

# Entanglement and Holography

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work with Han-Chih Chang, **Kristan Jensen**,  
Brandon Robinson and Christoph Uhlemann.

talk at IPMU, June 19 2014

# What is Entanglement?

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# What is an entangled state?

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Single particle wavefunctions are superpositions:

$$|\psi\rangle = \frac{1}{\sqrt{2}} |\text{cat}\rangle + \frac{1}{\sqrt{2}} |\text{dog}\rangle$$

# State for two spins: Product State

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State of particle 2

$$|\psi\rangle = (|\uparrow\rangle + |\downarrow\rangle)(|\uparrow\rangle + |\downarrow\rangle)$$

State of particle 1

Particle 1 is 50/50 up or down, so is particle 2.

**Probabilities are independent.**

# State for two spins: Entangled State

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$$|\psi\rangle = |\uparrow\rangle|\uparrow\rangle + |\downarrow\rangle|\downarrow\rangle$$

Particle 1 is 50/50 up or down, so is particle 2.  
**But they always point in the same direction.**

One particle measurement undetermined;  
Two particle measurement correlated.



# What is Entanglement?

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It's easy to recognize entangled wavefunction.

But wavefunction can not be directly observed.

**“Entanglement = Quantum Correlation”**

# Entanglement as Correlation

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For

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

one obtains

$$\langle \vec{S}_A \rangle = \langle \vec{S}_B \rangle = 0$$

**not possible in  
any single particle state**

Single Particle Measurement completely uncertain.<sup>7</sup>

# Entanglement as Correlation

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For

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

one obtains

$$\langle \vec{S}_A \cdot \vec{S}_B \rangle = -\frac{3}{4}$$

**perfect correlation**



# Entanglement as Correlation

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$$G = \langle \psi | O_A O_B | \psi \rangle - \langle \psi | O_A | \psi \rangle \langle \psi | O_B | \psi \rangle$$

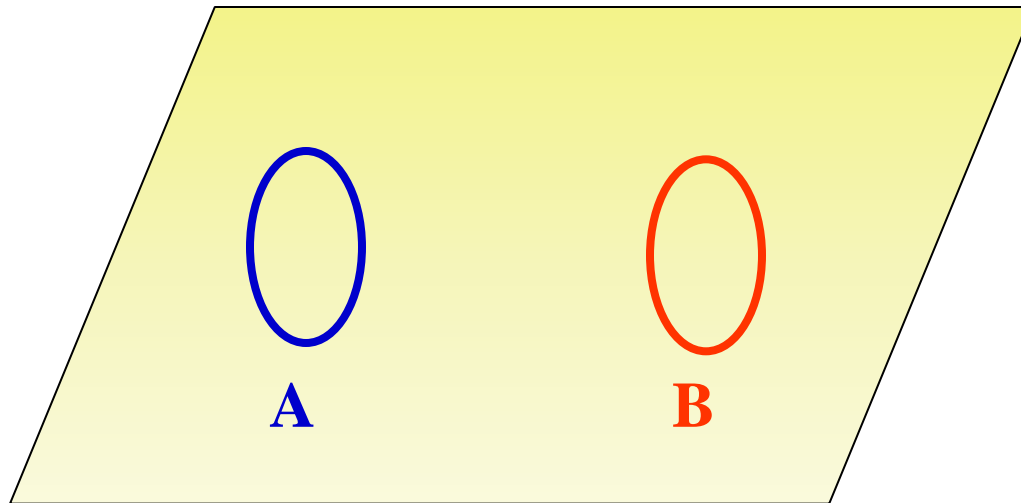
**Observable measuring entanglement  
are the disconnected correlation functions  
for any pair of operators in subsystem A and B.**

# Entanglement and Spacetime

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# Entanglement in a local QFT

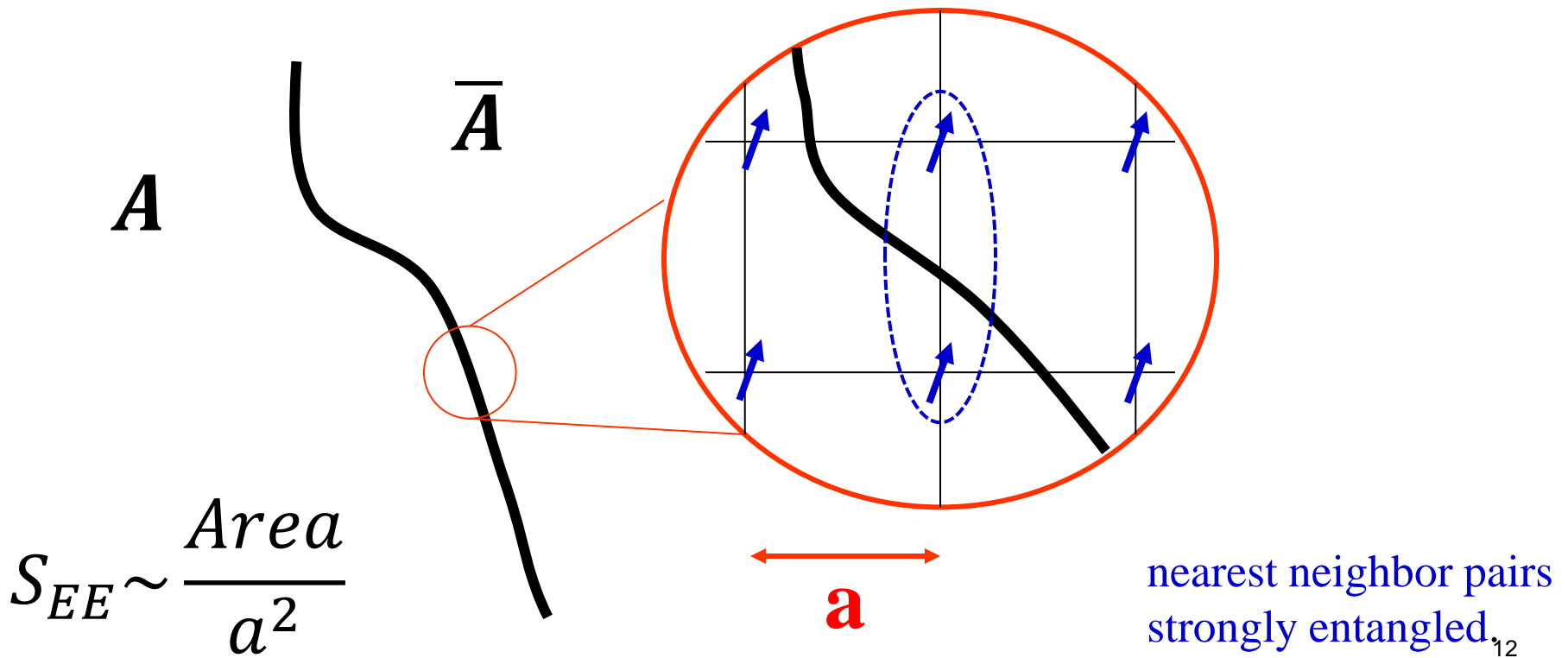
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In a local quantum field theory we can identify Subsystems **A** and **B** by their location in space.

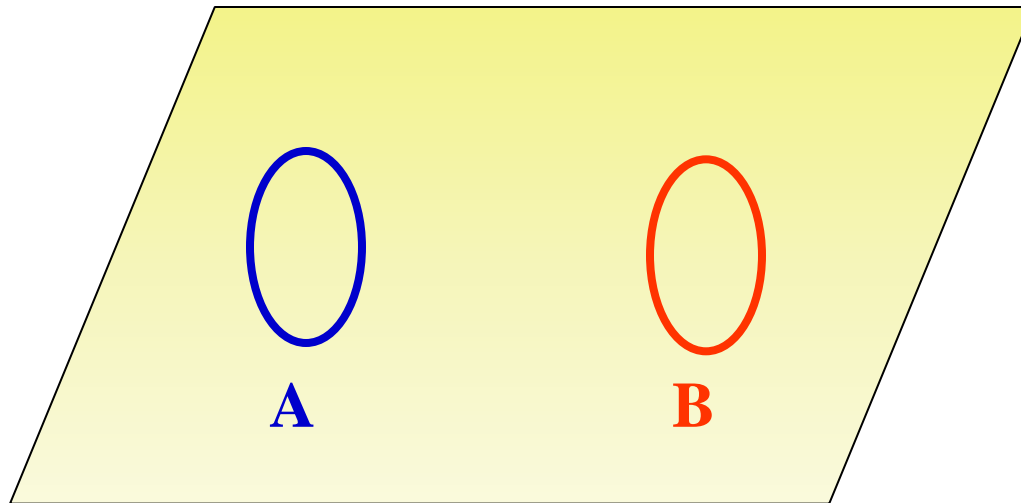
# Entanglement is **short range**

Clear Indication: Entanglement Entropy



# Entanglement defines distance!

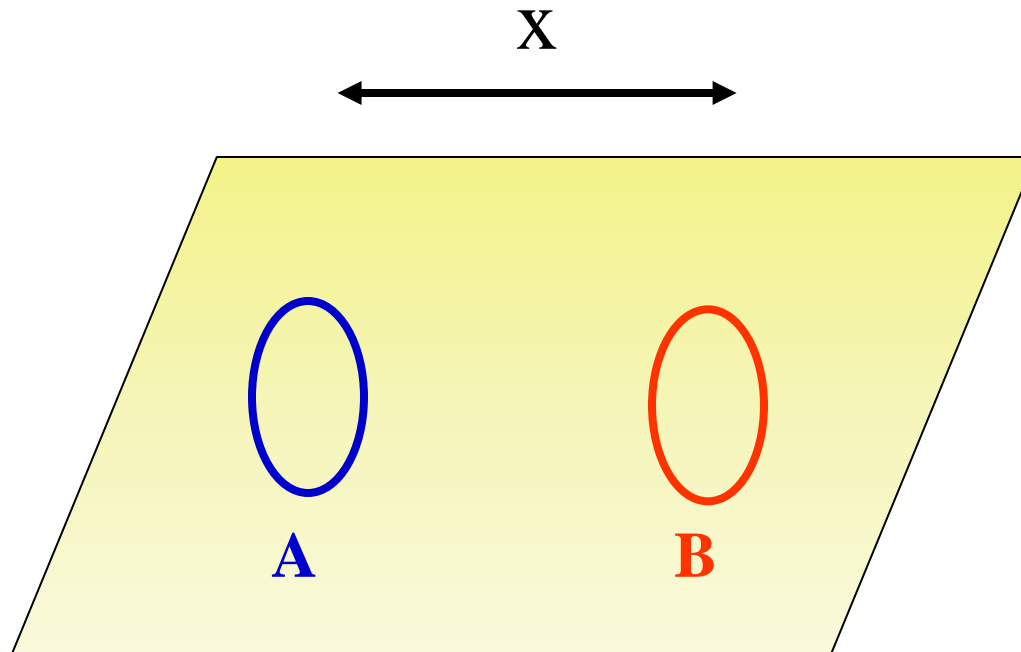
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In vacuum modes of the electromagnetic field in nearby regions A and B are entangled!

