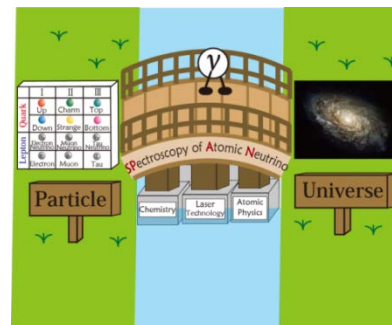


Towards Neutrino mass spectroscopy using atoms

M. Yoshimura @Okayama University
Experimental progress N. Sasao

- Why atoms for neutrino physics
- Unique way to distinguish Majorana from Dirac, and to determine the smallest neutrino mass
- Relic 1.9 K neutrino detection is feasible



SPAN project

Introduction

- What have been, and have not been, determined in neutrino experiments so far
- Remaining important questions on neutrino properties to probe physics beyond the standard theory and cosmology

Present status of neutrino physics

- Oscillation experiments

- Finite mass
- Flavor mixing
- Only mass-squared difference can be measured.

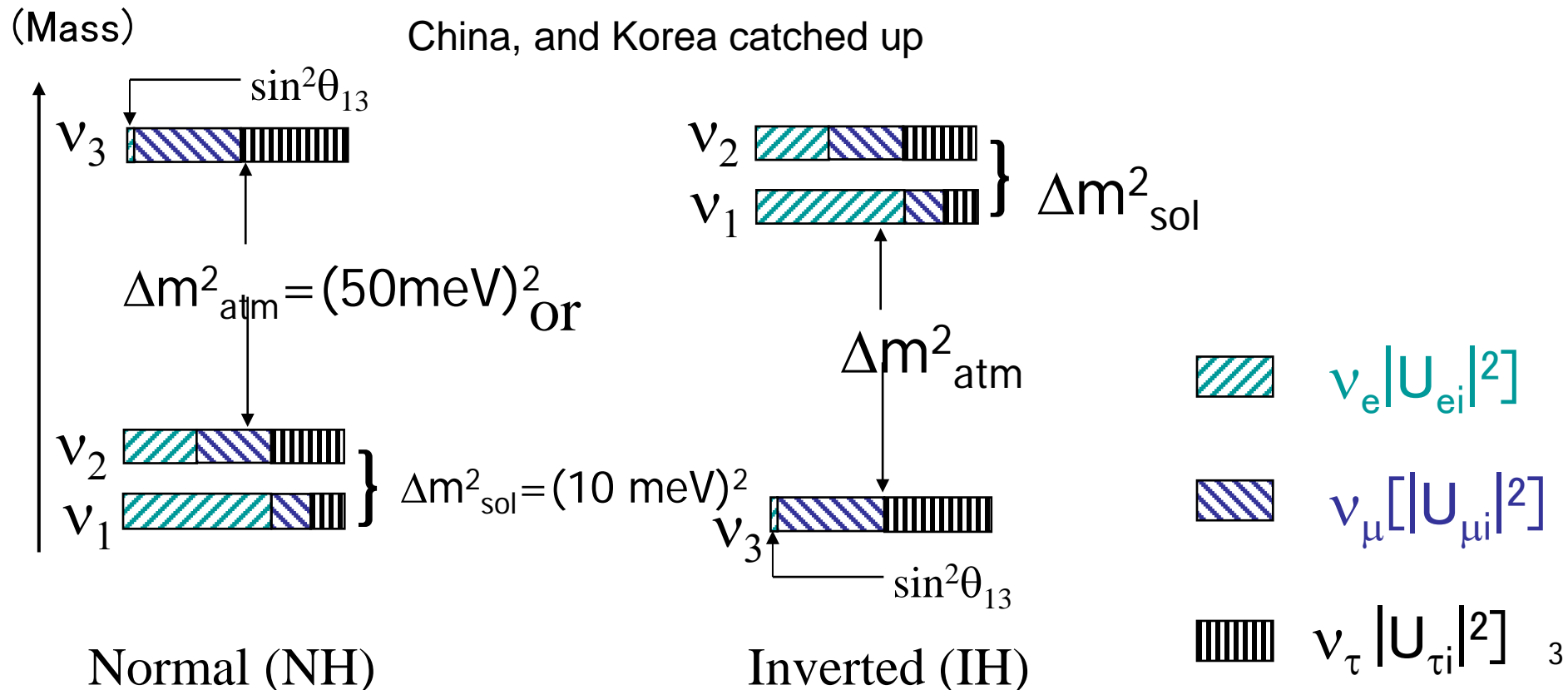
$$U = VP, \quad (A8)$$

where

$$V = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{23} + s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}, \quad (A9)$$

with $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$. The diagonal unitary matrix P may be expressed by

$$P = \text{diag.}(1, e^{i\alpha}, e^{i\beta}), \quad (A10)$$



Important questions left in neutrino physics

- Absolute mass scale and the smallest mass (oscillation experiments are sensitive to mass squared differences alone)
- Majorana vs Dirac distinction
- CPV phase (Majorana case has 2 extra phases)
 α, β, δ (KM – type)

These are relevant to explanation of matter-antimatter imbalance of universe.

We shall experimentally achieve all of these goals.

Our Okayama group has proposed an entirely new method using atoms, initiated R&D works and succeeded in establishing the huge rate enhancement in QED process, along with theoretical works.

Significance of Majorana neutrinos

- Plausible scenario of lepto-genesis

Heavy Majorana decay responsible for generation of lepton asymmetry, being converted to baryon asymmetry via strong electroweak B, L violation keeping B-L conserved.

Prerequisite: ordinary neutrinos are also Majorana.
New CPV sources related to heavy partners of mass \gg Fermi scale

- Seesaw mechanism and an important step for construction of grand unified theory

$$\frac{m^2}{M}$$

Lepto-genesis

- Leading theory to explain the matter-antimatter imbalance of our universe
- Prerequisite: lepton number violation or Majorana type of mass, CP violation
- Sensitivity to low energy parameters Davidsson-Ibarra, NPB648, 345(2003)
CP asymmetry in leptogenesis

$$\approx \frac{3y_1^2}{4\pi} \left(-2\left(\frac{m_3}{m_2}\right)^3 s_{13}^2 \sin 2(\delta + \alpha - \beta) + \frac{m_1}{m_2} \sin(2\alpha) \right)$$

+ (high energy phases inaccessible in low energy experiments)

Ours are sensitive to α , $\beta - \delta$; the same as in lepto - genesis

Majorana vs Dirac equations: chirally projected solutions



Majorana eq. : particle=antiparticle

$$(i\partial_t - i\vec{\sigma} \cdot \vec{\nabla})\varphi = im\sigma_2\varphi^*$$

$$\varphi_{\vec{p},h}(x) = c(\vec{p},h)e^{-ip\cdot x}u(\vec{p},h) + c^\dagger(\vec{p},h)e^{ip\cdot x}\sqrt{\frac{E_p + hp}{E_p - hp}}(-i\sigma_2)u^*(\vec{p},h),$$

$$u(\vec{p},h) = \frac{1}{2}\sqrt{\frac{E_p - hp}{pE_p(p + hp_3)}}\begin{pmatrix} p + hp_3 \\ h(p_1 + ip_2) \end{pmatrix}.$$

2 neutrino wave functions are anti-symmetrized

Dirac eq.: degenerate 2 Majorana

$$(i\partial_t - i\vec{\sigma} \cdot \vec{\nabla})\varphi = m\chi, \quad (i\partial_t + i\vec{\sigma} \cdot \vec{\nabla})\chi = m\varphi$$

2-component in weak process involved

$$\psi_D = (1 - \gamma_5)\psi/2$$

$$\psi_D = b(\vec{p},h)e^{-ip\cdot x}u(\vec{p},h) + d^\dagger(\vec{p},h)e^{ip\cdot x}\sqrt{\frac{E_p + hp}{E_p - hp}}(-i\sigma_2)u^*(\vec{p},h)$$

