Cosmological magnetic fields from inflation and their non-Gaussian imprints

Rajeev Kumar Jain

CP3 Origins
Cosmology & Particle Physics

University of Southern Denmark

Kavli IPMU

Nov. 7, 2014

Outline of the talk

- Observational evidence for cosmic magnetic fields
- Origin of cosmic magnetic fields - Primordial vs. Astrophysical
- Primordial magnetic fields from inflation
  - Backreaction and strong coupling
  - Deflationary magnetogenesis
  - Anisotropic constraints
- Non-gaussian cross-correlations with curvature perturbations
  - A *new* magnetic consistency relation
  - The squeezed limit and the flattened shape
- Conclusions

Our universe is magnetized!

- Large scale magnetic fields are present everywhere in the universe e.g. in our solar system, in stars, in galaxies, in clusters, in galaxies at high redshifts and also in the intergalactic medium.

- Galaxies: B ~ 1 – 10 μG with coherence length as large as 10 kpc.

  Clusters: B ~ 0.1 – 1 μG, coherent on scales up to 100 kpc.

  Filaments: B ~ 10^{-7} – 10^{-8} G, coherent on scales up to 1 Mpc (Kronberg 2010).

  Intergalactic medium: B > 10^{-16} G, coherent on Mpc scales, the lower bound arises due to the absence of extended secondary GeV emission around TeV blazars (Neronov & Vovk, 2010), or even more robust limits of B > 10^{-19}G (Takahashi et al. 2011).

What is the origin of such fields?

Various mechanisms

- Primordial (early time)
  - Inflation
  - Phase transitions (QCD, EW)
  - Second-order perturbation theory
- Astrophysical (late time)
  - Structure formation
  - Biermann battery
  - Dynamo mechanism/MHD turbulence

Primordial magnetic fields from inflation

- Standard EM action is conformally invariant - the EM fluctuations do not grow in any conformally flat background like FRW - need to break it to generate magnetic fields. (Turner & Widrow, 1988)

- Various possible couplings:
  - Kinetic coupling: $\lambda(\phi, \mathcal{R}) F_{\mu\nu} F^{\mu\nu}$
  - Axial coupling: $f(\phi, \mathcal{R}) F_{\mu\nu} \tilde{F}^{\mu\nu}$
  - Mass term: $M^2(\phi, \mathcal{R}) A_\mu A^\mu$

Primordial magnetic fields from inflation...

- Axial coupling: $f(\phi, \mathcal{R}) F_{\mu\nu} \tilde{F}^{\mu\nu}$
  - strong constraints from backreaction, final field strength not enough (Durrer, Hollenstein, **RKJ**, 2011; Byrnes, Hollenstein, **RKJ**, Urban, 2012)
- Mass term: $M^2(\phi, \mathcal{R}) A_\mu A^\mu$
  - negative mass-squared needed for generating relevant magnetic fields, breaks gauge invariance

Primordial magnetic fields from inflation...

- Gauge-invariant coupling: $\lambda(\eta)F_{\mu\nu}F^{\mu\nu}$
  - For $\lambda(\eta) \propto a^{2\alpha} \propto \eta^{2\gamma}$, the magnetic field spectrum is

$$\frac{d\rho_B}{d\,\mathrm{lnk}}(\eta, k) \propto \left(\frac{k}{aH}\right)^{4+2\delta}$$

where $\delta = \gamma$ if $\gamma \leq 1/2$ and $\delta = 1 - \gamma$ if $\gamma \geq 1/2$.

The tilt of the spectrum is $n_B = 4 + 2\delta$ and $n_B = 0$ for $\alpha = 2$ or $\gamma = -2$. However, $n_B = 0$ also for $\gamma = 3$ but then the electric field vary strongly and so not interesting.

Various constraints

- Background
  - Strong coupling
  - Backreaction
- Perturbations
  - Power spectrum
  - Induced bispectrum
- Energy scale of inflation (from B-modes)

Constraint from strong coupling

- Adding the EM coupling to the SM fermions

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{4}\lambda(\phi)F_{\mu\nu}F^{\mu\nu} - \bar{\psi}\gamma^{\mu}(\partial_{\mu} + ieA_{\mu})\psi\right]$$

- The physical EM coupling now is

$$e_{phys} = e/\sqrt{\lambda(\phi)}$$

- Since $\sqrt{\lambda} \propto a^{\alpha}$ then for $\alpha > 0$, the physical coupling decreases by a large factor during inflation, and must have been very large at the beginning of inflation.

- QFT out of control initially. (Demozzi et.al, 2009)

- Solutions ?? Speculations...(Caldwell & Motta, 2012, Ferreira, **RKJ** & Sloth, 2013)

Constraint from backreaction

- The magnetic fields should not backreact on the background dynamics of the universe i.e. $\rho_{\rm em} < \rho_{\rm inf}$

- In the non-strongly coupled regime (α<0)

$$\rho_{\rm em} \simeq d_\alpha\, H^4\, e^{-(2\alpha+4)(N_{\rm tot}-N_{\rm b})}, \quad d_\alpha \equiv -\frac{\Gamma^2(1/2-\alpha)}{2^{2\alpha+2}\,\pi^3(2\alpha+4)}$$

$N_{\rm tot} = N_{\rm min} + \Delta N$ - total amount of inflation

$N_{\rm min} = \ln(R) + \frac{1}{2}\ln\left(\frac{H}{H_0}\right) + \frac{1}{4}\ln(\Omega_r),$ - minimum no. of e-folds

