Recent advances in dS/CFT

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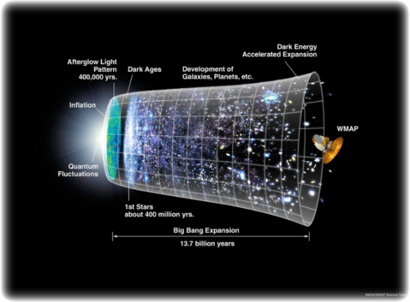
- Holography and cosmology (20 minute portion)
- Higher spin dS_4/CFT_3
  - Wavefunctional calculations
  - Extension of duality to parity-violating phases
- Entropy of the cosmological horizon via dS_3/CFT_2

Accelerating universe

Accelerated expansion applies both to

- the inflationary era (B-modes!!!!!!!!!!!!!!!)
- our current/late-time universe ("dark energy" domination)

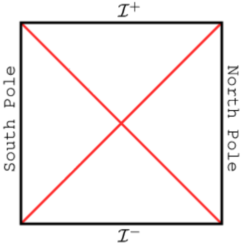


de Sitter spacetime

Both eras are well-approximated by the de Sitter geometry:

$$ds^2 = -dt^2 + e^{2Ht}dx_i^2 \, .$$

This represents an *exponentially* expanding spacetime.



Observer cosmological horizons obey thermodynamical laws, with $S = A/4$.

Fundamental questions: how do we understand quantum gravity "in" such a spacetime? What is the meaning of the horizon entropy? Proposal: Construct a holographic dual theory. [Maldacena; Strominger; Witten]

Holography

Historical intuition: black hole entropy $S = A/4$ scales like the area of the region as opposed to volume; same scaling as QFT in one lower dimension.

Proposal: perhaps quantum gravity can be reformulated as a QFT in one lower dimension ['t Hooft, Susskind].

Entropy bounds [Bousso] often lead to QFT "living" on an appropriate (conformal) boundary of the spacetime.

In most examples of holography renormalization group flow of QFT reconstructs emergent dimension; time is emergent in case of de Sitter.

Renormalization group flows are often between two fixed points; maybe in our universe these fixed points are the inflationary era and dark-energy domination! [Strominger]

Holography: AdS/CFT

AdS/CFT proposal [Maldacena; Gubser, Klebanov, Polyakov; Witten]:

$$\boxed{Z_{bulk} = Z_{CFT}}$$

Z defines a given theory by encoding all of its correlation functions.

Concrete proposal: Bulk theory is string theory in $\mathrm{AdS}_5 \times S^5$, boundary theory is four-dimensional $\mathcal{N} = 4$ super-Yang Mills.

Allows strongly coupled quantum gravity calculations through weakly coupled CFT calculations.

Duality often provides "microscopic" count of black hole entropy through a CFT_2 calculation.

But AdS is not our universe!

Holography: dS/CFT

Adapt success of AdS/CFT to dS. Hopeful due to analytic continuation $z \to i\eta$, $t \to iy$, $\ell_{AdS} \to i\ell_{dS}$ which connects AdS and dS:

$$ds^2 = \ell_{AdS}^2 \, \frac{-dt^2 + dz^2 + dx_i^2}{z^2} \longrightarrow \ell_{dS}^2 \, \frac{-d\eta^2 + dy^2 + dx_i^2}{\eta^2}$$

dS/CFT proposal [Maldacena; Strominger; Witten]:

$$\boxed{\Psi_{HH} = Z_{CFT}}$$

Weak form:

$$\log \Psi_{HH}[\phi(\vec{x}), \eta_c] = \sum_{n=1}^{\infty} \frac{1}{n!} \left(\int d^3x_1 \cdots \int d^3x_n \phi(\vec{x}_1) \cdots \phi(\vec{x}_n) \langle \mathcal{O}(\vec{x}_1) \cdots \mathcal{O}(\vec{x}_n) \rangle_{CFT} \right)$$

Strong form: Z_{CFT} non-perturbatively defines Ψ_{HH} for finite sources encoding geometry, topology, etc.

Holography: dS_4/CFT_3

Concrete conjectures for AdS/CFT (i.e. a specific bulk gravity theory and a boundary CFT which are equivalent) have existed for almost twenty years.

