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Natural inflation models in string-inspired supergravity

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In collaboration with

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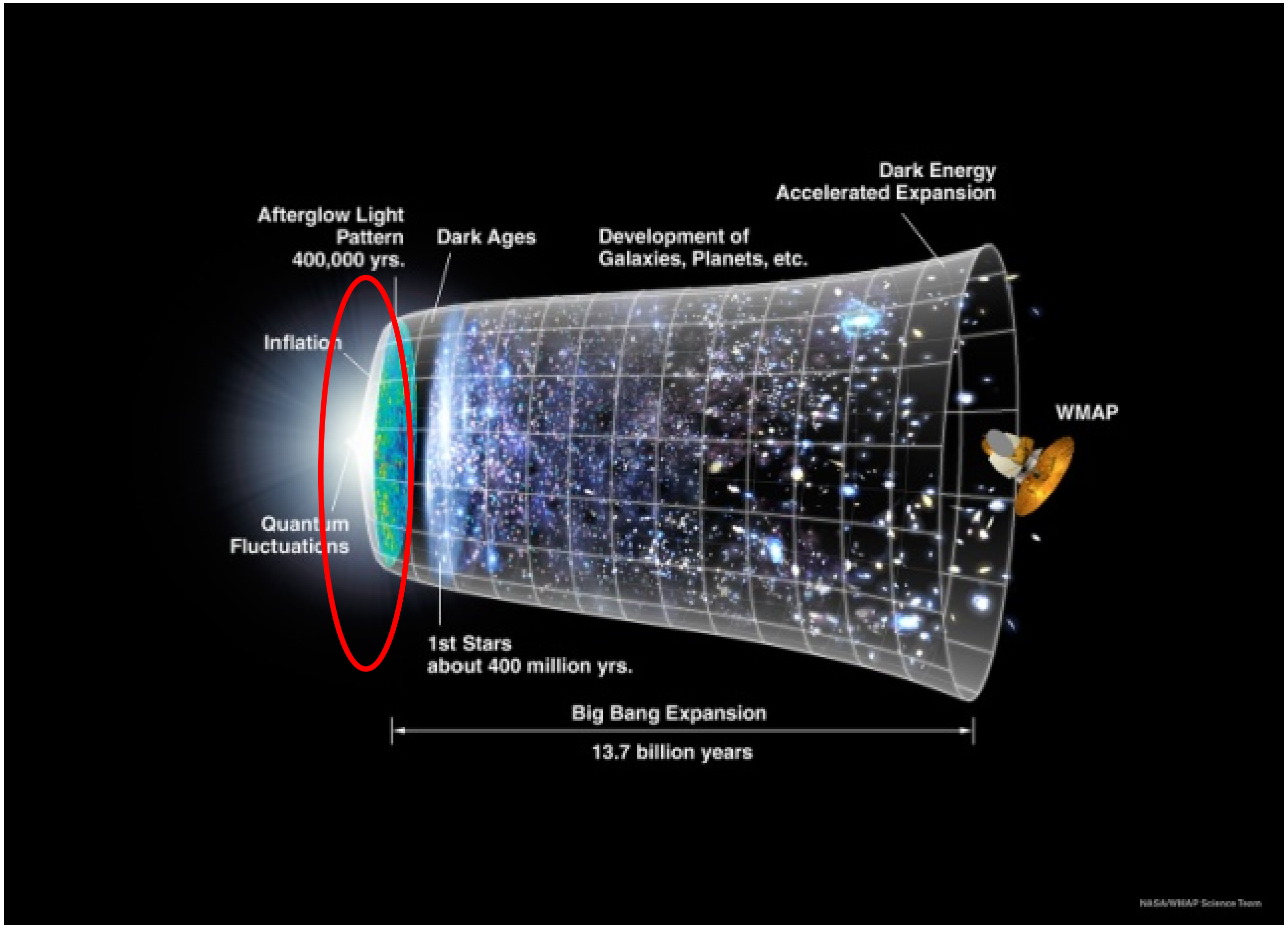
1403.0410, 1403.5883 and 1404.6923

Inflation

Accelerating expansion of the universe

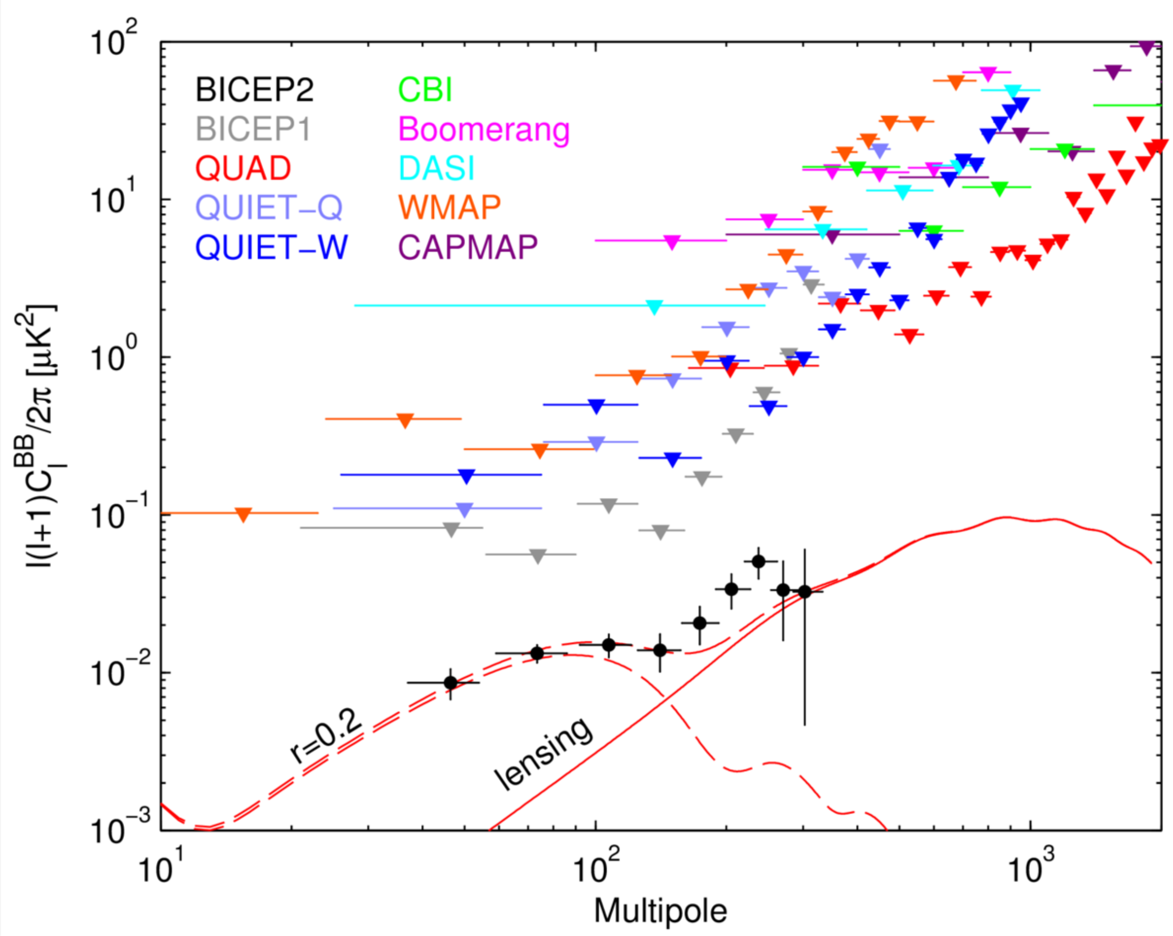


NASA

Primordial gravitational wave!?

$$r_{\text{BICEP2}} \simeq 0.2^{+0.07}_{-0.05}.$$

$$r \simeq \frac{(\Delta T/T)^2_{\text{tensor}}}{(\Delta T/T)^2_{\text{scalar}}}$$



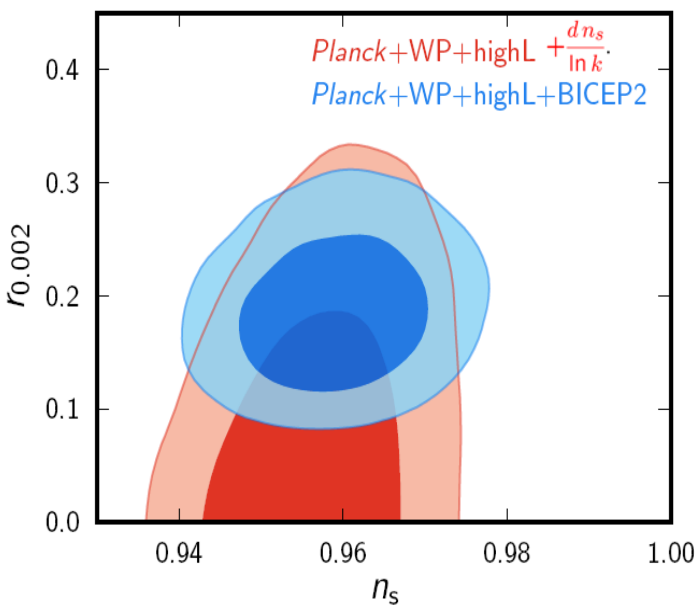
BICEP2 collaboration
1403.3985

Tension; needs more data

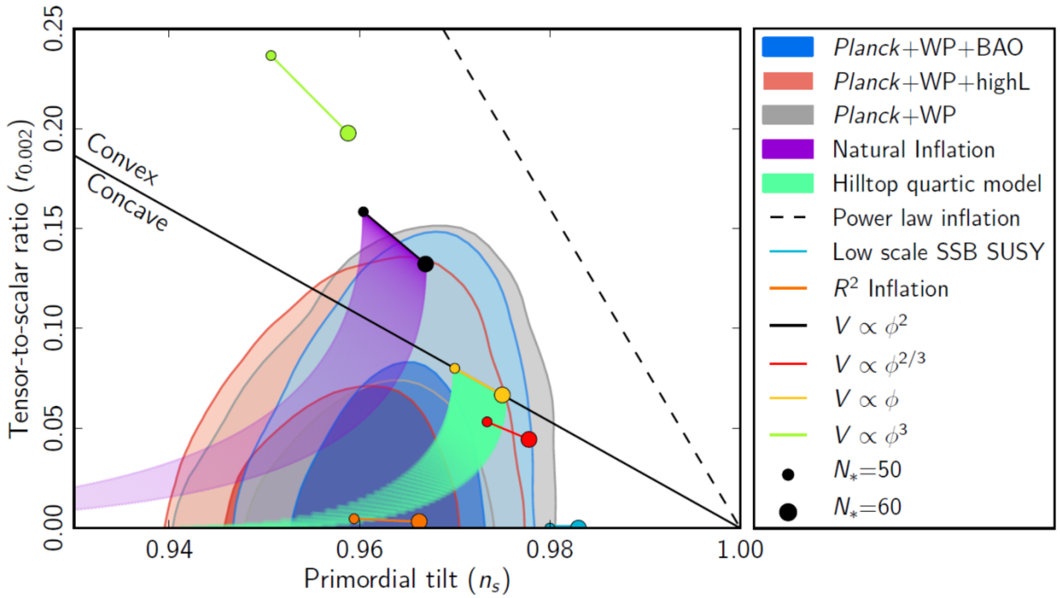
BICEP2: r ~ 0.2;
dust?

Mortonson et al; Flauger et al.

Planck: r < 0.11



BICEP2 collaboration



Planck collaboration

Motivation: Testing string theory

Future tensor mode confirmation = very high energy

$$V_{\mathrm{inf}} \simeq (2.0 \times 10^{16}\ \mathrm{GeV})^4 \cdot \left(\frac{r}{0.16}\right)$$

$$\Delta\phi > M_{\mathrm{Pl}}$$

Lyth bound

$$M_{\mathrm{string}} \sim 10^{17}\ \mathrm{GeV}$$

Inflaton: Ubiquitous heavy axion ϕ

Summary

- Natural inflation: Large tensor mode generated

$$V(\phi) = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right) \right]$$: Under good control and $f > M_{Pl}$

- String natural inflation: Many brane charges needed
  - Large decay constant:

$$f \sim \frac{N_{\text{brane}}}{\Delta} M_{\text{string}}$$

N_{brane}: # of D-branes, $\Delta < 1$: Alignment of winding/fluxes.

  - Severe tadpole condition?

Content

1. ~~Motivation~~
2. Observation and inflation
3. Axion inflation: (Multi-)natural inflation
4. Multi-natural inflation in supergravity
5. String/supergravity
6. Summary

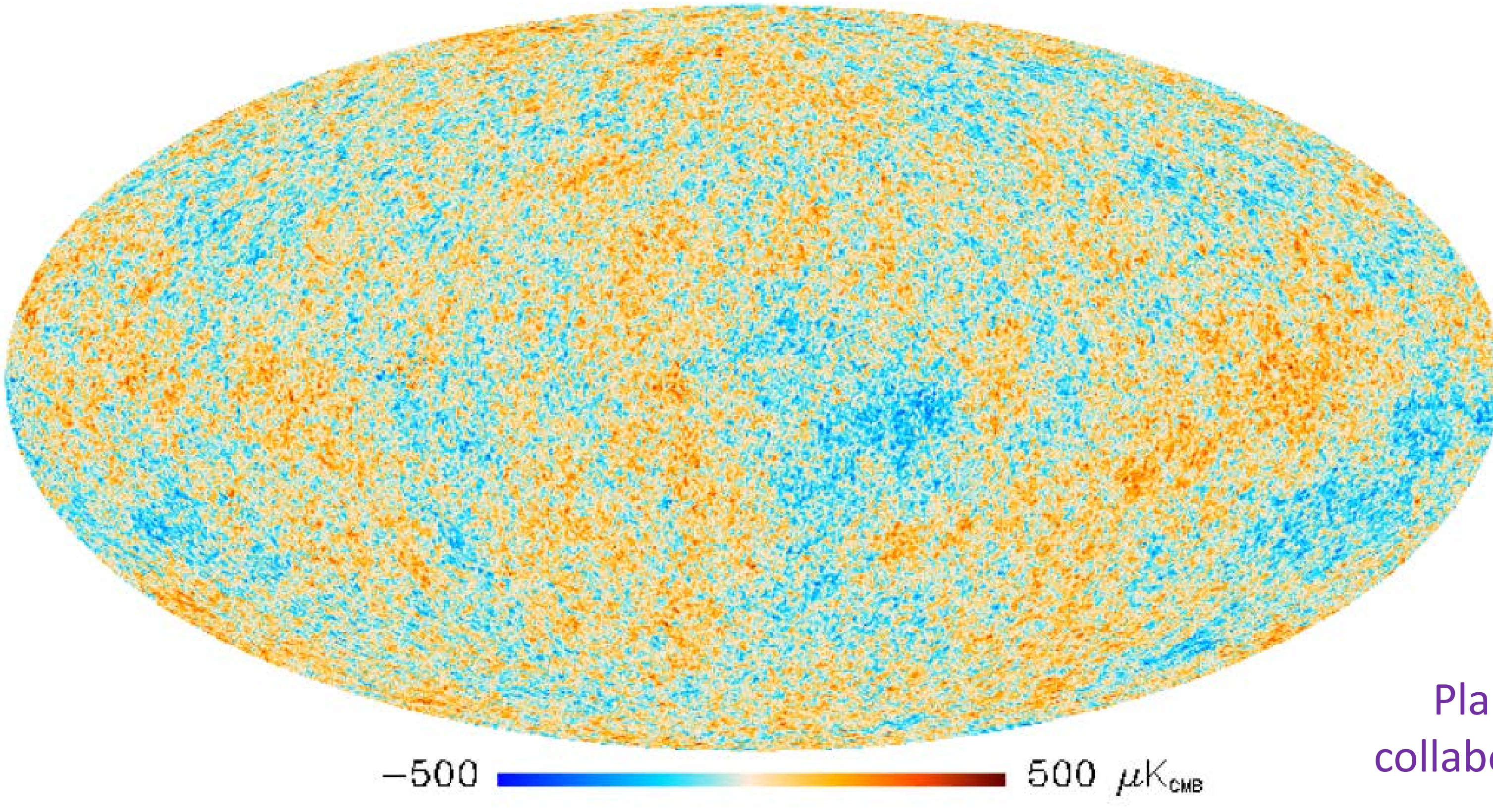
2. Observations and inflation

$M_{Pl} = 2.4 \times 10^{18}$ GeV = 1 will be used.