N_b - e-fold of conformal breaking

Constraints from backreaction

- The backreaction constraint translates into the inequality:

$$\left(\frac{H}{H_0}\right)^{-\alpha} < \frac{3\,\Omega_r^{(\alpha/2+1)}}{d_\alpha}\left(\frac{M_p}{H_0}\right)^2 e^{(2\alpha+4)(\Delta N - N_{\rm b})}$$

which implies

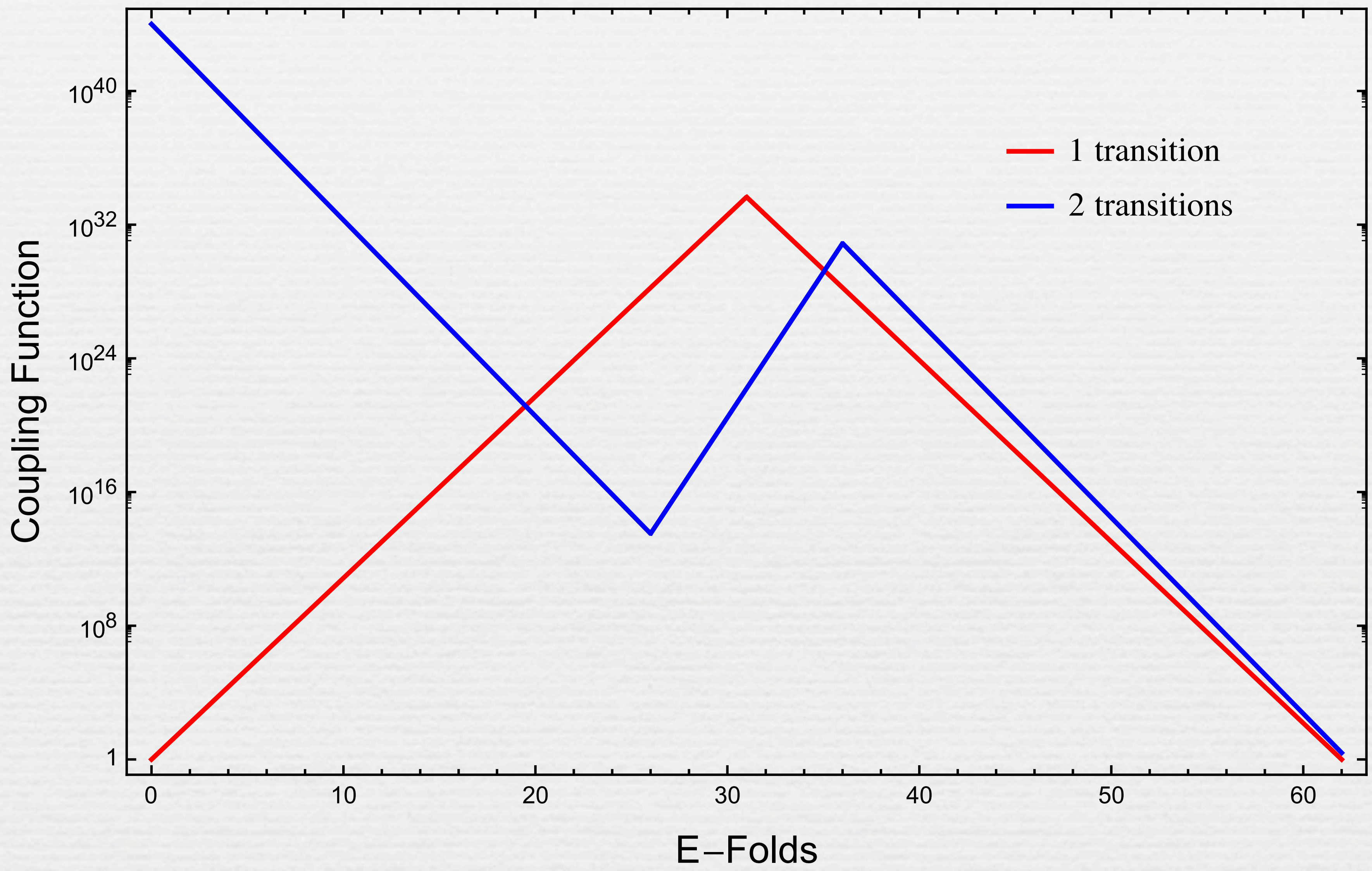
$$\alpha \gtrsim -2 + \frac{\ln\left(\frac{H}{M_p}\right)}{\frac{1}{2}\ln\left(\frac{H}{H_0}\Omega_r^{1/2}\right) + \Delta N - N_b}.$$

- Backreaction + Strong coupling constraints at most lead to B ~10^{-32} G today. (Demozzi et.al, 2009)

Approaches to circumvent this result

- Non-monotonic coupling function
  - A natural approach is to simply glue together the two scale invariant regimes with $\alpha = 2$ and $\alpha = -3$
- Minimize the redshift after inflation:
  - Effects of reheating
  - Lower the energy scale of inflation

Non-monotonic coupling function



No improvement greater than one order of magnitude!

Ferreira, **RKJ** & Sloth, 2013

Deflationary magnetogenesis

- Flux conservation leads to adiabatic decay of magnetic fields after inflation.
- Problem with modifying the inflationary part to generate even larger field strength during inflation.
- Rather, modify the post-inflationary evolution of magnetic fields until today.
- Consider prolonged reheating rather than instantaneous reheating.
- Deflation after inflation.

Ferreira, **RKJ** & Sloth, 2013

Deflationary magnetogenesis

- For radiation dominated universe immediately after inflation: $\rho_I/\rho_r = (a_0/a_f)^4$

- If the universe is instead dominated by a fluid with equation of state ω until the end of reheating:

$$\rho_I/\rho_r = (a_{reh}/a_f)^{3(1+\omega)}(a_0/a_{reh})^4$$

or $$\frac{a_0}{a_f} = \frac{1}{R}\left(\frac{\rho_I}{\rho_r}\right)^{\frac{1}{4}}$$

- Define the reheating parameter R as

$$\log(R) = \frac{-1+3\omega}{4}\log\left(\frac{a_{reh}}{a_f}\right)$$

Deflationary magnetogenesis

- The magnetic field spectrum today is

$$\left.\frac{d\rho_B}{d\log k}\right|_{a_0} = \left.\frac{d\rho_B}{d\log k}\right|_{a_f} \left(\frac{a_f}{a_0}\right)^4$$

- In terms of R, we get

$$B_k(\alpha, H) = \frac{\Gamma(-\alpha - 1/2)}{2^{3/2+\alpha}\pi^{3/2}} H^2 \left(R\,\Omega_r^{1/4}\right)^{-(1+\alpha)} \left(\frac{H_0}{H}\right)^{\frac{1}{2}(5+\alpha)} \left(\frac{k}{a_0 H_0}\right)^{3+\alpha}$$

- To get optimal values of the magnetic fields today, maximize in α and R.

