Large-scale structure formation with massive neutrinos and dynamical dark energy

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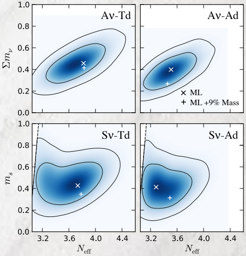
Motivation #1: Massive neutrinos

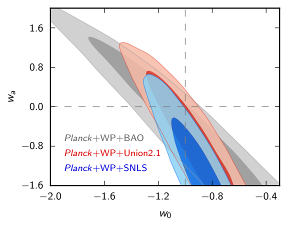
- Cosmology currently provides the best upper bound on the neutrino masses, $\Sigma m_\nu \leq 0.23$ eV.
- The lower bound $\Sigma m_\nu \geq 0.06$ eV will be reached by cosmological probes over the next several years, leading to a detection of massive neutrinos.
- Tensions between data sets could be hints of extra neutrino species.



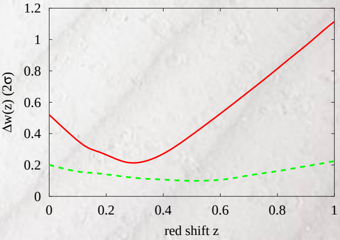
Wyman, Rudd, Vanderveld, Hu, PRL **112***:051302(2014)*

Motivation #2: Dynamical dark energy

- Accelerating expansion has been confirmed repeatedly.
- The simplest explanation (Λ) is highly tuned.
- Alternatives could give clues about the early universe, modifications to gravity, extra dimensions, etc.
- Models with $w(z) = P/\rho$ evolving rapidly are allowed but will be excluded (or detected) over the next 5-10 years.



Ade, *et al.*, arXiv:1303.5076



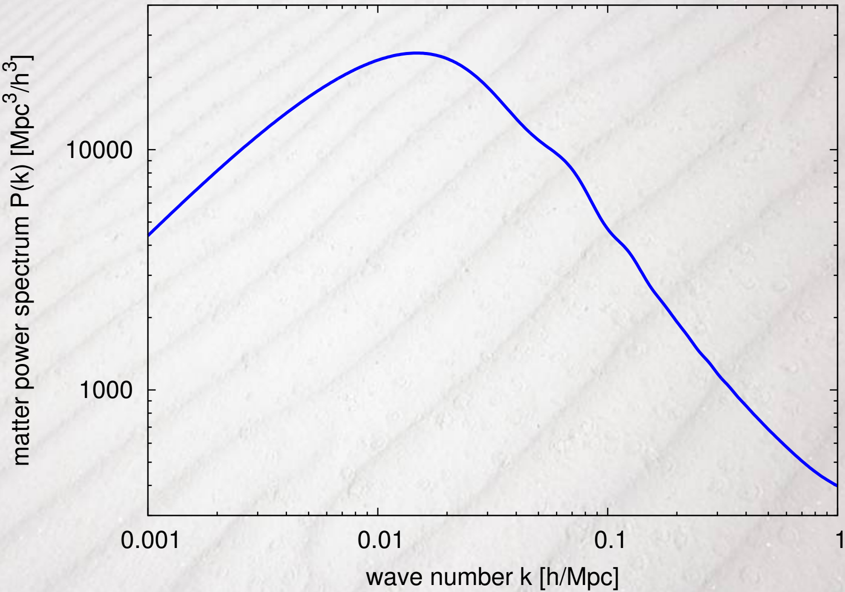
AU, Ishak, Steinhardt, PRD **72***:063501(2005)*

Outline

1. Motivation and introduction to large-scale structure
2. Cosmological perturbation theory at higher orders
    - Standard Perturbation Theory (SPT)
    - Time-Renormalization Group (Time-RG) perturbation theory
    - Massive neutrinos and dynamical dark energy
3. N-body dark matter simulations
    - Massive neutrinos in simulations
    - Tests of perturbation theory
4. Results
    - Parameter-dependence of the power spectrum: w_0, w_a, m_ν
    - Non-linear shift of the BAO peak
    - Applications and future work

AU, Biswas, Pope, Heitmann, Habib, Finkel, Frontiere,
PRD **89***:103515(2014)[arXiv:1309.5872]*

Power spectrum of large-scale structure



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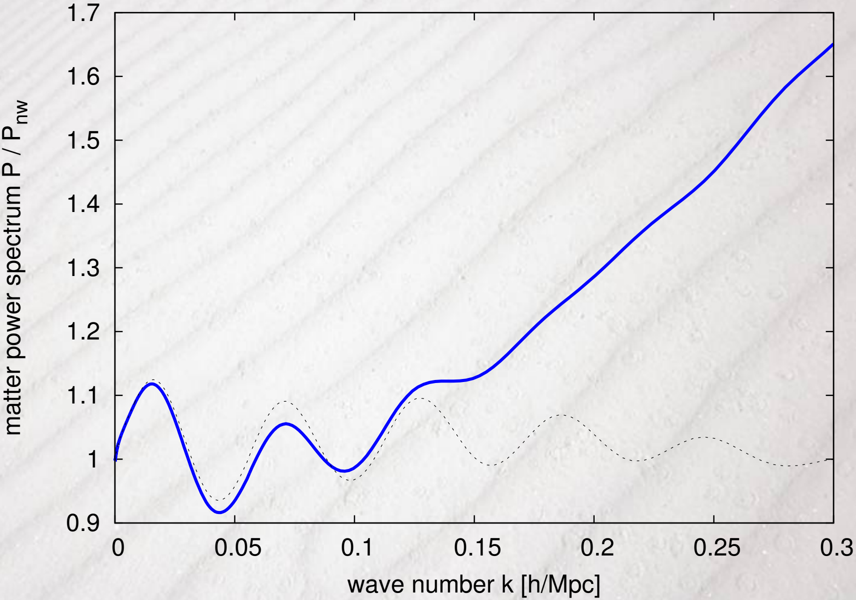
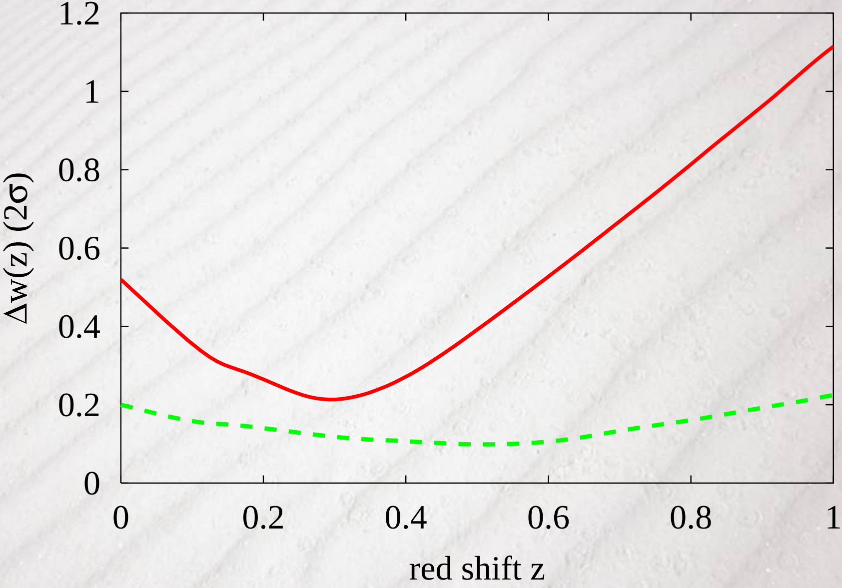


