Higgs: Naturalness and Some Other Issues

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November 26, 2014
Kavli IPMU, Japan

Plan of the talk

- **Naturalness, bottom-up**
- **Extended scalar sectors and naturalness**

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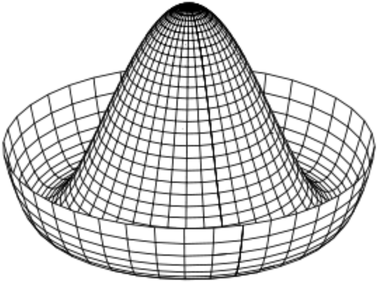
The Standard Model, based on the spontaneous breaking of $SU(2) \times U(1)$ by a complex scalar doublet, is now complete



$m_h = 125.3 \pm 0.6$ GeV

Q: Why the Higgs is here and not at the Planck scale?

CounterQ: Why should it be when all the other particles are at the EW scale or below?

The naturalness problem

- Definition:
  $A = B + C$, $B \to B + \delta B \Rightarrow A \to A + \delta A$
  If $|\delta A/A| \gg |\delta B/B|$, fine-tuning
- Fermion and gauge masses are protected by chiral and gauge symmetries
  Radiative correction $\sim \ln(\Lambda^2/m^2)$, Λ is the cut-off scale, $\sim 10^{19}$ GeV

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- $m^2\Phi^\dagger\Phi$ does not break any symmetry of the action
  Scalar mass not protected by any symmetry
  Quadratically divergent corrections

$$\underbrace{m^2_{\rm physical}}_{10^4} = m^2_{\rm bare} + \underbrace{a\Lambda^2}_{10^{38}}$$

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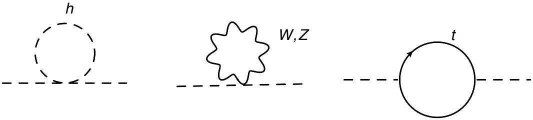

$$\delta m_h^2 = \frac{\Lambda^2}{16\pi^2}\left(6\lambda + \frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 6g_t^2\right) \equiv \frac{\Lambda^2}{16\pi^2} f_h\,,$$

Veltman Condition (Veltman 1981)

$$f_h = 0 \implies 2\lambda + \frac{1}{4}g_1^2 + \frac{3}{4}g_2^2 - 2g_t^2 = 0\,.$$

$$m_h^2 + 2m_W^2 + m_Z^2 - 4m_t^2 = 0\,.$$

$m_h = 316$ GeV, not satisfied in SM !

Two-loop: Hamada, Kawai, Oda (2012)

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The naturalness problem

Top-down approaches to naturalness:

- Supersymmetry (new d.o.f. cancelling the divergence)
- Extra dimensions (lowering Λ)

What if we do not know the beyond-SM dynamics?

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Bottom-up :
Claim that the VC is somehow satisfied and try to find the dynamics

Pre-top and Higgs days: both λ and g_t unknown. Second condition came from the stability of the cancellation.

Post-top but pre-Higgs days: only λ unknown, VC predicts $m_h \sim 316$ GeV.

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Does dimensional regularisation help?
Does not differentiate between log and quadratic divergences

$$\delta m_h^2 \propto \frac{1}{\epsilon}\left(6\lambda + \frac{1}{4}g_1^2 + \frac{3}{4}g_2^2 - 6g_t^2\right) .$$

Again, not satisfied in the SM — but the log-divergences have gone into this!

The naturalness problem

Q. Is a strict implementation of VC necessary as a guiding principle?

- Higher loop effects: suppressed by further powers of $\log(\Lambda^2/m^2)/16\pi^2$, subleading but definitely at the level of a few per cent.
- Can always accommodate some fine-tuning: 0.1%? 1%? 10%? Without any fine-tuning : $|\delta m_h^2| \leq m_h^2$

$$\left| m_h^2 + 2m_W^2 + m_Z^2 - 4m_t^2 \right| \leq \frac{16\pi^2}{3} \frac{v^2}{\Lambda^2} m_h^2 \,.$$