Ferreira, **RKJ** & Sloth, 2013

Deflationary magnetogenesis

- The backreaction constraint (during inflation) leads to an optimal value of α

$$\tilde{\alpha} = -2 + \frac{\ln\left(\frac{H_I}{M_p}\right)}{\frac{1}{4}\ln\left(\Omega_r \frac{H_I^2}{H_0^2}\tilde{R}^4\right)}$$

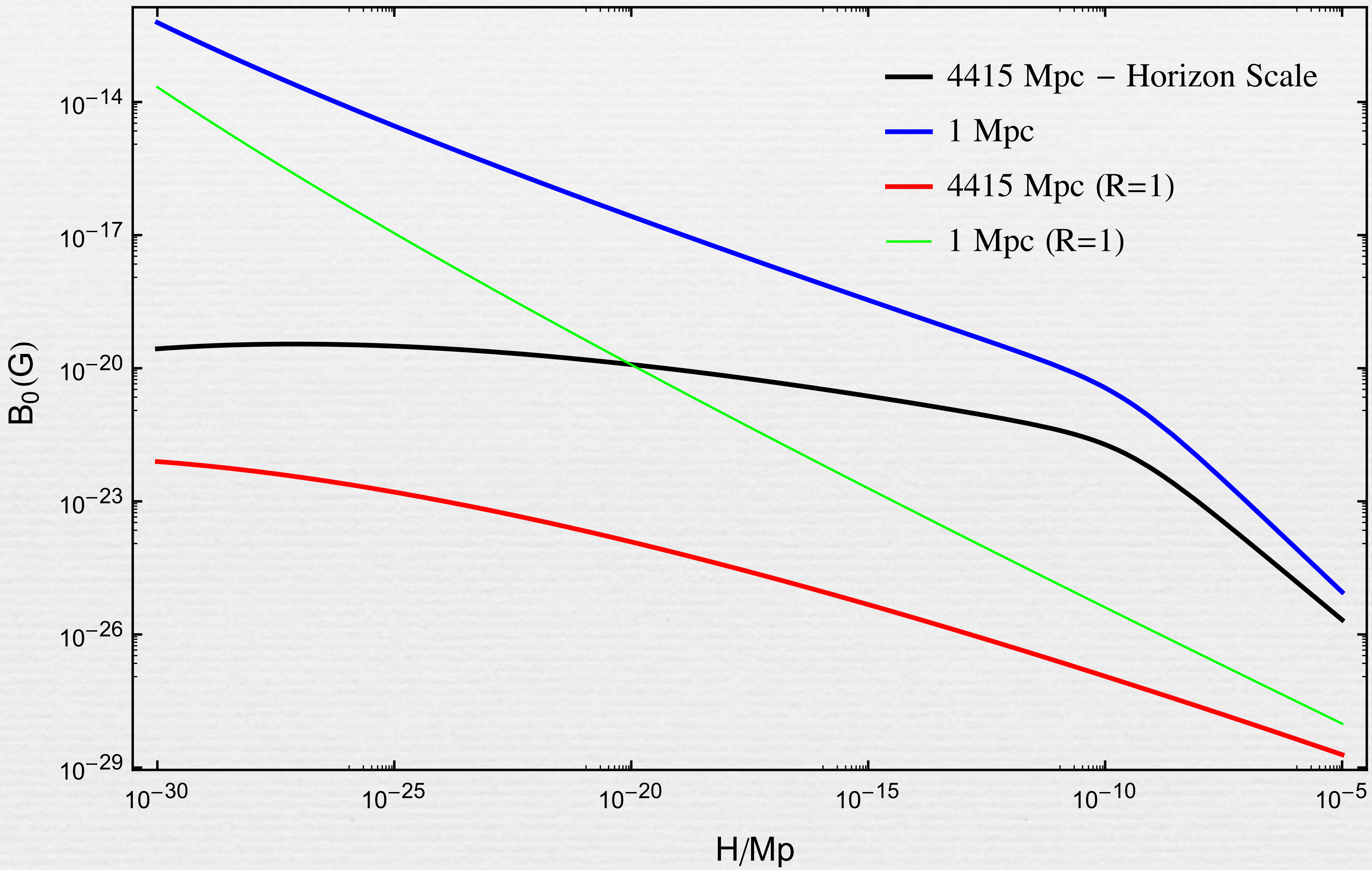
- The constraint $\rho_{nucl} < \rho_{reh} < \rho_I$ and the backreaction (after inflation)

$$\frac{\rho_B(t_{reh})}{\rho_{reh}} = \frac{\rho_B(t_0)}{\rho_r} < \gamma$$

leads to a maximal value of R

$$\tilde{R} \sim \left(\frac{1}{3\gamma^{1/4}}\left(\frac{H_I}{M_p}\right)^{1/2} + 10^{-41/6}\left(\frac{H_I}{M_p}\right)^{-1/6}\right)^{-1}$$

Final magnetic field strength



Ferreira, **RKJ** & Sloth, 2013

Constraints from perturbations

- Anisotropic constraints
  - Amplitude of induced curvature perturbations due to the EM field must be smaller than the observed power spectrum:

$$\mathcal{P}_{\zeta_{\rm em}}^{\rm max} < \mathcal{P}_{\zeta}^{\rm obs}$$

  - Non-gaussianity must be in agreement with Planck.
- B-modes
  - The (?) detection of tensor modes fixes the energy scale of inflation

Constraint from power spectrum

Fujita, Yokoyama, 2013, Ferreira, **RKJ** & Sloth, 2014

- The induced power spectrum

$$\mathcal{P}_{\zeta_{em}}(k) \simeq -\frac{16}{3(2\alpha+4)}\left(\frac{H^2 d_\alpha}{3\epsilon M_p^2}\right)^2 \left(e^{-(2\alpha+4)(N_{\rm tot}-N_{\rm b}-N_{\rm k})}-1\right)\left(e^{-(2\alpha+4)N_{\rm k}}-1\right)^2$$

- Requiring $\mathcal{P}_{\zeta_{\rm em}}^{\rm max} < \mathcal{P}_{\zeta}^{\rm obs}$ implies

$$\left(\frac{H}{H_0}\right)^{-2\alpha} \lesssim -\frac{3}{16}(2\alpha+4)\left(\frac{3\epsilon M_p^2}{d_\alpha H_0^2}\right)^2 \mathcal{P}_\zeta^{\rm obs}\,\Omega_r^{\alpha+2}\,e^{-2(2\alpha+4)N_{\rm b}}\left(e^{-(2\alpha+4)\Delta}-1\right)^{-1}$$

or

$$\alpha \gtrsim \frac{\ln\left(\frac{3}{16}\left(\frac{3M_p^2\epsilon}{H_0^2}\right)^2\mathcal{P}_\zeta^{\rm obs}\right)+2\ln\left(\Omega_r\right)+4\Delta-8N_b}{-2\ln\left(\frac{H}{H_0}\Omega_r^{1/2}\right)-2\Delta+4N_b}$$