How well can we do?



AU, Ishak, Steinhardt, PRD **72***:063501(2005)[arXiv:astro-ph/0411803]*

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Higher-order perturbation theory

Let's work in an Einstein-de Sitter cosmology.
(EdS: $\Omega_{\mathrm{m}} = 1$, nothing else in the universe)

Continuity and Euler equations:

- $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$

- $\rho \left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} \right) \vec{v} + \nabla \Phi = 0$

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  $\Rightarrow \frac{\partial \delta}{\partial t} + \theta = -\vec{\nabla} \cdot (\delta \vec{v})$ where $\delta = \rho/\bar{\rho} - 1$ and $\theta = \vec{\nabla} \cdot \vec{v}$
- $\rho \left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} \right) \vec{v} + \nabla \Phi = 0$
  $\Rightarrow \frac{\partial \theta}{\partial t} + H\theta + 4\pi G \bar{\rho} \delta = -\vec{\nabla} \cdot [(\vec{v} \cdot \vec{\nabla}) \vec{v}]$

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  $\Rightarrow \frac{\partial \delta(\vec{k})}{\partial \ln a} + \frac{\theta(\vec{k})}{aH} = -\frac{1}{aH} \int \frac{d^3p\, d^3q}{(2\pi)^3} \delta_{\mathrm{D}}(\vec{k} - \vec{p} - \vec{q}) \frac{\vec{k}\cdot\vec{p}}{p^2} \theta(\vec{p}) \delta(\vec{q})$
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  $- \frac{1}{aH} \int \frac{d^3p\, d^3q}{(2\pi)^3} \delta_{\mathrm{D}}(\vec{k} - \vec{p} - \vec{q}) \frac{k^2 (\vec{p}\cdot\vec{q})}{2p^2 q^2} \theta(\vec{p}) \theta(\vec{q})$

Standard perturbation theory (SPT)

- Linear theory: $\ddot{\delta} + 2H\dot{\delta} = 4\pi G\delta$ (growing mode: $\delta_{\mathrm{L}} \propto a$)

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  $\delta(\vec{k}, a) = \sum_{n=1}^{\infty} a^n \delta_n(\vec{k})$, $\qquad \theta(\vec{k}, a) = -aH \sum_{n=1}^{\infty} a^n \theta_n(\vec{k})$

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- Power spectrum: $(2\pi)^3 \delta_{\rm D}(\vec{k} + \vec{k}')P(k, a) = \left\langle \delta(\vec{k})\delta(\vec{k}') \right\rangle$
  $\Rightarrow P(k) = P_{\rm L}(k) + P^{(1,3)}(k) + P^{(2,2)}(k) + \ldots$

  $P^{(1,3)} = \frac{k^3 P_{\rm L}(k)}{1008\pi^2} \int_0^\infty dr P_{\rm L}(kr) \left[\frac{12}{r^2} - 158 + 100r^2 - 42r^4 + \frac{3(r^2-1)^3(7r^2+2)}{r^2} \ln\left| \frac{1+r}{1-r} \right| \right]$

  $P^{(2,2)} = \frac{k^3}{392\pi^2} \int_0^\infty dr P_{\rm L}(kr) \int_{-1}^{1} dx P_{\rm L}(k\sqrt{1 + r^2 - 2rx}) \frac{(3r+7x-10rx^2)^2}{(1+r^2-2rx)^2}$

  Makino, Sasaki, Suto, PRD ***46****:585(1992)*

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  Makino, Sasaki, Suto, PRD ***46****:585(1992)*
- Generalization beyond EdS:
  $\delta = \sum_{n=1}^{\infty} D(a)^n \delta_n(\vec{k})$, $\quad \theta = -aHf \sum_{n=1}^{\infty} D(a)^n \theta_n(\vec{k})$
  where $f = d\ln D / d\ln a$ is 1 in EdS

Generalization to other cosmologies

We developed SPT in Einstein-de Sitter, then generalized it to ΛCDM and wCDM through the simple replacement $a \to D(a)$. Why does this work so well?

First, a shorthand notation for the perturbations. $\eta = \ln(a/a_{\mathrm{in}})$ for some initial scale factor a_{in}, and define ψ_b, Ω_{bc}, and γ_{bcd} as

$\psi_0(\vec{k},\eta) = e^{-\eta}\,\delta(\vec{k},\eta)$,
$\psi_1(\vec{k},\eta) = -e^{-\eta}\,\theta/(aH)$,
$\Omega_{00} = -\Omega_{01} = 1$
$\Omega_{10}(\vec{k},\eta) = -\frac{3\Omega_{\mathrm{m}}H_0^2}{2a^3H^2} = -\frac{3}{2}\Omega_{\mathrm{m}}(a)$
$\Omega_{11}(\eta) = 3 + \frac{d\ln H}{d\eta}$
$\gamma_{010}(\vec{k},\vec{p},\vec{q}) = \gamma_{001}(\vec{k},\vec{q},\vec{p}) = \delta_{\mathrm{D}}(\vec{k}+\vec{p}+\vec{q})(\vec{p}+\vec{q})\cdot\vec{p}/(2p^2)$
$\gamma_{111}(\vec{k},\vec{p},\vec{q}) = \delta_{\mathrm{D}}(\vec{k}+\vec{p}+\vec{q})(\vec{p}+\vec{q})^2\vec{p}\cdot\vec{q}/(2p^2q^2)$

Then the evolution equations become

$$\frac{\partial}{\partial\eta}\psi_b(\vec{k}) = -\Omega_{bc}(\vec{k})\psi_c(\vec{k}) + e^{\eta}\gamma_{bcd}(\vec{k},-\vec{p},-\vec{q})\psi_c(\vec{p})\psi_d(\vec{q})$$