Particle annihilation

Anti-particle creation

Majorana phase dependence

- Pair emission current at cross thresholds

$$\begin{aligned} \langle (ip_1 h_1, jp_2 h_2) | j_\nu | 0 \rangle &= \xi_i^* \xi_j e^{i(p_1+p_2) \cdot x} v_1^\dagger \sigma u_2 - \xi_i \xi_j^* e^{i(p_1+p_2) \cdot x} v_2^\dagger \sigma u_1 \\ &= e^{i(p_1+p_2) \cdot x} \left(i \Im \xi_i^* \xi_j (v_1^\dagger \sigma u_2 + v_2^\dagger \sigma u_1) + \Re \xi_i^* \xi_j (v_1^\dagger \sigma u_2 - v_2^\dagger \sigma u_1) \right) \end{aligned}$$

$$\xi_i^* \xi_j = U_{ei}^* U_{ej} = c_{ij}^{(0)}, \quad U_{e1} = c_{12} c_{13}, \quad U_{e2} = s_{12} c_{13} e^{i\alpha}, \quad U_{e3} = s_{13} e^{i\beta}$$

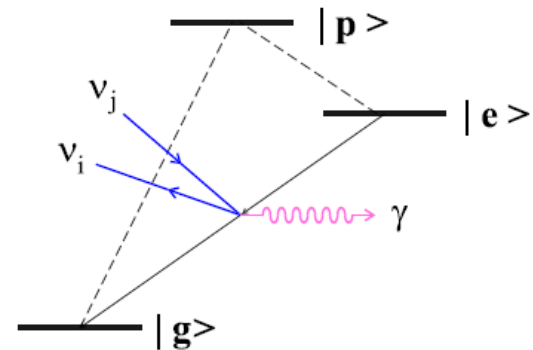
Unless $(v_1^\dagger \sigma u_2 + v_2^\dagger \sigma u_1)$ and $(v_1^\dagger \sigma u_2 - v_2^\dagger \sigma u_1)$ are orthogonal, T-reversal violation $\propto \Im \xi_i^* \xi_j \Re \xi_i^* \xi_j$ can be measured, and all Majorana phases α, β are measurable. Non-orthogonality holds for $i \neq j$, or $m_i \neq m_j$.

$$\cos(2\alpha), \quad \cos 2(\beta - \delta), \quad \text{at } (12), (13), (23) \text{ thresholds}$$

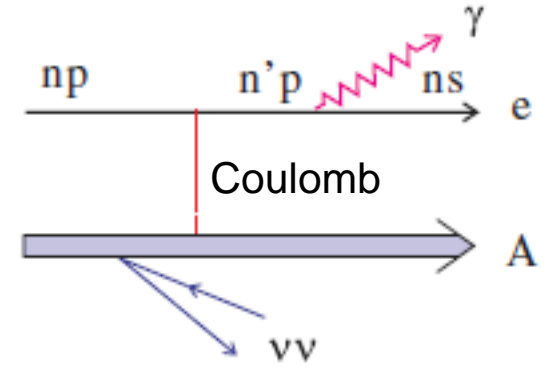
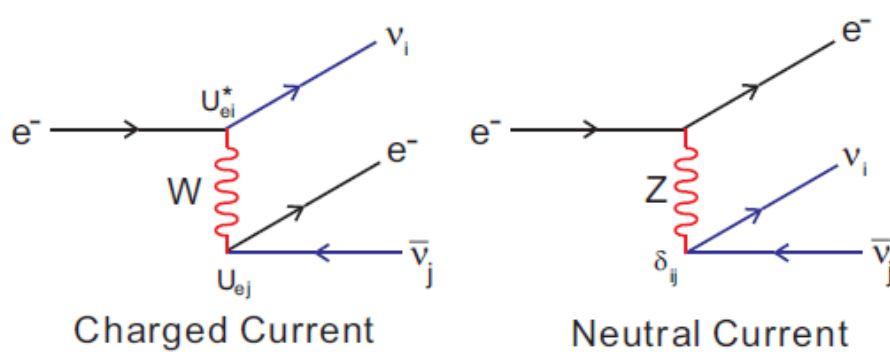
Relevant atomic process to us

Radiative Emission of Neutrino Pair (RENP) from metastable atomic levels

- Process undoubtedly existing in standard theory, assuming finite neutrino masses
- Possible to amplify otherwise small rates by developing macro-coherence of a twin process



Neutrino weak interaction with electron and quarks in standard electroweak theory



$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{i,j} \bar{\nu}_i \gamma^\mu (1 - \gamma_5) \nu_j \bar{e} \gamma_\mu (v_{ij} - a_{ij} \gamma_5) e,$$

$$\mathcal{H}_{2\nu}^q \sim \frac{G_F}{\sqrt{2}} \sum_i \nu_i^\dagger (1 - \gamma_5) \nu_i j_q^0,$$

$$v_{ij} = U_{ei}^* U_{ej} - \left(\frac{1}{2} - 2 \sin^2 \theta_W \right) \delta_{ij}, \quad a_{ij} = U_{ei}^* U_{ej} - \frac{1}{2} \delta_{ij}.$$

$$j_q^0 = -\frac{1}{2} j_n^0 + \frac{1}{2} (1 - 4 \sin^2 \theta_W) j_p^0,$$

Mixing in W-exchange

$$U = VP, \quad (\text{A8})$$

where

$$V = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}, \quad (\text{A9})$$

with $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$. The diagonal unitary matrix P may be expressed by

Rate amplification by macroscopic coherence

- Super-radiance coherent volume (Dicke)
 - In case of SR, coherent volume is proportional to $\lambda^2 L$.
 - Phase decoherence time (T_2) must be longer than T_{SR}

$$\text{Rate} \propto \left| \sum_j^N e^{i\vec{k} \cdot \vec{r}_j} M_{atm} \right|^2 \propto N^2 \quad \left(\text{for } |\vec{r}_j - \vec{r}_l| \leq \lambda \right)$$

- For a process with plural outgoing particles
 - Phase matching condition (momentum conservation) is satisfied.
 - Coherent volume is not limited by λ ., can be macroscopic.

$$\text{Rate} \propto \left| \sum_j^N e^{i(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \cdot \vec{r}_j} M_{atm} \right|^2 \propto N^2 \quad \left(\text{for } \vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0 \right)$$

Superradiance: 2 level and 1 photon case



Bob Dicke
1916—1997

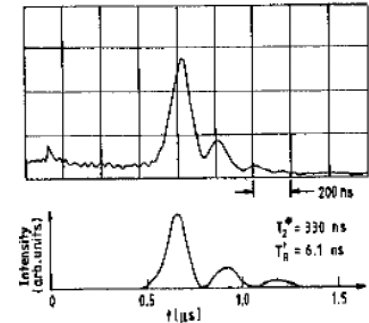
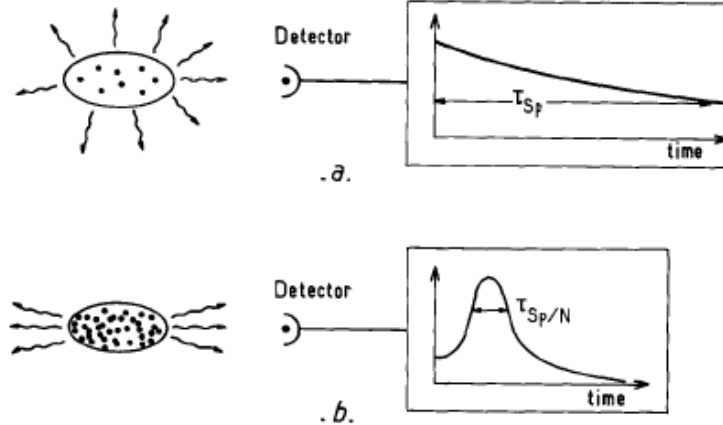
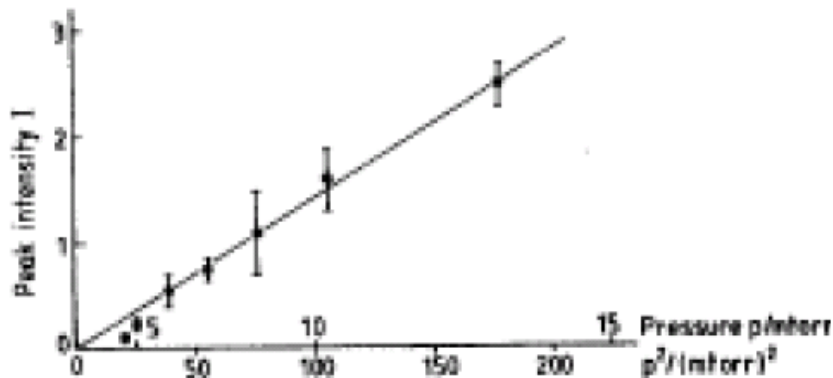