Not satisfied in the SM for $v^2/\Lambda^2 \leq 0.1 \Rightarrow \Lambda \geq 760$ GeV.
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Q: Where should the VC be valid?
Ideally, at all scales below the cut-off

$$\frac{d}{dt}\left(2\lambda + \frac{1}{4}g_1^2 + \frac{3}{4}g_2^2 - 2g_t^2\right) \sim \frac{v^2}{\Lambda^2}\,, \quad t = \log(Q^2/\mu^2)$$

$$1-\text{loop}: \quad 288\lambda^2 + 144g_t^2\lambda - 180g_t^4 - 36\lambda\left(g_1^2 + 3g_2^2\right)$$
$$+25g_1^4 - 15g_2^4 + 9g_1^2g_2^2 + g_t^2\left(192g_3^2 + 34g_1^2 + 54g_2^2\right) \sim \frac{v^2}{\Lambda^2}\,.$$

Two-loop?
In any generic Yukawa theory, $f_h = 0\,, df_h/dt = 0$ using 1-loop β-fns imply precisely the same condition as quadratic divergences at 2-loop to vanish (Einhorn and Jones 1992, Al-sarhi, Jack, and Jones 1992)

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Scalar extensions

- **Singlet**
  With or without Z_2, can be a DM candidate with Z_2 if does not mix with Φ. Need new fermions though
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Singlet scalar extension

Minimal extension, DM, solution to FT
(AK and Raychaudhuri 1996, Drozd, Grzadkowski, Wudka 2012, Chakraborty and AK 2013, Bazzocchi and Fabbrichesi 2013)

$$V(\Phi, S) = V_{\rm SM} + V_{\rm singlet} = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 - M^2S^2 + \tilde{\lambda}S^4 + aS^2(\Phi^\dagger\Phi)\,.$$

$Z_2 : S \to -S$. $\mu^2, M^2 > 0$ to start with. $\lambda, \tilde{\lambda} > 0$ for stability

Extra terms without Z_2:

$$V_{\not{Z}_2} = cS^3 + \underbrace{\alpha_1 S + \alpha_2\Phi^\dagger\Phi S}_{\rm tadpole}$$

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For N singlets with an $O(N)$ symmetry

$$V(\Phi, S_i) = V_{\rm SM} - M^2\sum_i S_i^2 + \tilde\lambda\left(\sum_i S_i^2\right)^2 + a(\Phi^\dagger\Phi)\sum_i S_i^2\,.$$

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Singlet scalar extension

Doublet VC with one singlet

$$\delta m_h^2 = \frac{\Lambda^2}{16\pi^2}\left(6\lambda + \frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 6g_t^2 + a\right) = \frac{\Lambda^2}{16\pi^2}\, f_h\,.$$

N identical singlets : $a \longrightarrow Na$.

$a = 4.17/\sqrt{N}$

Risk of hitting the Landau pole at a low energy!

Singlet VC

$$\delta m_S^2 = \frac{\Lambda^2}{16\pi^2}\left[(8+4N)\tilde{\lambda} + 4a\right].$$

$\tilde{\lambda} < 0$, unstable potential, ruled out !

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Singlet scalar extension

Introduce fermions vectorial under SU(2)

$$\mathcal{L}_{VF} = -m_F \bar{F} F - \zeta_F \bar{F} F S \,,$$

$\Rightarrow$ Does not couple with Φ: SM VC unaffected
$\Rightarrow M_{VF} = m_F + \zeta_F \langle S \rangle$, $S \to -S \Longrightarrow F \to i\gamma_5 F$, no bare mass term
$\Rightarrow$ For $\langle S \rangle = 0$, $M_{VF} = m_F$, expt: $m_F \geq 500$ GeV

Singlet VC

$$\delta m_S^2 = \frac{\Lambda^2}{16\pi^2} [(8+4N)\tilde{\lambda} + 4a - 4Z^2] \,,$$

$$Z^2 = \sum_i N_c \zeta_i^2 = \zeta_E^2 + \zeta_N^2 + 3(\zeta_U^2 + \zeta_D^2) = 8\zeta^2 \,,$$