A concrete conjecture for dS/CFT was only made a few years ago [Anninos, Hartman, Strominger], relating a four-dimensional higher-spin theory to a three-dimensional CFT of N symplectic scalars:

$$S = \frac{1}{2}\int d^3x\, \Omega_{ab}\partial_i\chi^a\partial_i\chi^b, \qquad \Omega_{ab} = \begin{pmatrix} 0 & 1_{N/2\times N/2} \\ -1_{N/2\times N/2} & 0 \end{pmatrix}.$$

- Construct wavefunctionals in this theory by calculation of Z_{CFT}.
- Generalize this example (and others) by adding Chern-Simons sector; connect to AdS cousins.

Holography: dS_3/CFT_2

There does not exist well-understood proposal for dS_3/CFT_2, but we may be able to make progress by symmetry principles and the constraints of two-dimensional CFTs.

I will argue that the observer-dependent horizon entropy can be accounted for by a count of local operators in the putative dual CFT, although this does not seem to carry over to four dimensions!

Open mathematical problems of relevance to quantum de Sitter

- Do there exist modular-invariant functions (or even modular forms) of the form
$$Z(\tau,\bar{\tau}) = \sum_i \rho(\Delta_i,\bar{\Delta}_i) q^{\Delta_i} q^{\bar{\Delta}_i}$$
with $\rho(\Delta_i,\bar{\Delta}_i) \in \mathbb{Z}^+$ with Δ_i and $\bar{\Delta}_i$ *complex*?
- Equations like $O(-N) = Sp(N)$ are understood mathematically and physically; what about analytic continuations to imaginary N, i.e. $O(iN)$? [Deligne]

Caffeine Time

$Sp(N)$ theory

Minimal parity-invariant Type-A Vasiliev theory with Neumann boundary conditions in bulk dual to free $Sp(N)$ theory of *anticommuting* scalars [Anninos, Hartman, Strominger]:

$$S = \frac{1}{2}\int d^3x\sqrt{g}\,\Omega_{AB}\left(\partial_i\chi^A\partial_j\chi^B g^{ij} + \frac{R[g]}{8}\chi^A\chi^B + m(x^i)\chi^A\chi^B\right).$$

$$\chi\cdot\chi \longleftrightarrow \phi \quad \text{with} \quad m^2 l_{dS}^2 = +2, \qquad T_{ij} \longleftrightarrow g_{\mu\nu}\,.$$

Need to restrict the theory to singlet sector, i.e. only $Sp(N)$ invariant operators in spectrum.

Conserved currents of the form $J^{(s)}_{i_1\cdots i_s} = \Omega_{ab}\chi^a\partial_{(i_1}\cdots\partial_{i_s)}\chi^b + \cdots$ with dimension $\Delta = s+1$ dual to bulk spin-s fields.

Dunne-Kirsten method

What to do with this theory? Begin by computing wavefunctionals!

Turn off sources for operators $J^{(s)}_{i_1\cdots i_s} = \Omega_{ab}\chi^a \partial_{(i_1}\cdots\partial_{i_s)}\chi^b + \cdots$ with dimension $\Delta = s+1$ dual to higher spin fields.

Preserve $SO(3)$ symmetry: on S^3 consider mass profile $m(\theta)$ and metric deformation $ds^2 = d\theta^2 + f(\theta)^2 \, \sin^2\theta \, d\Omega_2^2$.

Gaussian theory: zeta-regularized partition function computed with Dunne-Kirsten formula on $\mathbb{R}^3$:

$$\log\left(\frac{\det\left[-\nabla^2+\mu^2+\hat{m}(r)\right]}{\det\left[-\nabla^2+\mu^2\right]}\right) = \sum_{l=0}^{\infty}(2l+1)\left(\underbrace{\log T^{(l)}(\infty)}_{\text{Gelfand-Yaglom}} - \underbrace{\frac{\int_0^\infty dr\, r\, \hat{m}(r)}{2l+1}}_{\text{regularizer}}\right).$$

Constant mass on peanuts: divergence!