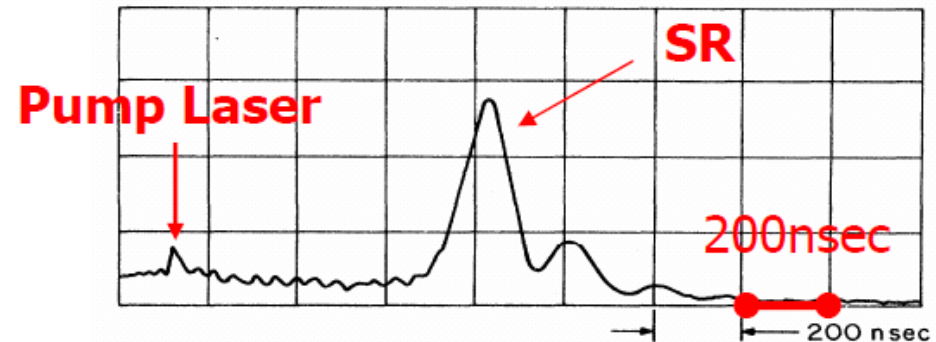
Figure 2.2. Oscilloscope trace of the super-radiance pulse observed by Skribanowitz *et al* [SHMP73] in HF gas at $84 \mu\text{m}$ ($J = 3 \rightarrow 2$), pumped by the $R_1(2)$ laser line, and the theoretical fit. The parameters are: pump intensity $i = 1 \text{ kW cm}^{-2}$, $p = 1.3 \text{ mTorr}$, $L = 100 \text{ cm}$. The small peak on the oscilloscope trace at $t = 0$ is the $3 \mu\text{m}$ pump pulse, highly attenuated.



Rate enhanced by N

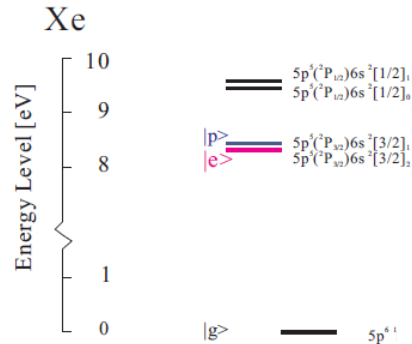
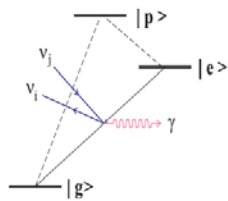
m.yoshimura 09/2014

(PRL30(1973)309)



Delayed enhanced signal
accompanied by ringing 12

Radiative emission of neutrino pair (RENP)



$$\Gamma = \Gamma_0 I(\omega) \eta(t)$$

Fig. 1 A-type atomic level for RENP $|e\rangle \rightarrow |g\rangle + \gamma + \nu_i \nu_j$ with ν_i a neutrino mass eigenstate. Dipole forbidden transition $|e\rangle \rightarrow |g\rangle + \gamma + \gamma$ may also occur via weak $M1 \times E1$ couplings to virtual intermediate state $|p\rangle$.

Six pair production thresholds of mass eigen states

$$\omega_{ij} = \frac{\epsilon_{eg}}{2} - \frac{(m_i + m_j)^2}{2\epsilon_{eg}}$$

Due to energy and momentum conservation
Resolved by precision of trigger lasers

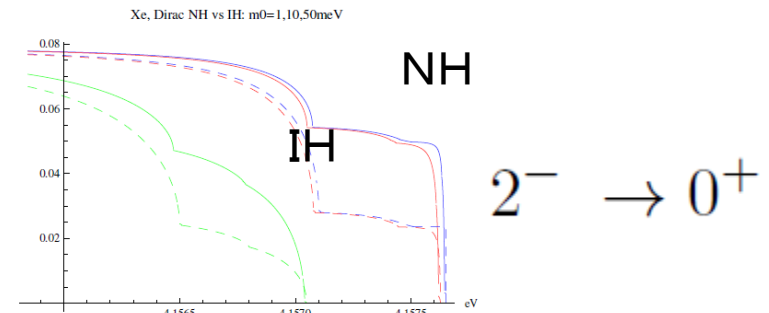
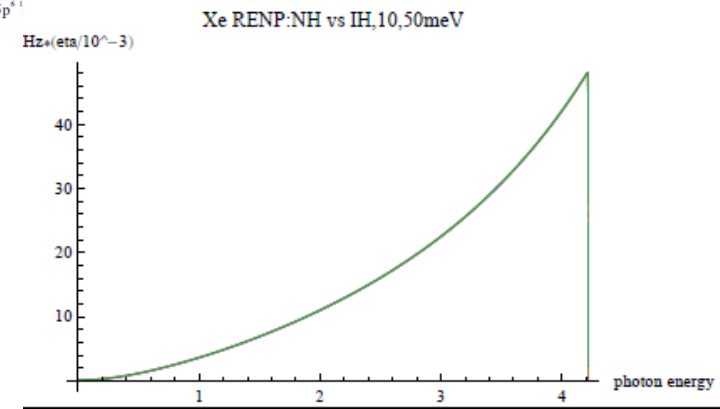


Fig. 2 RENP dimensionless spectrum function $I(\omega)$ near the neutrino pair emission thresholds from Xe level $5p^5(2P_{3/2})6s^2[3/2]_2$. Neutrinos of the smallest mass of 1, 10 and 50 meV are taken for the normal (solid curve) and the inverted (dashed curve) hierarchical mass pattern.

