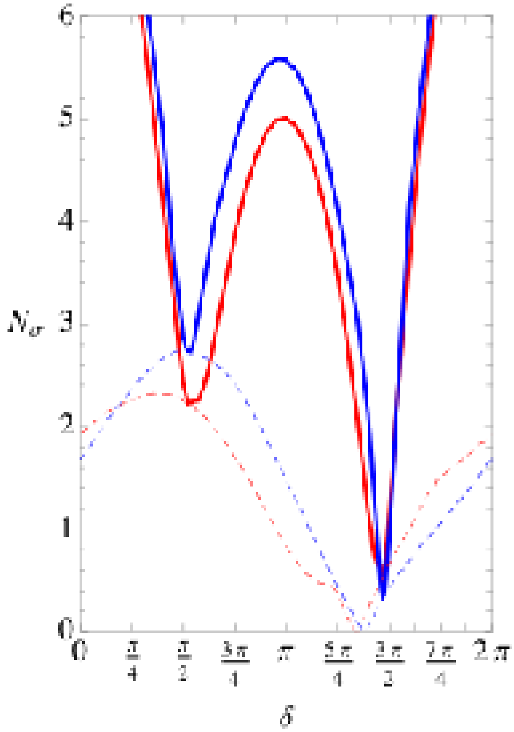


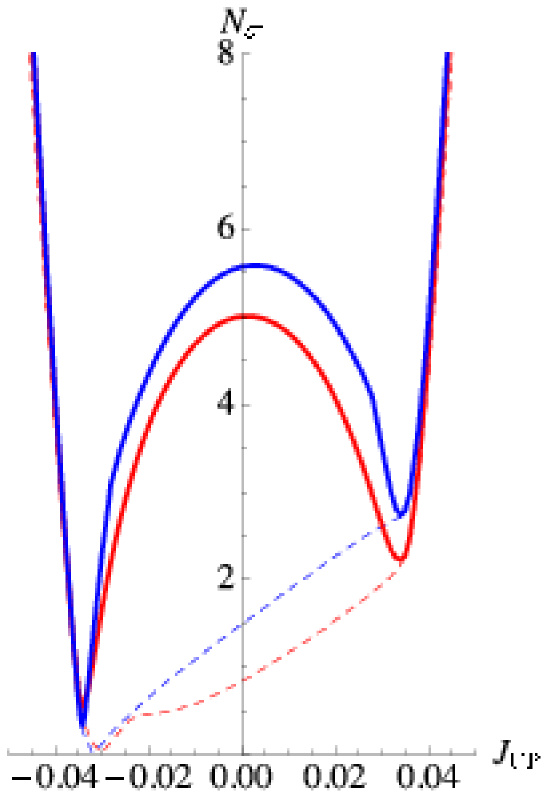
N_σ **as a function of** $\sin\theta_{13}$, $\sin^2\theta_{12}$, $\sin^2\theta_{23}$.

dashed lines - Fogli et al., solid lines - our analysis.

Blue lines - NH, red lines - IH; NH, IH: $\sin^2\theta_{23} \leq 0.5$ **at** $\sim 3.0\sigma$.

Standard Ordering - TBM





N_σ **as a function of** δ, J_{CP}. **Blue lines - NH, red lines - IH.**

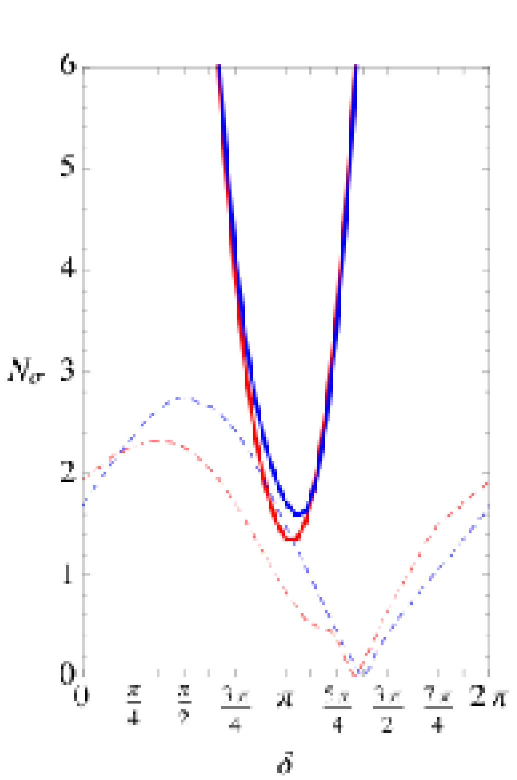
Dashed lines - Fogli et al., solid lines - our analysis.