Anninos, Denef, and Harlow computed Z_{CFT} for $m(x^i) = m_0$ on S^3.
Consider peanut deformation of geometry:

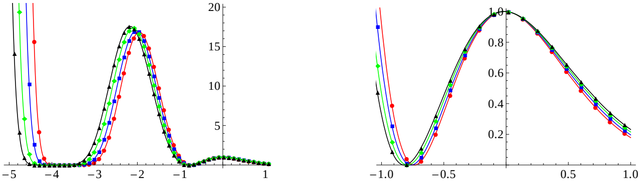


Figure : Left: $|\Psi_{HH}(\zeta, m)|^2$ for for $N = (\ell_{dS}/\ell_P)^2 = 2$ as a function of m_0 for peanut geometries ($l_{max} = 45$). Right: Zoomed in to de Sitter minimum.

$$\phi = \eta\, \nu(x^i) + \eta^2 \mu(x^i)$$

Spherical harmonics: killing the divergence

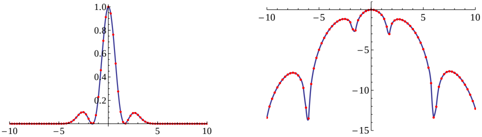


Figure : Left: Plot of $|\Psi_{HH}(A)|^2$ for the first harmonic mapped to $\mathbb{R}^3$. Right: Plot of $\log|\Psi_{HH}(A)|^2$.

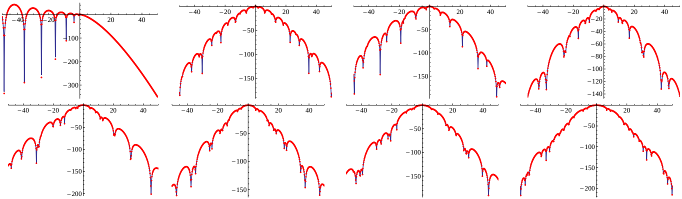


Figure : Plot of $\log|\Psi_{HH}(A)|$ for the first eight spherical harmonics mapped to $\mathbb{R}^3$.

More evidence and a conjecture

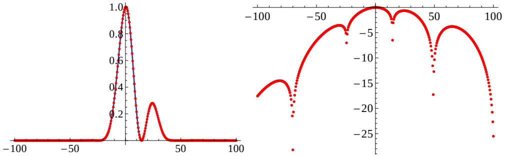


Figure : $|\Psi_{HH}(A)|^2$ (left) and $\log|\Psi_{HH}(A)|$ (right) as a function of A, the overall size of a Gaussian deformation $m(r) = A\,(e^{-r^2} - m_0(r))$ constructed to be orthogonal to the zero mode of the three-sphere.

Conjecture: *The partition function of any $SO(3)$-symmetric deformation for which the three-sphere zero mode harmonic is fixed is bounded.*

Evidence extends beyond conformal class of sphere: fixing zero mode on squashed sphere leads to normalizable wavefunction in squashing direction.

Extensions of higher-spin dS_4/CFT_3

Analytic continuation $z \to i\eta$, $t \to iy$, $\ell_{AdS} \to i\ell_{dS}$ connects AdS and dS:

$$ds^2 = \ell_{AdS}^2 \, \frac{-dt^2 + dz^2 + dx_i^2}{z^2} \longrightarrow \ell_{dS}^2 \, \frac{-d\eta^2 + dy^2 + dx_i^2}{\eta^2}$$

$Sp(N)$ theory discovered by taking $(\ell/\ell_P)^2 = N \to -N$ of AdS higher-spin duality with $O(N)$ theory. $O(-N) = Sp(N)$.

Zoo of duals to higher-spin gravities in the bulk:

- $Sp(N)$ Chern-Simons-ghost–boson $\leftrightarrow$ $O(N)$ Chern-Simons-boson
- $Sp(N)$ Chern-Simons-ghost–fermion $\leftrightarrow$ $O(N)$ Chern-Simons-fermion
- $U(N)$ Chern-Simons-ghost–boson $\leftrightarrow$ $U(N)$ Chern-Simons-boson
- $U(N)$ Chern-Simons-ghost–fermion $\leftrightarrow$ $U(N)$ Chern-Simons-fermion

On AdS side Chern-Simons-matter proposed to be dual to parity-violating bulk Vasiliev; do the dS “wrong-statistics” theories mirror this? Bulk maintains a simple map [Chang, Pathak, Strominger].