Degenerate VF generation: no extra contribution to S and T

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Singlet scalar extension

Singlet-doublet mixing

$V \supset -M^2 S^2 + \tilde{\lambda} S^4 + a S^2 (\Phi^\dagger \Phi)$, $M^2 > 0 \Rightarrow \langle S \rangle \neq 0$

Minimization

$$-\mu^2 + \lambda v^2 + a v'^2 = 0\,, \quad -M^2 + 2\tilde{\lambda} v'^2 + \frac{1}{2} a v^2 = 0\,,$$

Mass matrix

$$\begin{pmatrix} h & S \end{pmatrix} \mathcal{M} \begin{pmatrix} h \\ S \end{pmatrix} = \begin{pmatrix} h & S \end{pmatrix} \begin{pmatrix} \lambda v^2 & a v v' \\ a v v' & 4\tilde{\lambda} v'^2 \end{pmatrix} \begin{pmatrix} h \\ S \end{pmatrix} .$$

Real masses: $4\lambda\tilde{\lambda} \geq a^2$

Too large $\tilde{\lambda}$, nonperturbative, hits Landau pole quickly

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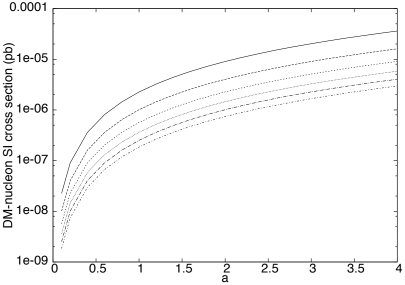
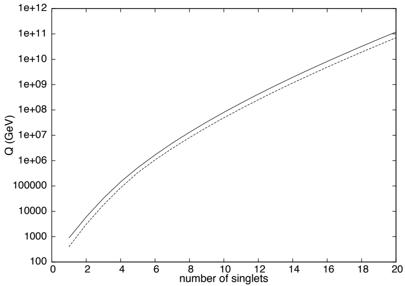
Possible DM candidate





m_S = 200-700 GeV (top to bottom) LP (top), pert. (bottom)

Two-Higgs doublet models

Φ_1 and Φ_2, both with $Y = 1$:

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \rho_1 + v_1 \end{pmatrix}, \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \rho_2 + v_2 \end{pmatrix}, \quad \tan\beta = \frac{v_2}{v_1}$$

Four types of 2HDM, based on no tree-level FCNC (Glashow and Weinberg 1977, Paschos 1977):

- Type-I : none with Φ_1, all with Φ_2
- Type-II : $T_3 = -\frac{1}{2}$ to Φ_1, $T_3 = +\frac{1}{2}$ to Φ_2
- Flipped : d_i to Φ_1, u_i, ℓ_i to Φ_2
- Lepton-specific : leptons to Φ_1, quarks to Φ_2

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Two-Higgs doublet models

Scalar potential

$$V = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \underbrace{m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1\right)}_{\not{Z}_2}$$
$$+ \frac{1}{2}\lambda_1 \left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{1}{2}\lambda_2 \left(\Phi_2^\dagger \Phi_2\right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right)$$
$$+ \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) + \frac{1}{2}\lambda_5 \left[\left(\Phi_1^\dagger \Phi_2\right)^2 + \left(\Phi_2^\dagger \Phi_1\right)^2\right] .$$

Mass eigenstates

$$h = \rho_2 \cos\alpha - \rho_1 \sin\alpha\,, \quad H = \rho_2 \sin\alpha + \rho_1 \cos\alpha\,.$$

$h(H)$ is SM-like if $\cos(\beta-\alpha)(\sin(\beta-\alpha)) \sim 0$

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Scalar potential

$$V = m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \underbrace{m_{12}^2\left(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right)}_{\not{Z}_2}$$
$$+\frac{1}{2}\lambda_1\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{1}{2}\lambda_2\left(\Phi_2^\dagger\Phi_2\right)^2 + \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right)$$
$$+\lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \frac{1}{2}\lambda_5\left[\left(\Phi_1^\dagger\Phi_2\right)^2 + \left(\Phi_2^\dagger\Phi_1\right)^2\right].$$