Generalization to other cosmologies

In EdS, Ω_{bc} takes a particularly simple form:

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In a cosmology with a homogeneous dark energy, if we change our time variable from $\ln(a/a_{\rm in})$ to $\ln(D/D_{\rm in})$ and $\psi_1 \rightarrow \psi_1/f$,

$$\mathbf{\Omega} = \begin{pmatrix} 1 & -1 \\ -\frac{3\Omega_{\rm m}(a)}{2f^2} & \frac{3\Omega_{\rm m}(a)}{2f^2} \end{pmatrix} \text{ with } f = \frac{d\ln D}{d\ln a}$$

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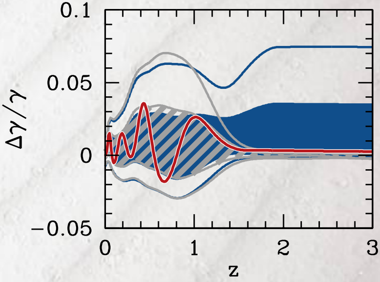
For a large range of cosmologies, $f(a) \approx \Omega_{\mathrm{m}}(a)^{0.55}$ to excellent precision. Then $\Omega_{\mathrm{m}}(a)/f(a)^2 \approx \Omega_{\mathrm{m}}(a)^{-0.1} \approx 1$ to $\approx 10\%$ since decoupling.

When does $f = \Omega_\mathrm{m}(a)^{0.55}$ fail?

Massive neutrinos behave as a warm dark matter component. In the late universe they cluster like matter for k below a free-streaming scale $k_\mathrm{fs}(a)$, and don't cluster for k much greater than k_fs. Growth becomes scale-dependent.

Exotic dark energy models such as early dark energy can change $f(a)$ to $\Omega_\mathrm{m}(a)^{\gamma(a)}$ with $\gamma \neq 0.55$:
Mortonson, Hu, Huterer,
PRD **81***:063007(2010)[arXiv:0912.3816]*



We need a perturbation theory that doesn't rely on this close cancellation!

Time Renormalization Group (TRG) perturbation theory

Rather than treating D as a time variable, integrate the evolution equations for the power spectrum directly:

$$\frac{\partial}{\partial\eta}\langle\psi_a\psi_b\rangle = -\Omega_{ac}\langle\psi_c\psi_b\rangle - \Omega_{bc}\langle\psi_a\psi_c\rangle + e^{\eta}\gamma_{acd}\langle\psi_c\psi_d\psi_b\rangle + e^{\eta}\gamma_{bcd}\langle\psi_a\psi_c\psi_d\rangle$$

$$\frac{\partial}{\partial\eta}\langle\psi_a\psi_b\psi_d\rangle = -\Omega_{ad}\langle\psi_d\psi_b\psi_c\rangle - \Omega_{bd}\langle\psi_a\psi_d\psi_c\rangle - \Omega_{cd}\langle\psi_a\psi_b\psi_d\rangle + e^{\eta}\gamma_{ade}\langle\psi_d\psi_e\psi_b\psi_c\rangle + e^{\eta}\gamma_{bde}\langle\psi_a\psi_d\psi_e\psi_c\rangle + e^{\eta}\gamma_{cde}\langle\psi_a\psi_b\psi_d\psi_e\rangle$$

...

Linear theory: Neglect the bispectrum $\sim\langle\psi_a\psi_b\psi_c\rangle$. Then the infinite tower of evolution equations truncates, and we can integrate to find the power spectrum.

Time-RG is the next level of approximation. Keep the bispectrum but neglect the trispectrum, the connected part of $\langle\psi_a\psi_b\psi_c\psi_d\rangle$. This includes the 1-loop terms and some of the 2-loop terms.

Pietroni, JCAP **810***:36(2008)[arXiv:0806.0971]*

Dark energy and neutrinos in Time-RG

Dark energy can be included trivially. Just use the actual linear evolution matrix $\mathbf{\Omega}$ rather than its EdS approximation.

Neutrinos modify the calculation several ways:

1. Homogeneous evolution $H(a)$ is modified;
2. Growth becomes scale-dependent, $D(a) \longrightarrow D(\vec{k}, a)$;
3. ψ_0 becomes $e^{-\eta}\,\delta_{\rm cb}$, the CDM+baryon density contrast, and ψ_1 becomes the CDM+baryon velocity divergence;
4. Ω_{10} changes from $-\frac{3}{2}\Omega_{\rm m}(a)$ to $-\frac{3}{2}\Omega_{\rm m}(a)\left[(1-f_\nu)+f_\nu\frac{\delta_\nu}{\delta_{\rm cb}}\right]$.

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PRD **89***:103515(2014)[arXiv:1309.5872]*

N-body simulations

The gravitational motion of cold, collision-less dark matter is described by the Vlasov-Poisson equations:

$$\frac{df}{dt} \equiv \frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} - \frac{\partial \Phi}{\partial \vec{x}} \frac{\partial f}{\partial \vec{v}} = 0$$

$$\nabla^2 \Phi(\vec{x}, t) = 4\pi G \int f(\vec{x}, \vec{v}, t) d\vec{v}$$

An N-body simulation is a Monte Carlo solution to this set of equations. In each time step of a Particle-Mesh (PM) simulation, the gravitational field Φ is computed on a mesh and then used to move the particles.

Our simulations use 512^3 particles on a 1024^3 grid in 1 Gpc boxes. For each model we average the results of 16 simulation runs in order to reduce noise at large scales.

Massive neutrinos in simulations

Massive neutrinos are warm dark matter and have a large velocity dispersion. Including them as particles in an N-body simulation is quite difficult.

