dRGT massive gravity and cosmology: the view of an outsider

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Outline

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- Massive gravity, which weakens gravity on very large scales seems a most natural and simple idea.

This motivated me to look into massive gravity theories.

A mass for the graviton

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- A mass term is proportional the the square of excitation w.r.t to some reference metric.
- In 1939 Pauli and Fierz showed that within linearized gravity, there is only one form for quadratic potential for $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ (mass term) which does not lead to a 'ghost', namely

$$U = \frac{m^2}{4}\left(h_{\mu\nu}h^{\mu\nu} - h^2\right) .$$

  In general, $h_{\mu\nu}$ has 6 degrees of freedom. 5 of them make up the massive spin-2 graviton and the 6th is a spin-0 ghost (i.e. its kinetic term has the wrong sign). The above form of the mass term ensures that one of these 6 degrees of freedom is not propagating but fixed by a constraint.

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- More precisely: consider the quadratic Lagrangian for $h_{\mu\nu}$,

$$\mathcal{L} = \frac{M_p^2}{2}\left[h_{\nu\mu}\mathcal{E}^{\mu\nu\alpha\beta}h_{\alpha\beta} - U\right] ,$$

$$\begin{aligned}\mathcal{E}^{\mu\nu\alpha\beta} \quad = \quad & -\frac{1}{2}\Big[\left(\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\nu}\eta^{\alpha\beta}\right)\Box + \\ & \left(\eta^{\mu\nu}\eta^{\alpha\rho}\eta^{\beta\sigma} + \eta^{\alpha\beta}\bar{\eta}^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\beta}\bar{\eta}^{\nu\rho}\eta^{\alpha\sigma} - \eta^{\alpha\nu}\eta^{\beta\rho}\eta^{\mu\sigma}\right)\partial_\rho\partial_\sigma\Big]\end{aligned}$$

  is the Lichnerowicz operator (on flat spacetime). The 'lapse function' h_{00} enters only linearly, in the form $h_{00}(\cdots)$, like a Lagrange multiplier. Hence its variation yields an additional constraint and removes one of the 6 degrees of freedom in h_{ij}. The remaining constraints determine h_{i0}.

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- This can be generalized to an arbitrary reference metric, $g_{\mu\nu} = f_{\mu\nu} + h_{\mu\nu}$ and remains true in the quadratic action.

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- This can be generalized to an arbitrary reference metric, $g_{\mu\nu} = f_{\mu\nu} + h_{\mu\nu}$ and remains true in the quadratic action.
- But whatever higher order covariant terms you add to U, h_{00} will no longer appear as a Langrange multiplier, the additional condition is lost and the 'ghost' will appear. $\Rightarrow$ Boulware Deser ghost (1972).

Van Dam-Veltman-Zakharov discontinuity (1970)

- Setting $\mathcal{D}_{\mu\nu} = \eta_{\mu\nu} - \frac{1}{m^2}\partial_\mu\partial_\nu$, the equation of motion for $h_{\alpha\beta}$ is

$$\left(\Box - m^2\right) h_{\mu\nu} = \frac{1}{M_P^2}\left[\mathcal{D}_{\mu(\alpha}\mathcal{D}_{\beta)\nu} - \frac{1}{3}\mathcal{D}_{\mu\nu}\mathcal{D}_{\alpha\beta}\right] T^{\alpha\beta}$$

so that the massive graviton propagator is

$$G^{(\mathrm{mass})}_{\mu\nu\alpha\beta} = \frac{F^{(\mathrm{mass})}_{\mu\nu\alpha\beta}}{\Box - m^2}\,, \qquad F^{(\mathrm{mass})}_{\mu\nu\alpha\beta} = \mathcal{D}_{\mu(\alpha}\mathcal{D}_{\beta)\nu} - \frac{1}{3}\mathcal{D}_{\mu\nu}\mathcal{D}_{\alpha\beta}\,.$$

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- The amplitude for graviton exchange between two sources becomes

$$\mathcal{A}^{(\mathrm{mass})}_{T,T'} = \int d^4x T'^{\,\mu\nu} G^{(\mathrm{mass})}_{\mu\nu\alpha\beta} T^{\alpha\beta}\,.$$