Mass eigenstates

$$h = \rho_2\cos\alpha - \rho_1\sin\alpha\,, \quad H = \rho_2\sin\alpha + \rho_1\cos\alpha\,.$$

$h(H)$ is SM-like if $\cos(\beta-\alpha)(\sin(\beta-\alpha)) \sim 0$

Two-Higgs doublet models

Stability of the potential

$$\lambda_1\,,\lambda_2 \geq 0\,,\quad \lambda_3 \geq -\sqrt{\lambda_1\lambda_2}\,,\quad \lambda_3+\lambda_4-|\lambda_5| \geq -\sqrt{\lambda_1\lambda_2}\,.$$

- λ_3, λ_4, and λ_5 can potentially be negative
- Can be more than one minimum, chance for false vacuum transition
- If there is a true minimum, charge or CP violating extrema can only be saddle points (Barroso et al. 2013)

Yukawa couplings

$$\mathcal{L}_Y = -\sum_{j=1}^{2}\left[Y_j^d\,\bar{Q}_L d_R \Phi_j + Y_j^u\,\bar{Q}_L u_R \tilde{\Phi}_j + Y_j^e\,\bar{L}_L l_R \Phi_j + \mathrm{h.c.}\right]\,,$$

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Two-Higgs doublet models

VC without Yukawa

$$\delta' m^2_{\rho_1} = \frac{\Lambda^2}{16\pi^2}\left[\left(\frac{9}{4}g_2^2 + \frac{3}{4}g_1^2\right) + 2\lambda_3 + 3\lambda_1 + \lambda_4\right] \equiv \frac{\Lambda^2}{16\pi^2} f'_{\rho_1}\,,$$

$$\delta' m^2_{\rho_2} = \frac{\Lambda^2}{16\pi^2}\left[\left(\frac{9}{4}g_2^2 + \frac{3}{4}g_1^2\right) + 2\lambda_3 + 3\lambda_2 + \lambda_4\right] \equiv \frac{\Lambda^2}{16\pi^2} f'_{\rho_2}\,.$$

No solution, but Yukawas are there (rules out type-I)

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Two-Higgs doublet models

VC with Yukawa

- Type II:

$$f_{\rho_1} = f'_{\rho_1} - 3\left(Y_1^b\right)^2 - \left(Y_1^\tau\right)^2 \,, \qquad f_{\rho_2} = f'_{\rho_2} - 3\left(Y_2^t\right)^2 \,.$$

- Lepton-specific:

$$f_{\rho_1} = f'_{\rho_1} - \left(Y_1^\tau\right)^2 \,, \qquad f_{\rho_2} = f'_{\rho_2} - 3\left(Y_2^b\right)^2 - 3\left(Y_2^t\right)^2 \,.$$

- Flipped:

$$f_{\rho_1} = f'_{\rho_1} - 3\left(Y_1^b\right)^2 \,, \qquad f_{\rho_2} = f'_{\rho_2} - 3\left(Y_2^t\right)^2 - \left(Y_2^\tau\right)^2 \,.$$

If $\lambda_1 \sim \lambda_2$, all Yukawas must be of the same order
$\Rightarrow$ Naturalness solution prefers large $\tan\beta$

Two-Higgs doublet models

Scanned over a large range of perturbative couplings (Chakraborty and AK 2014)

$\tan\beta$ ranges

- Type-II: 31 - 50, increases linearly with λ_1, $Y_t \sim Y_b$
- Flipped: 40 - 51, otherwise same as Type-II, $Y_t \sim Y_b$ but Y_τ on the other side
- Lepton-specific: $\tan\beta > 140$, as $Y_t \sim Y_\tau$. Makes potential unstable at a very low energy $\sim 1\,TeV$

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Triplet scalars

SM + complex triplet X of scalars ($Y = 2$)
Can generate neutrino mass through $\Delta L = 2$ terms

$$\langle\phi^0\rangle = \frac{v_1}{\sqrt{2}}\,, \quad \langle X^0\rangle = v_2\,, \quad X = \begin{pmatrix} X^+/\sqrt{2} & X^{++} \\ X^0 & -X^+/\sqrt{2} \end{pmatrix}.$$

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$v_2 \ll v_1$ for $\delta\rho \sim 0$: at most a few GeV
doublet-triplet mixing is negligible
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$$V_{\Delta L=2} = -if_{ab} L_a^T C^{-1} \tau_2 X L_b + \mathrm{h.c.}\,.$$

Take $f_{ab} = f\delta_{ab}$ for simplicity.

