Causality and Hyperbolicity of Lovelock Theories

Norihiro Tanahashi (Kavli IPMU/DAMTP)
with Harvey S. Reall, Benson Way

Based on arXiv:1406.3379 & work in progress

Causality and Hyperbolicity of Lovelock Theories

- Lovelock Theories
  = General Relativity + (higher-curvature corrections)

  - EoM up to 2nd-order derivatives → Avoids ghost instability
  - From string theory?

- GR: Gravitons propagate at the speed of light
- Lovelock: Faster/slower propagation than light

Causality in Lovelock theories?
Does EoM remain hyperbolic?

Causality and Hyperbolicity of Lovelock Theories

- Causality in Lovelock theories?
  - Can we define causality in this theory?
  - Can graviton escape from black hole interior?

- Does EoM remain hyperbolic?
  - Hyperbolic EoM = Wave equation
  - Determined by principal part of EoM
  - ➢ GR: EoM guaranteed to be hyperbolic
  - ➢ Lovelock: ?

Contents

1. Introduction
   - Lovelock theories
   - Characteristics
   - Hyperbolicity
2. Questions
   - Can graviton escape from black hole interior?
   - Propagation on *plane wave solutions*
   - Propagation *around black holes*
3. Summary

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Introduction: Lovelock theories

- Lovelock theories in d dimensions ($p \leq (d-1)/2$)

$$\mathcal{L} = R - 2\Lambda - \sum_{p\geq 2} 2k_p \delta^{c_1 \ldots c_{2p}}_{d_1 \ldots d_{2p}} R_{c_1 c_2}{}^{d_1 d_2} \ldots R_{c_{2p-1} c_{2p}}{}^{d_{2p-1} d_{2p}}$$

$$= R - 2\Lambda - 8k_2 \left(R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd} \right) + \cdots$$

$$\left(\delta^{c_1 \ldots c_n}_{d_1 \ldots d_n} \equiv n! \delta^{c_1}_{[d_1} \ldots \delta^{c_n}_{d_n]} \right)$$

- EoM = Einstein eq. + correction

$$0 = A^a{}_b \equiv G^a{}_b + \Lambda \delta^a{}_b + B^a{}_b$$

where

$$B^a{}_b = \sum_{p\geq 2} k_p \delta^{a c_1 \ldots c_{2p}}_{b d_1 \ldots d_{2p}} R_{c_1 c_2}{}^{d_1 d_2} \ldots R_{c_{2p-1} c_{2p}}{}^{d_{2p-1} d_{2p}}$$

Introduction: Characteristics

- Propagation of gravitational signals

  Propagate on *characteristic surface*

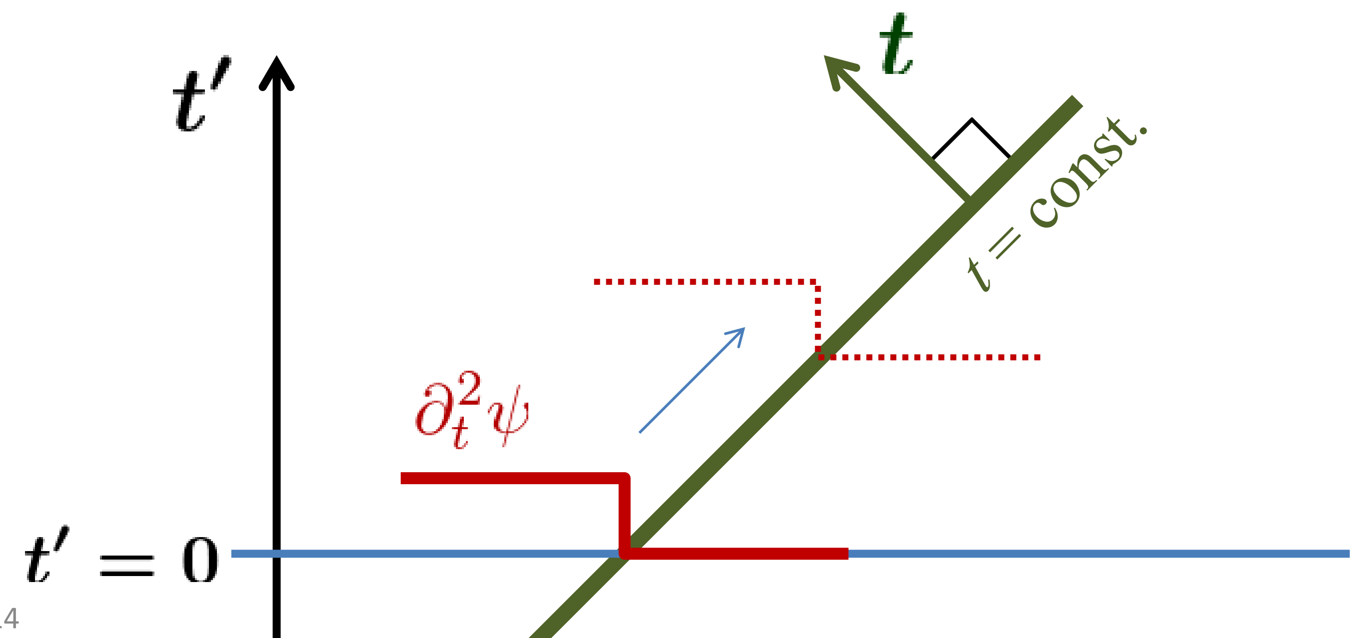
EoM of ψ: $$0 = E(\psi, \partial\psi, \partial^2\psi)$$
$$= \frac{\partial E}{\partial(\partial_t^2\psi)}\partial_t^2\psi + F(\partial_t\psi,\ \psi)$$

- $\frac{\partial E}{\partial(\partial_t^2\psi)} \neq 0$: $\partial_t^2\psi$ uniquely determined
  → usual time evolution
- $\frac{\partial E}{\partial(\partial_t^2\psi)} = 0$: $\partial_t^2\psi$ non-unique
  → t = const. surface is *characteristic*

Introduction: Characteristics

- $\dfrac{\partial E}{\partial(\partial_t^2\psi)} = 0:$ $\partial_t^2\psi$ non-unique
  → t = const. surface is *characteristic*

