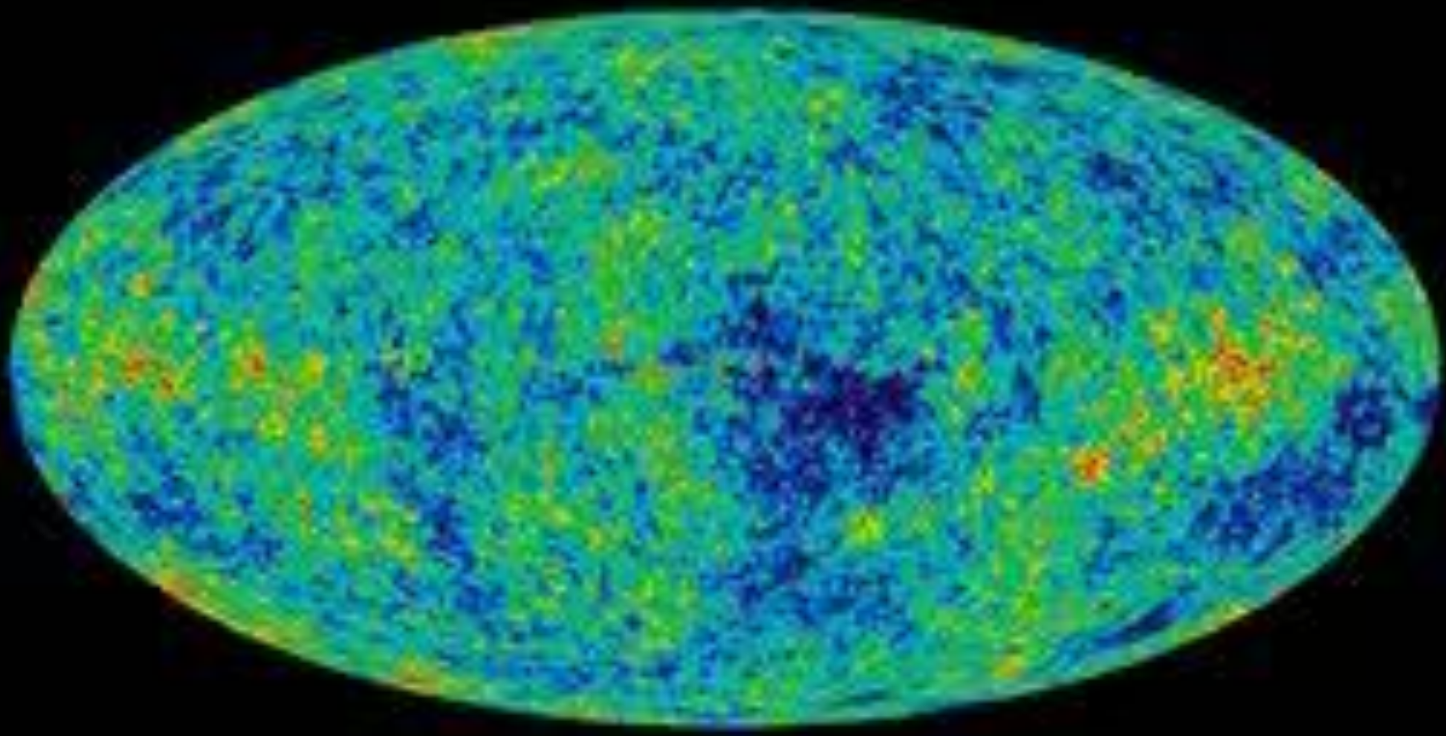
Clusters, voids and their profiles

Ravi K Sheth (Penn/ICTP)

- Halo abundances and clustering
  - Solving Press-Schechter (with Musso)
  - Excursion set peaks (with Paranjape); combines peak theory with Press-Schechter
  - Scale and k-dependent bias is generic
  - Tidal shear makes clustering anisotropic (with Chan, Scoccimarro, Papai)
- Voids
  - Excursion set troughs
  - Under-dense regions can have no large scale bias
  - Profiles (with Castorina, Massara, Varghese)

WMAP of Distant Universe



Cold Dark Matter

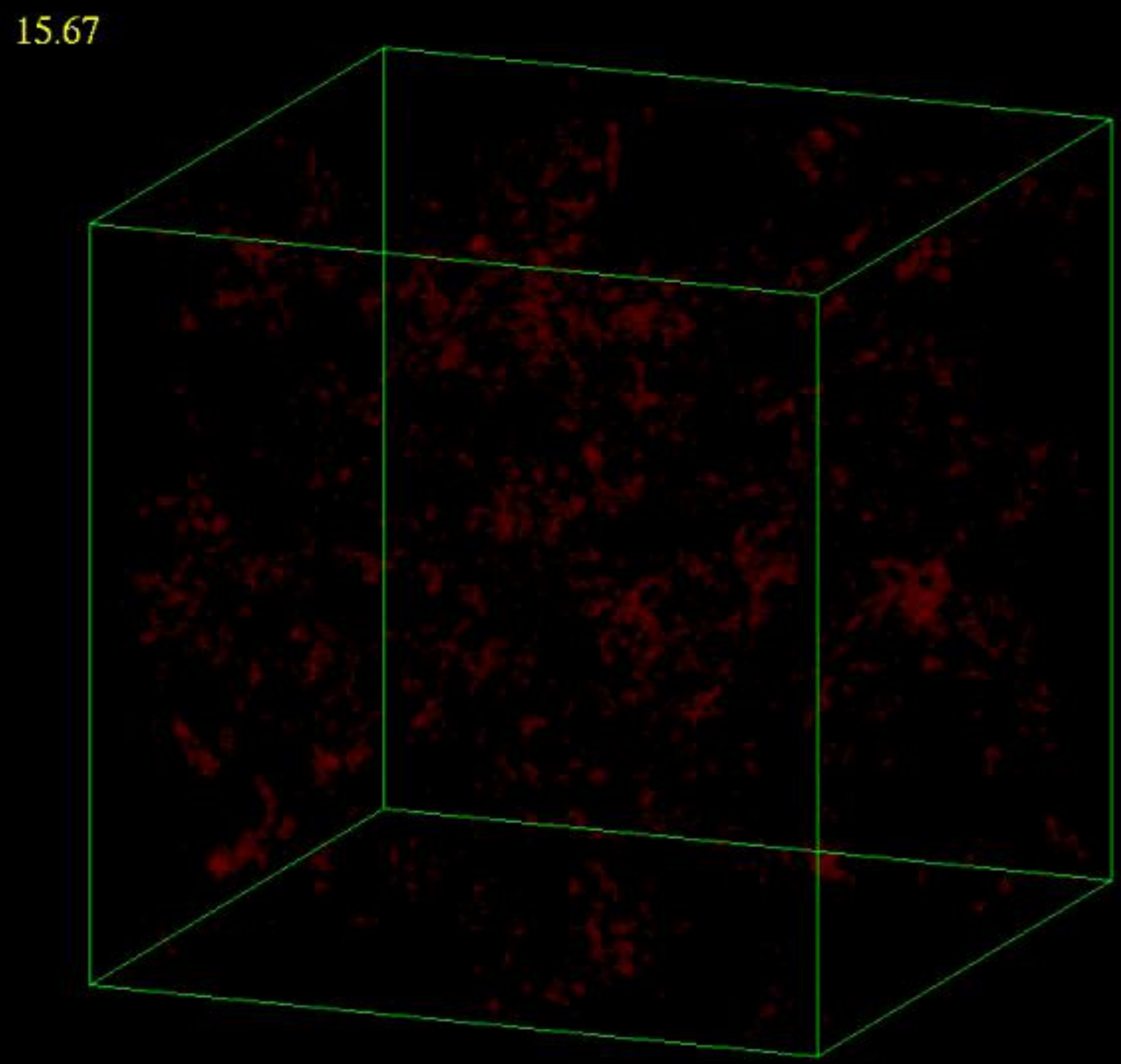
- Cold: speeds are non-relativistic
  - To illustrate, 1000 km/s ×10Gyr ≈ 10Mpc; from z~1000 to present, nothing (except photons!) travels more than ~ 10Mpc
- Dark: no idea (yet) when/where the stars light-up
- Matter: gravity the dominant interaction

Cold

Dark

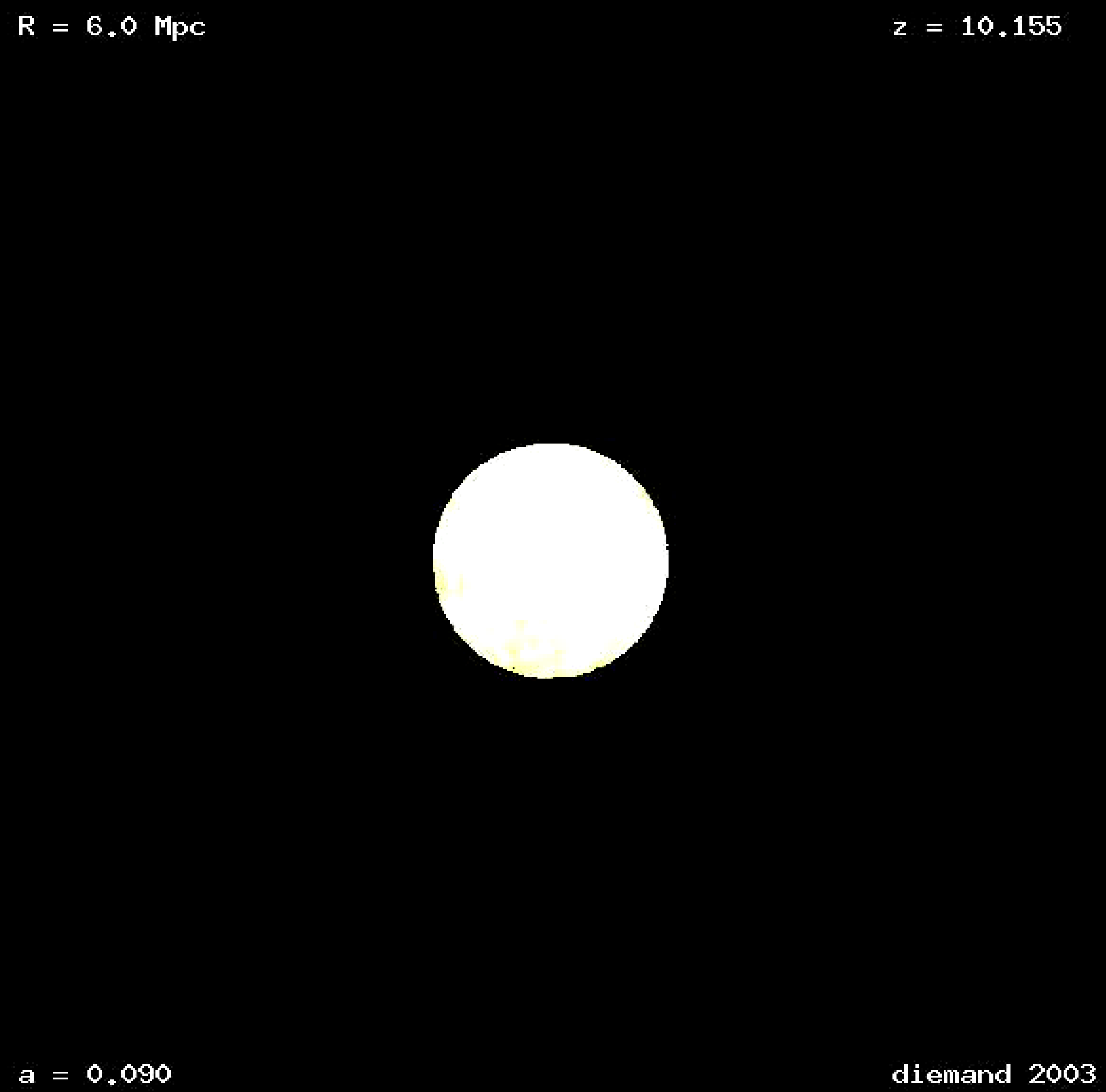
Matter

- Simulations include gravity only (no gas)
- Late-time field retains memory of initial conditions
- Cosmic capitalism



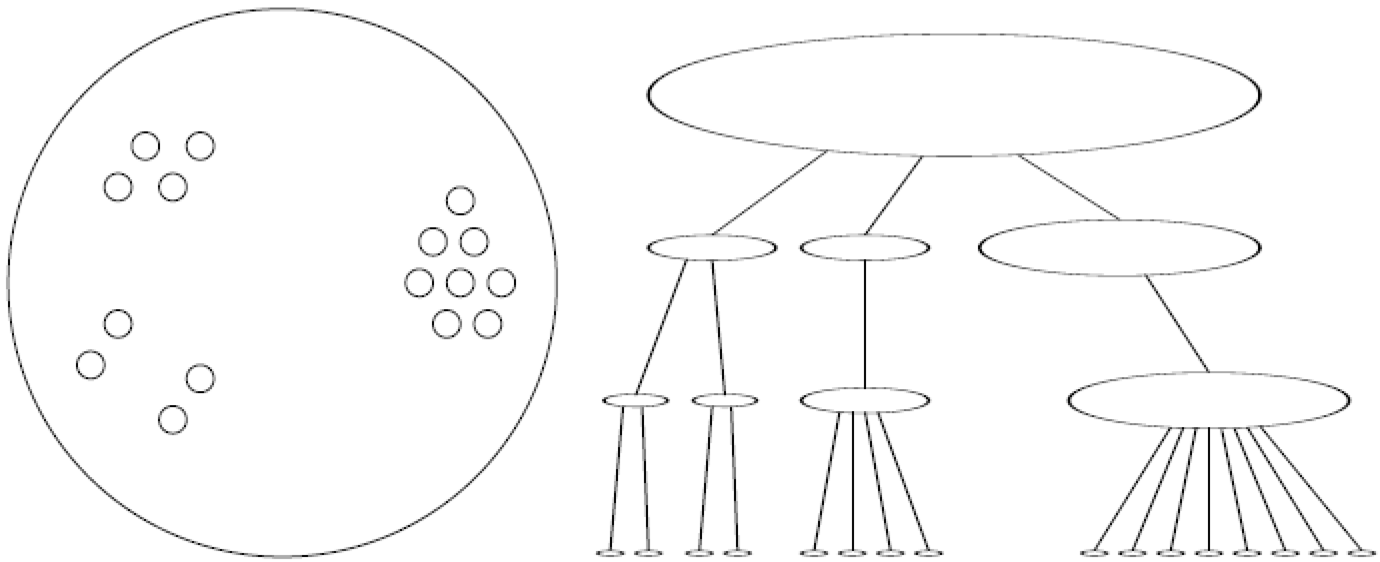
Co-moving volume ~ 100 Mpc/h

Halo formation



Initial conditions determine merger history

stochastic stochastic



(Mo & White 1996; Sheth 1996)

Birkhoff's theorem important

Halo-
model
≈

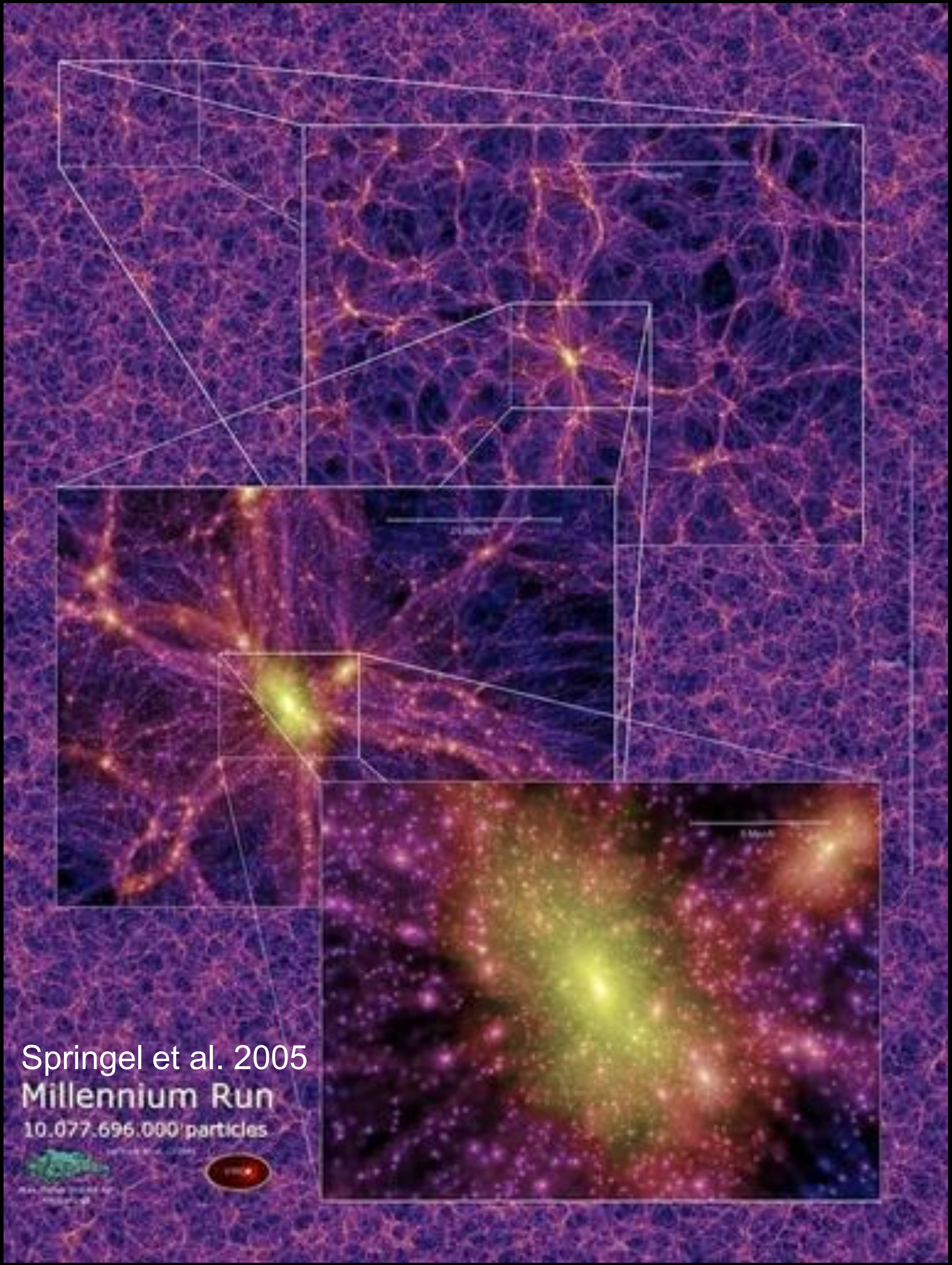
Circles in
circles

Hierarchical models

Dark matter 'haloes' are basic building blocks of 'nonlinear'structure

Galaxies form in the halos

Galaxy formation depends on halo formation



Springel et al. 2005

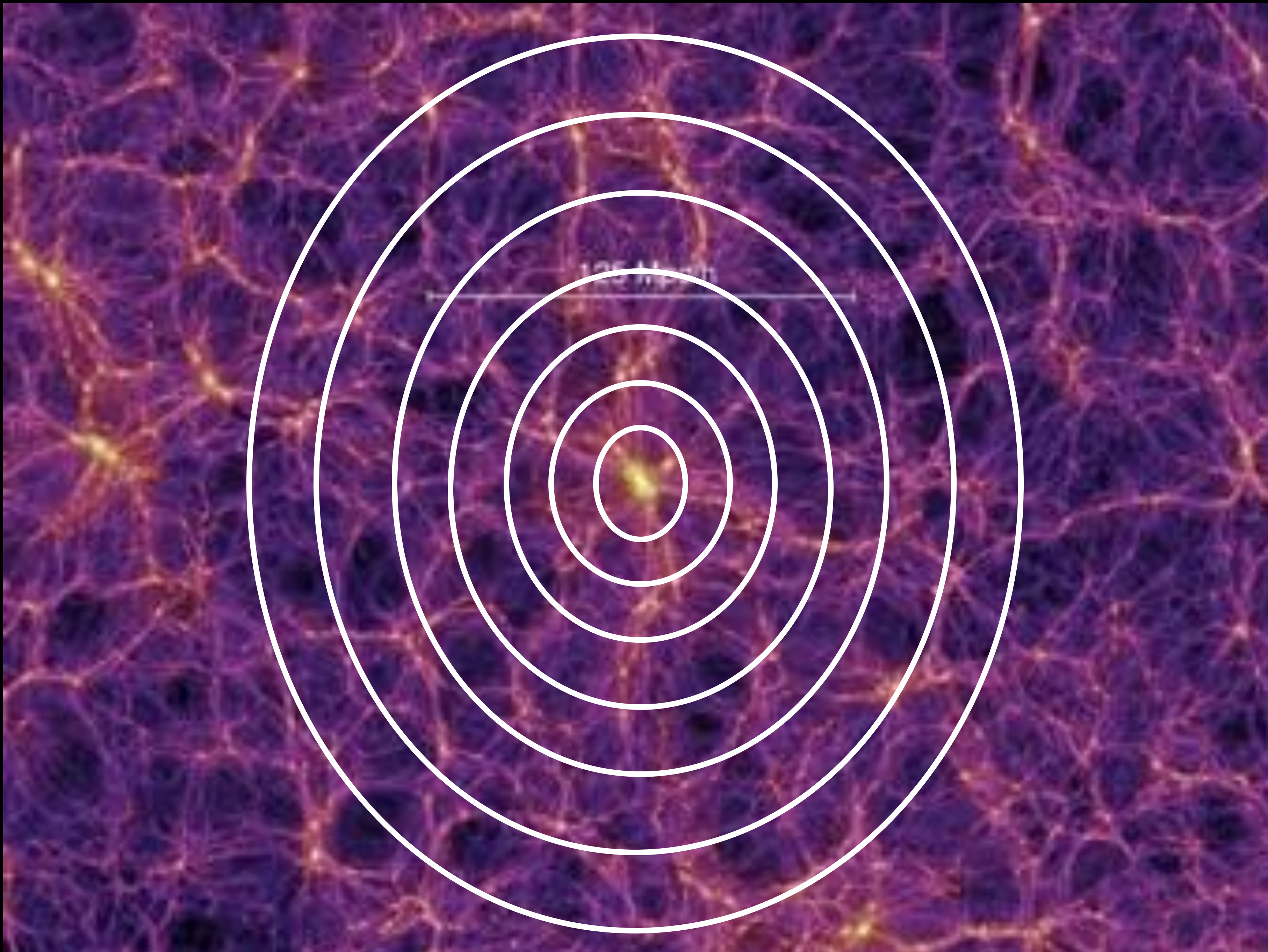
Models of halo abundances and clustering: Gravity in an expanding universe

Goal:

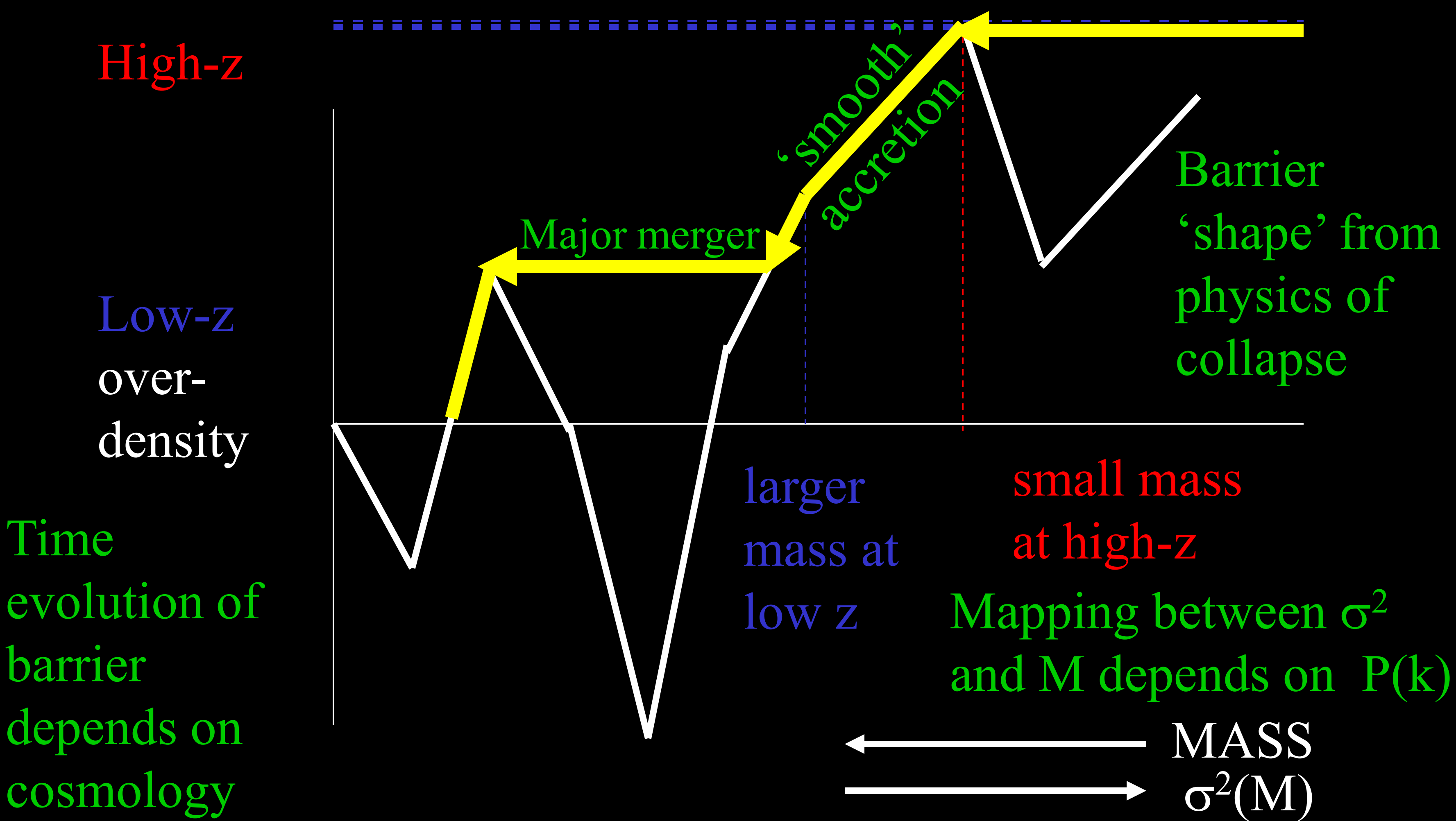
Use knowledge of initial conditions (CMB) to make inferences about late-time, nonlinear structures