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Triplet scalars

Scalar potential

$$
\begin{aligned}
V &= -\mu_1^2(\Phi^\dagger\Phi) + \mu_2^2(X^\dagger X) \\
&+a_0(\Phi\Phi X^\dagger) + h.c. \\
&+\lambda_1(\Phi^\dagger\Phi)^2 + \lambda_2(X^\dagger X)^2 + \lambda_3(\Phi^\dagger\Phi)(X^\dagger X) \\
&+\lambda_4(\Phi^\dagger\tau_i\Phi)(X^\dagger t_i X) + \lambda_5\left|X^T\tilde{C}X\right|^2,
\end{aligned}
$$

$$
\tilde{C} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}
$$

- Triplets can be heavy even with small v_2
- Without trilinear, global O(2) $\Rightarrow$ Goldstone in spectrum
- $X \to -X$ forbids the trilinear but also the $\Delta L = 2$ term

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Triplet scalars

Stability conditions

$$\lambda_1, \lambda_2 \geq 0\,, \lambda_2 + 2\lambda_5 \geq 0\,, \lambda_3 \pm \lambda_4 \geq -2\sqrt{\lambda_1 \lambda_2}$$

Doublet VC

$$6\lambda_1 + 3\lambda_3 + \frac{3}{4}g_1^2 + \frac{9}{4}g_2^2 - 6g_t^2 = 0 \quad \Rightarrow \lambda_3 \approx 1.4$$

Triplet VC

$$4\lambda_2 + \lambda_3 + 2\lambda_5 + \frac{1}{2}g_1^2 + g_2^2 - 3f^2 = 0$$

Normal hierarchy: $3f^2 \to f^2_{\rm normal}$, inv. hierarchy: $3f^2 \to f^2_{\rm inv}$
Like singlet, triplet models are also valid only up to $\sim 10^7$ GeV

(Chakraborty and AK 2014)

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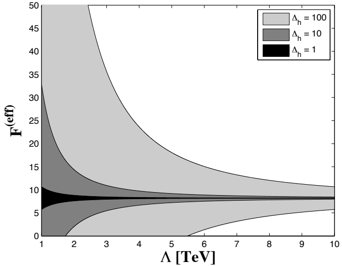
Naturalness in an effective theory

Suppose all new particles are heavy and there is only an EFT

$$\mathcal{L}_{\rm eff} = \sum_{n=1}^{\infty} \frac{1}{\Lambda^n} \sum_i C_i \mathcal{O}_i$$

$$\delta m_h^2 = \frac{\Lambda^2}{16\pi^2} \left| F^{\rm eff} - 8.2 \right| .$$

FT if $m_h^2 \ll \Lambda^2$ (Bar-shalom et al. 2014)



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Part II : Constraints on an Effective Theory

EFT and Scattering Unitarity

Dim-4 couplings can be modified

Effective Lagrangian

$$\mathcal{L}_{\rm eff} = g_t \frac{\sqrt{2}m_t}{v} h\bar{t}t + g_W \frac{2m_W^2}{v} hW_\mu^+ W^{\mu -} + g_Z \frac{m_Z^2}{v} hZ_\mu Z^\mu$$

$g_t, g_W, g_Z = 1$ in SM
LHC best fits are close to the SM values but $\sim 10\%$ deviations can be entertained

- Partial wave unitarity may get spoiled
- Vacuum stability may get affected

(Choudhury, Islam, AK 2014)