✓ *Characteristic surface* is a possible wave front



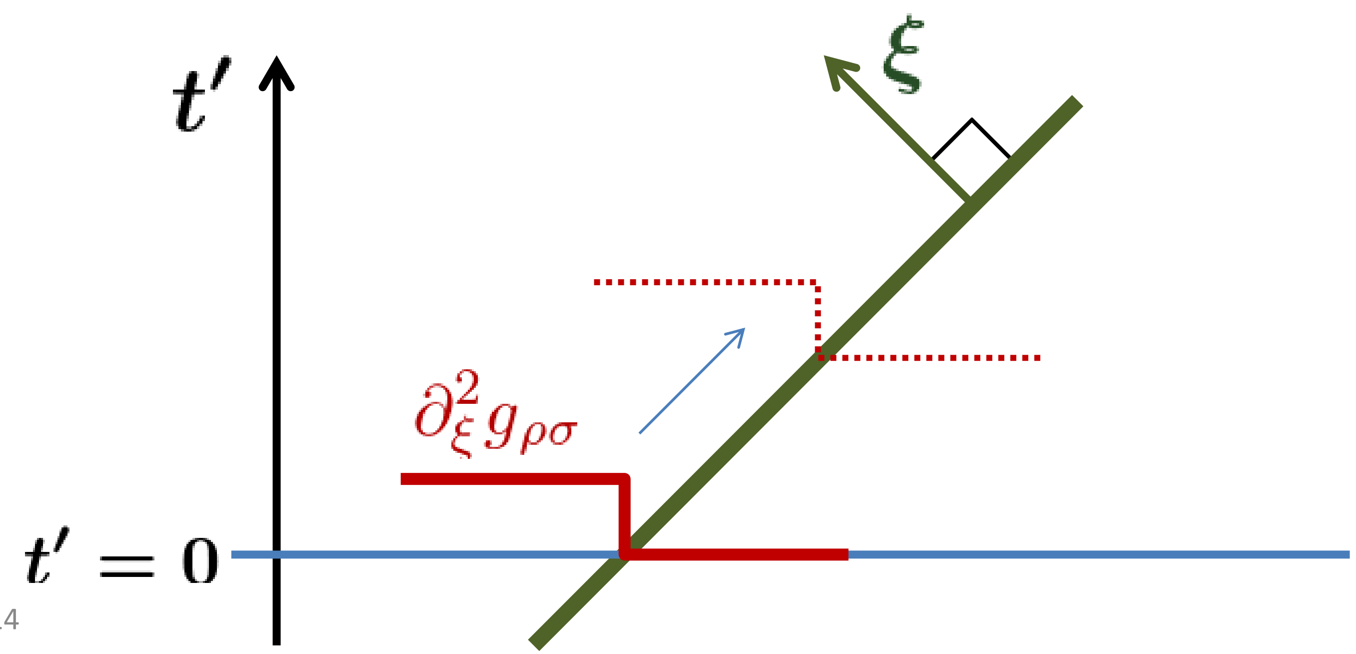
Introduction: Characteristics

- Characteristics in Lovelock theories

[Aragone '87]
[Choquet-Bruhat'88]

$$E_{ab} \equiv R_{ab} - \frac{2\Lambda}{d-2} g_{ab} + B_{ab} - \frac{1}{d-2} B^c{}_c g_{ab} = 0$$

$$P(x,\xi)_{\mu\nu}{}^{\rho\sigma} \equiv \frac{\delta E_{\mu\nu}}{\delta(\partial_t^2 g_{\rho\sigma})} = \frac{\delta E_{\mu\nu}}{\delta(\partial_\alpha \partial_\beta g_{\rho\sigma})} \xi_\alpha \xi_\beta$$



Introduction: Characteristics

- Characteristics in Lovelock theories

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$$E_{ab} \equiv R_{ab} - \frac{2\Lambda}{d-2} g_{ab} + B_{ab} - \frac{1}{d-2} B^c{}_c g_{ab} = 0$$

$$P(x,\xi)_{\mu\nu}{}^{\rho\sigma} \equiv \frac{\delta E_{\mu\nu}}{\delta(\partial_t^2 g_{\rho\sigma})} = \frac{\delta E_{\mu\nu}}{\delta(\partial_\alpha \partial_\beta g_{\rho\sigma})} \xi_\alpha \xi_\beta$$

✓ Characteristic $\Leftrightarrow$ $\det P = 0$

✓ $(P \cdot t)_{ab} = (P_{GR} \cdot t)_{ab} + (\mathcal{R} \cdot t)_{ab}$ (t_{ab}: symmetric)

$$(P_{GR} \cdot t)_{ab} = -\frac{1}{2}\xi^2 t_{ab} + \xi^c \xi_{(a} t_{b)c} - \frac{1}{2}\xi_a \xi_b t^c{}_c$$

$$(\mathcal{R} \cdot t)^a{}_b = -\sum_{p \geq 2} 2p k_p \delta^{ac_1 \ldots c_{2p}}_{bd_1 \ldots d_{2p}} \xi_{c_1} \xi^{d_1} t_{c_2}{}^{d_2} R_{c_3 c_4}{}^{d_3 d_4} \ldots R_{c_{2p-1} c_{2p}}{}^{d_{2p-1} d_{2p}}$$
$$+ \frac{1}{d-2} \delta^a_b \sum_{p \geq 2} 2p k_p \delta^{ec_1 \ldots c_{2p}}_{ed_1 \ldots d_{2p}} \xi_{c_1} \xi^{d_1} t_{c_2}{}^{d_2} R_{c_3 c_4}{}^{d_3 d_4} \ldots R_{c_{2p-1} c_{2p}}{}^{d_{2p-1} d_{2p}}$$

Introduction: Characteristics

- Characteristics in *GR*

$$\det P = 0 \Rightarrow (P_{GR} \cdot t)_{ab} = -\frac{1}{2}\xi^2 t_{ab} + \xi^c \xi_{(a} t_{b)c} - \frac{1}{2}\xi_a \xi_b t^c{}_c = 0$$

✓ Gauge modes: $(P\cdot t)$ invariant under

$$t_{ab} \rightarrow t_{ab} + \xi_{(a} X_{b)} \qquad \text{for any } X_a .$$

➢ If ξ is not null, $t_{ab} = \xi_{(a} X_{b)}$ for some X_a → Pure gauge modes $\times\, d$

➢ If ξ is null, $\xi^c \xi_{(a} t_{b)c} - \frac{1}{2}\xi_a \xi_b t^c{}_c = 0$

$$\rightarrow \quad \xi^b t_{ab} - \frac{1}{2}\xi_a t^c{}_c = 0 \quad \rightarrow \text{Constraints} \times d$$

∴ Physical modes with null ξ, $\frac{1}{2}d(d+1) - d - d = \frac{1}{2}d(d-3)$ modes

Introduction: Characteristics

- Characteristics in Lovelock theories [Aragone '87] [Choquet-Bruhat'88]

$$\det P = 0 \Rightarrow (P\cdot t)_{ab} = (P_{GR}\cdot t)_{ab} + (\mathcal{R}\cdot t)_{ab} = 0$$

➢ If ξ is not null, t_{ab} = (non-gauge part) + (gauge part)

$$t_{ab} = \hat{t}_{ab} + \xi_{(a}X_{b)} \quad \text{s.t.} \quad \xi^b \hat{t}_{ab} - \frac{1}{2}\xi_a \hat{t}^c{}_c = 0$$

$$\Rightarrow \quad \frac{1}{2}\xi^2 \hat{t} = \mathcal{R}(x,\xi)\cdot\hat{t}$$

➢ If ξ is null, solve in null coordinates $\xi_0 = 0 = \xi_i,\ \xi_1 = 1$

$$\begin{cases} \frac{1}{2}t_{00} + (\mathcal{R}\cdot t)_{01} = 0 & \frac{1}{2}t_{0i} + (\mathcal{R}\cdot t)_{1i} = 0 \\ (\mathcal{R}\cdot t)_{ij} = 0 & -\frac{1}{2}t_{ii} + (\mathcal{R}\cdot t)_{11} = 0 \end{cases}$$

Introduction: Hyperbolicity

- Hyperbolicity

  = "Initial value problem is *well-posed*"

