Position-dependent power spectrum and the application to the SDSS-III BOSS DR10 CMASS sample

(1403.3411 and 1412.xxxx)

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IPMU lunch seminar
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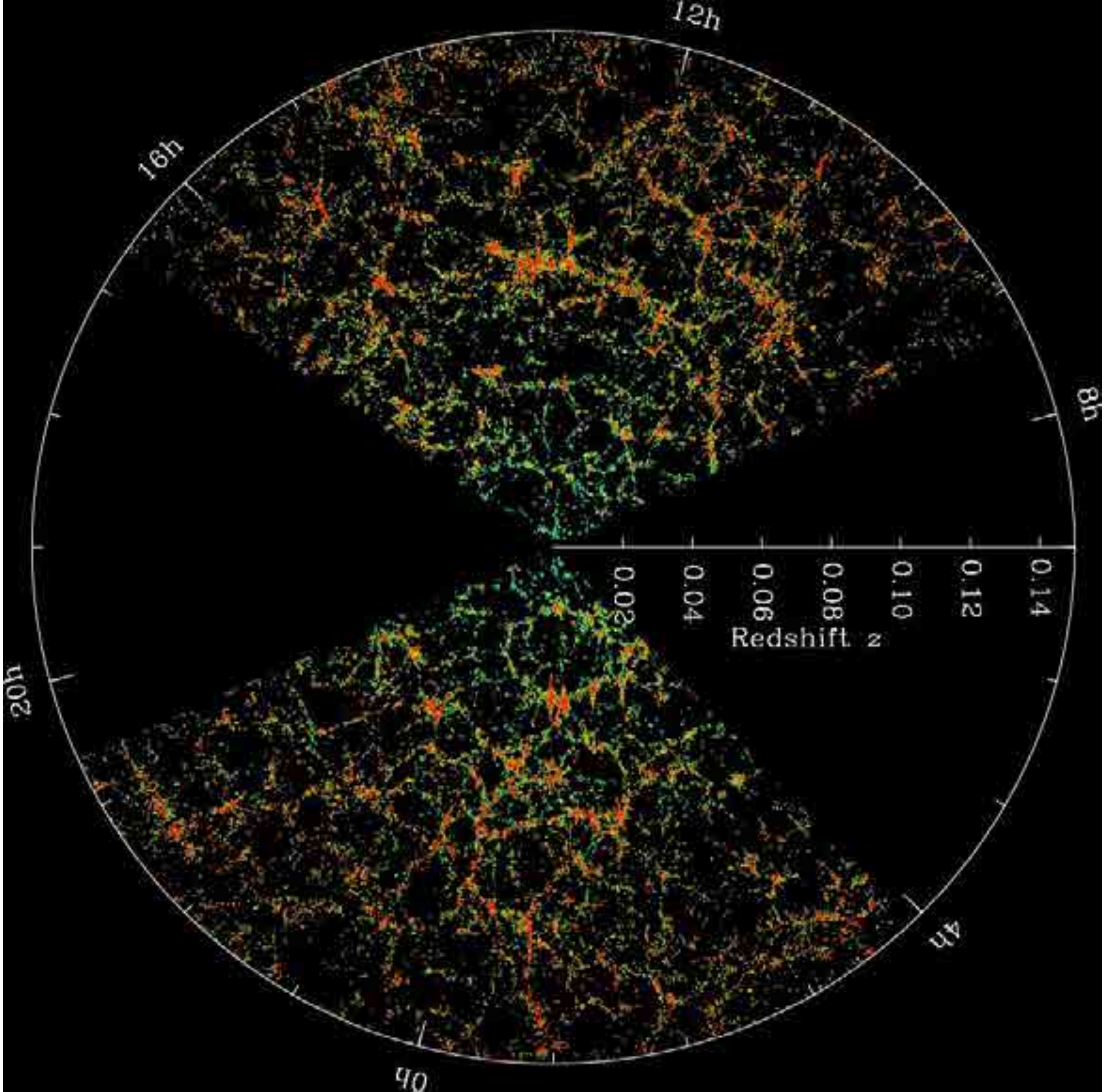

12h
16h
8h
20h
4h
0h
0.02
0.04
0.06
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0.10
0.12
0.14
Redshift z

What is the 2-point correlation function?

- density fluctuation: $\delta(\mathbf{r}) = \frac{n(\mathbf{r})}{\langle n \rangle} - 1$
- 2-point correlation function: $\langle \delta(\mathbf{r}_1)\delta(\mathbf{r}_2) \rangle = \xi(\mathbf{r}_1, \mathbf{r}_2)$
- probability of finding two particles at r_1 and r_2:

  $dP = \bar{n}^2[1 + \xi(\mathbf{r}_1, \mathbf{r}_2)]d^3r_1 d^3r_2$

- homogeneity (translational invariance): $\xi(\mathbf{r}_1 - \mathbf{r}_2)$
- isotropy (rotational invariance): $\xi(|\mathbf{r}_1 - \mathbf{r}_2|)$

What is the power spectrum?

- Fourier transformation of the 2-point correlation function:

$$P(k) = \int d^3r \; \xi(r) e^{-i\mathbf{r}\cdot\mathbf{k}}$$

or

$$\langle \delta(\mathbf{k}) \delta(\mathbf{k}') \rangle = (2\pi)^3 \delta_D(\mathbf{k} + \mathbf{k}') P(k)$$

(assuming homogeneity and isotropy)

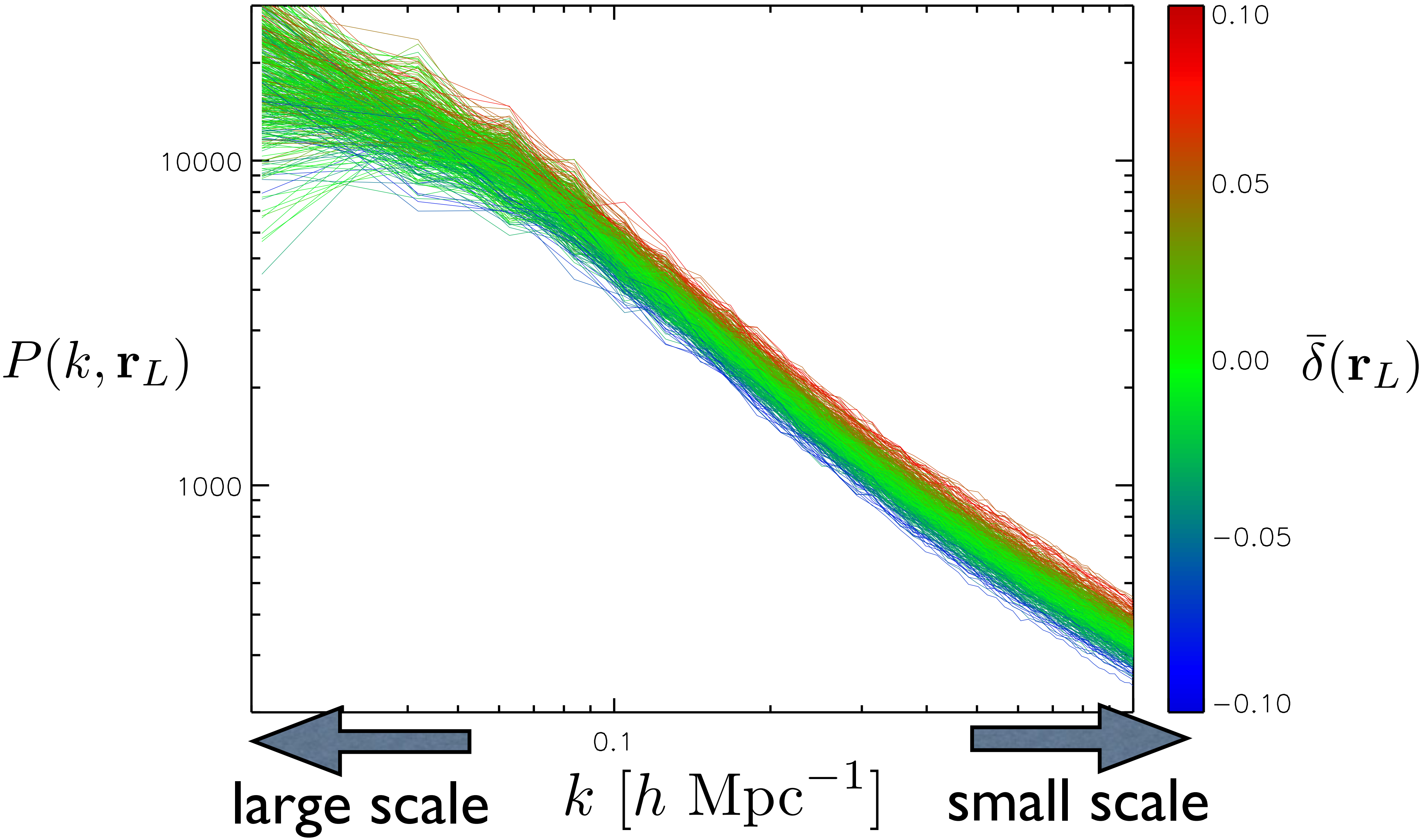
Question

- How does the small-scale power spectrum ***respond*** to the large-scale density fluctuations?

Answer

- The answer depends on the bispectrum (three-point function)!
- In the absence of the bispectrum, the small-scale power spectrum is independent of the large-scale density fluctuations.
- Therefore, we can measure the bispectrum by measuring the small-scale power spectrum as a function of the large-scale density fluctuations!

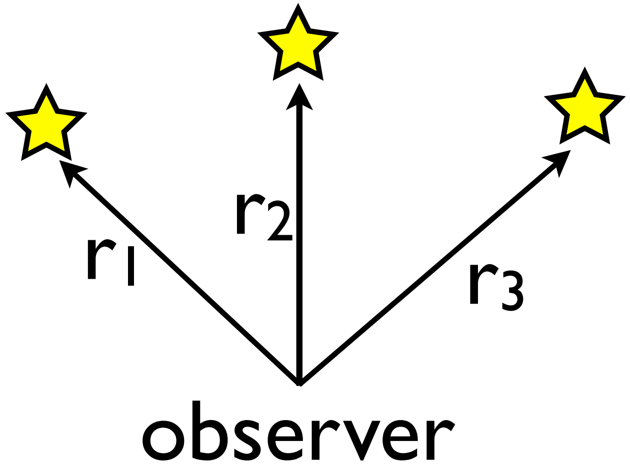
Small-scale power spectra depend on large-scale density fluctuations!



What is the 3-point correlation function and bispectrum?

- 3-point correlation function:

$$\langle\delta(\mathbf{r}_1)\delta(\mathbf{r}_2)\delta(\mathbf{r}_3)\rangle = \zeta(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$$

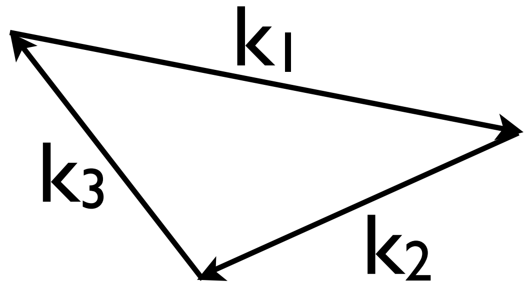


- probability of finding pairs:

$$dP = \bar{n}^3[1+\zeta(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)]d^3r_1 d^3r_2 d^3r_3$$

- bispectrum:

$$\langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle$$
$$= (2\pi)^3\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1, k_2, k_3)$$



Why measure the bispectrum?

- If the fluctuations follow Gaussian statistics, then one only needs the power spectrum (2-point correlation function) to describe their statistical properties.
- If not, then one needs higher-order statistics.
- Non-Gaussianities come from the non-linear evolution, and possibly from inflationary physics.

Why study position-dependent power spectrum?

- Direct measurement of the 3-point correlation function (bispectrum) is difficult when the survey geometry and selection function are not ideal.
- There are only a few measurements of the bispectrum for the current large-scale structure surveys (e.g. Gil-Marín et al 2014 for SDSS DR11).
- This motivates us to find a new observable which is easier to model and gives similar information (especially in the squeezed limit).

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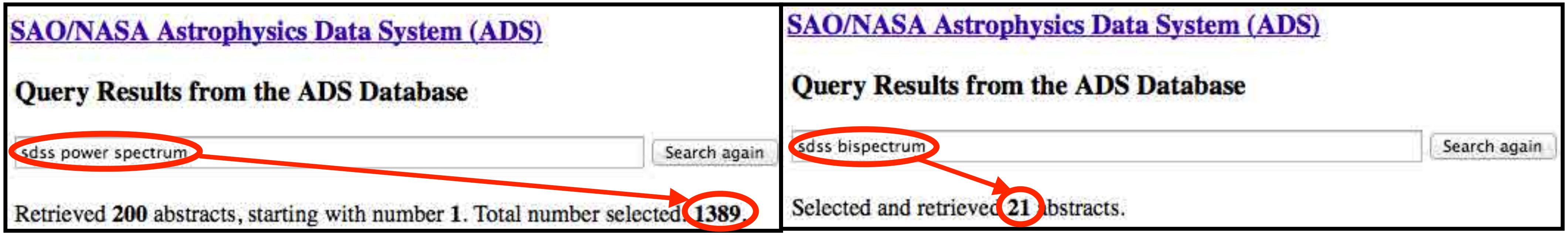
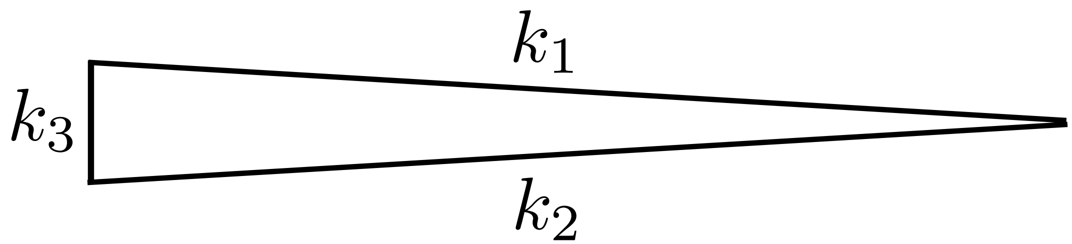


- This motivates us to find a new observable which is easier to model and gives similar information (especially in the squeezed limit).

What is the squeezed-limit bispectrum?