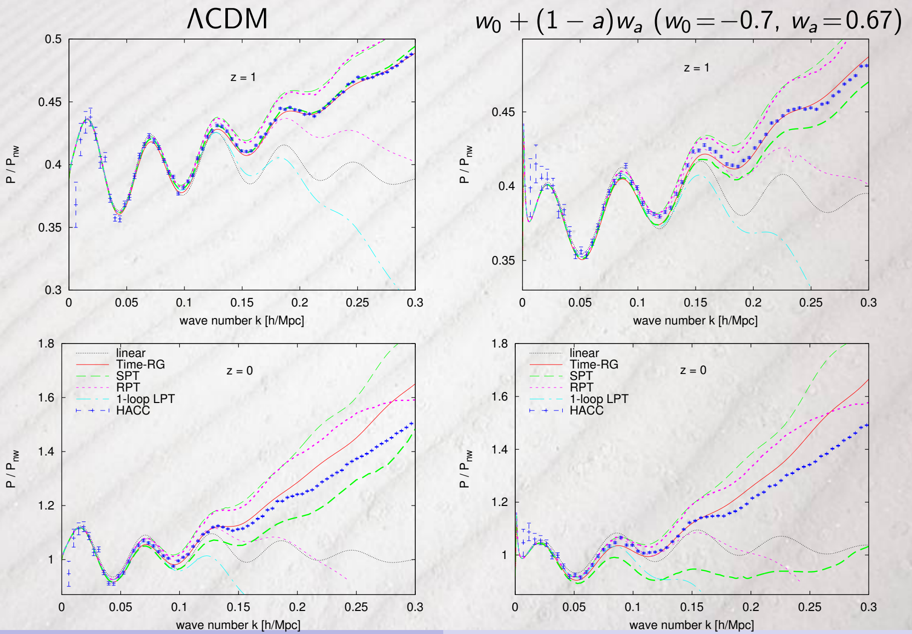
Our simulation used a linear approximation for massive neutrinos, neglecting their contribution to non-linear dark matter growth:

1. neutrinos were included in the evolution $H(z)$ and the scale-independent linear growth $D(a)$;
2. particles in the N-body simulation only represented cold matter (dark matter and baryons), not neutrinos;
3. the power spectrum was computed by adding the linear neutrino power and the cross term:

$$P = f_{\mathrm{cb}} P_{\mathrm{cb}} + 2 f_{\mathrm{cb}} f_{\nu} \sqrt{P_{\mathrm{cb}} P_{\nu}} + f_{\nu}^2 P_{\nu}$$

This approximation was proposed and tested in *Saito, Takada, Taruya, PRL* **100***:191301(2008)*. It works well for testing perturbation theory at low k.

How accurate is perturbation theory?



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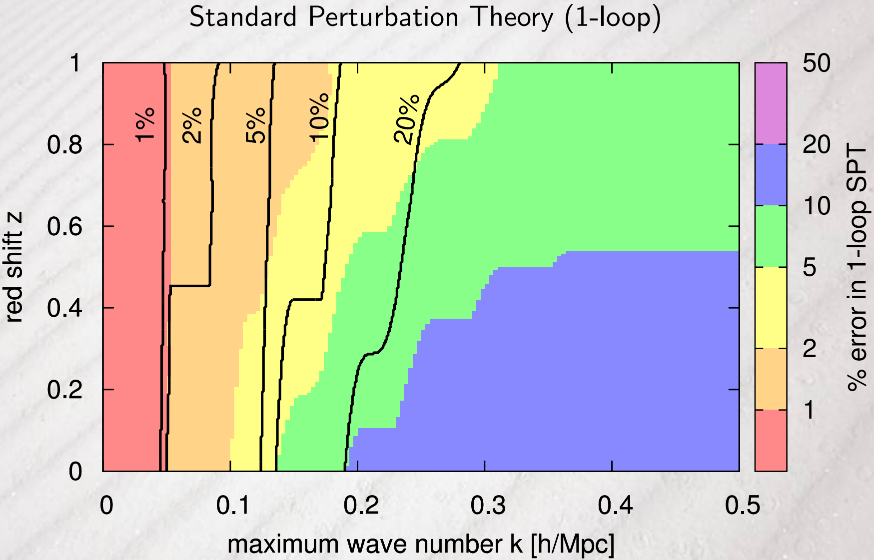

AU *et al.*, PRD **89**:103515(2014)

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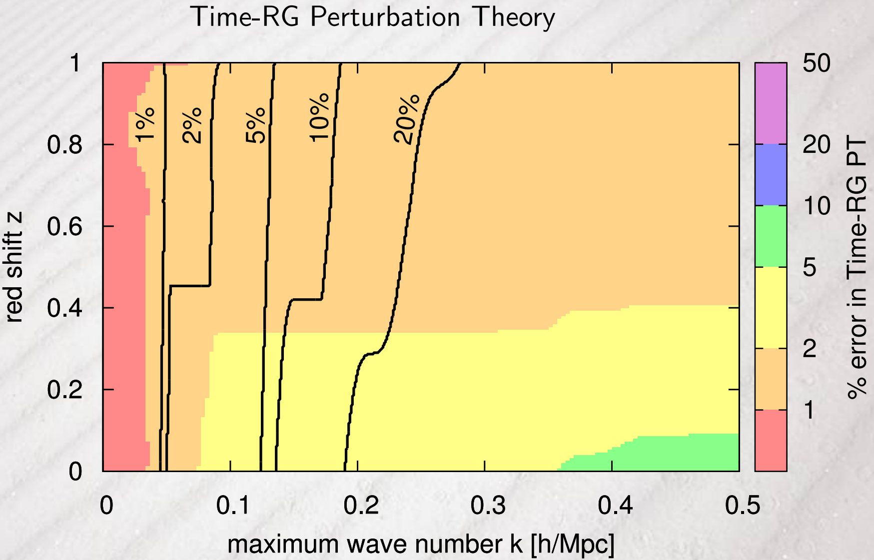

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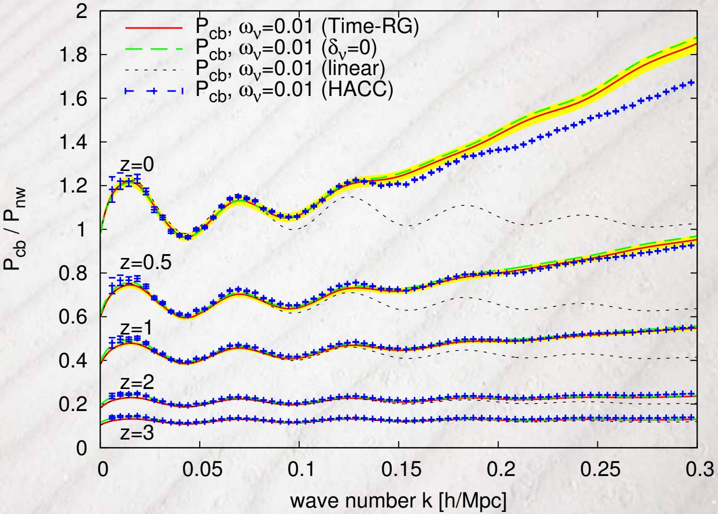
ΛCDM with massive neutrinos ($\Omega_\nu h^2 = 0.01$)



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Time-RG and massive neutrinos