Cosmic Microwave Background

CMB fluctuation: $\Delta T/T \sim 10^{-5}$

where $T \sim 2.7$ K



Planck collaboration

Why inflation?

Inflation = Accelerating expansion of the universe

- Generating density fluctuations in CMB

  = seeds of galaxies

- Gratitational wave

- Solutions for fine-tuning problems
  - Flatness problem: $\Omega_{curvature} << 1$
  - Horizon problem : T ～2.7K in CMB

Inflation driven by an inflaton ϕ

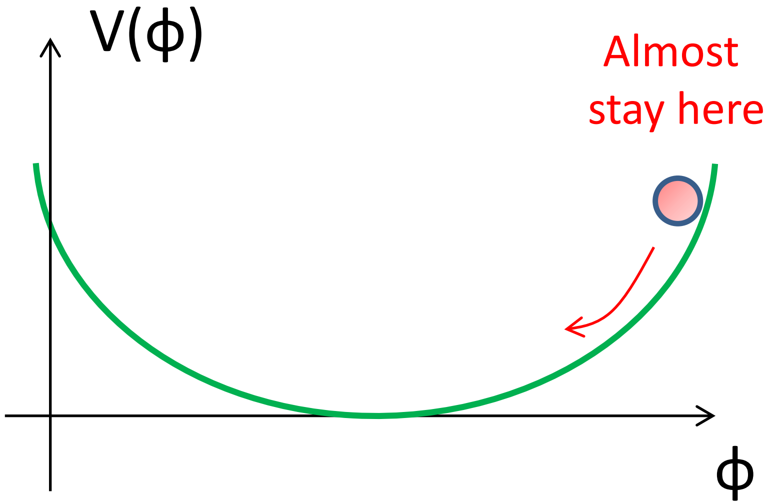
- EOM

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V = 0$$

- Friedman Eq.

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V\right)$$

- Slow-roll conditions

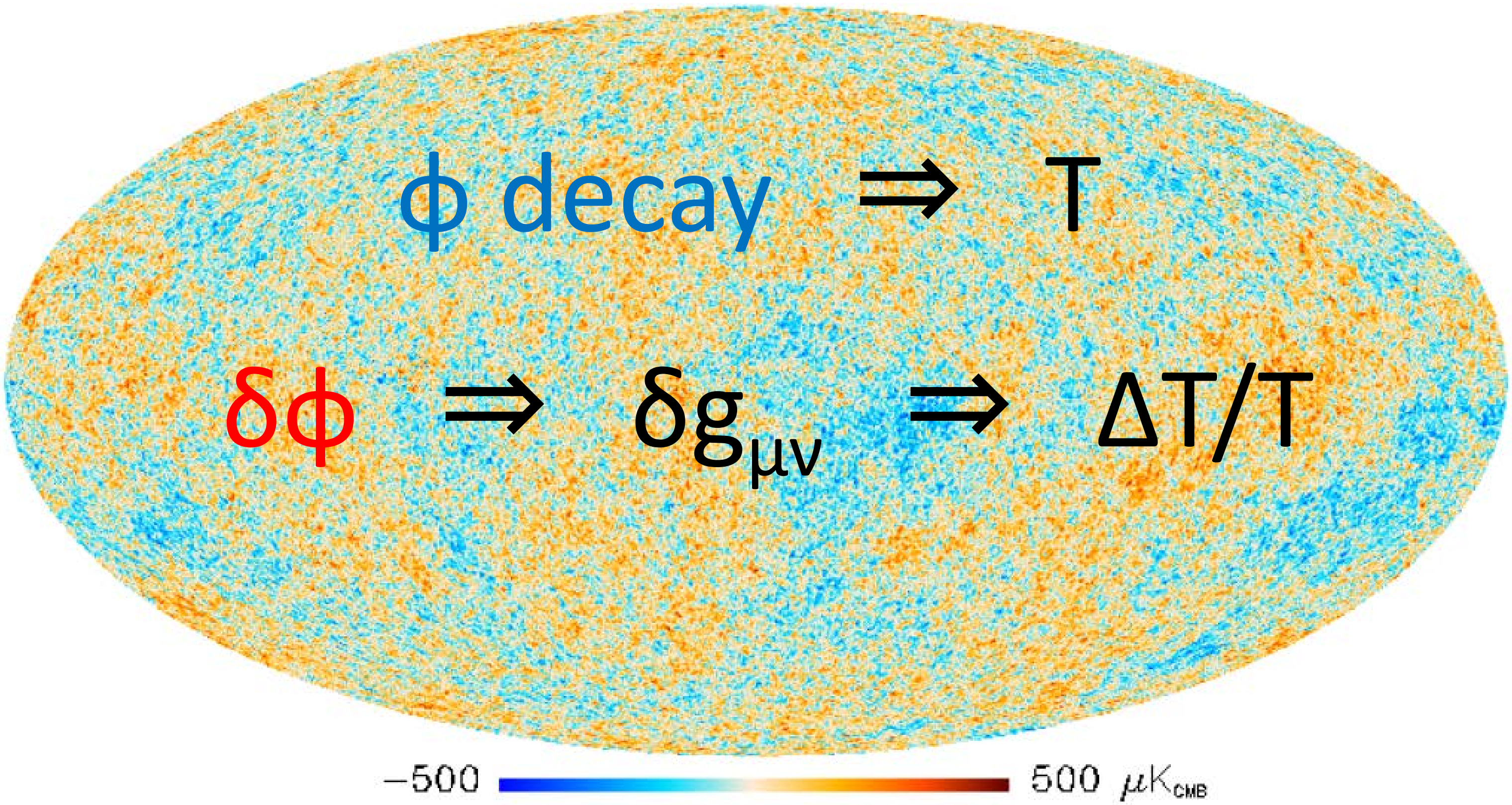


$$\frac{\dot{H}}{H^2} \ll 1, \quad \ddot{\phi} \ll H\dot{\phi} \quad \Rightarrow \quad \boxed{\epsilon = \frac{1}{2}\left(\frac{\partial_\phi V}{V}\right)^2 \ll 1, \quad \eta = \frac{\partial_\phi^2 V}{V} \ll 1}$$

Inflation!! $\boxed{a \propto e^{Ht}}$

a(t): Scale factor
H: Hubble parameter

CMB fluctuation generated by $\delta\phi$



$\delta\phi$: inflaton fluctuation

Metric perturbation

- Metric perturbations:

$$ds^2 = -dt^2 + a(t)^2 e^{\zeta(t,\vec{x})}[\delta_{ij} + h_{ij}(t,\vec{x})]dx^i dx^j$$

ζ: Scalar perturbation $\zeta \leftrightarrow \delta\phi \frac{H}{\dot{\phi}}$

h: Tensor perturbation (Gravitational wave)

→ B-mode polarization in CMB photon

Power spectrum and r

- Power spectrum $(\Delta T/T)^2$ from inflation:

$$P_\zeta = \frac{V_{\mathrm{inf}}}{24\pi^2\epsilon}\left(\frac{k}{k_0}\right)^{n_s-1}, \qquad P_h = \frac{2V_{\mathrm{inf}}}{3\pi^2}\left(\frac{k}{k_0}\right)^{n_t}$$

$k_0 = 0.002\mathrm{Mpc}^{-1}$

- Tensor to scalar ratio:

$$r = \frac{P_h}{P_\zeta} \simeq 16\epsilon.$$

Observations and potential shape

BICEP2;
Planck collaboration

$$r_{\text{BICEP2}} \simeq 16\epsilon \simeq 0.2.$$

$$P_\zeta \simeq \frac{V_{\text{inf}}}{24\pi^2\epsilon} \simeq 2.5 \times 10^{-9}$$

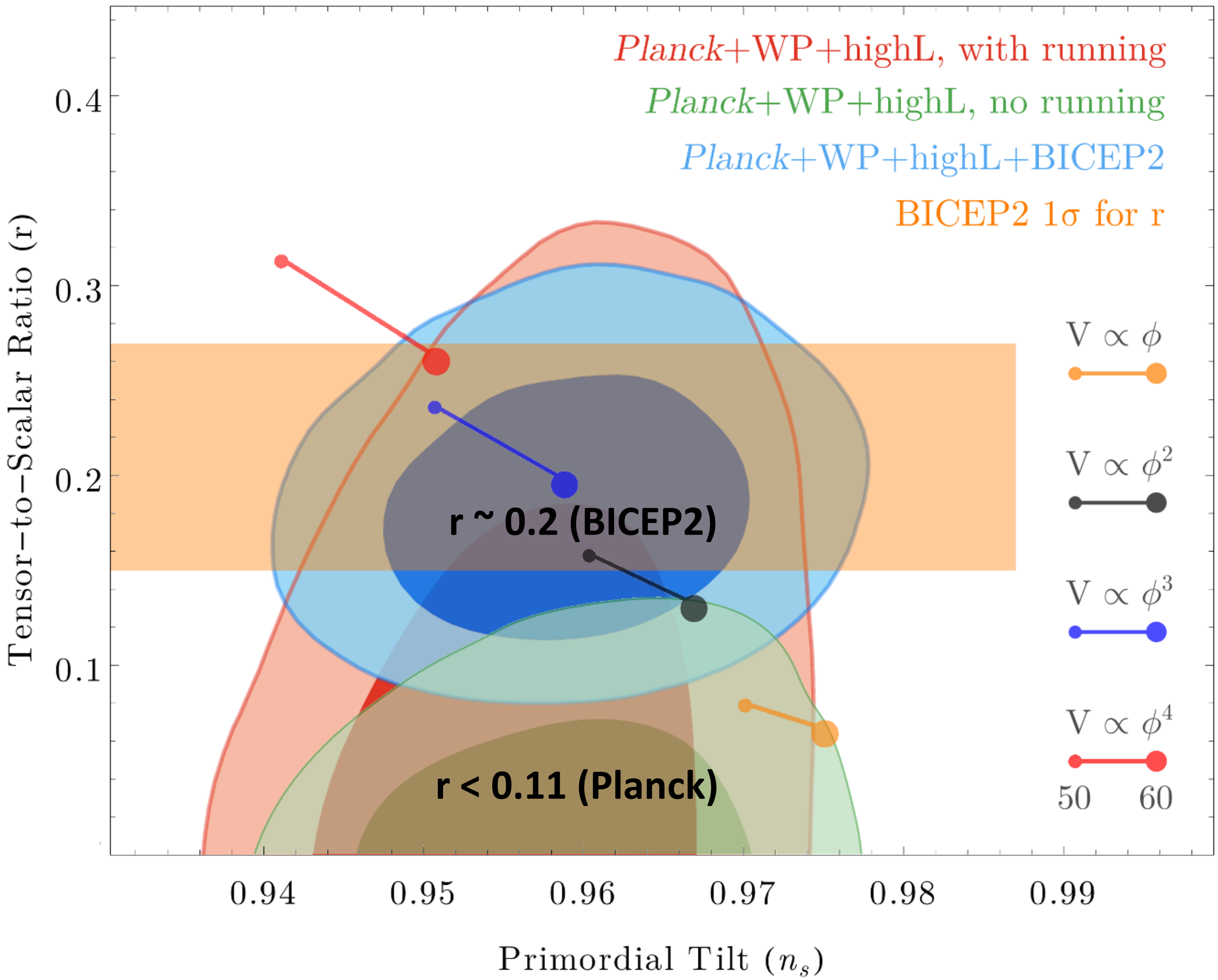
$$n_s \simeq 1 - 6\epsilon + 2\eta \simeq 0.96.$$

$$k_0 = 0.002\text{Mpc}^{-1}$$

$$\boxed{P_\zeta = \frac{V_{\text{inf}}}{24\pi^2\epsilon}\left(\frac{k}{k_0}\right)^{n_s-1} ; \ \epsilon = \frac{1}{2}\left(\frac{\partial_\phi V}{V}\right)^2 \ll 1, \quad \eta = \frac{\partial_\phi^2 V}{V} \ll 1}$$

n_s-r contour

1403.4584
De Raedt et al



High scale and long excursion

- Energy/Hubble scale:

$$V_{\rm inf} \simeq (2.0 \times 10^{16}\ {\rm GeV})^4 \cdot \left(\frac{r}{0.16}\right)$$

$$H_{\rm inf} \simeq (1.0 \times 10^{14}\ {\rm GeV}) \cdot \left(\frac{r}{0.16}\right)^{1/2}$$

$3H_{\rm inf}^2 = V_{\rm inf}$

- Distance during the inflation in field space

Lyth

$$\frac{\Delta\phi}{M_{\rm Pl}} \gtrsim 7.1 \cdot \left(\frac{r}{0.16}\right)^{1/2} \left(\frac{N_e}{50}\right).$$

N_e: e-folding number

$$N_e = \int H dt \simeq \int \frac{d\phi}{\sqrt{r/8}}.$$

3. Axion inflation:

(Multi-)natural inflation

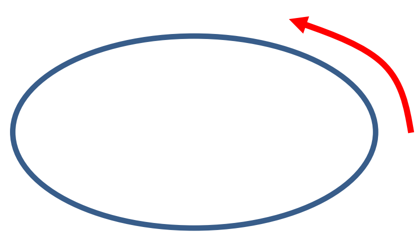
Inflation with an axion ϕ

Shift symmetry controls theory

$$\phi \rightarrow \phi + C$$

Almost flat potential over super-Planckian for ϕ

ϕ-space



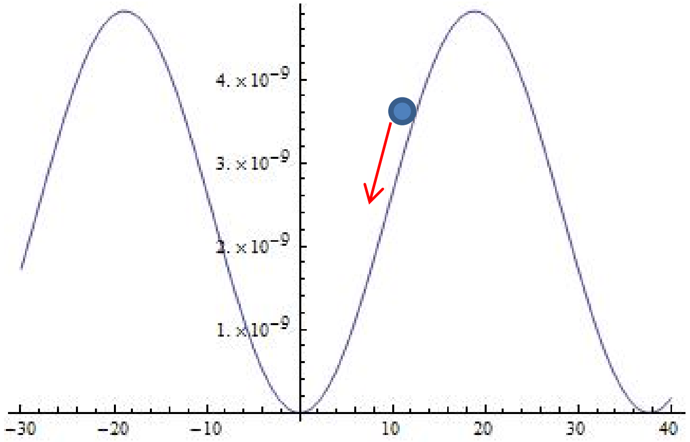
C

Cf: Pion as PNG boson

Natural inflation: Large field inflation

Freese et al

$$V_{\mathsf{natural}}(\phi) = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f} \right) \right].$$

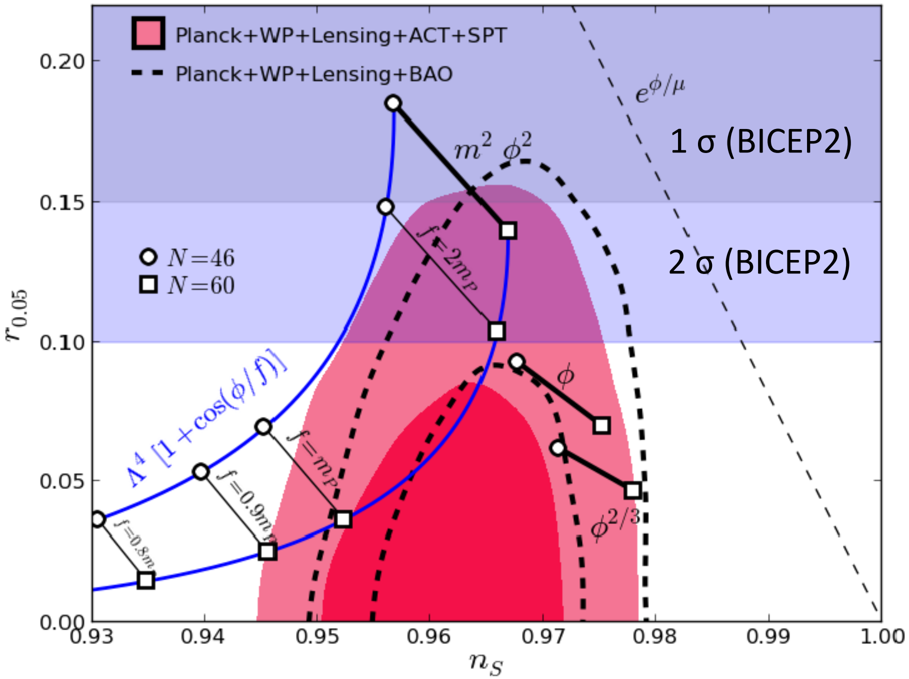


- Small shift-violation $\Lambda << M_{Pl}$
  - will always exist via a non-perturbative effect $\frac{\phi}{f_\phi}\langle F_{\mu\nu}\tilde{F}^{\mu\nu}\rangle$