- $\langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle = (2\pi)^3\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3)$
- squeezed limit: $k_1 \approx k_2 \gg k_3$



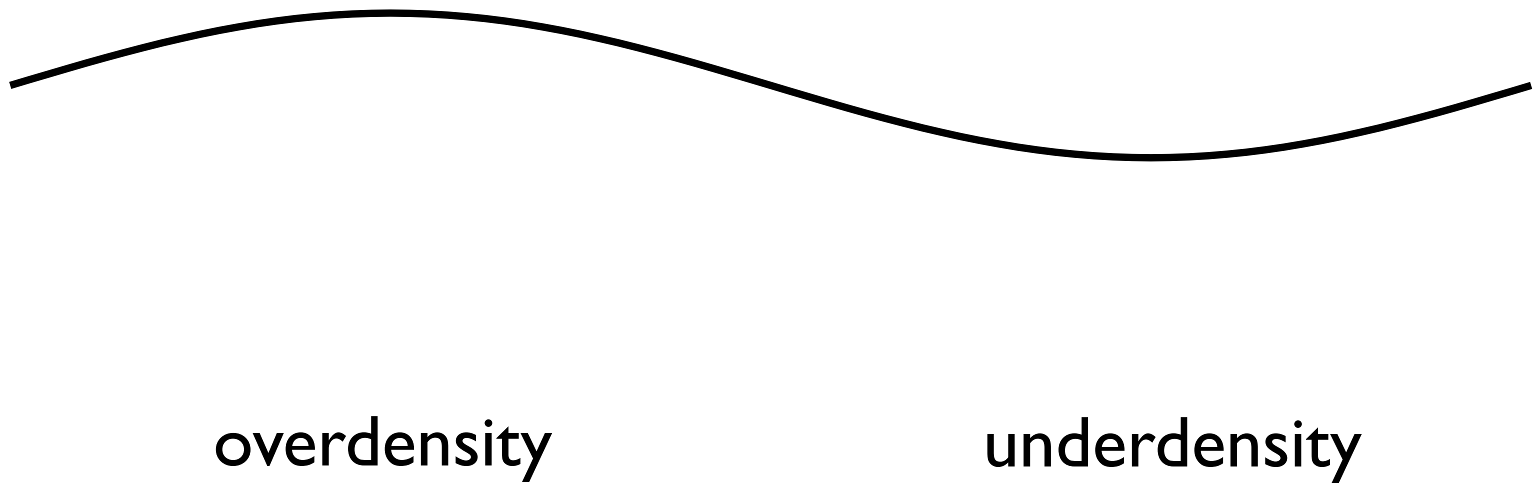
- Squeezed-limit bispectrum measures the correlation of one long-wavelength (small-wavenumber) fluctuation and two short-wavelength (large-wavenumber) fluctuations.

Why squzeed-limit bispectrum?

- Single field inflation predicts vanishing local-type non-Gaussianity in the primordial fluctuations.
- The current constraint from Planck is $f_{NL}=2.7\pm5.8$.
- Large-scale structure would improve the constraint in the foreseeable future.

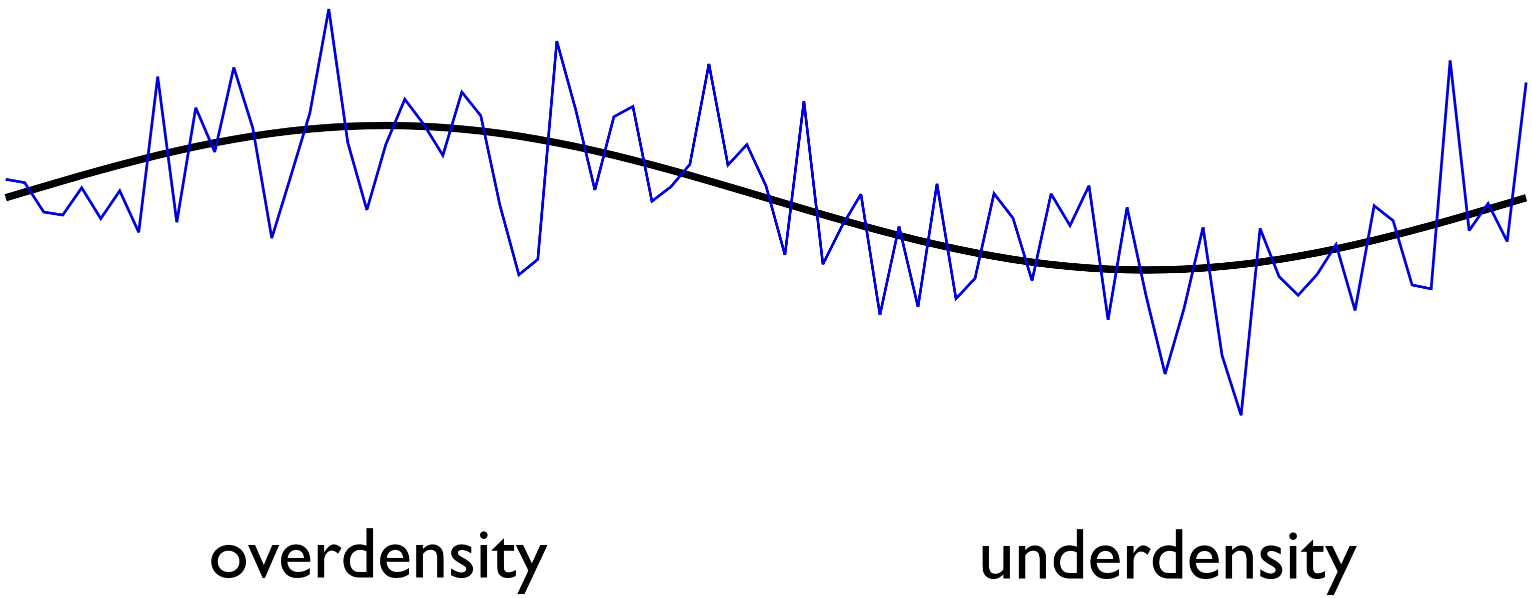
How is the position-dependent P(k,r) modulated by the bispectrum?

consider a long-wavelength density fluctuation...



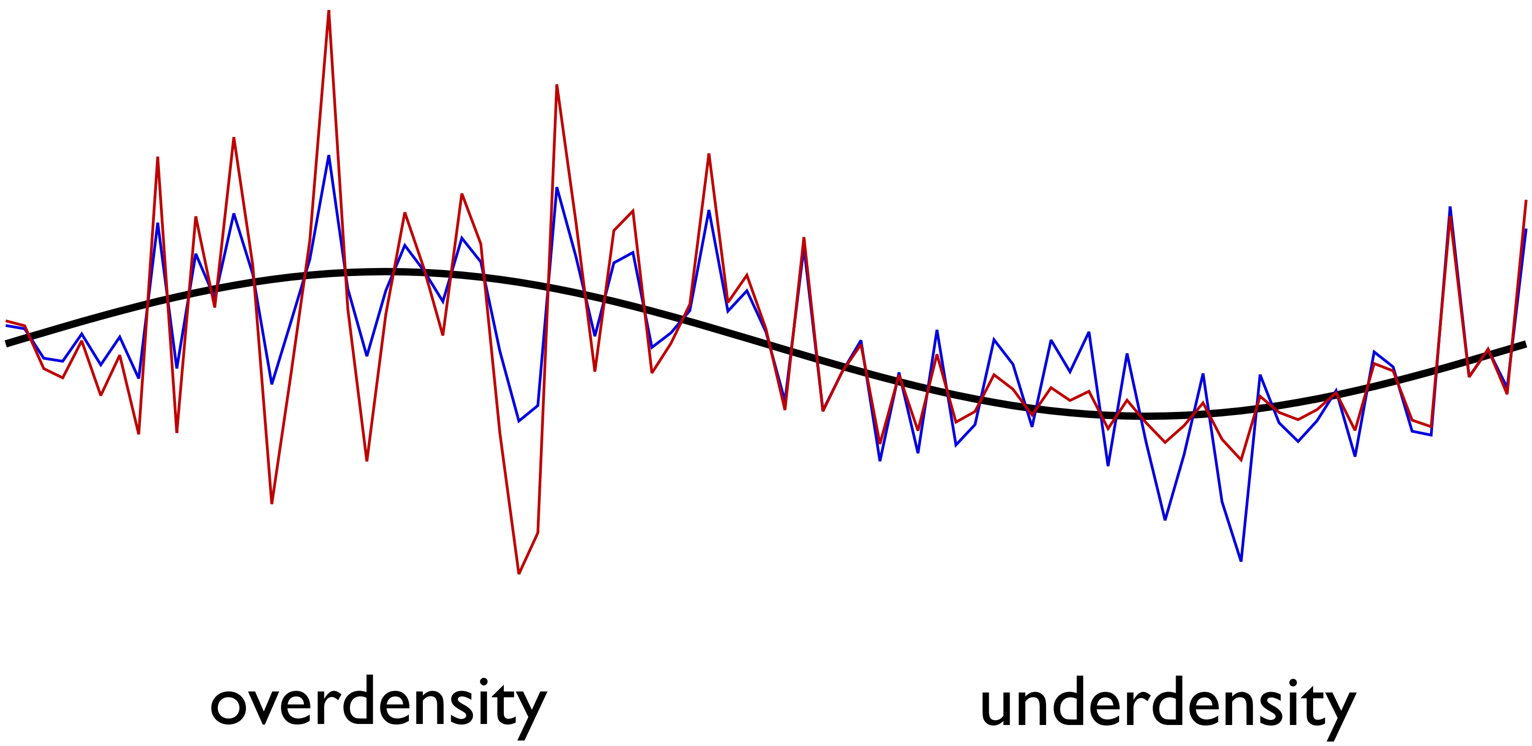
How is the position-dependent P(k,r) modulated by the bispectrum?

vanishing squeezed-limit bispectrum:
no correlation between power spectra and positions



How is the position-dependent P(k,r) modulated by the bispectrum?

non-vanishing squeezed-limit bispectrum:
correlation between power spectra and positions



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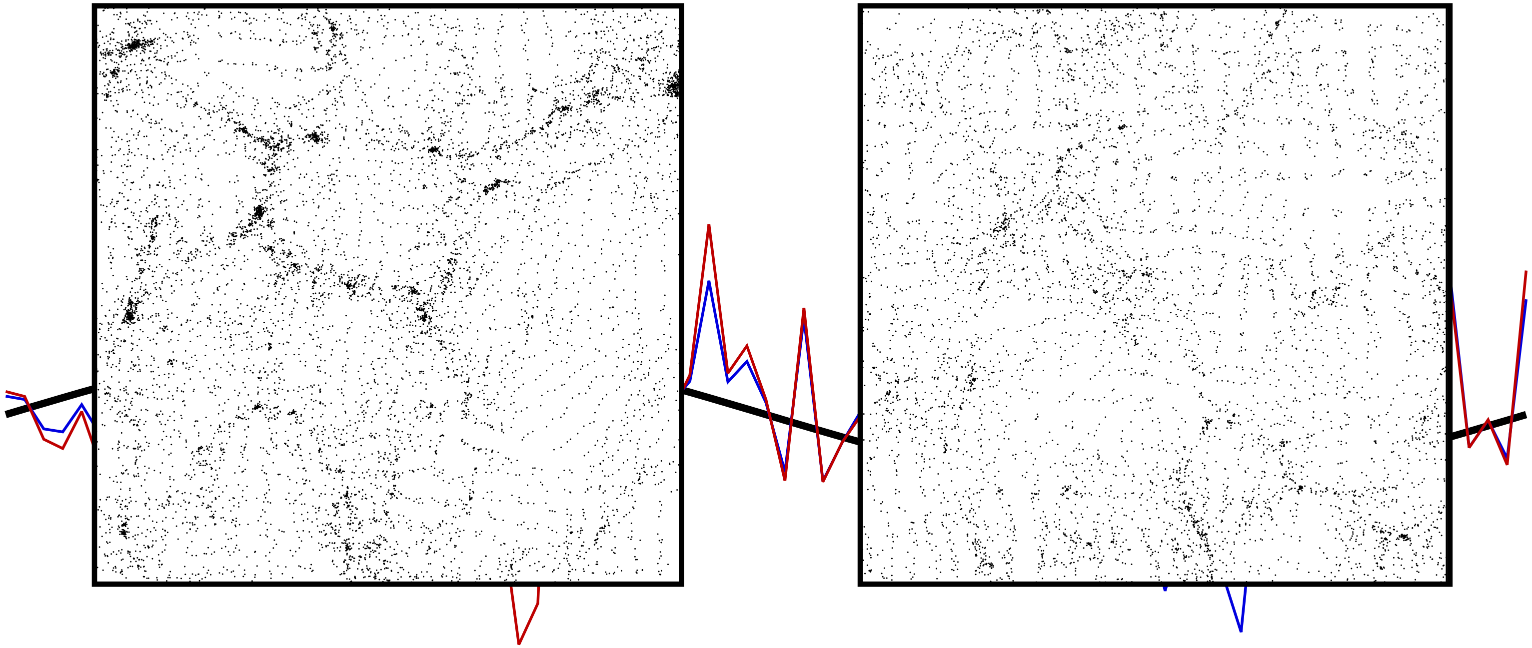


overdensity underdensity

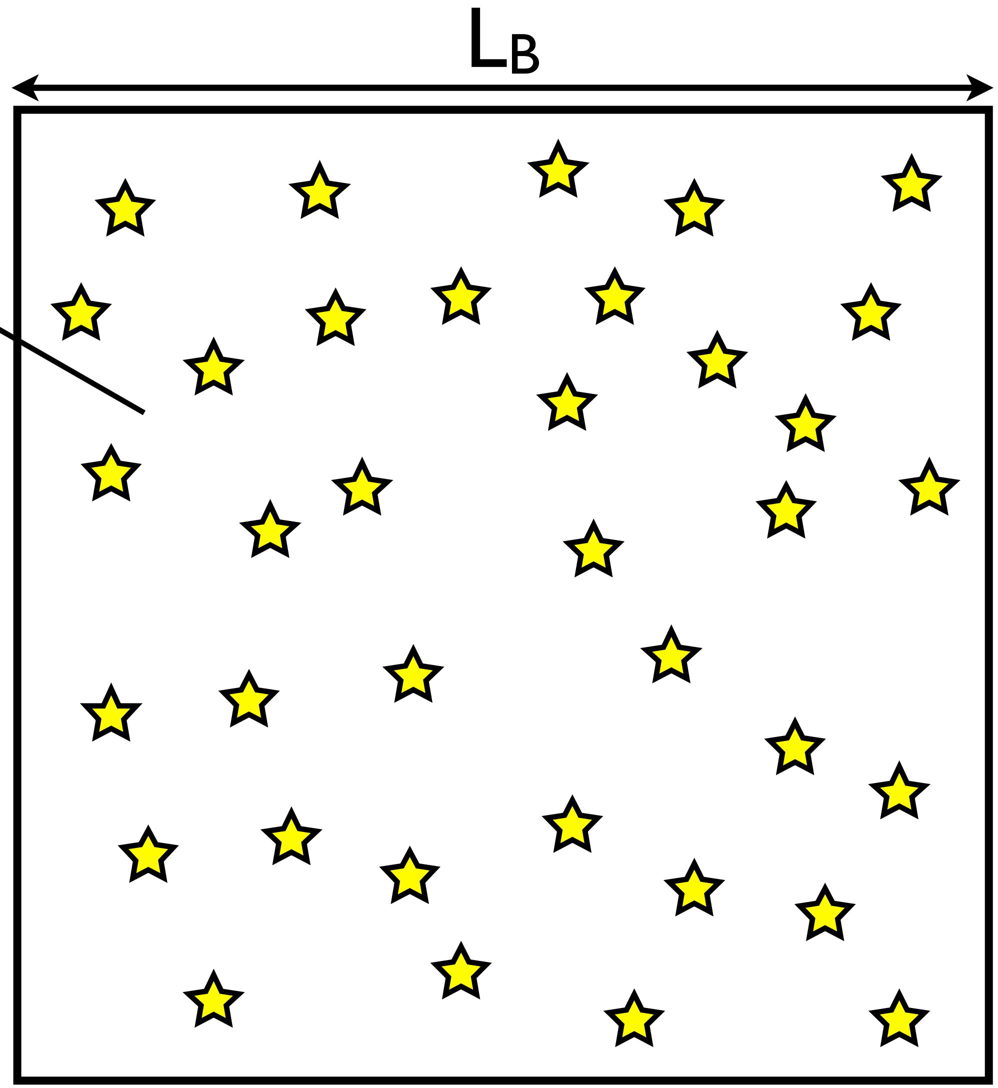
How to measure P(k)?