THE EXCURSION SET APPROACH

Halo abundances: Epstein (1983); Bond et al. (1991)
Hale mergers/formation: Lacey & Cole (1993)
Clustering/environment: Mo & White (1996)
Counts-in-cells: Sheth (1998); Lam & Sheth (2008)
Voids: Sheth & van de Weygaert (2004); Paranjape et al. (2011)
Filaments and sheets: Shen et al. (2006)
Correlated steps and peaks theory: Musso & Sheth (2012)



The excursion set approach



From Walks to Halos: Ansätze

- $f(\delta_c, s)ds$ = fraction of walks which first cross $\delta_c(z)$ at s

  ≈ fraction of initial volume in patches of comoving volume $V(s)$ which were just dense enough to collapse at z

  ≈ fraction of initial mass in regions which each initially contained $m = \rho V(1+\delta_c) \approx \rho V(s)$ and which were just dense enough to collapse at z (ρ is comoving density of background)

  $\approx dm\ m\ n(m, \delta_c)/\rho$

Simplification because…

- Everything local
- Evolution determined by cosmology (competition between gravity and expansion)
- Statistics determined by initial fluctuation field: since Gaussian, statistics specified by initial power-spectrum *P(k)*
- Fact that only very fat cows are spherical is a detail (*crucial* for precision cosmology); in excursion set approach, mass-dependent barrier height increases with distance along walk

Path integrals …

Press-Schechter: Want $\delta \geq \delta_c$

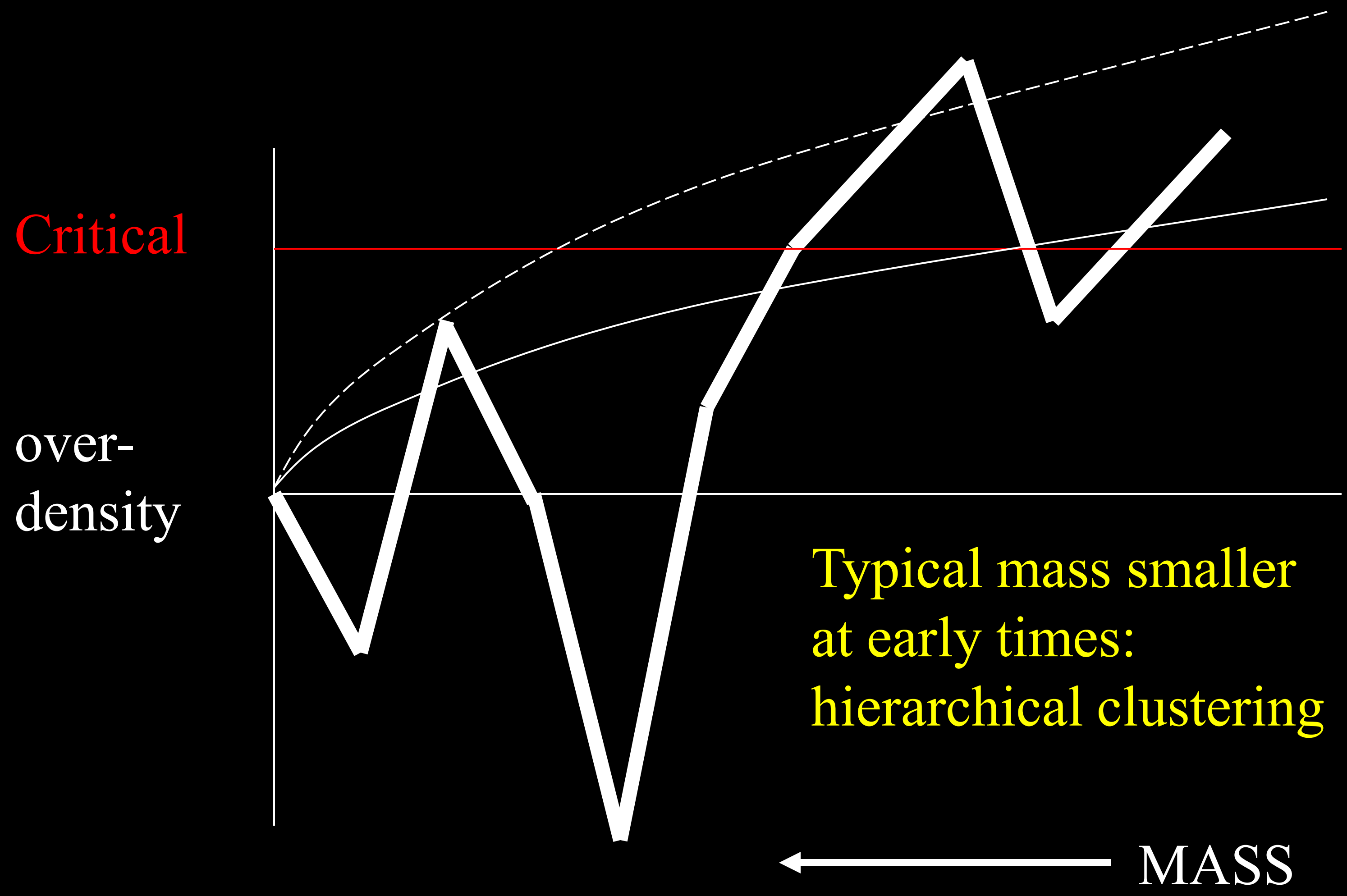
Bond, Cole, Efstathiou, Kaiser:
$\delta(s) \geq \delta_c$ and $\delta(S) \leq \delta_c$ for all $S \leq s$:

$$f(s)\Delta s = \int_{-\infty}^{\delta_c} d\delta_1 \cdots \int_{-\infty}^{\delta_c} d\delta_{n-1} \int_{\delta_c}^{\infty} d\delta_n \, p(\delta_1, \ldots, \delta_n)$$

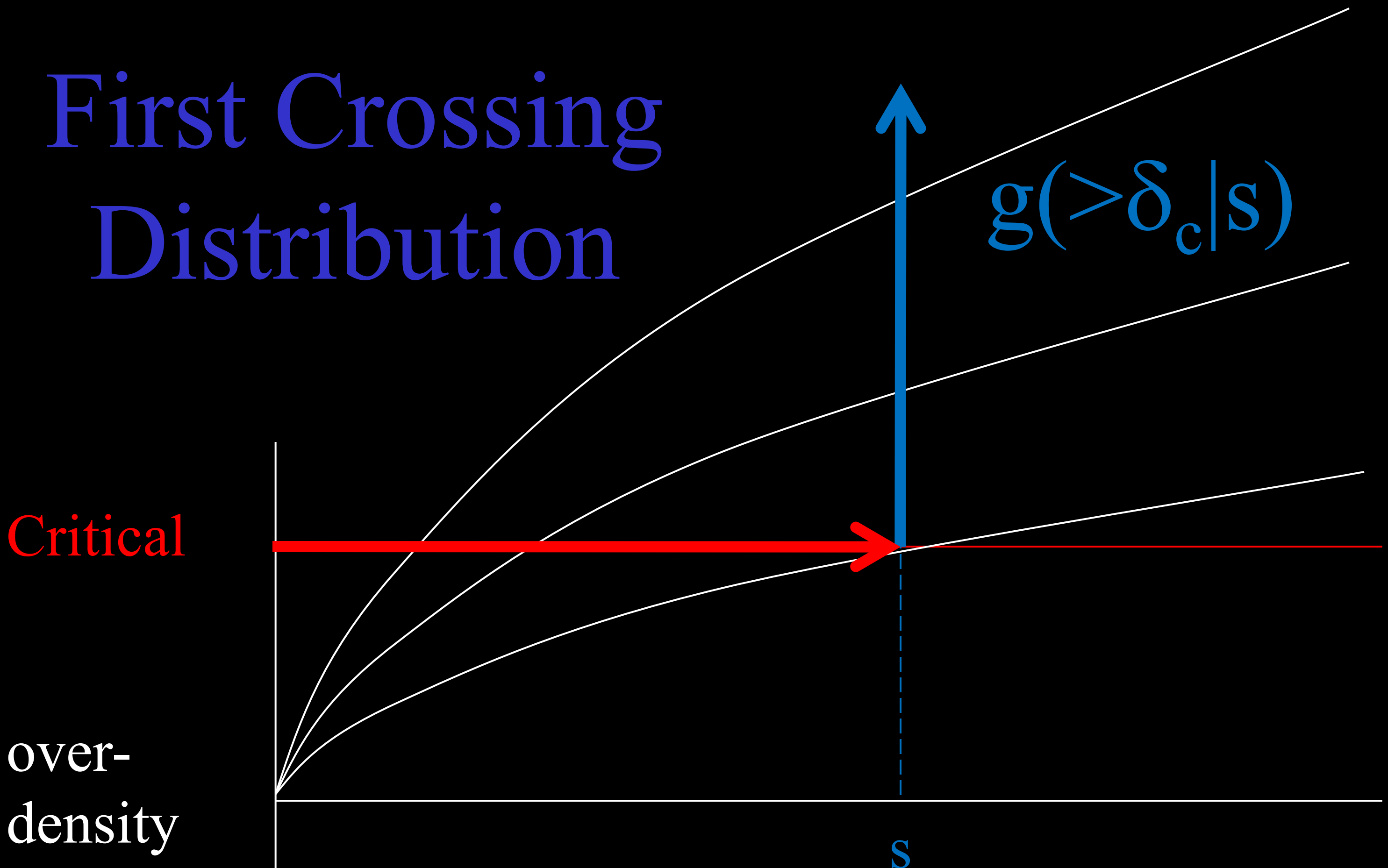
Since $s = n\Delta s$ this requires n-point distribution in limit as $n \to \infty$ and $\Delta s \to 0$.
(Best solved by Monte-Carlo methods.)

… yield little/no insight

Key insight: Think of walks with 'completely correlated' steps



First Crossing Distribution



$$ {}_0\!\int^{s} dS\, f(S)\, p(>\delta_c, s \mid \text{first } \delta_c \text{ at } S) $$
$$ \approx {}_0\!\int^{s} dS\, f(S)\, p(>\delta_c, s \mid \delta_c, S) $$

First crossing distributions

- Smooth walks: $p(>\delta_c, s | \delta_c, S, \text{first}) = 1$
- Uncorrelated steps: $p(>\delta_c, s | \delta_c, S, \text{first}) = ½$
  - This is the Press-Schechter factor of 2
  - $s\,f(s) = dg(>\delta|s)/d\ln s = \delta_c \exp(-\delta_c^2/2s) / \sqrt{2\pi s}$
  - Self-similar in units of $\nu = \delta_c/\sqrt{s}$
- Correlated steps somewhere in between
  - NB. Easy if $p(>\delta_c, s | \delta_c, S, \text{first})$ = separable function of s and S

For correlated steps

rather than thinking of a walk

as a list of heights

(i.e. the path integrals of Bond et al 1991),

it is more efficient to think of it

as a curve specified by

its height on one scale and

its derivatives

Correlated steps

Require walk below barrier on scale just larger than s, but above barrier on scale s:

$$f(s)ds \approx \int d\delta' \int d\delta \;\; p(\delta,\delta') \quad \text{where}$$

$$\delta_c < \delta < \delta_c + \Delta s\, \delta' \quad \text{and} \quad \delta' > 0$$

$$= \Delta s\; p(\delta_c, s) \int d\delta'\; p(\delta'|\delta_c)\; \delta'$$

Reduces problem from n >>1 dimensions, to just 2

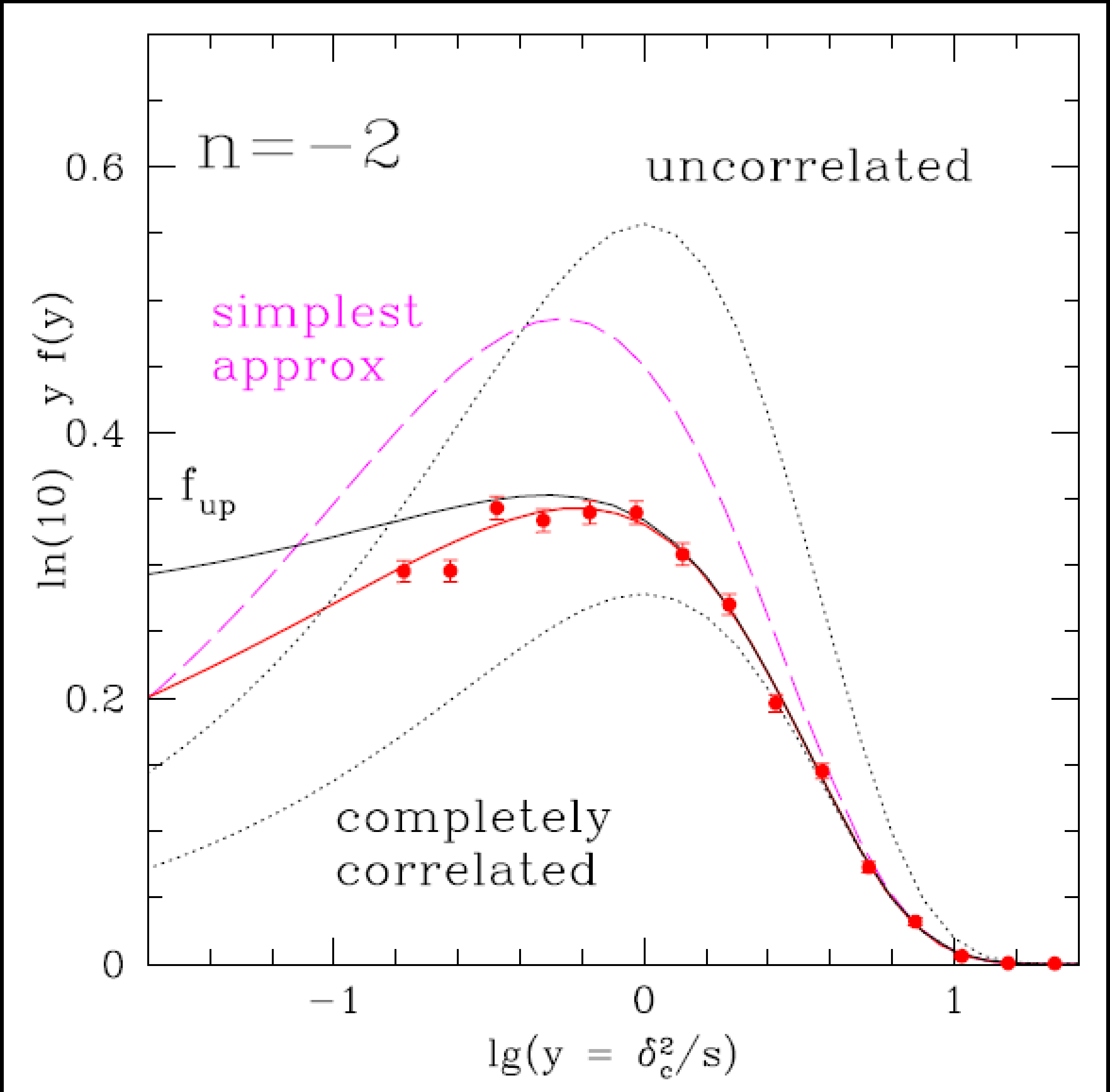
Generalizes trivially to any barrier shape and also for non-Gaussian fields (Musso & Sheth 2012, 2014)

Correlated steps (constant barrier)

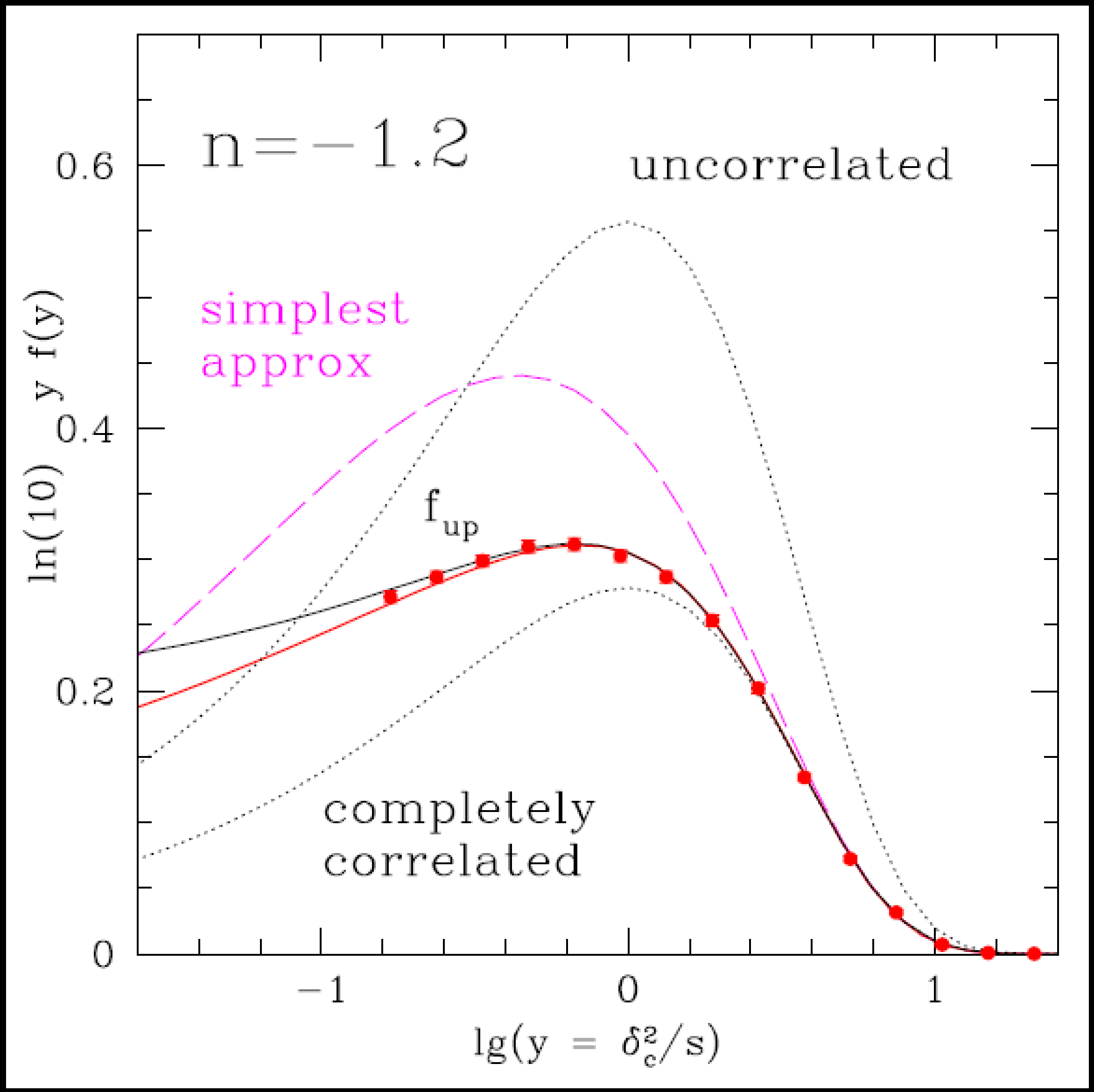
$$\nu f(\nu) = \frac{\nu\, \mathrm{e}^{-\nu^2/2}}{\sqrt{2\pi}} \left[\frac{1 + \mathrm{erf}(\Gamma\nu/\sqrt{2})}{2} + \frac{\mathrm{e}^{-\Gamma^2\nu^2/2}}{\sqrt{2\pi}\Gamma\nu} \right]$$

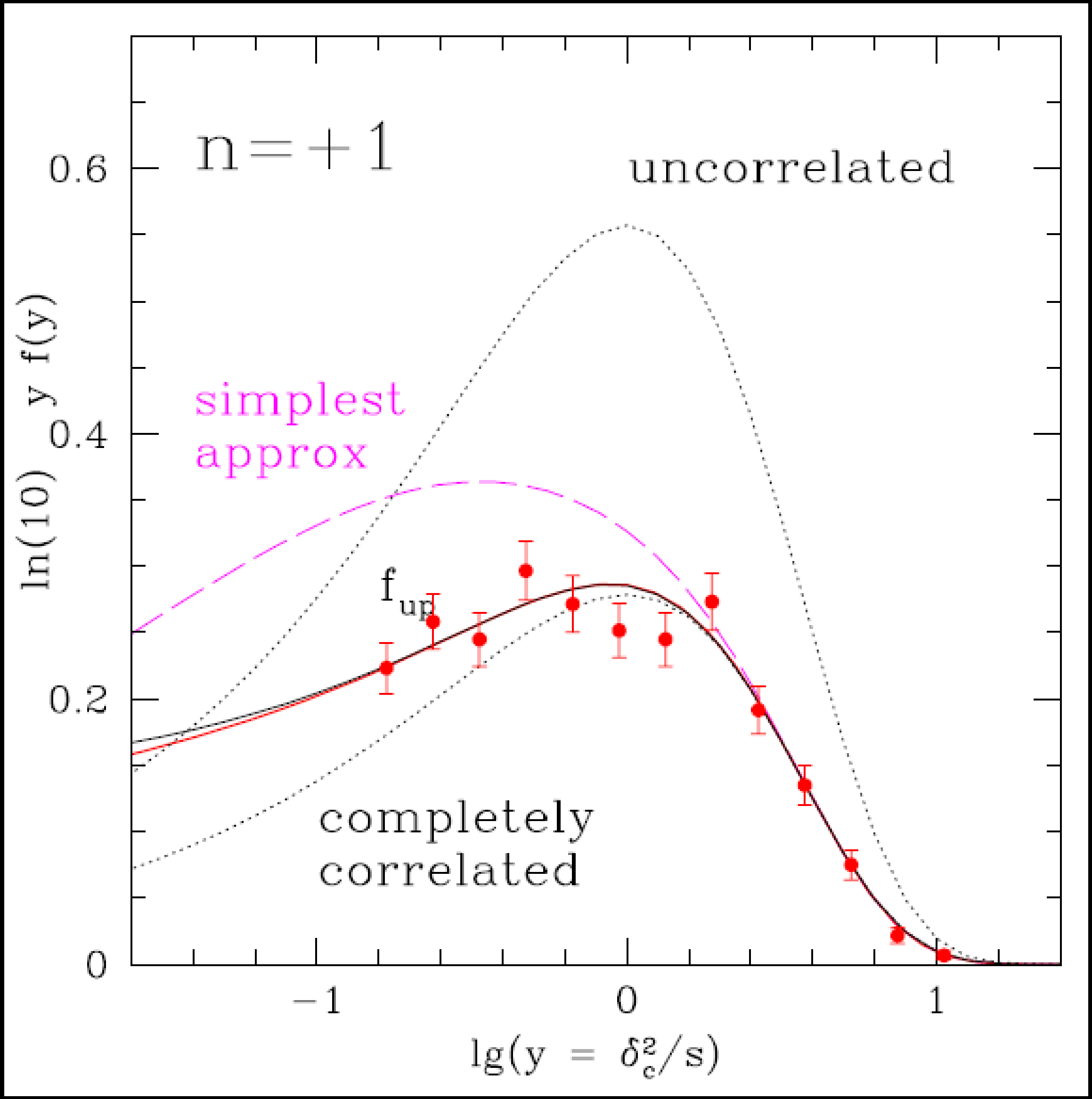
N.B. Not quite universal because of Γ:

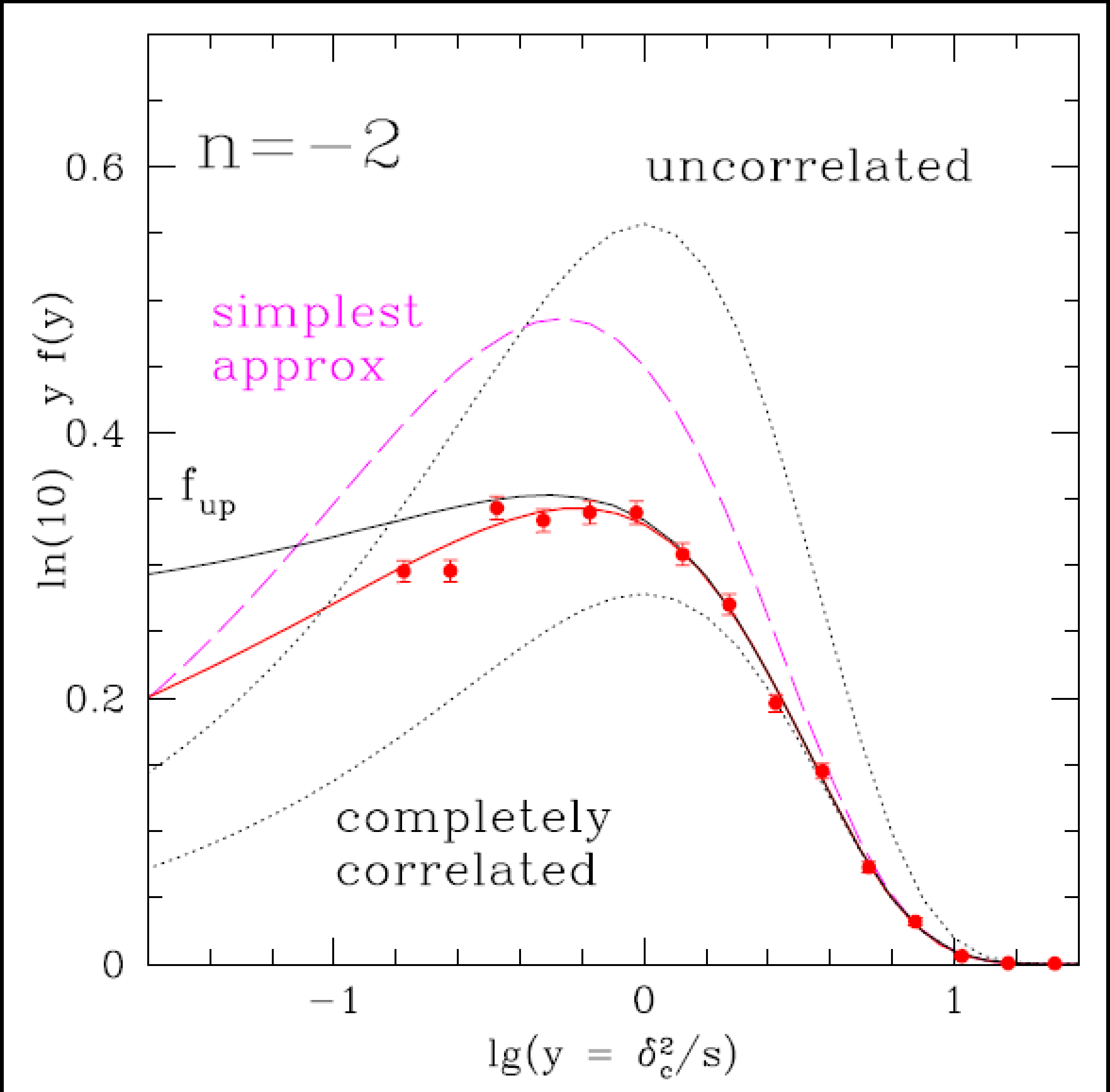
$$\gamma^2 \equiv \frac{\langle \delta\delta' \rangle^2}{\langle \delta^2 \rangle \langle \delta'^2 \rangle} \quad \text{and} \quad \Gamma^2 = \frac{\gamma^2}{1 - \gamma^2}$$