- Slow roll for $f > M_{Pl}$: $\epsilon \sim \eta \sim \left(\frac{M_{\mathsf{Pl}}}{f}\right)^2 \ll 1.$

Natural inflation and observation

- Parameters: $f \gtrsim 5M_{\mathrm{Pl}}$, $\Lambda \sim 10^{16}\,\mathrm{GeV}$ (BICEP2)



- Potential:

$$V(\phi) = \Lambda^4\left[1 - \cos\left(\frac{\phi}{f}\right)\right]$$

- Inflaton mass:

$$m_\phi \sim \frac{\Lambda^2}{f} \sim 10^{13}\,\mathrm{GeV}$$

1403.5277: Freese et al

$m_{\mathrm{Pl}} \simeq 1.2\times 10^{19}\ \mathrm{GeV} \simeq 5M_{\mathrm{Pl}}.$

Multi-Natural inflation

1401.5212 : Czerny, Takahashi

- Multi-corrections can exist:

$$V = v_0 - \Lambda_1^4 \cos\left(\frac{\phi}{f_1}\right) - \Lambda_2^4 \cos\left(\frac{\phi}{f_2} + \theta\right) + \cdots .$$

Multi-Natural inflation

1401.5212 : Czerny, Takahashi

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$$V = v_0 - \Lambda_1^4 \cos\left(\frac{\phi}{f_1}\right) - \Lambda_2^4 \cos\left(\frac{\phi}{f_2} + \theta\right) + \cdots .$$

$$\uparrow$$

$$\frac{\phi}{f_1'}\langle F_{\mu\nu}\tilde{F}^{\mu\nu}\rangle + \frac{\phi}{f_2'}\langle G_{\mu\nu}\tilde{G}^{\mu\nu}\rangle + \cdots$$

Multi-Natural inflation

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- Multi-corrections can exist:

$$V = v_0 - \Lambda_1^4 \cos\left(\frac{\phi}{f_1}\right) - \Lambda_2^4 \cos\left(\frac{\phi}{f_2} + \theta\right) + \cdots .$$

- Wide parameter space:

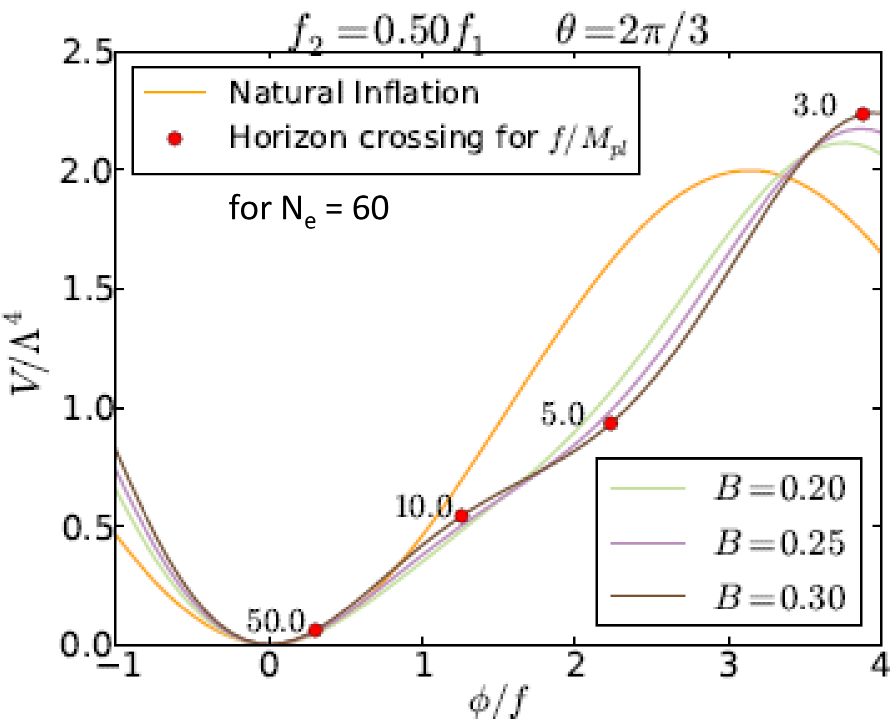
1403.0410, 1403.5883 :
Czerny, TH, Takahashi

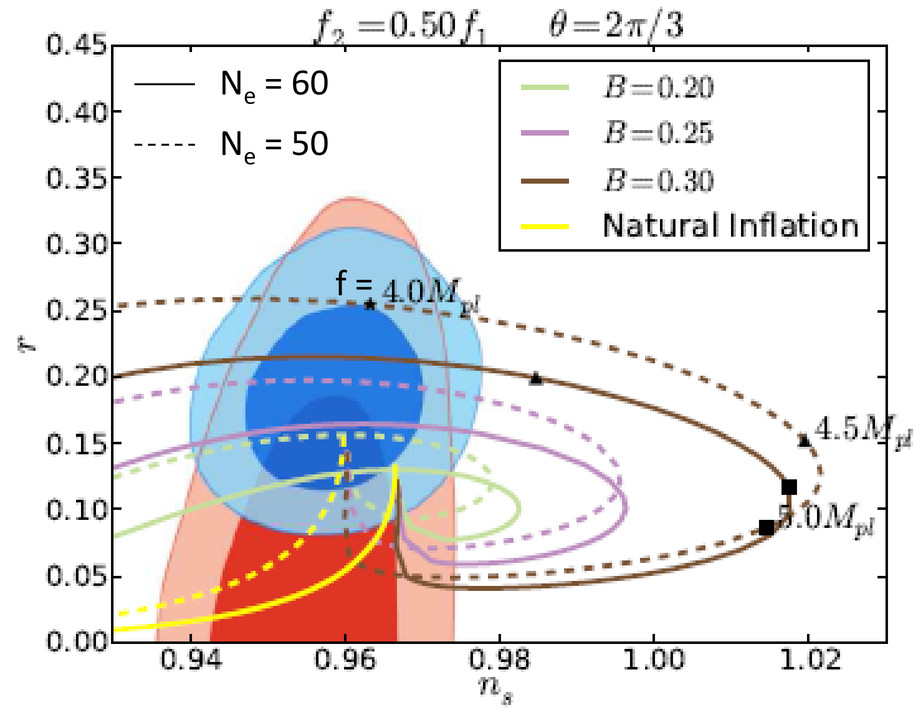
  – Large field inflation with r ∼0.1

1403.4589:
Czerny, Kobayashi, Takahashi

  – Running n_s with modulations
  – Also small field inflation with r << 0.1 for Planck.

1403.5883 : Czerny, TH, Takahashi





$$\boxed{f_2 = 0.5 f_1, \quad \Lambda_2^4 = B\Lambda_1^4, \quad \theta = \frac{2\pi}{3}}$$

$$V = v_0 - \Lambda_1^4 \cos\left(\frac{\phi}{f_1}\right) - \Lambda_2^4 \cos\left(\frac{\phi}{f_2} + \theta\right). \qquad m_\phi \sim \frac{\Lambda_1^2}{f_1} \sim \frac{\Lambda_2^2}{f_2} \sim 10^{13} \text{ GeV}.$$

Large decay constant for natural inflation

$$f > M_{\mathsf{Pl}}.$$

4. Multi-natural inflation in supergravity

1403.0410, 1403.5883 :
Czerny, TH, Takahashi

Why supergravity?

The theory controlled:

1. Toy model: Embedding inflation into string theory

2. Study of Non-perturbative effects

3. Stable; no tachyons

Note: **No relation to TeV scale SUSY!!**

Scalar potential in SUGRA

$$V = e^K[|DW|^2 - 3|W|^2].$$

$$|DW|^2 = K^{i\bar{j}}(D_iW)(\overline{D_jW}), \quad DW = (\partial K)W + \partial W, \quad K^{i\bar{j}} = (\partial_i\overline{\partial_j}K)^{-1}$$

K: Kähler potential, W: Superpotential

4d N=1 effective supergravity

- K: Symmetric under $\Phi \to \Phi + iC$

$$K = \frac{f^2}{2}(\Phi + \Phi^\dagger)^2, \qquad f \lesssim 1.$$

- W: Const. + gaugino condensations Λ^3

$$W = W_0 + Ae^{-a\Phi} + Be^{-b\Phi},$$

- Spontaneous SUSY-breaking:

$$\Delta V = 3e^{2K/3}|W_0|^2.$$

4d N=1 effective supergravity

- K: Symmetric under $\Phi \to \Phi + iC$

$$K = \frac{f^2}{2}(\Phi + \Phi^\dagger)^2, \qquad f \lesssim 1.$$

Canonical normalization:

$$\Phi = \sigma + i\phi; \qquad \sigma \to \frac{\sigma}{\sqrt{2}f}, \qquad \phi \to \frac{\phi}{\sqrt{2}f}.$$

$$f^2(\partial\sigma)^2 + f^2(\partial\phi)^2 \quad \to \quad \frac{1}{2}(\partial\sigma)^2 + \frac{1}{2}(\partial\phi)^2.$$

4d N=1 effective supergravity

- W: Const. + gaugino condensations Λ^3; N-vacua

$$W = W_0 + Ae^{-a\Phi} + Be^{-b\Phi},$$

$$a \sim b \sim 10^{-2}, \quad A \sim B \sim 10^{-(6-7)}, \quad W_0 \sim 10^{-3}.$$

Exponent = (1/decay constant) after canonical normalization:

$$W \supset W_0 + Ae^{-i\frac{a}{\sqrt{2}f}\phi} + Be^{-i\frac{b}{\sqrt{2}f}\phi},$$

4d N=1 effective supergravity

- Spontaneous SUSY-breaking:

$$\Delta V = 3e^{2K/3}|W_0|^2.$$

- Heavier saxion σ: $V_{\rm saxion}(\sigma) \simeq |W_0|^2\sigma^2;$ $\langle\sigma\rangle = 0.$
- SUSY-breaking in true vacuum: $W_0 \sim m_{3/2}.$
- Fine-tuning of CC; other type possible KKLT

Axion potential = Multi-natural

- Axion potential for $\sigma = 0$

$$V_{\text{axion}}(\phi) \simeq 6AW_0\left[1-\cos\left(\frac{\phi}{f_1}\right)\right] + 6BW_0\left[1-\cos\left(\frac{\phi}{f_2}+\theta\right)\right]$$

$$\theta = \text{Arg}(1/B)$$

$$AW_0 \sim (\text{dynamical scale})^3 \times (\text{SUSY breaking})$$

Large decay constant for small a or b:

$$f_1 \equiv \frac{\sqrt{2}f}{a}, \quad f_2 \equiv \frac{\sqrt{2}f}{b} \quad \gg f \text{ for } a^{-1} \sim b^{-1} \sim 10^2$$

Saxion σ decoupled from inflation

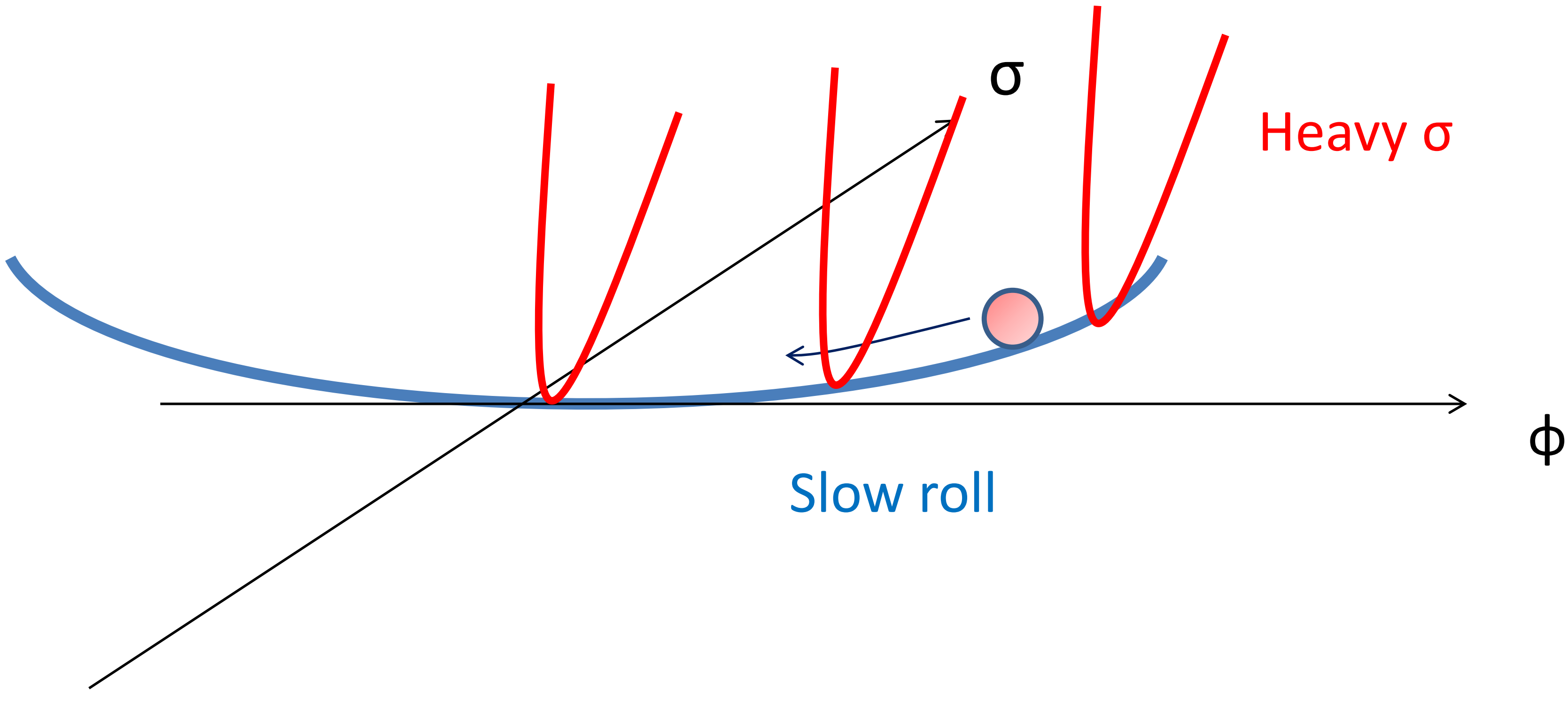
- Decoupling condition for successful inflation:

$$\boxed{AW_0 \ll W_0^2 \quad \Longrightarrow \quad H_{\mathrm{inf}} \ll m_\sigma}$$

$$m_\sigma \sim W_0, \qquad H_{\mathrm{inf}} \sim \sqrt{V_{\mathrm{inf}}} \sim \sqrt{AW_0};$$