- In the limit $m \to 0$ this tends to

$$\mathcal{A}^{(m\to 0)}_{T,T'} = \int d^4x T'^{\,\mu\nu}\frac{1}{\Box}\left(T_{\mu\nu} - \frac{1}{3}T\eta_{\mu\nu}\right) \neq \mathcal{A}^{(m=0)}_{T,T'} = \int d^4x T'^{\,\mu\nu}\frac{1}{\Box}\left(T_{\mu\nu} - \frac{1}{2}T\eta_{\mu\nu}\right)\,.$$

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- This problem is solved by the Vainshtein mechanism (1972): In the presence of massive sources, the kinetic term of the scalar graviton mode gets strongly enhanced so that it cannot be excited for
$r < r_* = M/(4\pi M_P^2 m^2))^{1/3} = (\lambda_m^2 r_s)^{1/3} \simeq 100$pc for $M = M_\odot$.

De Rham-Gabadaze-Tolley (dRGT) ghost free massive gravity

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- The proof of this statement has in the meantime been given in several different ways. Note that even if there are only 5 degrees of freedom left, it is not clear that these are 'healthy' in all physically relevant situations. In the following I shall show that especially in cosmology they are usually not.

De Rham-Gabadaze-Tolley (dRGT) ghost free massive gravity

The general potential is given by

$$
\begin{aligned}
U_0(\mathcal{K}) &= \epsilon^{\mu\nu\alpha\beta}\epsilon_{\mu\nu\alpha\beta} = 4! \\
U_1(\mathcal{K}) &= \epsilon^{\mu\nu\alpha\beta}\epsilon_{\mu'\nu\alpha\beta}\mathcal{K}^{\mu'}{}_{\mu} = 3![\mathcal{K}]\,, \\
U_2(\mathcal{K}) &= \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\epsilon_{\mu'\nu'\alpha\beta}\mathcal{K}^{\mu'}{}_{\mu}\mathcal{K}^{\nu'}{}_{\nu} = \frac{1}{2}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right)\,, \\
U_3(\mathcal{K}) &= \frac{1}{6}\epsilon^{\mu\nu\alpha\beta}\epsilon_{\mu'\nu'\alpha'\beta}\mathcal{K}^{\mu'}{}_{\mu}\mathcal{K}^{\nu'}{}_{\nu}\mathcal{K}^{\alpha'}{}_{\alpha} = \frac{1}{6}\left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right)\,, \\
U_4(\mathcal{K}) &= \frac{1}{24}\epsilon^{\mu\nu\alpha\beta}\epsilon_{\mu'\nu'\alpha'\beta'}\mathcal{K}^{\mu'}{}_{\mu}\mathcal{K}^{\nu'}{}_{\nu}\mathcal{K}^{\alpha'}{}_{\alpha}\mathcal{K}^{\beta'}{}_{\beta} \\
&= \frac{1}{24}\left([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]\right) = \det(\mathcal{K})\,.
\end{aligned}
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where $\mathcal{K} = \mathbb{I} - \sqrt{g^{-1}f}$ and $[\mathcal{M}] = \mathrm{Tr}\mathcal{M}$.

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where $\mathcal{K} = \mathbb{1} - \sqrt{g^{-1}f}$ and $[\mathcal{M}] = \mathrm{Tr}\mathcal{M}$.
U_n is the n-th order term appearing in the characteristic polynomial of $\mathcal{K}$,

$$\det(\lambda\mathbb{1} - \mathcal{K}) = \sum_{n=0}^{4}\frac{\lambda^n}{n!}U_{4-n}(\mathcal{K})$$

and the potential for g is given by

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This somewhat unwieldy non-analytic potential for $g_{\mu\nu}$ becomes much simpler if given in terms of vier-beins for g and f.
The vier-beins are normalized 1-forms such that

$$g_{\mu\nu}dx^\mu dx^\nu = \eta_{\alpha\beta}\theta^\alpha\theta^\beta \text{ and } f_{\mu\nu}dx^\mu dx^\nu = \eta_{\alpha\beta}\vartheta^\alpha\vartheta^\beta$$