Evidence for extensions on $\mathbb{R}^3$

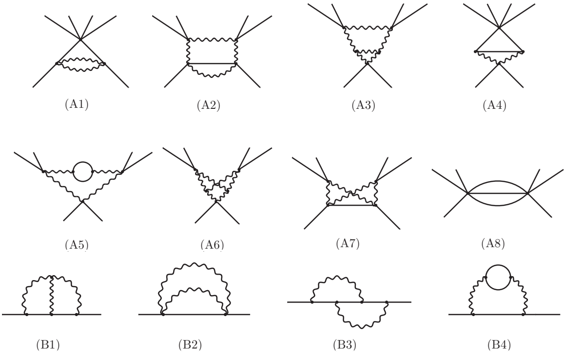
Consider the $Sp(N)$ Chern-Simons-matter bosonic theory deformed by marginal triple-trace interaction:

$$S_{CS} = -\frac{ik}{8\pi}\int d^3x\, \epsilon^{\mu\nu\rho}\left(A^a_\mu \partial_\nu A^a_\rho + \frac{1}{3} f^{abc} A^a_\mu A^b_\nu A^c_\rho\right),$$

$$S_B = \int d^3x \left(\Omega_{ij}(D_\mu \chi)_i (D^\mu \chi)_j + N\frac{\lambda_6^b}{3!}\left(\frac{\Omega_{ij}\chi_i\chi_j}{N}\right)^3\right), \quad D_\mu \equiv \partial_\mu + A_\mu.$$

We can calculate the beta functions of this theory as a function of $\lambda = N/k$, $\lambda_6 = g_6 N^2$. CS level k quantized and does not run.

Evidence for extensions on $\mathbb{R}^3$



$$\beta_{\lambda_6} = \frac{1}{16\pi^2 N^2}\left(12\lambda^4(\pm N - 1) - 20\lambda^2\lambda_6(\pm N - 1) + \lambda_6^2(\pm 3N + 22)\right)$$

Path-integral argument

In Euclidean light-cone gauge $A_- = (A_1 + iA_2)/\sqrt{2} = 0$, action is quadratic in A_3^a and linear in A_+^a. A_+^a thus serves as a constraint for A_3^a, and we can rewrite partition function solely in terms of the matter fields.

Schematically, we have

$$Z_B \sim \int [d\chi] \exp\left\{-\bar{\chi}_i(-\partial^2)\chi_i \mp N^{-1}\lambda(\bar{\chi}_i\chi_i)^2 - N^{-2}(\lambda^2+\lambda_6)(\bar{\chi}_i\chi_i)^3\right\}.$$

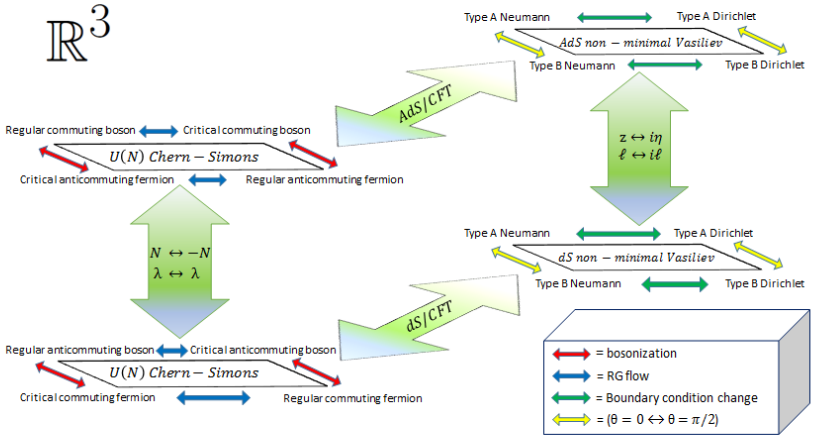
Perform a Hubbard-Stratonovich transformation:

$$1 = \int [d\gamma\, d\mu] \exp\left\{-i\mu\left(\gamma - \bar{\chi}_i\chi_i\right)\right\}.$$

Integrate out the χ fields to get

$$Z_B \sim \int [d\gamma\, d\mu] \exp\left\{\pm N \log\det(-\partial^2 - i\mu) \mp N^{-1}\lambda\gamma^2 - N^{-2}(\lambda^2+\lambda_6)\gamma^3 - i\mu\gamma\right\}.$$

Results on $\mathbb{R}^3$



An analogous set of dualities exists for the minimal Vasiliev theories.