Nuclear monopole rates

- Pair emission from nucleus (**monopole**) gives the largest rates

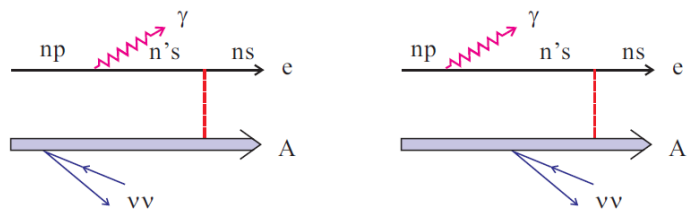


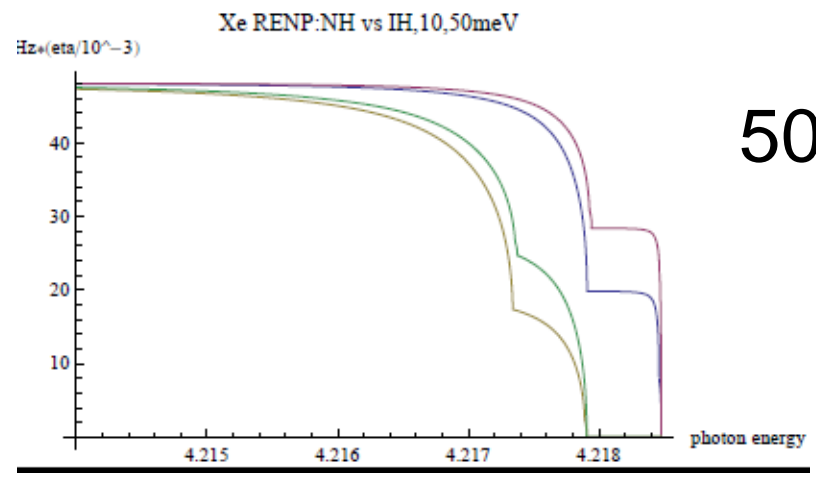
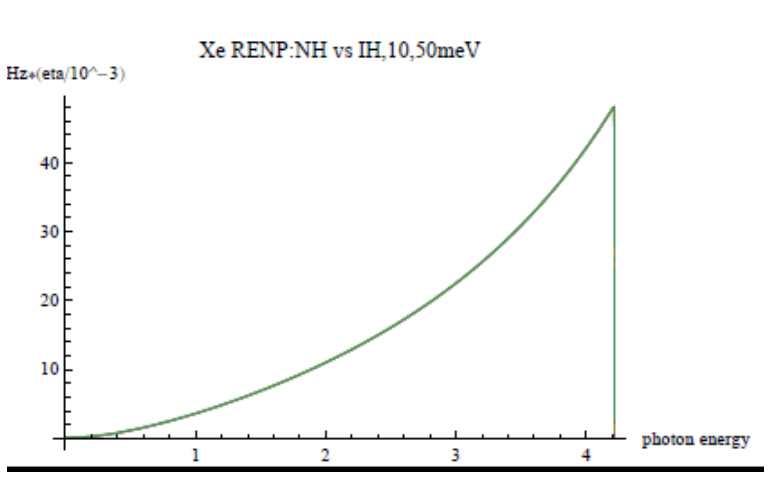
Figure 3: RENP diagrams 3 for alkali atoms.

$$Q_W^2 Z^{8/3}$$

$$Q_w \sim N - 0.044Z$$

Nuclear coherence effect

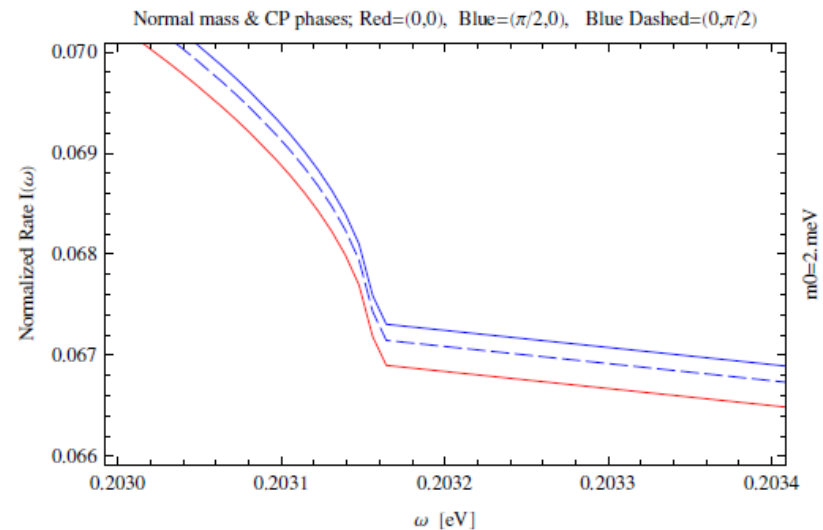
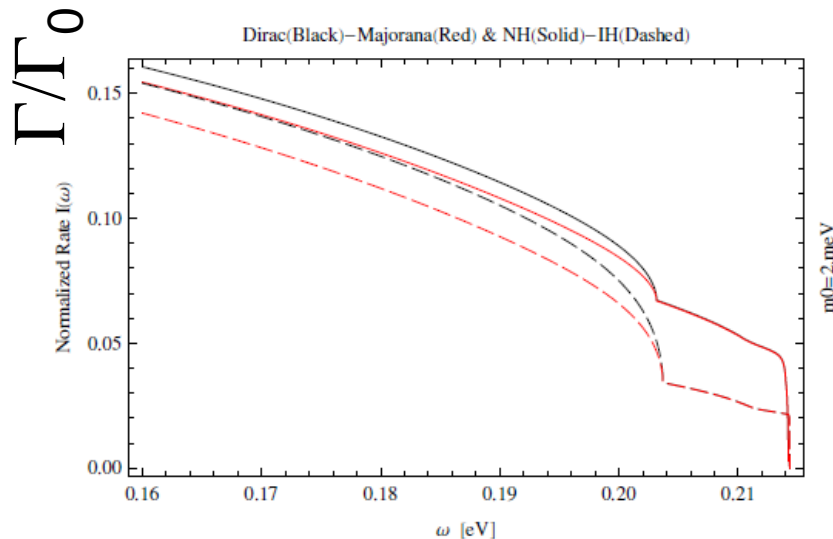
Spectrum rates for **gas** Xe



50 Hz

Dirac vs Majorana & CP phases

We need to go to the lower energy (smaller level spacing) to see CP phases.



E_γ [eV] $E_{eg} = 0.429$ eV
 $E_{pg} = 0.446$ eV

E_γ [eV]

Parity violating effects and asymmetric rates calculated: proof of involved weak interaction and important to increase S/N ratio

1. asymmetry under the magnetic field reversal,
2. asymmetry under the reversal of
trigger photon circular polarization

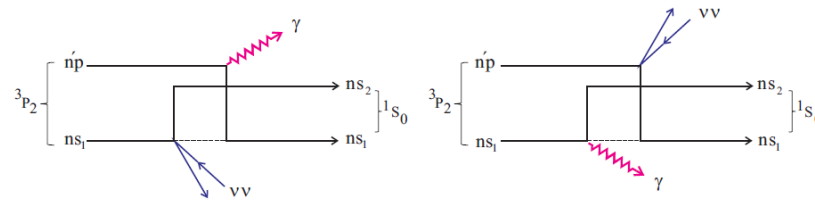
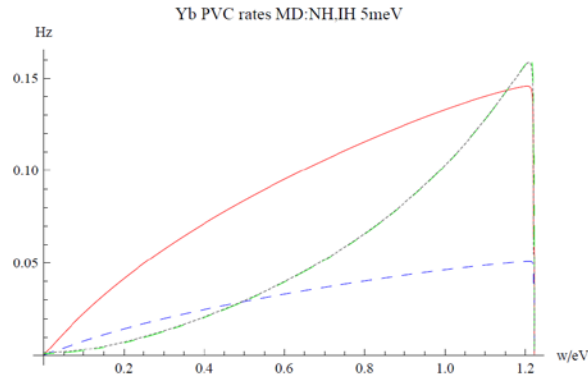
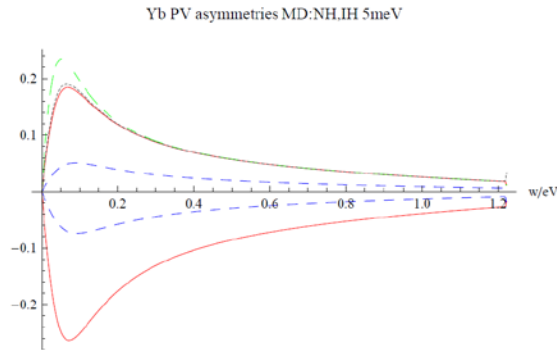


Figure 1: Parity odd contribution of valence electron exchange. Neutrino pair emission contains the PO part of vertex, as described in the text.



PV rate much smaller than PC rate

Figure 8: $^3P_2, J = 2, M_J = 1$ Yb PC rates, PV rate differences. Zeeman mixing amplitude 5×10^{-6} (corresponding to the magnetic field **), $\eta_\omega(t) = 1$, $n = 10^{22}\text{cm}^{-3}$, and 10^2cm^3 are assumed. Majorana NH PV in solid red, M-IH PV in dashed blue, M-NH PC rate divided by 50 in dash-dotted green, and M-IH/50 in dotted black (degenerate with M-NH PC).



MD distinction possible by
measurement of PV asymmetry

Figure 11: ^3P_2Yb PV asymmetries vs photon energy. Zeeman mixing amplitude 5×10^{-6} , $\eta_\omega(t) = 1$, $n = 10^{22}\text{cm}^{-3}$, and a target volume 10^2cm^3 assumed. In the positive side the Majorana case of PV asymmetry under polarization reversal for NH is depicted in solid red, M-IH case in dashed blue, D-NH in dash-dotted green and the Dirac case for IH in dotted black. In the negative side PV asymmetry under the field reversal is plotted; M-NH and D-NH in solid red, and M-IH and D-IH in dashed blue, all assuming the smallest neutrino mass 5 meV.