# Entanglement defines distance!

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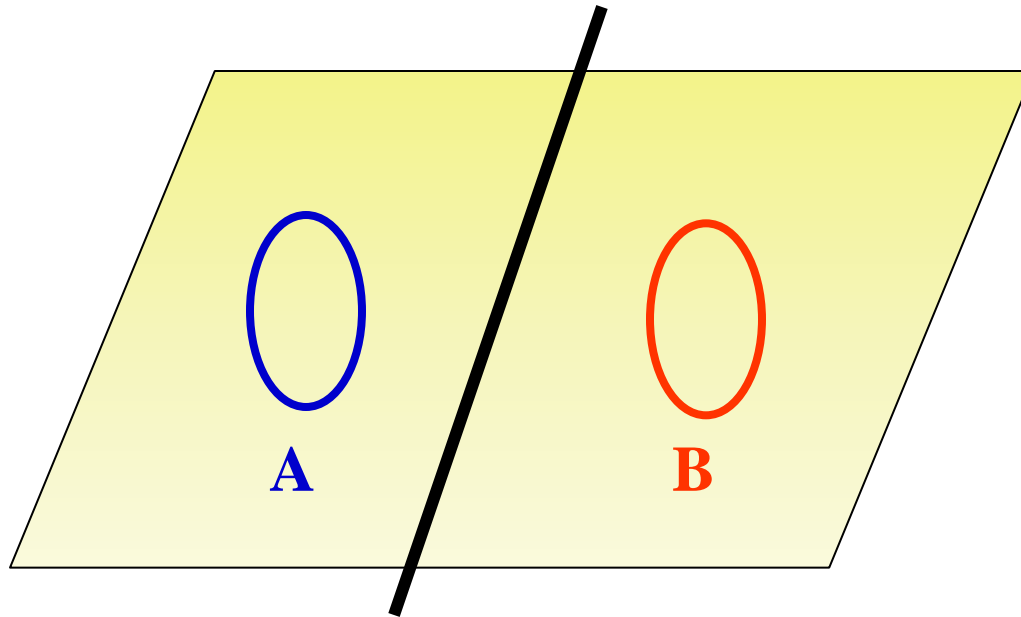


Entanglement decreases  
with distance!

$$\langle O_A O_B \rangle \sim 1/x$$

# Entanglement defines distance!

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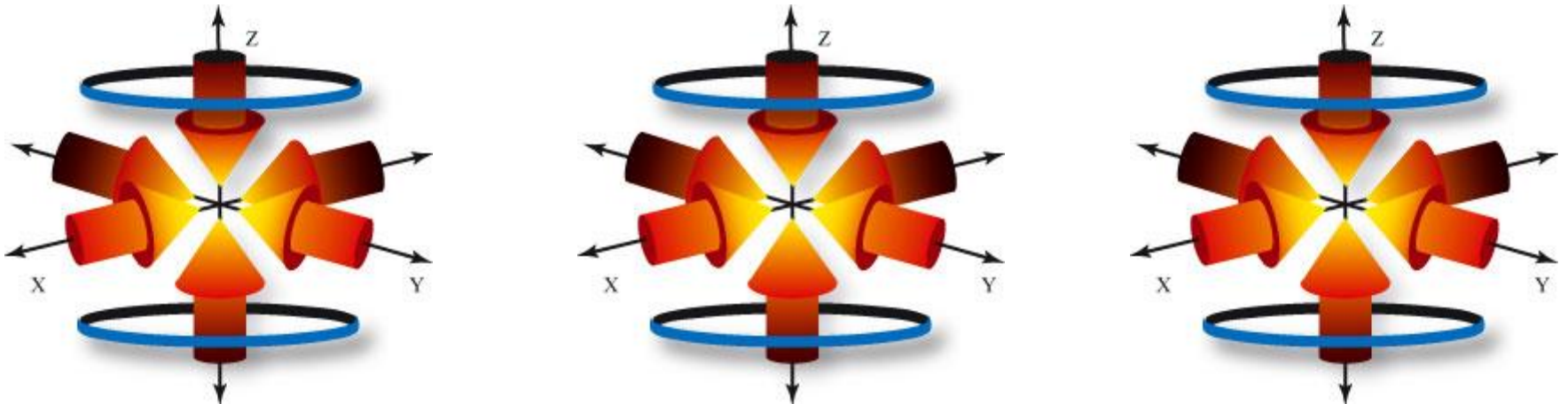


Metal plate (Dirichlet bc) disentangles the modes.

**A** and **B** are “disconnected”.

# Entanglement = Connectedness

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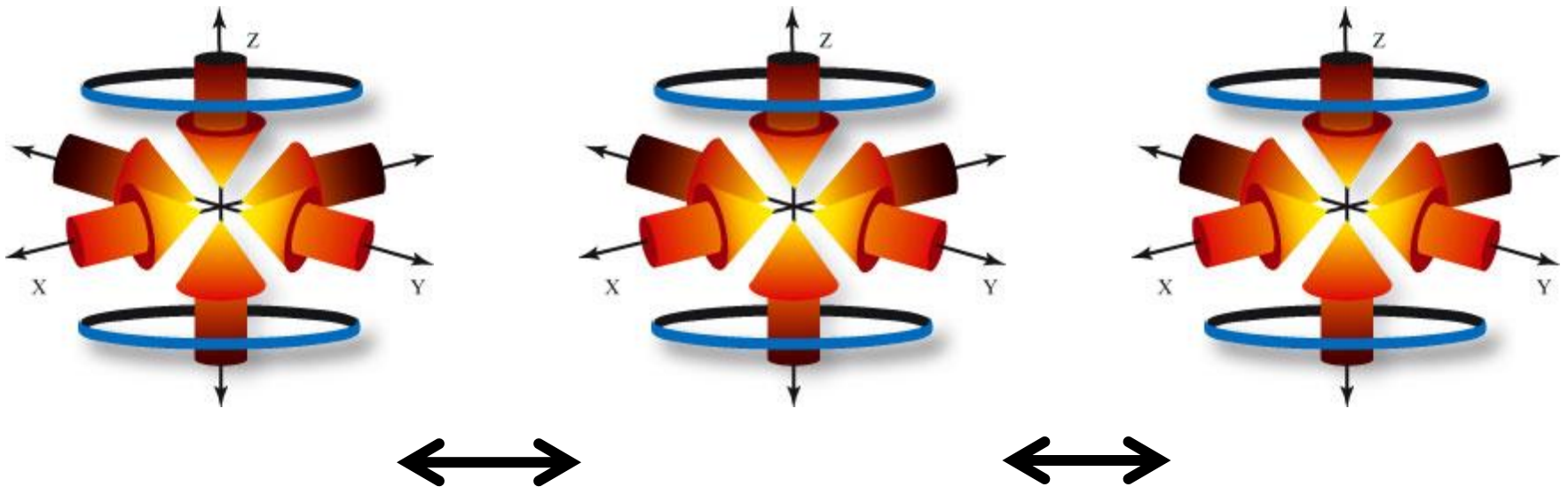


3 atoms in 3 traps; disconnected.



# Entanglement = Connectedness

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Entanglement “connects” the three into a line!

Entanglement defines notion of distance!

# Entanglement and Holography

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# Emergent Dimensions

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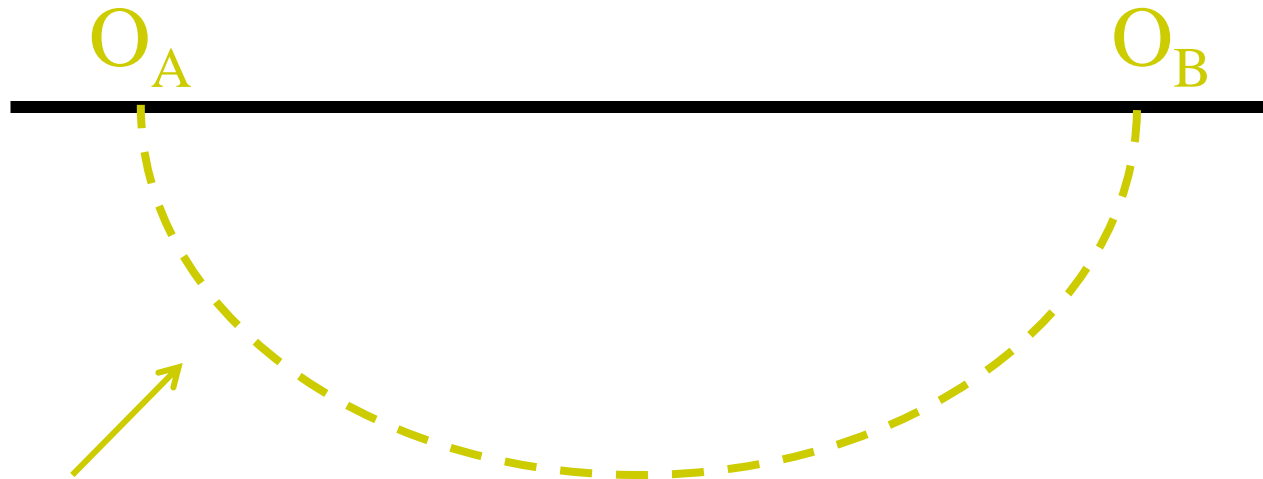
These ideas can be made very concrete in holography.

- Entanglement encodes bulk geometry  
(van Raamsdonk)
- Correlations are given by classical propagators. Finite Correlation implies finite spatial distance.

# AdS/CFT = Classical Bulk

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$$G = \langle \psi | O_A O_B | \psi \rangle - \langle \psi | O_A | \psi \rangle \langle \psi | O_B | \psi \rangle$$



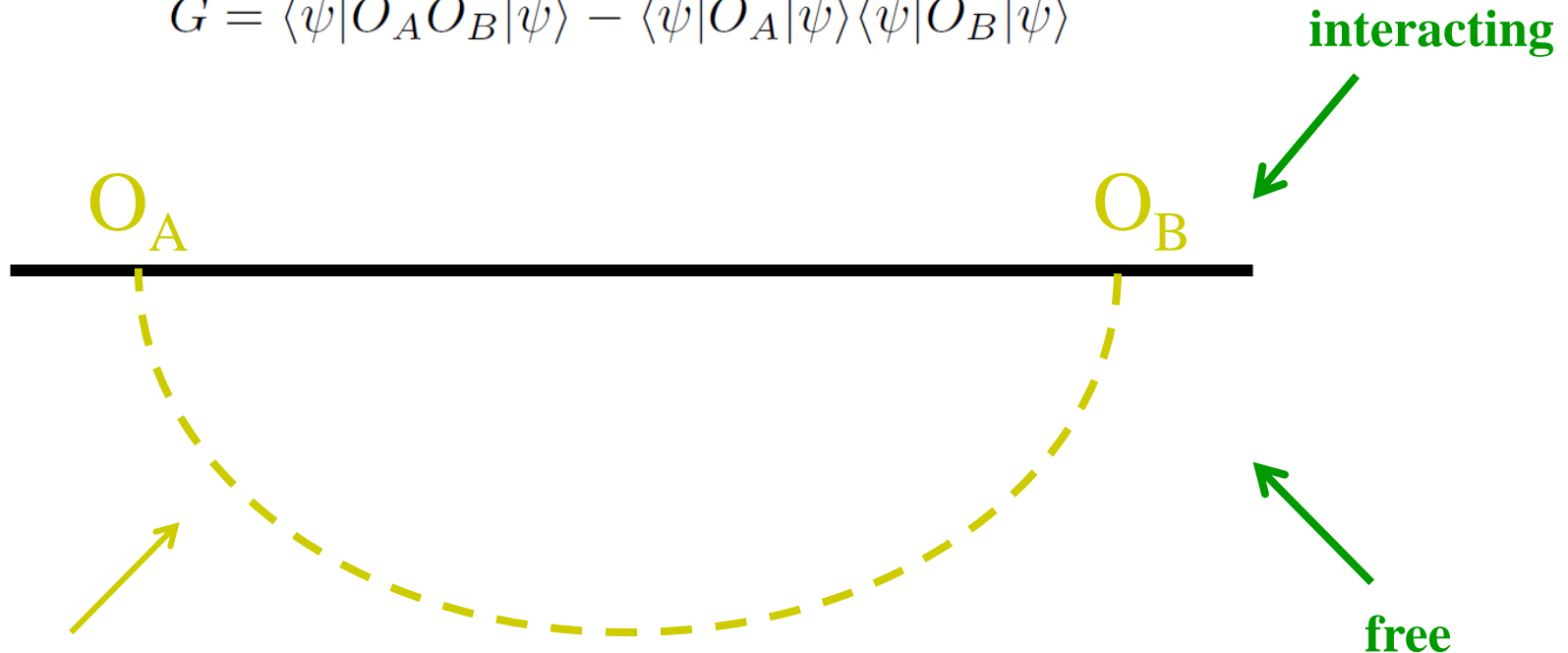
classical bulk  
Green's function

**finite correlation  
only possible for finite distance!**

# AdS/CFT = Classical Bulk

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$$G = \langle \psi | O_A O_B | \psi \rangle - \langle \psi | O_A | \psi \rangle \langle \psi | O_B | \psi \rangle$$