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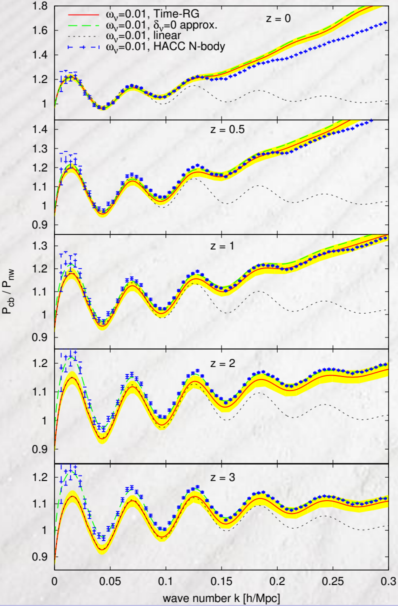
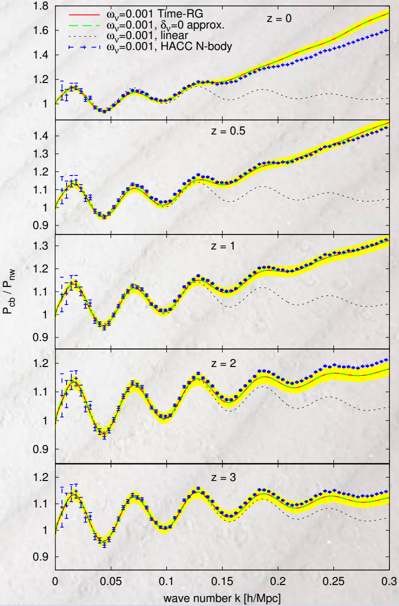
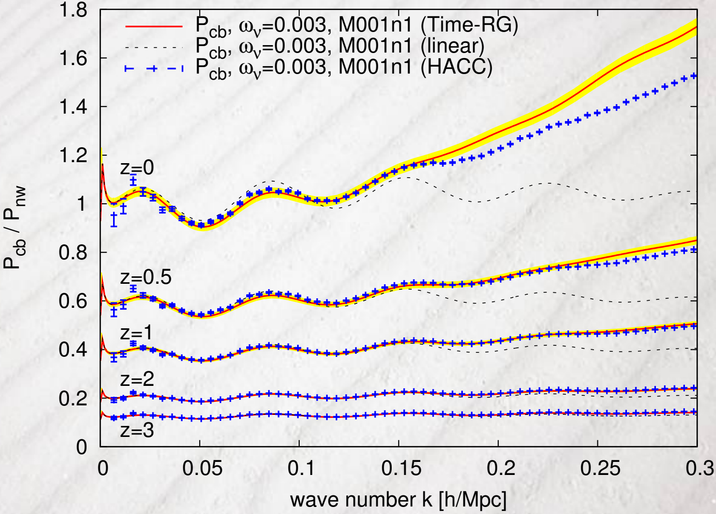


ΛCDM with $\Omega_\nu h^2 = 0.001$



Time-RG, massive neutrinos, and early dark energy

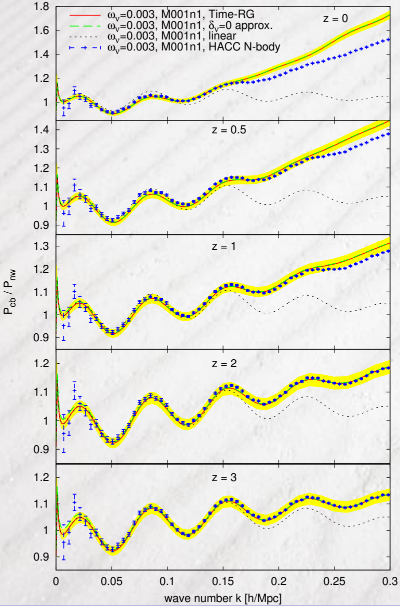
wCDM ($w(a) = w_0 + w_a(1-a)$, $w_0 = -0.7$, $w_a = 0.67$) with massive neutrinos ($\Omega_\nu h^2 = 0.003$)



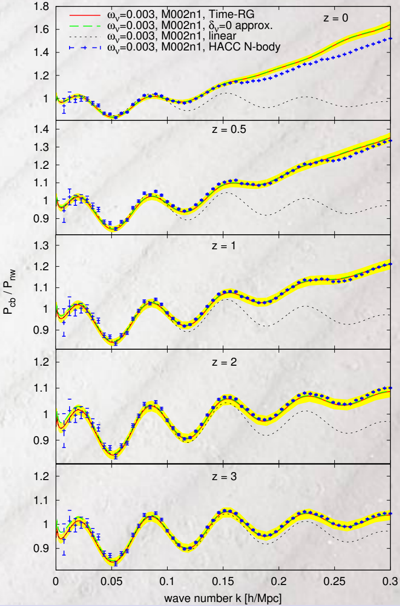
AU *et al.*, PRD **89**:103515(2014)

Time-RG, massive neutrinos, and early dark energy

$w_0 = -0.7,\ w_a = 0.67,\ \Omega_\nu h^2 = 0.003$



$w_0 = -1.17,\ w_a = 1.15,\ \Omega_\nu h^2 = 0.003$



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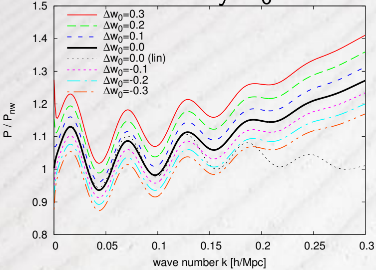
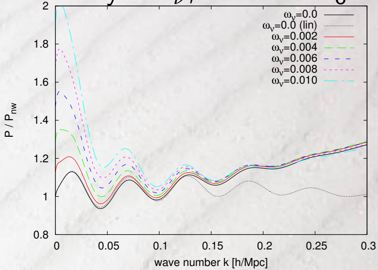
Varying w_0, w_a, and Σm_ν in Time-RG at $z = 1$