Saxion σ decoupled from inflation

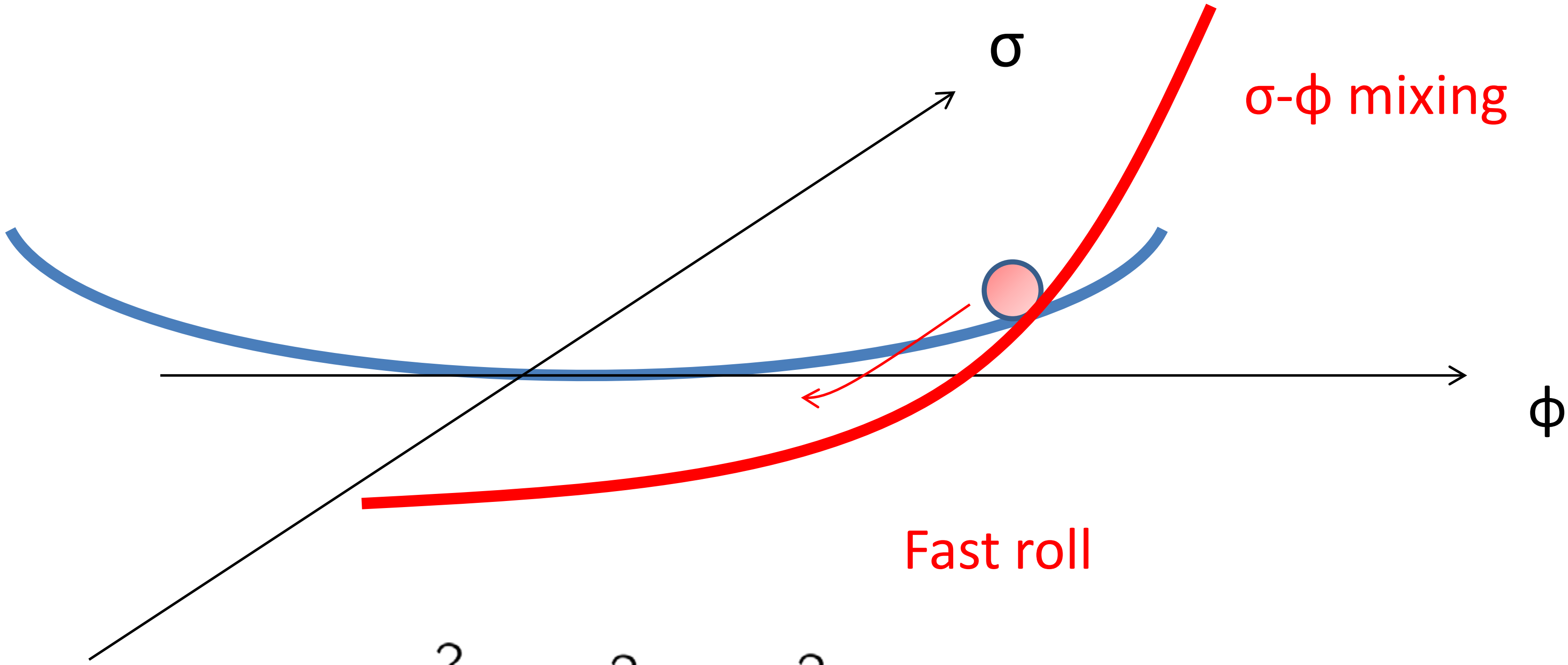
- Successful inflation



$$H_{\text{inf}} \ll m_\sigma; \quad A \ll W_0$$

Saxion σ decoupled from inflation

- Otherwise, no slow-roll inflation due to mixing



$$\eta \sim \frac{m_\phi^2}{V} \sim \frac{m_\sigma^2}{V} \sim \frac{W_0^2}{W_0^2} \sim 1 \quad \text{for} \quad A \sim W_0.$$

Two lessons

- Large decay constant = a << 1 in exponent

$$f_1 \propto \frac{1}{a} f.$$

$$W \sim e^{-a\Phi}$$

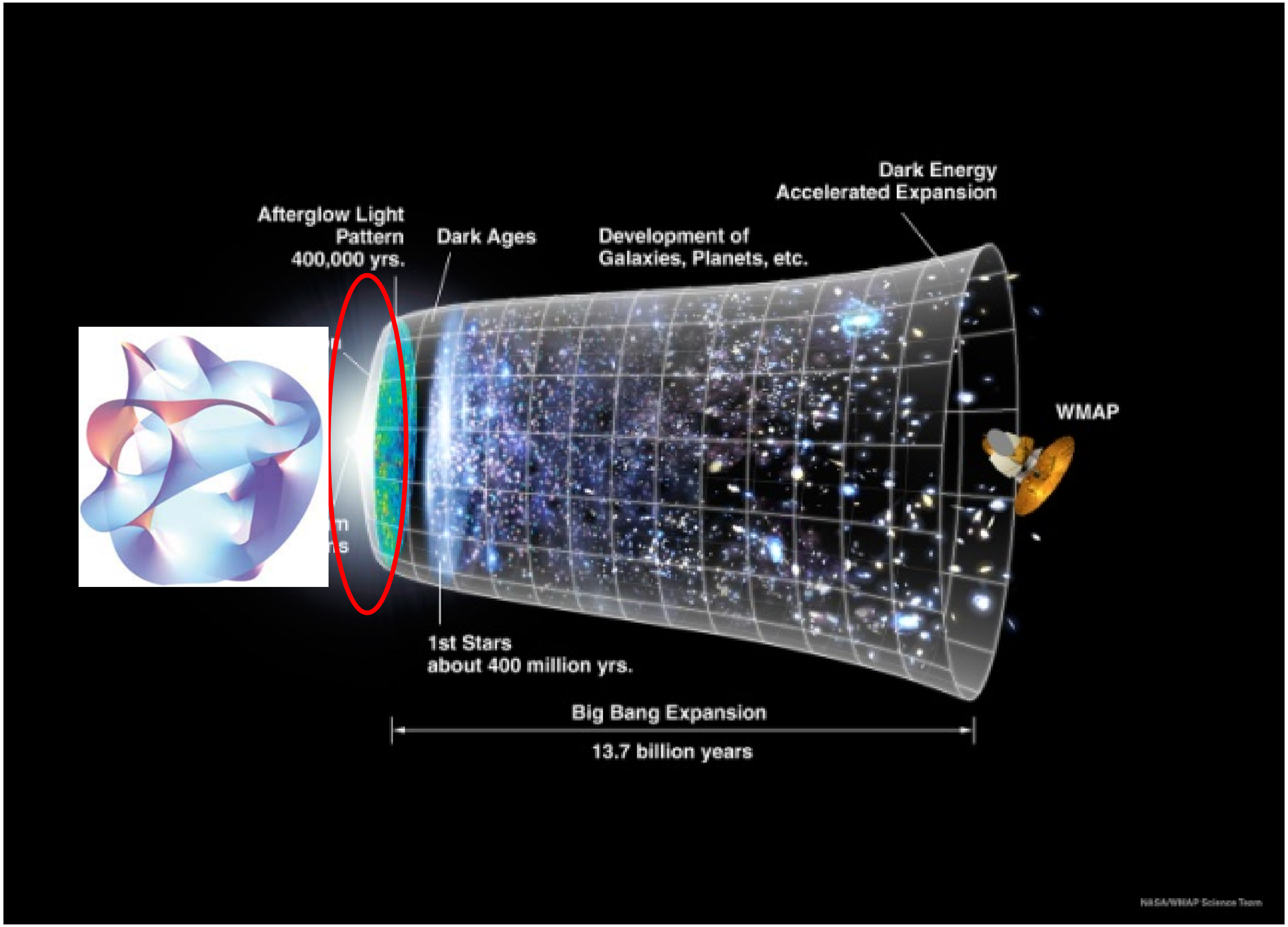
$$\mathcal{L}_{kin} = f^2 (\partial \phi)^2$$

- Moduli σ should be much heavier

$$H_{\text{inf}} \ll m_\sigma$$

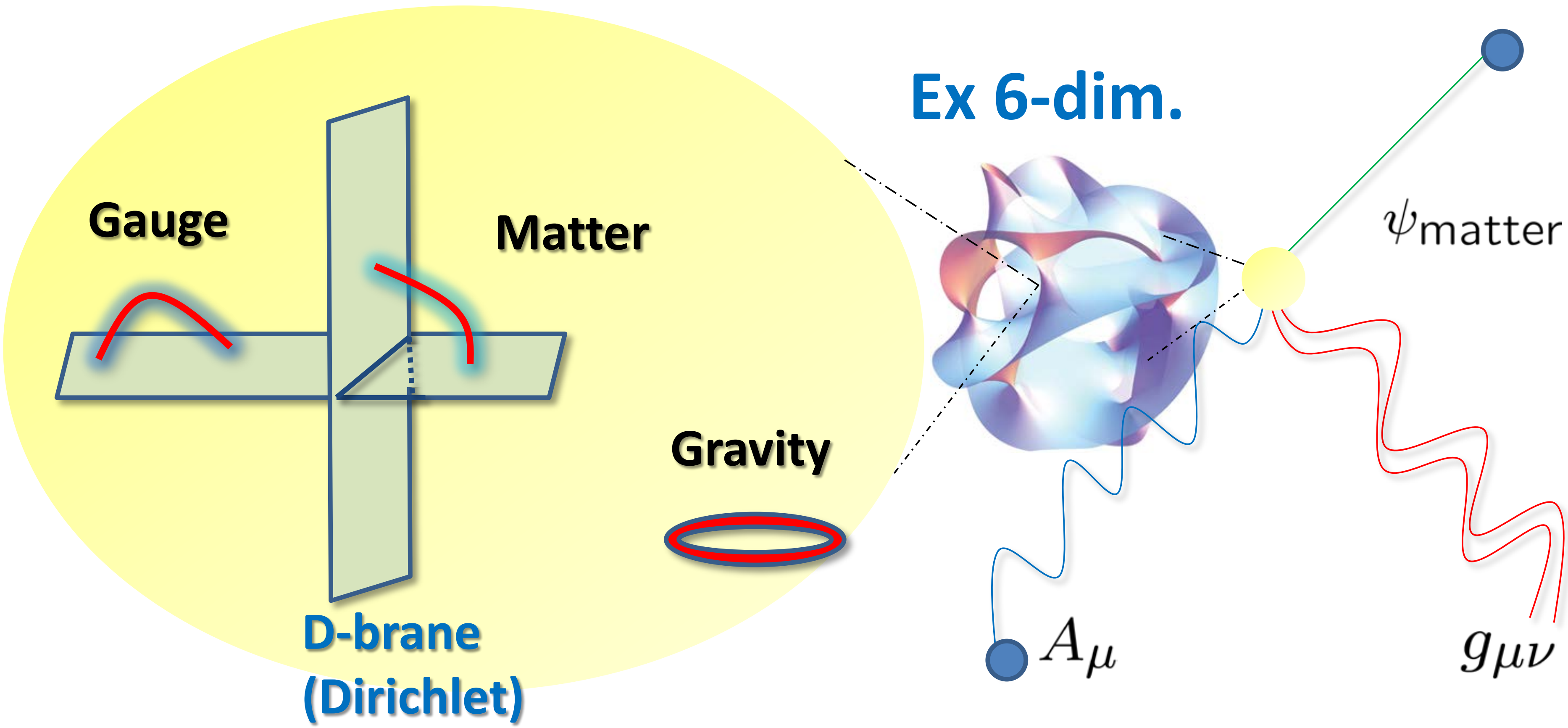
5. String theory/Supergravity

Axion inflation in string theory



Motivation for string axion inflation

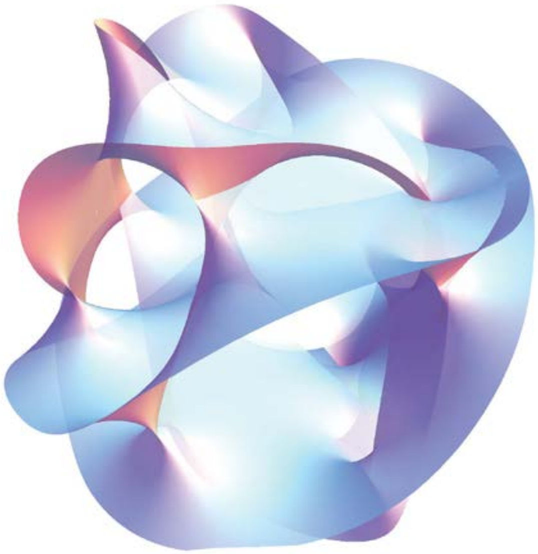
1. Unified theory; SU(N) on N × D-branes



Motivation for string axion inflation

1. Unified theory; SU(N) on N × D-branes
2. Extra dimension = Hidden sector:

Multiple axions from gauge field of branes/strings



$$a(x) = \int C_n^{\mathsf{RR}}, \quad b(x) = \int B_2^{\mathsf{NS}}.$$

$$\mathsf{Ex}: b = B_{56}.$$

Axion shift symmetry
= remnant of gauge symmetry

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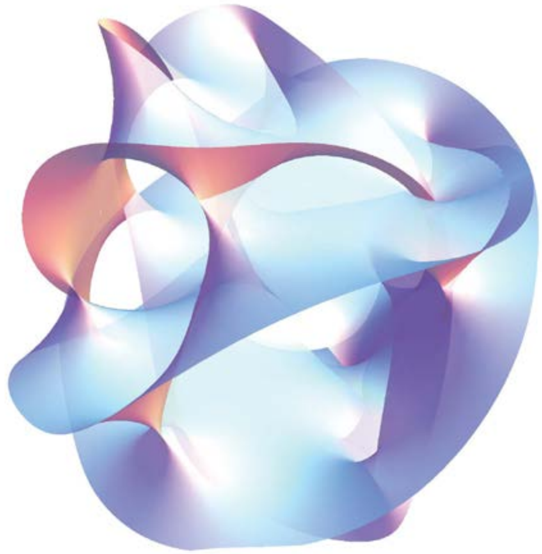


$$\phi(x) = \int C_4^{\mathrm{RR}} \quad \text{or} \quad \int C_2^{\mathrm{RR}}.$$

Axion shift symmetry
= remnant of gauge symmetry

Motivation for string axion inflation

1. Unified theory; SU(N) on N × D-branes
2. Extra dimension = Hidden sector:

Multiple axions from gauge field of branes/strings

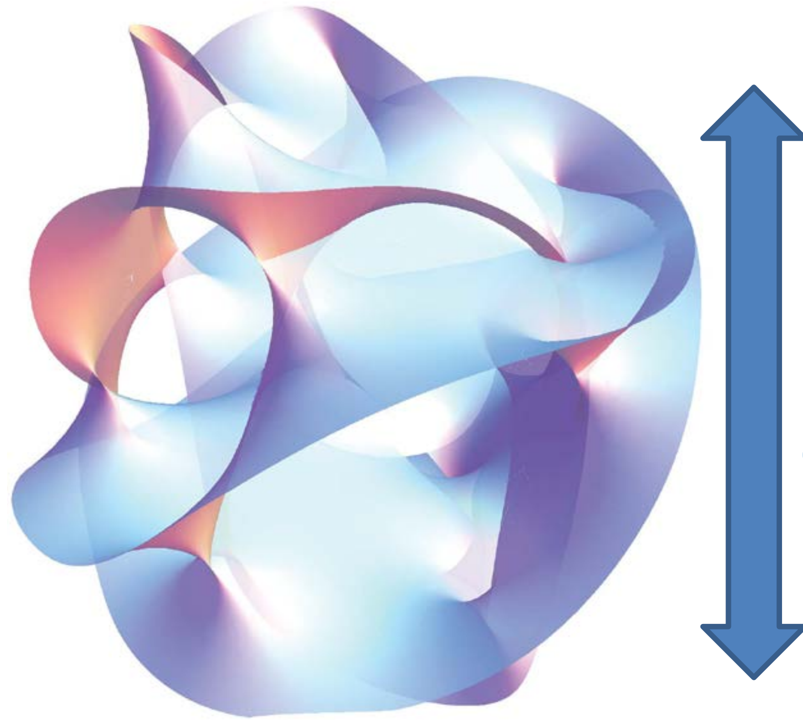
$$\phi(x) = \int C_4^{\mathrm{RR}} \quad \text{or} \quad \int C_2^{\mathrm{RR}}.$$