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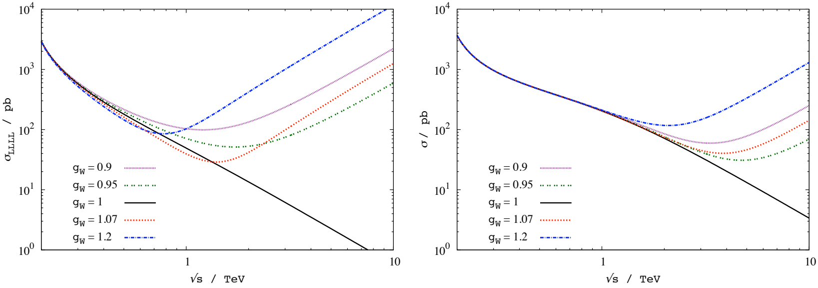
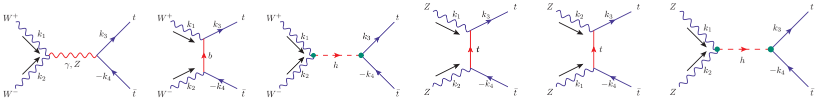


Unpolarized $WW \to WW$ (L) and $W_L W_L \to W_L W_L$ (R) x-sec

EFT and Scattering Unitarity

$V_L V_L \to t\bar{t}$

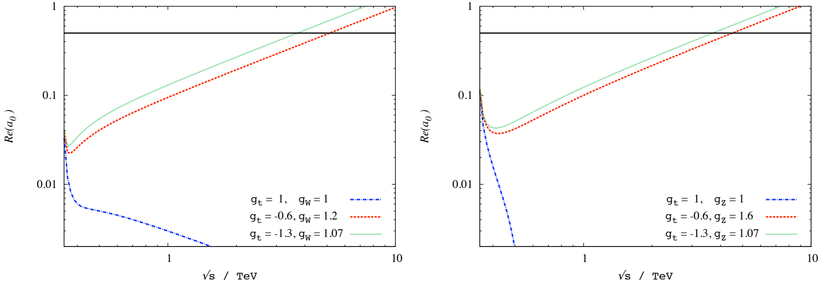


$$a_\ell \equiv \frac{1}{32\,\pi}\int_{-1}^{1} d\cos\theta\; P_\ell(\cos\theta)\;\mathcal{M}(s,\cos\theta;\{m_i,g_i\})\,, \quad |Re(a_\ell)| < \frac{1}{2}\,, \;\; \forall\ell$$

Most sensitive $a_0(0,0,1,1) \equiv a_0(W_L^+ W_L^- \to t_+\bar{t}_+)$

EFT and Scattering Unitarity

x-sec falls if and only of $g_t g_W = 1$ ($WW \to t\bar{t}$) and $g_t g_Z = 1$ ($ZZ \to t\bar{t}$)



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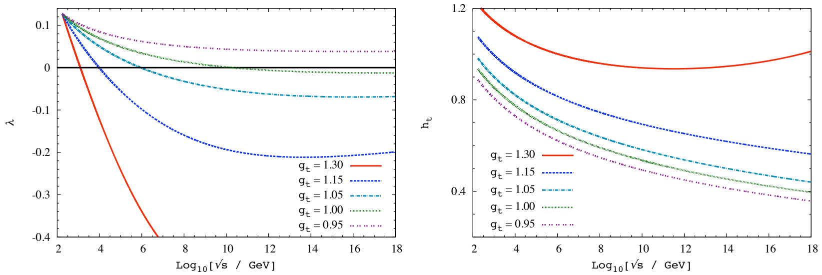
EFT and Scattering Unitarity

If $g_t \neq 1$, the vacuum may get unstable at a much lower scale!



RG running of λ_1 (L) and top Yukawa (R)

Conclusions

- The demand for naturalness can put strong constraints on any BSM theory
- As an example, we show that it leads to new bosonic d.o.f.
- Discussed consequences in three typical scalar extensions: singlet, 2HDM, complex triplet
- Singlets: More than one favoured, possible DM candidate
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Arigatou !