Detection of relic neutrinos of 1.9 K

Recent work with N. Sasao and M. Tanaka
arXiv: 1409.3648

- Direct remnant at a few seconds after the big bang
- Prove that neutrinos were in thermal equilibrium, giving the important basis of light element synthesis such as 4He
- T differs from 2.7K of microwave, because electron-positron annihilation occurred after the neutrino decoupling at a few MeV, heating up matter in equilibrium
- Prediction is firm: $(4/11)^{(1/3)} 2.7 \text{ K} = 1.9 \text{ K}$, 110cm^{-3}

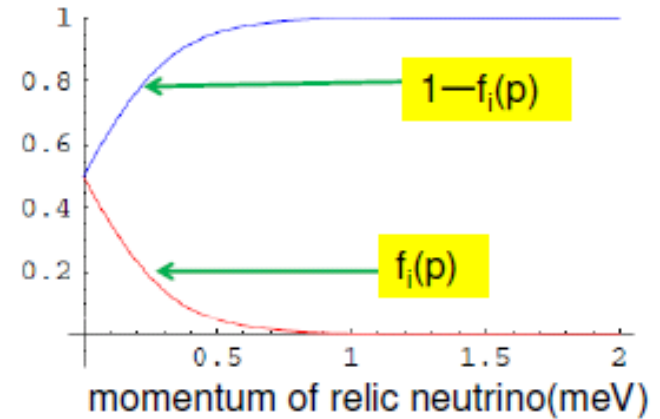
- Spectrum distortion by the Pauli blocking caused by ambient relic neutrinos

Neutrino distribution function

$$f(p) = \frac{1}{\zeta e^{\sqrt{p^2+m^2}/(z_d+1)^2/T} + 1} \approx \frac{1}{\zeta e^{p/T} + 1}$$

$$\zeta = e^{-\mu_d/T_d}, \quad z_d = O(10^{10})$$

Blocking given by $1-f(p)$



$$F_{ij}^A(\omega; T_\nu) = \frac{1}{8\pi\omega} \int_{E_-}^{E_+} dE_1 g_{ij}^A(E_1) \cdot \left(1 - f(\sqrt{E_1^2 - m_i^2})\right) \left(1 - \bar{f}(\sqrt{(\epsilon_{eg} - \omega - E_1)^2 - m_j^2})\right),$$

$$g_{ii}^M(E) = -E^2 + (\epsilon_{eg} - \omega)E + \frac{1}{2}m_i^2 - \frac{1}{4}\epsilon_{eg}(\epsilon_{eg} - 2\omega) + \delta_M \frac{m_i^2}{2},$$

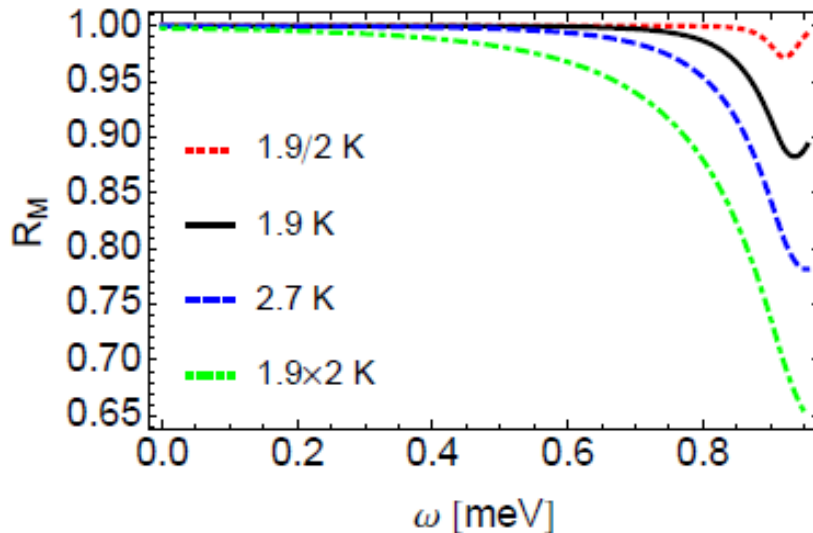
$$g_{ij}^S(E) = -\frac{1}{3}E^2 + \frac{1}{3}(\epsilon_{eg} - \omega)E + \frac{1}{12}\epsilon_{eg}(\epsilon_{eg} - 2\omega) - \frac{1}{12}(m_i^2 + m_j^2) - \delta_M \frac{m_i m_j}{2},$$

$$E_{\pm} = \frac{1}{2} \left((\epsilon_{eg} - \omega) \left(1 + \frac{m_i^2 - m_j^2}{\epsilon_{eg}(\epsilon_{eg} - 2\omega)}\right) \pm \omega \Delta_{ij}(\omega) \right), \quad \Delta_{ij}(\omega) = \left\{ \left(1 - \frac{(m_i + m_j)^2}{\epsilon_{eg}(\epsilon_{eg} - 2\omega)}\right) \left(1 - \frac{(m_i - m_j)^2}{\epsilon_{eg}(\epsilon_{eg} - 2\omega)}\right) \right\}^{1/2}.$$

Temperature measurement possible for RENP ?

Ratio of rates: with to without Pauli blocking

with/without Pauli blocking



Difference of distortions
for 1.9 and 2.7 K

10% level

For small level spacing, temperature measurement seems possible. Less sensitive than the inverse process.

Effect of chemical potential

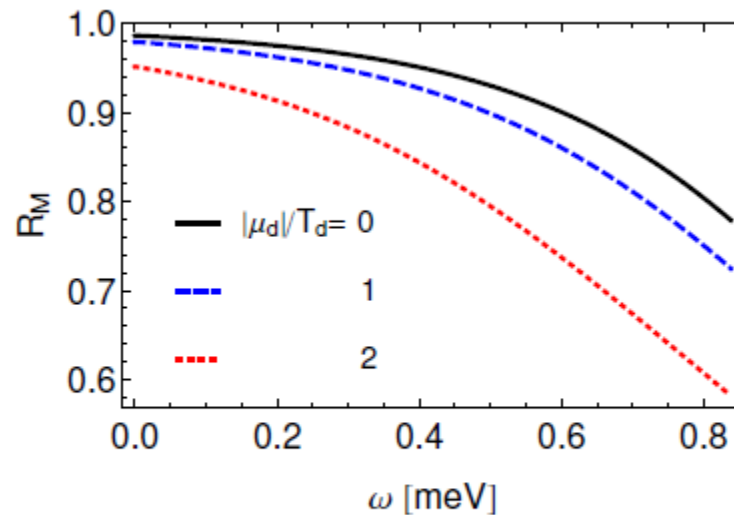
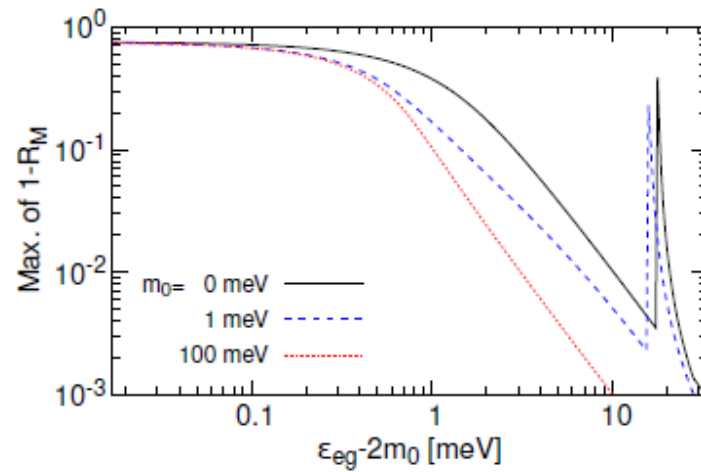
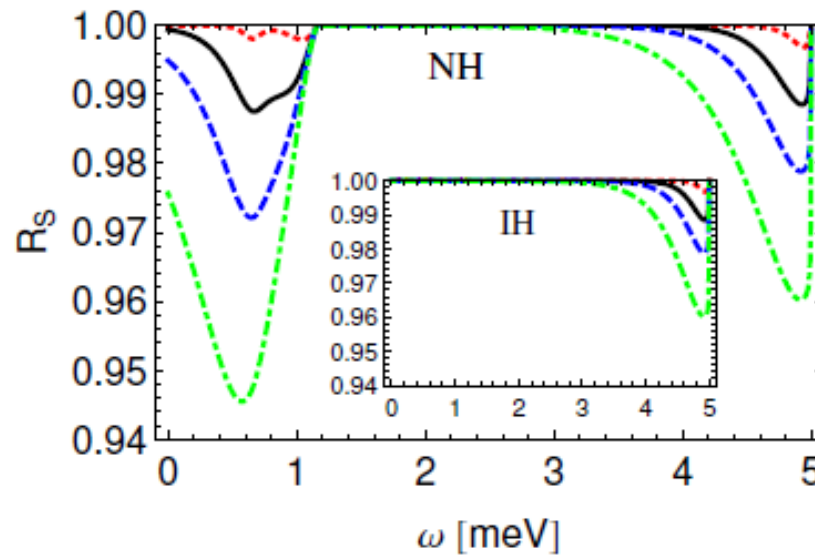