$$\delta(\mathbf{r}) = \frac{n(\mathbf{r})}{\langle n \rangle} - 1$$

Fourier transform

$$\delta(\mathbf{k}) = \int d^3r\ \delta(\mathbf{r}) e^{-i\mathbf{r}\cdot\mathbf{k}}$$

$$P(\mathbf{k}) = \frac{1}{V_B} |\delta(\mathbf{k})|^2$$



How to measure P(k,r$_L$)?

local mean

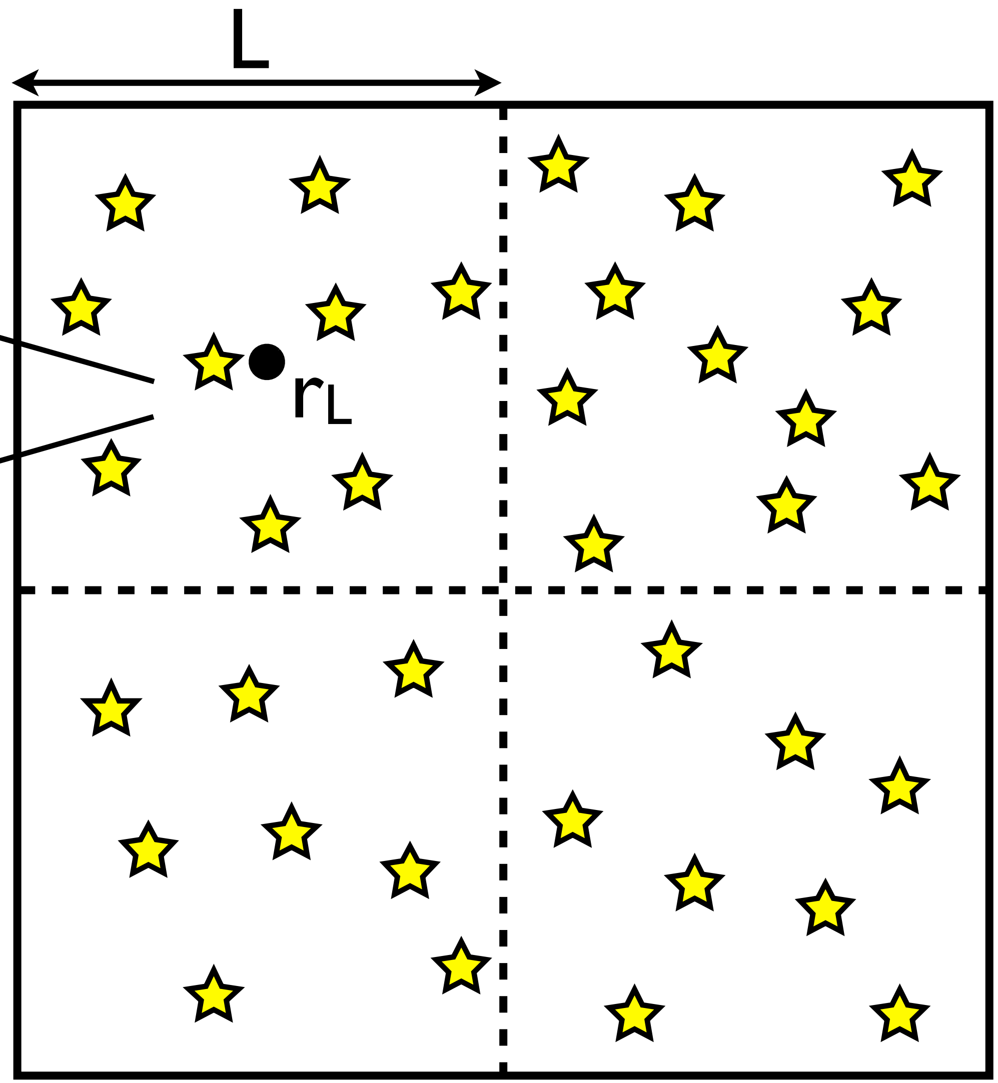
$$\bar{\delta}(\mathbf{r}_L) = \frac{\bar{n}(\mathbf{r}_L)}{\langle n \rangle} - 1$$

global mean

$$P(\mathbf{k}, \mathbf{r}_L) = \frac{1}{V_L} |\delta(\mathbf{k}, \mathbf{r}_L)|^2$$

local Fourier transform

$$\langle P(\mathbf{k}, \mathbf{r}_L)\bar{\delta}(\mathbf{r}_L)\rangle$$



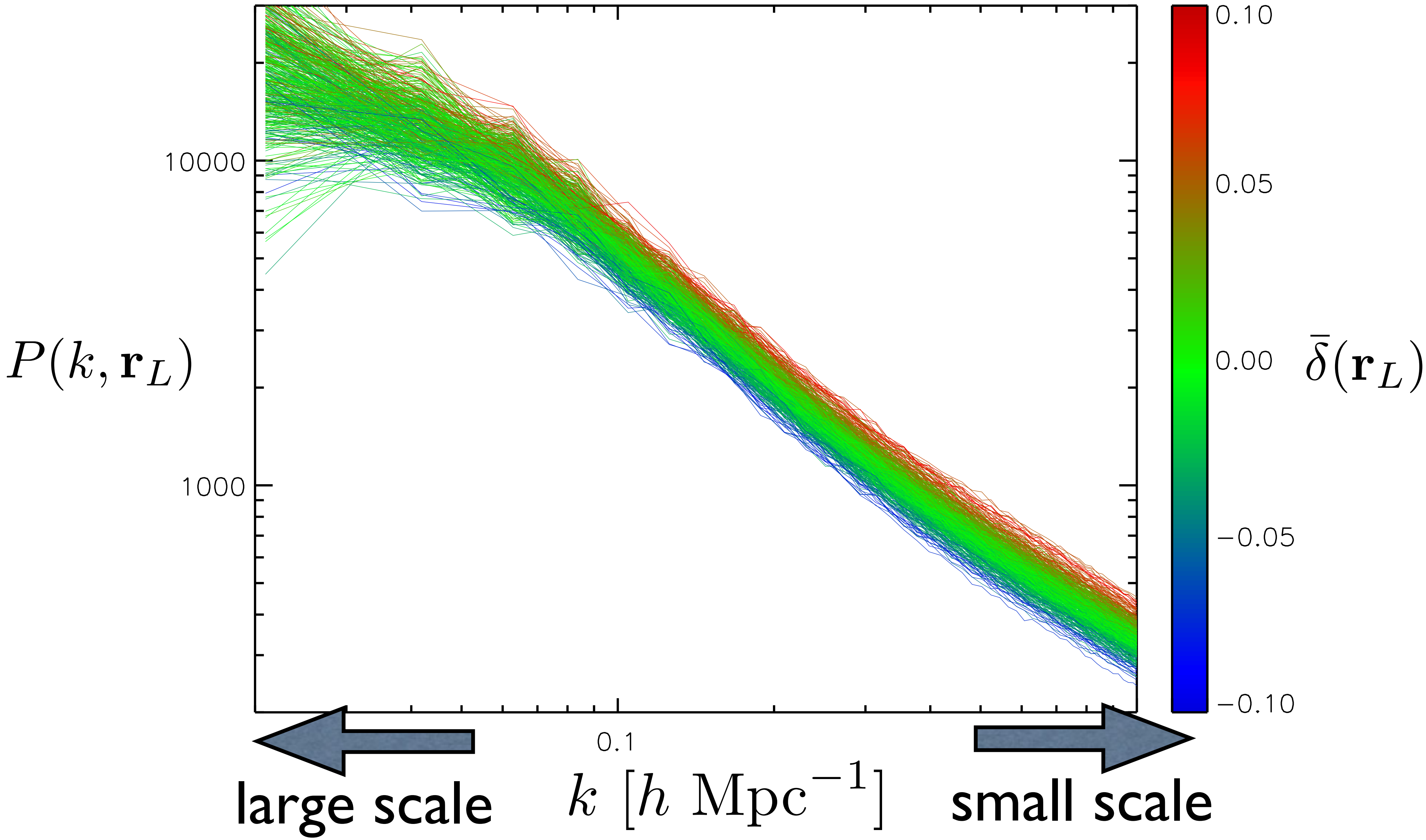
Integrated bispectrum iB(k)

- $$iB(\mathbf{k}) = \langle P(\mathbf{k}, \mathbf{r}_L)\bar{\delta}(\mathbf{r}_L)\rangle$$
  $$= \frac{1}{V_L^2}\int\frac{d^3q_1}{(2\pi)^3}\int\frac{d^3q_2}{(2\pi)^3}\; B(\mathbf{k}-\mathbf{q}_1, -\mathbf{k}+\mathbf{q}_1+\mathbf{q}_2, -\mathbf{q}_2)$$
  $$\times W_L(\mathbf{q}_1)W_L(-\mathbf{q}_1-\mathbf{q}_2)W_L(\mathbf{q}_2)$$
- $$W_L(\mathbf{k}) = V_L\ \mathrm{sinc}\left(\frac{k_xL}{2}\right)\mathrm{sinc}\left(\frac{k_yL}{2}\right)\mathrm{sinc}\left(\frac{k_zL}{2}\right)$$
- squeezed limit: $k \gg q_1 \sim q_2 \sim 1/L$ (e.g. k=0.3 h Mpc^{-1} and L=300 $h^{-1}Mpc$)
- focus on angular average of $\mathbf{k}$

N-body simulations

- 160 simulations of ΛCDM cosmology with Gaussian initial conditions
  - Box size: $(2400\ h^{-1}\mathrm{Mpc})^3$
  - Particle mass: $2.3\times10^{12}\ h^{-1}M_{sun}$
- Divide the simulation box into sub-volumes
  - cut=4: 64 sub-volumes of $(600\ h^{-1}\mathrm{Mpc})^3$
  - cut=8: 512 sub-volumes of $(300\ h^{-1}\mathrm{Mpc})^3$
  - cut=20: 8000 sub-volumes of $(120\ h^{-1}\mathrm{Mpc})^3$

P(k,r$_L$) measured in 512 sub-volumes of (300 h^{-1}Mpc)^3



Normalized integrated bispectrum iB(k)

- $$iB(k) = \int \frac{d^2\Omega_{\hat{\mathbf{k}}}}{4\pi}\ iB(\mathbf{k}) = \langle P(k, \mathbf{r}_L)\bar{\delta}(\mathbf{r}_L)\rangle$$
$$= \frac{1}{V_L^2}\int \frac{d^2\Omega_{\hat{\mathbf{k}}}}{4\pi}\int \frac{d^3q_1}{(2\pi)^3}\int \frac{d^3q_2}{(2\pi)^3}\ B(\mathbf{k}-\mathbf{q}_1, -\mathbf{k}+\mathbf{q}_1+\mathbf{q}_2, -\mathbf{q}_2)$$
$$\times W_L(\mathbf{q}_1)W_L(-\mathbf{q}_1-\mathbf{q}_2)W_L(\mathbf{q}_2)$$

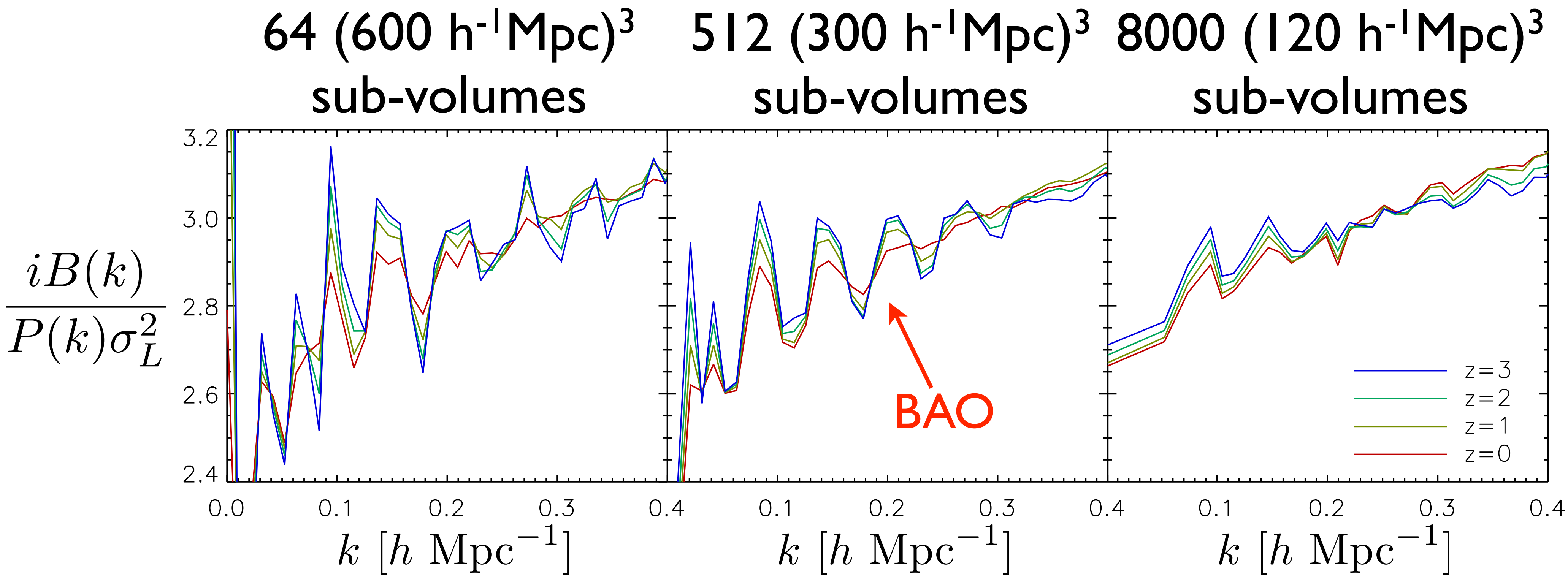
- in the squeezed limit ($k \gg q_1 \sim q_2 \sim 1/L$):

$$\int \frac{d^2\Omega_{\hat{\mathbf{k}}}}{4\pi}\ B(\mathbf{k}-\mathbf{q}_1, -\mathbf{k}+\mathbf{q}_1+\mathbf{q}_2, -\mathbf{q}_2) \to f(k)P(k)P(q_2)$$

- normalized integrated bispectrum:

$$\frac{iB(k)}{P(k)\sigma_L^2} \quad \text{where} \quad \sigma_L^2 = \frac{1}{V_L^2}\int \frac{d^3q_2}{(2\pi)^3}\ |W_L(\mathbf{q}_2)|^2 P(q_2)$$

Measured normalized iB(k)



- BAO is damped more with smaller sub-volume size
- Nonlinear evolution also damps BAO
- Broad-band power is slightly larger on small scales

How to model iB(k)?

- Model the bispectrum and calculate the 8-d integral
  - standard perturbation theory
  - bispectrum fitting formula
- Separate universe approach
  - linear, halofit, Coyote emulator, halo model
- Modeling does not change with the size of the sub-volumes, so we only show the predictions for (300 h^{-1}Mpc)^3 sub-volumes.