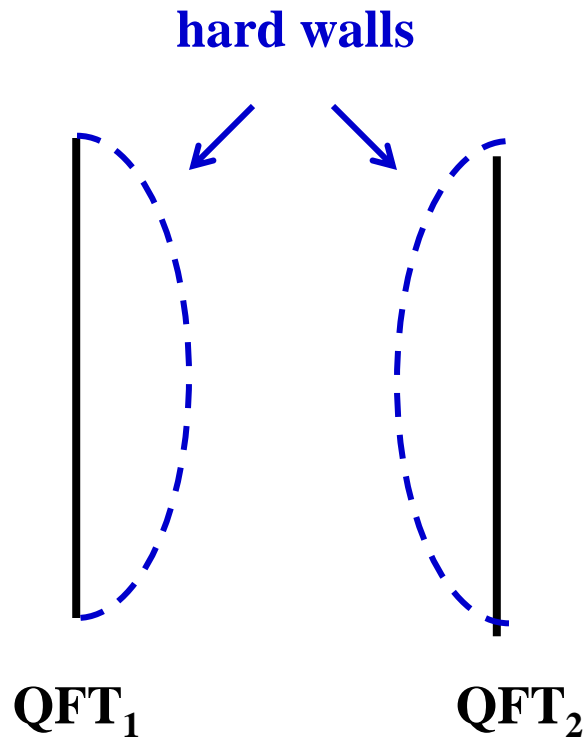


classical bulk  
Green's function

finite correlation  
only possible for finite distance!

# Example: 2 copies of a QFT

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**No Entanglement**

Product State

# Example: 2 copies of a QFT

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Even though the two QFTs are decoupled, we can still consider entangled states!

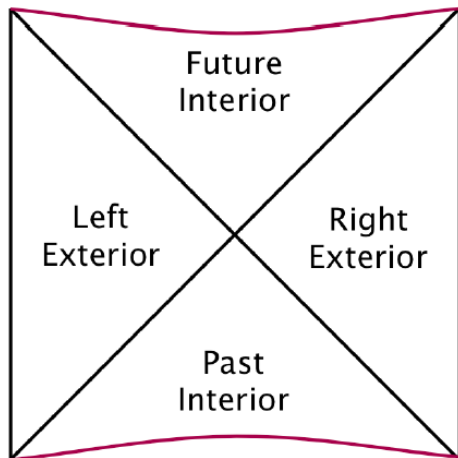
$$|\Psi\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\beta E_n/2} |E_n\rangle_1 \times |E_n\rangle_2$$

**Entangled state  
(Thermofield Double)**

- Tracing over QFT<sub>1</sub> gives thermal density matrix for QFT<sub>2</sub>
- Used for “real time” finite temperature calculations.

# Example: 2 copies of a QFT

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$$|\Psi\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\beta E_n/2} |E_n\rangle_1 \times |E_n\rangle_2$$

**Entangled state  
(Thermofield Double)**

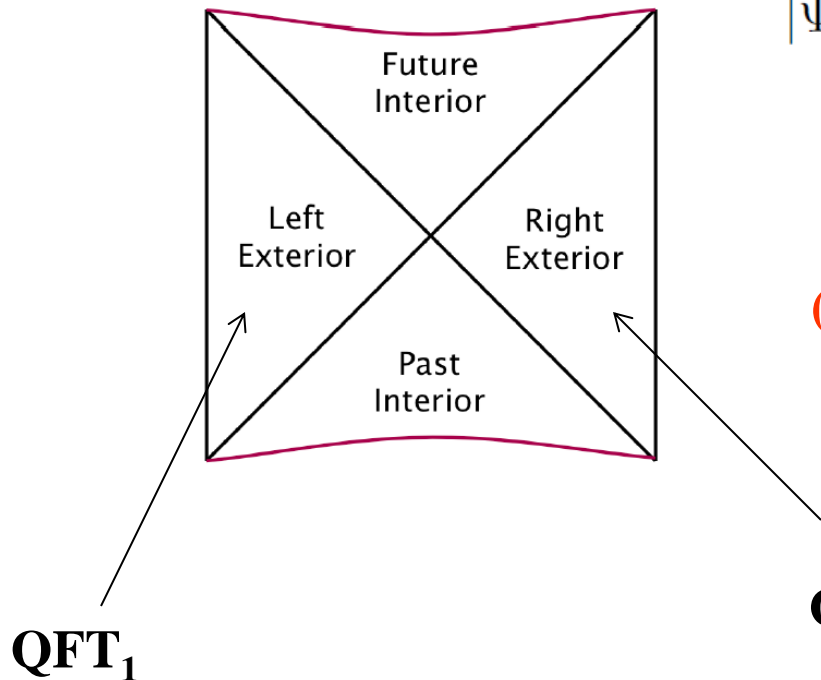
Holographic dual: Eternal AdS black hole

(Maldacena)



# Example: 2 copies of a QFT

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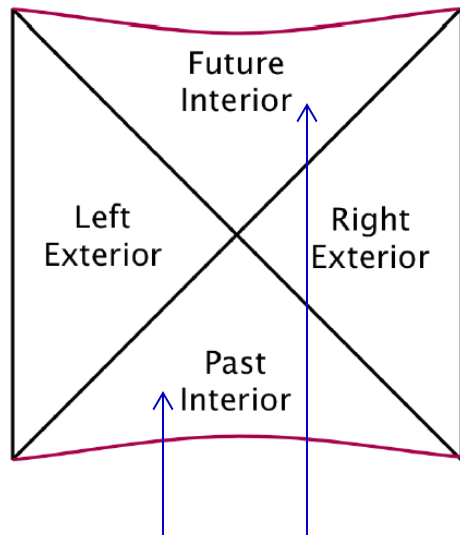
$$|\Psi\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\beta E_n/2} |E_n\rangle_1 \times |E_n\rangle_2$$

**Entangled state  
(Thermofield Double)**

**QFT<sub>2</sub>**

# Example: 2 copies of a QFT

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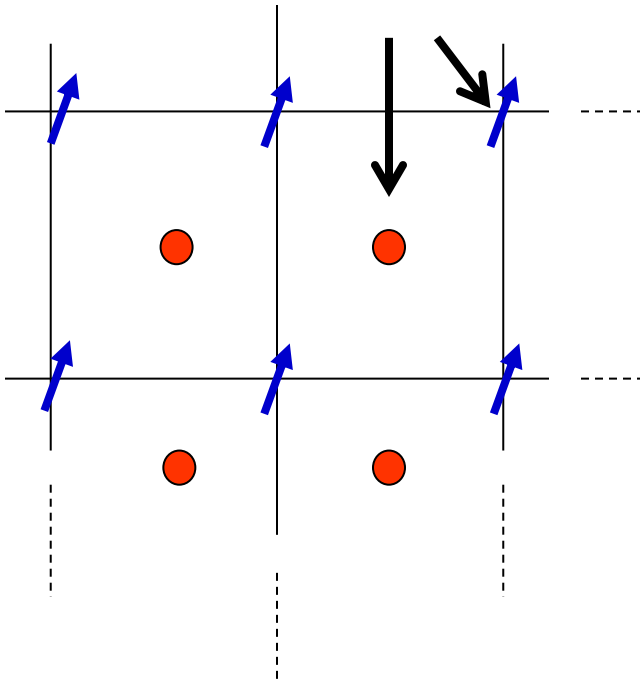
$$|\Psi\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\beta E_n/2} |E_n\rangle_1 \times |E_n\rangle_2$$

**Entangled state  
(Thermofield Double)**

**These two regions encode the correlation implied by the entanglement between the two QFTs.**

# Entanglement defines Distance

Entangled!



“space” static. or no space.  
**abstract quantum system.**  
here, e.g: **2d space**

 =earth



 =moon

**gravity**  
3d space and time are  
emergent concepts

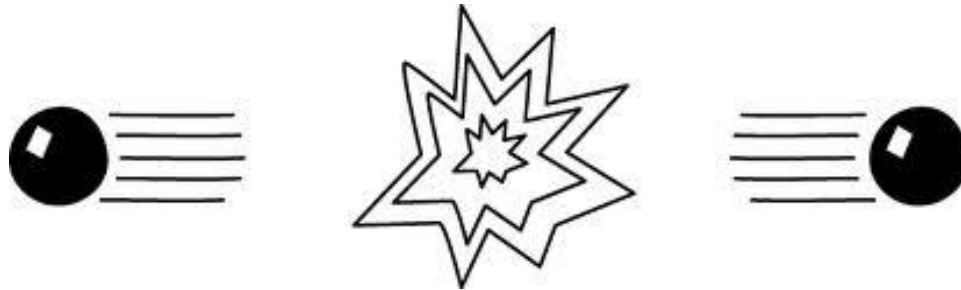
# The EPR pair

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# The EPR pair.

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Extreme example of entanglement: **EPR pair**



E.g.: **anti-quark**

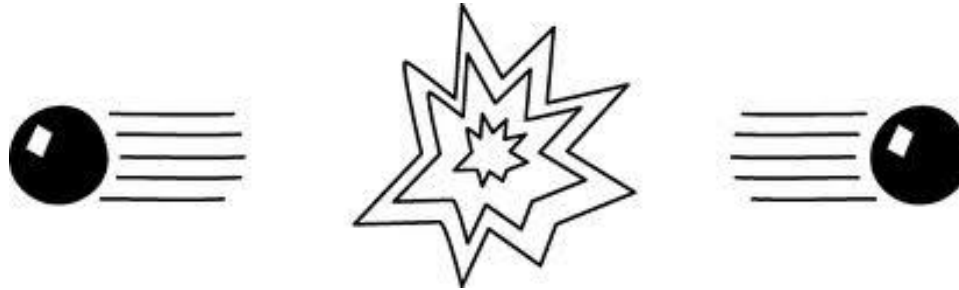
**quark**

**pair created in  
background electric field**

# The EPR pair:

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spin 0 in initial state



**We all know how to write the EPR wavefunction:**

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

# Two particle correlation

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$$\langle \vec{S}_{(1)} \cdot \vec{S}_{(2)} \rangle = -1$$

100% likelihood for spins to point  
in opposite directions!

- this is an equal time correlator
- in EPR pair this is non-zero despite of **lack of causal contact**.