3. Large decay constant

$$f > M_{\mathrm{Pl}}.$$

Cf: $M_{Pl} > M_{string}$

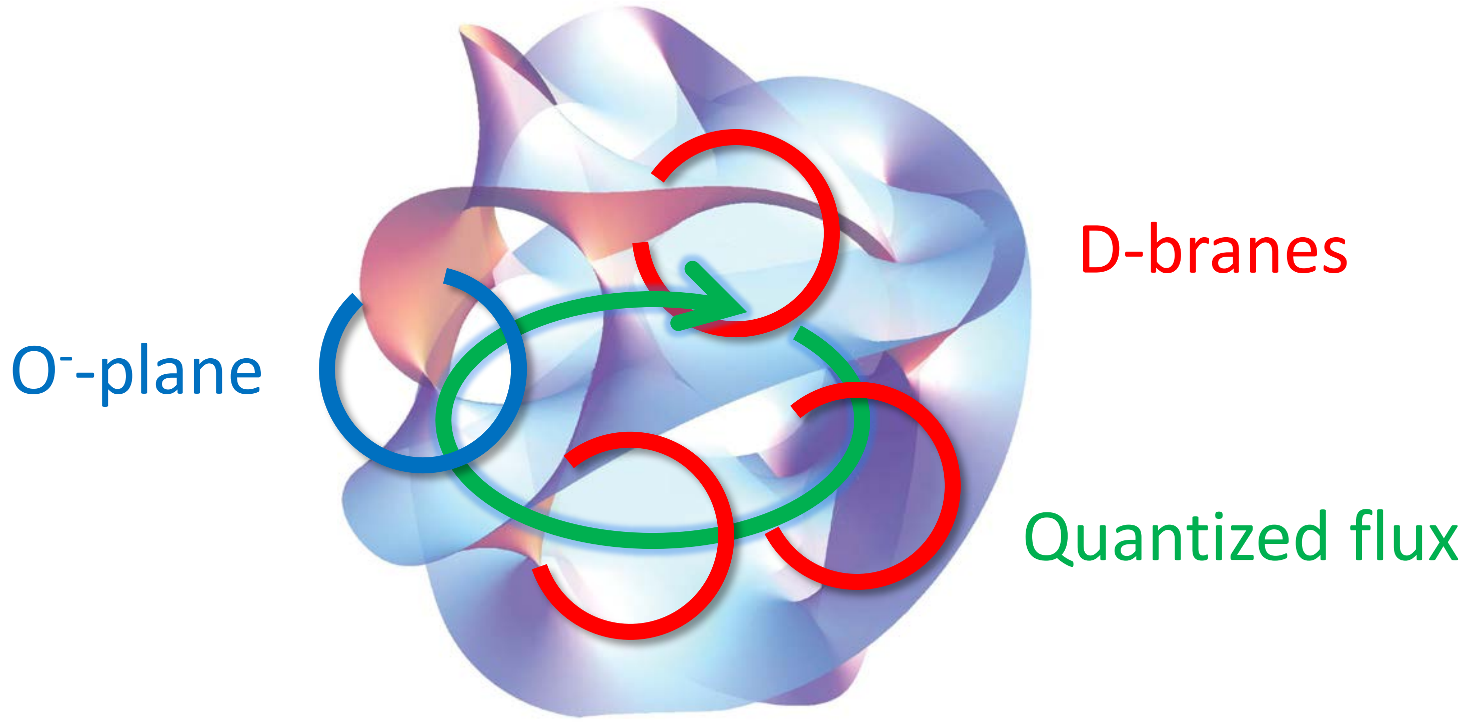
Moduli stabilization = Scales fixed



Fixing
a size of
extra dimension

Scales of inflation, EW, SUSY, …

Moduli stabilization = Scales fixed

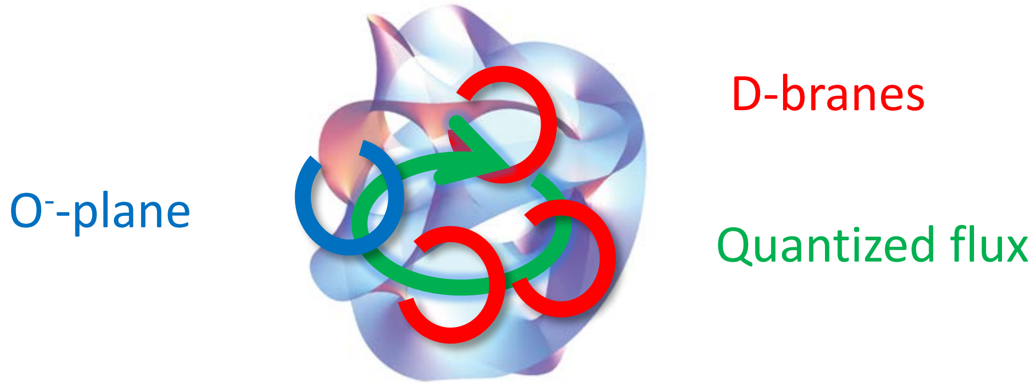


Scales of inflation, EW, SUSY, …

IIB Effective action for volume-axion T

$$K = -2\log(\mathcal{V}), \qquad W = W_0 + \sum_n A_n e^{-\sum_i a_{ni} T_i}.$$

- No-scale Kähler potential $K = -3\log(T + T^\dagger)$; no $T - T^\dagger$!
- W_0: Flux stabilized heavy moduli $\int \langle F_n^{\mathrm{RR}} \rangle \neq 0, \quad \int \langle H_3^{\mathrm{NS}} \rangle \neq 0.$
- $\exp[-aT]$: Gaugino condensations on D-branes



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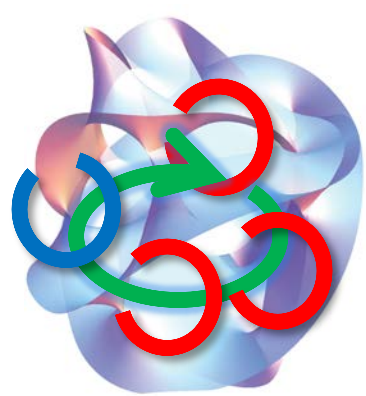
- No-scale Kähler potential $K = -3\log(T + T^\dagger);\ \text{no}\ T -$
- W_0: Flux stabilized heavy moduli $\int \langle F_n$
- $\exp[-aT]$: Gaugino condensations on D

Decay constant:

Exp[iaϕ]: f ~ 1/a



4D Gauge coupling = "Volume"

- Dynamical scale of SU(N) on N × D7-branes:

$$W \sim e^{-\frac{2\pi}{N}T},$$

$$T \equiv \text{volume} - i\int_{D7} C_4.$$

∵ Gauge coupling on D7-brane having 4 Ex-dim:

$$\int d^4x \int d^4y (F_{MN})^2 \quad \Rightarrow \quad \frac{4\pi}{g^2} = \text{volume}.$$

$$\int_{4D}\int_y C_4 \wedge F_2 \wedge F_2 \quad \Rightarrow \quad \frac{\theta}{2\pi} \equiv \int_{D7} C_4.$$

4D Gauge coupling = "Volume"

- Dynamical scale of SU(N) on N × D7-branes:

$$W \sim e^{-\boxed{\frac{2\pi}{N}}T},$$

Decay const.:
f ∝ N

$$T \equiv \text{volume} - \boxed{i\int_{D7} C_4.}$$

Inflaton candidate

∵ Gauge coupling on D7-brane having 4 Ex-dim:

$$\int d^4x \int d^4y (F_{MN})^2 \qquad \frac{4\pi}{g^2} = \text{volume}.$$

$$\int_{4D}\int_y C_4 \wedge F_2 \wedge F_2 \qquad \Rightarrow \qquad \frac{\theta}{2\pi} \equiv \int_{D7} C_4.$$

Wrapping branes

- A single stack of N × D7-branes (N=1: Instanton)

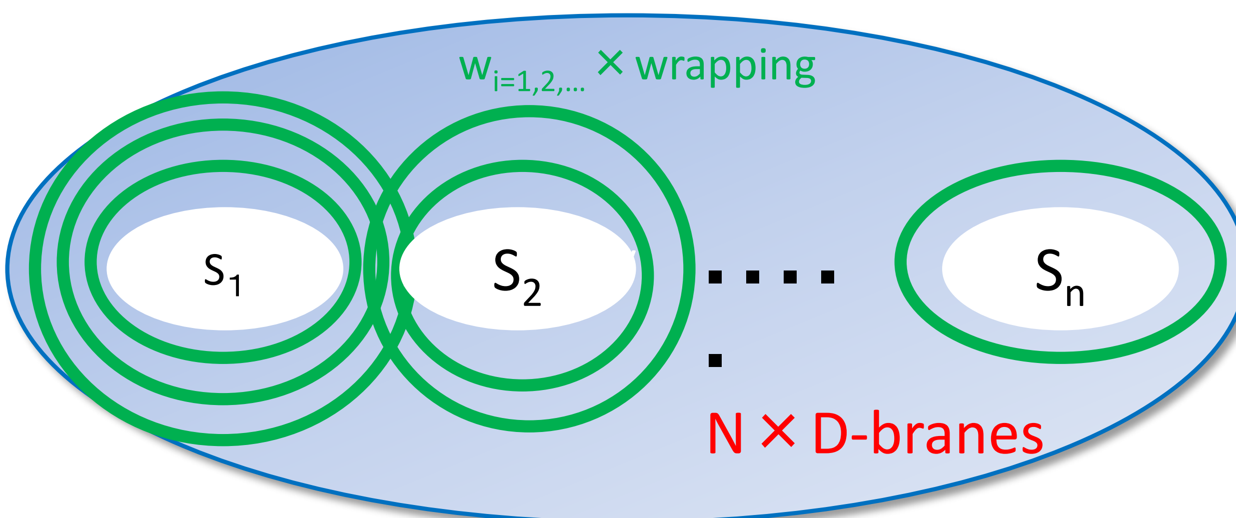
$$W \sim e^{-\frac{2\pi}{N}(w_1 T_1 + w_2 T_2 + \cdots)}$$

$$T = w_1 T_1 + w_2 T_2 + \cdots$$

Extra dimension

T_i: Volume of S_i
(Real part)

$h^{1,0}(w_i S_i) = h^{2,0}(w_i S_i) = 0$



Wrapping branes

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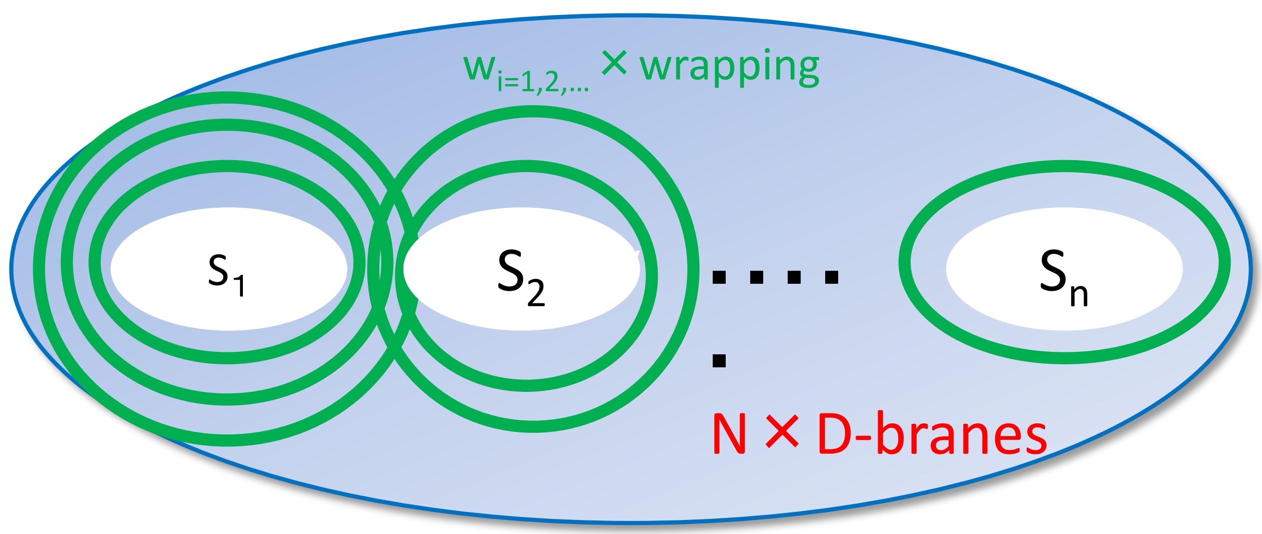
Extra dimension



T_i: Volume of S_i
(Real part)

$h^{1,0}(w_i S_i) = h^{2,0}(w_i S_i) = 0$

Flux corrections to gauge coupling

- Corrections in the presence of gauge flux F

$$\mathcal{F} = \frac{1}{2\pi}\int_{D7} dy^5 dy^6 F_{56} \in \mathbb{Z}$$

$$\delta\left(\frac{4\pi}{g_h^2}\right) = \mathcal{F}\,G + \cdots$$

G: Two cycle (Kähler) moduli

$$G = (e^{-\phi} - iC_0^{\text{RR}})\int B_2^{\text{NS}} + i\boxed{\int C_2^{\text{RR}}}.$$

Inflaton candidate

Magnetized branes

- A stack of N × D7-branes (N=1: Instanton)

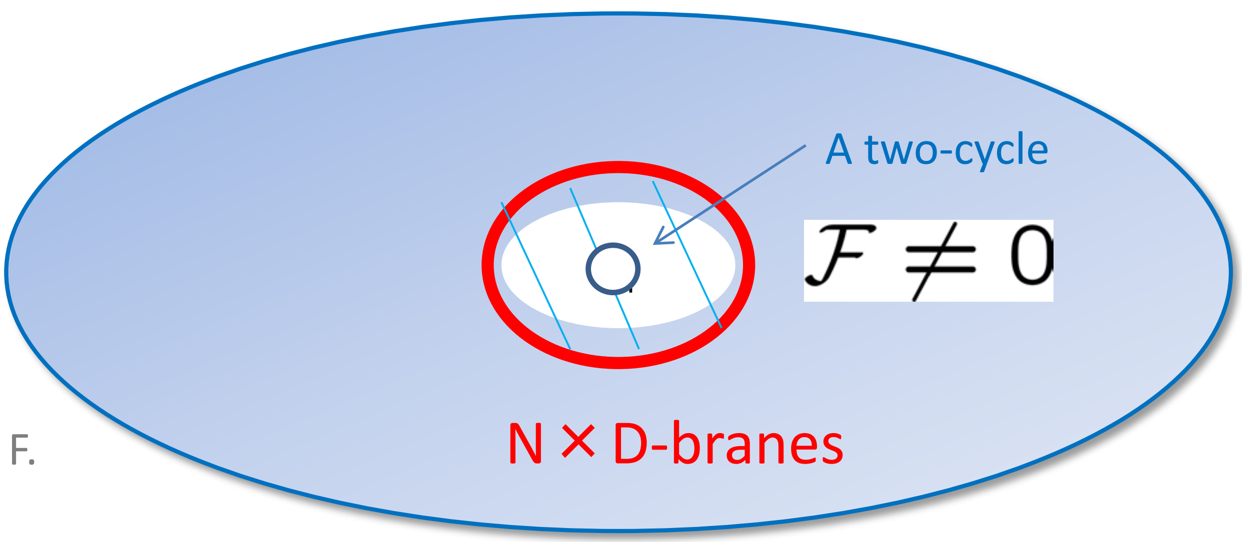
$$W \sim e^{-\frac{2\pi}{N}(T+\mathcal{F}G)}$$

$$T \to T + \mathcal{F}G$$

Extra dimension

T_i: Volume of S_i
(Real part)

Zero modes stabilized by flux F.
(Extra adjoint fileds)



Magnetized branes

- A stack of N × D7-branes (N=1: Instanton)

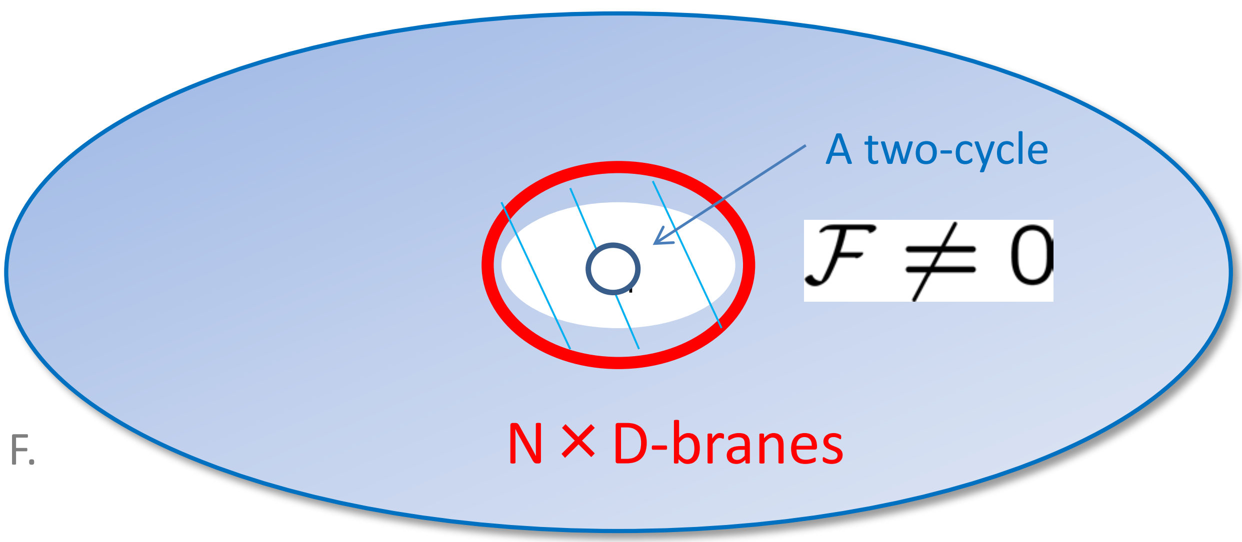
$$W \sim e^{-\frac{2\pi}{N}(T+\mathcal{F}G)}$$

$$T \to T + \mathcal{F}G$$

Extra dimension

T_i: Volume of S_i
(Real part)

Zero modes stabilized by flux F.
(Extra adjoint fileds)



Large decay constant for $V(\phi) \sim \cos\left(\frac{\phi}{f}\right)$

$$\boxed{f \sim \frac{N_{\text{brane}}}{\Delta(\mathcal{F}, w)} f_a}$$

N_{brane}: # of D-branes from Gaugino condensation

Yonekura;
Harigaya, Ibe: U(1) charge N;

Δ: Alignment of w or F with multiple axions

Kim-Nilles-Peloso

f_a: Decay constant in $(\partial\phi)^2$: M_{string}, M_{KK} and $M_{winding}$.