Summary for dS_4/CFT_3

- Perturbative calculations in Landau gauge for bosonic theories and path-integral calculations in Euclidean light-cone gauge for *all* theories (bosonic/fermionic, minimal/non-minimal) indicate $N \to -N$ with λ and λ_6 fixed, consistent with the bulk [Chang, Pathak, Strominger]. This allows evidence for higher-spin AdS/CFT dualities to be extended to evidence for higher-spin dS/CFT dualities.
- $N \to -N$ and bosonization maps break on $S^1 \times S^2$.
  - Issues on nontrivial topology need to be understood! [Banerjee, Hellerman, Maltz, Shenker; Banerjee, Belin, Hellerman, Lepage-Jutier, Maloney, Radicevic, Shenker].
- Though higher harmonics for mass-deformation on sphere are normalizable, zero-mode divergence needs to be explained: maybe turning on a finite λ helps.
- Need Einstein-gravity version!

dS_3/CFT_2 duality?

There is no well-understood proposal for a dS_3/CFT_2 duality (but see [Ouyang]).

Forget higher spins.

Proceed by symmetries and general principles: can this account for de Sitter entropy?

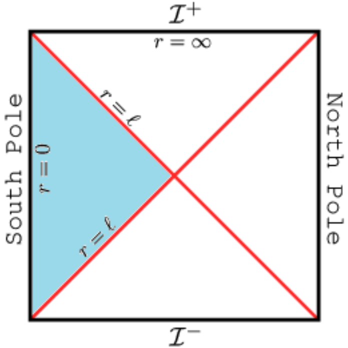
Assumptions: modular-invariant CFT dual to Einstein gravity bulk, $|c| = 3\ell/2G$, spectrum given by conical defects with $(\Delta, \bar{\Delta}) = (iM + J, iM - J)$. Light fields can also be included: $\Delta = \bar{\Delta} = 1 \pm \sqrt{1 - m^2\ell^2}$. CFT is non-unitary!

de Sitter static patch

Consider static patch of de Sitter

$$-(1-r^2/\ell^2)dt^2 + \frac{dr^2}{1-r^2/\ell^2} + r^2 d\phi^2, \qquad \phi \sim \phi + 2\pi\,.$$

Horizon at $r = \ell$ obeys thermodynamic properties; is there statistical count of thermodynamic entropy?



Black hole entropy via Cardy formula

"Microscopic" derivations of black hole entropy often performed by application of Cardy formula:

$$S = \frac{\pi^2}{3}(c_L T_L + c_R T_R) = 2\pi\left(\sqrt{\frac{c_L \Delta}{6}} + \sqrt{\frac{c_R \bar{\Delta}}{6}}\right)$$

where

$$\left(\frac{\partial S_{CFT}}{\partial \Delta}\right) = \frac{1}{T_L}, \qquad \left(\frac{\partial S_{CFT}}{\partial \bar{\Delta}}\right) = \frac{1}{T_R}$$
$$\Delta = M - J, \qquad \bar{\Delta} = M + J$$

In any unitary, modular-invariant 2D CFT, Cardy formula applies asymptotically in Δ or T.

Technique: find locally AdS_3 factor, compute T_L and T_R, use $c_L = c_R = 3\ell/2G$ (Einstein gravity), and plug in. Try BTZ black hole!

de Sitter entropy via Cardy formula?

Dual to 3D de Sitter generically non-unitary, as shown by complex weights of massive fields and conical defects.