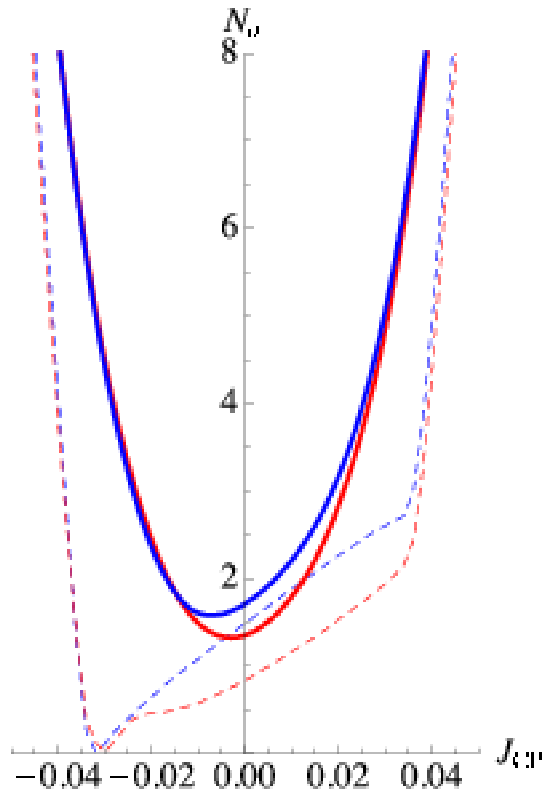
$J_{CP} \neq 0$ **at** $\sim 5\sigma$; **b.f.v.:** $J_{CP} \cong -0.034$, **NH,IH;**

at 3σ, **NH:** $0.032 \lesssim J_{CP} \lesssim 0.036$ **or** $-0.038 \lesssim J_{CP} \lesssim -0.028$;

at 3σ, **IH:** $0.027 \lesssim J_{CP} \lesssim 0.037$ **or** $-0.039 \lesssim J_{CP} \lesssim -0.024$.

Standard Ordering - BM, LC





N_σ as a function of δ, J_{CP}. **Blue lines - NH, red lines - IH.**

Dashed lines - Fogli et al., solid lines - our analysis.

NH, IH b.f.v.: $J_{CP} \cong 0$; **NH, IH, at** 3σ: $-0.026 \lesssim J_{CP} \lesssim 0.022$.