Natural inflation models

No decompactification problem

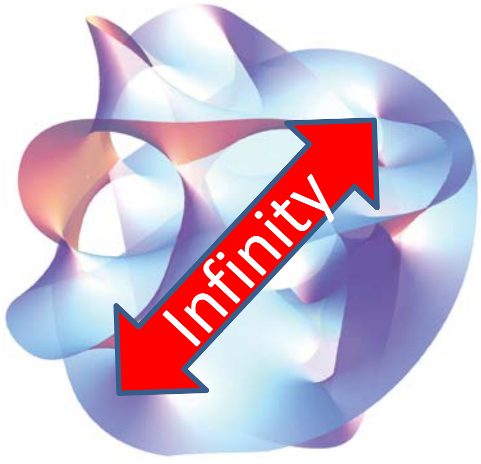
Inflation does not disturb moduli stabilization

$$\boxed{H_{\text{inf}} \lesssim m_{\text{moduli}} \sim m_{3/2}}$$

Our case: due to axion shift symmetry

Cf: Kallosh et al.
for $m_{\text{moduli}} > m_{3/2}$

Otherwise runaway volume:
Sensible 4D lost.

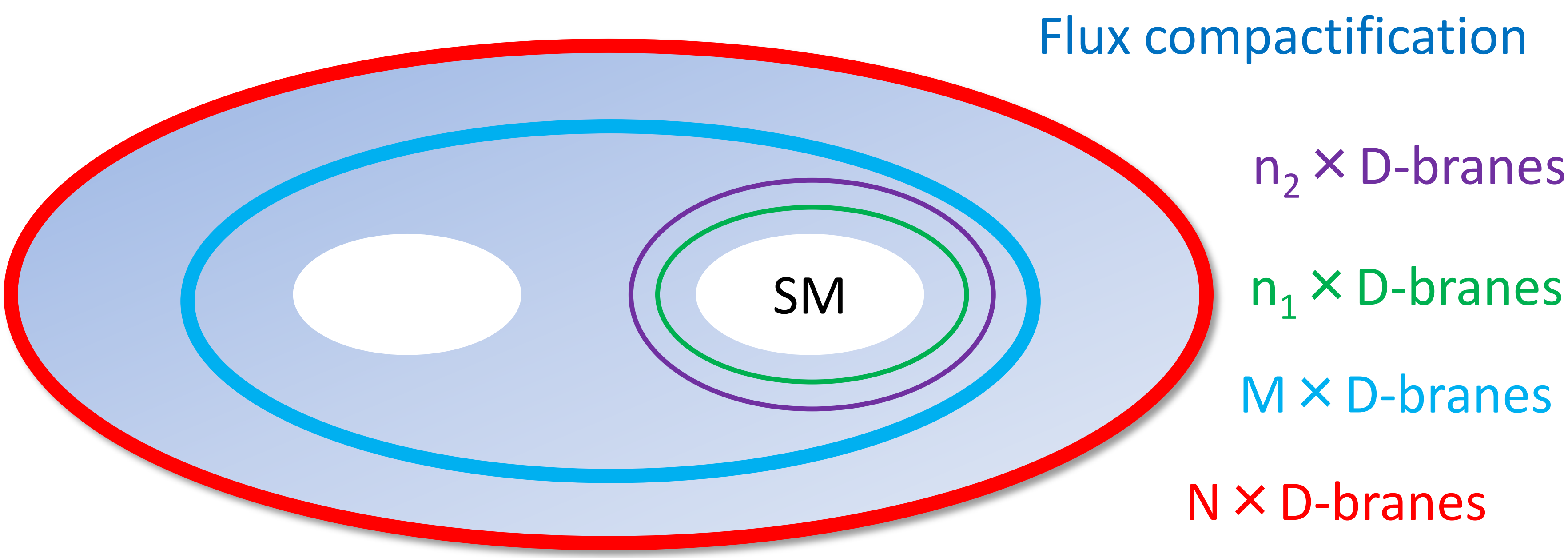


1. Model with O(10-100) D7-branes

- Effective action on CY with two holes:

$$K = -2\log(t_0^{3/2} - t_1^{3/2} - t_2^{3/2}); \quad t_i = (T_i + T_i^\dagger) \quad \text{for } i = 0, 1, 2,$$

$$W = W_0 - Ce^{-\frac{2\pi}{N}T_0} - De^{-\frac{2\pi}{M}(T_1+T_2)} + Ae^{-\frac{2\pi}{n_1}T_2} + Be^{-\frac{2\pi}{n_2}T_2},$$



1. Model with O(10-100) D7-branes

- Effective action on CY wit

Inflaton potential

$$K = -2\log(t_0^{3/2} - t_1^{3/2} - t_2^{3/2}); \quad t_i = (T_i + \bar{T}_i) \text{ for } i = 0, 1, 2,$$

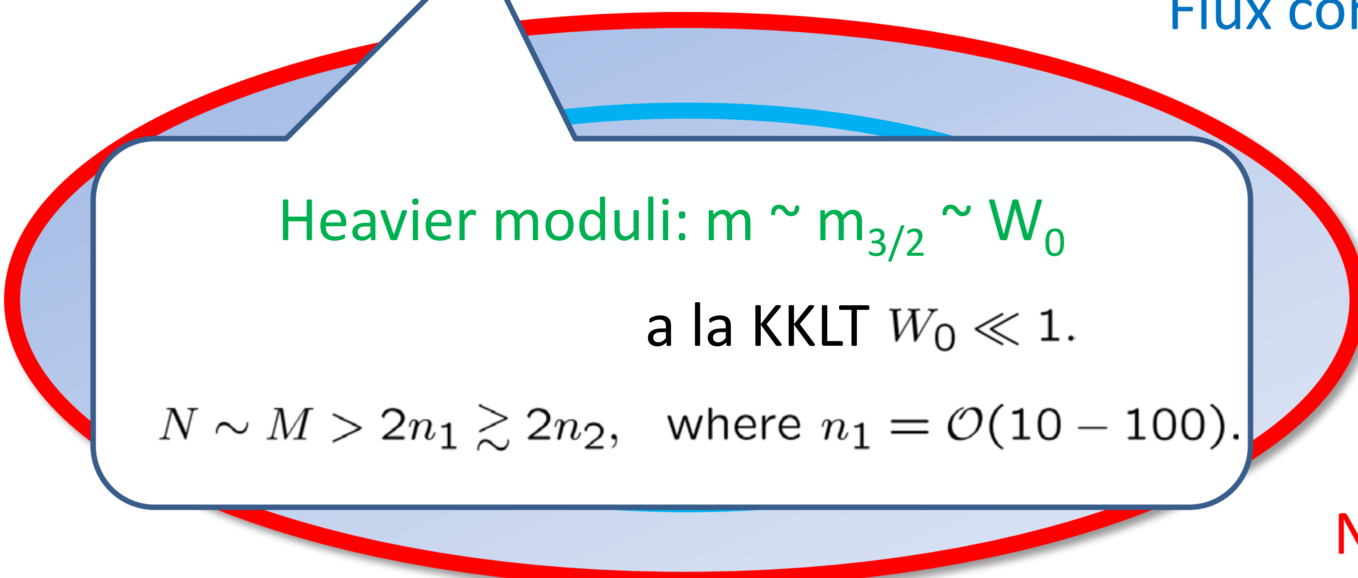
$$W = \boxed{W_0 - Ce^{-\frac{2\pi}{N}T_0} - De^{-\frac{2\pi}{M}(T_1+T_2)}} + \boxed{Ae^{-\frac{2\pi}{n_1}T_2} + Be^{-\frac{2\pi}{n_2}T_2},}$$

Flux compactification

Heavier moduli: m ~ $m_{3/2}$ ~ W_0

a la KKLT $W_0 \ll 1.$

$N \sim M > 2n_1 \gtrsim 2n_2,$ where $n_1 = \mathcal{O}(10-100).$



n_2 × D-branes

n_1 × D-branes

M × D-branes

N × D-branes

1. Low energy action: Multi-natural

- Effective action for axion multiplet Φ = $-T_1 + T_2$

$$K_L \approx \frac{f^2}{2}(\Phi + \Phi^\dagger)^2$$

$$W_L \approx m_{3/2} + \hat{A}e^{-\frac{\pi}{n_1}\Phi} + \hat{B}e^{-\frac{\pi}{n_2}\Phi}$$

$$f^2 \equiv \frac{3}{2\sqrt{2}\sqrt{t}\mathcal{V}} \lesssim 1, \qquad m_{3/2} \sim W_0, \qquad \hat{B} \lesssim \hat{A} \ll m_{3/2} \ll 1.$$

Here $\hat{A} \sim (m_{3/2})^{M/2n_1}$, $\hat{B} \sim (m_{3/2})^{M/2n_2}$, $M > 2n_1 \gtrsim 2n_2$, and $n_1 = \mathcal{O}(10 - 100)$.

1. Low energy action: Multi-natural

- Effective action for axion multiplet $\Phi = -T_1 + T_2$

$$\frac{f^2}{2}(\Phi + \Phi^\dagger)^2$$

$$m_{3/2} + \hat{A}e^{-\frac{\pi}{n_1}\Phi} + \hat{B}e^{-\frac{\pi}{n_2}\Phi}$$

$f \sim M_{KK} \sim M_{string}$

$V \sim Re(T_0^{3/2})$; $t = Re(T_1+T_2)$

$$f^2 \equiv \frac{3}{2\sqrt{2}\sqrt{t}\mathcal{V}} \lesssim 1, \qquad m_{3/2} \sim W_0, \qquad \hat{B} \lesssim \hat{A} \ll m_{3/2} \ll 1.$$

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1. Large decay constant = Many branes

- Axion potential for Re(Φ) = 0: $\phi \propto \mathrm{Im}(-T_1 + T_2)$

$$V_{\rm axion}(\phi) \simeq 6\hat{A} m_{3/2}\left[1-\cos\left(\frac{\phi}{f_1}\right)\right] + 6\hat{B} m_{3/2}\left[1-\cos\left(\frac{\phi}{f_2}+\theta\right)\right]$$

Large decay constant ∝ $N_{\rm brane}$:

$$\boxed{f_1 = n_1 \frac{\sqrt{2}f}{\pi}, \quad f_2 = n_2 \frac{\sqrt{2}f}{\pi}; \quad f_1 \sim 50f \quad \text{for } n_1 = 100.}$$

$$\Rightarrow \quad f_1 \sim 5M_{\rm Pl} \quad \text{for } f \sim 0.1 M_{\rm Pl}.$$

$\theta = \mathrm{Arg}(1/\hat{B})$

2. Alignment mechanism with axions

hep-ph/0409138: Kim-Nilles-Peloso

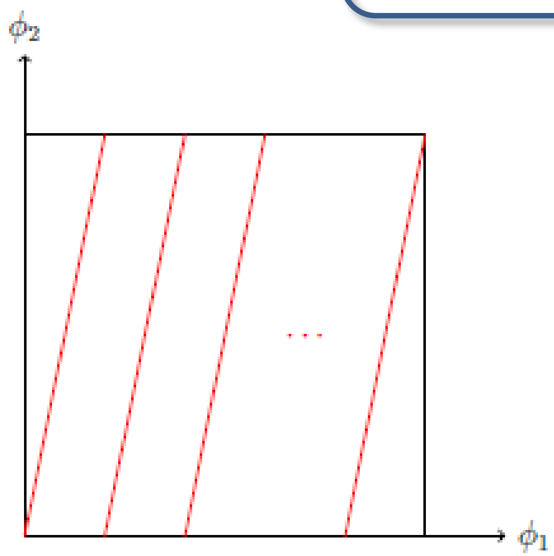
- Two axion: Large decay constant for ψ

$$V \sim \cos\left(\frac{\phi_1}{f_1}+\frac{\phi_2}{f_2}\right)+\epsilon\cos\left(\frac{\phi_1}{f_1}n_1+\frac{\phi_2}{f_2}n_2\right)$$

$$\psi = \frac{\phi_1}{f_2}-\frac{\phi_2}{f_1};$$

$$\boxed{f_\psi \sim \frac{n_1 f}{|n_1-n_2|} \to \infty \quad \text{for} \quad n_2 \to n_1.}$$

$\phi_2 \to 2\pi f_2$



$\phi_1 \to 2\pi f_1$

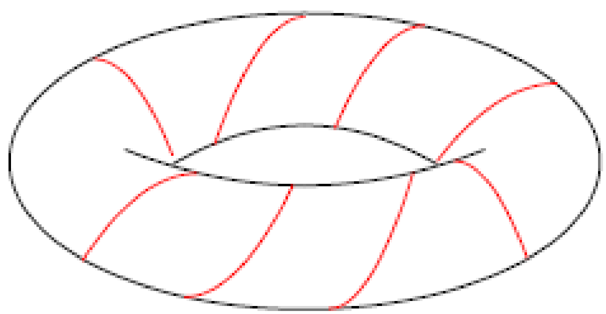


Figure from 1404.6209

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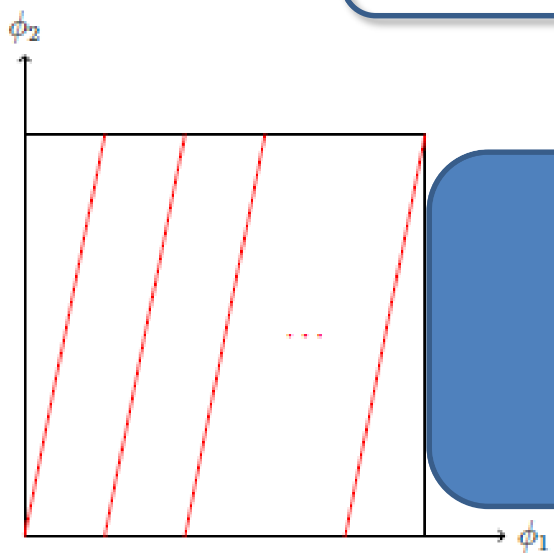


$\phi_1 \to 2\pi f_1$

D-brane wrapping/flux;