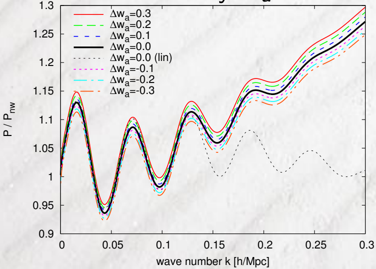
vary w_0



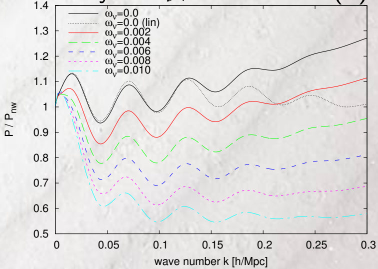
vary Σm_ν, constant σ_8



vary w_a



vary Σm_ν, constant $P(0)$



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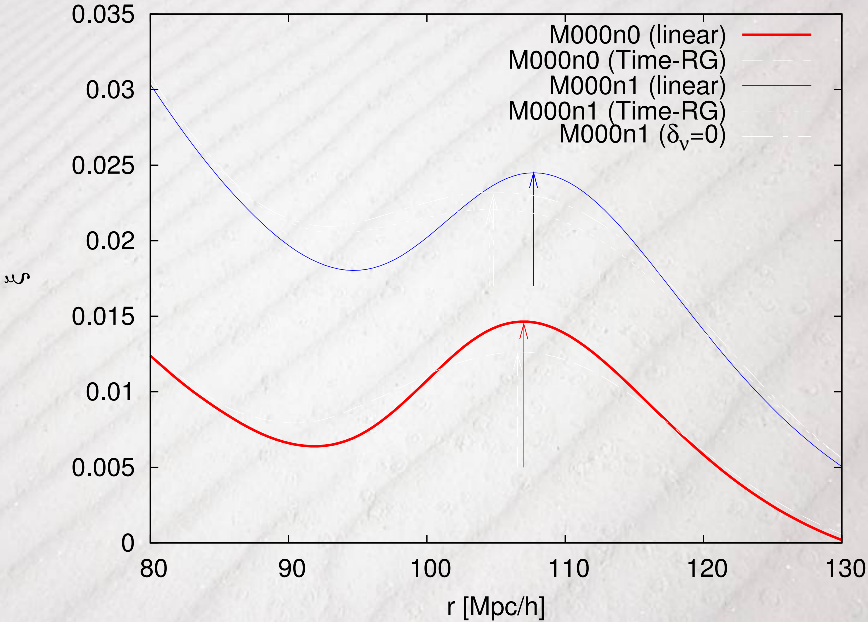
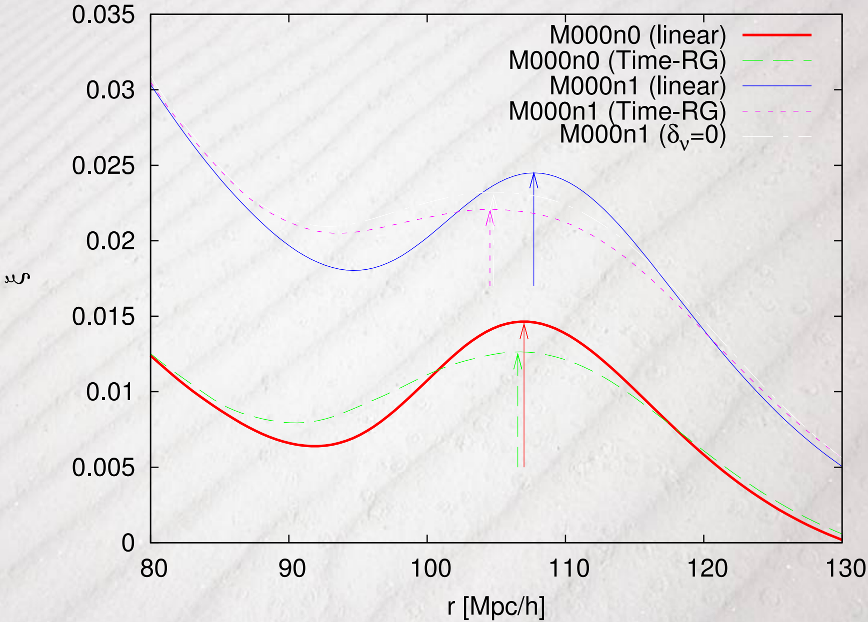
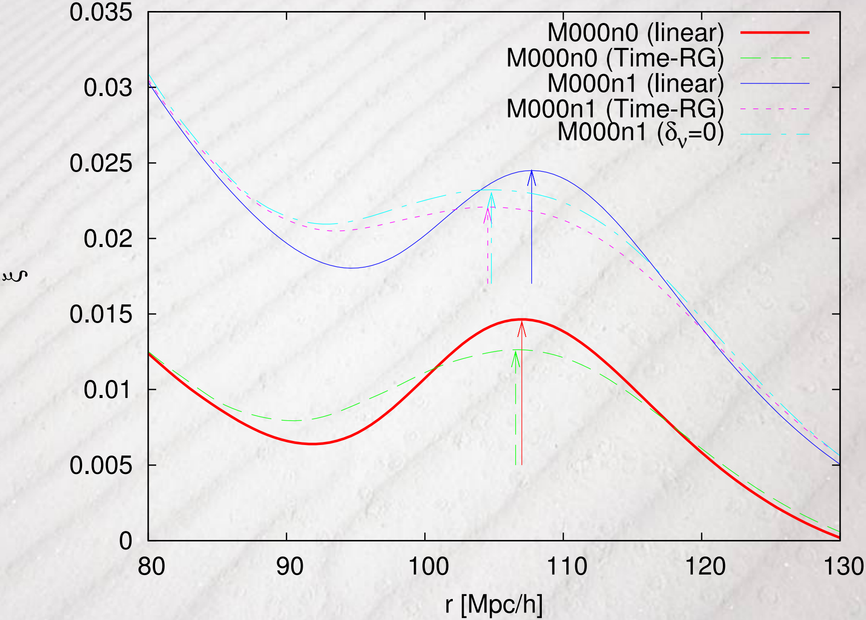
Effect of non-linearity on the BAO peak