# Operational definition EPR pair.

From now on we define an **EPR** pair to be an **entangled** pair **not in causal contact**.



# Holographic EPR pair

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EPR pair = non-vanishing correlators,  
but no causal contact.

(Holography): Finite spatial distance between  
the two points in the bulk, but no  
causal contact.

# Holographic EPR pair

---

EPR pair = non-vanishing correlators,  
but no causal contact.

**(Holography):** Finite spatial distance between  
the two points in the bulk, but no  
causal contact.

**This defines an ER bridge / wormhole!**

# ER=EPR

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So almost by definition, the holographic dual of an EPR pair is a non-traversable ER wormhole.

(Maldacena-Susskind)

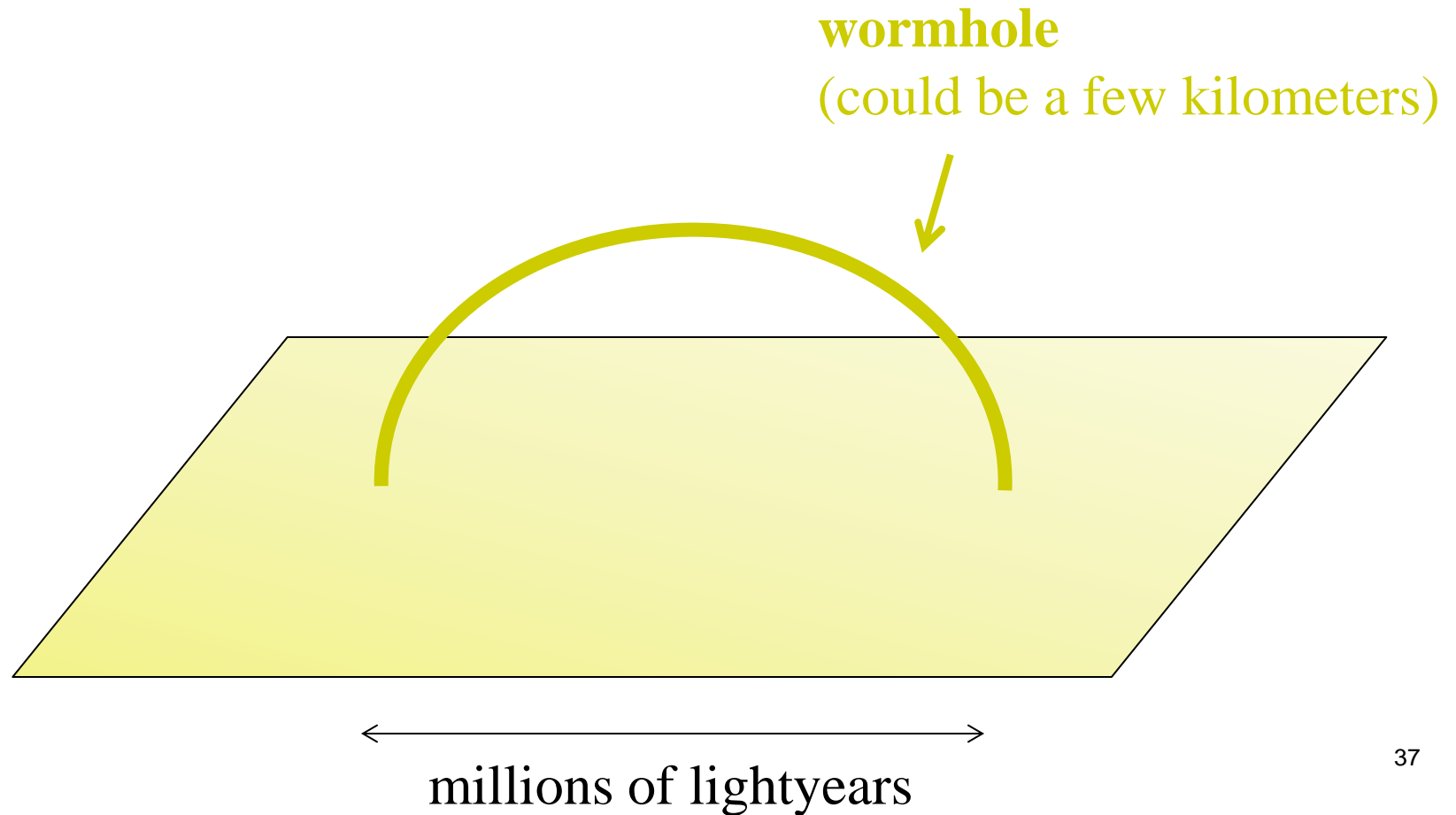
(MS make slightly stronger statement.  $EPR=ER$ .  
Crisp statement: ER and EPR give rise to identical physical consequences = correlators)

# The ER Bridge

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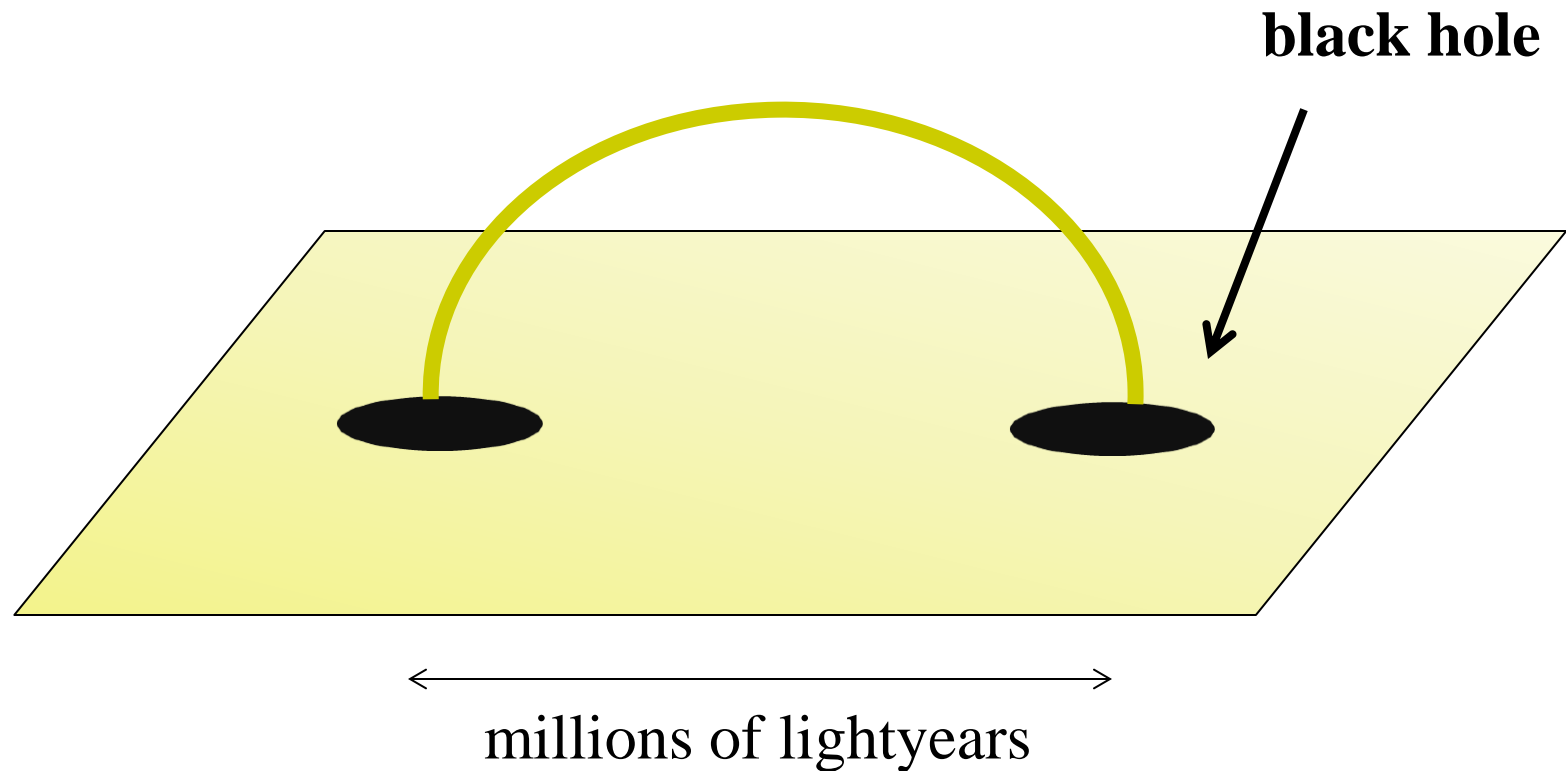
# Einstein-Rosen Bridge

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# Einstein-Rosen Bridge

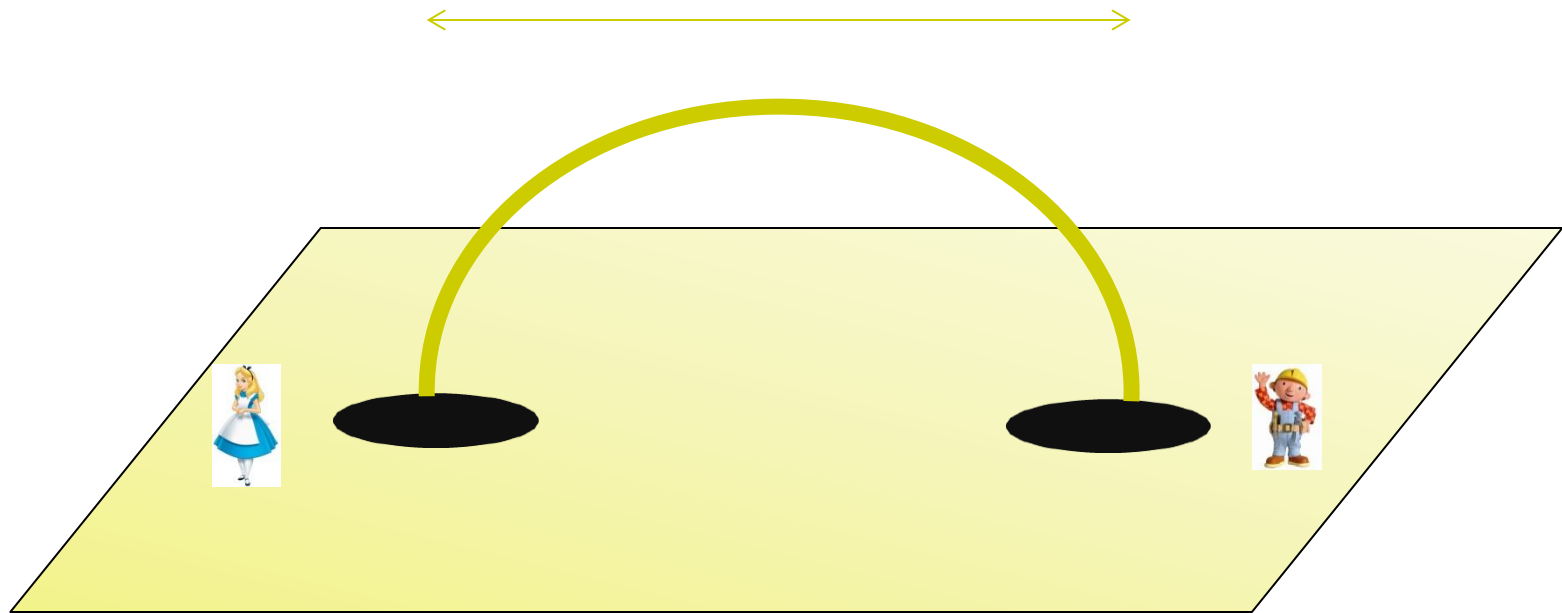
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# Einstein-Rosen Bridge