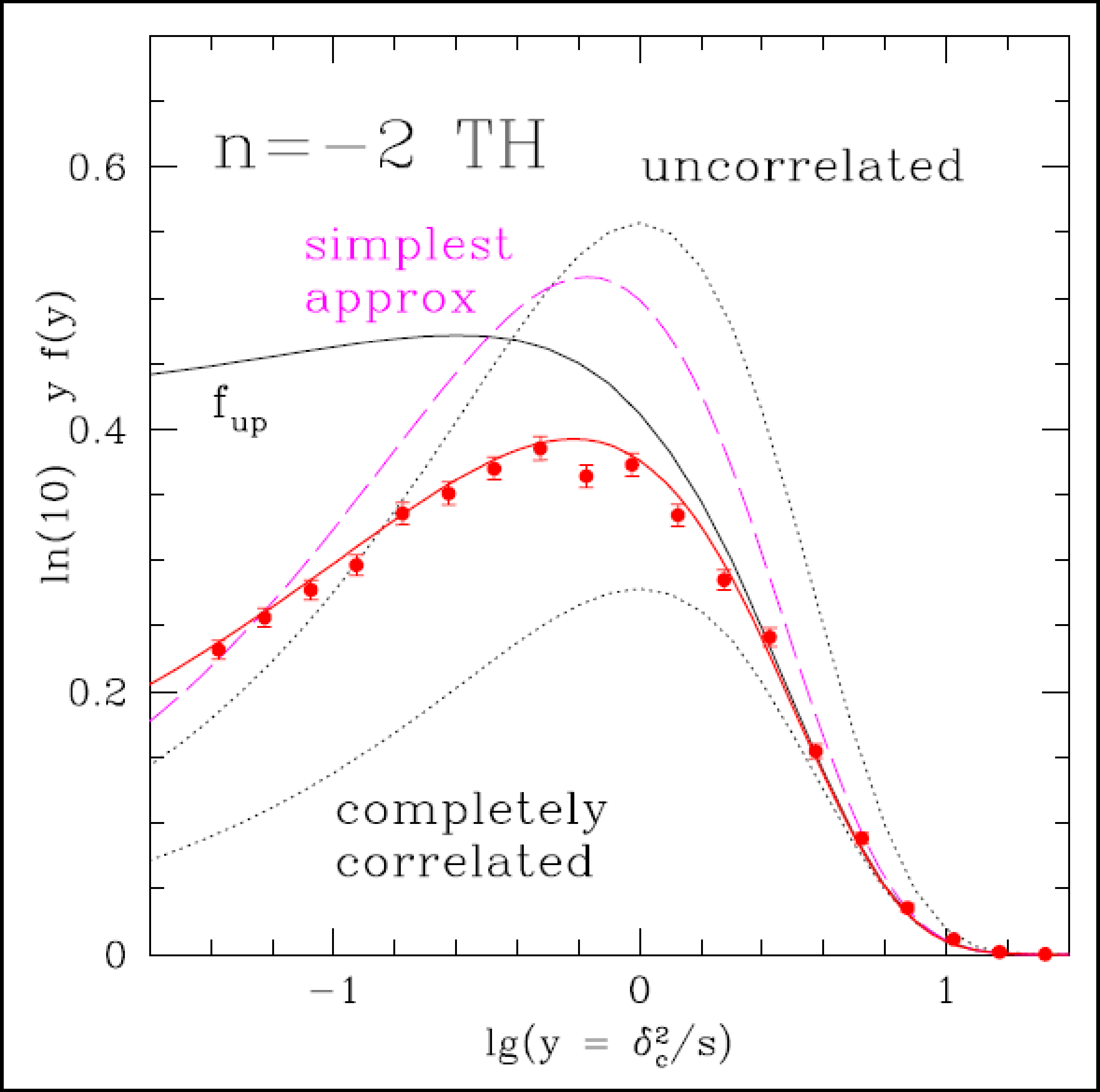
n = −2
uncorrelated
simplest
approx
f_up
completely
correlated
0.6
0.4
0.2
0
−1
0
1
ln(10) y f(y)
lg(y = δ_c²/s)


n=−1.2
uncorrelated
simplest
approx
f_up
completely
correlated
0.6
0.4
0.2
0
−1
0
1
ln(10) y f(y)
lg(y = δ_c²/s)


n=+1
uncorrelated
simplest
approx
f_{up}
completely
correlated
0.6
0.4
0.2
0
−1
0
1
$\ln(10)\ y\ f(y)$
$\lg(y = \delta_c^2/s)$



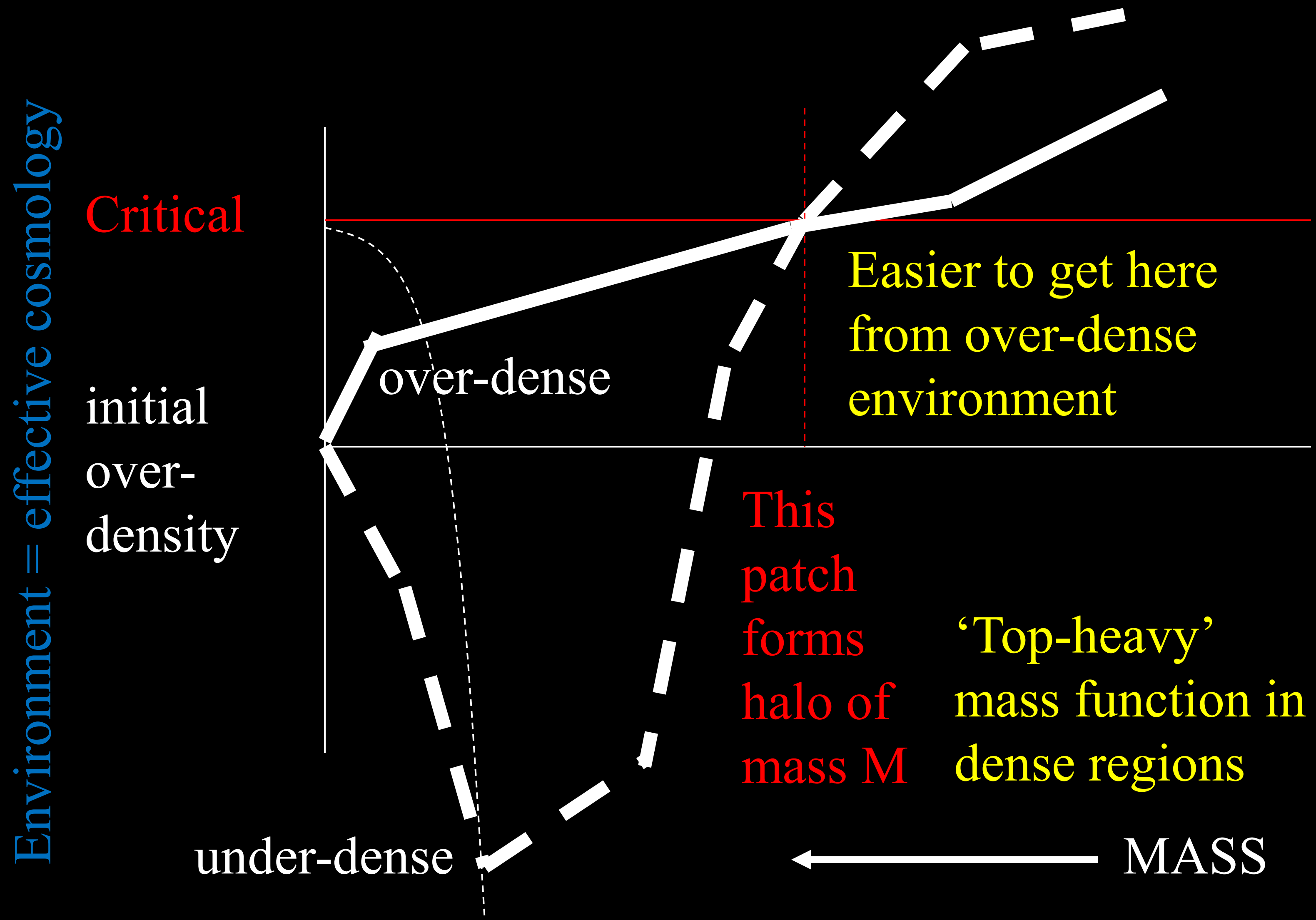
n = −2
uncorrelated
simplest
approx
f_up
completely
correlated
ln(10) y f(y)
lg(y = δ_c²/s)
0.6
0.4
0.2
0
−1
0
1


n=−2 TH
uncorrelated
simplest
approx
f_{up}
completely
correlated
ln(10) y f(y)
$\lg(y = \delta_c^2/s)$
0.6
0.4
0.2
0
−1
0
1

The first crossing distribution,
for arbitrary barriers and arbitrary correlation structures,
is now a solved problem.

(Musso & Sheth 2014)

Correlations with environment



Constrained walks with correlated steps easy:

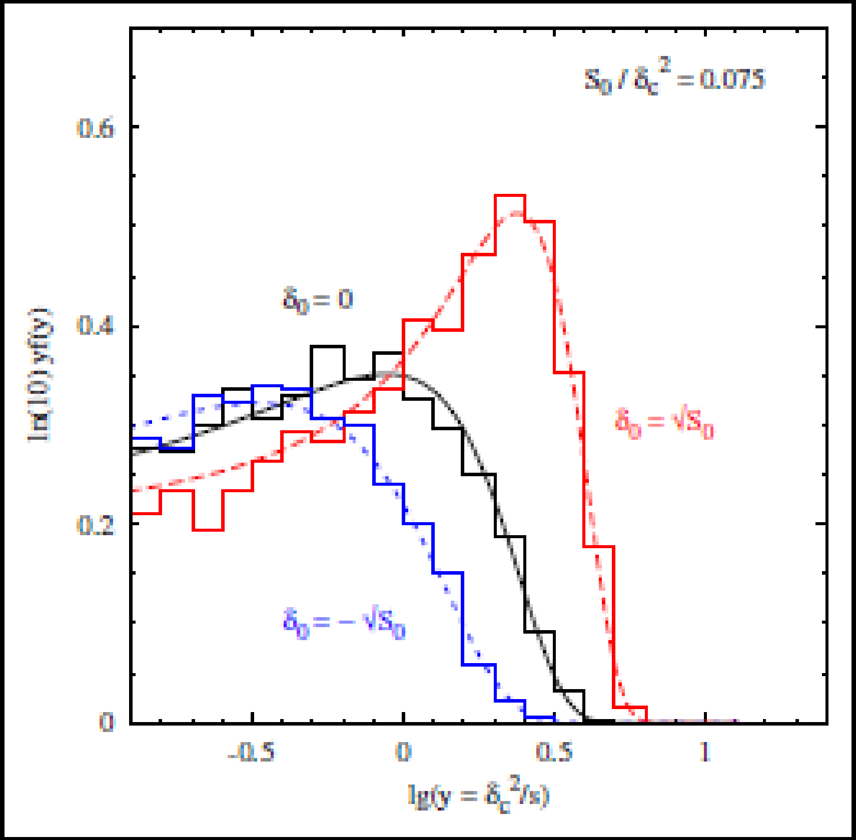
$$f(s|\delta_0,S)ds \approx \int dv \int d\delta \; p(\delta,v|\delta_0)$$

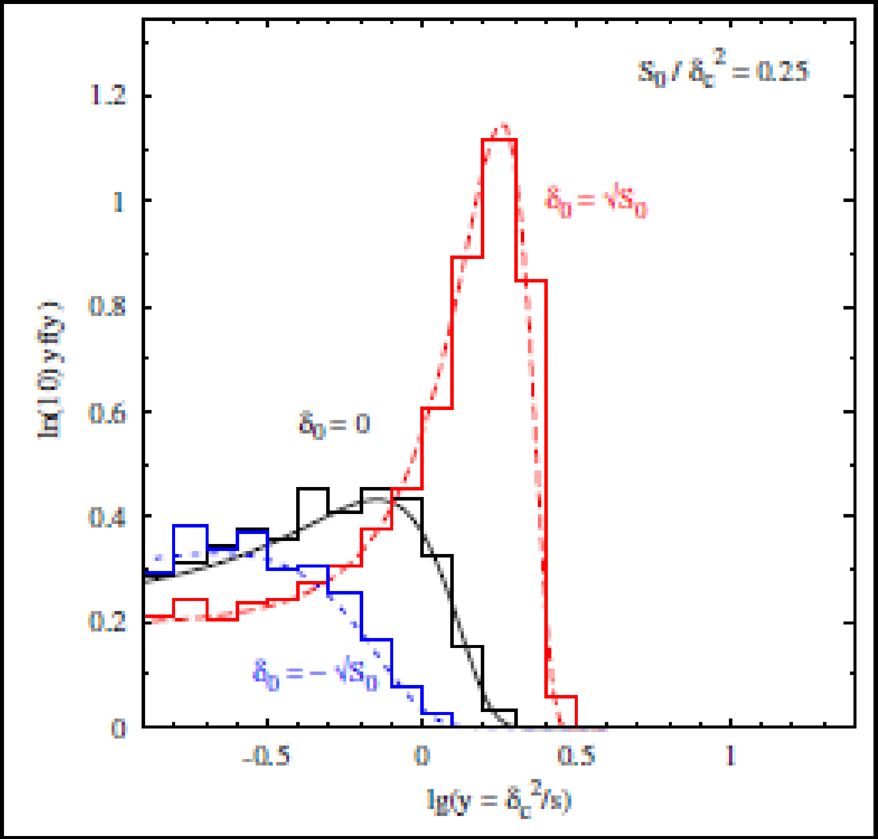
over $\delta_c < \delta < \delta_c + \Delta s\, v$ and $v > 0$

$$= \Delta s\; p(\delta_c|\delta_0) \int dv\; p(v|\delta_c,\delta_0)\; v$$

$$= \Delta s\; p(\delta_c|\delta_0) \langle v|\delta_c,\delta_0 \rangle$$

Constrained walks easy …

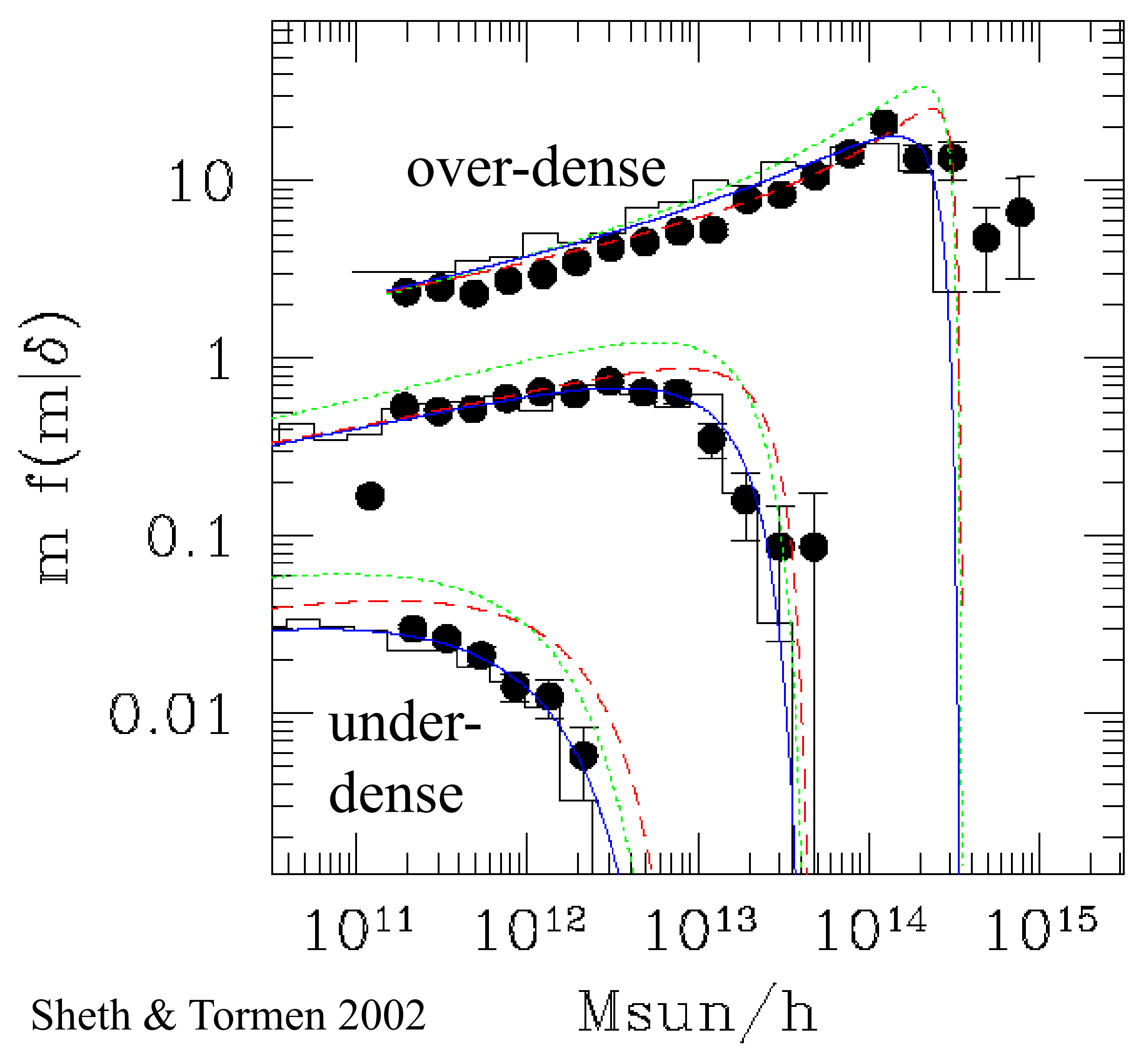




… and accurate (Musso, Paranjape, Sheth 2012)

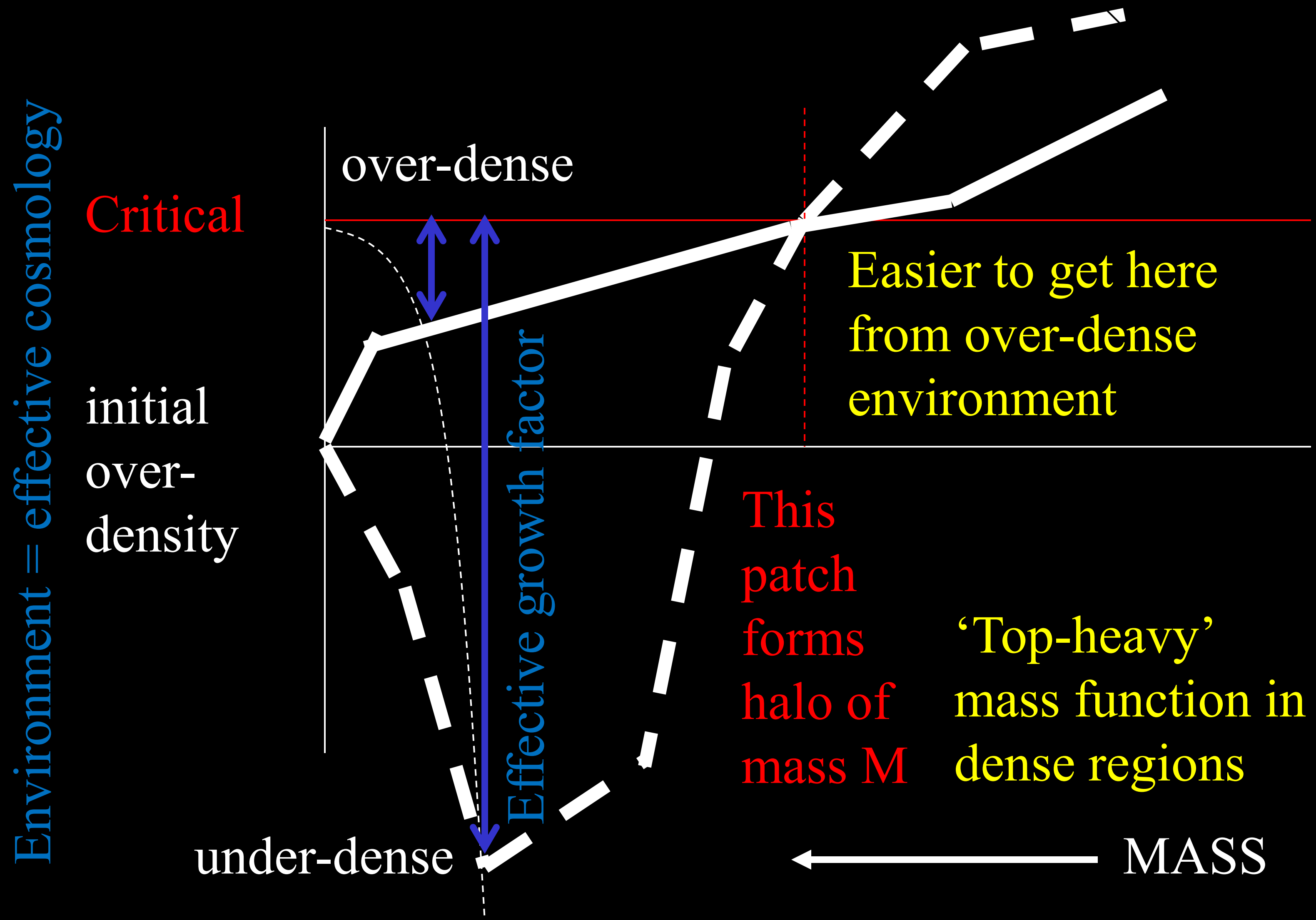
Most massive halos populate densest regions

Key to understand galaxy biasing



$$n(m|\delta) = [1 + b(m)\delta]\, n(m) \neq [1 + \delta]\, n(m)$$

Correlations with environment



Mon. Not. R. Astron. Soc. **394,** 2109–2112 (2009) doi:10.1111/j.1365-2966.2009.14467.x

On the equivalence between the effective cosmology and excursion set treatments of environment

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ABSTRACT

In studies of the environmental dependence of structure formation, the large-scale environment is often thought of as providing an effective background cosmology: for example the formation of structure in voids is expected to be just like that in a less dense universe with appropriately modified Hubble and cosmological constants. However, in the excursion set description of structure formation which is commonly used to model this effect, no explicit mention is made of the effective cosmology. Rather, this approach uses the spherical evolution model to compute an effective linear theory growth factor, which is then used to predict the growth and evolution of non-linear structures. We show that these approaches are, in fact, equivalent: a consequence of Birkhoff's theorem. We speculate that this equivalence will not survive in models where the gravitational force law is modified from an inverse square, potentially making the environmental dependence of clustering a good test of such models.

Key words: methods: analytical – dark matter – large-scale structure of Universe.