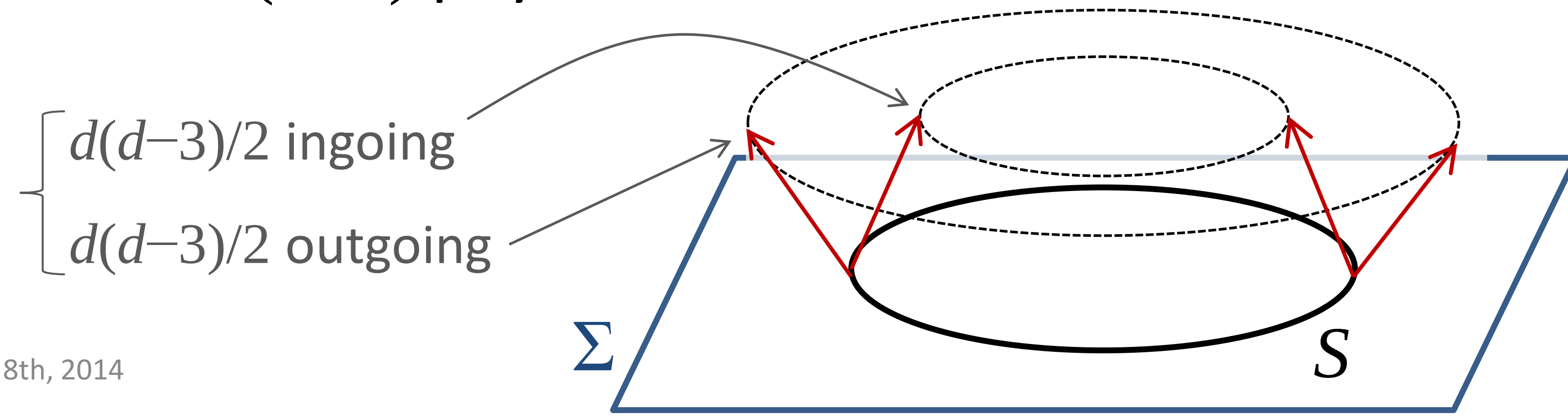
  = "*Unique solution exists locally* for good initial data"

  "Solution depends on initial data continuously"

  = Σ: $(d-1)$-dim. initial surface

  "Any $(d-2)$-dim. surface S in Σ has

  $d(d-3)$ physical characteristic surfaces from S"



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Questions

1. Can graviton escape from black hole interior?

2. Propagation on *plane wave solutions*
3. Propagation *around black holes*

- Does it obey causality?
- Is hyperbolicity maintained?

Summary

- ◆ Characteristics in Lovelock theories

1. Can graviton escape from black hole interior?

   → No: Killing horizon is characteristic surface

2. Propagation on *Ricci-flat type N spacetimes*

   ✓ Characteristics = Null w.r.t. effective metrics

   ✓ Causality w.r.t. the largest cone

3. Propagation *around black holes*

   ✓ Characteristics = Null w.r.t. effective metrics

   ✓ Hyperbolicity violation near small BH horizons

?: Does ~~hyperbolicity~~ occur in generic time evolution?

?: Propagation of discontinuity in this theory
→ Shock formation due to nonlinearity?

1. “Can graviton escape from black hole interior?”

⇔ “Is an event horizon characteristic for any mode?”

≈ “Is a Killing horizon characteristic for any mode?”

✓ GR: All characteristics are null

→ Killing horizon is a characteristic

✓ GR + Gauss-Bonnet correction:

Killing horizon shown to be a characteristic

[Izumi ‘14]

✓ Lovelock: ?

1. "Can graviton escape from black hole interior?"

$\Leftrightarrow$ "Is an event horizon characteristic for any mode?"

$\approx$ "Is a Killing horizon characteristic for any mode?"

- Killing horizon $\Rightarrow$ $R_{0i0j} = R_{0ijk} = 0$ in null coordinates
- Assuming null ξ, count the number of solutions of

$$\begin{cases} \frac{1}{2}t_{00} + (\mathcal{R}\cdot t)_{01} = 0 & \frac{1}{2}t_{0i} + (\mathcal{R}\cdot t)_{1i} = 0 \\ (\mathcal{R}\cdot t)_{ij} = 0 & -\frac{1}{2}t_{ii} + (\mathcal{R}\cdot t)_{11} = 0 \end{cases}$$

✓ Assume $t_{00} = t_{0\mathrm{i}} = 0$ $\Rightarrow$ Only $-\frac{1}{2}t_{ii} + (\mathcal{R}\cdot t)_{11} = 0$ remains

$$\therefore \quad \frac{1}{2}d(d+1) - d - (1+d-2) - 1 = \frac{1}{2}d(d-3) \text{ modes}$$

$\therefore$ A Killing horizon is characteristic for any mode.

2. Propagation on *plane wave solutions*

More generally, we consider

Ricci-flat type N spacetimes

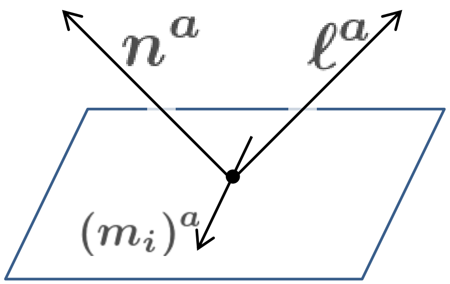
as backgrounds.

- Null basis

$$\begin{cases} (e_0)^a = \ell^a \\ (e_1)^a = n^a \\ (e_i)^a = m^a \end{cases}$$

$$\ell \cdot \ell = 0 = n \cdot n$$

$$\ell \cdot n = 1$$



- *Ricci-flat type N spacetimes*:

  Only non-vanishing component of Riemann tensor is

$$R_{1i1j} \equiv \Omega_{ij}$$

symmetric traceless

2. Propagation on *plane wave solutions*

- *Ricci-flat type N spacetimes*:

  Only non-vanishing component of Riemann tensor is

$$R_{1i1j} \equiv \Omega_{ij}$$

symmetric traceless

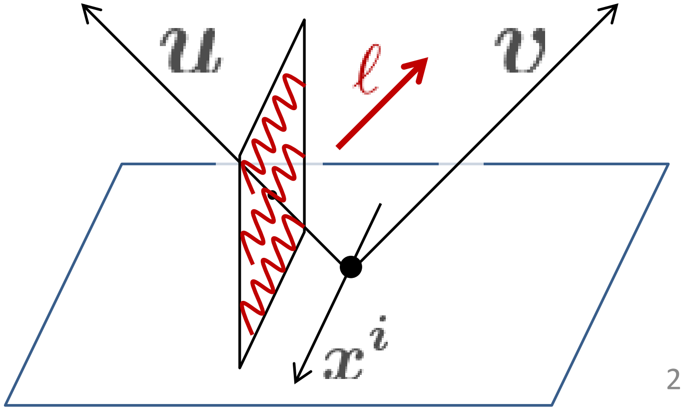
✓ Solution of Lovelock theories if $\Lambda = 0$

✓ Example: Plane wave solution [Boulware-Deser '85]

$$ds^2 = a_{ij}(u) x^i x^j du^2 + 2du dv + \delta_{ij} dx^i dx^j$$

$a_{ij}(u)$: Symmetric traceless

$$(e_0)^a = \ell^a = (\partial/\partial v)^a$$



2. Propagation on *plane wave solutions*

- *Ricci-flat type N spacetimes*:
  Only non-vanishing component of Riemann tensor is

$$R_{1i1j} \equiv \Omega_{ij}$$

symmetric traceless

✓ Solution of Lovelock theories if $\Lambda = 0$

✓ Example: Plane wave solution [Boulware-Deser '85]

$$ds^2 = a_{ij}(u) x^i x^j du^2 + 2dudv + \delta_{ij} dx^i dx^j$$

➢ Assume a_{ij} to be constant for simplicity

$$\Rightarrow \ R_{1i1j} \propto a_{ij}$$

2. Propagation on *Ricci-flat type N spacetimes*

Proposition:

Characteristic surfaces are null w.r.t. “effective metrics”:

$$G_I^{ab} = g^{ab} + \omega_I \ell^a \ell^b \quad (I = 1, \ldots, d(d-3)/2)$$

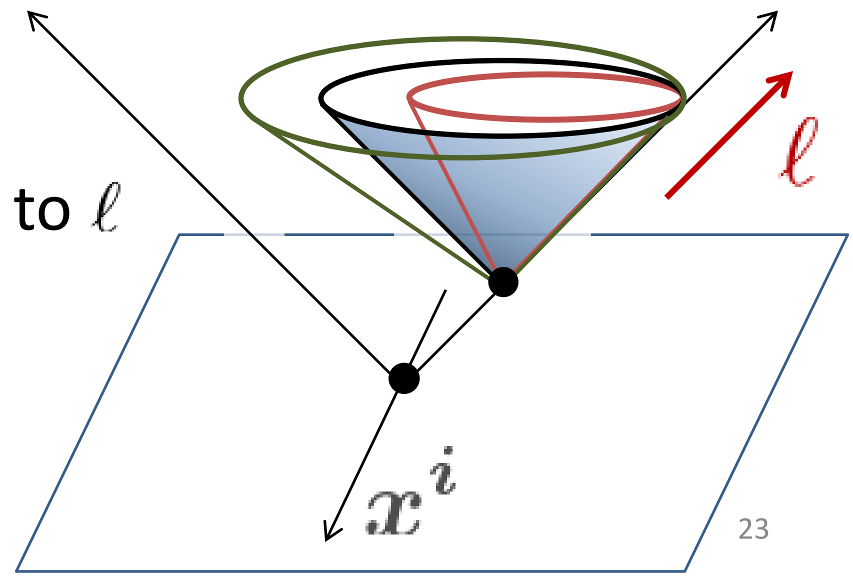
✓ ω_I : Functions of Ω_{ij}

✓ ℓ : null w.r.t. G_I

$\Rightarrow$ Characteristic cones tangent to ℓ

✓ Nested characteristic cones

✓ Causality w.r.t. the largest cone



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Key points:

- $(\mathcal{R} \cdot t)^\mu{}_\nu$ simplifies:

$$(\mathcal{R} \cdot t)^\mu{}_\nu = 16k_2 \left(-\delta^{\mu\rho_1\rho_2 1i}_{\nu\sigma_1\sigma_2 0j} \xi_{\rho_1} \xi^{\sigma_1} t_{\rho_2}{}^{\sigma_2} \Omega_{ij} + \frac{1}{d-2} \delta^\mu_\nu \delta^{k\rho_1\rho_2 1i}_{k\sigma_1\sigma_2 0j} \xi_{\rho_1} \xi^{\sigma_1} t_{\rho_2}{}^{\sigma_2} \Omega_{ij} \right)$$

- Non-null characteristics satisfies $\frac{1}{2}\xi^2 \hat{t} = \mathcal{R}(x,\xi) \cdot \hat{t}$

$\Rightarrow$ Eigenvalue eq. $\mathcal{R}(x,\xi) \cdot \hat{t} = T^{ab} \xi_a \xi_b \, \hat{t}$ gives

$$0 = \xi^2 - T^{ab}\xi_a\xi_b = \left(g^{ab} - T^{ab}\right)\xi_a\xi_b$$

2. Propagation on *Ricci-flat type N spacetimes*

- Eigenvalue eq. for $\mathcal{R}(x,\xi)\cdot t$

  - Gauge modes: $t_{ab} = \xi_{(a}X_{b)}$
  - Zero eigenvalue modes: $\begin{cases} t_{ab} = \ell_{(a}X_{b)} \\ t_{ij} = \hat{t}_{ij} + \alpha\delta_{ij}, \quad t_{0\mu} = 0 = t_{1\mu} \end{cases}$
  - Non-zero eigenvalue modes:

$$t_{ab} = 2t_{01}\ell_{(a}n_{b)} + t_{ij}m_{ia}m_{ib} \qquad (t_{ii} = 0)$$

$$\Rightarrow \begin{cases} (\mathcal{R}\cdot t)_{01} = \dfrac{16k_2(d-4)}{d-2}\left(\dfrac{1}{2}t_{01}\xi^i\xi^j + \xi_0^2 t^{ij}\right)\Omega_{ij} \\ (\mathcal{R}\cdot t)_{ij} = 16k_2\,\xi_0^2\,\mathcal{O}(t)_{ij} \end{cases}$$

$$\left(\mathcal{O}(t)_{ij} = t_{ik}\Omega_{kj} + t_{jk}\Omega_{ki} - \frac{2}{d-2}t_{kl}\Omega_{kl}\delta_{ij}\right)$$

2. Propagation on *Ricci-flat type N spacetimes*

- Eigenvalue eq. for $\mathcal{R}(x,\xi)\cdot t$

$$\mathcal{O}(t)_{ij} = \nu_I\, t_{ij} \quad \Rightarrow \quad (\mathcal{R}\cdot t)_{ij} = -\frac{1}{2}\xi_0^2\, \omega_I\, t_{ij}$$

$$(I = 1, \ldots, d(d-3)/2) \qquad (\omega_I = -32 k_2 \nu_I)$$

➢ Non-zero eigenvalue modes:

$$t_{ab} = 2t_{01}\ell_{(a}n_{b)} + t_{ij}m_{ia}m_{ib} \qquad (t_{ii} = 0)$$

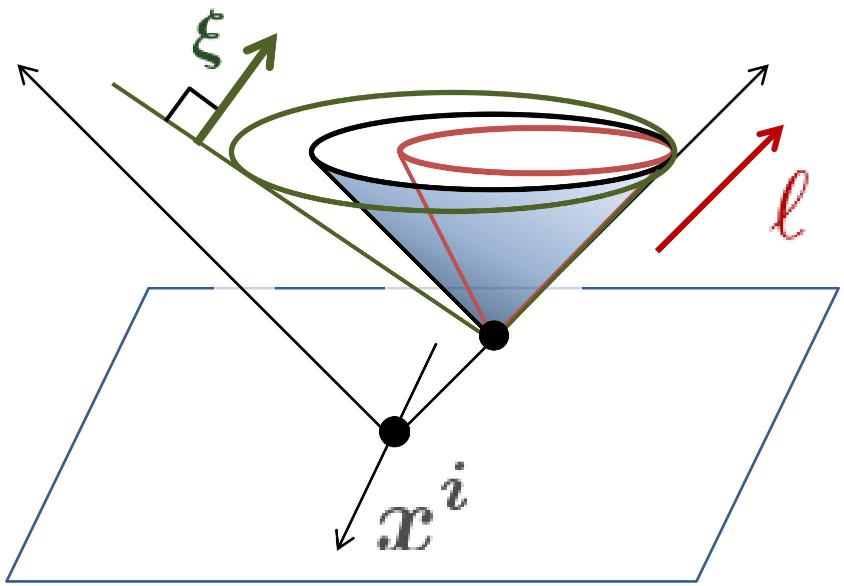
$$\Rightarrow \begin{cases} (\mathcal{R}\cdot t)_{01} = \dfrac{16k_2(d-4)}{d-2}\left(\dfrac{1}{2}t_{01}\xi^i\xi^j + \xi_0^2 t^{ij}\right)\Omega_{ij} \\ (\mathcal{R}\cdot t)_{ij} = 16k_2\, \xi_0^2\, \mathcal{O}(t)_{ij} \end{cases}$$

$$\left[\mathcal{O}(t)_{ij} = t_{ik}\Omega_{kj} + t_{jk}\Omega_{ki} - \frac{2}{d-2}t_{kl}\Omega_{kl}\delta_{ij}\right]$$