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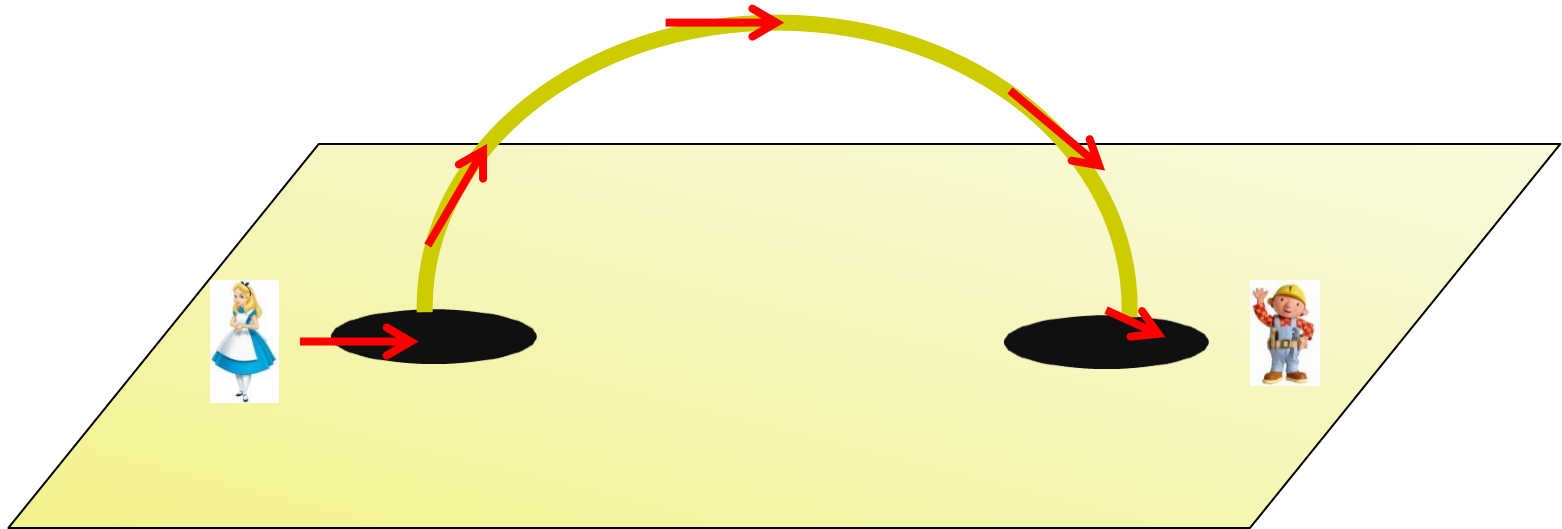
distance (through wormhole) just  
a couple of kilometers  
(finite spatial distance)



# Einstein-Rosen Bridge

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Alice can send signal into wormhole,  
but signal can't reach Bob.  
(Causally disconnected).

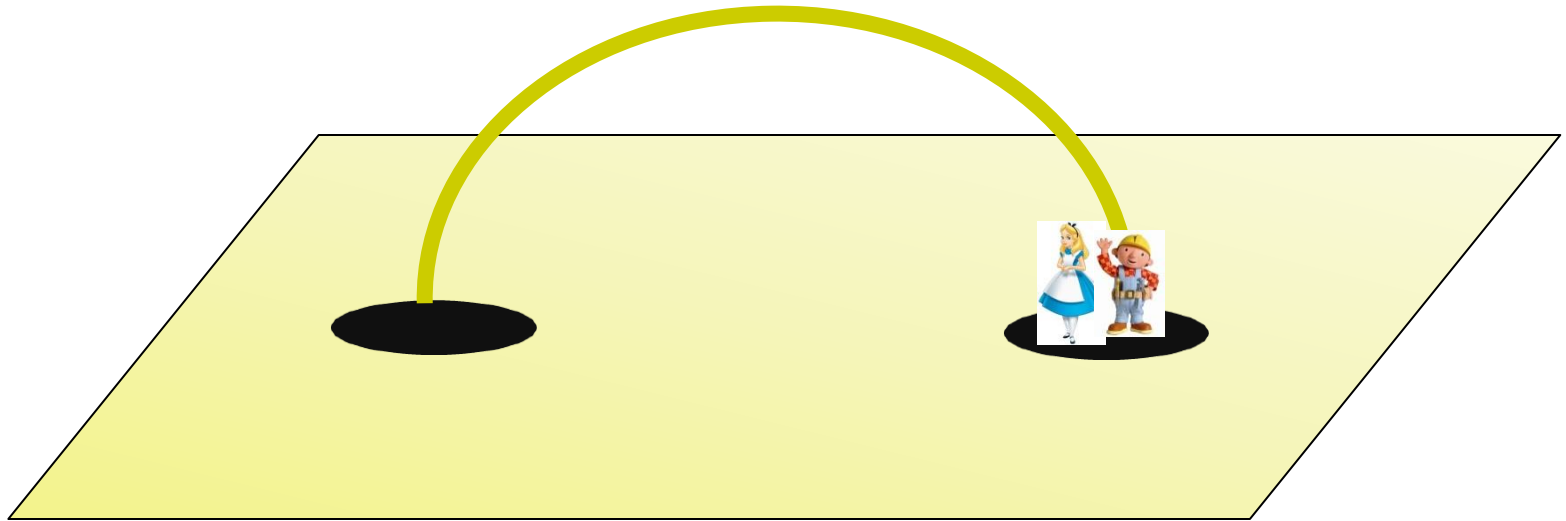




# Einstein-Rosen Bridge

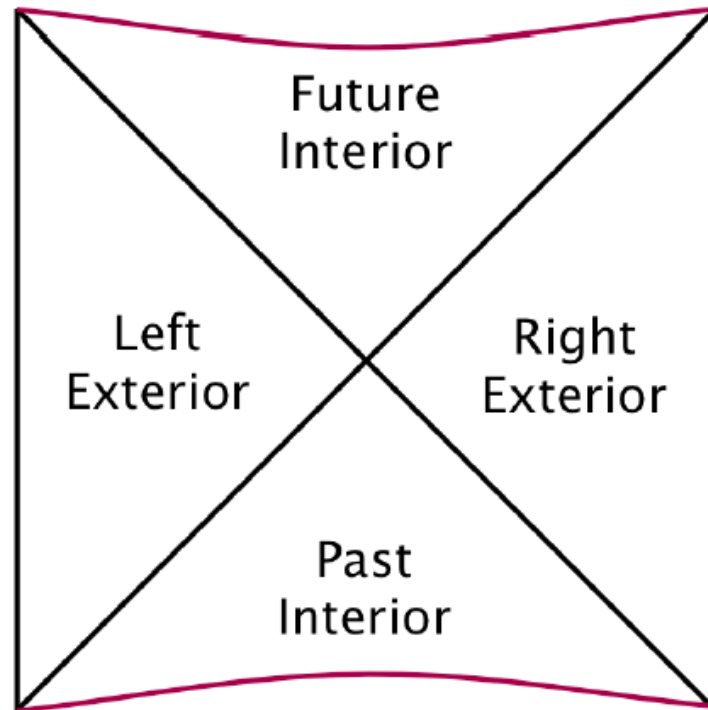
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But if **both** jump into their respective black hole, they can easily meet (but never come back out)



# Example: Eternal Black Hole

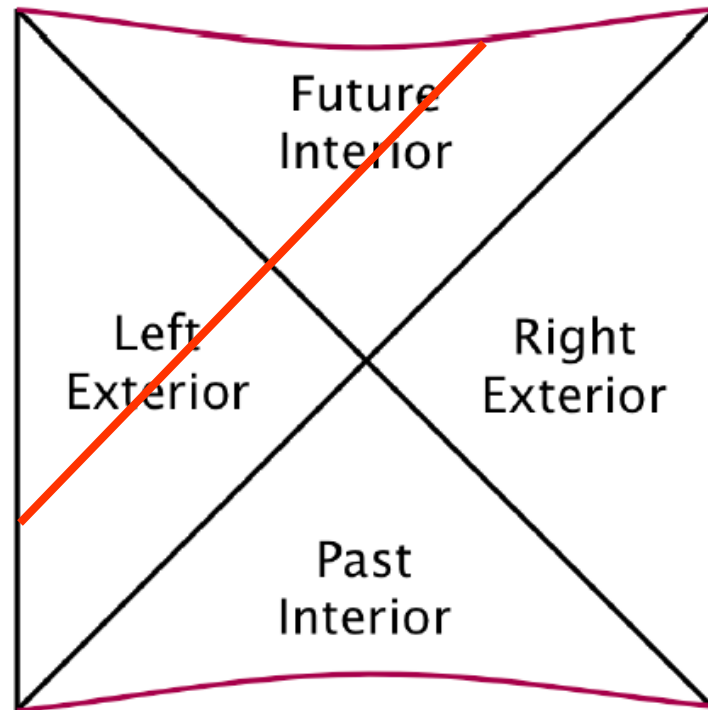
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**Think of this as describing two far away entangled black holes**

# Example: Eternal Black Hole

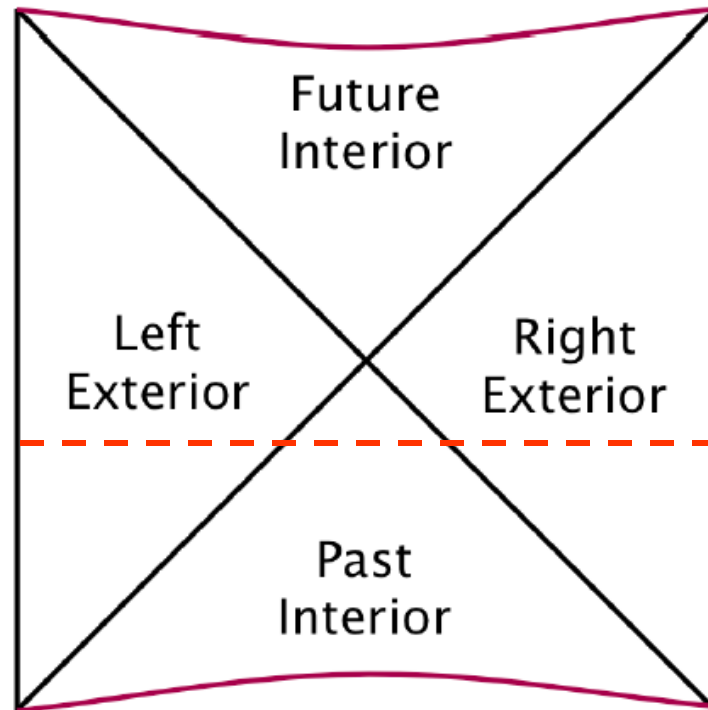
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**no causal contact between the two asymptotic regions...**

# Example: Eternal Black Hole

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**... but finite spatial distance.**

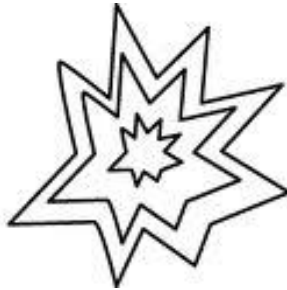
# Holographic EPR pair

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# Example: holographic EPR (Jensen-AK)