Constraint from induced bispectrum

Fujita, Yokoyama, 2013, Nurmi, Sloth, 2013, Ferreira, **RKJ** & Sloth, 2014

- 3-point function in the squeezed limit

$$f_{NL}^{\rm em} = -\frac{20}{27\,(2\alpha+4)}\left(\mathcal{P}_\zeta^{\rm obs}\right)^{-2}\left(\frac{2d_\alpha H^2}{3\epsilon M_p^2}\right)^3\left(e^{-(2\alpha+4)(N_{\rm min}-N_{\rm b}-N_{\rm k})}-1\right)e^{-3(2\alpha+4)N_{\rm k}}$$

- Requiring $f_{NL}^{\rm em} < f_{NL}^{\rm loc}$, we find

$$\left(\frac{H}{H_0}\right)^{-3\alpha} \lesssim -\frac{27}{20}(2\alpha+4)f_{NL}^{\rm loc}\left(\frac{3M_p^2\epsilon}{2d_\alpha H_0^2}\right)^3\left(\mathcal{P}_\zeta^{\rm obs}\right)^2\Omega_r^{3/2(\alpha+2)}e^{-3(2\alpha+4)N_{\rm b}}\left(e^{-(2\alpha+4)\Delta}-1\right)^{-1}$$

or $$\alpha \gtrsim \frac{\ln\left(\frac{27}{20}f_{NL}^{\rm loc}\left(\mathcal{P}_\zeta^{\rm obs}\right)^2\left(\frac{3M_p^2\epsilon}{2H_0^2}\right)^3\right)+3\ln\left(\Omega_r\right)+4\Delta-12N_b}{-3\ln\left(\frac{H}{H_0}\Omega_r^{1/2}\right)-2\Delta+6N_b}$$

$f_{NL}^{\rm loc} < 8.5$ from Planck

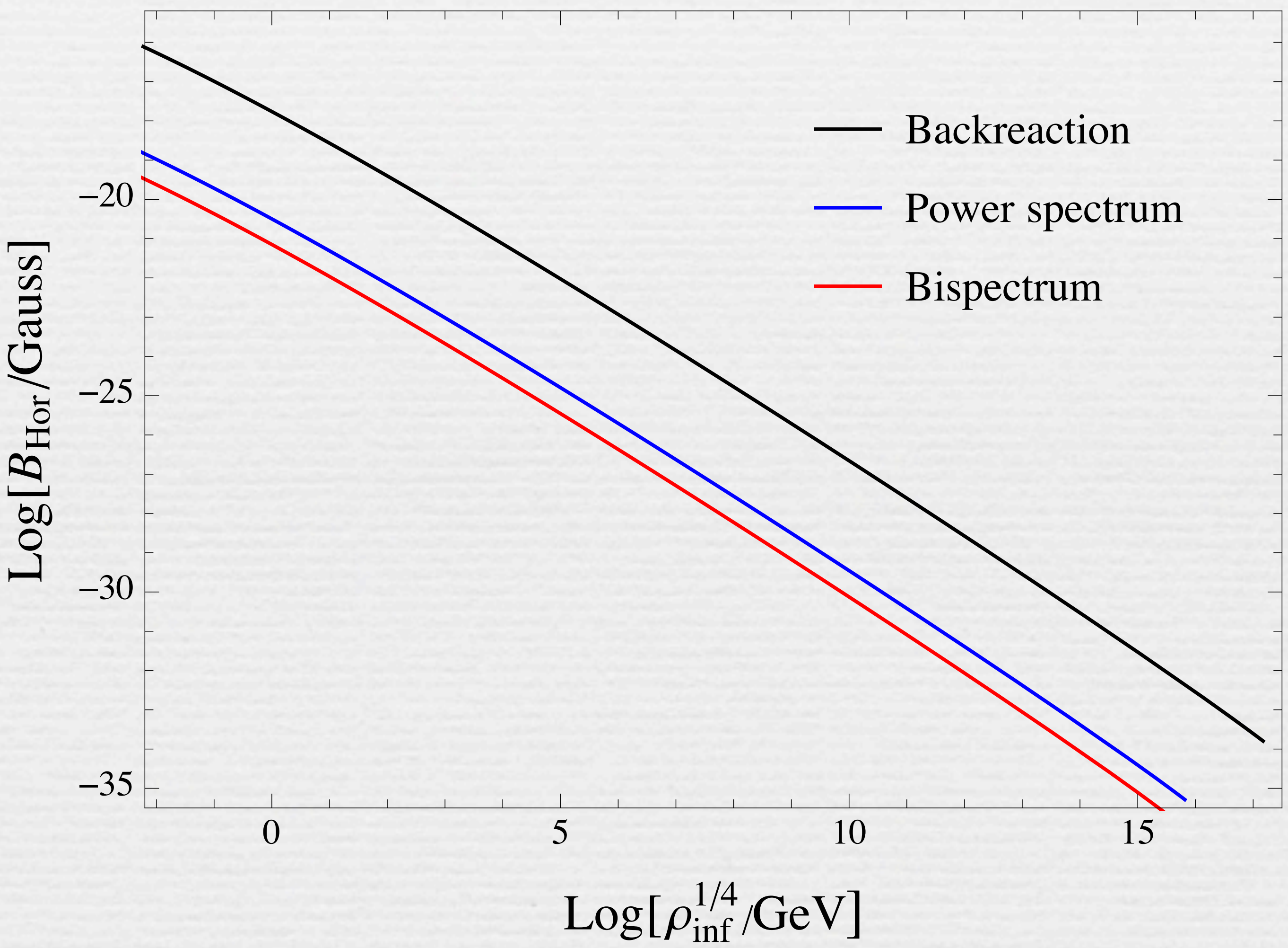
Backreaction vs. Anisotropic constraints

- Long enough inflation -> backreaction is the strongest constraint.

  Fujita, Yokoyama, 2013

- If inflation lasts closer to the minimum required, the hierarchy of constraints is reversed.