Environmental effects

- In hierarchical models, close connection between evolution and environment (dense region ~ dense universe ~ more evolved)
- Gastrophysics determined by formation history of halo
- Observed correlations with environment test hierarchical galaxy formation models – all environmental effects because massive halos populate densest regions

Large scale bias coefficients from Taylor series around δ_0

$$f(s|\delta_0,S) = p(\delta_c|\delta_0) \langle v|\delta_c,\delta_0 \rangle$$

Bias gets additional contribution from dependence of mean v on large scale δ_0

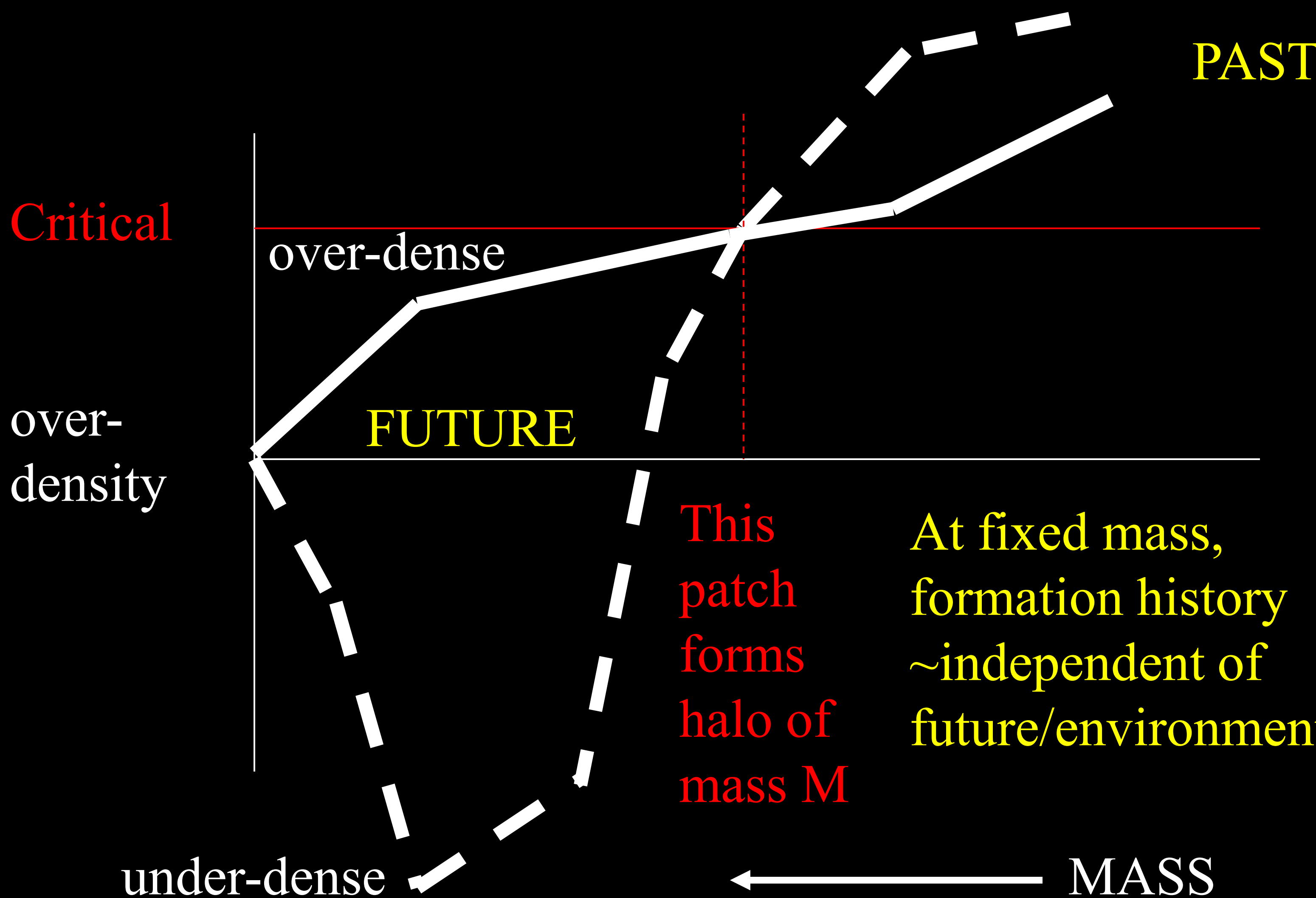
- Dependence on ν makes bias factor k-dependent, because ν means derivative with respect to smoothing filter; e.g. $W = \exp(- k^2 R^2/2)$:

$$\mathrm{bias}(k) = (b_{10} + b_{01}\, k^2) W(kR_h)$$

This is generic.

- Coefficients depend on halo mass; there are consistency relations between coefficients

Correlations with environment



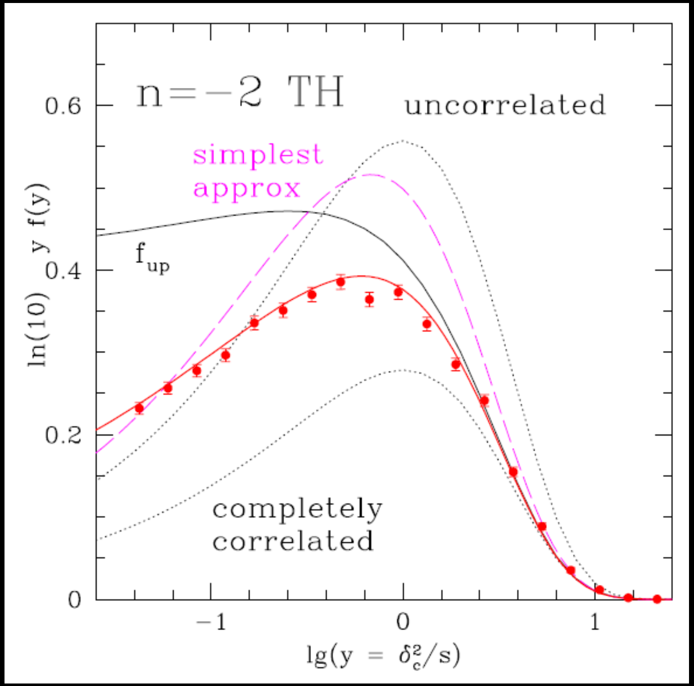
This
patch
forms
halo of
mass M

At fixed mass,
formation history
~independent of
future/environment

Assembly bias

- At fixed mass, formation history independent of future/environment *if walks are Markovian* i.e. have uncorrelated steps (White 1996)
- In simulations, at fixed mass, formation history *does* correlate with environment (Sheth & Tormen 2004; Gao et al. 2005; etc.)
- A simple 'Markov Velocities' model captures most of this effect (Musso & Sheth 2014)

Markov Velocities …



- (Old) independent steps
  = Markov heights

  $$p(\delta|\Delta,\Delta,\Delta,\ldots) = p(\delta|\Delta)$$

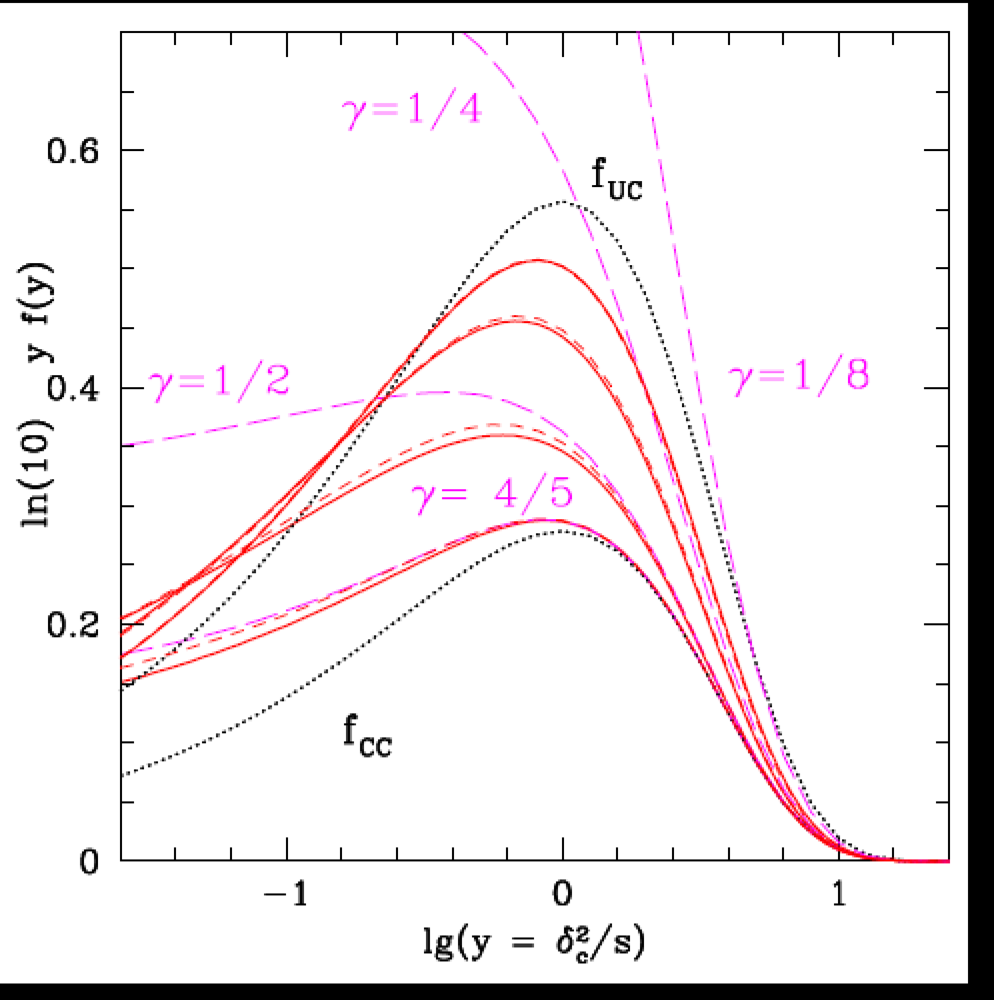
- (New) Markov velocities = correlated steps but

  $$p(\delta|\Delta,\Delta,\Delta,\ \ldots) = p(\delta|\Delta,\Delta')$$ and similarly

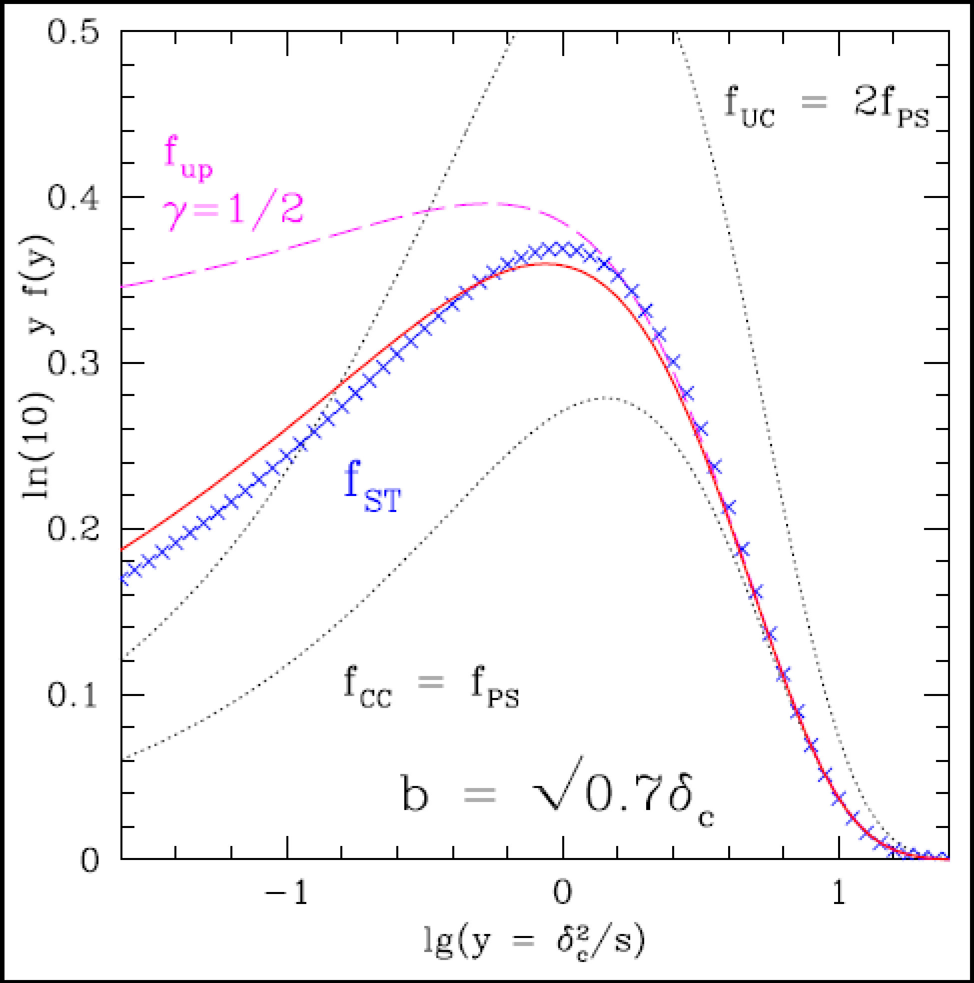
  $$p(\Delta|\delta,\delta',\ \delta,\delta',\ \delta,\delta',\ \ldots) = p(\Delta|\delta,\delta')$$

… have simplest realistic Assembly Bias built-in

Large family of models with different correlation structures



Can provide good description of formulae used to fit halo counts in simulations, provided …



From walks to halos …

- Use first crossing distribution as physically motivated fitting formula in terms of $a\nu$ and fit for a
- I.e., find that a for which

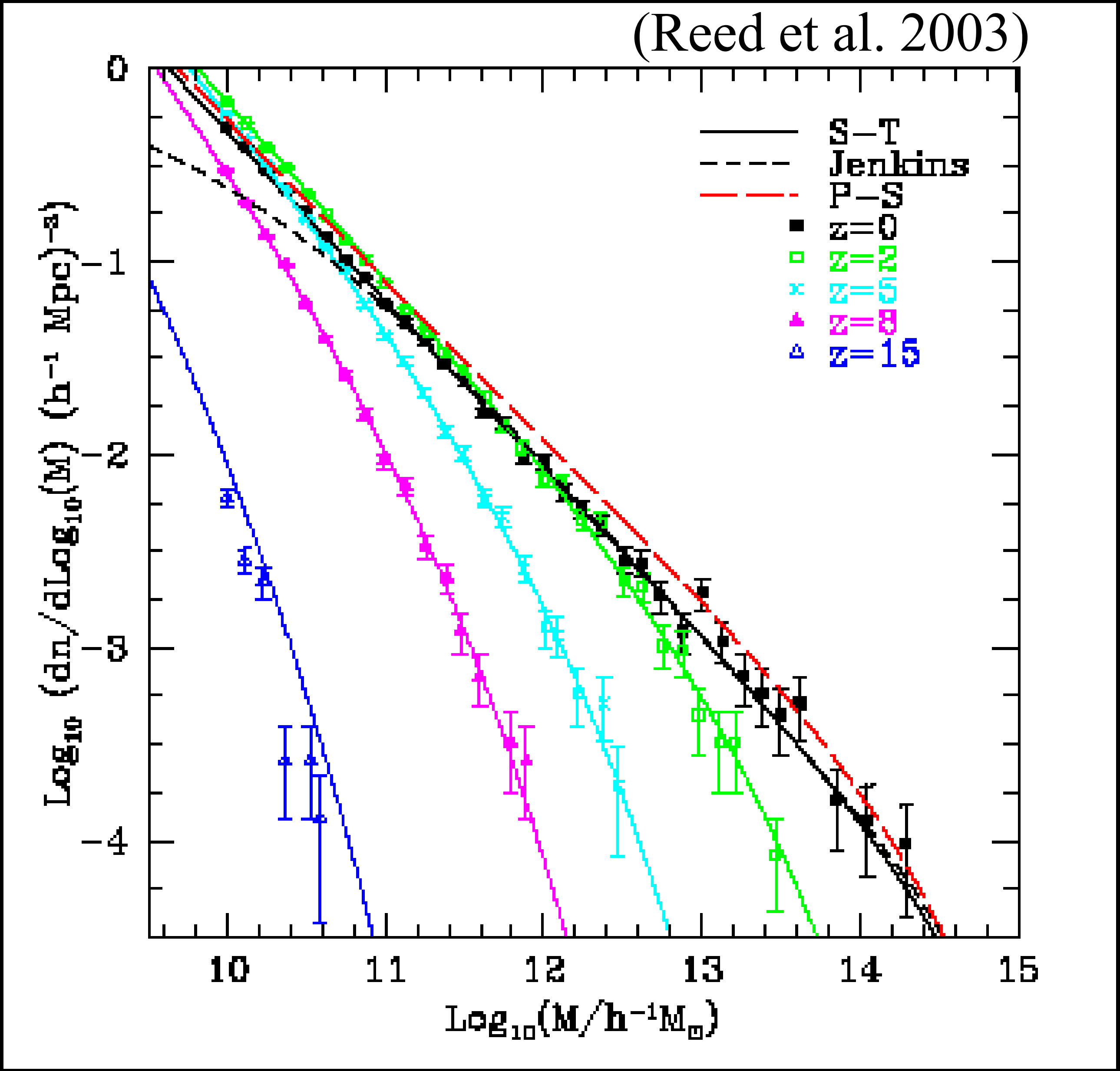
$$f(\nu)d\nu = f(m,z)dm = (m/\rho)\, dn(m,z)/dm\, dm$$

where dn/dm is comoving number density of halos of mass m at z

- It happens that $a\sim0.85$ approximately independent of cosmology and z

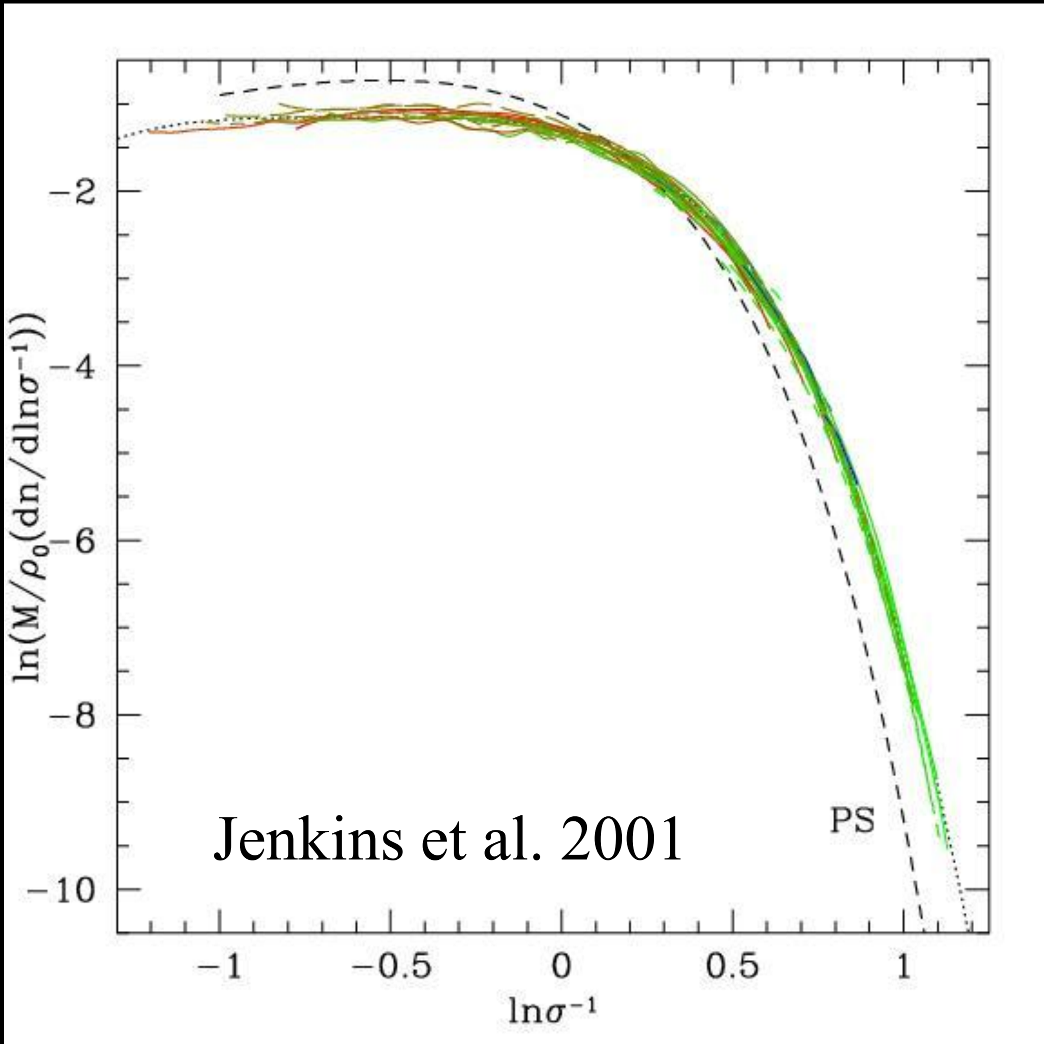
The Halo Mass Function

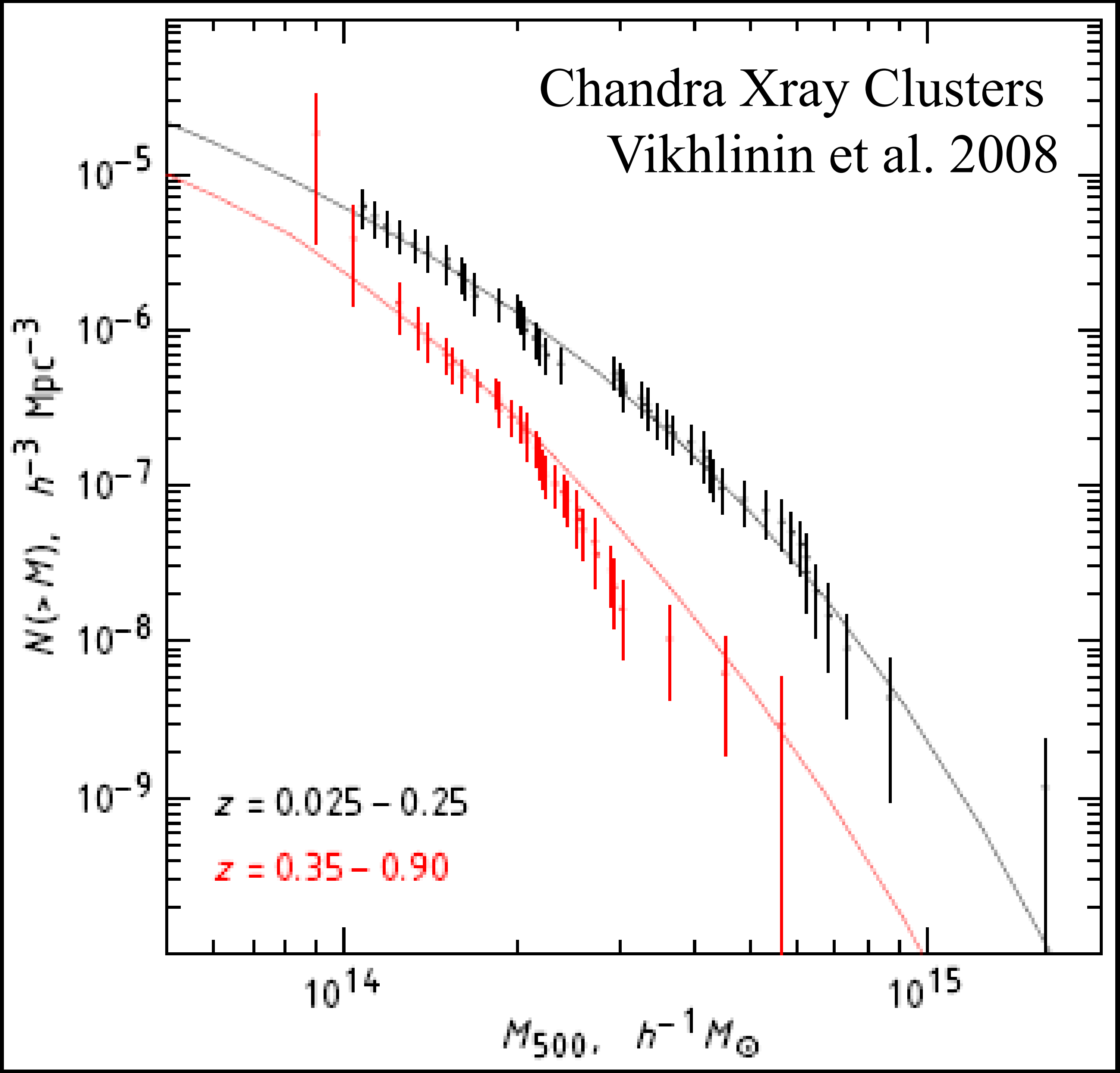
- Small halos collapse/virialize first
- Can also model halo spatial distribution
- Massive halos more strongly clustered



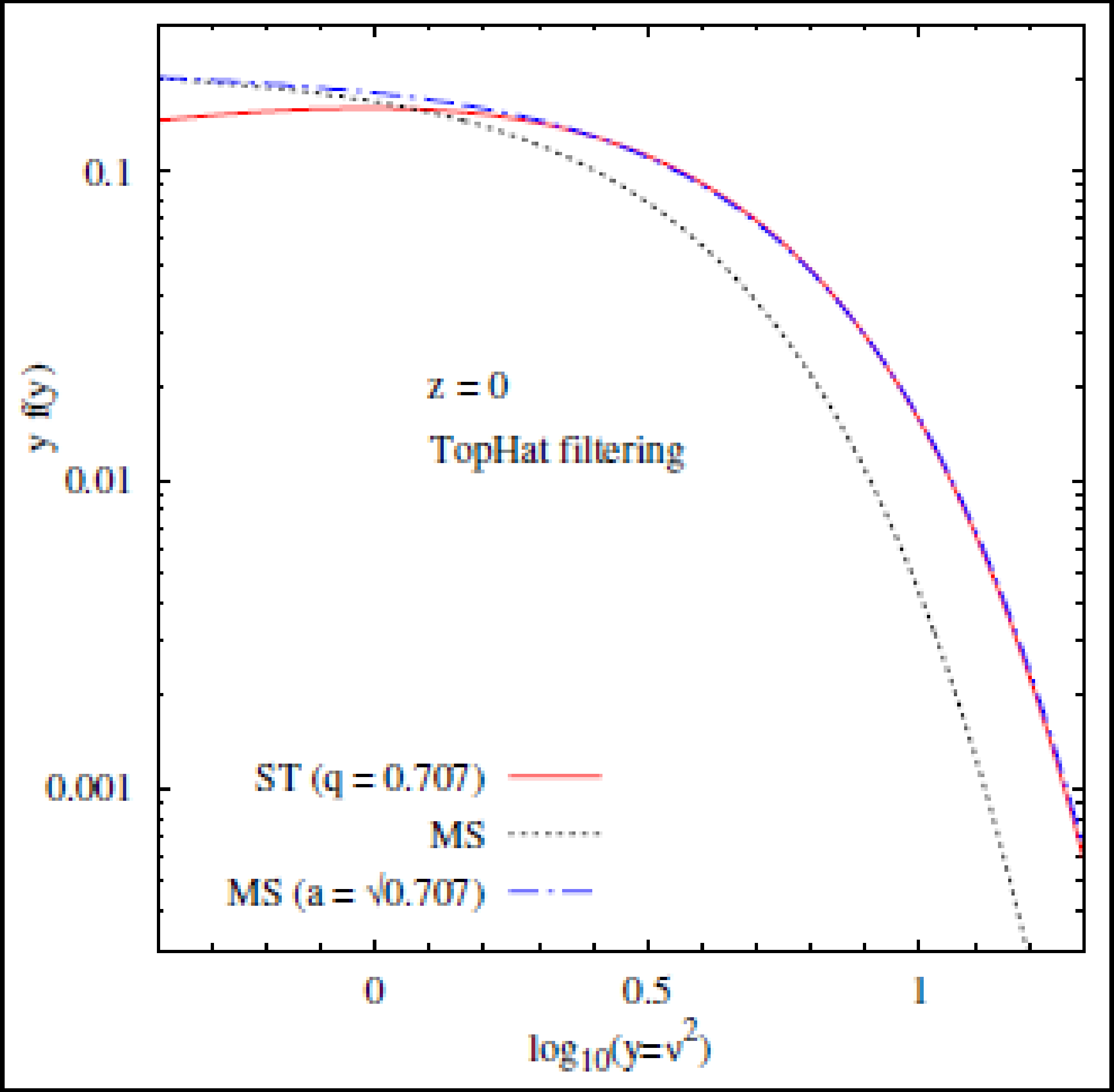
Universal form?