Direct integration

$$iB(k) = \int \frac{d^2\Omega_{\hat{\mathbf{k}}}}{4\pi}\, iB(\mathbf{k}) = \langle P(k, \mathbf{r}_L)\bar{\delta}(\mathbf{r}_L)\rangle$$

$$= \frac{1}{V_L^2}\int \frac{d^2\Omega_{\hat{\mathbf{k}}}}{4\pi}\int \frac{d^3q_1}{(2\pi)^3}\int \frac{d^3q_2}{(2\pi)^3}\, B(\mathbf{k}-\mathbf{q}_1, -\mathbf{k}+\mathbf{q}_1+\mathbf{q}_2, -\mathbf{q}_2)$$

$$\times W_L(\mathbf{q}_1)W_L(-\mathbf{q}_1-\mathbf{q}_2)W_L(\mathbf{q}_2)$$

input whatever bispectrum you like

Standard perturbation theory

- $B_{\rm SPT}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = 2[F_2(\mathbf{k}_1,\mathbf{k}_2)P_l(\mathbf{k}_1)P_l(\mathbf{k}_2) + (2\text{ cyclic})]$

  where $F_2(\mathbf{k}_1,\mathbf{k}_2) = \frac{5}{7} + \frac{\mathbf{k}_1\cdot\mathbf{k}_2}{2k_1k_2}\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right) + \frac{2}{7}\left(\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1k_2}\right)^2$

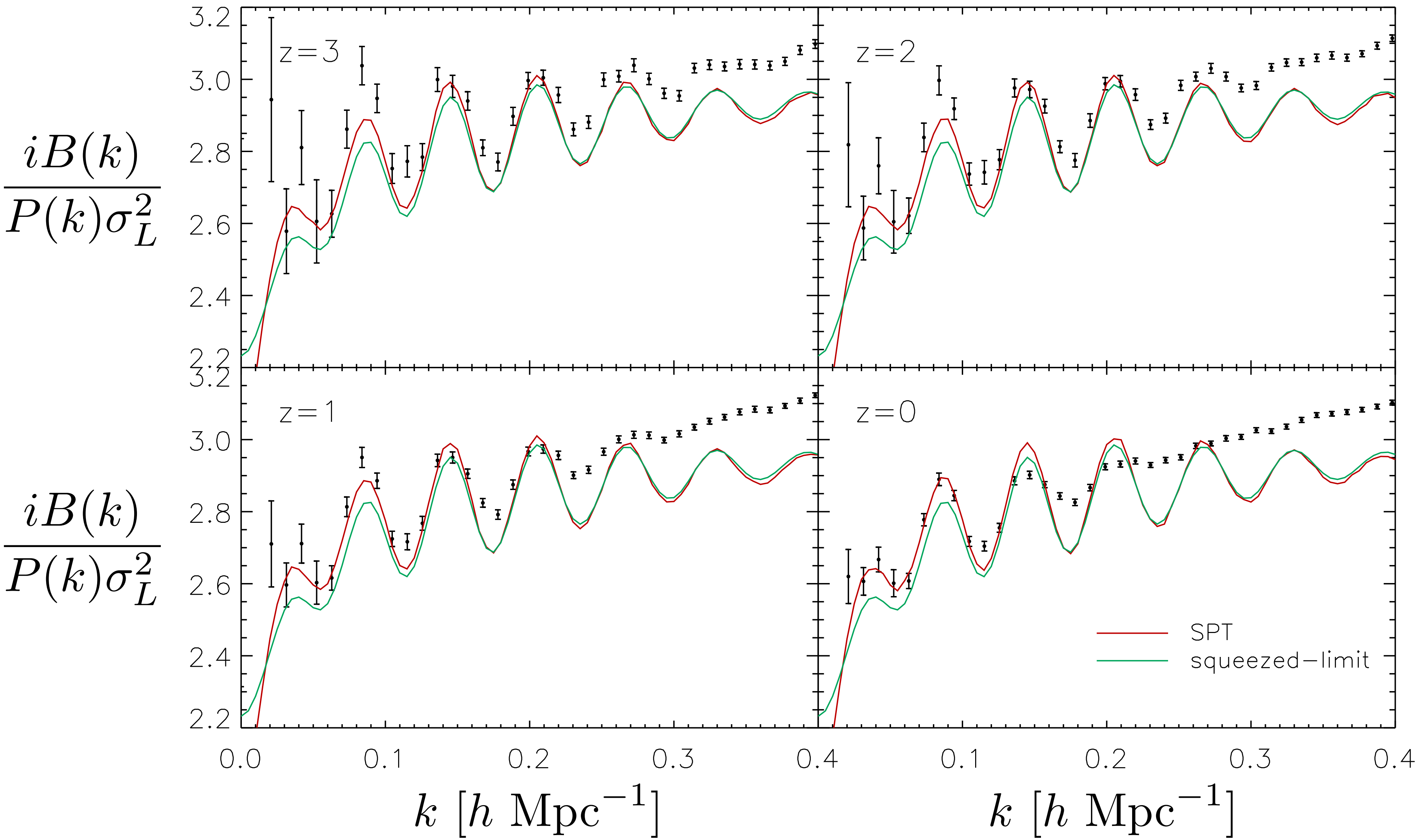
- in the squeezed limit:

$$\int \frac{d^2\Omega_{\hat{\mathbf{k}}}}{4\pi}\; B_{\rm SPT}(\mathbf{k}-\mathbf{q}_1, -\mathbf{k}+\mathbf{q}_1+\mathbf{q}_2, -\mathbf{q}_2)$$

$$= \left[\frac{68}{21} - \frac{1}{3}\frac{d\ln k^3 P_l(k)}{d\ln k}\right] P_l(k)P_l(q_2) + \mathcal{O}\left(\frac{q_{1,2}}{k}\right)^2$$

  and thus $\frac{iB_{\rm SPT}(k)}{P_l(k)\sigma_L^2} \to \left[\frac{68}{21} - \frac{1}{3}\frac{d\ln k^3 P_l(k)}{d\ln k}\right]$

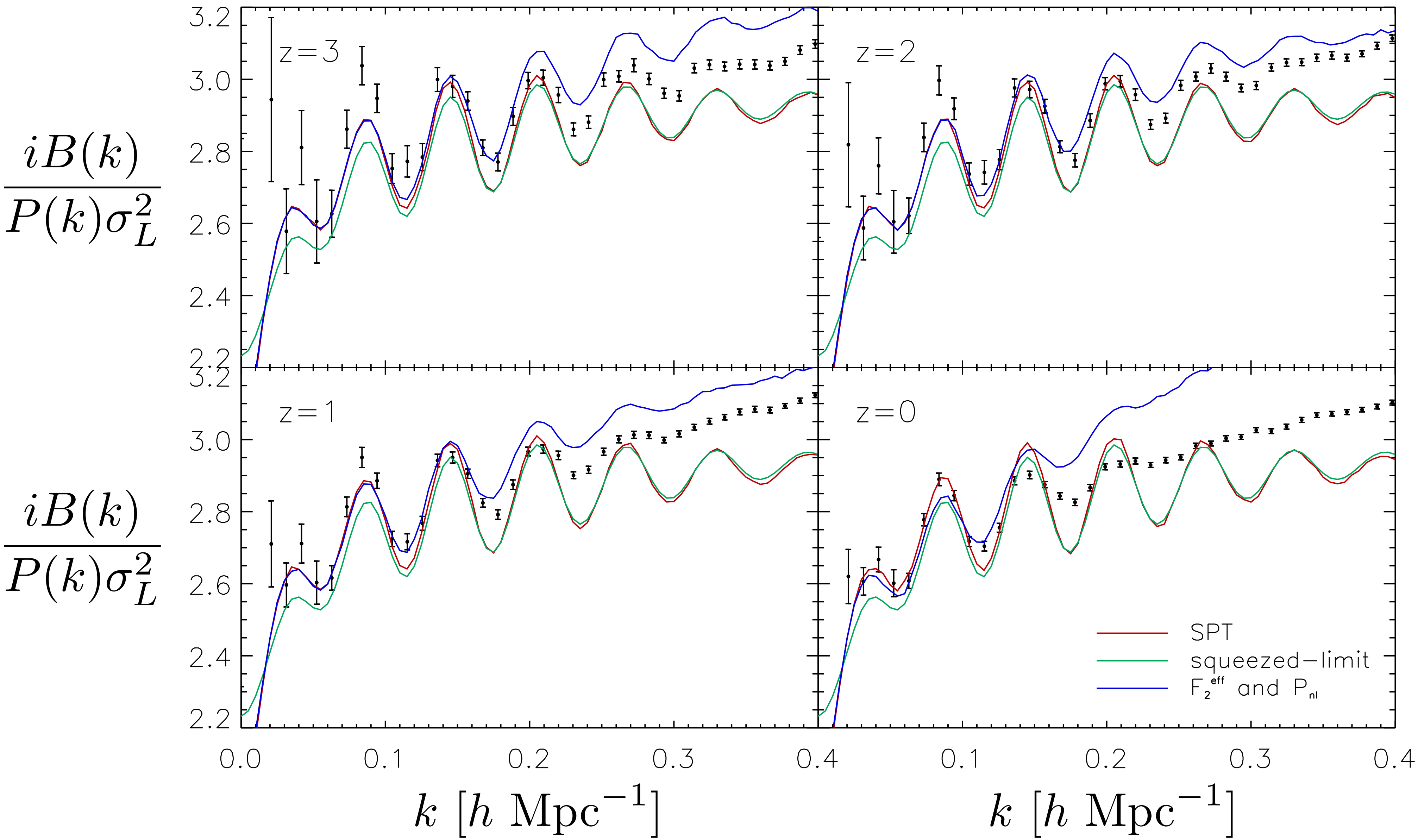
SPT modeling



Bispectrum fitting formula

- First proposed in Scoccimarro and Couchman 2001 and further improved in Gil-Marin et al. 2012
- $B_{\rm eff}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 2[F_2^{\rm eff}(\mathbf{k}_1, \mathbf{k}_2) P_{nl}(\mathbf{k}_1) P_{nl}(\mathbf{k}_2) + (2 \text{ cyclic})]$
- The fitting formula contains 9 fitting parameters.

F_2^{eff} modeling



Separate universe approach

- Squeezed-limit bispectrum measures the correlation of large-scale fluctuations and small-scale structures.

- The power spectrum in the universe with an infinite-wavelength density perturbation δ_0 is

$$P(k|\delta_0) = P(k)|_{\delta_0=0} + P(k)\frac{d\ln P(k)}{d\delta_0}\Big|_{\delta_0=0}\delta_0 + \mathcal{O}(\delta_0^2)$$

- One can then consider the normalized integrated bispectrum as how the power spectrum responds to the density perturbation δ_0, i.e.

$$\frac{iB(k)}{P(k)\sigma_L^2} = \frac{d\ln P(k)}{d\delta_0}$$

Mapping the universe

Ω_m

Ω_Λ

Ω_k

H_0

ρ_m

(e.g. Sirko 2005, Baldauf et al 2011, Sherwin & Zaldarriaga 2012, Li et al, 2014)

Mapping the universe

Ω_m	Ω_m
Ω_Λ	Ω_Λ
Ω_k	Ω_k
H_0	H_0
ρ_m	$\rho_m(1+\delta_0)$

(e.g. Sirko 2005, Baldauf et al 2011, Sherwin & Zaldarriaga 2012, Li et al, 2014)

Mapping the universe

Ω_m	Ω_m		$\tilde{\Omega}_m$
Ω_Λ	Ω_Λ		$\tilde{\Omega}_\Lambda$
Ω_k	Ω_k	$\longrightarrow$	$\tilde{\Omega}_k$
H_0	H_0		$\tilde{H}_0$
ρ_m	$\rho_m(1+\delta_0)$		ρ_m

(e.g. Sirko 2005, Baldauf et al 2011, Sherwin & Zaldarriaga 2012, Li et al, 2014)

Flat ΛCDM universe with δ_0

change of cosmology
in linear order of δ_0

$$\tilde{H}_0 = H_0 \left[1 + \delta_H\right]$$

$$\delta_H = \left(-\frac{1}{2}\Omega_m - \frac{1}{3}f_0\right)\delta_0$$

$$\tilde{\Omega}_m = \Omega_m \left[1 - 2\delta_H\right]$$

$$\tilde{\Omega}_K = -\left(\Omega_m + \frac{2}{3}f_0\right)\delta_0$$

$$\tilde{\Omega}_\Lambda = \Omega_\Lambda \left[1 - 2\delta_H\right]$$

change of time and scale
in linear order of δ_0

$$\tilde{a}(t) = a(t)\left[1 - \frac{1}{3}\frac{D(t)}{D(t_0)}\delta_0\right]$$

$$\tilde{\mathbf{x}}(t) = \mathbf{x}(t)\left[1 - \frac{1}{3}\frac{D(t)}{D(t_0)}\delta_0\right]$$

$$\tilde{D}[\tilde{a}(t)] = D(a)\left[1 + \frac{13}{21}\delta_0\right]$$

↙ exact only in Einstein-de Sitter universe
but depends very mildly on cosmology
(~0.1% fractional difference at z=0)

Change of P(k) with δ_0

- change in scale:

$$\tilde{P}(k,t) \to P(k,t)\left[1 - \frac{1}{3}\frac{d\ln k^3 P(k,t)}{d\ln k}\frac{D(t)}{D(t_0)}\delta_0\right]$$

- change in scale factor (and growth factor):

$$\tilde{P}(k,t) \to P\left(k, a\left[1 - \frac{1}{3}\frac{D(t)}{D(t_0)}\right]\right)$$

- change in normalization ($\rho \to \rho(1+\delta_0)$):

$$\tilde{P}(k,t) \to (1+2\delta_0)P(k,t)$$

- total change:

$$\tilde{P}\left(k, a\left[1 - \frac{1}{3}\frac{D(t)}{D(t_0)}\delta_0\right]\right)\left[1 + \left(2 - \frac{1}{3}\frac{d\ln k^3 P(k,a)}{d\ln k}\right)\frac{D(t)}{D(t_0)}\delta_0\right]$$

Change of linear P(k) with δ_0

- linear power spectrum is proportional to D^2

$$\tilde{P}_l\left(k, a\left[1-\frac{1}{3}\frac{D(a)}{D(a_0)}\delta_0\right]\right) = \left(\frac{\tilde{D}\left(a\left[1-\frac{1}{3}\frac{D(a)}{D(a_0)}\delta_0\right]\right)}{D(a)}\right)^2 P_l(k,a)$$

- change of growth factor:

$$\tilde{D}\left(a\left[1-\frac{1}{3}\frac{D(a)}{D(a_0)}\delta_0\right]\right) = D(a)\left[1+\frac{13}{21}\frac{D(a)}{D(a_0)}\delta_0\right]$$