  Ferreira, **RKJ** & Sloth, 2014



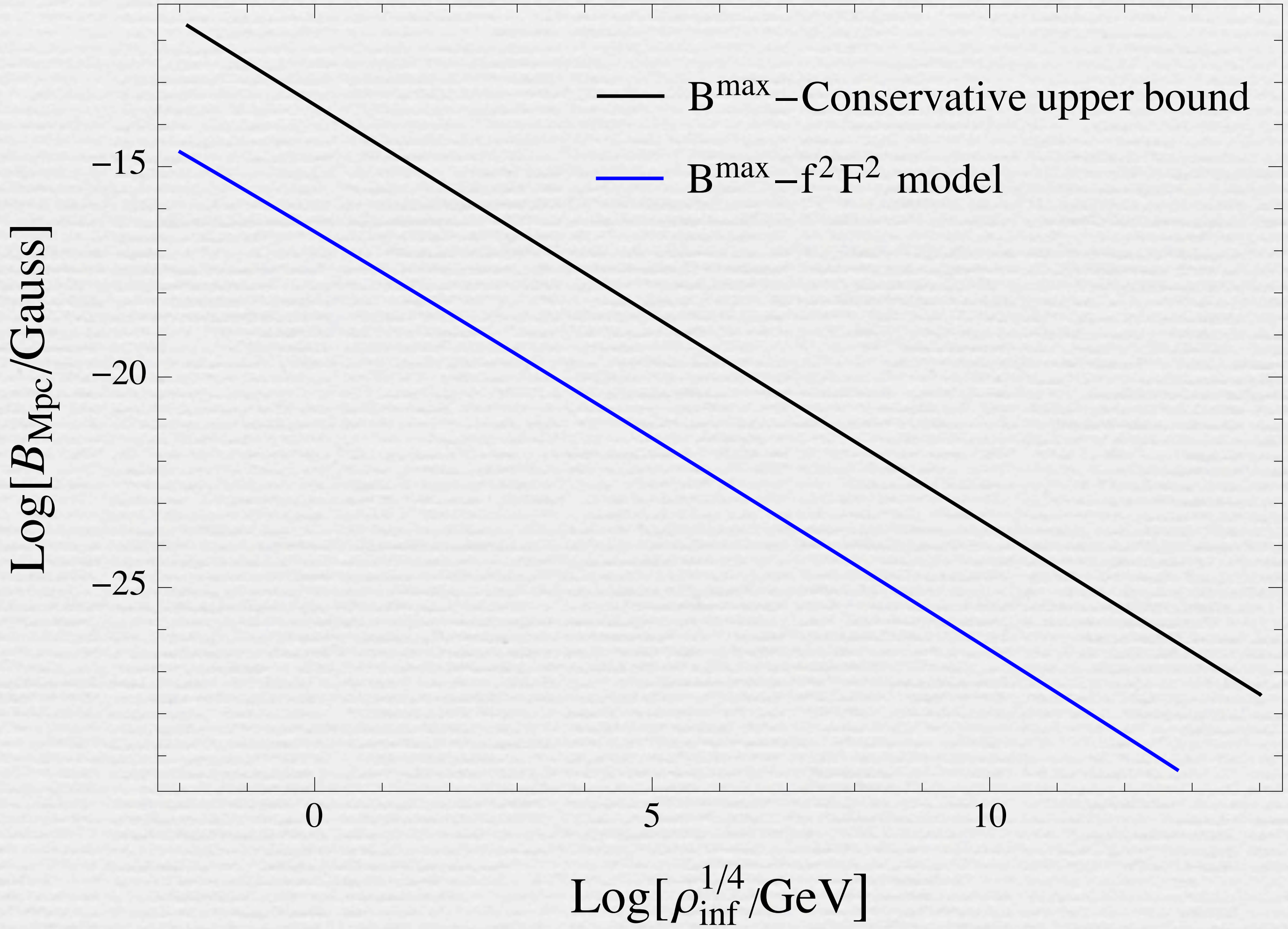
Model independent bound vs. kinetic coupling model

Fujita, Yokoyama, 2013, Ferreira, **RKJ** & Sloth, 2014

- Model independent bound (MIB)

$$\rho_{\rm inf}^{1/4} \; < \; 29.3\,{\rm GeV}\left(\frac{k}{1\,{\rm Mpc}^{-1}}\right)^{5/4}\left(\frac{B_0}{10^{-15}\,{\rm G}}\right)^{-1}$$

- Optimal scenario of kinetic coupling model lies about 3 orders of magnitude below the MIB.