With these

$$\begin{aligned}\sqrt{-g}U(\mathcal{K})d^4x &= -\frac{m^2}{4}\epsilon_{\mu\nu\alpha\beta}\Big[c_0\theta^\mu\wedge\theta^\nu\wedge\theta^\alpha\wedge\theta^\beta + c_1\theta^\mu\wedge\theta^\nu\wedge\theta^\alpha\wedge\vartheta^\beta \\ &+\frac{c_2}{2}\theta^\mu\wedge\theta^\nu\wedge\vartheta^\alpha\wedge\vartheta^\beta + \frac{c_3}{6}\theta^\mu\wedge\vartheta^\nu\wedge\vartheta^\alpha\wedge\vartheta^\beta + \frac{c_4}{24}\vartheta^\mu\wedge\vartheta^\nu\wedge\vartheta^\alpha\wedge\vartheta^\beta\Big]\end{aligned}$$

The term $\propto c_0$ is simply a cosmological constant and the term $\propto c_1$ is a tadpole which we shall neglect. The constant c_2 can be absorbed in the definition of m where as c_3 and c_4 are genuinely new, giving rise to new physical phenomena.

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We shall also use that

$$\begin{aligned}U &= -\frac{m^2}{2}\left(U_2(\mathcal{K}) + c_3U_3(\mathcal{K}) + c_4U_4(\mathcal{K})\right) \\ &= -m^2\left[a_0 + a_1U_1(\sqrt{g^{-1}f}) + a_2U_2(\sqrt{g^{-1}f}) + a_3U_3(\sqrt{g^{-1}f})\right]\end{aligned}$$

with

$$a_0 = 6 + 4c_3 + c_4, \qquad a_1 = -(3 + 3c_3 + c_4)$$
$$a_2 = 1 + 2c_3 + c_4, \qquad a_3 = -c_3 - c_4.$$

Fluctuations

- To study solutions of a given theory, we need not only to find them, but we also have to analyse their stability. Even if a theory of modified gravity has no ghosts a priori, we need to check whether physically interesting solutions of GR are still viable, i.e. stable. To this aim we have studied the perturbations of massive gravity around a fixed background metric $\bar{g}$. We set $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ and write the Langrangian to 2nd order in $h_{\mu\nu}$.

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- The kinetic term $\sqrt{-g}R$ is expanded as usual,

$$\sqrt{-g}R = \sqrt{-\bar{g}}\bar{R} + h_{\mu\nu}\mathcal{E}^{\mu\nu\alpha\beta}(\bar{g})h_{\alpha\beta} - 2h_{\mu\nu}\bar{G}^{\mu\nu} + \partial_\mu V^\mu$$

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- The mass term is more tricky. Since for non-commuting matrices $\sqrt{AB} \neq \sqrt{A}\sqrt{B}$, we cannot simply expand $\sqrt{g^{-1}f} = \sqrt{(\mathbb{I} + h)^{-1}\bar{g}^{-1}f}$ in $h \equiv (h^\mu{}_\nu) \equiv (\bar{g}^{\mu\alpha}h_{\alpha\nu})$. We use the following trick: Setting $t_i = U_i(\sqrt{g^{-1}f})$ and $s_i = U_i(g^{-1}f)$ one easily verifies the relations

$$t_1^2 = s_1 + 2t_2\,, \quad t_2^2 = s_2 - 2\sqrt{s_4} + 2t_1t_3\,, \quad t_3^2 = s_3 + 2t_2\sqrt{s_4}\,.$$

- Now we can expand the s_i and with them the t_i to second order in $h_{\mu\nu}$.