Metric for conical defects (the analog of BTZ black holes):

$$-(M^2 - r^2/\ell^2)dt^2 + \frac{dr^2}{M^2 - r^2/\ell^2} + r^2 d\phi^2, \qquad \phi \sim \phi + 2\pi.$$

Naïve application of Cardy formula works [Bousso, Maloney, Strominger]:

$$S_{con} = \frac{2\pi^2 \ell}{3} |c| T_{con} = \frac{A}{4},$$

where the matching extends to conical defects with angular momentum.

Why should it work?

What is even being counted in the Euclidean, non-unitary theory?

Proposal: Modular invariant partition function dual to three-dimensional de Sitter has the spectral decomposition

$$Z(\tau,\bar{\tau}) = \sum_i \rho(\Delta_i,\bar{\Delta}_i) q^{\Delta_i} q^{\bar{\Delta}_i}$$

with $\rho(\Delta,\bar{\Delta})$ a positive integer while Δ, $\bar{\Delta}$ generically complex.
$S = \log \rho(\Delta,\bar{\Delta})$ interpreted as the degeneracy of *local operators*.

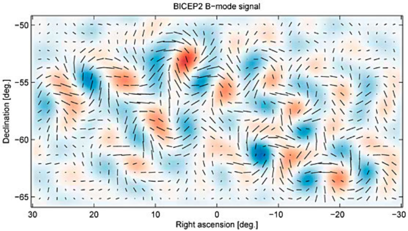
Check on proposal: Cardy formula can be proven for non-unitary theory with complex weights for conical defects $(\Delta,\bar{\Delta}) = (iM + J, iM - J)$, as long as $c = -3i\ell/2G$.

Outlook

- Due to tight constraints on modular-invariant 2D CFTs, one can possibly "bootstrap" the problem: Do there exist modular-invariant functions of the proposed form? Do they lead to a unitary bulk theory?
- Cardy formula uses asymptotia of $Z(\tau) = Z(-1/\tau)$; one can also use the fixed point of the map $\tau = i$ to constrain the theory [Hellerman].

de Sitter Outlook

- Holographic cosmology is entering a concrete, calculational phase with many dS_4/CFT_3 dualities to play with.
  - All dualities are for higher-spin bulk theories.
  - Not understood on higher genus surfaces.
- Wavefunctionals at finite N can be computed via the concrete dualities.
  - Though an infinite class of wavefunctionals are well-behaved, the divergence of the zero mode needs to be understood.
- General principles seem to suggest the dS_3 entropy can be explained as a count of local operators in the dual CFT.
  - Interpretation of local operator count not obviously extendible to four dimensions.



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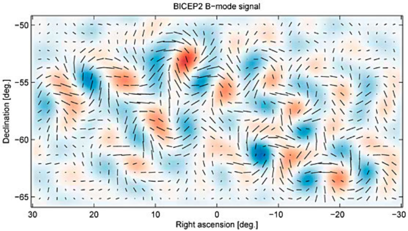

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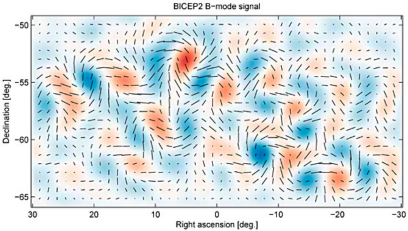

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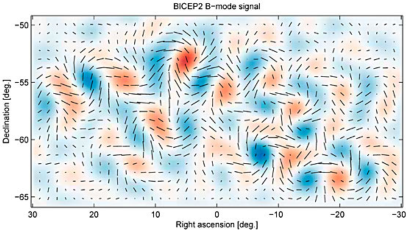

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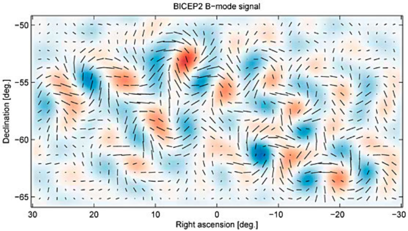

Cardy failure in 4D

$$S_{dS} = \log |\Psi_{HH}[S^3]|^2 = \log |Z_{CFT}[S^3]|^2$$

but $Z_{CFT}[S^3]$ is not counting local operators; indeed, even for purely topological theories void of local operators altogether (like pure Chern-Simons theories), $Z_{CFT}[S^3]$ is nonzero.