Effect of non-linearity on the BAO peak

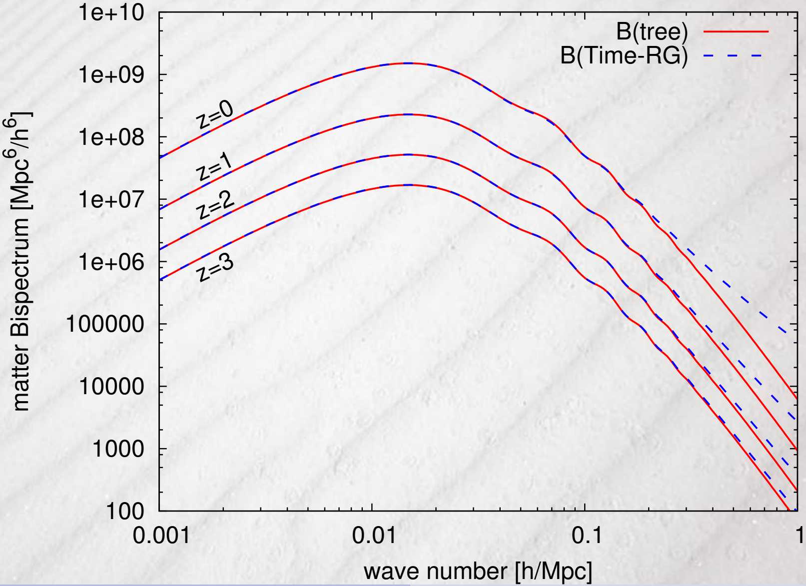


Effect of non-linearity on the BAO peak



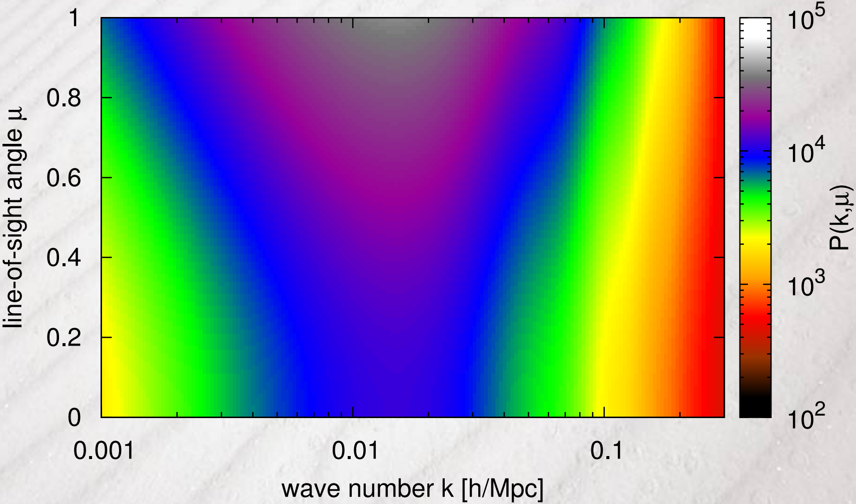
Future work: Bispectrum in Time-RG

Equilateral component of the bispectrum

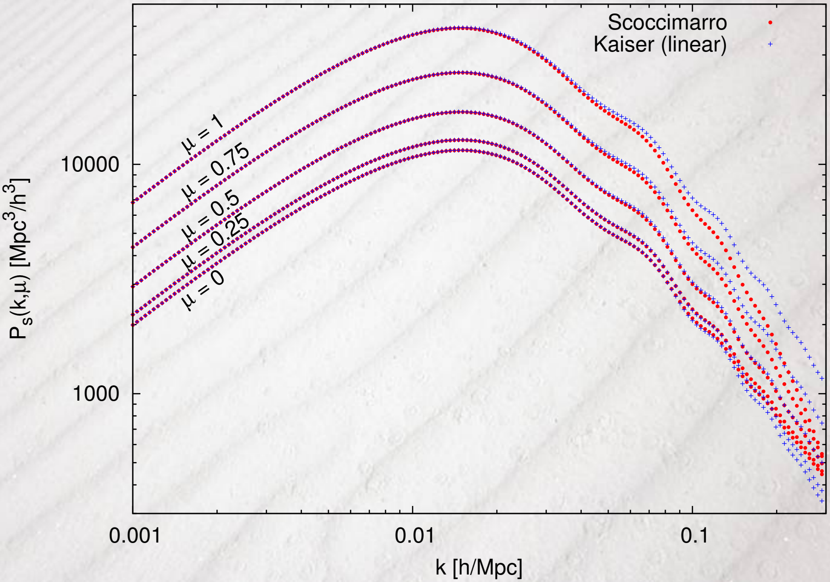


Future work: Redshift-space distortions

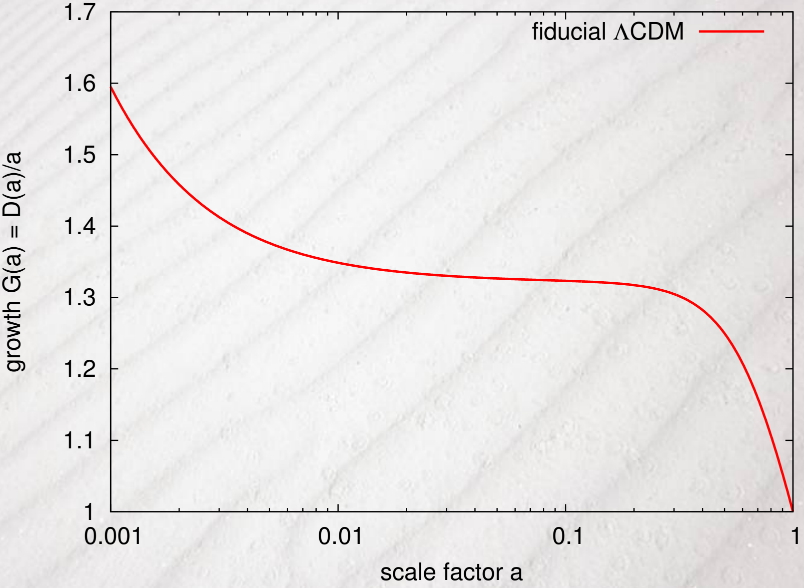
Scoccimarro ansatz: $P_s(k,\mu) = e^{-(fk\mu\sigma_v)^2}(P_{\delta\delta} - 2\mu^2 P_{\delta\theta} + \mu^4 P_{\theta\theta})$