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**anti-quark**



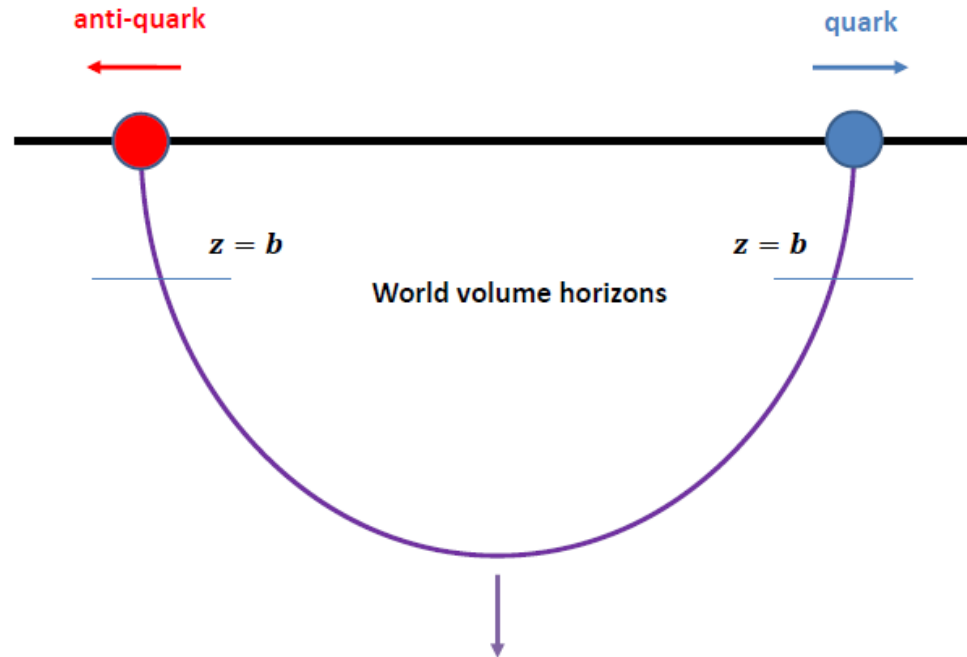
**quark**



**pair created in  
background electric field**

We can do this in N=4 SYM.  
What is the holographic dual?

# Holographic EPR



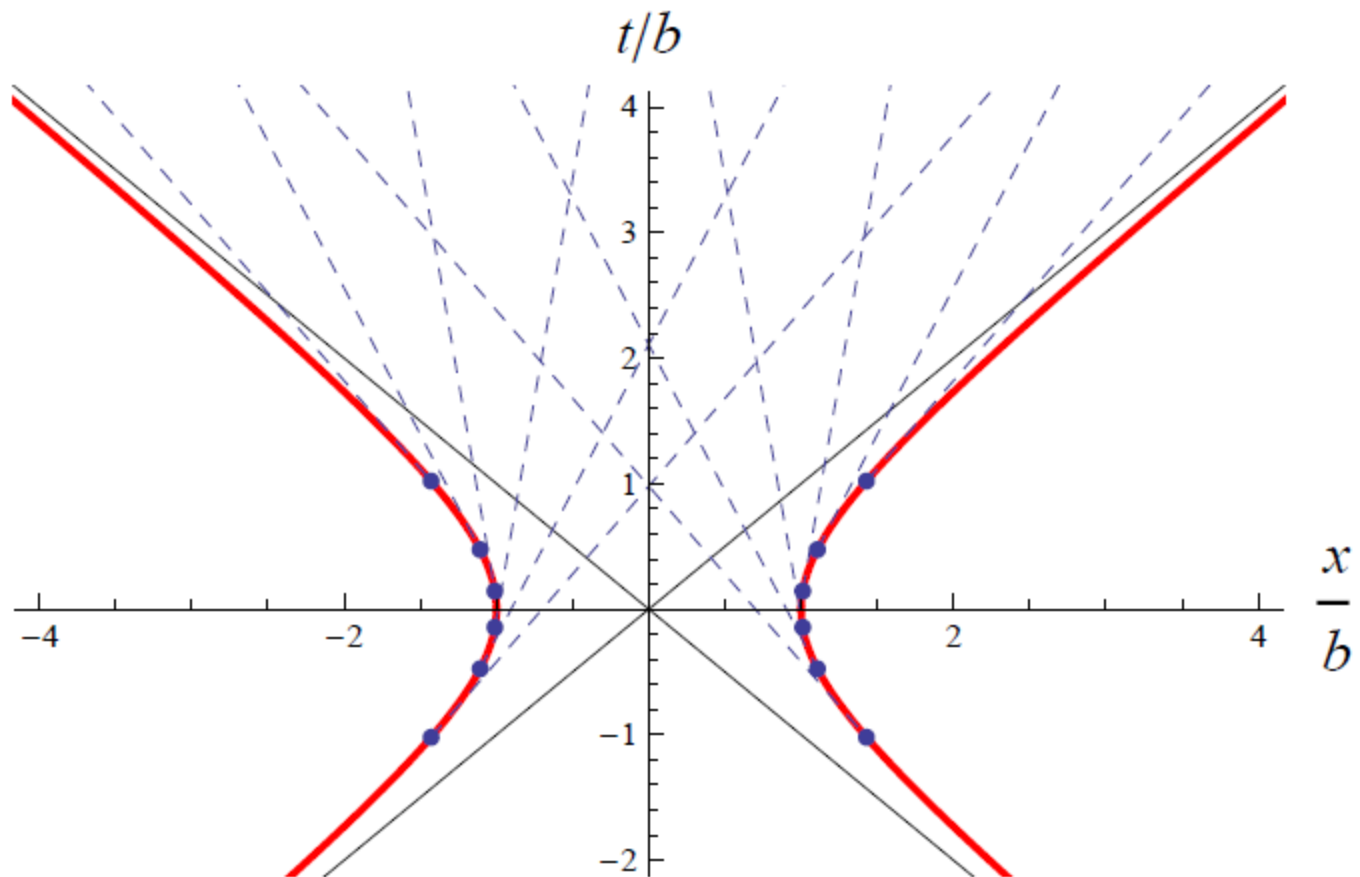
$$x^2 = t^2 + b^2 - z^2 .$$

(Xiao)

**Worldsheet = ER bridge (finite distance, no causal connection)** 47

# Holographic EPR: geodesics

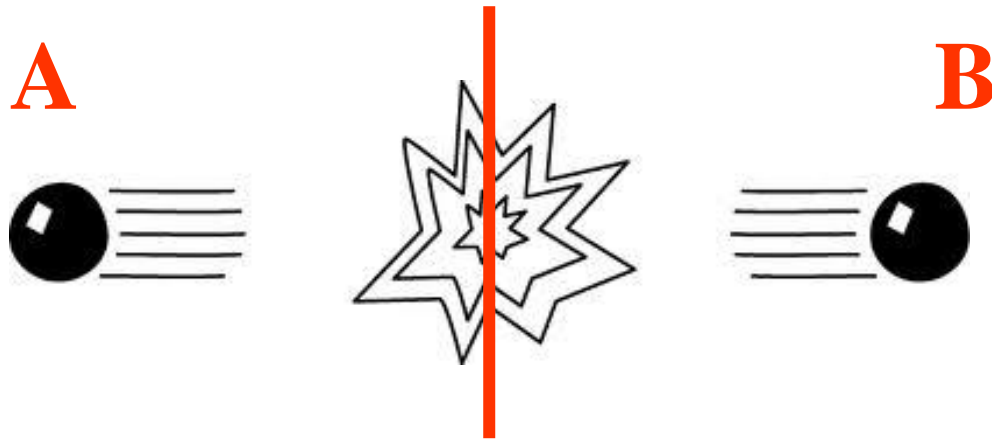
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# $S_{EE}$ for the holographic EPR pair

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Finally: calculate EE for a probe brane!

$$S_{EE} = \sqrt{\lambda}/3$$

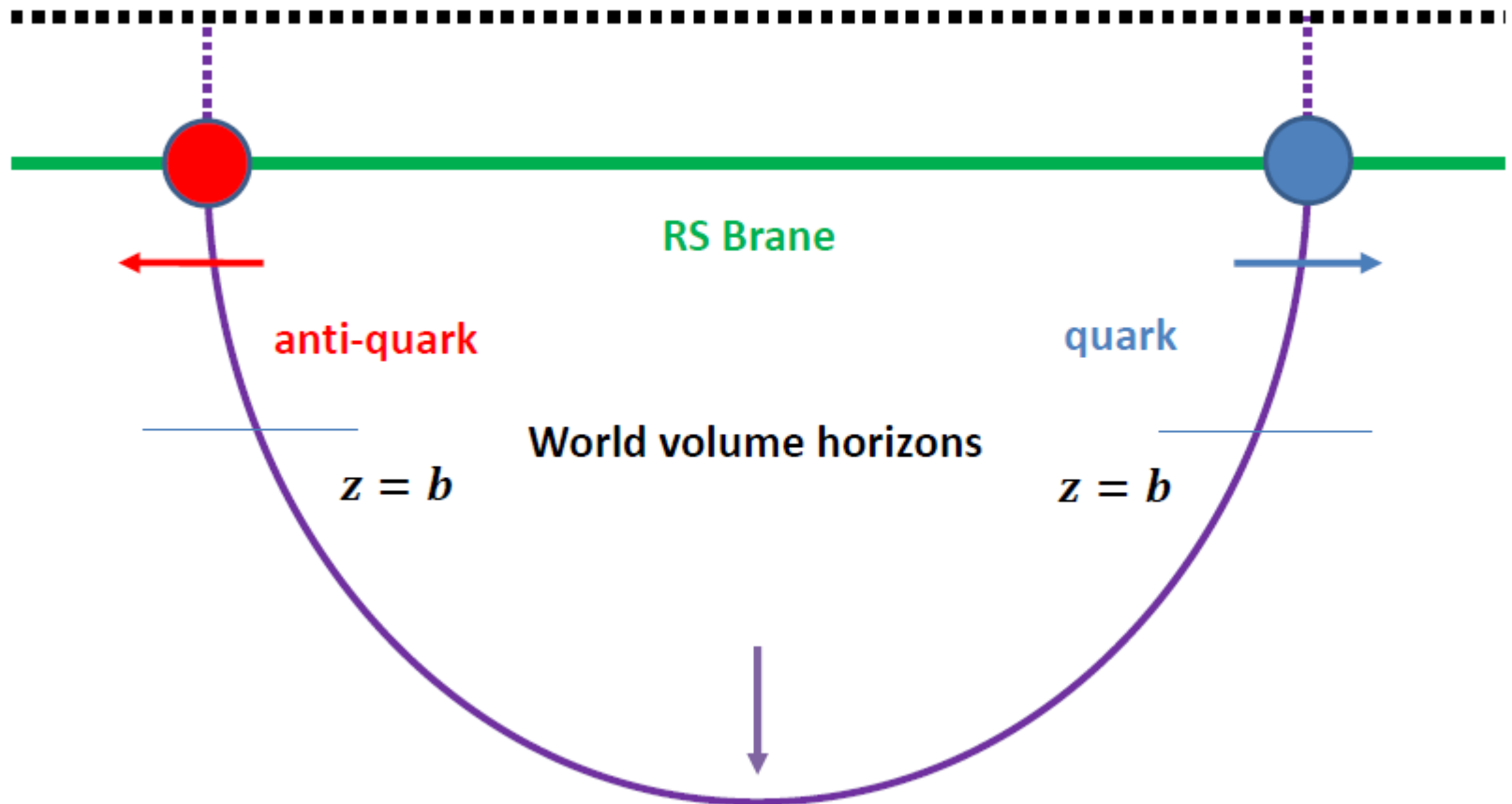
quark not just a single parton,  $\sqrt{\lambda}$  gluons part of quasi-particle <sup>49</sup>

# Generalizations

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- Including Dynamical Gravity
- Holographic Hawking Pairs

# 1) Dynamical Gravity via RS



# Bottom Line: No Change

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RS-holography:

**CFT with UV cutoff  $a$   
+ dynamical gravity with  
 $G_N$  induced by matter loops**      =      **Brane located at fixed radial  
position; 5d fluctuations induce  
a localized 4d mode.**

Only change: Horizon disappears when  
q-qbar separation is less than  $a$ .