- Spherical evolution (Press & Schechter 1974; Bond et al. 1991)
- Ellipsoidal evolution (Sheth & Tormen 1999; Sheth, Mo & Tormen 2001)
- Simplifies analysis of cluster abundances (e.g. X-ray, SZ, Opt)
- Small departures from universality now seen




Chandra Xray Clusters
Vikhlinin et al. 2008
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
$N(>M),\ h^{-3}\ \mathrm{Mpc}^{-3}$
$z = 0.025 - 0.25$
$z = 0.35 - 0.90$
10^{14}
10^{15}
$M_{500},\ h^{-1} M_{\odot}$

Yet another stretch factor in cosmology?



The real cloud-in-cloud problem:

When spheres are no longer concentric

In concentric spheres problem progress from thinking of nearby scales, and so derivatives with respect to scale

For non-concentric spheres, think of next nearby position: taking derivatives wrt position leads to … Peaks Theory

Resulting Excursion Set Peaks model is marriage of two 20 yr old literature streams

Excursion set peaks

$$f(s|\text{peak})ds \approx \int dv \int d\delta \;\; p(\delta,v)\, q_{peak}(v)$$

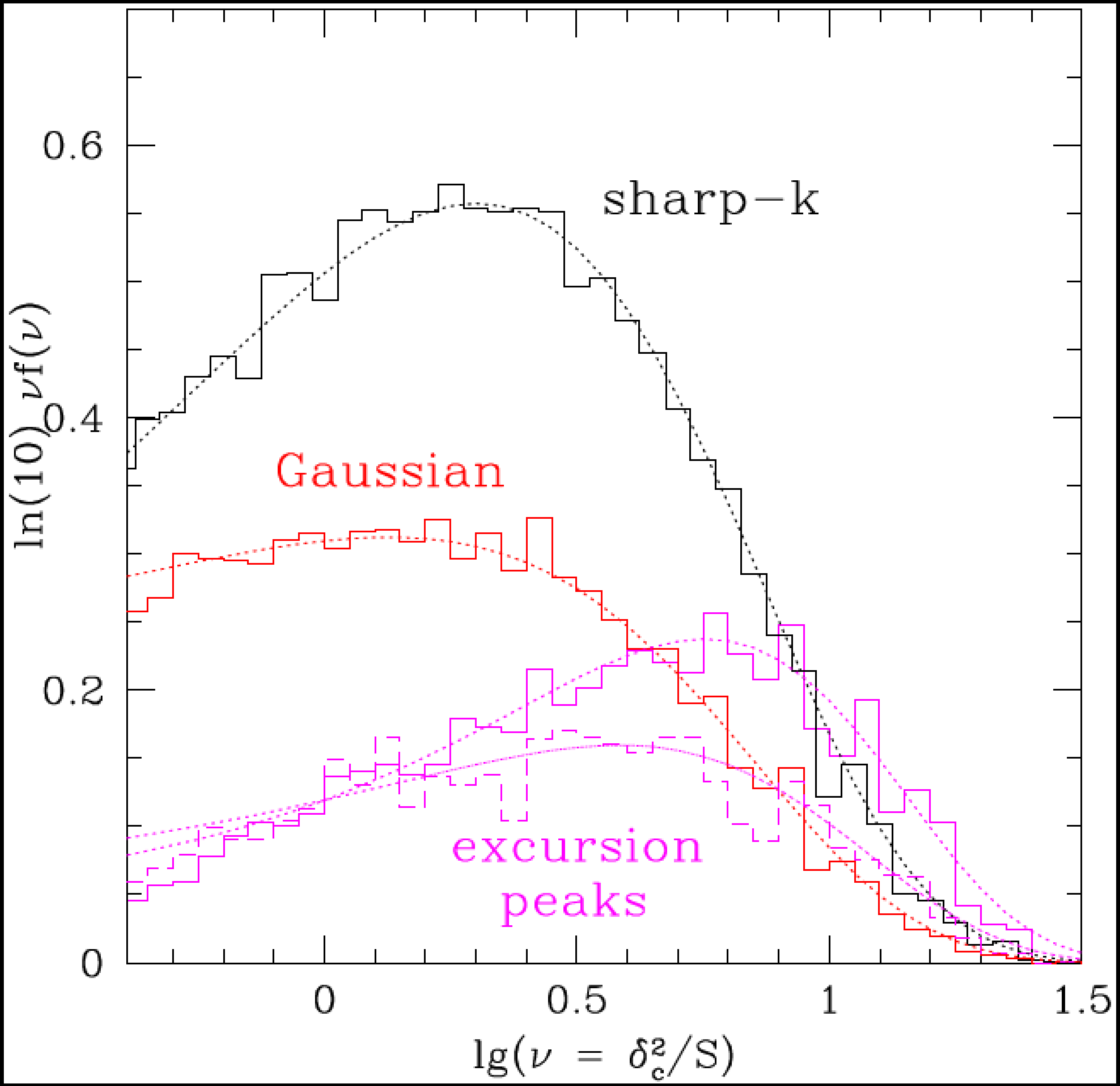
over $\delta_c < \delta < \delta_c + \Delta s\, v$ and $v > 0$

$$= \Delta s\, p(\delta_c, s) \int dv\, p(v\,|\delta_c)\, q_{peak}(v)\, v$$

$$= \Delta s\, p(\delta_c, s) \langle v\,|\delta_c, \text{peak}\rangle$$

Correlated walks? No.

Better choice of ensemble over which to average? Yes.


sharp−k
Gaussian
excursion
peaks
0.6
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0.2
0
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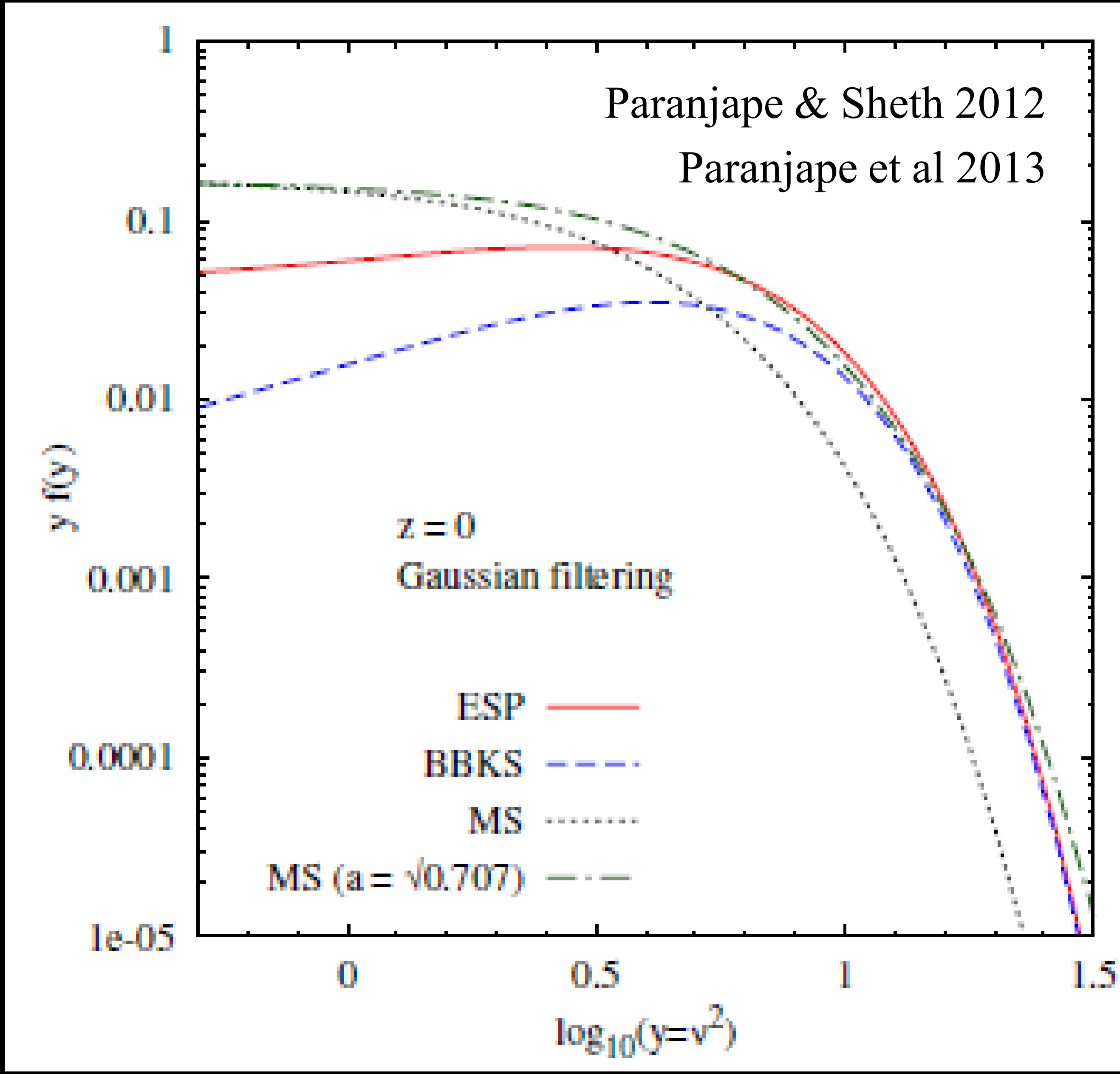
1
1.5
ln(10) νf(ν)
lg(ν = δ$_c^2$/S)

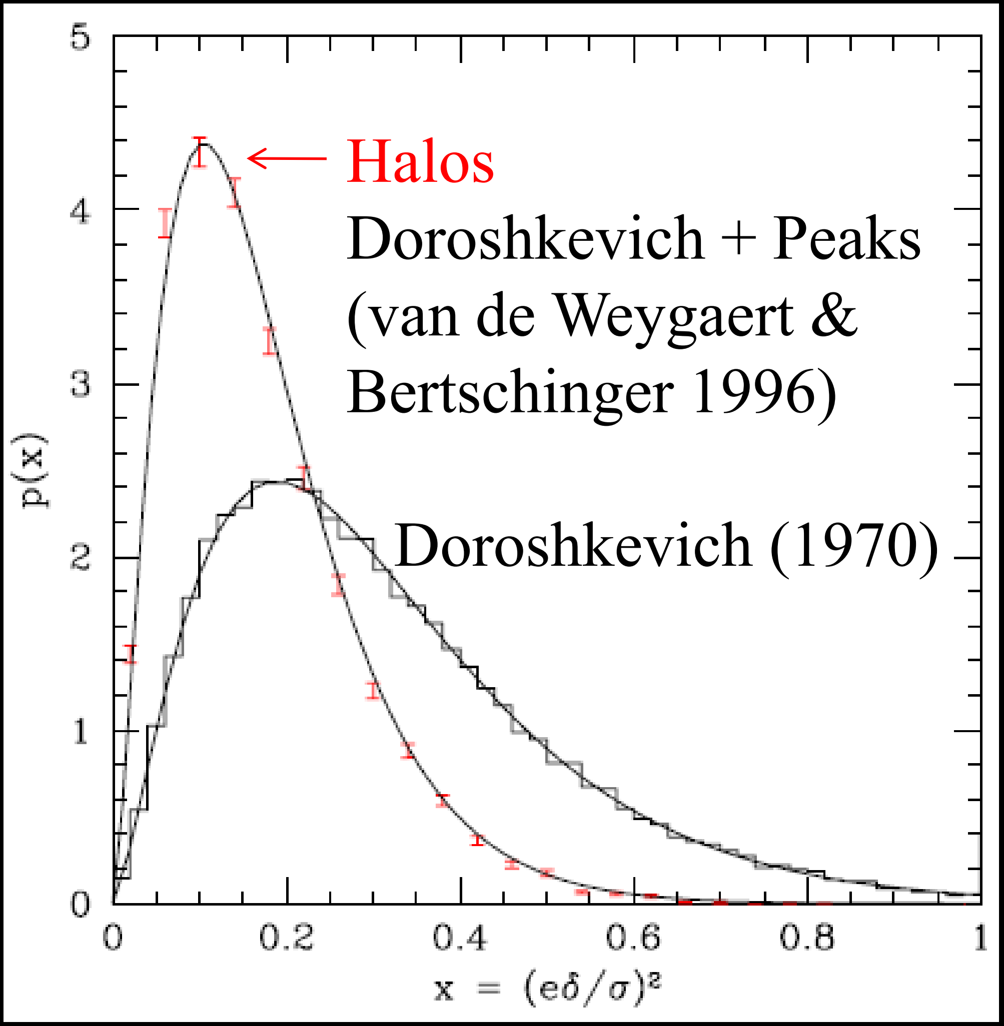

Paranjape & Sheth 2012
Paranjape et al 2013
1
0.1
0.01
0.001
0.0001
1e-05
y f(y)
z = 0
Gaussian filtering
ESP
BBKS
MS
MS (a = √0.707)
0
0.5
1
1.5
log10(y=ν2)

More direct evidence from statistics of initial patches

- For EC, need $p(\delta,e,p) = p(\delta)\, p(e,p|\delta)$
- For random patches, Doroshkevich (1970) shows $p(e,p|\delta)$ same for all δ, and distribution of $(\delta e)/\sigma(m) \sim (\lambda_1-\lambda_3)/\sigma(m)$ is universal
- In simulations, $p(\delta e/\sigma)$ indeed universal, but with smaller variance ~ like distribution around peaks in δ.

Essentially all previous analyses averaged over an ensemble of randomly placed walks.

Therefore they implicitly assumed that the statistics of center-of-mass walks are the same as those in random positions. This is wrong (Sheth, Mo, Tormen 2001).



Despali, Tormen, Sheth 2013

Recall: Large scale bias from Taylor series around δ_0;
Bias gets additional contribution from dependence of mean v on large scale δ_0:

$$f(s|\delta_0,S) = p(\delta_c|\delta_0) \langle v|\delta_c ,\delta_0 \rangle$$

- Dependence on ν makes bias factor k-dependent:

$$\text{bias}(k) = (b_{10} + b_{01}\, k^2)W(kR_h)$$

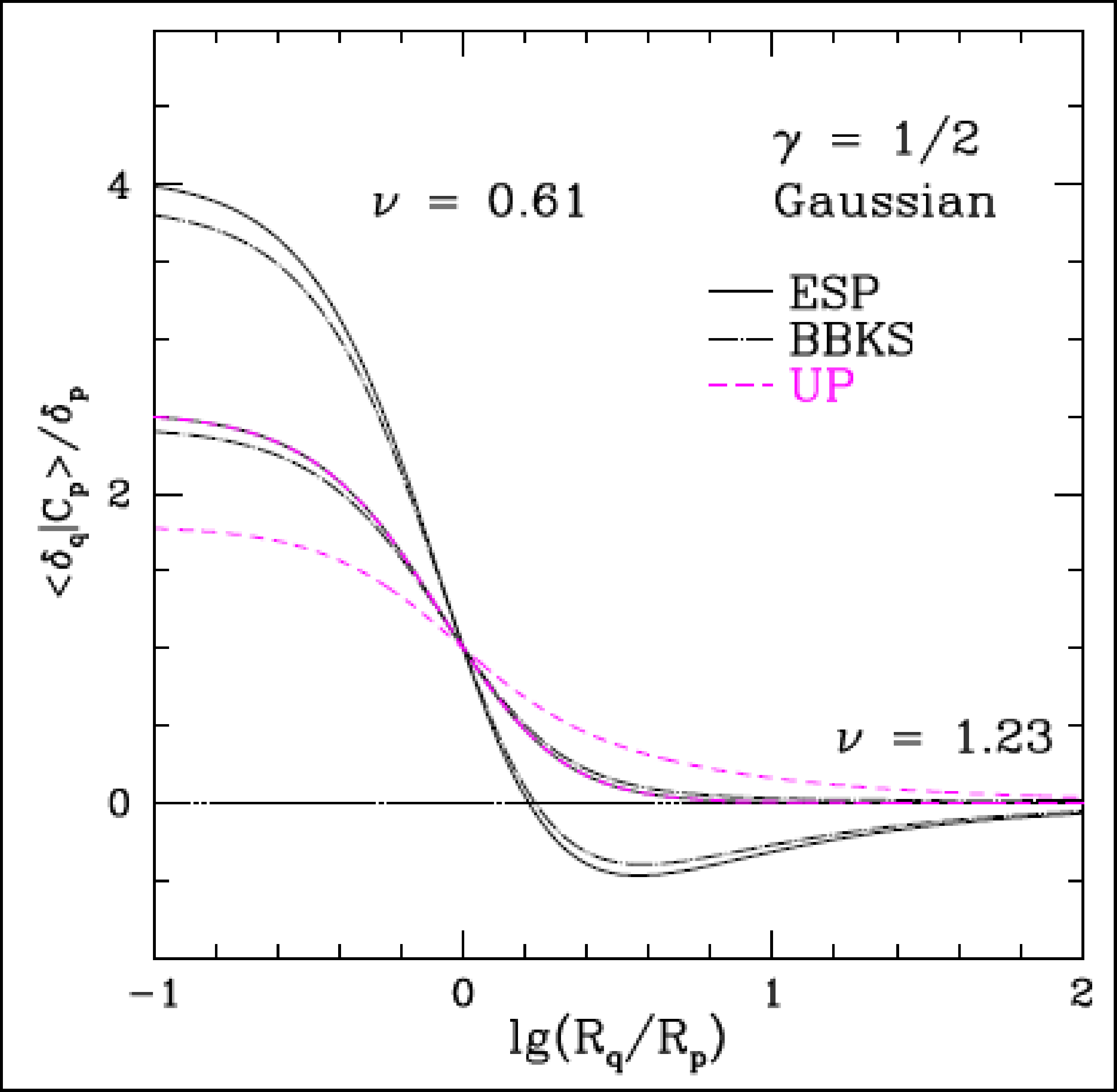
This is generic

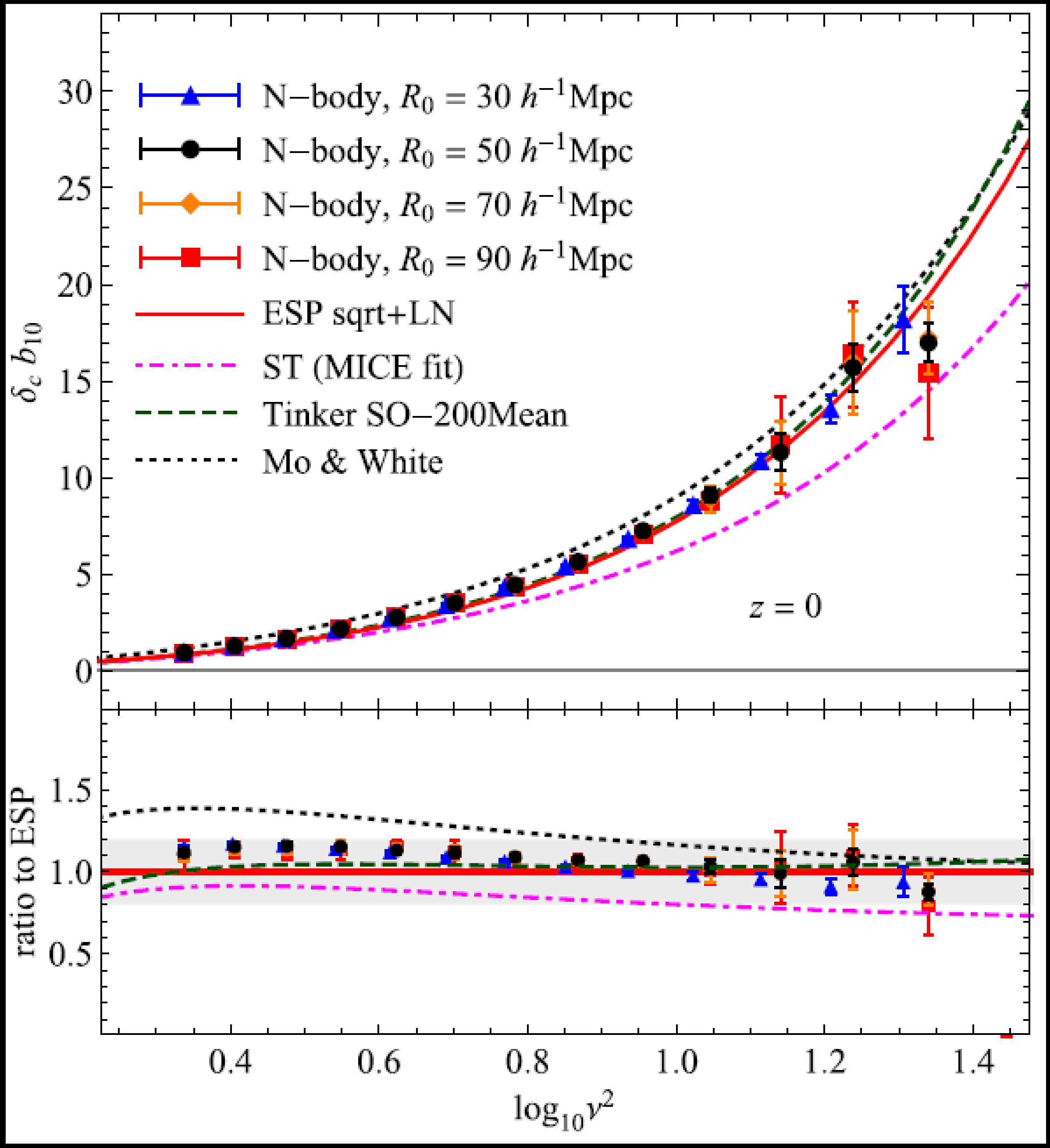
- Coefficients depend on halo mass

- Since peaks have different ν's but otherwise same structure, peak bias has same structure but different coefficients

Density profile = cross correlation between peak and mass

Generic: Low mass = more concentrated

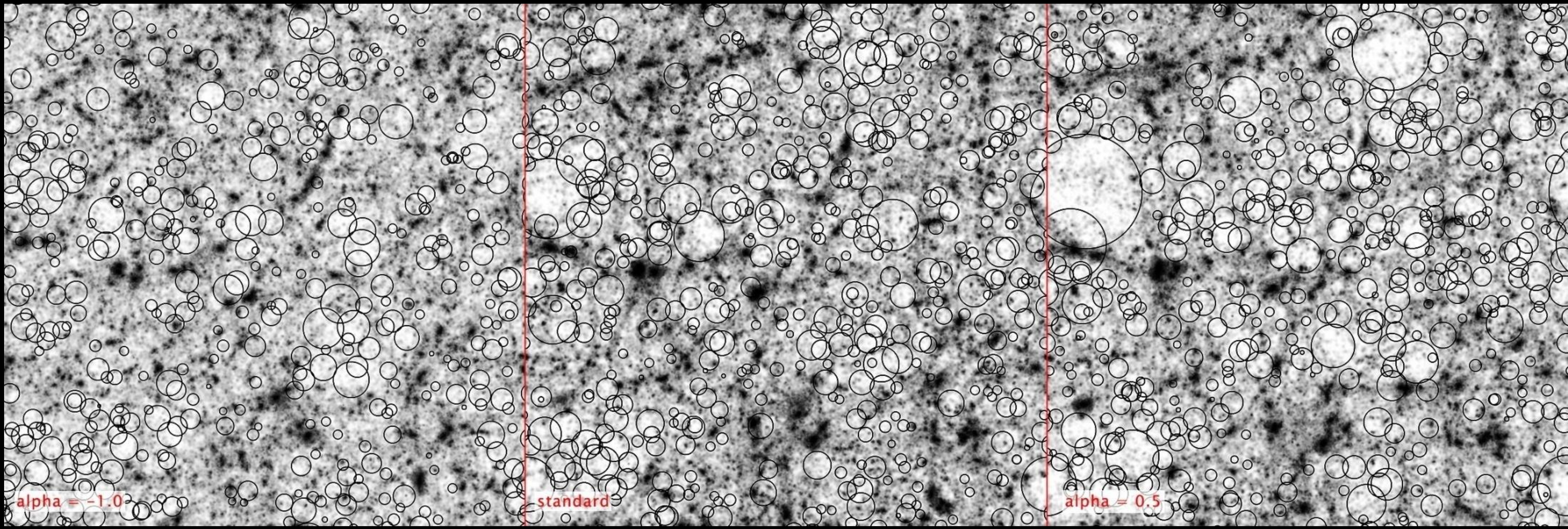




N−body, $R_0 = 30\ h^{-1}$Mpc
N−body, $R_0 = 50\ h^{-1}$Mpc
N−body, $R_0 = 70\ h^{-1}$Mpc
N−body, $R_0 = 90\ h^{-1}$Mpc
ESP sqrt+LN
ST (MICE fit)
Tinker SO−200Mean
Mo & White
$z = 0$
$\delta_c\, b_{10}$
ratio to ESP
$\log_{10}\nu^2$
0
5
10
15
20
25
30
0.5
1.0
1.5
0.4
0.6
0.8
1.0
1.2
1.4