Figure 4: Spectrum distortion $R_M(\omega)$ for magnitudes of neutrino degeneracy $|\mu_d|/T_\nu = 0$ meV in solid black, 1 in dashed blue, and 2 in dotted red. The lightest neutrino mass $m_0 = 0$ meV. $\epsilon_{eg} = 10T_\nu \sim 1.7$ meV chosen.



monopole



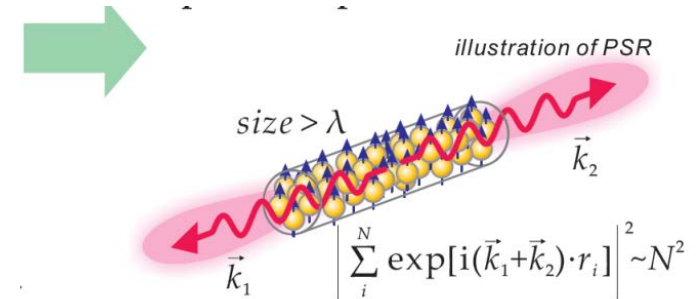
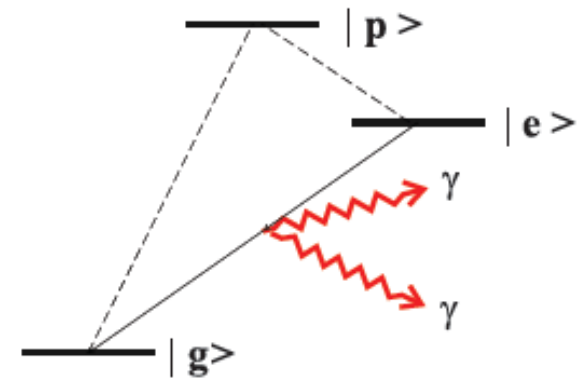
spin

Twin process: Paired Super-Radiance (PSR) important to develop large rates for RENP (also to prove the principle of macro-coherence)

- Macro-coherent amplification
 - A new type of coherent phenomena
 - Should be established experimentally
- Two photon emission process

$$|e\rangle \rightarrow |g\rangle + \gamma + \gamma$$

- Paired Super-Radiance
 - QED instead of weak process
 - Good experimental signature; i.e. back-to-back radiations with same color.



Effective 2-level model for trigger and medium evolution

2 level interaction with field

$$\frac{d}{dt} \begin{pmatrix} c_e \\ c_g \end{pmatrix} = -i\mathcal{H} \begin{pmatrix} c_e \\ c_g \end{pmatrix}, \quad -\mathcal{H} = 2 \begin{pmatrix} \mu_{ee} & 2e^{i\epsilon_{eg}} \mu_{ge} \\ 2e^{-i\epsilon_{eg}} \mu_{ge} & \mu_{gg} \end{pmatrix} E^2$$

Stark shifts : 2×2 Hamiltonian

$$\text{Ba} \quad \begin{pmatrix} 6.0 & 2.1 \\ 2.1 & 16 \end{pmatrix} \text{GHz} \frac{|E|^2}{10^6 \text{Wmm}^{-2}}$$

$$\text{Yb} \quad \begin{pmatrix} -1.2 \times 10^{-7} & 1.7 \times 10^{-4} \\ 1.7 \times 10^{-4} & 6.6 \end{pmatrix} \text{GHz} \frac{|E|^2}{10^6 \text{Wmm}^{-2}}$$

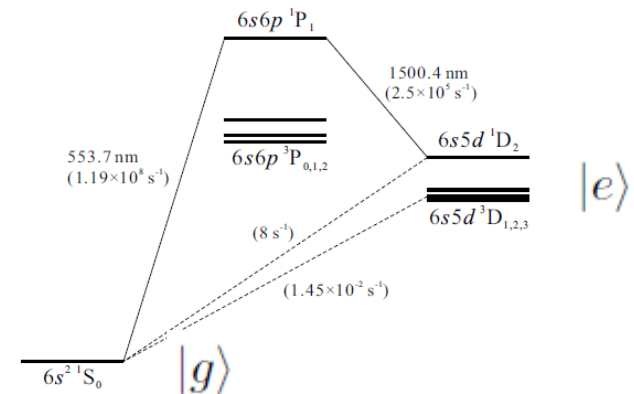
$$\text{Xe} \quad \begin{pmatrix} -8.4 \times 10^{-11} & 1.5 \times 10^{-5} \\ 1.5 \times 10^{-5} & 0.22 \end{pmatrix} \text{GHz} \frac{|E|^2}{10^6 \text{Wmm}^{-2}}$$

$$\text{Ca}^+ \quad \begin{pmatrix} 10 & 2.5 \\ 2.5 & 4.5 \end{pmatrix} \text{GHz} \frac{|E|^2}{10^6 \text{Wmm}^{-2}}$$

$$\text{pH}_2 \quad \begin{pmatrix} 0.27 & 0.16 \\ 0.16 & 0.22 \end{pmatrix} \text{GHz} \frac{|E|^2}{10^6 \text{Wmm}^{-2}}$$

$$\mu_{ee} = 2 \sum_j \frac{d_{je}^2 E_{je}}{E_{je}^2 - \omega^2}, \quad \mu_{gg} = 2 \sum_j \frac{d_{jg}^2 E_{jg}}{E_{jg}^2 - \omega^2}, \quad d_{ij} = \sqrt{3\pi \frac{\gamma_{ij}}{E_{ij}^3}}$$

$$\mu_{eg} = \sum_j \frac{d_{je} d_{jg}}{E_c - \delta\omega}, \quad \mu_{ge} = \sum_j \frac{d_{je} d_{jg}}{E_c + \delta\omega}.$$



Ba

Maxwell-Bloch equation for PSR simulations: 1+1 dim

Bloch equation for medium

$$\vec{R} = \text{tr } \rho \vec{\sigma} = \langle \psi | \vec{\sigma} | \psi \rangle$$

$$\partial_t R_1 = (\mu_{ee} - \mu_{gg}) E^+ E^- R_2 - i \mu_{ge} (e^{i\epsilon_{eg}} E^+ E^+ - e^{-i\epsilon_{eg}} E^- E^-) R_3 - \frac{\kappa_1}{T_2},$$

$$\partial_t R_2 = -(\mu_{ee} - \mu_{gg}) E^+ E^- R_1 + \mu_{ge} (e^{i\epsilon_{eg}} E^+ E^+ + e^{-i\epsilon_{eg}} E^- E^-) R_3 - \frac{R_2}{T_2},$$

$$\partial_t R_3 = \mu_{ge} (i(e^{i\epsilon_{eg}} E^+ E^+ - e^{-i\epsilon_{eg}} E^- E^-) R_1 - (e^{i\epsilon_{eg}} E^+ E^+ + e^{-i\epsilon_{eg}} E^- E^-) R_2) - \frac{R_3 + n}{T_1}.$$