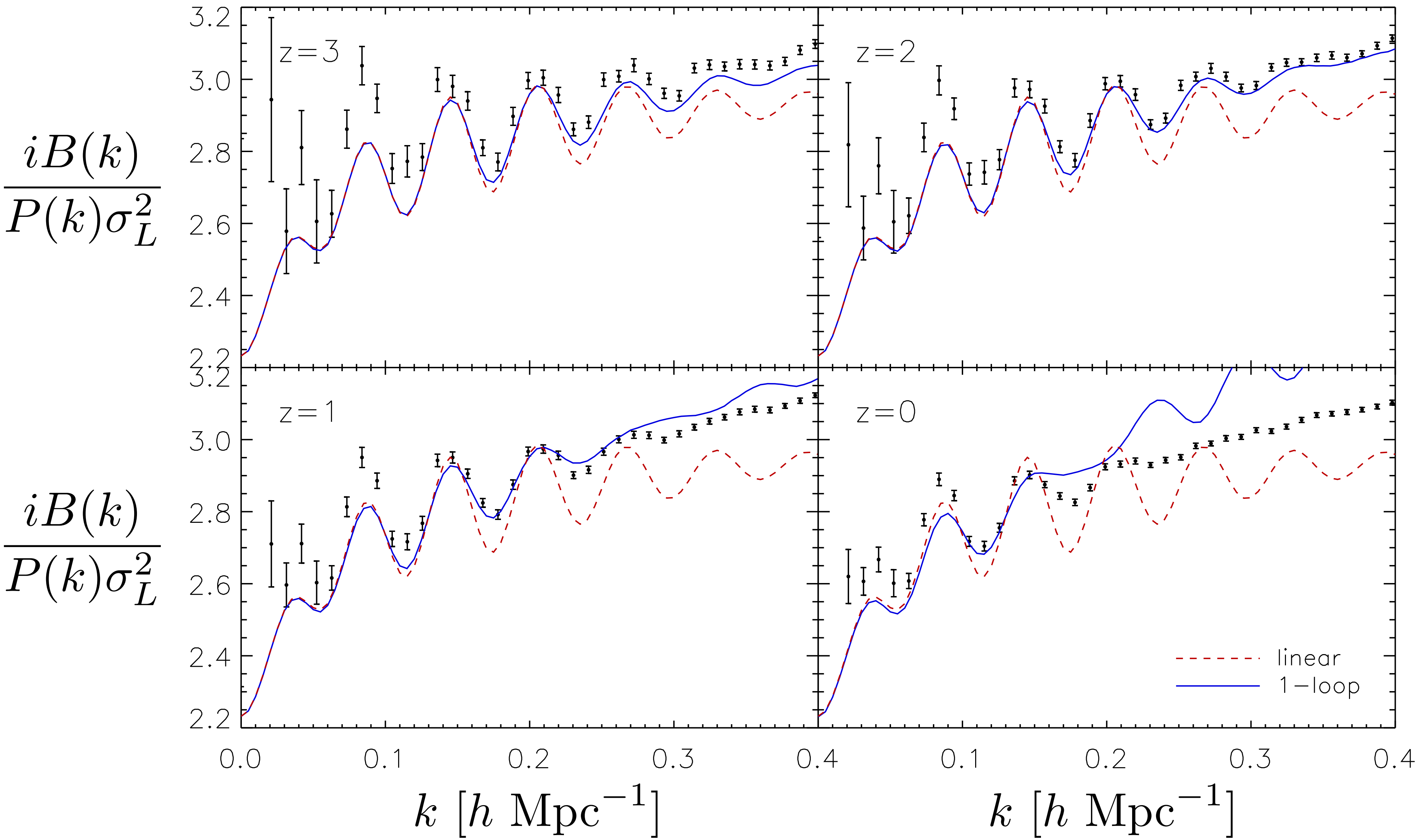
- finally, $\dfrac{d\ln P_l(k,t)}{d\delta_0} = \dfrac{D(t)}{D(t_0)}\left[\dfrac{68}{21} - \dfrac{1}{3}\dfrac{d\ln k^3 P_l(k,t)}{d\ln k}\right]$

agree with SPT (F_2 kernel) result!

SPT 1-loop P(k)

- $$P(k,a) = P_l(k,a) + P_{22}(k,a) + 2P_{13}(k,a)$$
  $$= D^2(a)P_l(k) + D^4(a)[P_{22}(k) + 2P_{13}(k)]$$
- $$\tilde{D}[\tilde{a}(t)] = D(a)\left[1 + \frac{13}{21}\delta_0\right]$$
- $$\frac{d\ln P(k,a)}{d\delta(a)} = \frac{68}{21} - \frac{1}{3}\frac{d\ln k^3 P(k,a)}{d\ln k} + \frac{26}{21}\frac{P_{22}(k,a) + 2P_{13}(k,a)}{P(k,a)}$$

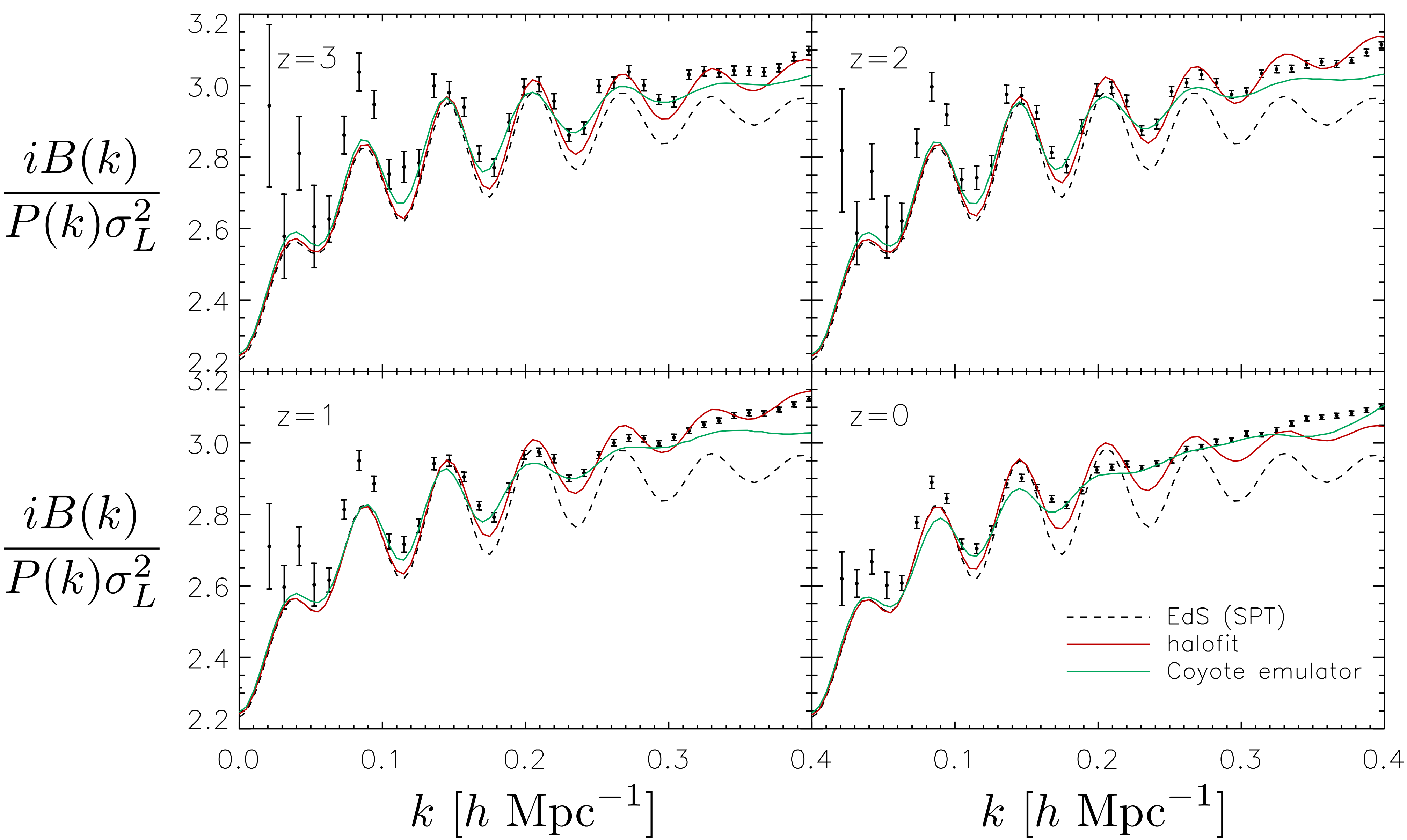
SPT 1-loop P(k) modeling



halofit and Coyote emulator

- Mimic the effect of δ_0 by changing the cosmological parameters in the machineries of calculating non-linear power spectrum.
- halofit prescription (Smith et al. 2003) and Coyote emulator (Heitmann et al. 2013)

halofit and Coyote emulator modeling



Summary

- Without directly measuring the bispectrum, the position-dependent power spectrum P(k,r) gives similar information.
- Pros:
  - Easy to measure, especially with non-ideal survey geometry.
  - Procedures developed for power spectrum estimation can be directly applied to position-dependent power spectrum (e.g. removing window function effect).
  - iB(k) depends only on one wavenumber k (unlike the bispectrum which depends on two wavenumbers and one angle), so the covariance matrix would be easier to model.
- Cons: Use only a subset of information.
- Separate universe approach is a powerful way to model the small-scale power spectrum in the presence of δ_0.

Application to the SDSS-III BOSS DR10 CMASS sample

- Correlation function is easier to measure than power spectrum for galaxy surveys.
- What can we learn from the galaxy 3-point correlation function?

How to measure ξ(r,r$_L$)?

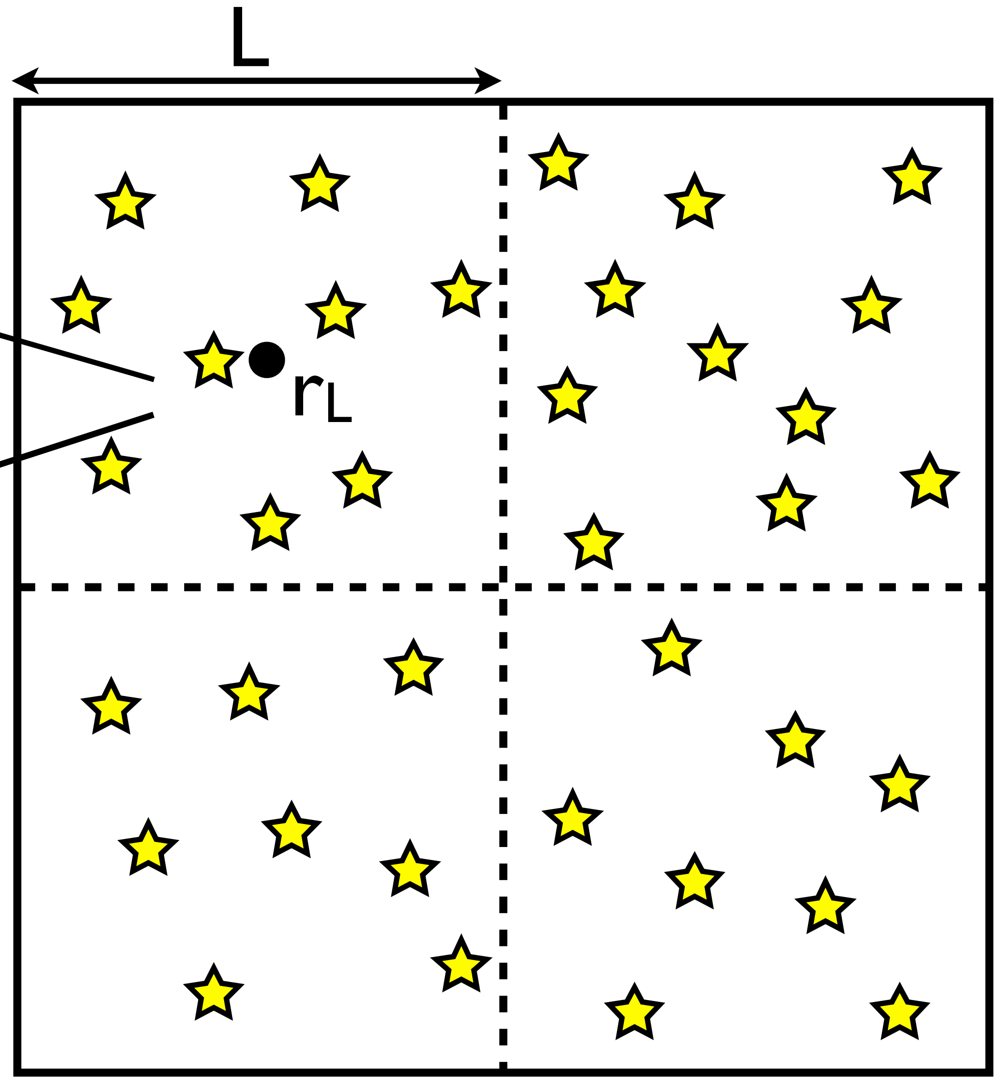
local mean

$$\bar{\delta}(\mathbf{r}_L) = \frac{\bar{n}(\mathbf{r}_L)}{\langle n \rangle} - 1$$

global mean

$$\xi(\mathbf{r}, \mathbf{r}_L)$$

$$\langle \xi(\mathbf{r}, \mathbf{r}_L) \bar{\delta}(\mathbf{r}_L) \rangle$$



Integrated 3-point function

- $i\zeta(r) = \langle \xi(r, \mathbf{r}_L)\bar{\delta}(\mathbf{r}_L)\rangle$
$$= \frac{1}{V_L^2}\int \frac{d^2\hat{r}}{4\pi}\int d^3x_1 \int d^3x_2\ \zeta(\mathbf{r}+\mathbf{x}_1+\mathbf{r}_L, \mathbf{x}_1+\mathbf{r}_L, \mathbf{x}_2+\mathbf{r}_L)$$
$$\times W_L(\mathbf{r}+\mathbf{x}_1)W_L(\mathbf{x}_1)W_L(\mathbf{x}_2)$$
- $W_L(\mathbf{r})=1$ if $|r_i|\leq L/2$ and 0 otherwise
- Galaxy (halo) 3-point function: $\zeta_g = b_1^3\zeta_{\rm SPT} + b_1^2 b_2\zeta_{b_2}$
- ζ_{SPT}: standard perturbation theory
- ζ_{b2} : nonlinear bias

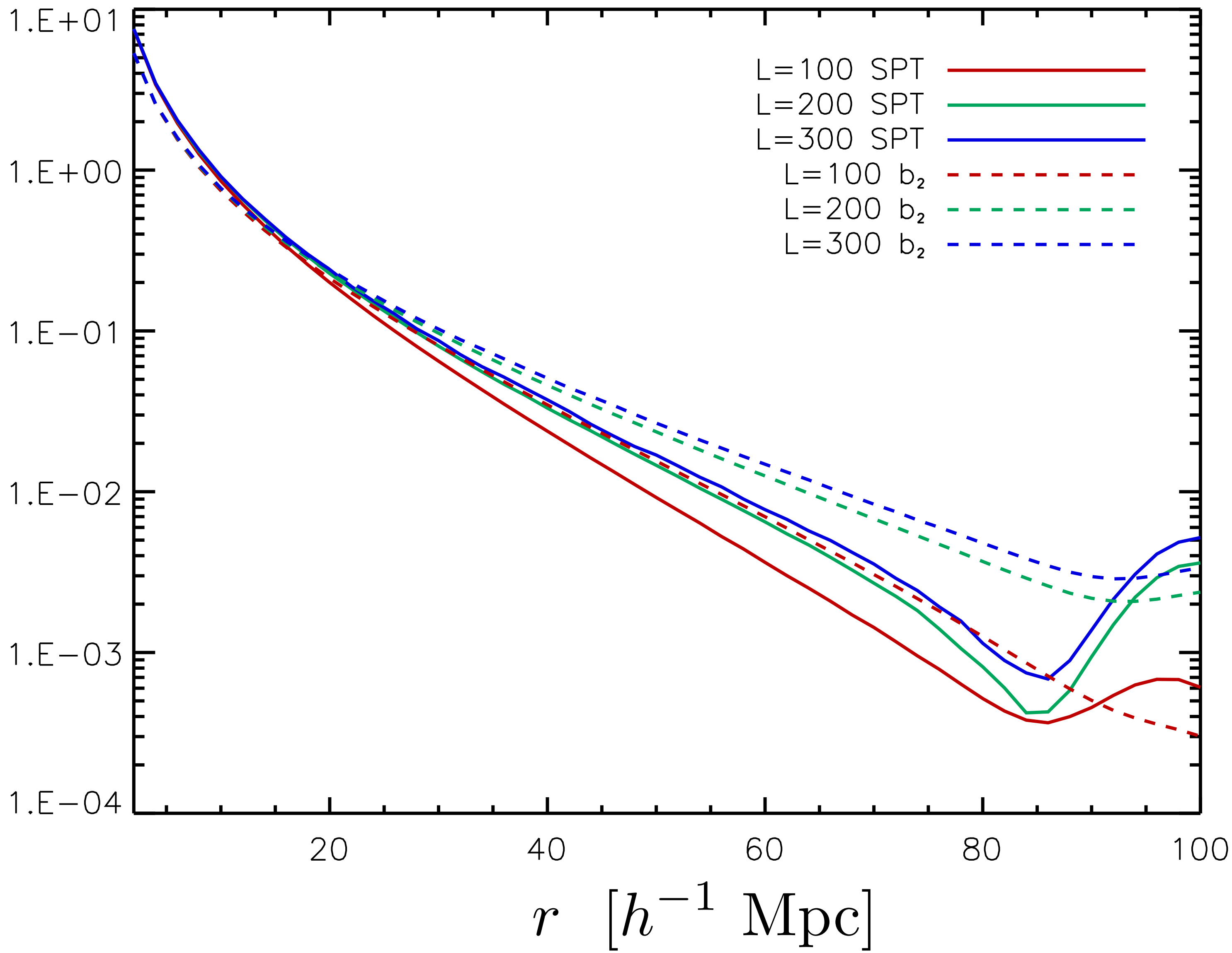
$$\delta_g(\mathbf{r}) = b_1\delta_m(\mathbf{r}) + \frac{b_2}{2}\delta_m^2(\mathbf{r})$$