The mass term

After a lengthy calculation we end up with

$$
\begin{aligned}
\sqrt{-\det g}\,U(f,g) &= \sqrt{-\det \bar g}\left[U(f,\bar g) - 2\mathcal{M}^{\mu\nu}h_{\mu\nu} + \mathcal{M}^{\mu\nu\alpha\beta}(f,\bar g)h_{\mu\nu}h_{\alpha\beta}\right] + \mathcal{O}(h^3) \quad \mathrm{v} \\
\mathcal{M}^{\mu\nu\alpha\beta} &= -m^2\left[a_0\mathcal{M}_0^{\mu\nu\alpha\beta} + a_1\mathcal{M}_1^{\mu\nu\alpha\beta} + a_2\mathcal{M}_2^{\mu\nu\alpha\beta} + a_3\mathcal{M}_3^{\mu\nu\alpha\beta}\right], \\
\mathcal{M}_0^{\mu\nu\alpha\beta} &= \frac{1}{4}\bar g^{\mu\nu}\bar g^{\alpha\beta} - \frac{1}{4}\left(\bar g^{\mu\alpha}\bar g^{\nu\beta} + \bar g^{\mu\beta}\bar g^{\nu\alpha}\right) \\
\mathcal{M}_j^{\mu\nu\alpha\beta} &= \bar t_j\mathcal{M}_0^{\mu\nu\alpha\beta} + \frac{1}{2}\left(\bar g^{\mu\nu}t_j^{\alpha\beta} + \bar g^{\alpha\beta}t_j^{\mu\nu}\right) + 2t_j^{\mu\nu\alpha\beta}, \quad 1 \le j \le 3, \\
\mathcal{M}^{\mu\nu} &= -2m^2(a_1t_1^{\mu\nu} + a_2t_2^{\mu\nu} + a_3t_3^{\mu\nu}) \\
t_j^{\mu\nu} &= \left.\frac{\partial t_j}{\partial g_{\mu\nu}}\right|_{g=\bar g}, \qquad t_j^{\mu\nu\alpha\beta} = \frac{1}{2}\left.\frac{\partial^2 t_j}{\partial g_{\mu\nu}\partial g_{\alpha\beta}}\right|_{g=\bar g}.
\end{aligned}
$$

with very cumbersome expressions for the $t_j^{\mu\nu}$ and even more so for $t_j^{\mu\nu\alpha\beta}$ (see P. Guarato and RD arXiv:1309.2245).

Application to cosmology: same conformal time

Let us assume that f is a Friedmann-Lemaître geometry and we have found a cosmological solutionb $\bar{g}$ with the same conformal time,

$$\begin{aligned} \bar{g}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu &= a^2(t)(-\mathrm{d}t^2 + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j), \\ f_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu &= b^2(t)(-\mathrm{d}t^2 + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j). \end{aligned}$$

In this case the mass term becomes

$$\begin{aligned} \mathcal{M}^{\mu\nu\alpha\beta}(f,\bar{g}) &= -m^2\left[\alpha\bar{g}^{\mu\nu}\bar{g}^{\alpha\beta} + \frac{\beta}{2}\left(\bar{g}^{\mu\alpha}\bar{g}^{\nu\beta} + \bar{g}^{\mu\beta}\bar{g}^{\nu\alpha}\right)\right]. \\ \alpha(t) &= \frac{1}{4}\left[1 + (1-r)\left\{(5-r) + c_3(4-2r) + c_4(1-r)\right\}\right], \quad r = \frac{b}{a} \\ -\beta(t) &= \frac{1}{4}\left[1 + (1-r)\left\{(11-4r) + c_3\left(8-7r+r^2\right) + c_4(1-r)(2-r)\right\}\right] \end{aligned}$$

In the cosmological situation α and β depend only on time, but the expressions below in terms of $r(t) = b(t)/a(t)$ are always correct when the two metrics $\bar{g}$ and f are conformally related by $f = r^2\bar{g}$.
For $\alpha \neq -\beta$ this metric has a ghost with mass (Jaccard, Maggiore & Mitsou)

$$m^2_{\text{ghost}} = \frac{(\alpha + 4\beta)}{2(\alpha+\beta)}m^2 .$$

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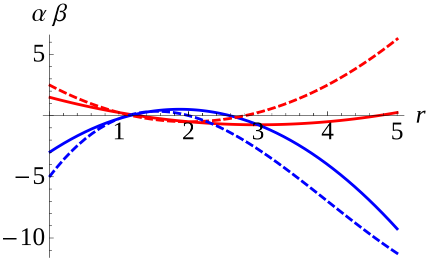
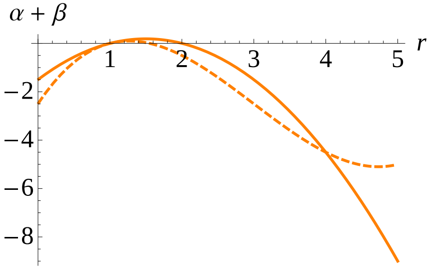




The functions $\alpha(r)$ (red) and $\beta(r)$ (blue) for two cases:
$c_3 = c_4 = 0$ (solid) and $c_3 = 1\,,\ c_4 = 0$ (dashed)
(from P. Guarato & RD arXiv:1309.2245).