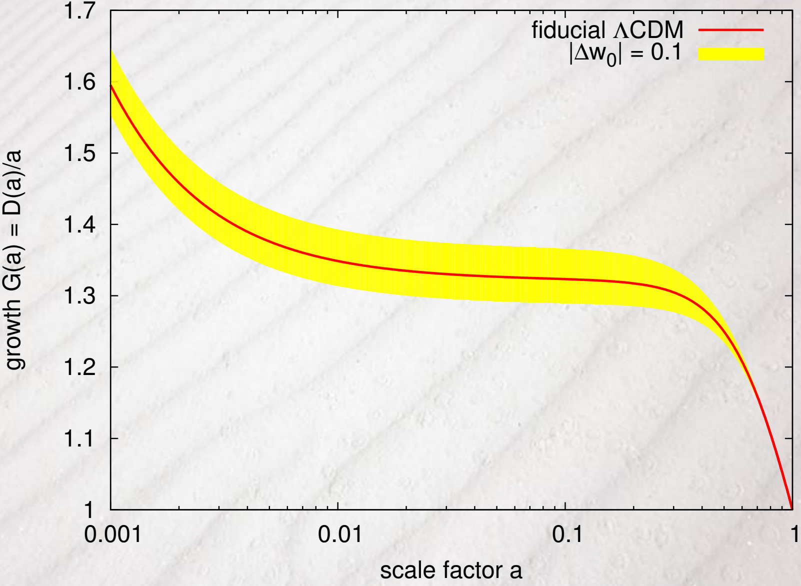
Future work: Redshift-space distortions

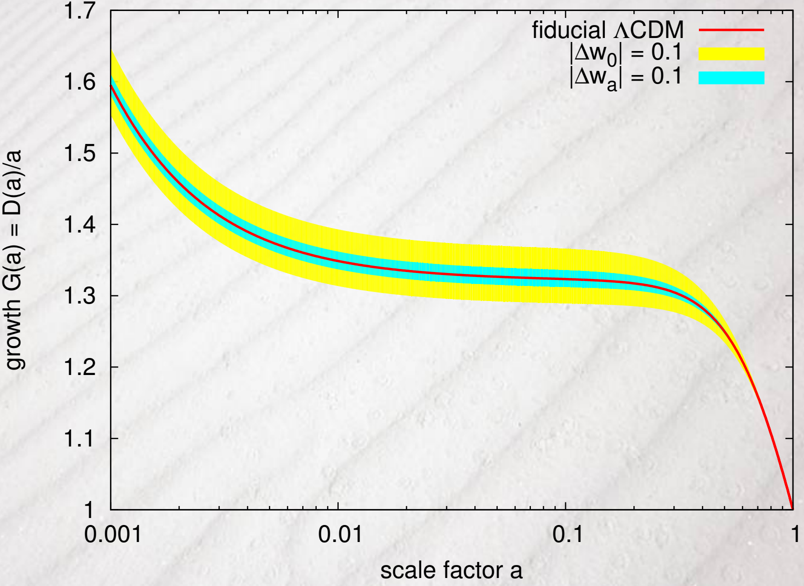


Future work: Forecasts on w_0, w_a, Ω_ν

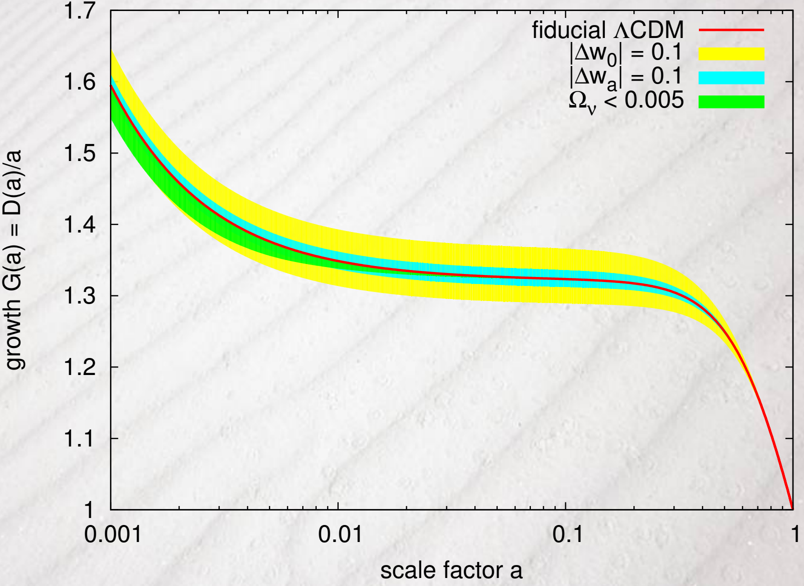


Future work: Forecasts on w_0, w_a, Ω_ν

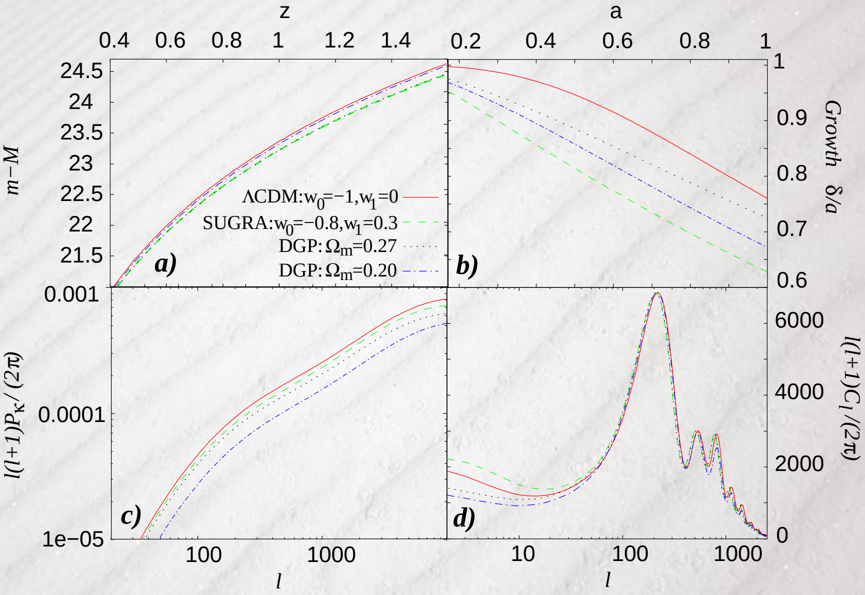



fiducial ΛCDM
|Δw0| = 0.1
|Δwa| = 0.1
growth G(a) = D(a)/a
scale factor a
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0.001
0.01
0.1
1

Future work: Forecasts on w_0, w_a, Ω_ν



Future work: Coupled dark energy (modified gravity)



*Ishak, AU, Spergel, PRD **74**:043513(2006)[arXiv:astro-ph/0507184]*

Conclusions

1. Higher-order perturbation theory improves significantly upon the linear computation of the power spectrum of large-scale structure.
2. Time-RG perturbation theory allows for scale-dependent growth, and agrees well with N-body simulations for early dark energy models with massive neutrinos.
3. w_0 and w_a mainly affect the power spectrum through the linear growth rate, while Σm_ν causes a scale-dependent suppression of power.
4. Although non-linearities shift the BAO peak scale, the contribution of neutrinos to this shift is small in the current generation of surveys.
5. Over the next decade, perturbation theory and N-body simulations will be powerful tools for interpreting galaxy surveys and constraining fundamental physics.
   The next 5-10 years will be very exciting!