$i\zeta_{SPT}(r)$ and $i\zeta_{b2}(r)$

$$\frac{i\zeta(r)}{\sigma_L^2}$$

$$\downarrow$$

$$\langle \bar{\delta}(\mathbf{r}_L)^2 \rangle$$



Squeezed-limit approximation

- Squeezed-limit: $r \ll L$
- $\xi(\mathbf{r}, \mathbf{r}_L) = \xi(\mathbf{r})|_{\bar{\delta}=0} + \left.\dfrac{d\xi(\mathbf{r})}{d\bar{\delta}}\right|_{\bar{\delta}=0} \bar{\delta} + \mathcal{O}(\bar{\delta}^2)$
- $i\zeta(\mathbf{r}) = \langle \xi(\mathbf{r}, \mathbf{r}_L)\bar{\delta}(\mathbf{r}_L)\rangle = \dfrac{d\xi(\mathbf{r})}{d\bar{\delta}} \langle \bar{\delta}^2 \rangle + \mathcal{O}(\bar{\delta}^3)$

$$= \left[\frac{d}{d\bar{\delta}} \int \frac{d^3k}{(2\pi)^3} \, P(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}}\right] \langle \bar{\delta}^2 \rangle + \mathcal{O}(\bar{\delta}^3)$$

$$= \int \frac{d^3k}{(2\pi)^3} \, iB(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}} + \mathcal{O}(\bar{\delta}^3)$$

Redshift-space distortion

- Standard perturbation theory at tree-level
- $B_{\rm SQ1} = b_1^3 \sum_{i=1}^{3} \beta^{i-1} B_{{\rm SQ1},i}$ $B_{\rm SQ2} = b_1^3 \beta \sum_{i=1}^{3} \beta^{i-1} B_{{\rm SQ2},i}$ $B_{\rm NLB} = b_1^2 b_2 \sum_{i=1}^{3} \beta^{i-1} B_{{\rm NLB},i}$

  $B_{\rm FOG} = b_1^4 \beta [B_{{\rm FOG},1} + \beta (B_{{\rm FOG},2} + B_{{\rm FOG},3}) + \beta^2 (B_{{\rm FOG},4} + B_{{\rm FOG},5}) + \beta^3 B_{{\rm FOG},6}]$

- $B_{\rm SQ1} \propto F_2(\mathbf{k}_1, \mathbf{k}_2)$ $B_{\rm SQ2} \propto G_2(\mathbf{k}_1, \mathbf{k}_2)$ $\beta = f/b_1$
- $$iB_{{\rm x},i}(k) = \frac{1}{V_L^2} \int \frac{d^2\hat{k}}{4\pi} \int \frac{d^3 q_1}{(2\pi)^3} \int \frac{d^3 q_2}{(2\pi)^3} B_{{\rm X},i}(\mathbf{k} - \mathbf{q}_1, -\mathbf{k} + \mathbf{q}_1 + \mathbf{q}_3, -\mathbf{q}_3) \times W_L(\mathbf{q}_1) W_L(-\mathbf{q}_1 - \mathbf{q}_3) W_L(\mathbf{q}_3)$$

Scoccimarro, Couchman, Frieman 1998

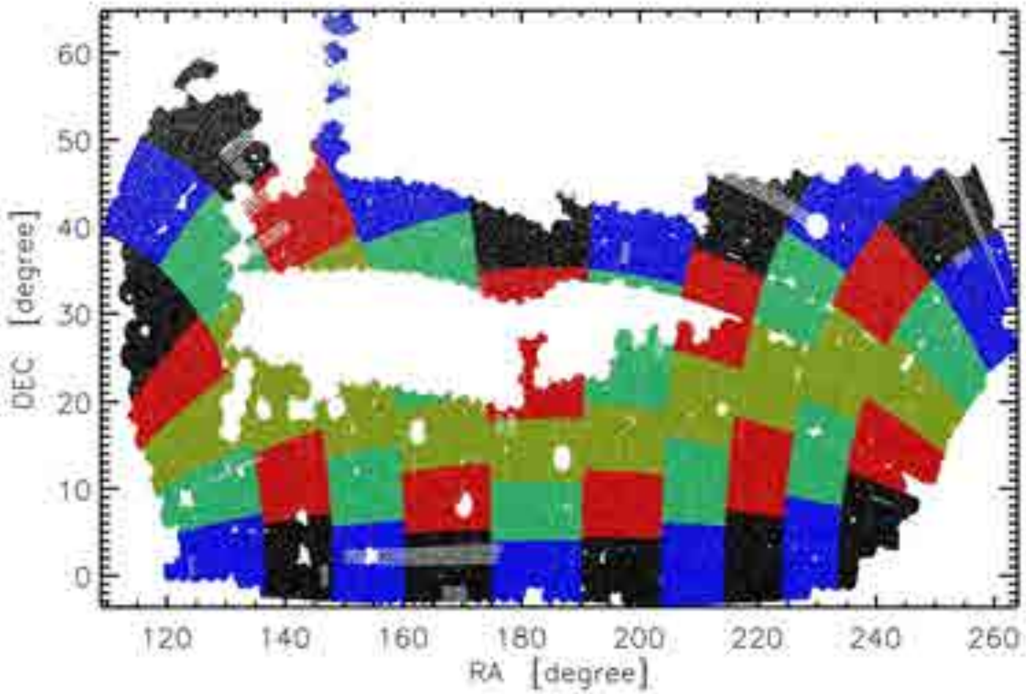
Modeling iζ(r) in zspace

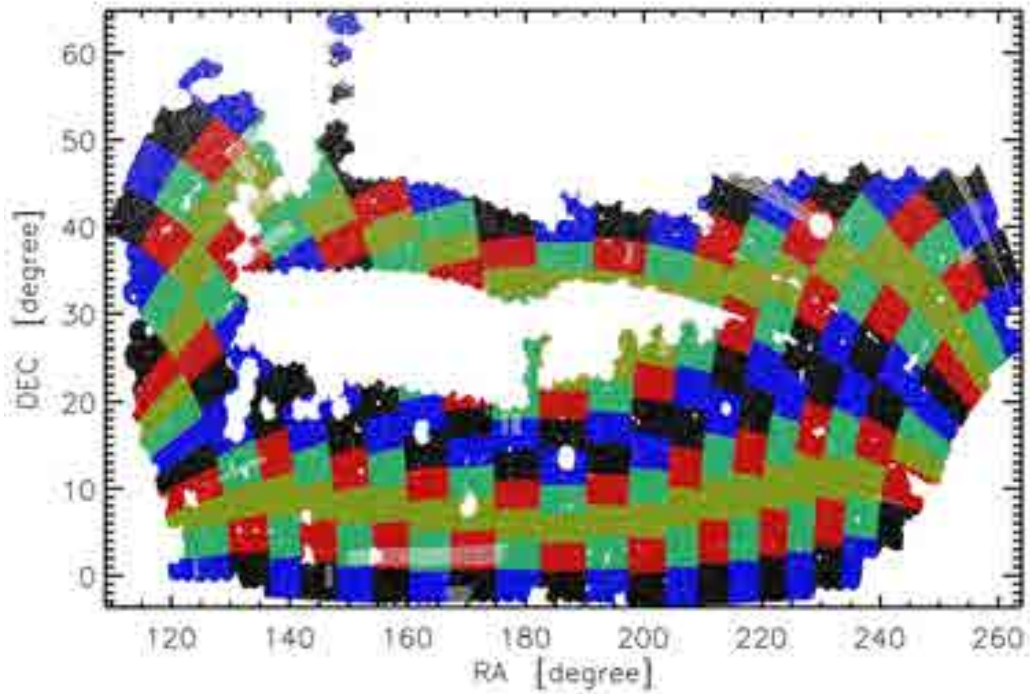
- Problem: We don't have a complete description for the tree-level three-point correlation function in the redshift space.
- Solution: In the squeezed limit, iζ(r) is simply the Fourier transformation of iB(k).
- Results: This simple modeling can describe the ratio of iζ_z(r)/iζ_r(r) for the mock catalogs in the quasi-squeezed-limit.

SDSS-III DR10 CMASS sample (north galactic cap)

- number of galaxies: ~400,000
- redshift range: $0.43<z<0.7$
- volume: ~ 2 h^{-3} Gpc^3 (4,892 deg^2)
- weighting: $w_{\rm BOSS} = (w_{cp} + w_{zf} - 1) w_{\rm star} w_{\rm see}$
- 600 PTHalo mock catalogs (Manera et al 2014)

Divide the sample





zcut=3
150 subvolumes
220 h^{-1} Mpc

zcut=5
1000 subvolumes
120 h^{-1} Mpc

Quantities in subvolumes

- $\bar{\delta}_i = \frac{1}{\alpha}\frac{w_{g,i}}{w_{r,i}} - 1 \qquad \alpha = \frac{\sum_{i=1}^{N_s} w_{g,i}}{\sum_{i=1}^{N_s} w_{r,i}} = \frac{w_{g,\mathrm{tot}}}{w_{r,\mathrm{tot}}}$
- $\xi_{\mathrm{LS}}(r,\mu) = \frac{DD(r,\mu)/DD_{\mathrm{norm}}}{RR(r,\mu)/RR_{\mathrm{norm}}} - 2\frac{DR(r,\mu)/DR_{\mathrm{norm}}}{RR(r,\mu)/RR_{\mathrm{norm}}} + 1$
- $\xi_i(r) = (1+\bar{\delta}_i)^2 \xi_{\mathrm{LS},i}(r) + \bar{\delta}_i^2$
- $\langle g_i \rangle = \frac{1}{w_{r,\mathrm{tot}}} \sum_{i=1}^{N_s} g_i w_{r,i}$
- $\sigma_L^2 = \left\langle \bar{\delta}_i^2 - \frac{P_{\mathrm{shot},i}}{V_{L,i}} \right\rangle$

Mocks in rspace

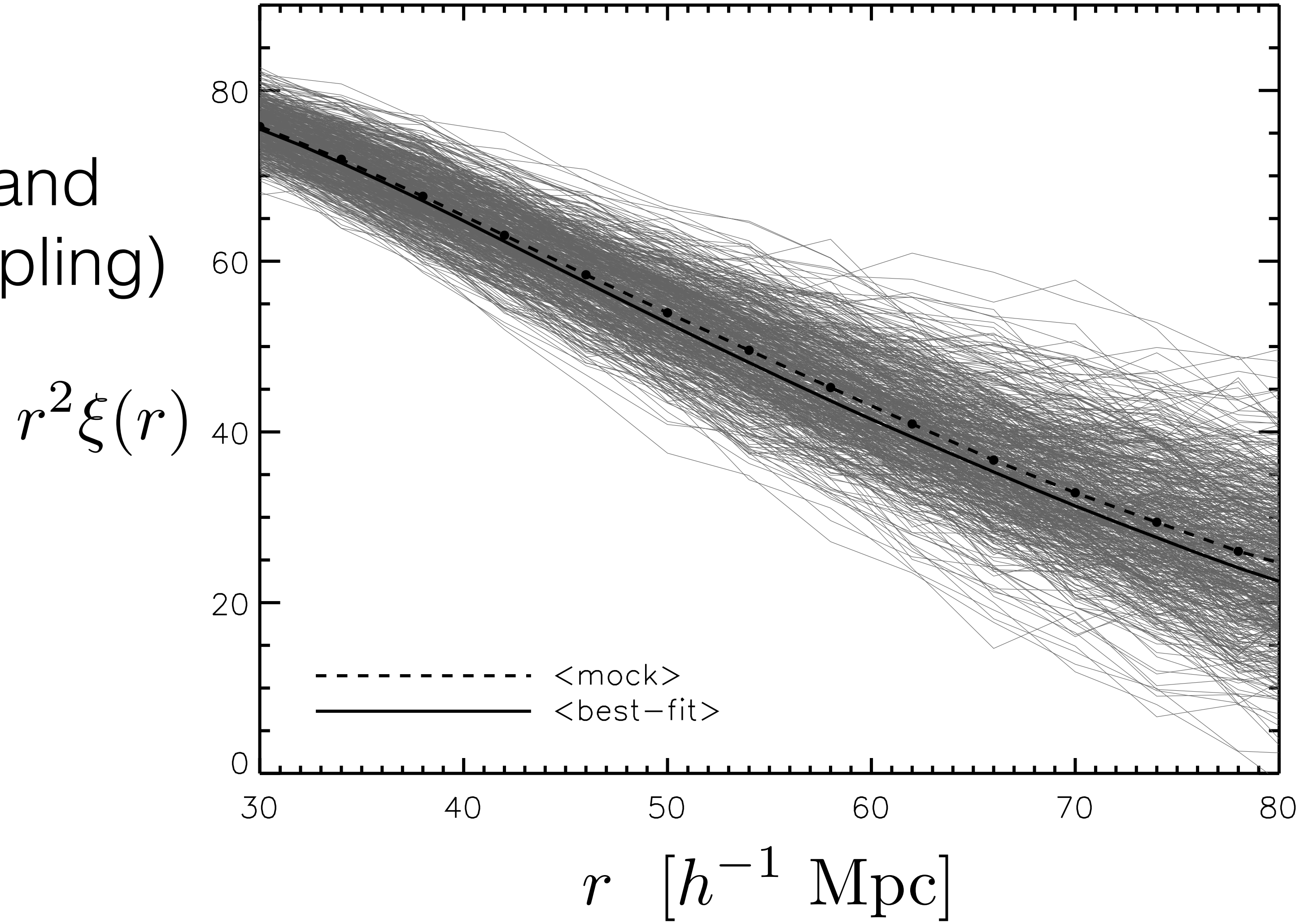
- Joint fit $i\zeta(r)/\sigma_L^2$ (for 2 sizes) and $\xi(r)$
- Models:

$$P_g(k) = b_1^2[P_l(k)e^{-k^2\sigma_v^2} + A_{\rm MC}P_{\rm MC}(k)]$$

$$\frac{i\zeta_g(r)}{\sigma_L^2} = \frac{b_1 i\zeta_{\rm SPT}(r)}{\sigma_{L,l}^2} + \frac{b_2 i\zeta_{b_2}(r)}{\sigma_{L,l}^2}$$