In general the physical metric will not have the same conformal time as the reference metric f and we expect

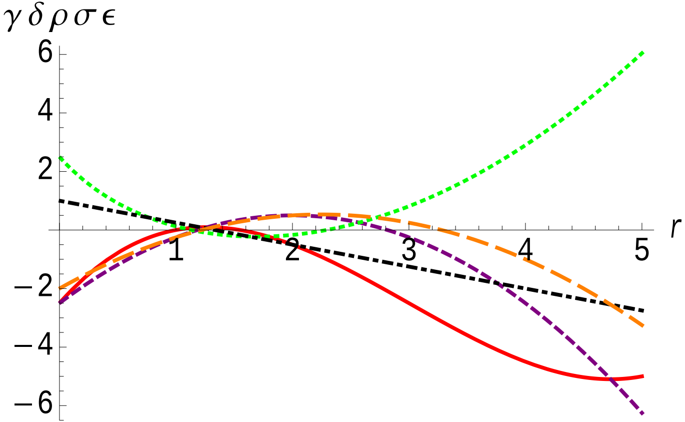
$$\begin{aligned} f_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu &= -\mathrm{d}\tau^2 + b^2(\tau)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j\,, \\ \bar{g}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu &= -N^2(\tau)\mathrm{d}\tau^2 + a^2(\tau)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j\,. \end{aligned}$$

with some unknown lapse function N. In this case the mass term is of the more complicated form

$$\begin{aligned} \mathcal{M}^{0000} &= -m^2\gamma(\tau), \qquad \mathcal{M}^{ij00} = -m^2\delta(\tau)\bar{g}^{ij}, \qquad \mathcal{M}^{i0j0} = -m^2\epsilon(\tau)\bar{g}^{ij}, \\ \mathcal{M}^{ijkl} &= -m^2\left\{\rho(\tau)\bar{g}^{ij}\bar{g}^{kl} + \frac{\sigma(\tau)}{2}\left[\bar{g}^{ik}\bar{g}^{jl} + \bar{g}^{il}\bar{g}^{jk}\right]\right\}. \end{aligned}$$

where δ, γ, ϵ, ρ, and σ are polynomials of r and N^{-1}.
The Fierz-Pauli tuning corresponds to $N = 1$ and $\gamma = 0$, $\rho = -\delta = -\sigma = 1/4$, $\epsilon = 1/8$.
It is reached when $N = 1$ and $a = b$.

Application to cosmology



The functions $\gamma(r)$ (red), $\delta(r)$ (purple), $\epsilon(r)$ (green), $\rho(r)$ (black), and $\sigma(r)$ (orange) for the case $c_3 = 1$, $c_4 = 0$ and $N = 1$.

Background cosmology

In the presence of a mass term Einstein's equation take the form

$$G_{\mu\nu} + \mathcal{M}_{\mu\nu} = M_P^{-2} T_{\mu\nu}.$$

The covariant conservation of $G_{\mu\nu}$ is a geometrical identity and the conservation of $T_{\mu\nu}$ is a consequence of the matter equation (we assume that these are not modified). Hence we also have ${\mathcal{M}^{\mu\nu}}_{;\nu} = 0$. These 4 additional equations are necessary since we have lost diffeomorphism invariance when $f_{\mu\nu}$ is fixed (One could re-install it with the Stückelberg trick which we are not doing here).