‘Modified’ gravity theories

Martino & Sheth 2009



weaker gravity on large scales stronger gravity

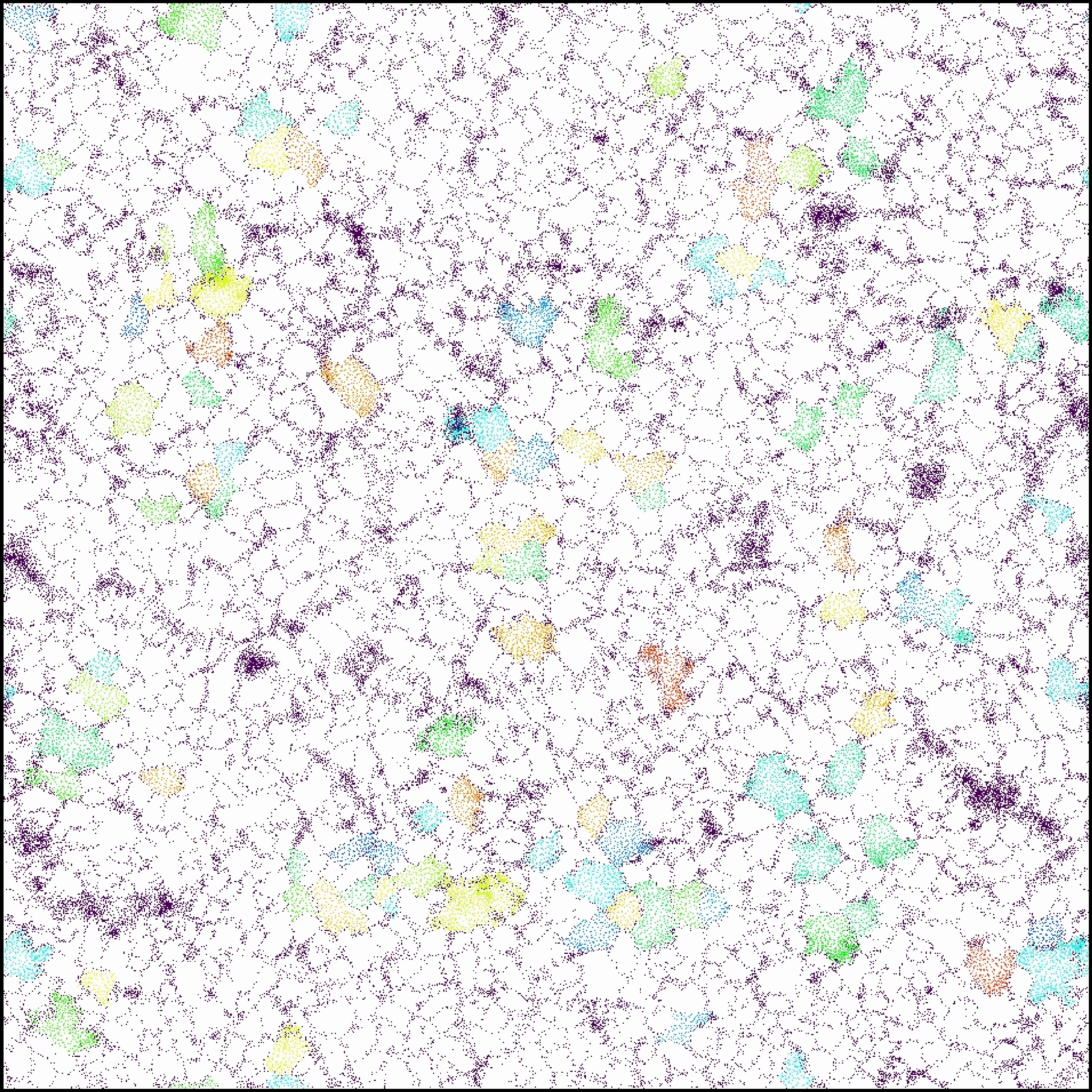
Voids/clusters/clustering are useful indicators

Voids

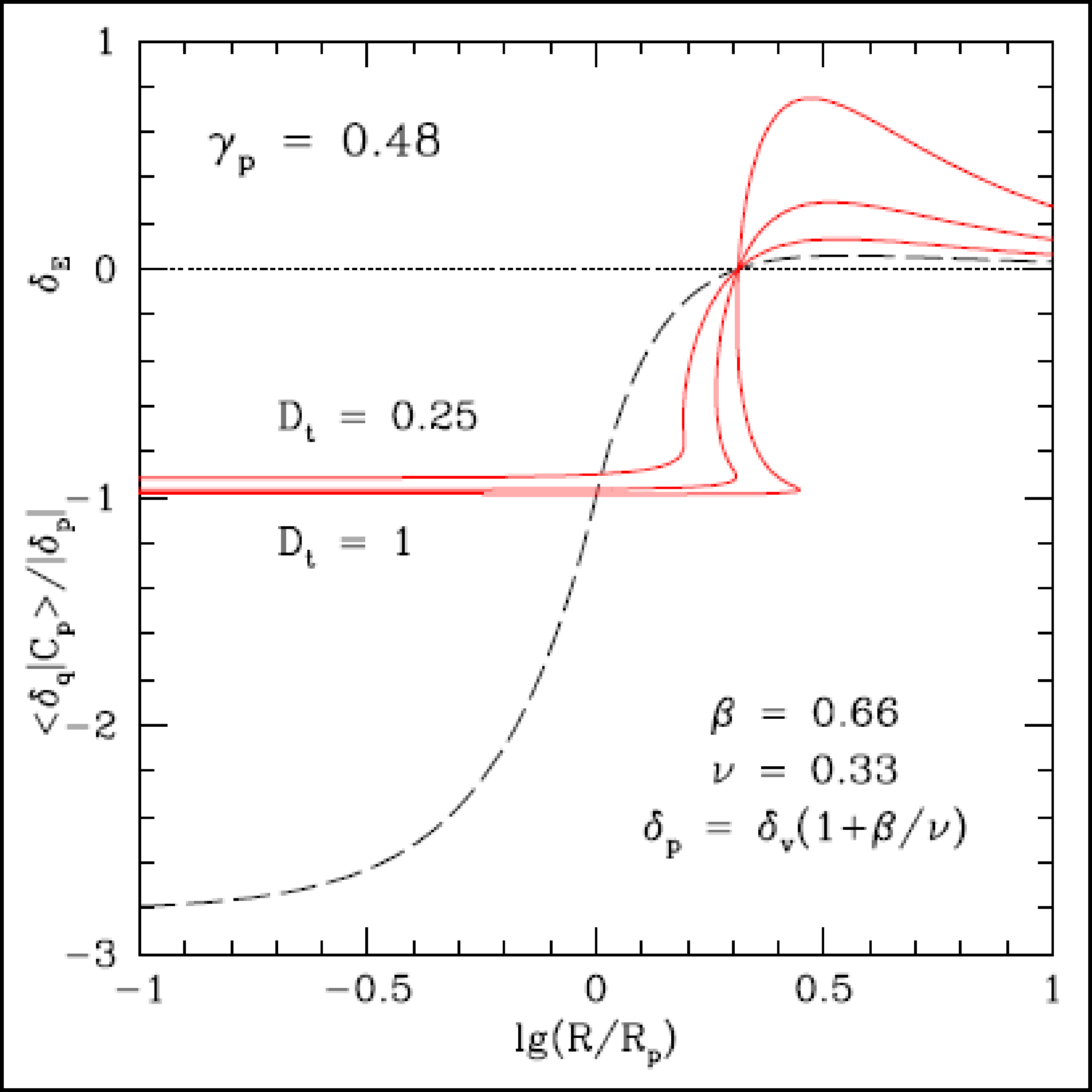
- Just change sign, so can do almost same cosmology with voids as with clusters
- Must be a little careful since small voids can be crushed if surroundings sufficiently overdense (Sheth, van de Weygaert 2004)
- Change of sign interesting because

  $b^E = 1 + b^L$ can equal zero for certain voids

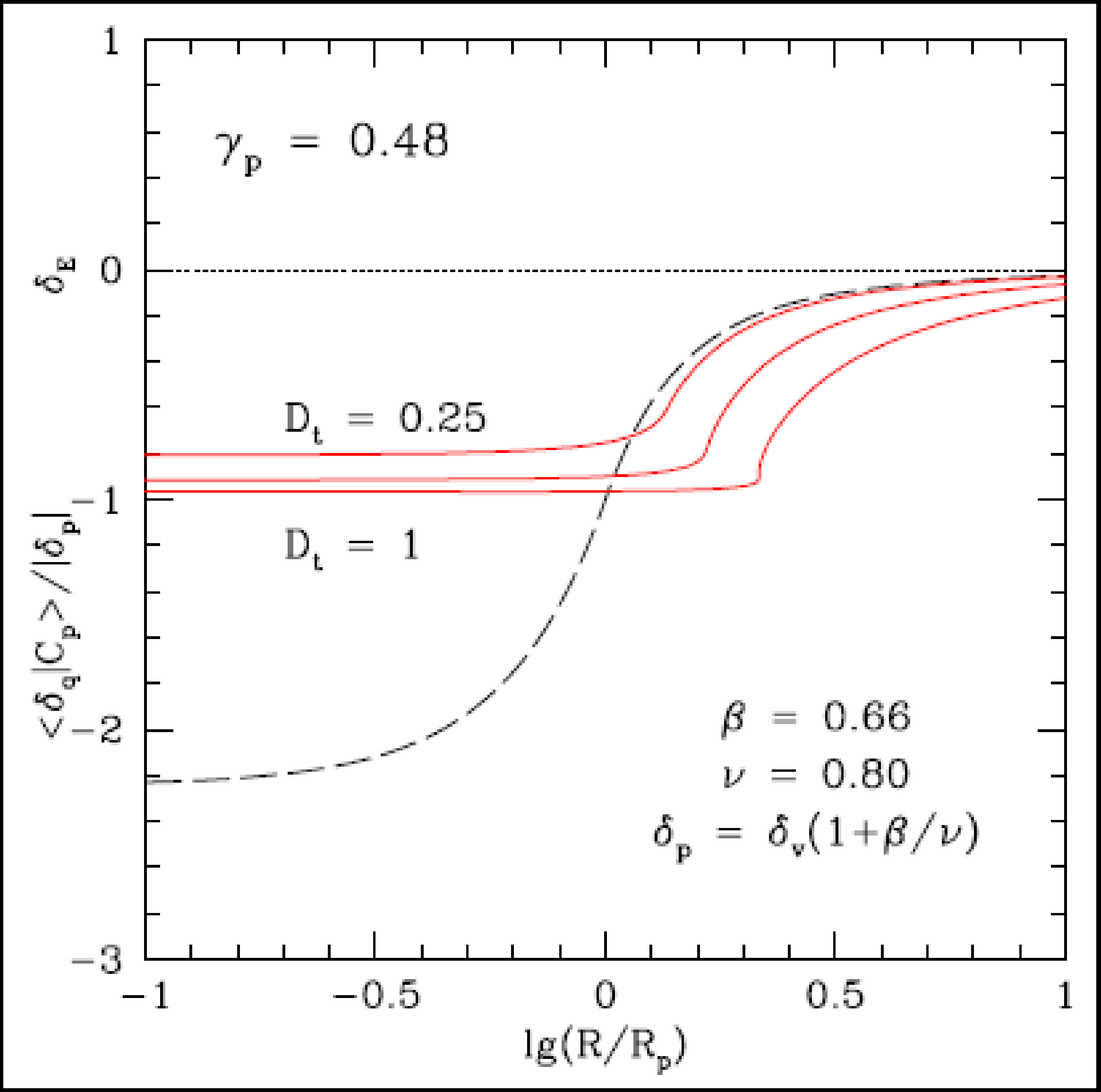
  whereas $b^E > 0$ for halos.



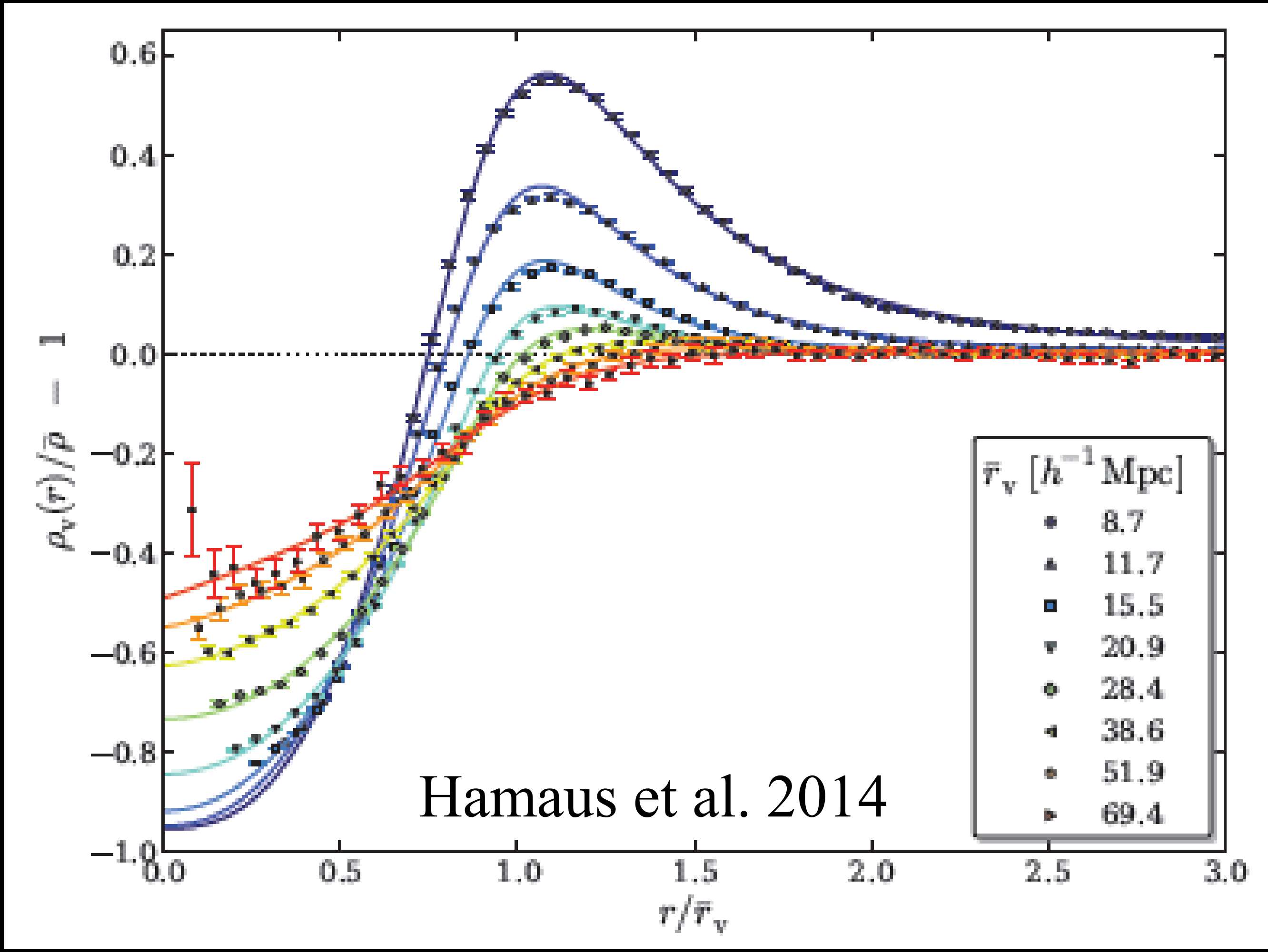
Small voids will have obvious walls and bias > 0



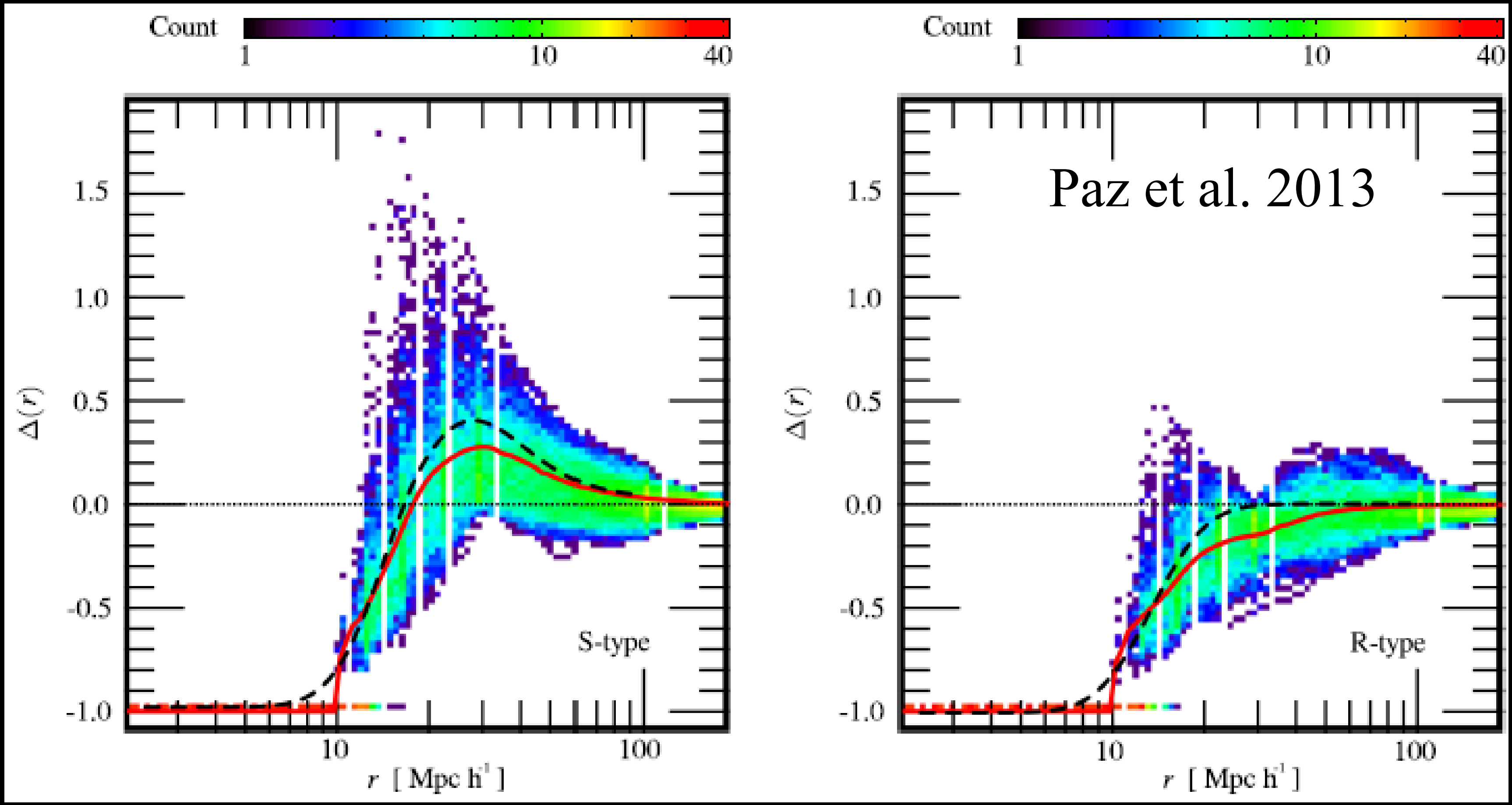
Big voids will have bias < 0 and less obvious walls



Seen in simulations …



… and in data

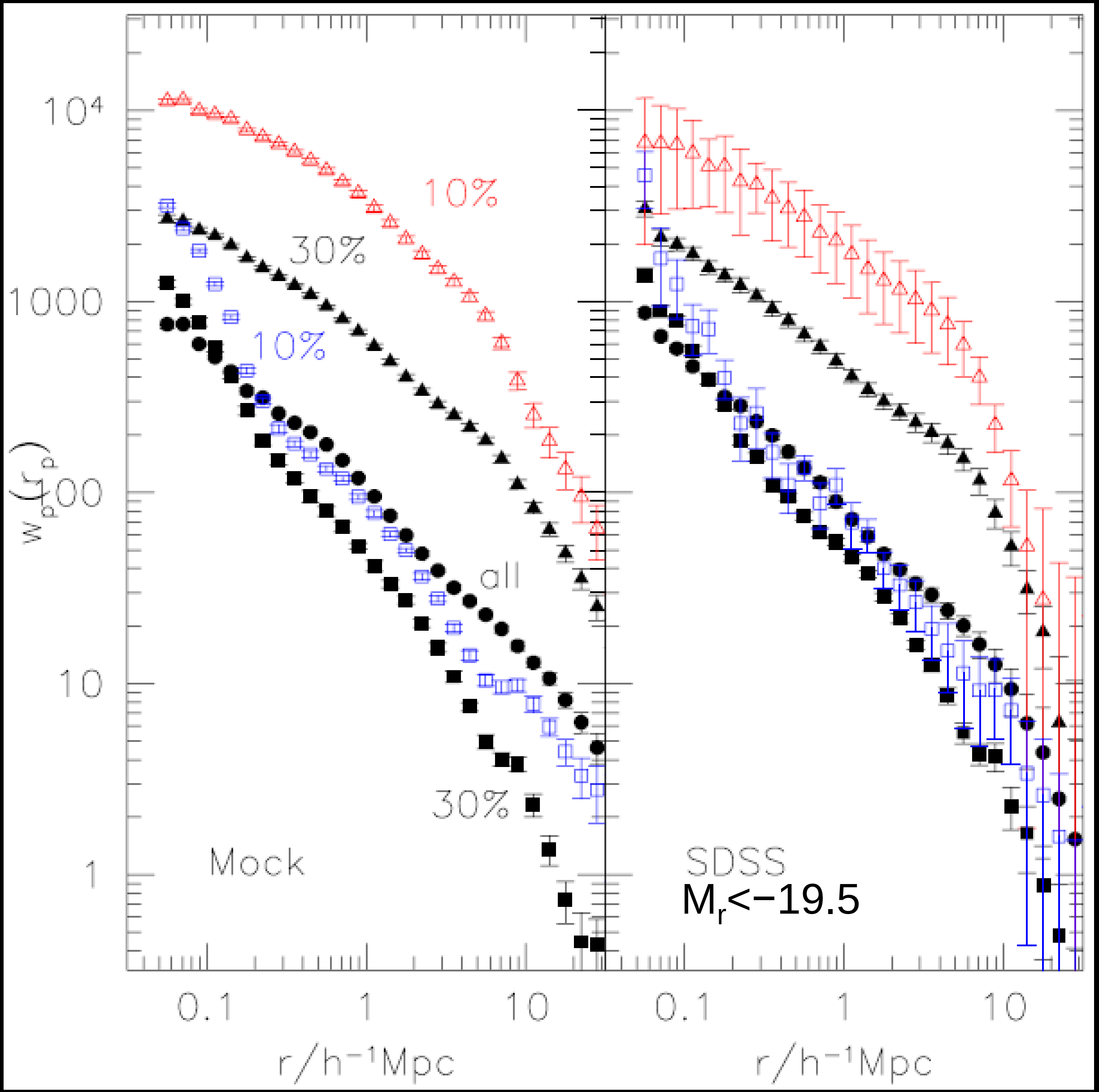


Work in progress to see if model also quantitatively OK

Since some voids have b>0
and others b<0,
some 'voids' have bias = 0.

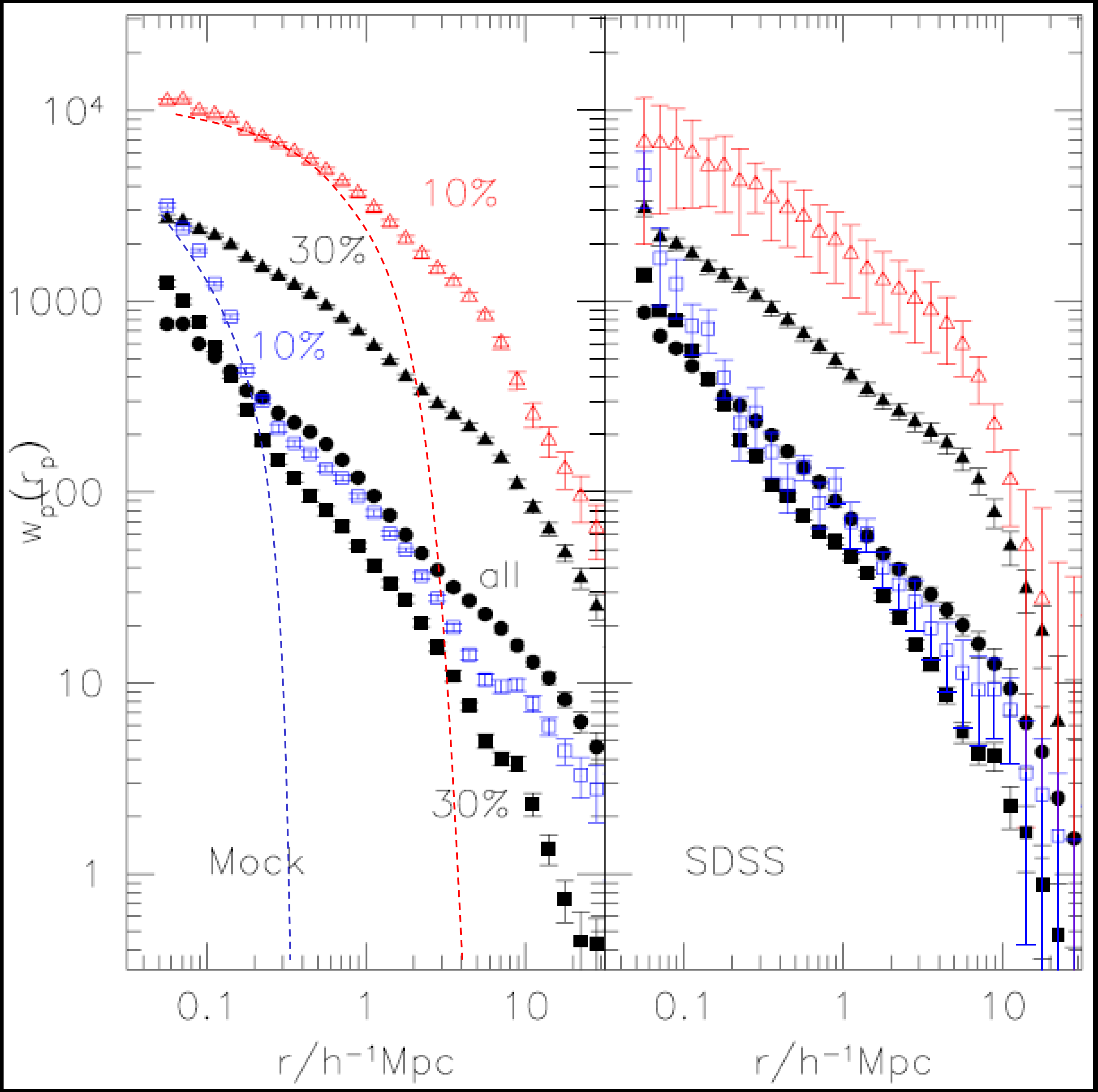
Generically, bias=0 is possible for sufficiently large sufficiently underdense regions.