2. Propagation on *Ricci-flat type N spacetimes*

- Non-null characteristics satisfies $\frac{1}{2}\xi^2\hat{t} = \mathcal{R}(x,\xi)\cdot\hat{t}$
- $(\mathcal{R}\cdot t)_{ij} = -\frac{1}{2}\xi_0^2\ \omega_I\ t_{ij}$

$$\Rightarrow \quad 0 = \xi^2 + \omega_I \xi_0^2 = (g^{ab} + \omega_I \ell^a \ell^b)\xi_a\xi_b$$
$$\equiv G_I^{ab}\xi_a\xi_b \qquad (I = 1, \ldots, d(d-3)/2)$$



3. Propagation *around black holes*

- Static, maximally symmetric black holes

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Sigma^2$$

- ➢ Σ : $(d-2)$-dim space with constant curvature $\kappa = +1, 0, -1$
- ➢ $f(r) = \kappa - r^2\psi(r)$
- ➢ $\psi(r)$ satisfies an algebraic equation

$$W[\psi] \equiv -\sum_{p\geq 2}\left[2^{p+1}k_p\left(\prod_{k=1}^{2p-2}(d-2-k)\right)\psi^p\right] + \psi - \frac{2\Lambda}{(d-1)(d-2)} = \frac{\mu}{r^{d-1}}$$

3. Propagation *around black holes*

- Static, maximally symmetric black holes

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Sigma^2$$

- Orthonormal basis

$$\begin{cases} e_0 = -f^{1/2}dt \\ e_1 = f^{-1/2}dr \\ e_i = \text{(Orthonormal in } \Sigma) \end{cases}$$

- $$\begin{cases} R_{IJKL} = R_1(r)\left(\eta_{IK}\eta_{JL} - \eta_{IL}\eta_{JK}\right) \\ R_{IiJj} = R_2(r)\ \eta_{IJ}\delta_{ij} \\ R_{ijkl} = R_3(r)\left(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}\right) \end{cases}$$

η_{IJ} : 2 dim Minkowski

δ_{ij} : metric of Σ

3. Propagation *around black holes*

Proposition:

Characteristic surfaces are null w.r.t. "effective metrics":

$$G^{A}_{\mu\nu}dx^{\mu}dx^{\nu} = -f(r)dt^2 + f(r)^{-1}dr^2 + \frac{r^2}{c_A(r)}d\Sigma^2$$

- ✓ A : Tensor, Vector, Scalar modes
- ✓ $c_A(r)$: (Propagation speed)^2 in Σ directions

$$0 = \det P(x,\xi) = \left(G_S^{ab}(x)\xi_a\xi_b\right)^{p_S}\left(G_V^{cd}(x)\xi_c\xi_d\right)^{p_V}\left(G_T^{ef}(x)\xi_e\xi_f\right)^{p_T}$$

$$p_S + p_V + p_T = d(d-3)/2$$

3. Propagation *around black holes*

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- ✓ A : Tensor, Vector, Scalar modes
- ✓ $c_A(r)$: (Propagation speed)^2 in Σ directions

✓ Read out from perturbation equations

$$0 = \frac{\delta E_{\mu\nu}}{\delta(\partial_\alpha\partial_\beta g_{\rho\sigma})}\partial_\alpha\partial_\beta\delta g_{\rho\sigma} + \cdots \quad \Rightarrow \quad \frac{\delta E_{\mu\nu}}{\delta(\partial_\alpha\partial_\beta g_{\rho\sigma})}\xi_\alpha\xi_\beta = P(x,\xi)$$

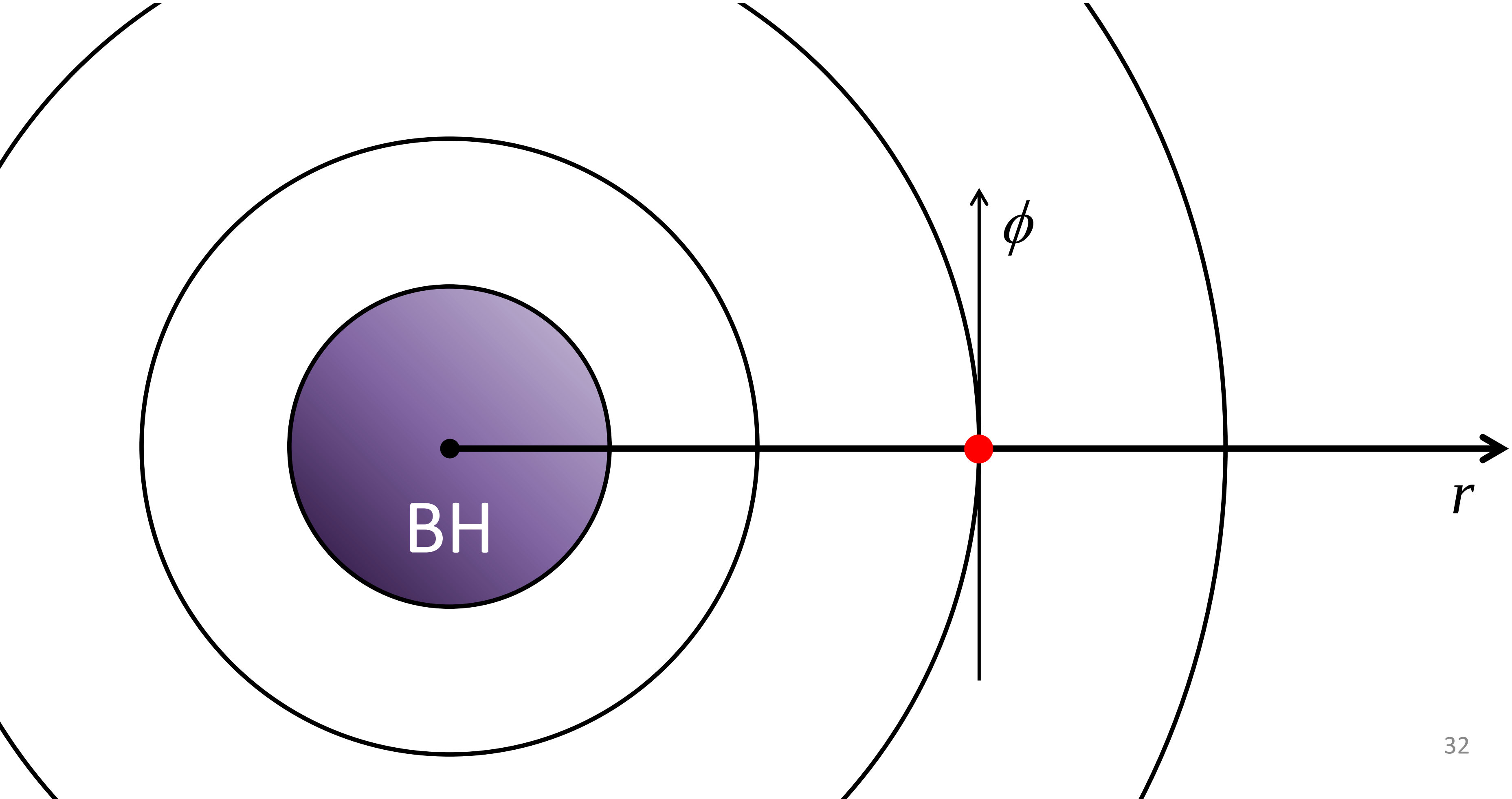
$$\left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} - V_l(r)\right)\Psi_l(t,r) = 0 \quad \Rightarrow \quad \left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} + \frac{f(r)c_A(r)D^2}{r^2}\right)\Psi \equiv f(r)G_A^{\mu\nu}\partial_\mu\partial_\nu\Psi$$