## 2) Hawking Pairs

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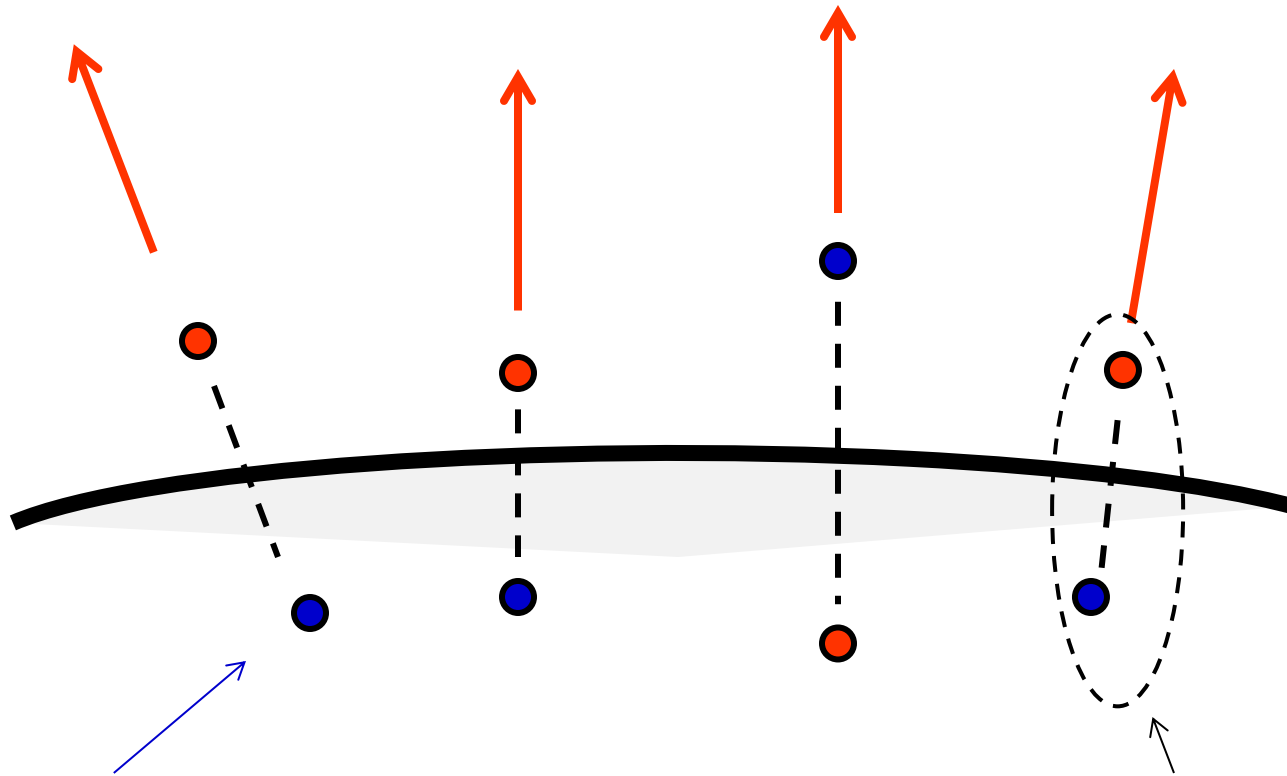
AdS/CFT allows us to study the CFT on any background metric  $g_0$

$g_0$  sets the near boundary behavior of the bulk metric. Need to solve Einstein equations with that boundary behavior (Fefferman-Graham).

If we chose  $g_0$  to be a black hole, we can study a **Hawking pair** (= EPR pair separated by horizon).

# Hawking Radiation:

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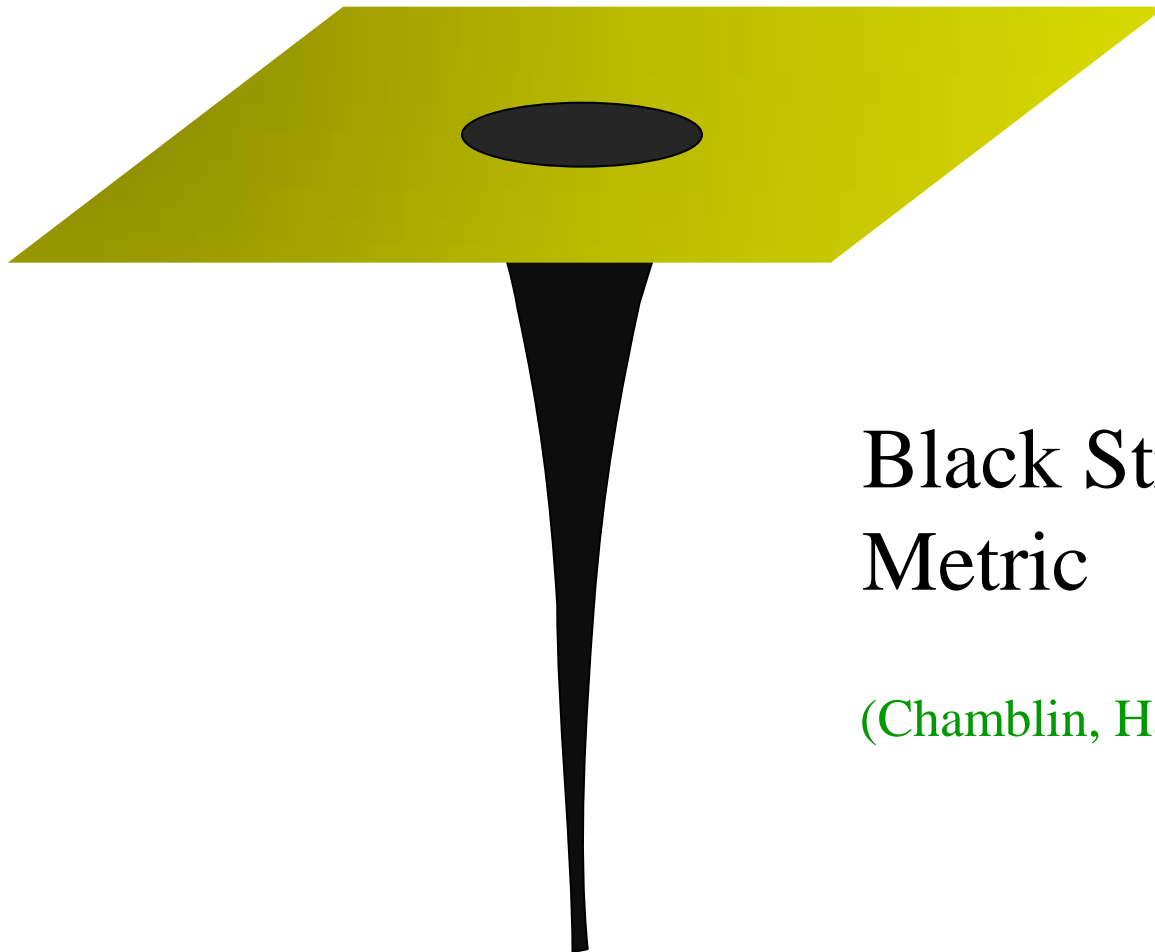


**Quark in flavored  
N=4 SYM.**

**What's the holographic  
dual to a Hawking pair?**

# N=4 on Schwarzschild Black Hole

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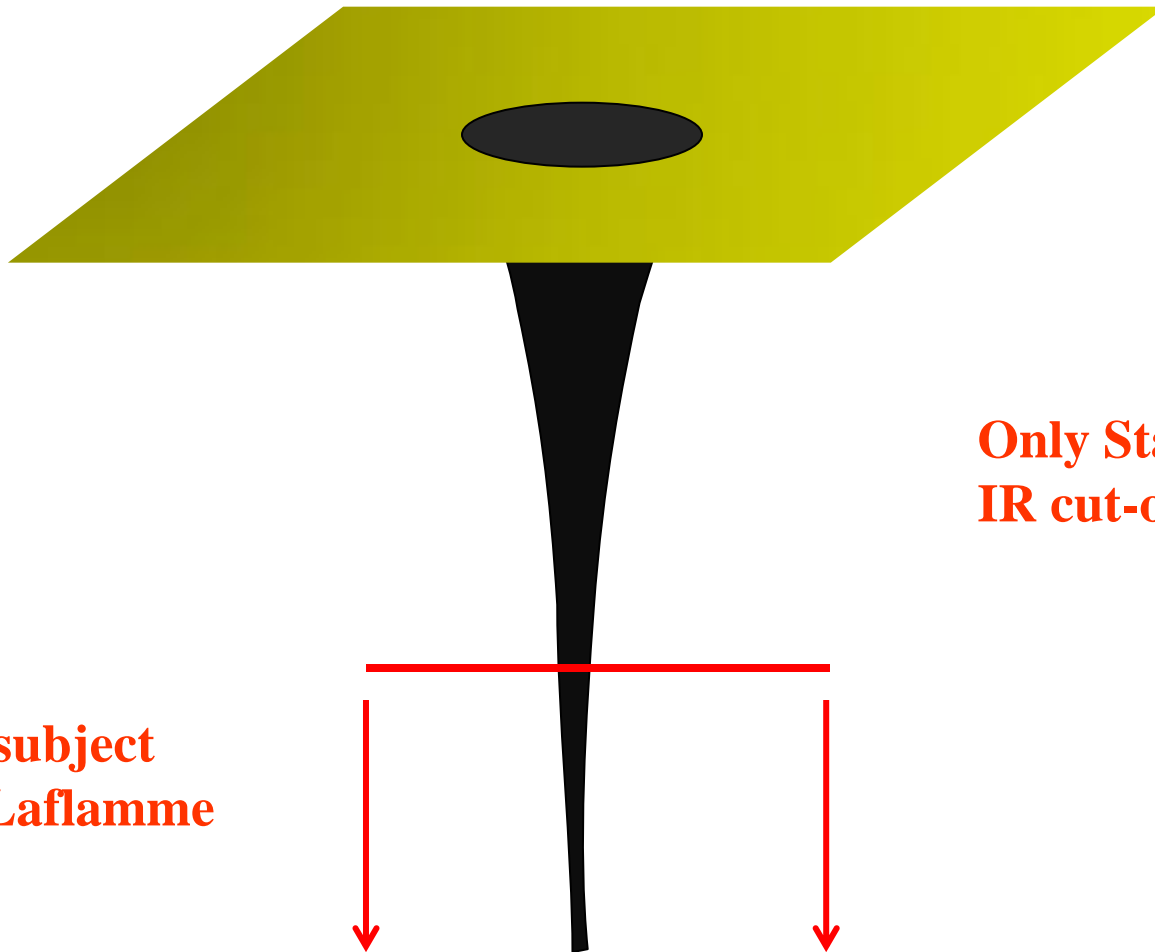


Black String  
Metric

(Chamblin, Hawking, Reall)

# N=4 on Schwarzschild Black Hole

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**Thin String subject  
to Gregory-Laflamme  
Instability!**

**Only Stable with  
IR cut-off (hard wall).**



# Unstable vacuum on BH

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The unstable state can still be studied.