- Assume cosmology → halo profiles, halo abundance, halo clustering
- Calibrate *g(m)* by matching n_{gal} and $\xi_{gal}(r)$ of full sample
- Make mock catalog assuming same *g(m)* for all environments
- Measure clustering in sub-samples defined similarly to SDSS



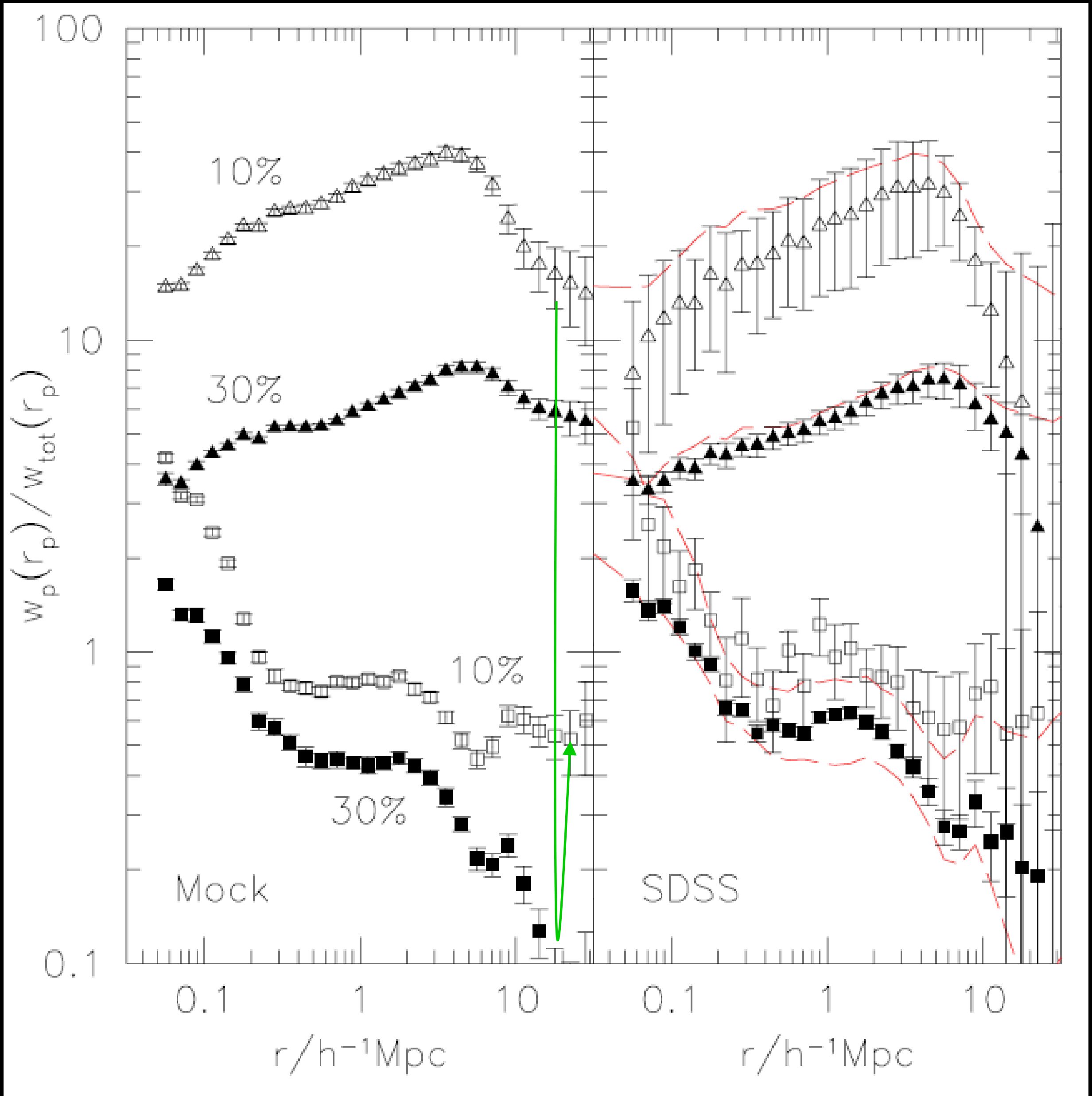
Abbas & Sheth 2007

- Environment = neighbours within 8 Mpc
- Clustering stronger in dense regions
- Dependence on density NOT monotonic in less dense regions
- Same seen in mock catalogs; little room for extra effects

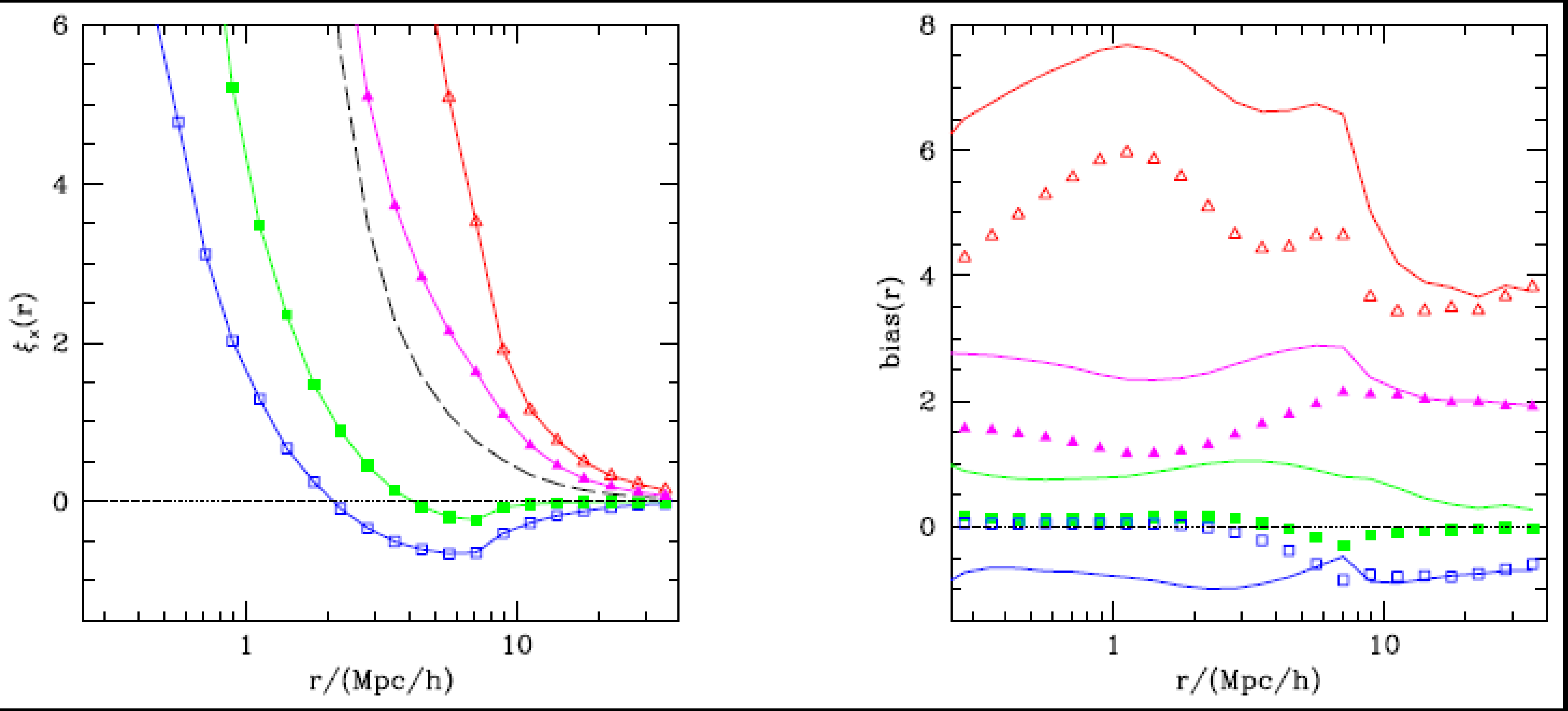


Abbas & Sheth 2007

- Galaxy distribution remembers that, in Gaussian random fields, high peaks and low troughs cluster similarly (but with opposite signs)



Auto-correlation only sees b^2



Bias from cross correlation $\propto$b is indeed monotonic, and crosses 0

Some interest in using b=0 objects as standard rods (Hamaus et al. 2013)

- These will depend on tracer population.
- SDSS Main Galaxy sample in Abbas-Sheth had b~1, so underdense patches of size 8Mpc/h in this sample had b=0.
- In LRG sample, b=0 for voids of size 20 Mpc/h.

Nonlocal bias

- Bias is generically expected to be k-dependent
- Should we expect angular dependence as well?

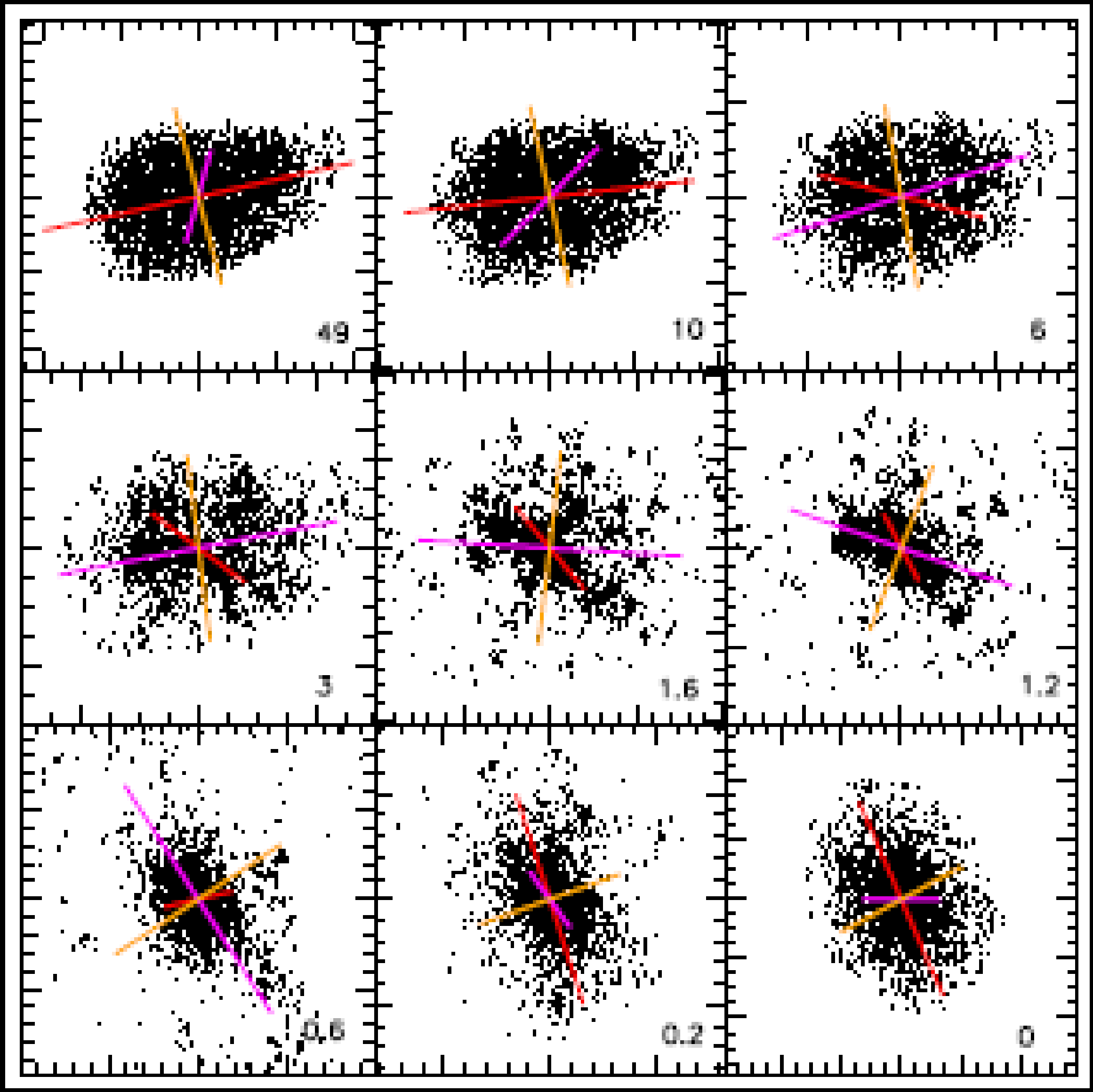
Triaxial collapse

Halos identified using ellipsoidal overdensity

Spherical overdensity masses OK to ~ 10%

Shapes differ by ~ 40%

Despali, Tormen, Sheth 2013

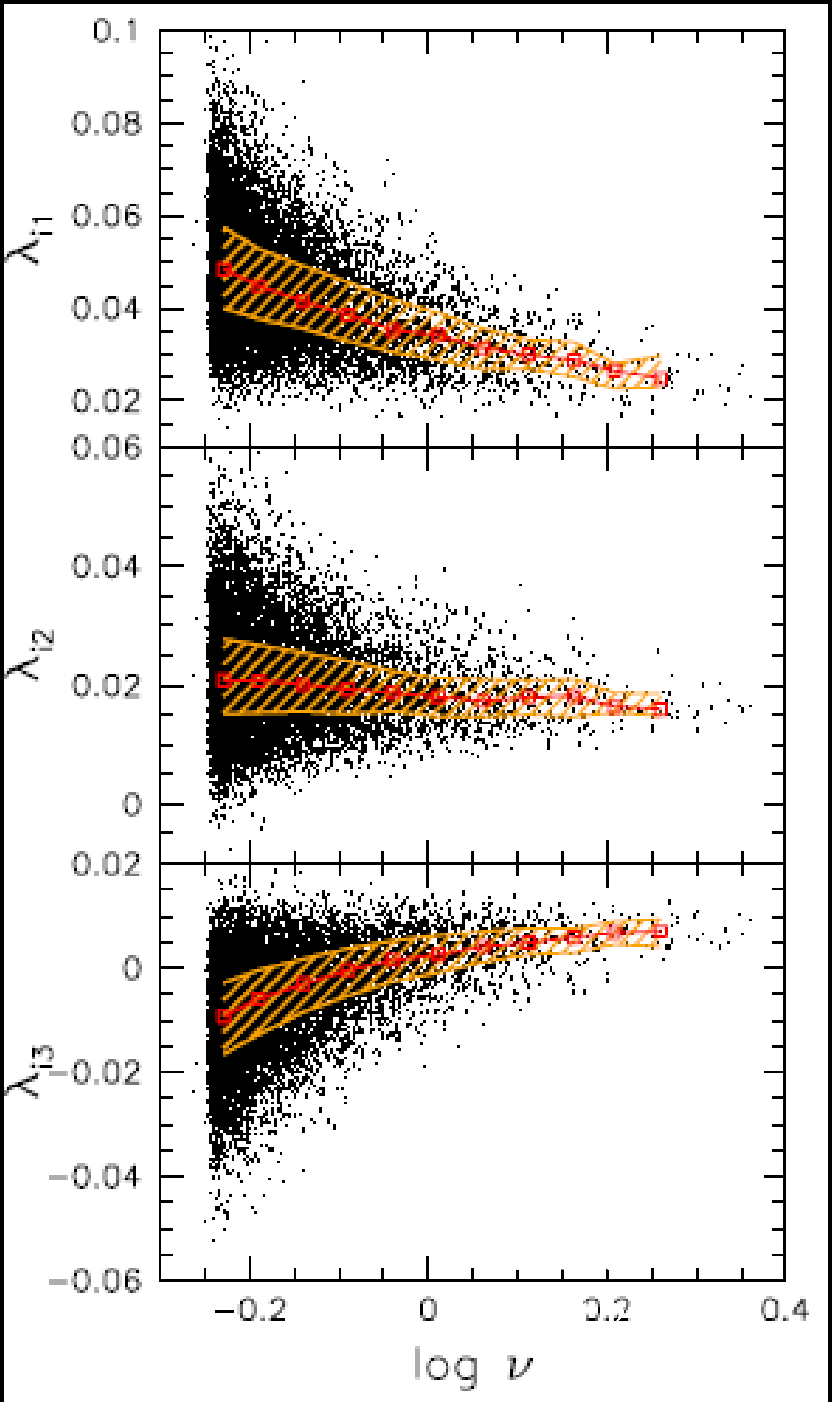


Halo formation depends on more than trace of Deformation tensor

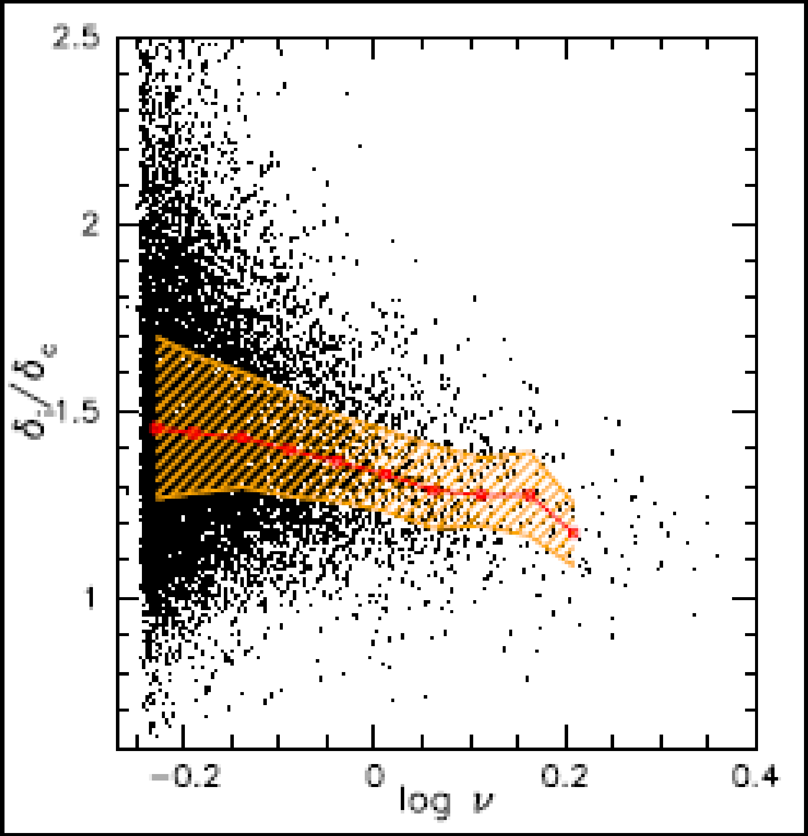
Bond & Myers 1996;
Sheth, Mo, Tormen 2001

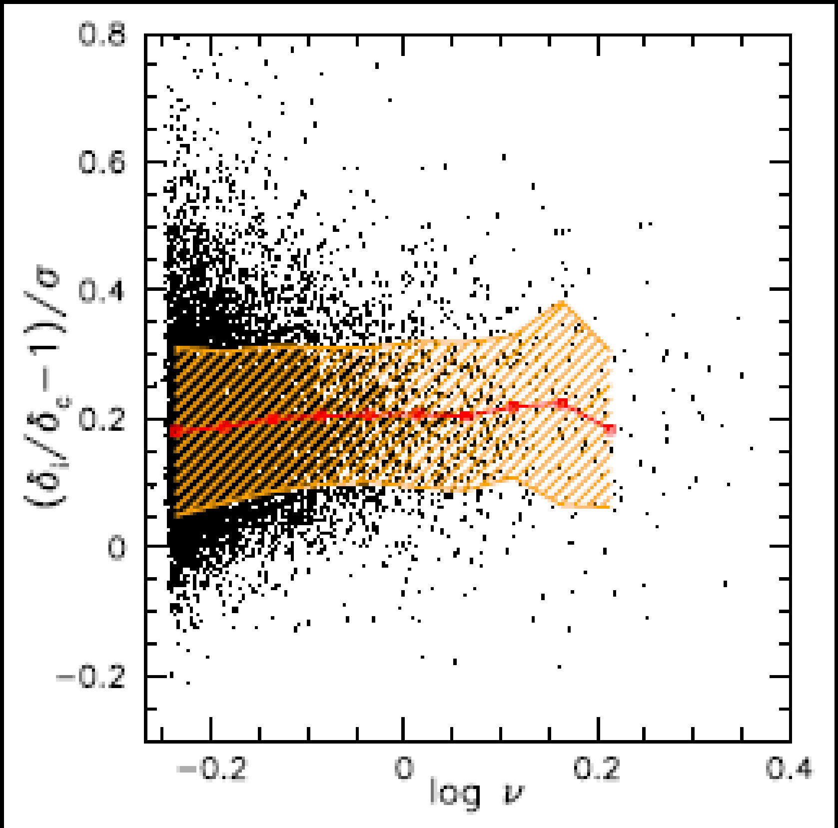
- Not all eigenvalues have same sign

Despali, Tormen, Sheth 2013



More massive protohaloes are rounder (virialized haloes are not)

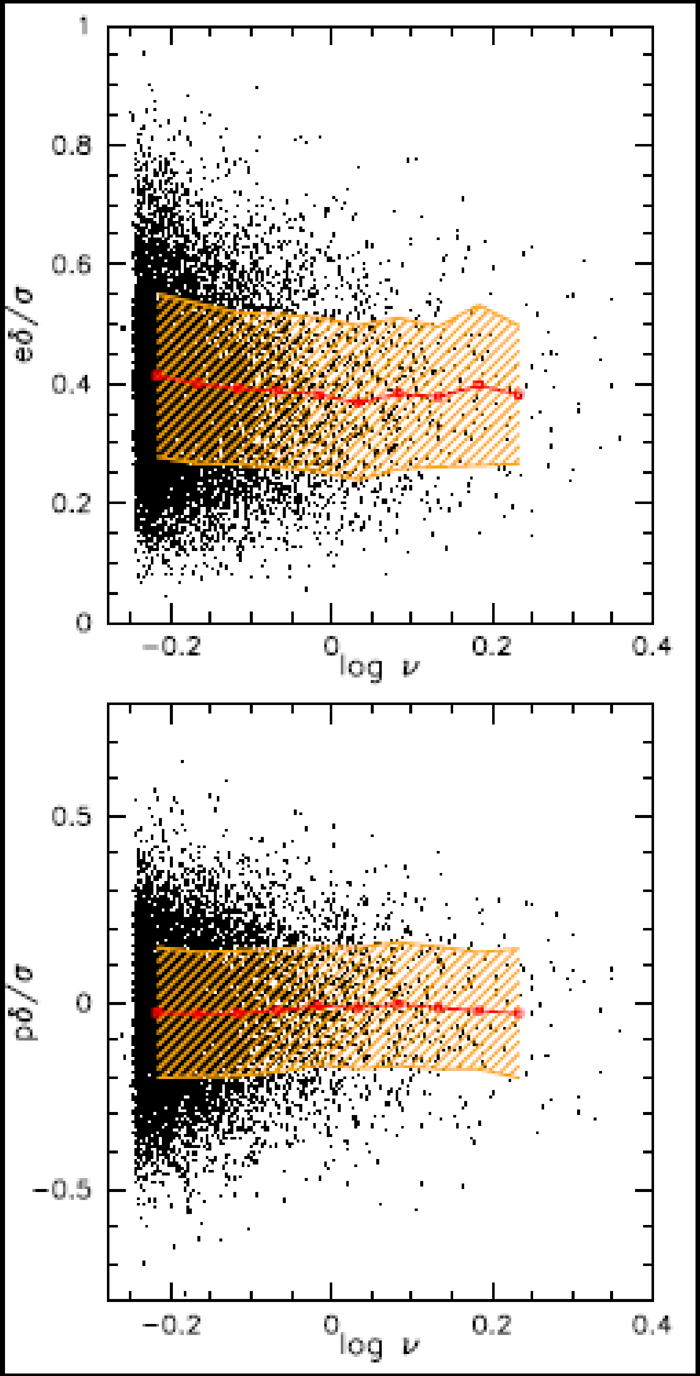




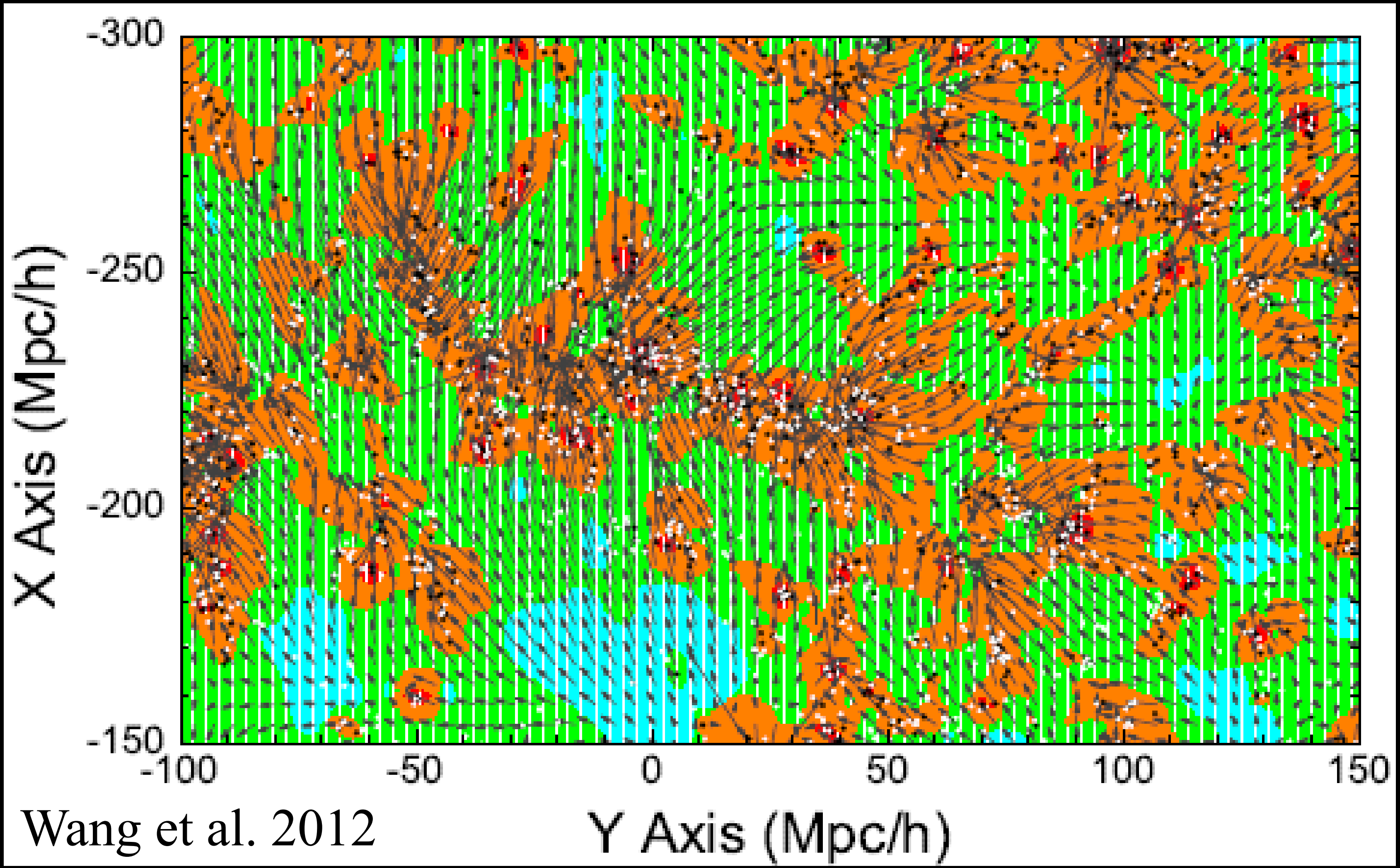
Despali, Tormen, Sheth 2013

Overdensity, ellipticity, prolateness all scale with σ, so decrease at large mass

Despali, Tormen, Sheth 2013



Can infer density, velocity, tidal fields



Can we model this?