[Dotti-Gleiser '05]
[Konoplya-Zhidenko '08]
[Takahashi-Soda '09, '10]

$$V_l(r) \ni \frac{l^2}{r^2} \simeq -\frac{1}{r^2}D^2$$

3. Propagation *around black holes*

$$G^{A}_{\mu\nu}dx^{\mu}dx^{\nu} = -f(r)dt^2 + f(r)^{-1}dr^2 + \frac{r^2}{c_A(r)}d\Sigma^2$$



3. Propagation *around black holes*

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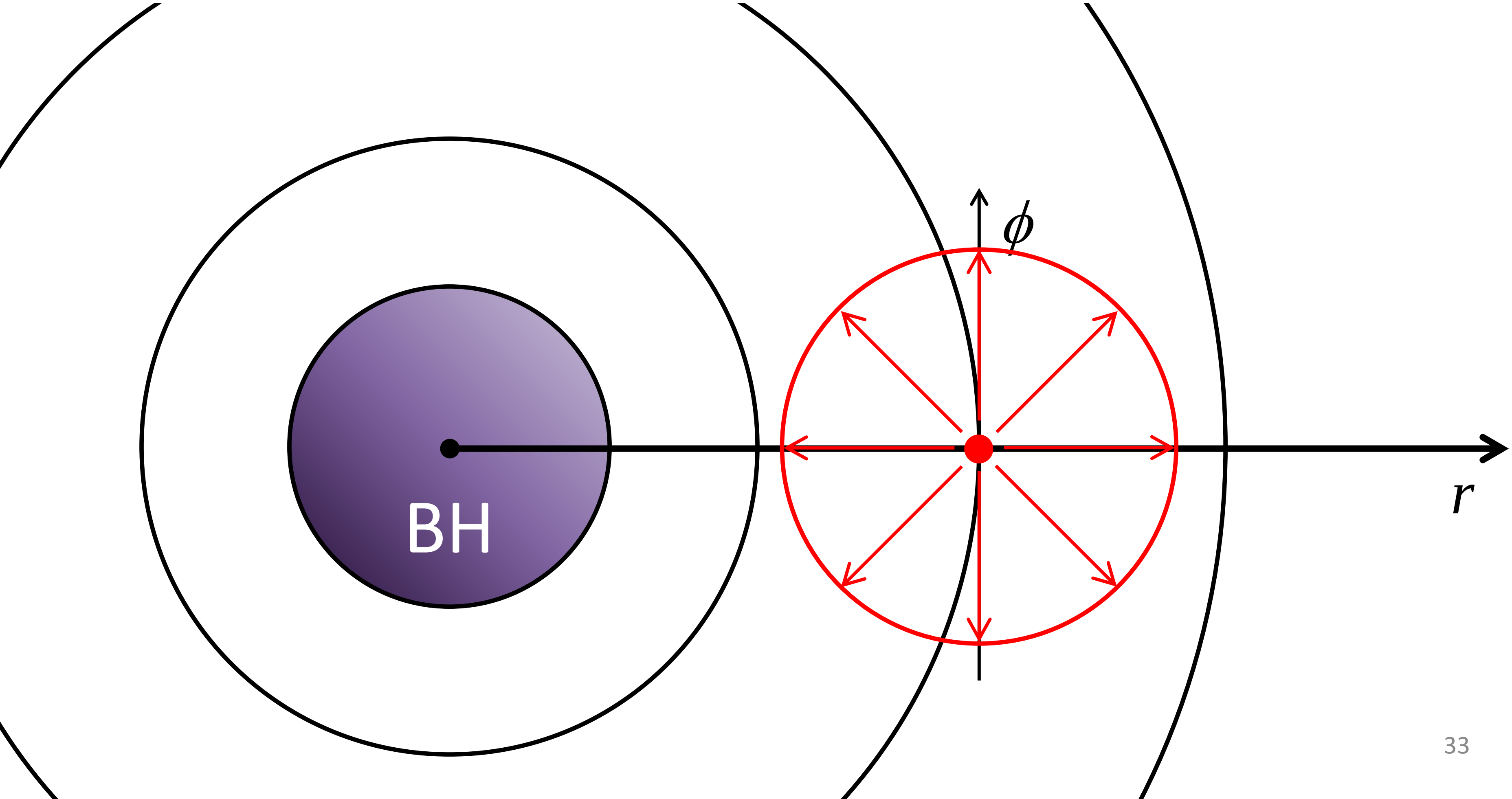
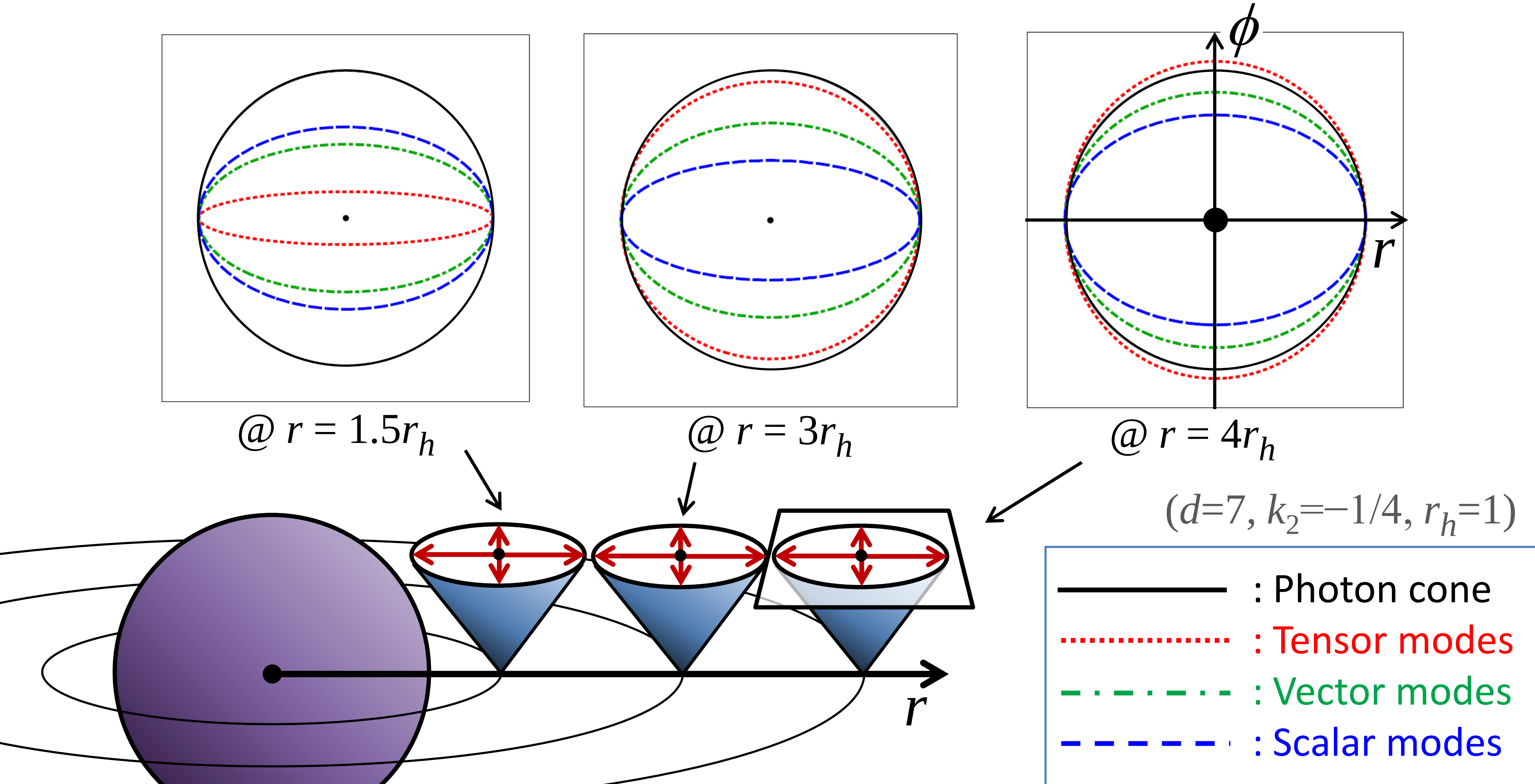
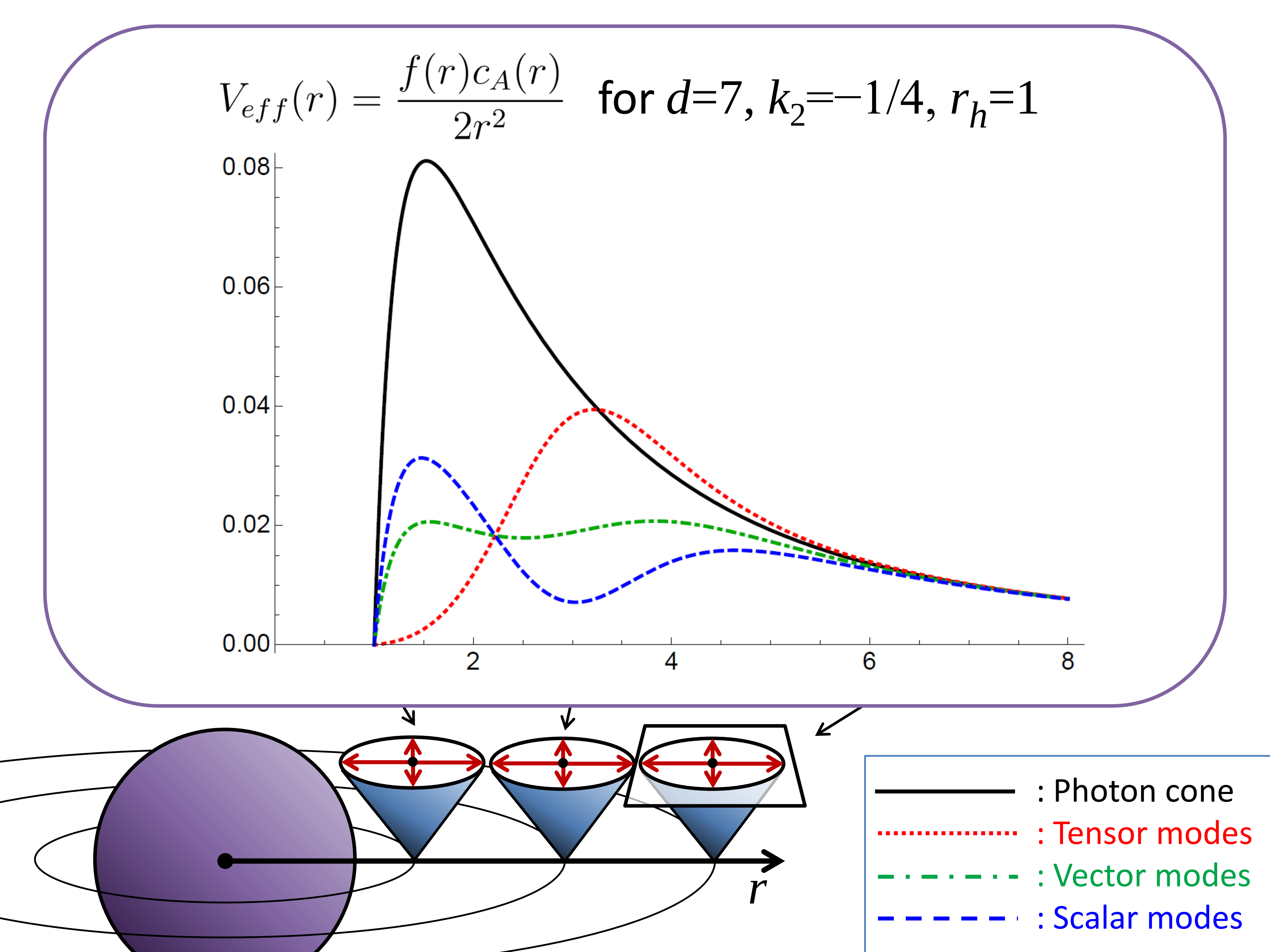