Field equation

$$(\partial_t^2 - \vec{\nabla}^2) \vec{E} = \vec{\nabla}^2 \mathcal{D} \vec{E},$$
$$-\mathcal{D} \vec{E}^+ = \left(\frac{\mu_{ee} + \mu_{gg}}{2} n + \frac{\mu_{ee} - \mu_{gg}}{2} R_3 \right) \vec{E}^+ + \mu_{ge} e^{-i\epsilon_{eg} t} (R_1 - i R_2) \vec{E}^-.$$

SVEA (Slowly Varying Envelope Approximation)

$$E = \frac{1}{2} \left(e^{-i\omega_1(t-x)} E_R + e^{-i\omega_2(t+x)} E_L + (\text{h.c.}) \right), \quad \omega_1 + \omega_2 = \epsilon_{eg}$$

complex amplitudes $E_R(x, t), E_L(x, t)$ slowly varying in 1+1 spacetime

- Coupled system of field and medium polarization highly non-linear

PSR simulations for two counter-propagating modes and soliton-condensates

Explosive PSR with initial coherence

para-H2 $n = 1 \times 10^{21} \text{ cm}^{-3}$, $L = 30 \text{ cm}$, $T_1 = 1 \mu\text{s}$, $T_2 = 10 \text{ ns}$

Coherent initial state: $r_1 = 1$, $r_2 = r_3 = 0$

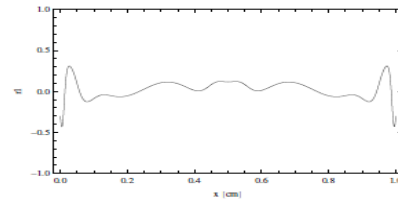
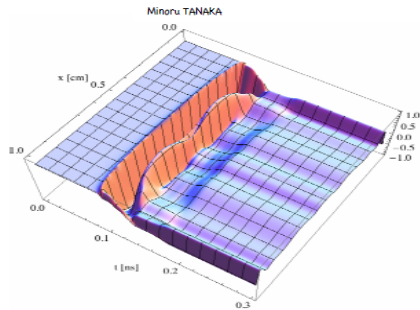
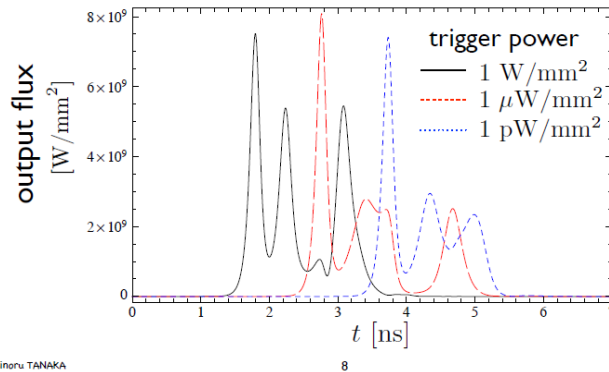


Figure 6: Spacetime profile of r_1 for the 1 Wmm^{-2} case of Fig(3).

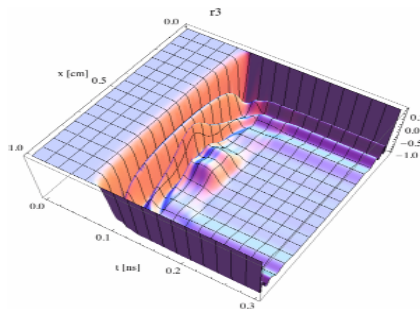


Figure 8: Spacetime profile of r_3 for the 1 Wmm^{-2} case of Fig(3).

Figure 7: Spatial profile of r_1 at the latest time, 0.3 ns after trigger irradiation, of Fig(6).

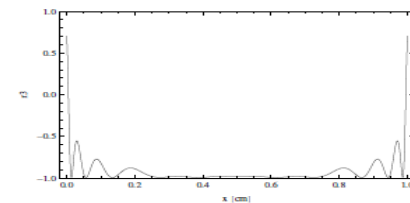
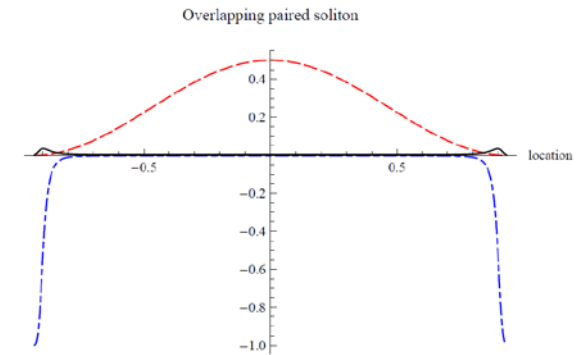
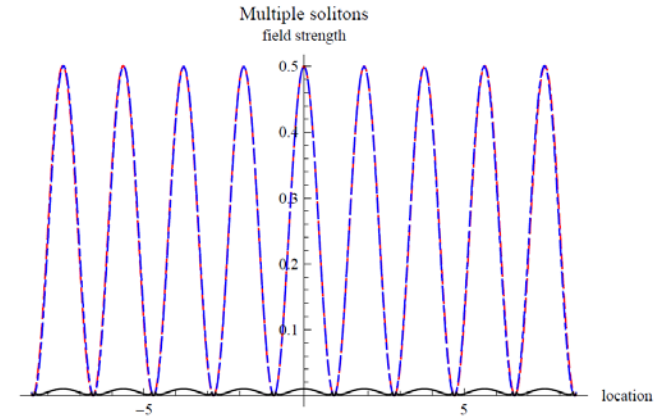


Figure 9: Spatial profile of r_3 at the latest time, 0.3 ns after trigger irradiation, of Fig(8).