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The Friedmann equations now become

$$H^2 + \frac{1}{a^2 K_g} = \frac{1}{3}\left(\rho_{\mathcal{M}} + \rho\right), \qquad 2\dot{H} + 3H^2 + \frac{1}{a^2 K_g} = \left(P_{\mathcal{M}} + P\right).$$

where $H = \dot{a}/(Na)$, $\quad r = b/a \quad$ and

$$\begin{aligned} -\rho_{\mathcal{M}} &= N^{-2}\Big[r^3\left(c_3 + c_4\right) - r^2\left(6c_3 + 3c_4 + 3\right) + r\left(9c_3 + 3c_4 + 9\right) - 4c_3 - c_4 - 6\Big], \\ P_{\mathcal{M}} &= 6 + 4c_3 + c_4 + \left(2c_3 + c_4 + 1\right) r^2 - 2\left(3c_3 + c_4 + 3\right) r - \\ & \quad N^{-1}\Big[\left(c_3 + c_4\right) r^2 + 3c_3 + c_4 + 3 - 2\left(2c_3 + c_4 + 1\right) r\Big]. \end{aligned}$$

We interpret $\mathcal{M}_{\mu\nu}$ as a gravity mass fluid. Its covariant conservation implies

$$\left[3c_4(r-1)^2 + 3c_3(r-3)(r-1) + 9 - 6r\right]\left(N^{-1}\dot{a} - \dot{b}\right) = 0$$

Hence either $[\cdots] = 0$ which implies $r = b/a =$constant or $N = \dot{b}/\dot{a}$, so that $Ha = \dot{b} = H_f b, \quad H_f =\equiv \dot{b}/b$. In the first case the evolution of the physical scale factor is fixed by the reference metric b and only N depends on the matter content.

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Hence the Friedmann equations become algebraic relations which relate a and N to b and the matter content of the Universe.

Fluctuations

We have studied the scalar sector of fluctuations in detail for the special solution $\rho = P = 0, \quad N = 1$ and

$$r = \frac{3 + c_3}{1 + c_3} = r_c \, .$$

It is easy to verify that this is the only solution with $r =$constant and $N = 1$ which solves all background equations (apart from the trivial case $r = 1$).
We define the most general scalar perturbation Fourier mode by

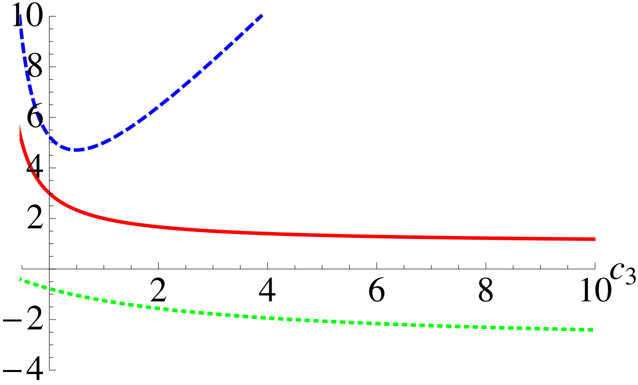
$$h_{\mu\nu}(\mathbf{k}) \equiv \delta g_{\mu\nu} = \begin{pmatrix} -2\phi & iak_jB \\ iak_iB & 2a^2(\psi\delta_{ij} - k_ik_jE) \end{pmatrix} \, .$$

We can eliminate ϕ and B from the perturbation equations using the constraints. For ψ and $\mathcal{E} = m^2 E$ we then obtain coupled linear equations of motion,

$$\frac{d^2}{d\tau^2} \begin{pmatrix} \psi \\ \mathcal{E} \end{pmatrix} = \left(m^2 A_0 + k^2 A_2\right) \begin{pmatrix} \psi \\ \mathcal{E} \end{pmatrix} \, ,$$

where $A_0(c_3, c_4)$ and $A_2(c_3, c_4)$ are 2×2 matrices given by rational functions of c_3 and c_4. A_2 has one vanishing eigenvalue, one of the two scalar modes does not propagate.

Fluctuations



The eigenvectors λ_{01} (red), λ_{02} (blue) of A_0 and λ_{22} (green) of A_2 as functions of $-0.5 < c_3 < 10$ for the case $c_4 = 0$.

A positive eigenvalue indicates an instability.

Conclusions

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- One possibility to evade these conclusions is to consider bi-gravity i.e. add a kinetic term also for the f metric. This re-installs diffeomorphism invariance and removes the Higuchi ghost.
- Whether it also cures the kind of instabilities discussed here is still unclear.