Constraint from B-modes

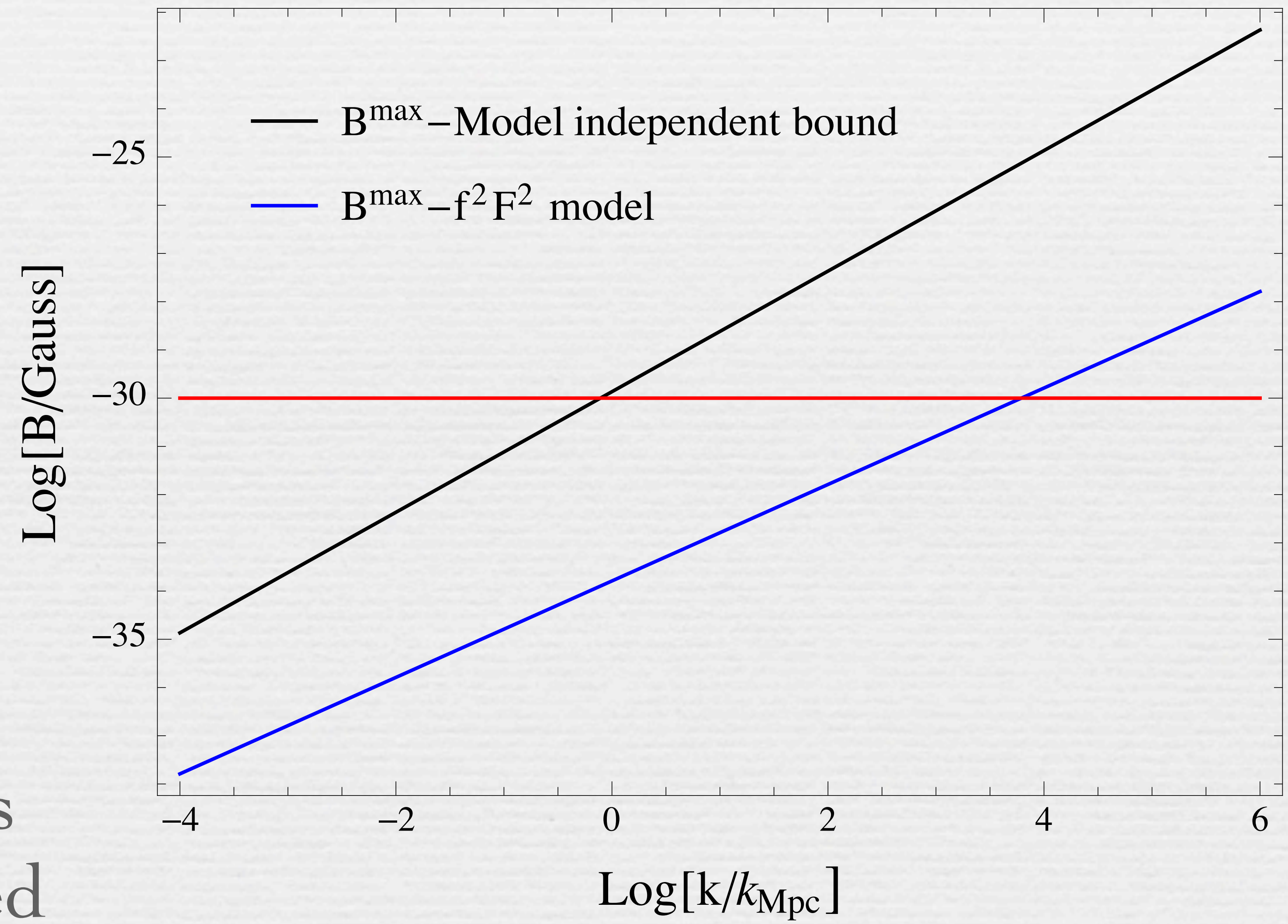
Ferreira, **RKJ** & Sloth, 2014

- B-modes fix the energy scale of inflation

$$\rho_{\rm inf}^{1/4} \simeq 2.2 \times 10^{16}\,{\rm GeV}$$

- Void magnetic fields (>10^{-16} G) are possibly excluded.

- Seed magnetic fields still possible although at scales below Kpc but still allowed by the MIB.



Non-Gaussiam imprints of primordial magnetic fields

Magnetic non-Gaussianity

- If magnetic fields are produced during inflation, they are likely to be correlated with the primordial curvature perturbations.
- Such cross-correlations are non-Gaussian in nature and it is very interesting to compute them in different models of inflationary magnetogenesis.
- We consider the following correlation here:

$$\langle \zeta(k_1) \mathbf{B}(k_2) \cdot \mathbf{B}(k_3) \rangle$$

(Ordinary) non-Gaussianity

- The primordial perturbations are encoded in the two-point function or the power spectrum

$$\langle \zeta_k \zeta_{k'} \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k'}) P_\zeta(k)$$

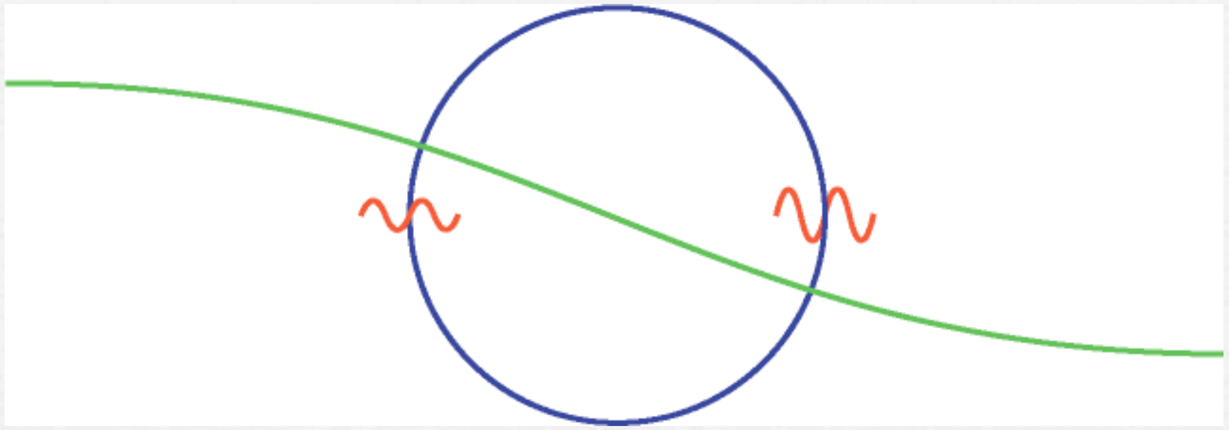
- A non-vanishing three-point function $\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle$ is a signal of NG.

- Introduce f_{NL} as a measure of NG.

$$f_{NL} \sim \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle / P_\zeta(k_1) P_\zeta(k_2) \; + \; perm.$$

(semi)Classical estimate

(for squeezed limit)



- Consider $\langle\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle$ in the squeezed limit i.e.
  - The long wavelength mode rescales the background for short wavelength modes

$$ds^2 = -dt^2 + a^2(t)\, e^{2\zeta(t,\mathbf{x})} d\mathbf{x}^2$$

  - Taylor expand in the rescaled background

$$\langle\zeta_{k_2}\zeta_{k_3}\rangle_{\zeta_1} = \langle\zeta_{k_2}\zeta_{k_3}\rangle + \zeta_1 \frac{\partial}{\partial\zeta_1}\langle\zeta_{k_2}\zeta_{k_3}\rangle + \ldots$$

$$\langle\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle_{\zeta_1} \approx \left\langle\zeta_{k_1}\langle\zeta_{k_2}\zeta_{k_3}\rangle_{\zeta_1}\right\rangle \sim \langle\zeta_{k_1}\zeta_{k_1}\rangle\, k\frac{d}{dk}\langle\zeta_{k_2}\zeta_{k_3}\rangle$$

$$\langle\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle \sim -(n_s - 1)\,\langle\zeta_{k_1}\zeta_{k_1}\rangle\,\langle\zeta_{k_2}\zeta_{k_3}\rangle$$

(Maldacena, 2002)

Non-gaussian cross-correlation

- Define the cross-correlation bispectrum of the curvature perturbation with magnetic fields as

$$\langle\zeta(\mathbf{k}_1)\mathbf{B}(\mathbf{k}_2)\cdot\mathbf{B}(\mathbf{k}_3)\rangle \equiv (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_{\zeta BB}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

- Introduce the magnetic non-linearity parameter

$$B_{\zeta BB}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \equiv b_{NL}P_\zeta(k_1)P_B(k_2)$$

- *Local* resemblance between f_{NL} and b_{NL}

$$\zeta = \zeta^{(G)} + \frac{3}{5}f_{NL}^{local}\left(\zeta^{(G)}\right)^2$$

$$\mathbf{B} = \mathbf{B}^{(G)} + \frac{1}{2}b_{NL}^{local}\zeta^{(G)}\mathbf{B}^{(G)}$$

RKJ & Sloth, 2012

A *new* magnetic consistency relation

- Use the same semi-classical argument to derive the consistency relation.