ξ(r) of mocks in rspace

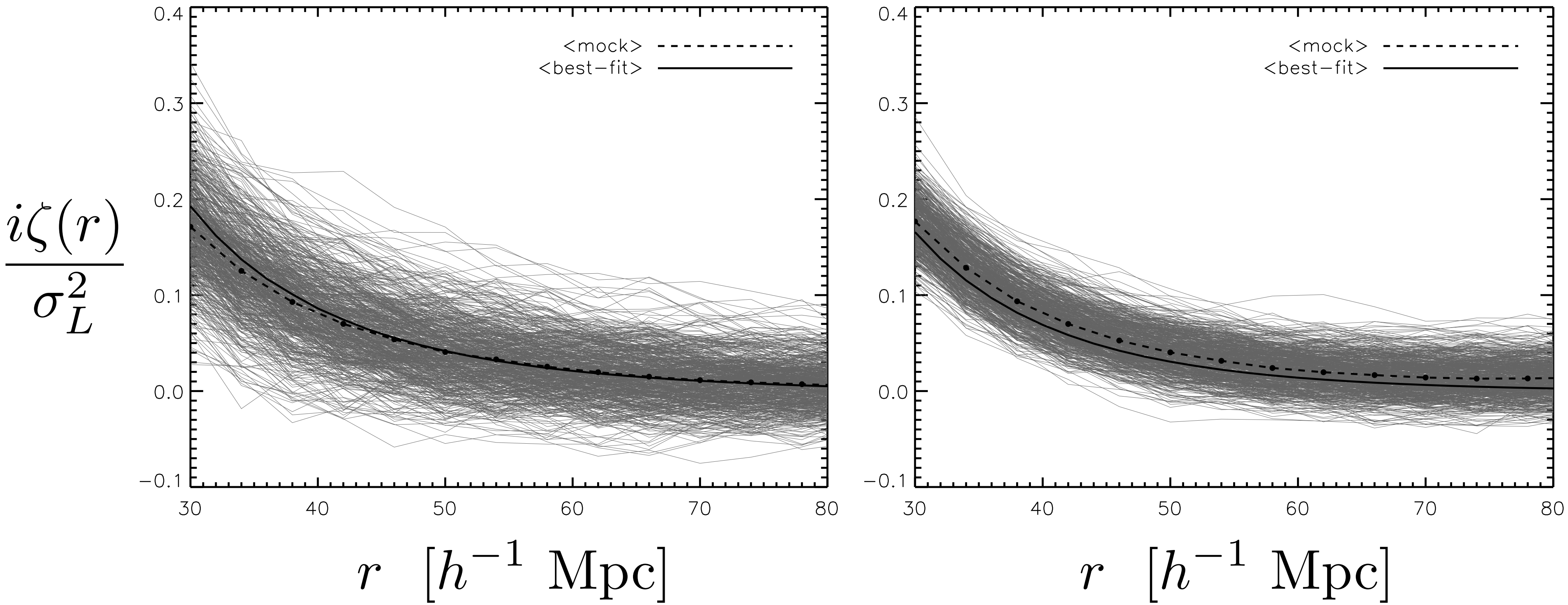
parameters:
b_1 (linear bias) and
A_{MC} (mode-coupling)



iζ(r) of mocks in rspace

parameters: b1 (linear bias) and b2 (non-linear bias)

220 h^{-1} Mpc 120 h^{-1} Mpc



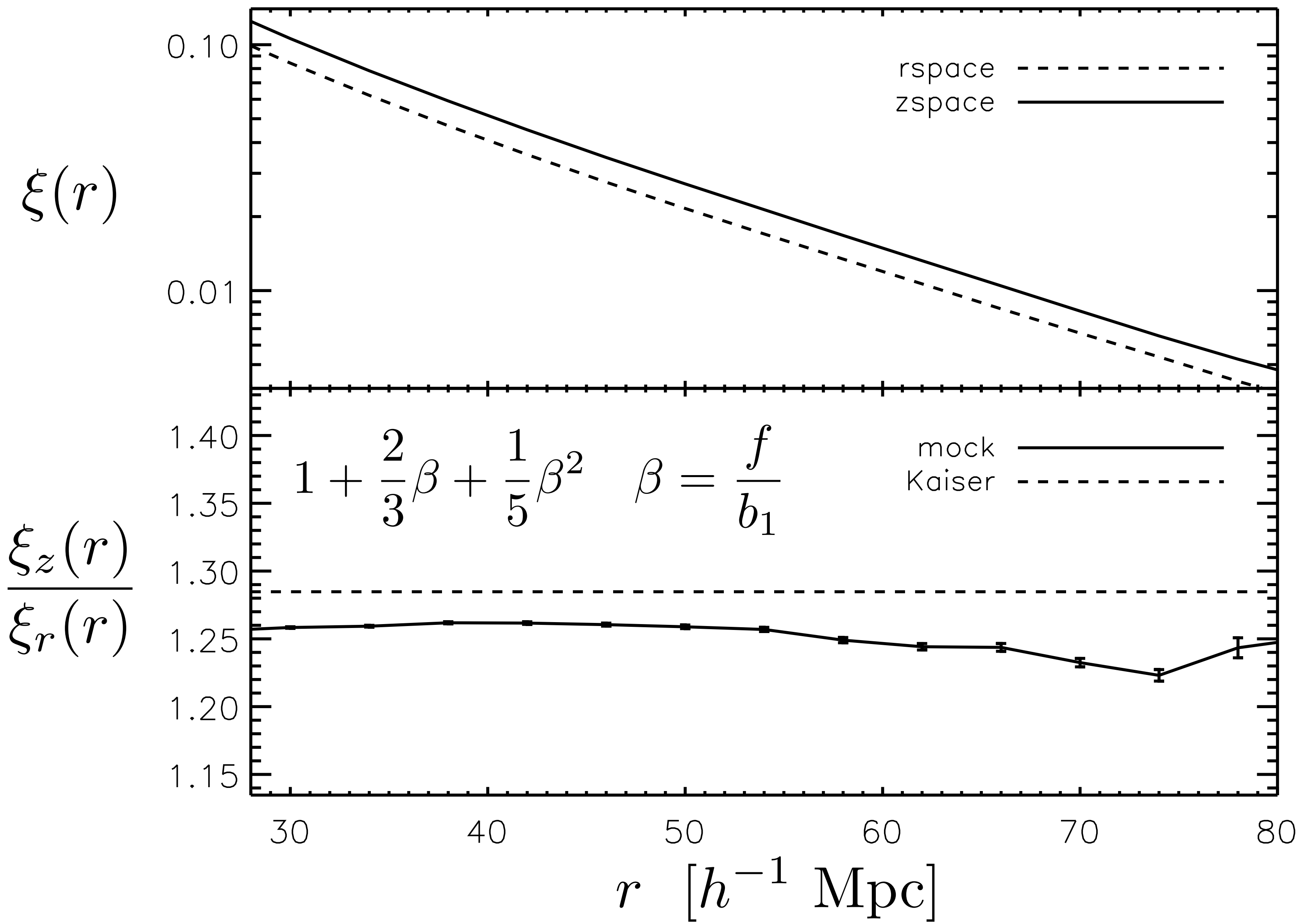
Mocks in zspace

- zspace to rspace ratio
- $\xi(r)$, σ_L^2: Kaiser effect $1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2 \quad \beta = \frac{f}{b_1}$
- $i\zeta(r)/\sigma_L^2$:

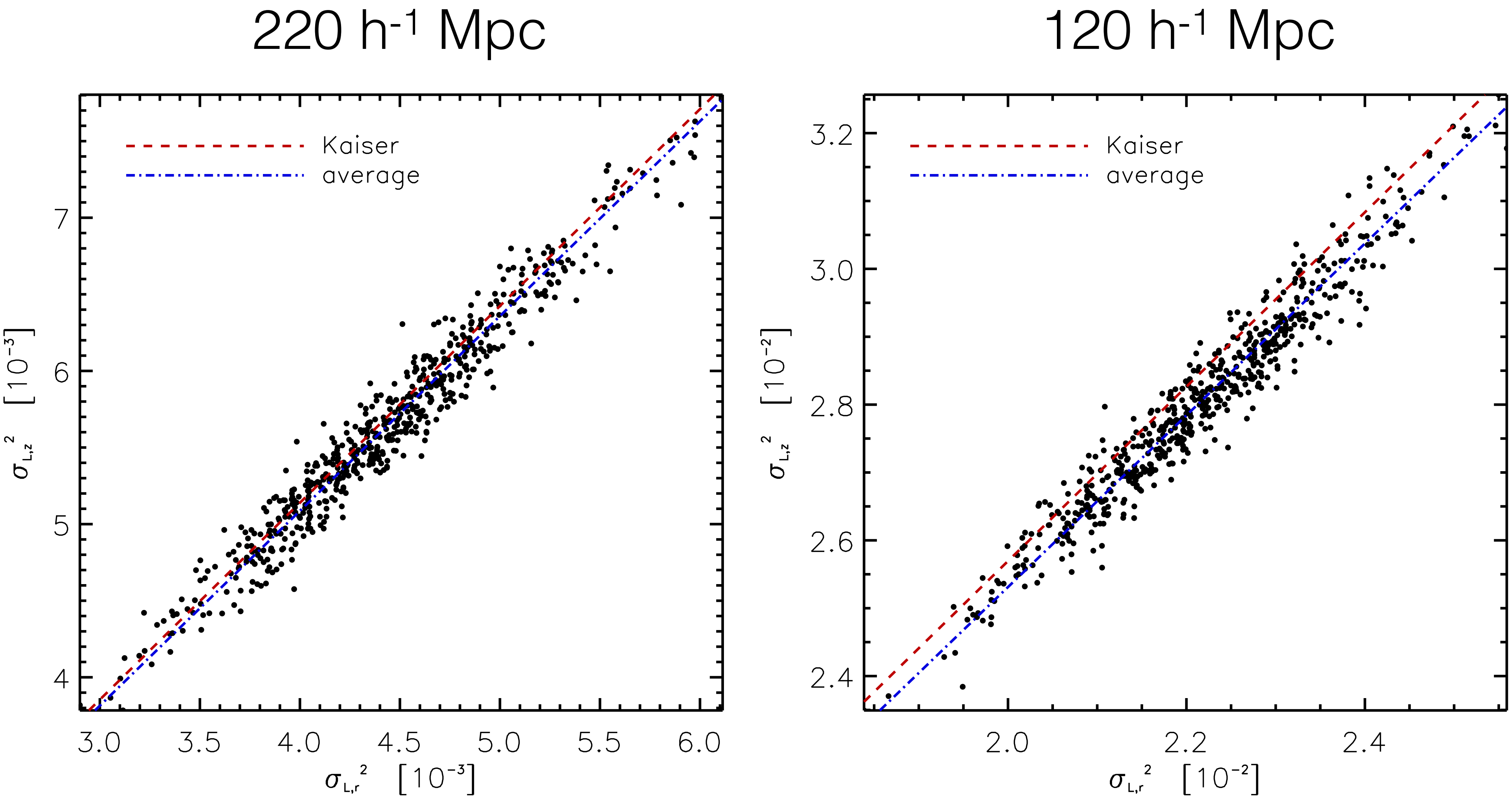
$$\frac{i\zeta_{\mathrm{SQ1}}(r) + i\zeta_{\mathrm{SQ2}}(r) + i\zeta_{\mathrm{NLB}}(r) + i\zeta_{\mathrm{FOG}}(r)}{b_1^3 i\zeta_{\mathrm{SQ1,1}}(r) + b_1^2 b_2 i\zeta_{\mathrm{NLB,1}}(r)} \quad \frac{1}{1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2}$$

$$i\zeta_{\mathrm{X}}(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3}\ iB_{\mathrm{X}}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}}$$