$\Delta = (n_1 - n_2)/n_1$

$\Delta = 0.1$ for $n_1 = 10$, $n_2 = 9$

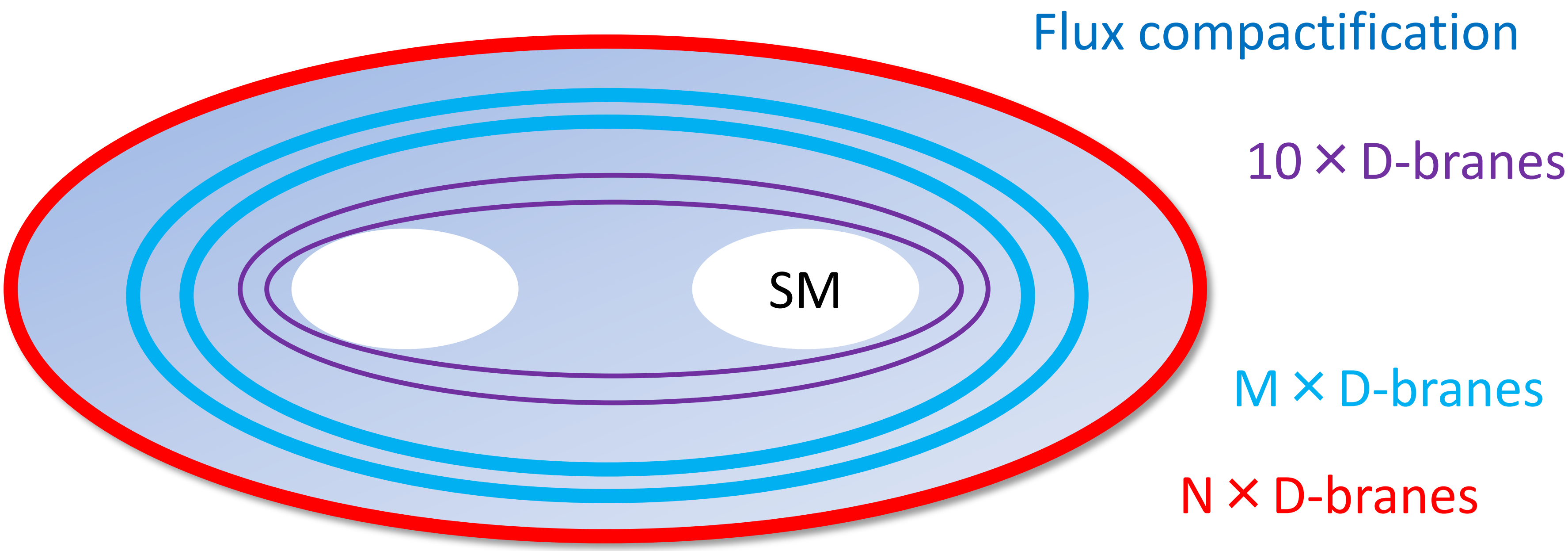
from 1404.6209

2. Model with wrapping branes

- Effective action on CY with 2 holes:

$$K = -2\log(t_0 - t_1^{3/2} - t_2^{3/2}) \qquad t_i = (T_i + T_i^\dagger)$$

$$W = W_0 - Ce^{-\frac{2\pi}{N}T_0} - Be^{-\frac{2\pi}{M}(T_1+10T_2)} + Ae^{-\frac{2\pi}{10}(T_1+11T_2)}$$



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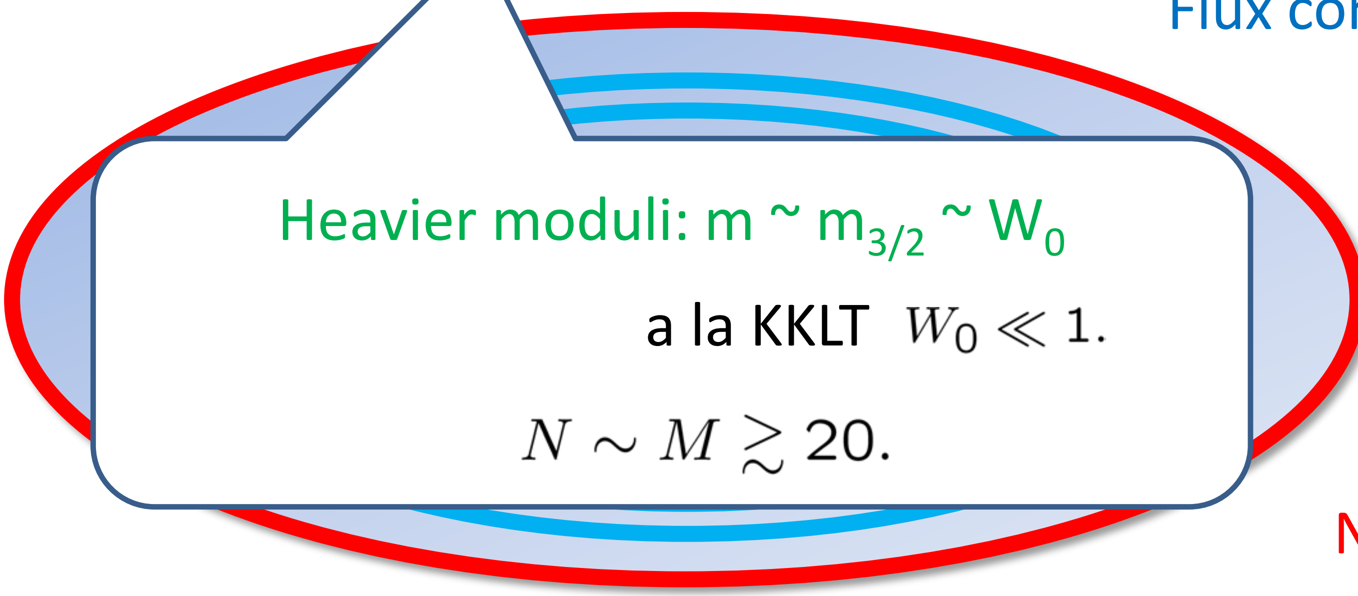
Inflaton potential

Flux compactification

Heavier moduli: m ~ $m_{3/2}$ ~ W_0

a la KKLT $W_0 \ll 1.$

$N \sim M \gtrsim 20.$



2. Model with wrapping branes

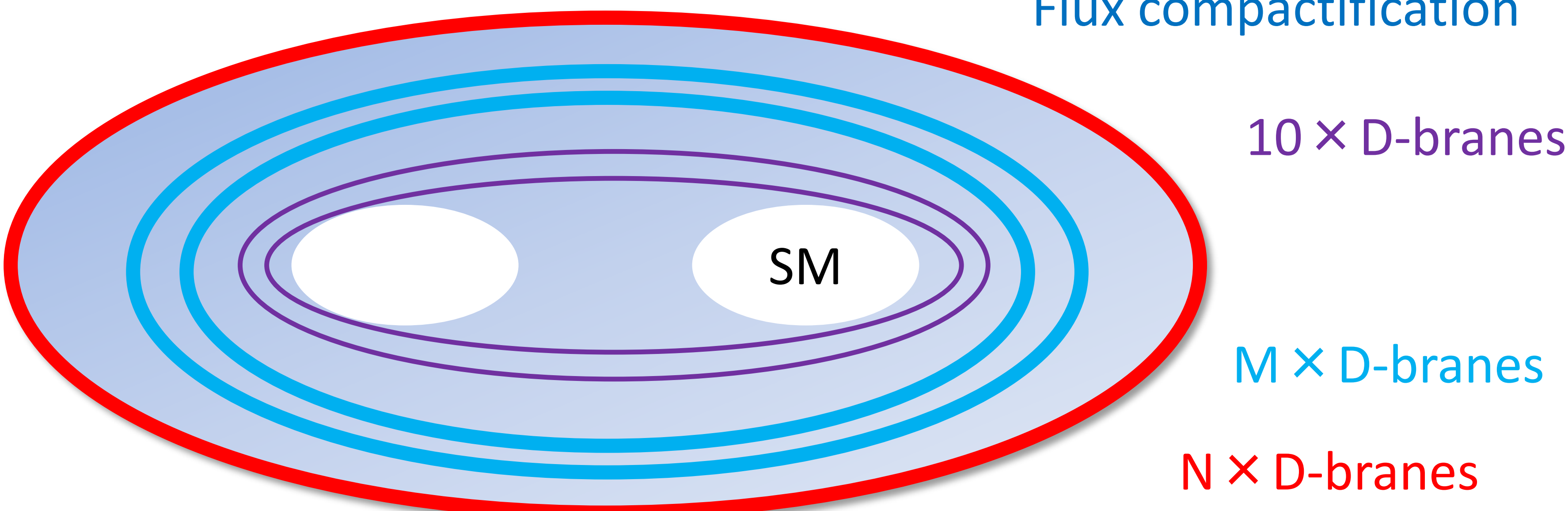
- Effective action on CY w

$$K = -2\log(t_0 - t_1^{3/}$$

Aligned to each other:
Δ ~ 1/10

$$W = W_0 - Ce^{-\frac{2\pi}{N}T_0} - Be^{-\frac{2\pi}{M}(T_1+\boxed{10}T_2)} + Ae^{-\frac{2\pi}{10}(T_1+\boxed{11}T_2)}$$

Flux compactification



2. Many times wrapping branes

- Natural inflation: $\phi \propto \mathrm{Im}(-10T_1 + T_2)$

$$V_{\rm axion}(\phi) \simeq 6\hat{A}m_{3/2}\left[1 - \cos\left(\frac{\phi}{f_1}\right)\right]$$

$$\boxed{f_1 \sim 227f \sim \frac{n_{\rm brane}}{\Delta}f \sim \frac{10}{\frac{1}{10}}f.}$$

$$\Rightarrow \quad f_1 \sim 5M_{\rm Pl} \quad \text{for } f \sim 0.02M_{\rm Pl}.$$

$$f \sim \frac{1}{\mathcal{V}^{1/2}}, \quad \hat{A} \sim Ae^{-\frac{111\pi}{505}\langle T\rangle} \ll m_{3/2}.$$

$$\mathcal{V} \sim t_0^{3/2}, \quad T = T_1 + 10T_2$$

3. Model with two cycle Kähler moduli

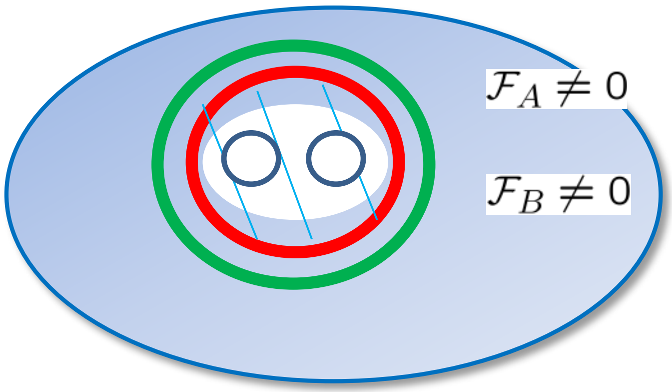
1404.7852: Long, McAllister, McGuirk

- 2 volumes $T_{1,2}$ + 2 two-cycle moduli $G_{1,2}$:

$$K = -2\log\left[t_0^{3/2} - (t_1 + g_1^2 + 3g_2^2/4)^{3/2}\right] \qquad t_i = T_i + T_i^\dagger,\ g_i = (G_i + G_i^\dagger).$$

$$W \supset W_0 + Ae^{-\frac{2\pi}{35}(T_1+4G_1-9G_2)} + Be^{-\frac{2\pi}{40}(T_1+5G_1-\frac{21}{2}G_2)}.$$

Swiss cheese CY with
2 two-cycle on a four-cycle



2 stacks of (35 + 40) magnetized
(+ 1 stack of unmagnetized branes;
also D-terms stabilizing T_0, T_1)

3. Model with two cycle Kähler moduli

1404.7852: Long, McAllister, McGuirk

- Low energy model after volume $T_{1,2}$ decoupled

$$K_L = f^2(G_1 + G_1^\dagger)^2 + f^2\frac{3}{4}(G_2 + G_2^\dagger)^2,$$

$$W_L = W_0 + \boxed{A e^{-\frac{8\pi}{35}(G_1 - \frac{9}{4}G_2)}} + B e^{-\frac{2\pi}{8}(G_1 - \frac{21}{10}G_2)}.$$

$$f^2 = 3\frac{t_1^{1/2}}{\mathcal{V}} \simeq M_{\text{wind}}^2 \sim M_{\text{string}}^2.$$

$$\mathcal{V} = (t_0^{3/2} - t_1^{3/2})/6.$$

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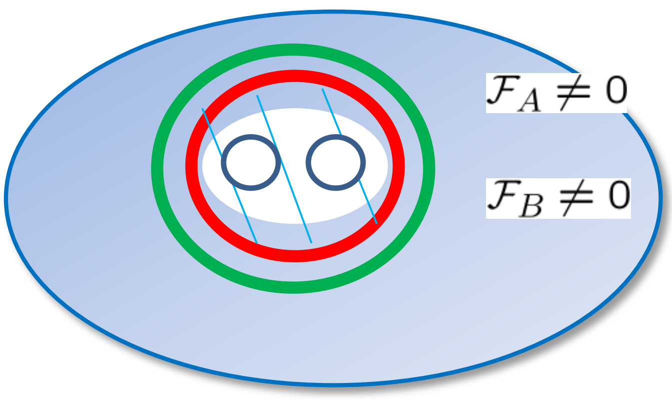

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$$W_L = W_0 + \boxed{Ae^{-\frac{8\pi}{35}(G_1 - \frac{9}{4}G_2)}} + Be^{-\frac{2\pi}{8}(G_1 - \frac{21}{10}G_2)}.$$

$$f^2 = 3\frac{t_1^{1/2}}{\mathcal{V}} \simeq M_{\text{wind}}^2 \sim M_{\text{string}}^2.$$

$$\mathcal{V} = (t_0^{3/2} - t_1^{3/2})/6.$$

$$\boxed{f_1 \sim \frac{N}{\Delta}f \sim \frac{10}{\frac{1}{10}}f \sim 100f \; : \quad \text{Aligned gauge flux}}$$

3. Model with two cycle Kähler moduli

1404.7852: Long, McAllister, McGuirk

- Multi-natural inflation: $\phi \sim \mathrm{Im}(G_1 - 2.5G_2).$

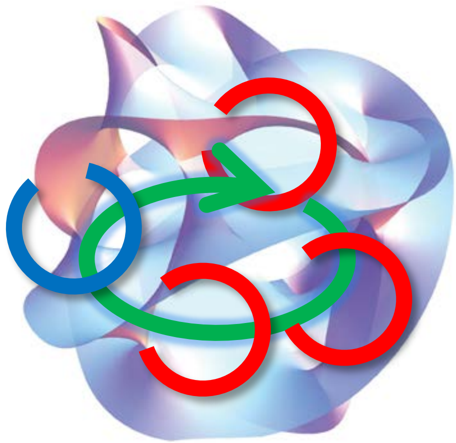
$$V \sim C - \Lambda^4 \left[\cos\left(\frac{\phi}{f_1}\right) + 0.1 \cdot \cos\left(\frac{\phi}{f_2}\right) \right]$$

$$C \sim \Lambda^4 \sim 10^{-8}, \;\; f_1 \sim 100 M_{\mathrm{Pl}}, \;\; f_2 \sim M_{\mathrm{Pl}}.$$

$$\text{Volume}: \;\; t_0 = 49, \quad t_1 = 9.$$

Anomaly/Tadpole condition

- Might be possible condition



$$\sum_i N_{\mathrm{D7}} w = Q^{\mathrm{O7}} = \mathcal{O}(10).$$

16 for toroidal cases

$$\sum_{\mathrm{brane+brane'}} \mathcal{F}_i = 0.$$

$$N_{\mathrm{D3}} + N_{\mathrm{flux}} = \frac{\chi(CY_4)}{24} = \mathcal{O}(10 - 10^4)$$

16 for toroidal cases

Anomaly/Tadpole condition

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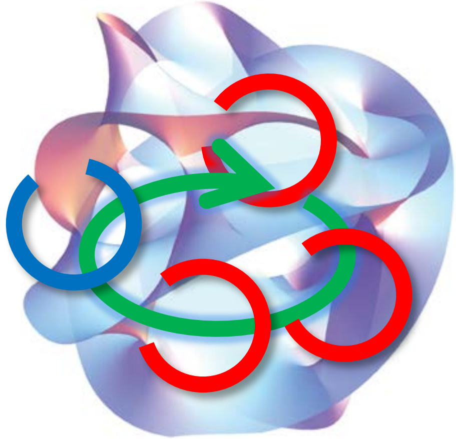