3. Propagation *around black holes*

$$G^{A}_{\mu\nu}dx^{\mu}dx^{\nu} = -f(r)dt^2 + f(r)^{-1}dr^2 + \frac{r^2}{c_A(r)}d\Sigma^2$$

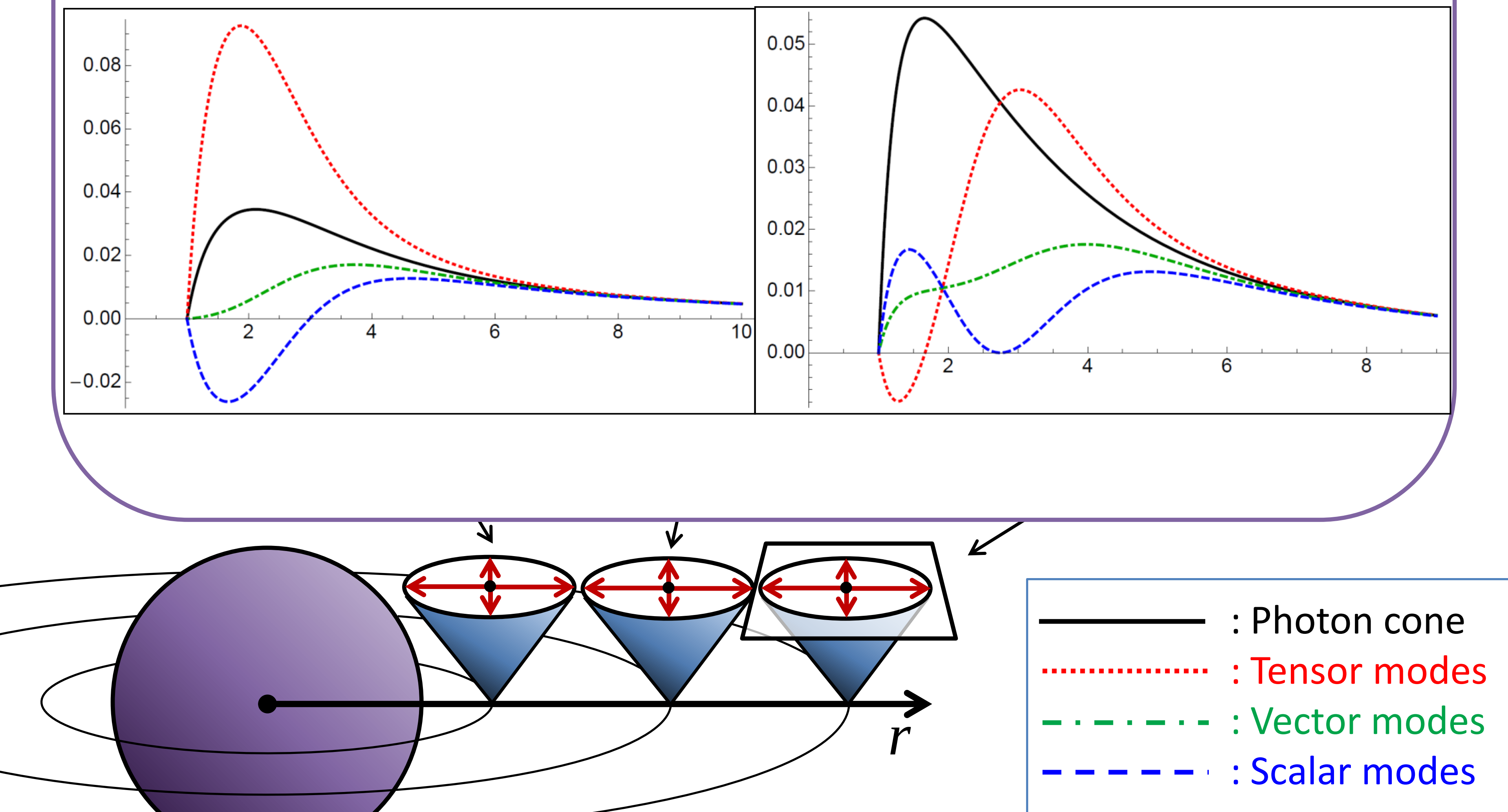




$V_{eff}(r) = \frac{f(r)c_A(r)}{2r^2}$ for d=7, k_2=−1/4, r_h=1
0.08
0.06
0.04
0.02
0.00
2
4
6
8
r
: Photon cone
: Tensor modes
: Vector modes
: Scalar modes

For d=5: $c_{\text{scalar}}(r)<0$ near horizon

For d=6: $c_{\text{tensor}}(r)<0$ near horizon



3. Propagation *around black holes*

- Small BH limit ($r_h \to 0$, k_n fixed)

P: Highest order of Lovelock term ($d \geq 2P+1$)

$d = 2P+1$ ⇨

$$\begin{cases} c_T(r_0) = \dfrac{3}{2P-3} + \mathcal{O}(r_0^2) \\ c_V(r_0) = \mathcal{O}(r_0^4) \qquad < 0 \\ c_S(r_0) = -\dfrac{3}{2P-1} + \mathcal{O}(r_0^2) \qquad < 0 \end{cases}$$

$d \neq 2P+1$ ⇨

$$\begin{cases} c_T(r_h) = \dfrac{d-1-3P}{(d-4)P} + \mathcal{O}(r_h^2) \qquad < 0 \text{ for } d < 1+3P \\ c_V(r_h) = \dfrac{d-1-2P}{(d-3)P} + \mathcal{O}(r_h^2) \\ c_S(r_h) = \dfrac{d-1-P}{(d-2)P} + \mathcal{O}(r_h^2) \end{cases}$$

3. Propagation *around black holes*

- $c_A < 0 \Rightarrow$ Violation of hyperbolicity

$$G^{A}_{\mu\nu}dx^\mu dx^\nu = -f(r)dt^2 + f(r)^{-1}dr^2 + \frac{r^2}{c_A(r)}d\Sigma^2$$

$$\Leftrightarrow \quad \left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} + \frac{f(r)c_A(r)D^2}{r^2}\right)\Psi \equiv f(r)G_A^{\mu\nu}\partial_\mu\partial_\nu\Psi$$

$D^2 \simeq -l^2$

- Interpretations

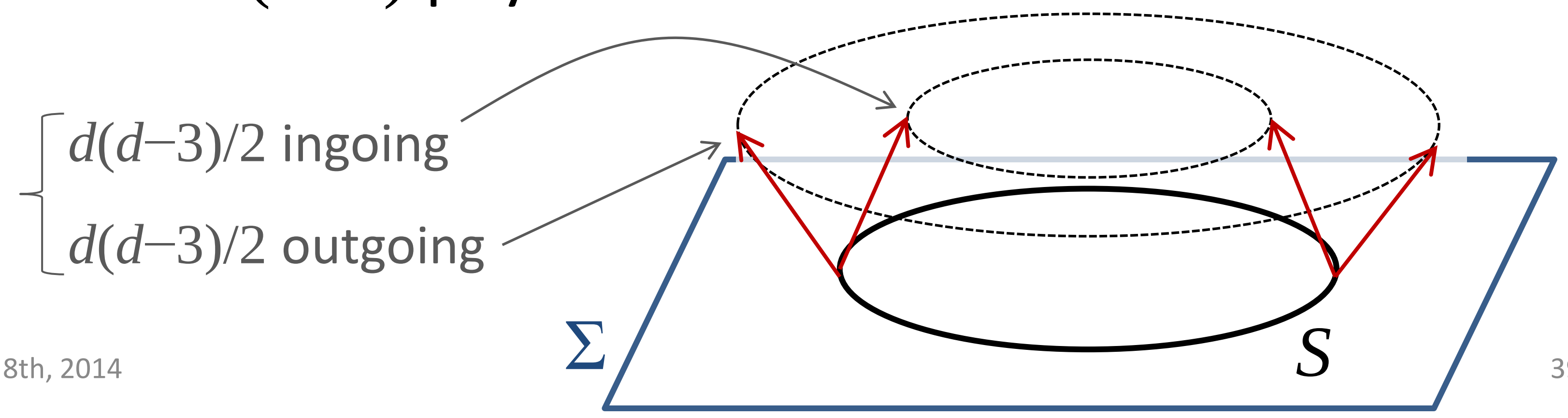
1. $\omega^2 = -\alpha^2 l^2 \Rightarrow$ Instability $\propto \exp(\alpha l t)$
2. Initial value problem is not well-posed

$$\delta g_{\mu\nu}(t,r,x) \sim e^{-\sqrt{l}}e^{\alpha l t} = \begin{cases} \bullet\, t = 0 \Rightarrow \delta g, \partial^n \delta g = 0 \\ \bullet\, t > 0 \Rightarrow \delta g \to \infty \end{cases}$$

with $l \to \infty$

Introduction: Hyperbolicity

- Hyperbolicity
  = "Initial value problem is *well-posed*"
  = "*Unique solution exists locally* for good initial data"
  "Solution depends on initial data continuously"
  = Σ: $(d-1)$-dim. initial surface
  "Any $(d-2)$-dim. surface S in Σ has
  $d(d-3)$ physical characteristic surfaces from S"



Summary

- Characteristics in Lovelock theories
  1. Can graviton escape from black hole interior?

     → No: Killing horizon is characteristic surface

  2. Propagation on *Ricci-flat type N spacetimes*

     ✓ Characteristics = Null w.r.t. effective metrics

     ✓ Causality w.r.t. the largest cone

  3. Propagation *around black holes*

     ✓ Characteristics = Null w.r.t. effective metrics

     ✓ Hyperbolicity violation near small BH horizons

?: Does ~~hyperbolicity~~ occur in generic time evolution?

?: Propagation of discontinuity in this theory

→ Shock formation due to nonlinearity?

$$V_{eff}(r) = \frac{f(r)c_A(r)}{2r^2}$$ for d=5, k_2=−1/80, r_h=1, κ=0, AdS with $\ell = 1$