- Consider $\langle\zeta(\tau_I,\mathbf{k}_1)A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\rangle$ in the squeezed limit.

- The effect of the long wavelength mode is to shift the background of the short wavelength mode.

$$\lim_{k_1\to 0}\langle\zeta(\tau_I,\mathbf{k}_1)A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\rangle = \left\langle\zeta(\tau_I,\mathbf{k}_1)\left\langle A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\right\rangle_B\right\rangle$$

- Since the vector field only feels the background through the coupling, all the effects of the long wavelength mode is indeed captured by

$$\lambda_B = \lambda_0 + \frac{d\lambda_0}{d\ln a}\delta\ln a + \cdots = \lambda_0 + \frac{d\lambda_0}{d\ln a}\zeta_B + \ldots\ .$$

RKJ & Sloth, 2012

A *new* magnetic consistency relation

- Compute the two point function of the vector field in the modified background

$$\langle A_i(\tau,\mathbf{x}_2)A_j(\tau,\mathbf{x}_3)\rangle_B = \left\langle \frac{1}{\lambda_B} v_i(\tau,\mathbf{x}_2)v_j(\tau,\mathbf{x}_3)\right\rangle$$
$$\simeq \frac{1}{\lambda_0}\langle v_i(\tau,\mathbf{x}_2)v_j(\tau,\mathbf{x}_3)\rangle - \frac{1}{\lambda_0^2}\frac{d\lambda}{d\ln a}\zeta_B\langle v_i(\tau,\mathbf{x}_2)v_j(\tau,\mathbf{x}_3)\rangle$$

  where $v_i = \sqrt{\lambda}\,A_i$ is the linear canonical vector field.

- One finally finds

$$\lim_{k_1\to 0}\langle\zeta(\tau_I,\mathbf{k}_1)A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\rangle$$
$$\simeq -\frac{1}{H}\frac{\dot{\lambda}}{\lambda}\langle\zeta(\tau_I,\mathbf{k}_1)\zeta(\tau_I,-\mathbf{k}_1)\rangle_0\langle A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\rangle_0$$

RKJ & Sloth, 2012

A *new* magnetic consistency relation

- In terms of magnetic fields, the correlation becomes

$$\langle \zeta(\tau_I, \mathbf{k}_1)\mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3)\rangle$$
$$= -\frac{1}{H}\frac{\dot{\lambda}}{\lambda}(2\pi)^3\delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)P_\zeta(k_1)P_B(k_2)$$

- With the coupling $\lambda(\phi(\tau)) = \lambda_I(\tau/\tau_I)^{-2n}$, we obtain

$$\boxed{b_{NL} = n_B - 4}$$

- For scale-invariant magnetic field spectrum, $n_B = 0$ and therefore, $b_{NL} = -4$

- Not so small......compared to $b_{NL} \sim \mathcal{O}(\epsilon, \eta)$

RKJ & Sloth, 2012

A *new* magnetic consistency relation

- In the squeezed limit $k_1 \ll k_2, k_3 = k$, we obtain a new *magnetic consistency relation*

$$\langle \zeta(k_1) \mathbf{B}(k_2) \cdot \mathbf{B}(\mathbf{k_3}) \rangle = (n_B - 4)(2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_B(k)$$

with $b_{NL}^{\text{local}} = (n_B - 4)$

- Compare with Maldacena's consistency relation

$$\langle \zeta(k_1) \zeta(k_2) \zeta(k_3) \rangle = -(n_s - 1)(2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_\zeta(k)$$

with $f_{NL}^{\text{local}} = -(n_s - 1)$

The full in-in calculation

- One has to cross-check the consistency relation by doing the full in-in calculation

$$\langle\Omega|\,\mathcal{O}(\tau_I)\,|\Omega\rangle = \langle 0|\,\bar{\mathrm{T}}\left(e^{i\int_{-\infty}^{\tau_I} d\tau H_{\rm int}}\right)\mathcal{O}(\tau_I)\mathrm{T}\left(e^{-i\int_{-\infty}^{\tau_I} d\tau H_{\rm int}}\right)|0\rangle$$

- The result is

$$\begin{aligned}\langle\zeta(\tau_I,\mathbf{k}_1)A_i(\tau_I,\mathbf{k}_2)A_j(\tau_I,\mathbf{k}_3)\rangle \;=\; & \frac{1}{H}\frac{\dot{\lambda}_I}{\lambda_I}(2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)|\zeta^{(0)}_{k_1}(\tau_I)|^2|A^{(0)}_{k_2}(\tau_I)||A^{(0)}_{k_3}(\tau_I)| \\ & \times\left[\left(\delta_{il}-\frac{k_{2,i}k_{2,l}}{k_2^2}\right)\left(\delta_{lj}-\frac{k_{3,l}k_{3,j}}{k_3^2}\right)\left(k_2k_3\,\tilde{\mathcal{I}}^{(1)}_n+\mathbf{k}_2\cdot\mathbf{k}_3\,\tilde{\mathcal{I}}^{(2)}_n\right)\right. \\ & \left.-\left(\delta_{il}-\frac{k_{2,i}k_{2,l}}{k_2^2}\right)k_{3,l}\left(\delta_{jm}-\frac{k_{3,j}k_{3,m}}{k_3^2}\right)k_{2,m}\,\tilde{\mathcal{I}}^{(2)}_n\right]\end{aligned}$$