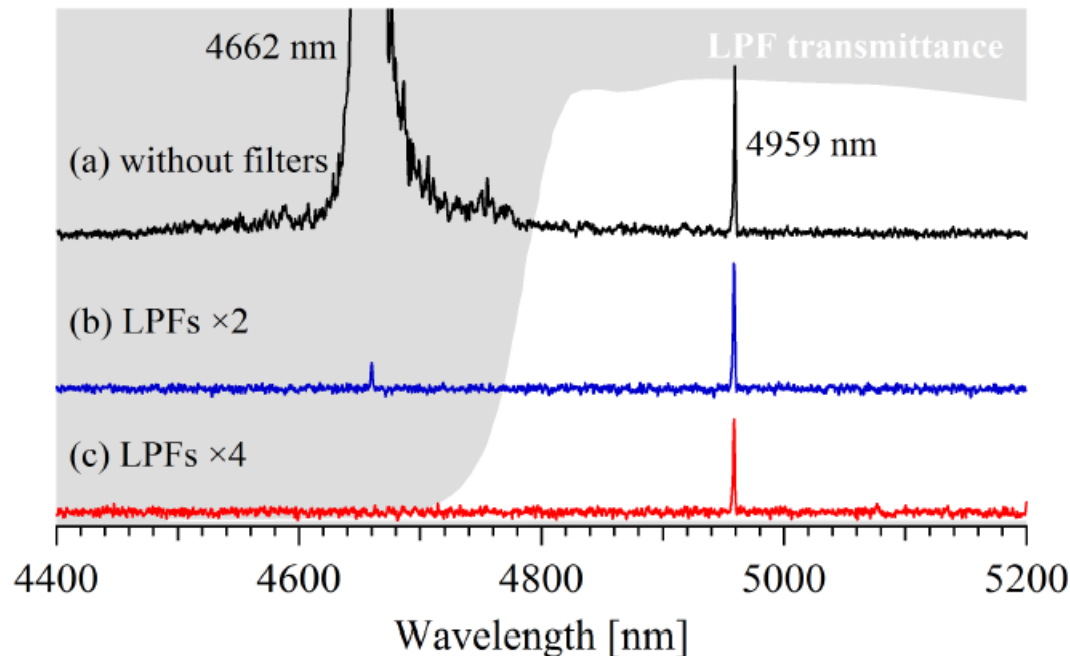
para-H₂ $v=1 \rightarrow 0$



Soliton-condensates stable against two-photon emission, unstable for RENP

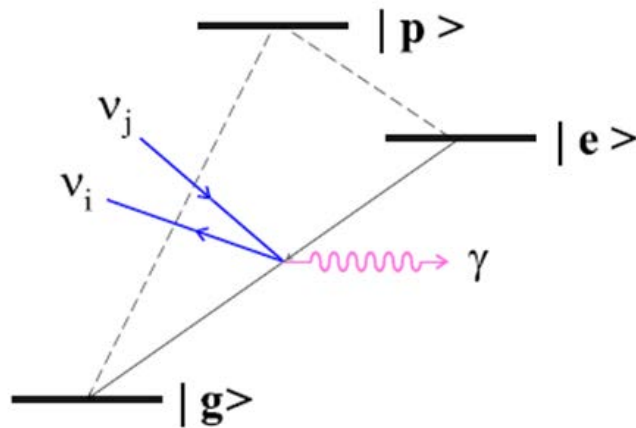
Experimental result (see Sasao)

- Linear growth region for two pair modes in the same direction
- Exponential time growth prop. to $n * \text{coherence}$, cut by de-coherence T_2

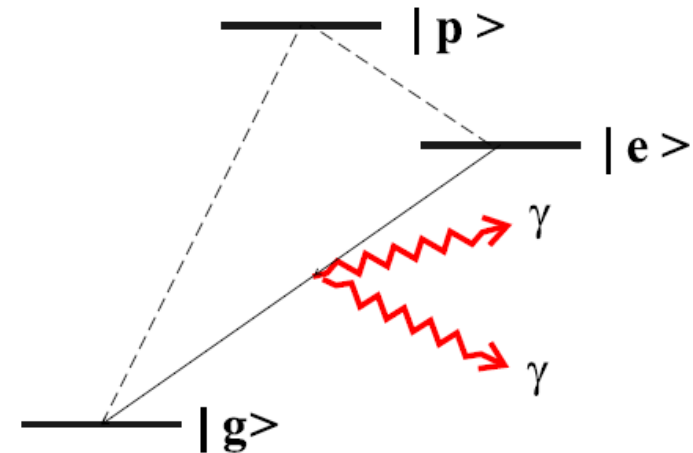


Twin process and controlled switching

RENPN uses large medium polarization and stored fields by PSR, but two processes have different selection rules



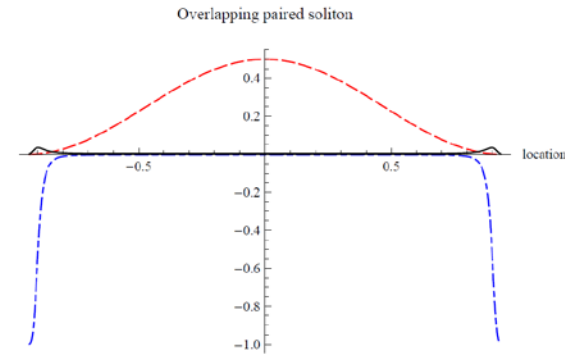
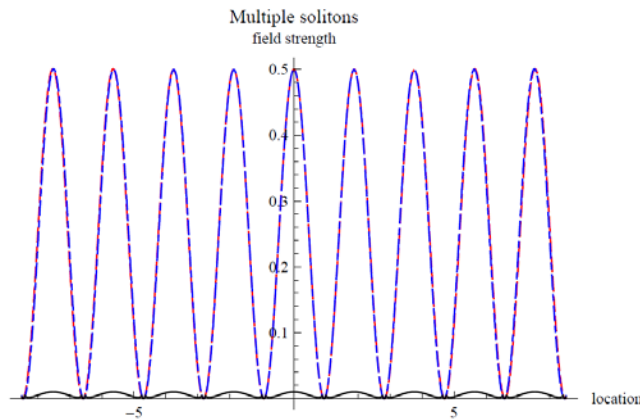
RENPN: $(E0 \text{ or } M1) \times E1$



PSR: $E1 \times E1$

PSR-RENPN switching is achieved
by application of modulated E

Ideal state for RENP after PSR activity



Soliton-condensates stable
against two-photon emission,
unstable for RENP

Analogue of
stopped light
polariton in cavity QED
Realized by two counter-propagating
trigger PSR modes

Soliton-condensate formulated by non-linear eigenvalue problem

- Stationary solutions are derived from dynamical master eq. for PSR

$$\left(-E - \frac{d^2}{d\xi^2} - \mathcal{V}(e_i) \right) \begin{pmatrix} e_R \\ e_L^* \end{pmatrix} = 0, \quad \text{Non-linear eigenvalue equation}$$

$$\mathcal{V}(e_i) = \begin{pmatrix} \frac{\gamma_-}{2}(r_3(e_i) + 1) & r_T^*(e_i) \\ r_T(e_i) & \frac{\gamma_-}{2}(r_3(e_i) + 1) \end{pmatrix} \quad \text{Forming potential well}$$

$$r_1(e_i) = -\frac{4\tau_2}{D} \left(\Im(e_R e_L) + 2\tau_2 \gamma_- \Re(e_R e_L) (|e_R|^2 + |e_L|^2) \right),$$

$$r_2(e_i) = -\frac{4\tau_2}{D} \left(\Re(e_R e_L) - 2\tau_2 \gamma_- \Im(e_R e_L) (|e_R|^2 + |e_L|^2) \right),$$

$$r_3(e_i) = -\frac{1 + 4\gamma_-^2 \tau_2^2 (|e_R|^2 + |e_L|^2)^2}{D}, \quad D = 1 + 4\gamma_-^2 \tau_2^2 (|e_R|^2 + |e_L|^2)^2 + 16\tau_1 \tau_2 |e_R e_L|^2,$$

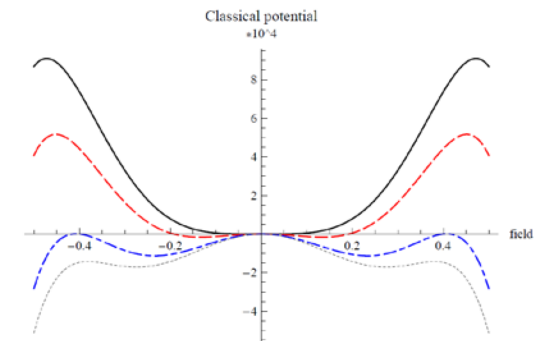
Equivalent to particle motion in 2 dim.

$$\vec{r} = (e_R, e_L)$$

$$\vec{r}_\perp = (e_L, -e_R)$$

$$\frac{d^2 \vec{r}}{d\xi^2} = \left(-E + \frac{r^2(r^2 - a^2 \cos \theta_0)}{1 + r^4} \right) \vec{r} + \frac{a^2 r^2 \cos \theta_0}{1 + r^4} \vec{r}_\perp,$$

$$a^2 = 8 \frac{\tau_2}{\tau_1} \sqrt{\tau_2(\tau_1 + \gamma_-^2 \tau_2)}, \quad \tan \theta_0 = \frac{1}{4\gamma_- \tau_2},$$



Experimental strategy towards neutrino mass spectroscopy

- 1st stage: proof of macro-coherence principle using QED (PSR)
- 2nd stage: control of PSR and soliton formation, switching between PSR and RENP modes, study of solid targets
- 3rd stage: discovery of the RENP process, measurements of mass matrix

Solid target: doped ions in ferro-electrics

- Large target number density required for PV measurements
- PSR \leftrightarrow RENP mode switching effective
- Collaboration with specialists to be started

Summary

- Systematic neutrino mass spectroscopy is made possible when macro-coherence is realized and PSR is controlled by formation of soliton-condensate
- Not a joke, since the macro-coherent QED process (PSR) has been experimentally observed (next talk)