Model 1: N_{D7} = O(10-100) : Too many branes?

$$\sum_i N_{\mathrm{D7}} w = Q^{\mathrm{O7}} = \mathcal{O}(10).$$

16 for toroidal cases

$$\sum_{\mathrm{brane+brane'}} \mathcal{F}_i = 0.$$

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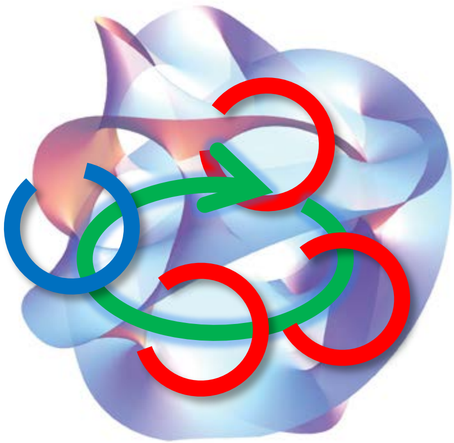
16 for toroidal cases

Anomaly/Tadpole condition

- Might be possible condition

Model 2: N_{D7} = O(10); w = O(10): Too many charge?



$$\sum_i N_{\mathrm{D7}} w = Q^{\mathrm{O7}} = \mathcal{O}(10).$$

16 for toroidal cases

$$\sum_{\mathrm{brane+brane'}} \mathcal{F}_i = 0.$$

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Anomaly/Tadpole condition

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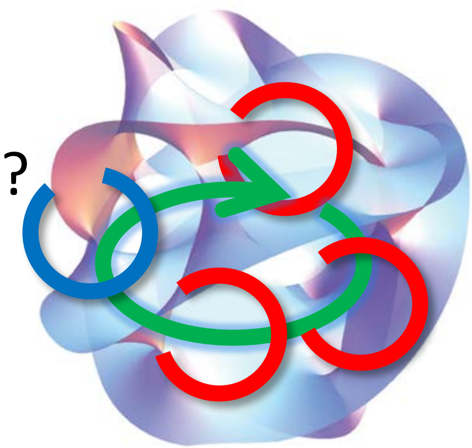


Model 3: N_{D7} = O(10); F = O(10): Marginally possible?

$$\sum_i N_{\mathrm{D7}} w = Q^{\mathrm{O7}} = \mathcal{O}(10).$$

16 for toroidal cases

$$\sum_{\mathrm{brane}+\mathrm{brane}'} \mathcal{F}_i = 0.$$

$$N_{\mathrm{D3}} + N_{\mathrm{flux}} = \frac{\chi(CY_4)}{24} = \mathcal{O}(10-10^4)$$

F^2

16 for toroidal cases

6. Conclusion

Summary

- Natural inflation: Large tensor mode generated

$$V(\phi) = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right)\right]$$: Under good control and $f > M_{Pl}$

- String natural inflation: Many brane charges needed
  - Large decay constant:

$$f \sim \frac{N_{\text{brane}}}{\Delta} M_{\text{string}}$$

N_{brane}: # of D-branes, $\Delta < 1$: Alignment of winding/fluxes.

  - **Two cycle axion = inflaton** via severe tadpole condition?

Two form axion = inflation?

$$\phi(x) = \int C_2^{\mathrm{RR}} \qquad ?$$

Discussion

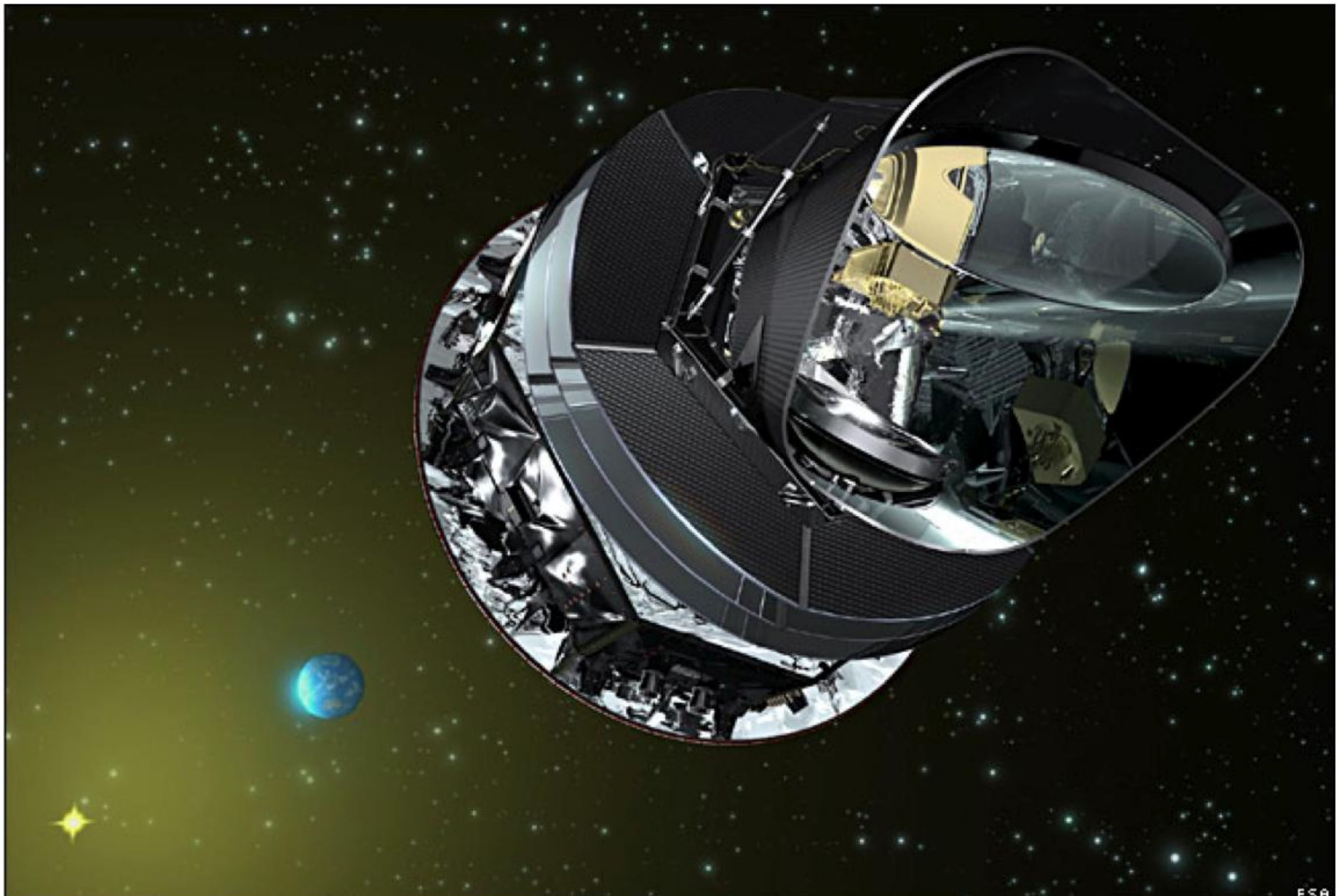
- Explicit model?
  - Non-perturbative effects, flux, The SM, consistency,…

- Light Axion mass protected by shift symmetry

$\Leftrightarrow$

Stability of our brane universe (RR-charge) protected by gauge symmetry?

Wait for Planck polarization data



Backups

Figures

Moduli and parameters

Heavy moduli: Flux potential in IIB

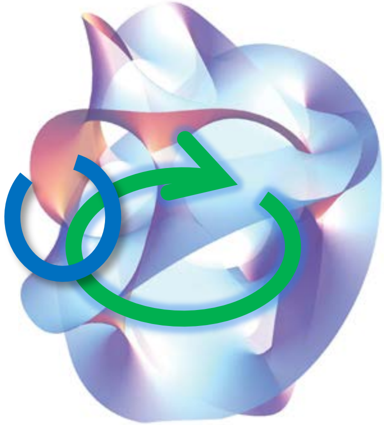
- Mass term for dilaton and complex structure:

Ω : holomorphic 3-form

$$W_{\text{flux}} = \int (F_3 - iSH_3) \wedge \Omega$$
$$= f_0 + f_1 U + f_2 U^2 + f_3 U^3 + S(h_0 + h_1 U + h_2 U^2 + h_3 S U^3)$$

- Constant term in W at low scales via DW = 0

$$\langle W_{\text{flux}} \rangle \equiv W_0$$



Kähler potential = No scale

- Single modulus case:

$$K = -2\log(\mathcal{V}) = -3\log(T + T^{\dagger}),$$

- Moduli : Free volume deformation = No-potential

$$K = -3\log(T + T^{\dagger}), \qquad W = W_0$$

$$\Rightarrow \quad V \equiv 0$$

Moduli stabilization in model 1

Model with O(100) D7-branes

- Moduli stabilization with $V_{\rm up} = \hat{\epsilon}\, e^{2K/3}; \quad \hat{\epsilon} = \mathcal{O}(W_0^2),$

$$\frac{2\pi}{N} T_0 \simeq \frac{2\pi}{M} T \simeq \log\left[\log(1/W_0)/W_0\right] \gg 1, \quad {\rm Re}[\Phi] = 0.$$

$$T = T_1 + T_2, \qquad \Phi = -T_1 + T_2 = \sigma + i\phi.$$

- Moduli masses: W_0 ~ gravitino mass $m_{3/2}$

$$m_{T_0} \simeq m_T \simeq \log(M_{\rm Pl}/m_{3/2}) m_{3/2},$$

$$m_\sigma \simeq \sqrt{2} m_{3/2},$$

$$\boxed{m_{3/2} = W_0/\mathcal{V},}$$

$$\mathcal{V} = t_0^{3/2} - \frac{t^{3/2}}{\sqrt{2}},$$

Saxion σ decoupled from inflation

- Decoupling condition for successful inflation:

$$\boxed{AW_0 \ll W_0^2 \quad \Longrightarrow \quad H_{\rm inf} \ll m_\sigma}$$

$$m_\sigma \sim W_0, \qquad H_{\rm inf} \sim \sqrt{V_{\rm inf}} \sim \sqrt{AW_0};$$

$$m_\phi \sim \frac{\sqrt{AW_0}}{f_1} \lesssim H_{\rm inf} \ll m_\sigma.$$

No decompactification

- Moduli decoupled from inflation:

$$\boxed{H_{\mathrm{inf}} \sim m_{3/2} \cdot (m_{3/2})^{(M-2n_1)/4n_1} \ll m_{3/2}.}$$

$$m_\phi \sim \frac{H_{\mathrm{inf}}}{f_1}, \quad m_{\mathrm{moduli}} \gtrsim m_{3/2}.$$

for $M > 2n_1 \gtrsim 2n_2$, where $n_1 = \mathcal{O}(10-100)$.

KNP with many axions

f >> 1 with multiple-axions alignment

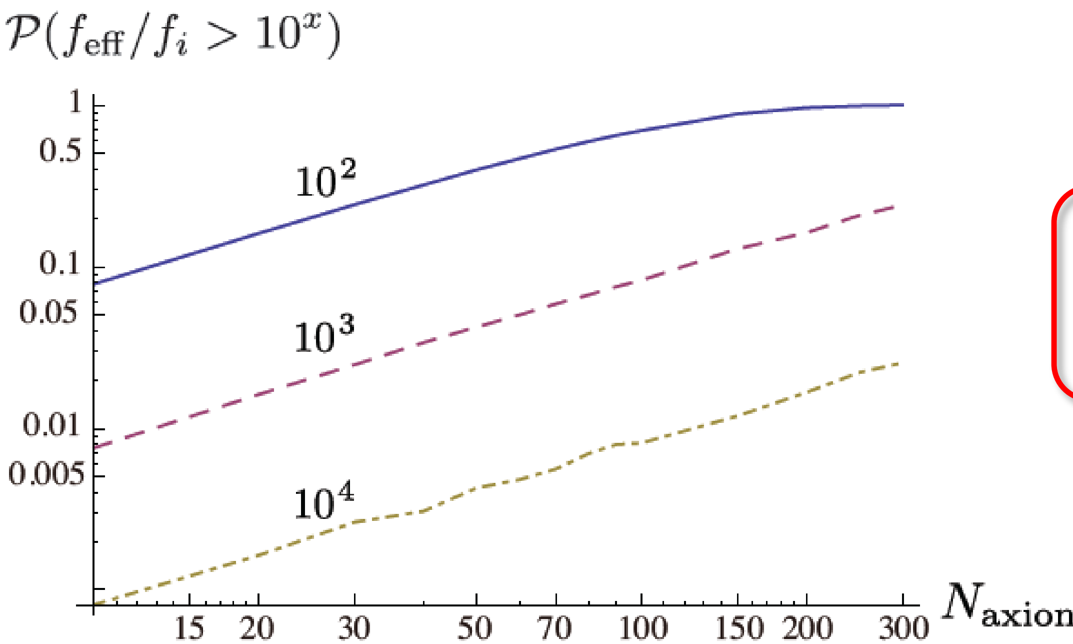
1404.6923: TH, Takahashi

- Many axions supporting a large decay constant

$$V(\phi_i) = \sum_{i=1}^{N_{\rm source}} \Lambda_i^4 \cos\left(\sum_{j=1}^{N_{\rm axion}} a_{ij}\frac{\phi_j}{f_j} + \theta_i\right) + V_0$$

$N_{\rm axion} = N_{\rm source}$

a_{ij} : Random integers with $|a_{ij}| \leqq 2$



Probablity to obtain $f_{\rm eff}$ > f.

$$\boxed{\mathcal{P}(f_{\rm eff}/f_i) \sim N_{\rm axion}\left(\frac{f_i}{f_{\rm eff}}\right)}$$

See also 1404.6209: Choi et al

Reheating

Reheating via axion coupling to A_μ

- Reheating temperature estimated:

$$T_R \sim 4 \times 10^{10}\,\mathrm{GeV} \cdot \left(\frac{m_\phi}{10^{13}\ \mathrm{GeV}}\right)^{3/2} \left(\frac{f_a/c}{10^{17}\ \mathrm{GeV}}\right)^{-1}.$$

$$\mathcal{L} = c\frac{\phi}{f_a} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Leptonegesis possible if $m_\phi > 2M_{RH\nu}$

Dark matter?

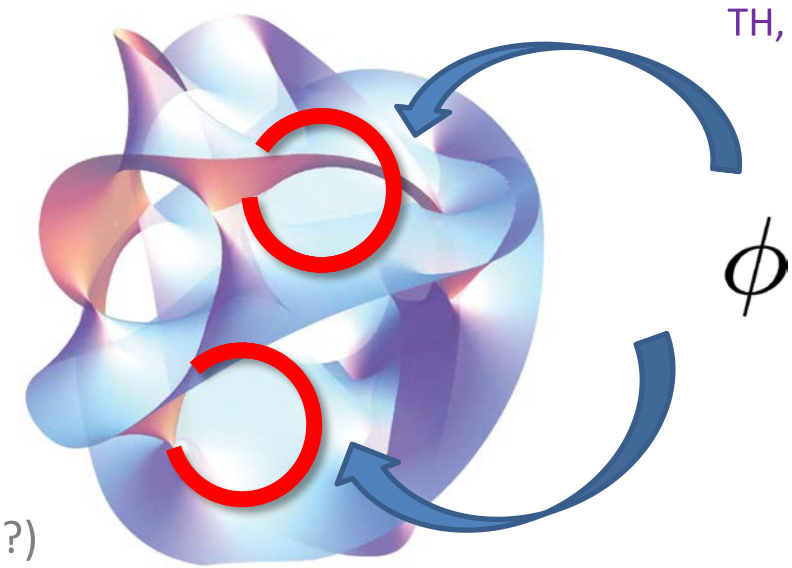
- **No light neutralino:** $H_{inf} < m_{3/2}$ for decompactification

(depends on uplifting (SUSY-breaking))

- **Mirror dark matter**: “DM-genesis” via mirror RHν’

1302.2516 and in preparation:
TH, Jeong, Takahashi

The SM-branes



The SM’-branes
= DM sector
(homologous to SM brane?)

Reheating/
Leptogenesis/
DM-genesis