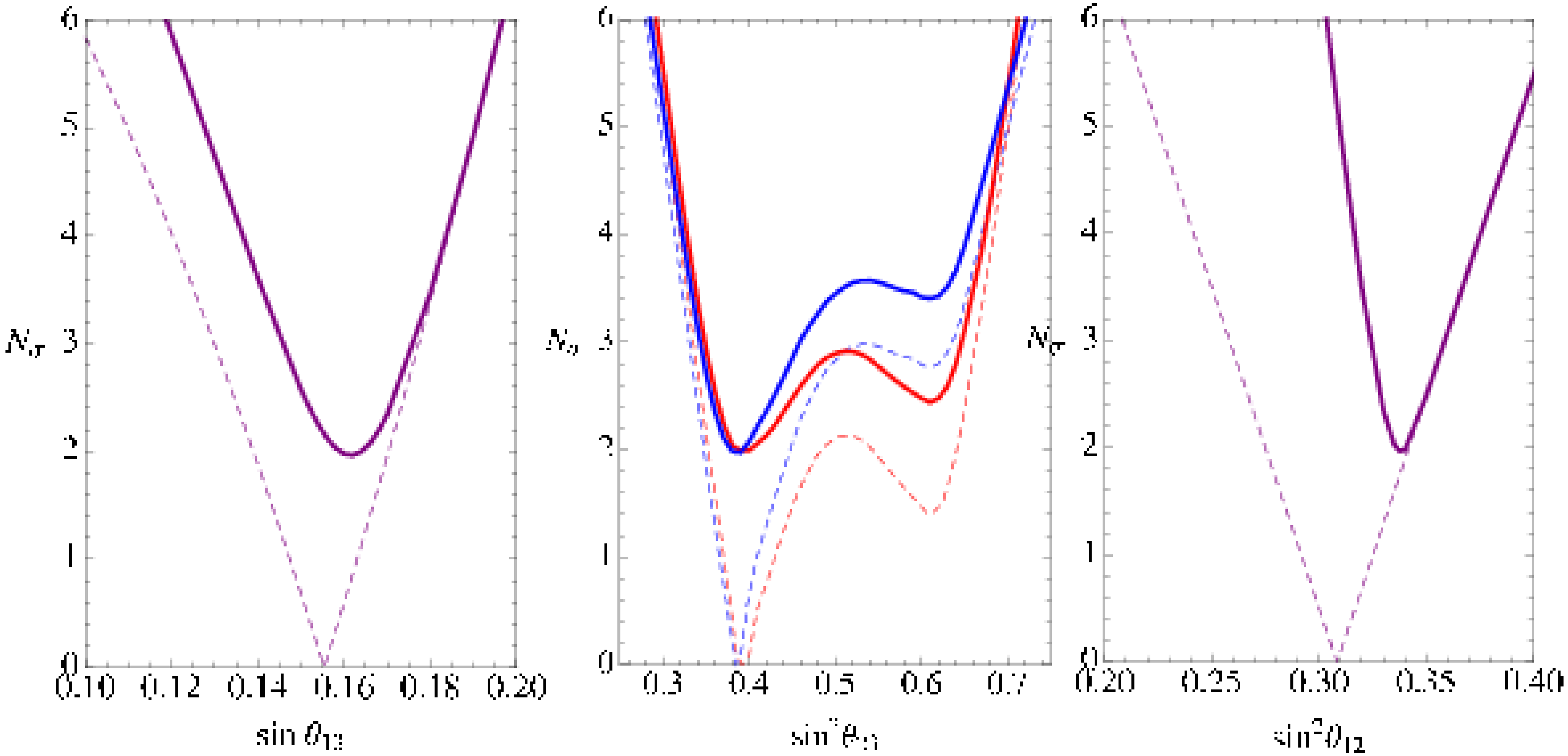
Inverse Ordering

$U = O_{23}^T(\theta_{23}^\ell) O_{12}^T(\theta_{12}^\ell) \mathrm{diag}(1, e^{i\tilde{\phi}}, e^{i\varphi}) O_{23}(\theta_{23}^\nu) O_{12}(\theta_{12}^\nu) P$

Now $\theta_{12}, \theta_{23}, \theta_{13}, \delta$ - functions of $\theta_{23}^\ell, \tilde{\phi}, \varphi, \theta_{12}^\ell$.

TBM: results as in Fogli et al.;
BM: tension (allowed at 2σ).

Inverse Ordering - BM



N_σ **as a function of** $\sin\theta_{13}$, $\sin^2\theta_{12}$, $\sin^2\theta_{23}$.

Dashed lines - Fogli et al., solid lines - our analysis.

Blue lines - NH, red lines - IH.

Conclusions.

- We have considered a simple scheme for obtaining $\sin^2\theta_{13} \cong 0.16$ and $\sin^2\theta_{23} \cong 0.4$ from TBM, BM, LC from the charged lepton corrections. $U_{\rm lep} \cong U_{\rm lep}(\theta^\ell_{12}, \theta^\ell_{23})$ required.
- The results depend strongly on the ordering of the 1-2 and 2-3 rotations in $U_{\rm lep}$.
- Interesting results in the case of "Standard Ordering", $O_{23}(\theta^\ell_{23})O_{12}(\theta^\ell_{12})$:

i) new "sum rules": $\delta = \delta(\theta_{12}, \theta_{23}, \theta_{13})$;

ii) BM, LC: $\delta \cong \pi$;

iii) TBM: $\delta \cong 3\pi/2$ (hints from data), $J_{CP} \neq 0$ at 5σ, b.f.v. $J_{CP} = -0.034$;

iv) $\sin^2\theta_{23} \leq 0.5$ at $\sim 2\sigma$ ($\sim 3\sigma$), TBM+NH (BM, LC).

Standard Ordering of U_{lep}, TBM, BM, LC: the predictions for $\sin^2\theta_{23}$, δ and J_{CP} will be tested by the ν-oscillation experiments able to determine whether $\sin^2\theta_{23} \leq 0.5$ or $\sin^2\theta_{23} > 0.5$, and in the experiments searching for CP violation in neutrino oscillations.