A generic result

RKJ & Sloth, 2013

Cross-correlation with magnetic fields

- Using this relation

$$\langle \zeta(\tau_I, \mathbf{k}_1) \mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3) \rangle = -\frac{1}{a_0^4} \left(\delta_{ij} \mathbf{k}_2 \cdot \mathbf{k}_3 - \mathbf{k}_{2,i} \mathbf{k}_{3,j} \right) \langle \zeta(\tau_I, \mathbf{k}_1) A_i(\tau_I, \mathbf{k}_2) A_j(\tau_I, \mathbf{k}_3) \rangle$$

- The cross-correlation with magnetic fields is

$$\begin{aligned} \langle \zeta(\tau_I, \mathbf{k}_1) \mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3) \rangle = & -\frac{1}{H} \frac{\dot{\lambda}_I}{\lambda_I} (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) |\zeta_{k_1}^{(0)}(\tau_I)|^2 |A_{k_2}^{(0)}(\tau_I)| |A_{k_3}^{(0)}(\tau_I)| \\ & \times \left[\left(\mathbf{k}_2 \cdot \mathbf{k}_3 + \frac{(\mathbf{k}_2 \cdot \mathbf{k}_3)^3}{k_2^2 k_3^2} \right) k_2 k_3 \tilde{\mathcal{I}}_n^{(1)} + 2 (\mathbf{k}_2 \cdot \mathbf{k}_3)^2 \tilde{\mathcal{I}}_n^{(2)} \right] . \end{aligned}$$

- The two integrals can be solved exactly for different values of n.

RKJ & Sloth, 2013

The integrals...

- The two integrals are

$$\tilde{\mathcal{I}}_n^{(1)} = \frac{\pi^3}{2}\frac{2^{-2n-1}}{\Gamma^2(n+1/2)}(-k_2\tau_I)^{n+1/2}(-k_3\tau_I)^{n+1/2}$$
$$\times \operatorname{Im}\Bigg[(1+ik_1\tau_I)e^{-ik_1\tau_I}H^{(1)}_{n+1/2}(-k_2\tau_I)H^{(1)}_{n+1/2}(-k_3\tau_I)$$
$$\times \int^{\tau_I} d\tau\tau(1-ik_1\tau)e^{ik_1\tau}H^{(2)}_{n-1/2}(-k_2\tau)H^{(2)}_{n-1/2}(-k_3\tau)\Bigg]$$

$$\tilde{\mathcal{I}}_n^{(2)} = \frac{\pi^3}{2}\frac{2^{-2n-1}}{\Gamma^2(n+1/2)}(-k_2\tau_I)^{n+1/2}(-k_3\tau_I)^{n+1/2}$$
$$\times \operatorname{Im}\Bigg[(1+ik_1\tau_I)e^{-ik_1\tau_I}H^{(1)}_{n+1/2}(-k_2\tau_I)H^{(1)}_{n+1/2}(-k_3\tau_I)$$
$$\times \int^{\tau_I} d\tau\tau(1-ik_1\tau)e^{ik_1\tau}H^{(2)}_{n+1/2}(-k_2\tau)H^{(2)}_{n+1/2}(-k_3\tau)\Bigg]$$

and the integrals again...

- For n=2, we get

$$\tilde{\mathcal{I}}_2^{(1)} = \frac{-1}{(k_2k_3)^{3/2}k_t^2} \times \left[-k_1^3 - 2k_1^2(k_2+k_3) - 2k_1(k_2^2+k_2k_3+k_3^2) - (k_2+k_3)(k_2^2+k_2k_3+k_3^2)\right]$$

$$\tilde{\mathcal{I}}_2^{(2)} = \frac{-1}{(k_2k_3)^{5/2}k_t^2} \times \left[(k_1+k_2)^2(-3k_1^3-3k_1^2k_2-k_2^3)+(k_1+k_2)(-9k_1^3-6k_1^2k_2-2k_2^3)k_3 + (-9k_1^3-6k_1^2k_2-2k_1k_2^2-2k_2^3)k_3^2 - 2(2k_1^2+k_1k_2+k_2^2)k_3^3 - 2(k_1+k_2)k_3^4 - k_3^5 + 3k_1^3k_t^2(\gamma+\ln(-k_t\tau_I))\right]$$

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The flattened shape

- In this limit, $k_1 = 2k_2 = 2k_3$, the second integral dominates

$$\tilde{\mathcal{I}}_2^{(2)} \simeq -\frac{3k_1^3}{(k_2 k_3)^{5/2}} \ln(-k_t \tau_I)$$

- The cross-correlation thus becomes

$$\langle \zeta(\tau_I, \mathbf{k}_1) \mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3) \rangle \simeq 96 \ln(-k_t \tau_I) P_\zeta(k_1) P_B(k_2)$$

- For the largest observable scale today, $\ln(-k_t \tau_I) \sim -60$,

$$\left| b_{NL}^{flat} \right| \sim 5760$$

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The squeezed limit

- In this limit, the integrals are

$$\tilde{\mathcal{I}}_n^{(1)} = \pi \int^{\tau_I} d\tau \tau J_{n-1/2}(-k\tau) Y_{n-1/2}(-k\tau)$$

$$\tilde{\mathcal{I}}_n^{(2)} = \tilde{\mathcal{I}}_{n+1}^{(1)} \ .$$

- The cross-correlation now becomes

$$\langle \zeta(\tau_I, \mathbf{k}_1) \mathbf{B}(\tau_I, \mathbf{k}_2) \cdot \mathbf{B}(\tau_I, \mathbf{k}_3) \rangle = -\frac{1}{H} \frac{\dot{\lambda}_I}{\lambda_I} (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) P_\zeta(k_1) P_B(k_2)$$

with $b_{NL} = -\frac{1}{H}\frac{\dot{\lambda}_I}{\lambda_I} = n_B - 4$ in agreement with the magnetic consistency relation.

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Conclusions

- Origin of cosmic magnetic fields is still poorly understood.
- Inflationary + deflationary magnetogenesis can produce the observed fields on large scales without the backreaction and strong coupling problem.
- Low scale inflationary magnetogenesis is still a viable possibility.
- Primordial non-Gaussianities induced by magnetic fields are very interesting.
- Violation of the consistency relation will rule out an important class of inflationary magnetogenesis models.
- The magnetic non-Gaussianity parameter is quite large in the flattened limit and can have non-trivial phenomenological consequences.

Thank you for your attention