Halo formation depends on more than trace of DefTensor

- In triaxial collapse models critical density for collapse depends on e,p (SMT2001)
- Will study simpler case in which new parameter is traceless shear q (this is the quadrupole in perturbation theory)
- Ask for largest scale on which

$$\delta > \delta_c (1 + q/q_0)$$

‘Stochastic’ barrier

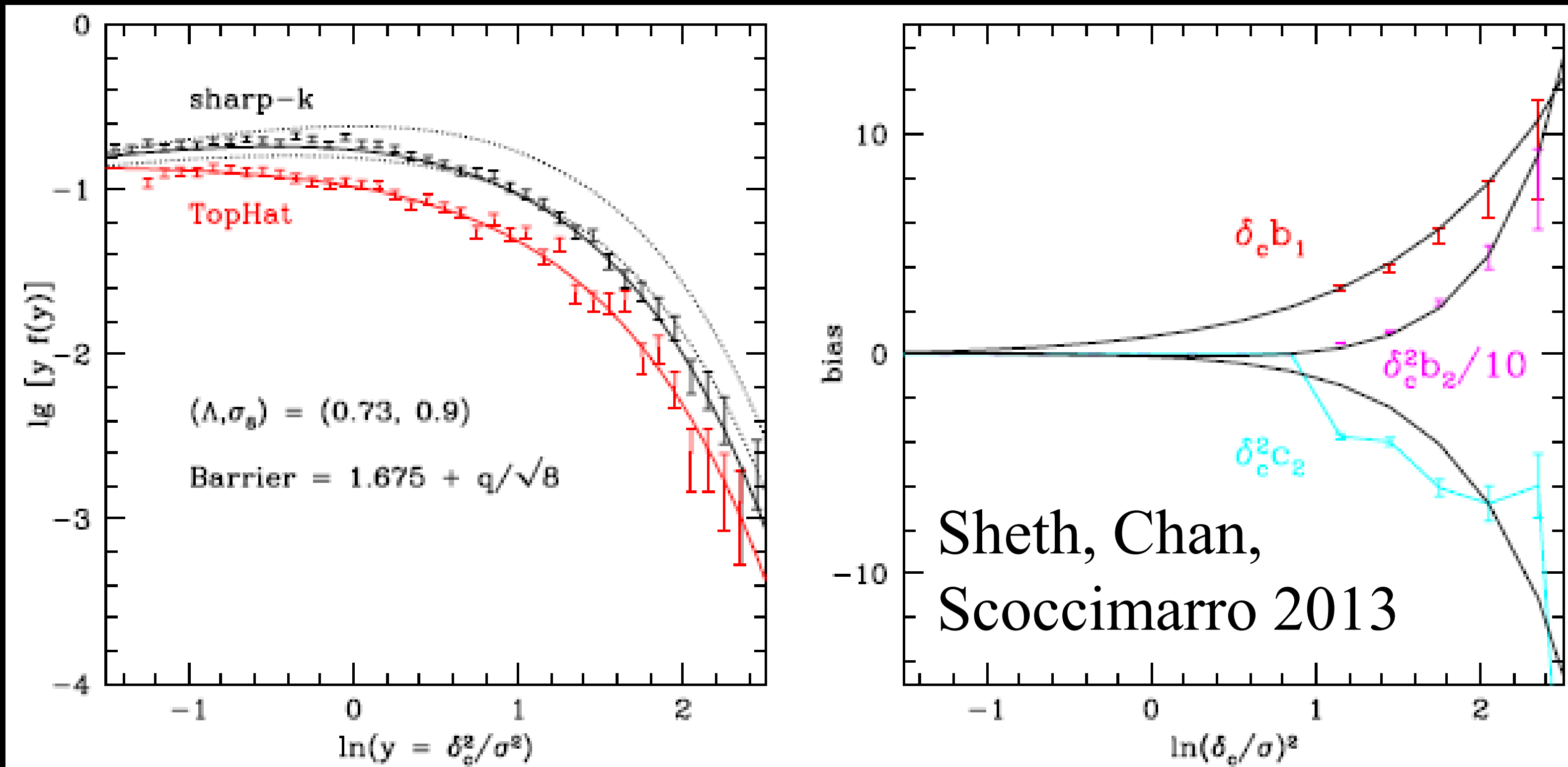
- Traceless shear q is non-Gaussian, $\chi^2(5)$, that is uncorrelated with δ
- Asking for largest scale on which

$$\delta > \delta_c\,(1 + q/q_0)$$

is like doing barrier crossing problem for 6d walks

- Can think of this as a stochastic barrier, whose height is different at each step (because of q) (Sheth-Tormen 2002)

6d walks with correlated steps



$$\nu f(\nu) \approx \frac{\nu\, e^{-(\nu+\sqrt{\delta_c^2/q_c^2})^2/2}}{\sqrt{2\pi}} \left[1 - \frac{\mathrm{erfc}(\Gamma\nu/\sqrt{2})}{2} + \frac{e^{-\Gamma^2\nu^2/2}}{\sqrt{2\pi}\,\Gamma\nu}\right]$$

Can also do as 1d nonGaussian (δ-q) walks (Musso, Sheth 2014)

Large scale bias

- The local bias model

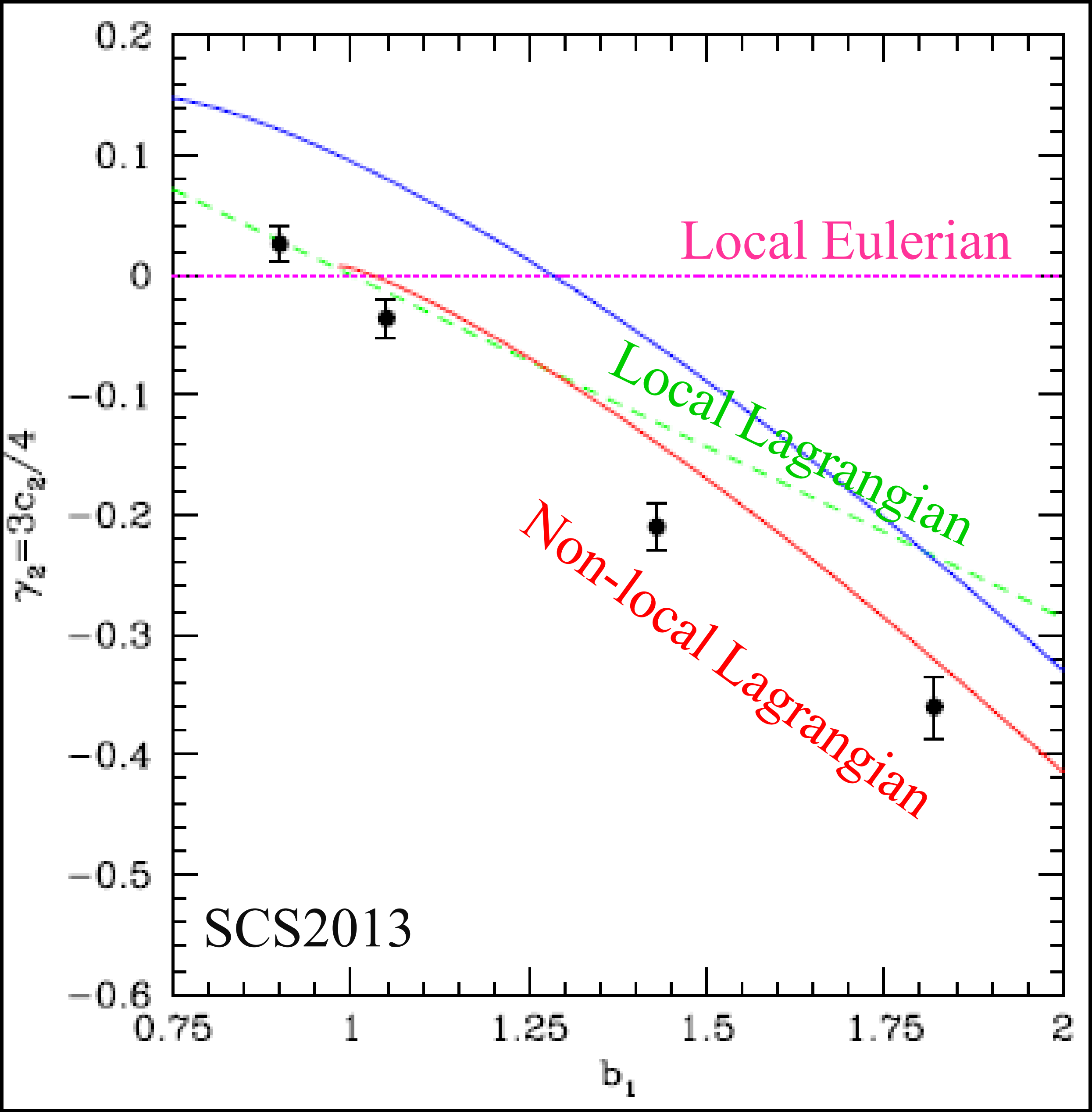
$$1 + \delta_h = f(s|\delta_L(\delta))/f(s)$$
$$= 1 + b_1\, \delta_L(\delta) + b_2\, \delta_L(\delta)^2 + \ldots$$

- 'Nonlocal' bias means things other than δ matter
- Even if $f(s|\delta_L(\delta))/f(s)$ depends only on δ_L, then bias with respect to δ will seem nonlocal because mapping between δ_L and δ is 'nonlocal'.
- If q matters, even Lagrangian bias is 'nonlocal'.

Account for additional nonlocality from contribution of tidal term to nonlinear evolution $\delta(\delta_0, q_0)$ to get Eulerian bias.

$$1 + \delta_\mathrm{h}^\mathrm{E}(\delta, \mathrm{q}^2) = (1 + \delta)(1 + \delta_\mathrm{h}^\mathrm{L})$$

$$\begin{aligned}
&= (1 + \delta)\left(1 + b_1^\mathrm{L}\,\delta_0 + b_2^\mathrm{L}\,\frac{\delta_0^2}{2} + c_2^\mathrm{L}\,\frac{q_0^2}{2} + \dots\right) \\
&= 1 + b_1^\mathrm{L}\,\delta_0 + b_2^\mathrm{L}\,\frac{\delta_0^2}{2} + c_2^\mathrm{L}\,\frac{q_0^2}{2} + \delta + b_1^\mathrm{L}\,\delta_0\delta \\
&= 1 + \delta\,(b_1^\mathrm{L} + 1) + \frac{\delta^2}{2}(8b_1^\mathrm{L}/21 + b_2^\mathrm{L}) \\
&\qquad + \frac{q_0^2}{2}(c_2^\mathrm{L} - 8b_1^\mathrm{L}/21).
\end{aligned}$$

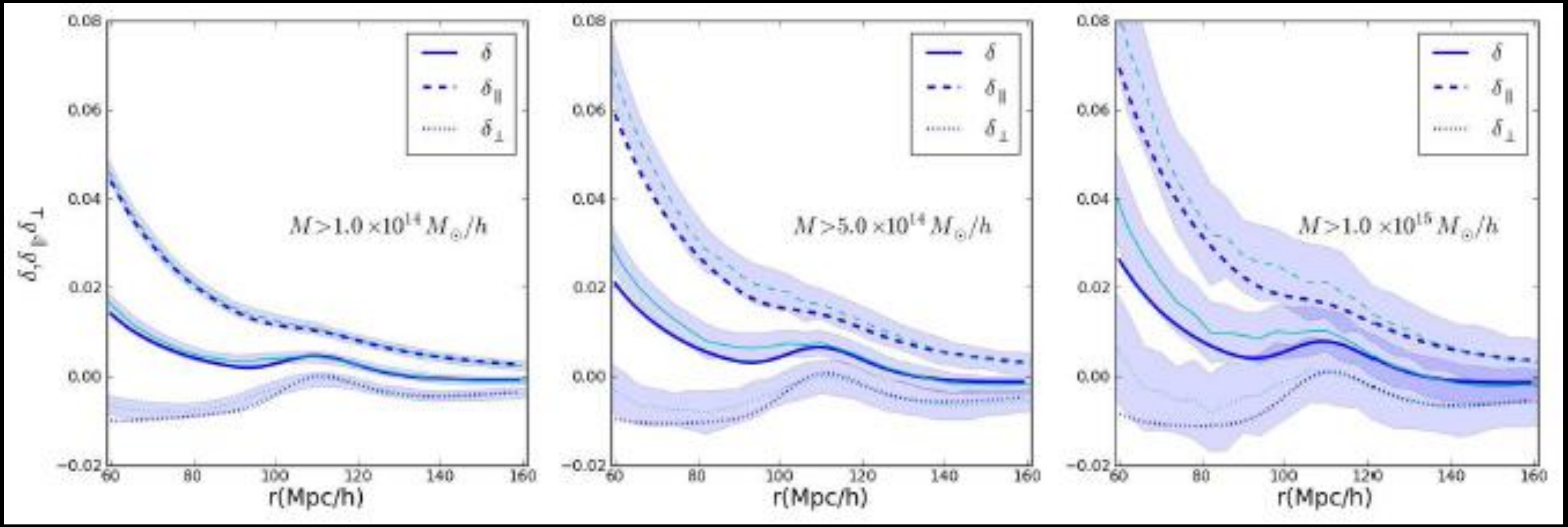


Local Eulerian
Local Lagrangian
Non-local Lagrangian
SCS2013
$\gamma_2 = 3c_2/4$
b_1
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
0.75
1
1.25
1.5
1.75
2

Summary

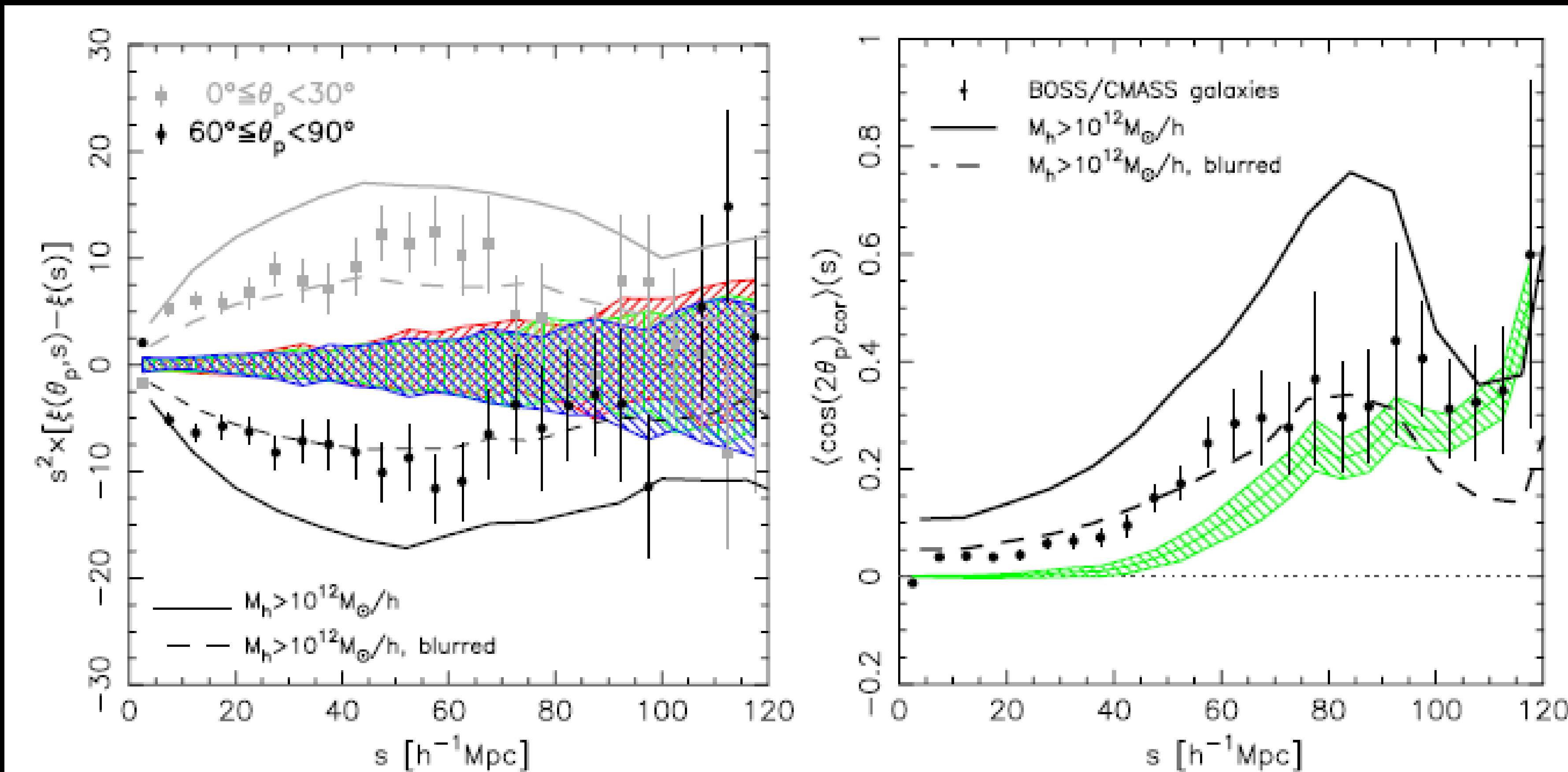
- Getting closer to a model which includes nonlocal, nonspherical effects, and reconciles peaks/halos (Castorina-Sheth 2013)
- These generate k-dependent bias (monopole), as well as anisotropic bias (e.g. quadrupole), even in real-space
- Nonlocal bias matters at high mass
- Useful for making physically motivated 'fitting formulae' which simplify data analysis

Halos aligned with LSS



- Measurements in sims from Faltenbacher et al. (2012)
- Model assumes alignment with large-scale shear field generates quadrupolar signal proportional to same q which makes nonlocal bias (Papai & Sheth 2013)

Also seen in CMASS



- Halos more strongly aligned than galaxies (modeling this is work in progress)

Summary

Its always good to step up; first passage problem with correlated steps ‘solved’ using only 3 variables.

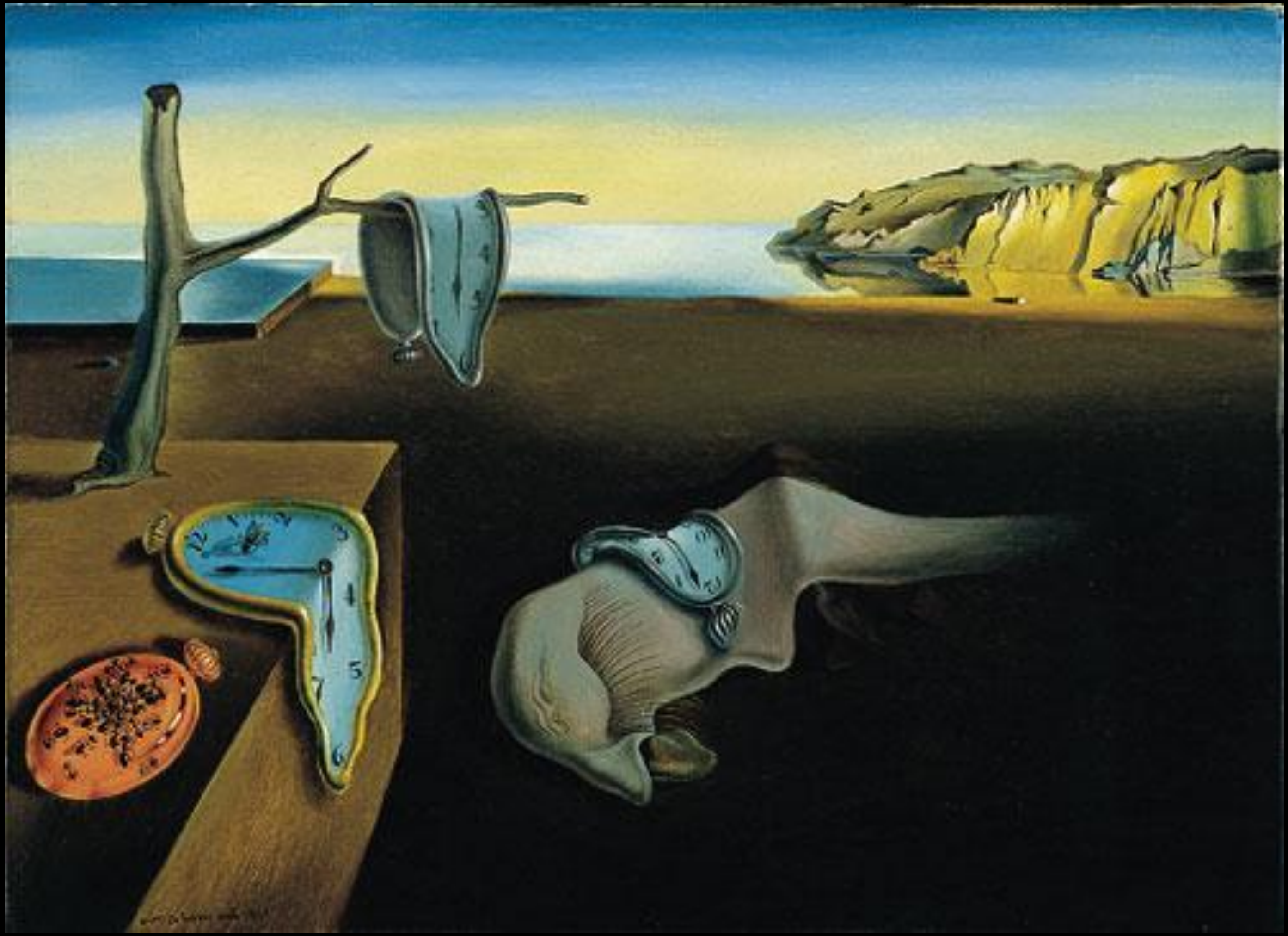
Markov velocities are the next most natural (Ising-model like) generalization of the usual Markov heights model; good approximation for CDM-like P(k)!

Self-consistent model must only average over special subset of walks. Peaks are a good choice, for which closed form expressions are now available.

Must incorporate stochasticity in halo formation from tidal field and (mis-alignment with!) proto-halo shapes.

Tidal field leaves signature on halo abundances, clustering, especially in higher-order statistics of highly clustered objects (typically high-mass halos).

Hierarchical clustering in GR



= the persistence of memory