Puzzle: **No Hawking radiation!**

$$\langle T_{\mu\nu} \rangle = 0$$

Potential Resolution:

at strong coupling, only color neutral  
Hawking radiation!

# Sharpening the Puzzle

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Or maybe we just shouldn't over-interpret the unstable state?

Puzzle can be sharpened:

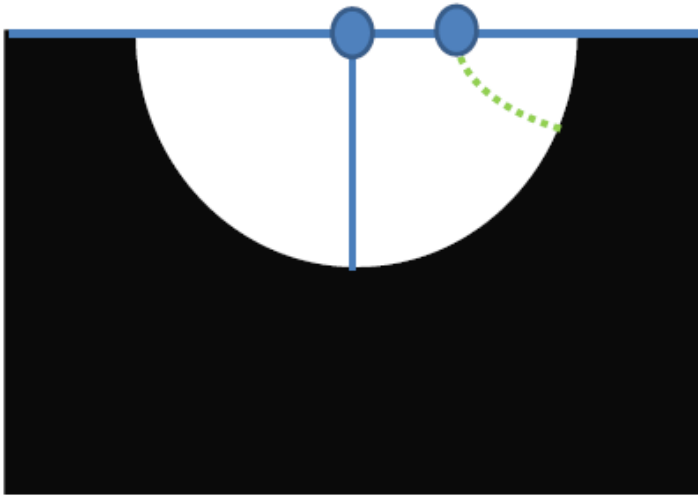
Black String for  $\text{AdS}_4$ -black hole is stable!

Still no Hawking Radiation.

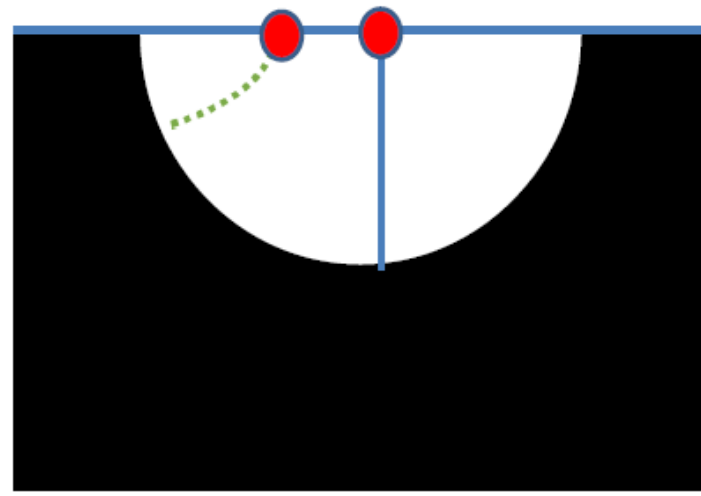
(Chamblin, AK; Gregory, Ross, Zegers)

# AdS<sub>4</sub> Black String Metric

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mSYM on two joined  
copies of AdS<sub>4</sub>  
black holes



thermofield double

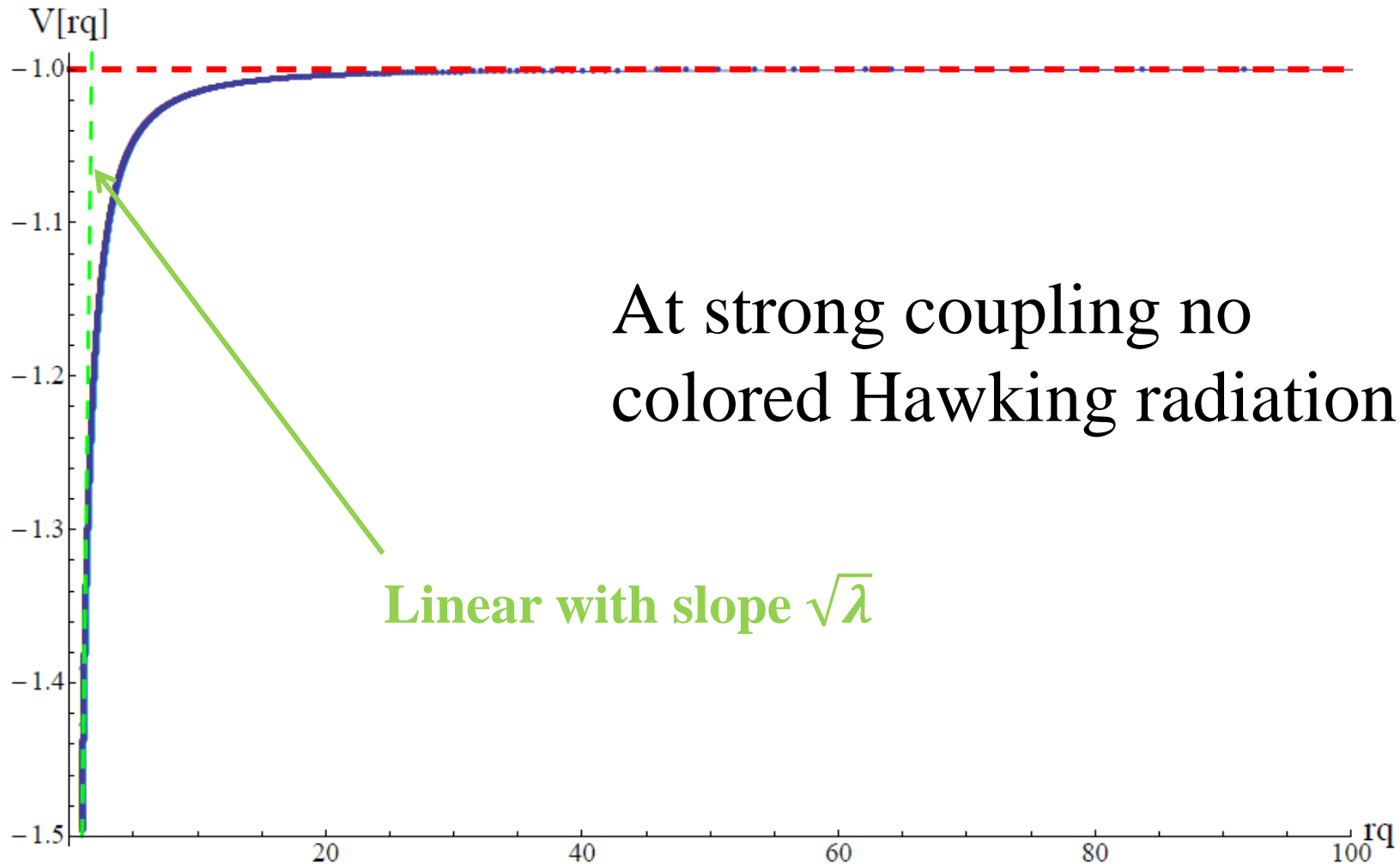
# Hawking pair dual to ER

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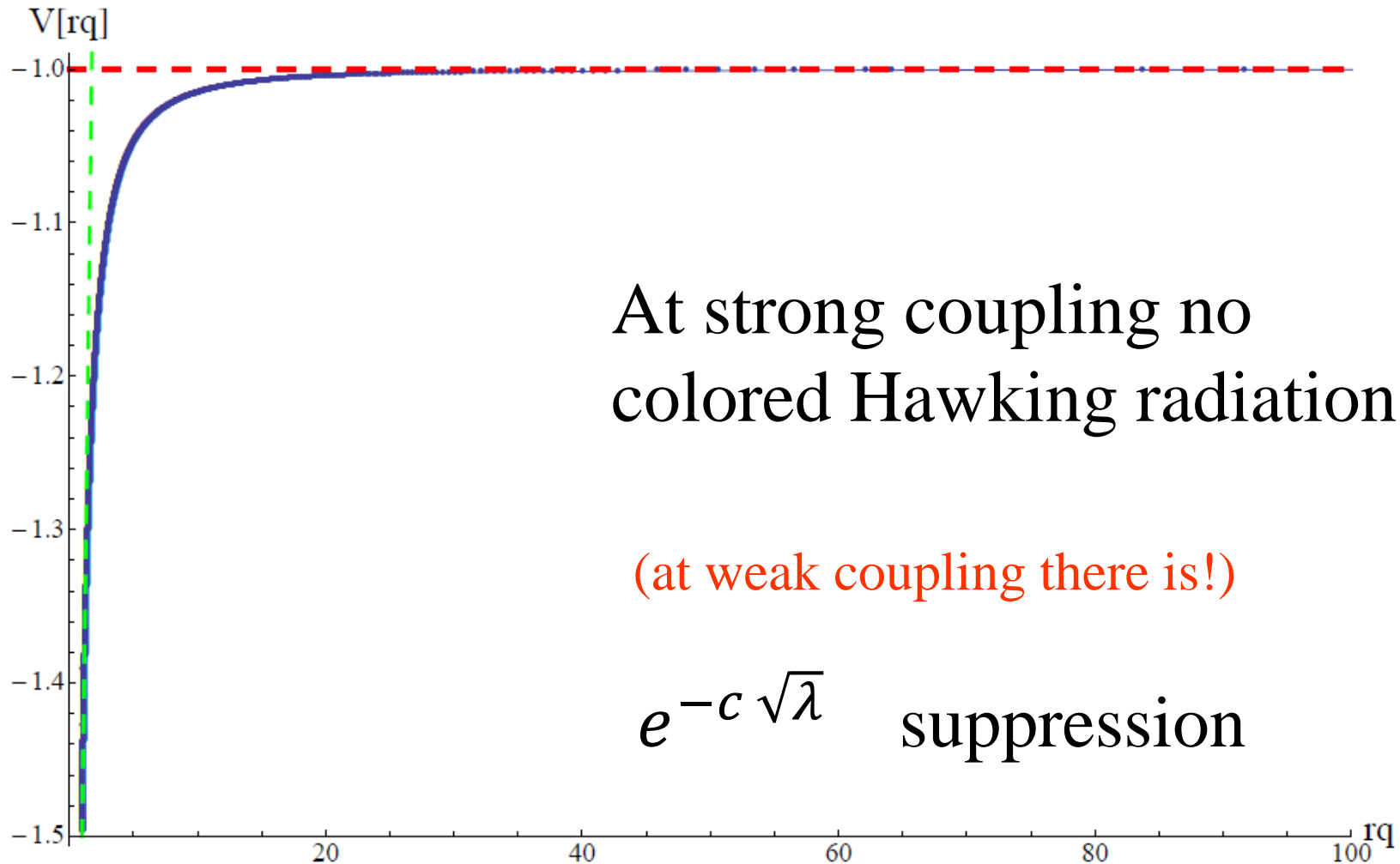
All our strings for all time  
cross the bulk horizon through the  
bifurcation point, they inherit  
the causal structure of the bulk ER bridge.

Worksheet metric = ER bridge.

# And Confinement!



# And Confinement!



# Conclusions

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The holographic dual of an EPR pair has to have the geometric structure of an ER bridge.

ER=EPR true almost by definition.



# Conclusions

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Bigger insight: Entanglement encodes  
Geometry holographically.





# Conclusions

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Can study consequences of Entanglement  
in strongly coupled systems.

E.g: Is the ridge in heavy ion collisions  
related to entanglement?

# Conclusions

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Side Benefit:

Absence of Hawking radiation in strongly coupled mSYM indeed due to confinement!

Fitzpatrick, Randall, Wiseman were right!