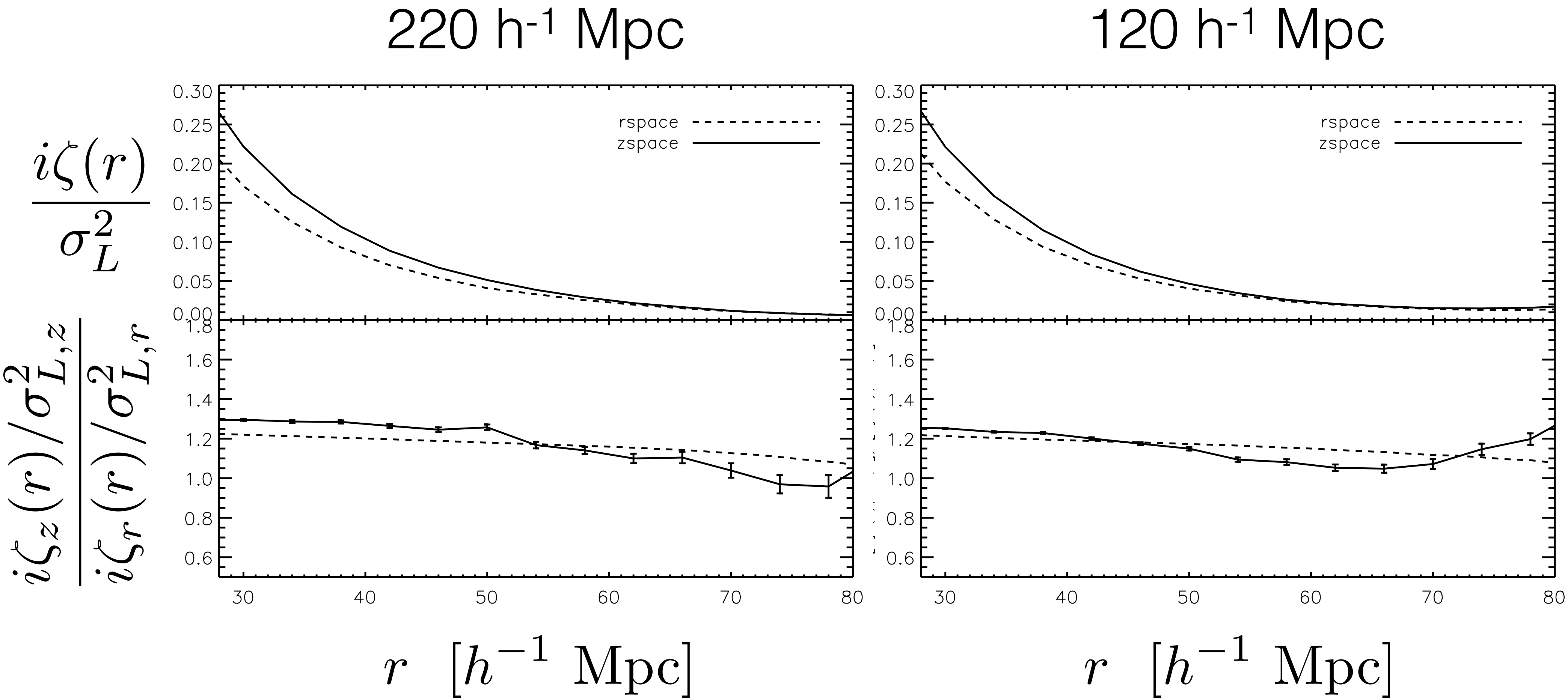
ξ(r) of mock in zspace



σ_L^2 of mock in zspace



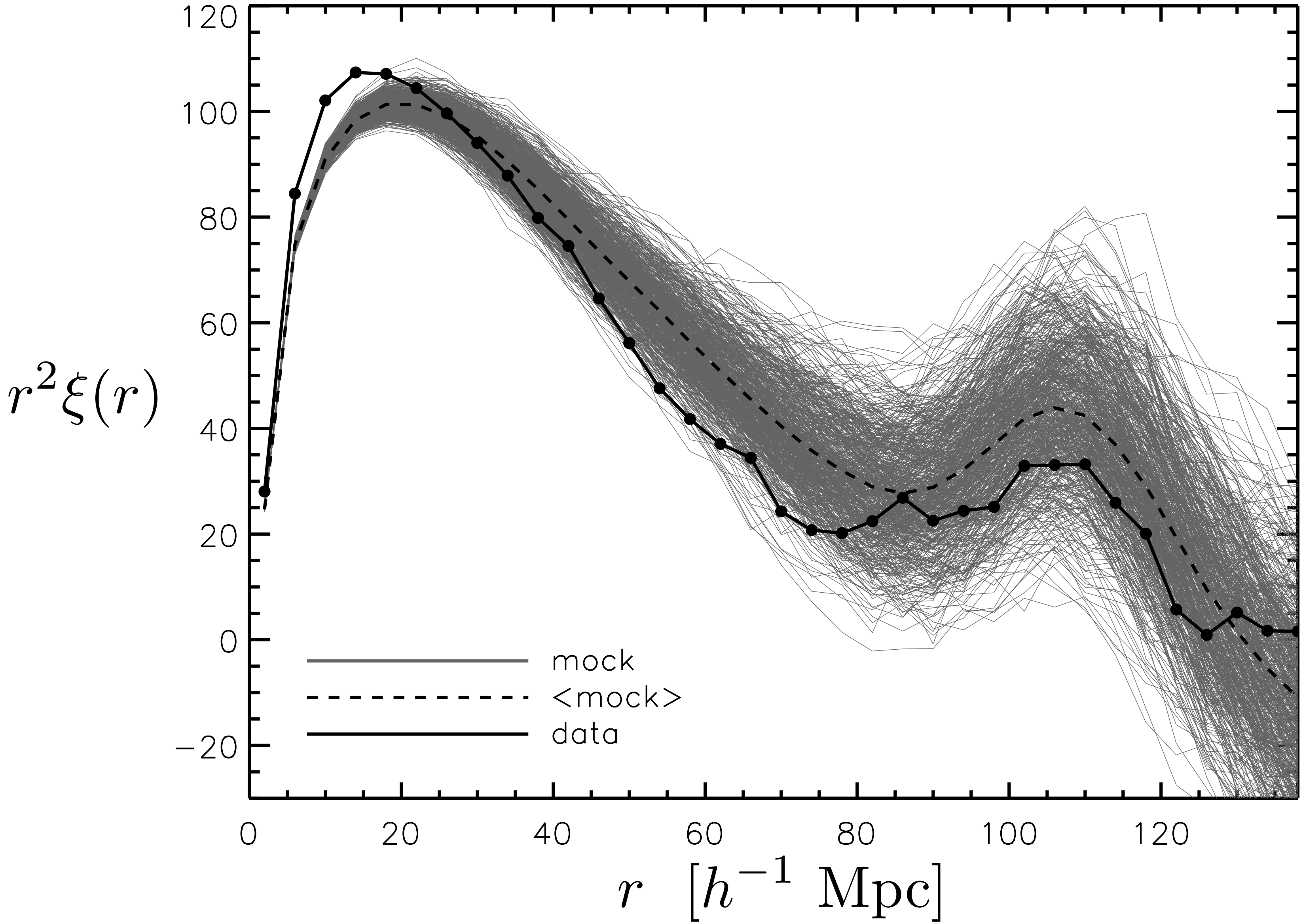
iζ(r) of mocks in zspace



SDSS-III BOSS DR10 CMASS sample

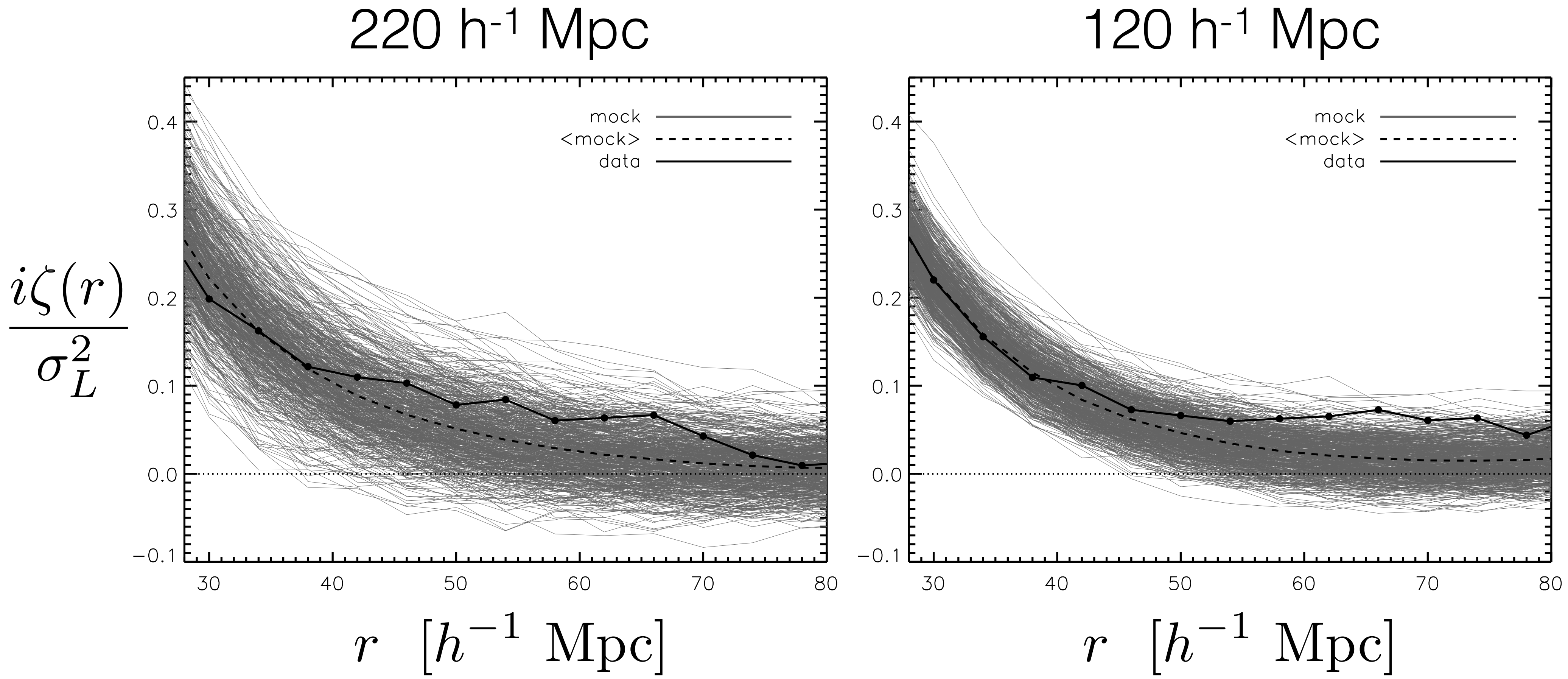
- data is always in zspace
- How well do the mock catalogs represent the data?

ξ(r) of data



120
100
80
60
40
20
0
−20
$r^2\xi(r)$
0
20
40
60
80
100
120
r [h^{-1} Mpc]
mock
<mock>
data

iζ(r) of data

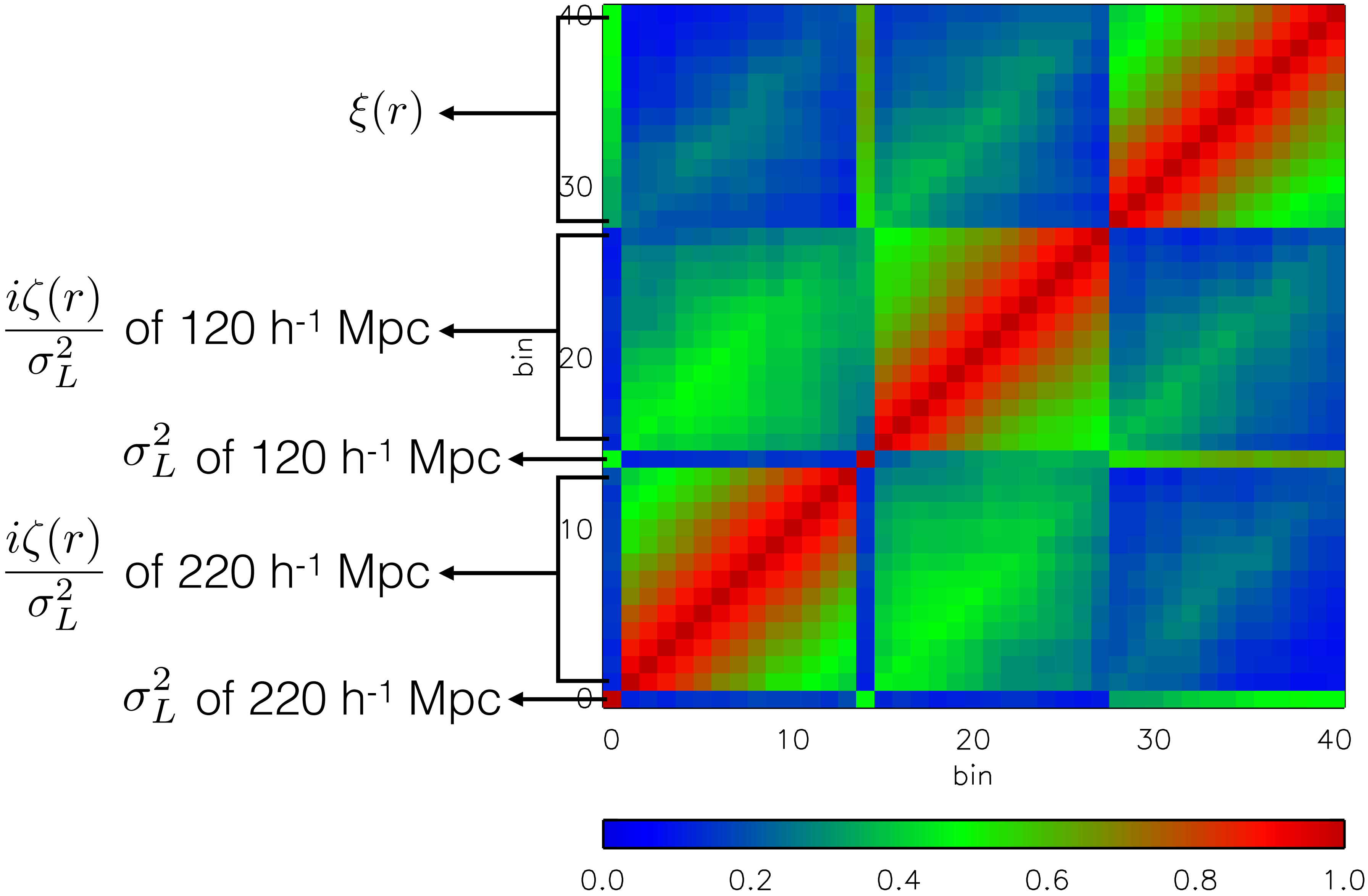


220 h-1 Mpc
120 h-1 Mpc
mock
<mock>
data
0.4
0.3
0.2
0.1
0.0
−0.1
30
40
50
60
70
80
$\frac{i\zeta(r)}{\sigma_L^2}$
$r\ [h^{-1}\ \mathrm{Mpc}]$

Fitting the amplitudes

- Models: mean of the mock catalogs
- Amplitudes: $i\zeta(r)/\sigma_L^2$, $\xi(r)$, σ_L^2
- Joint fit the three amplitudes

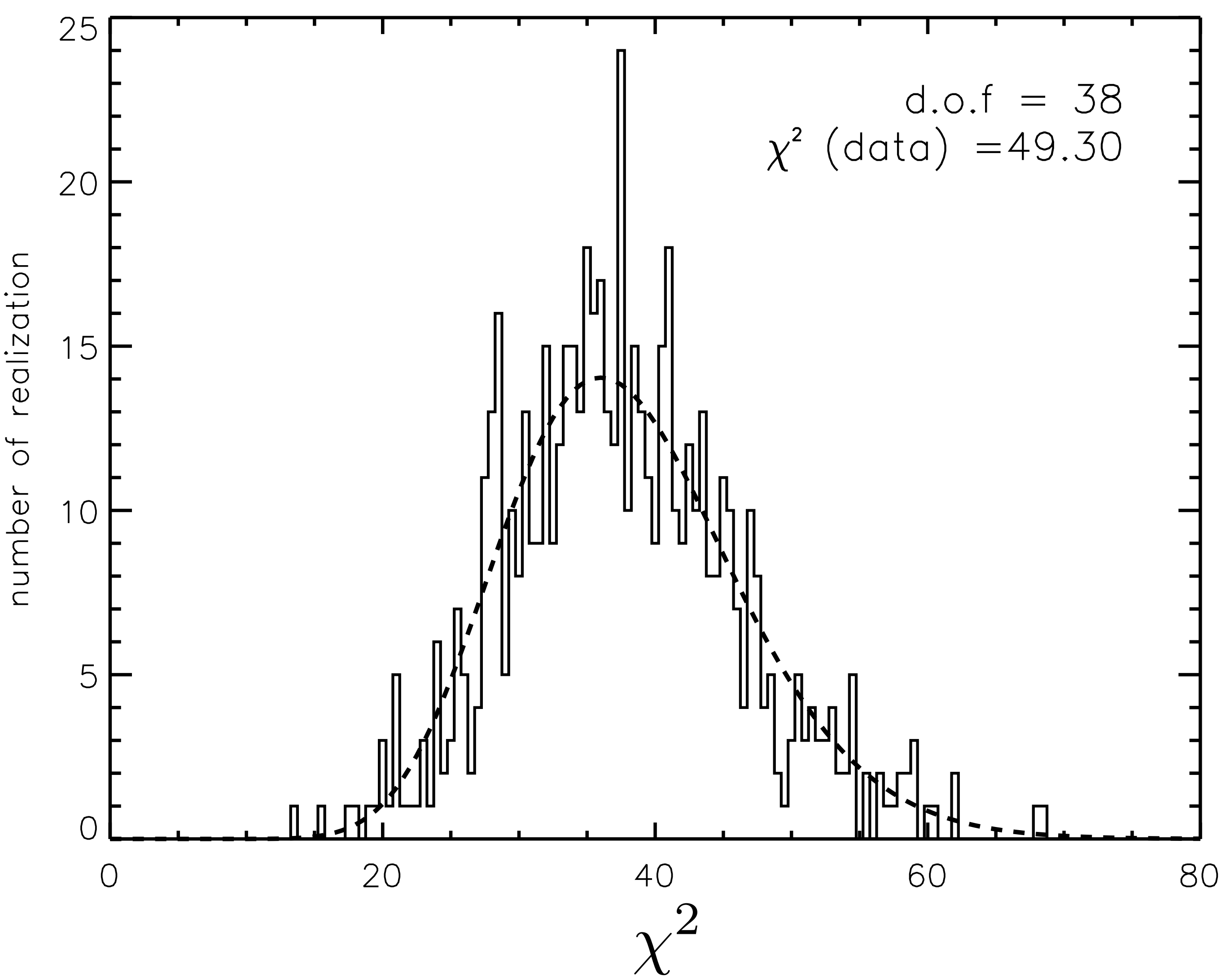
Covariance matrix



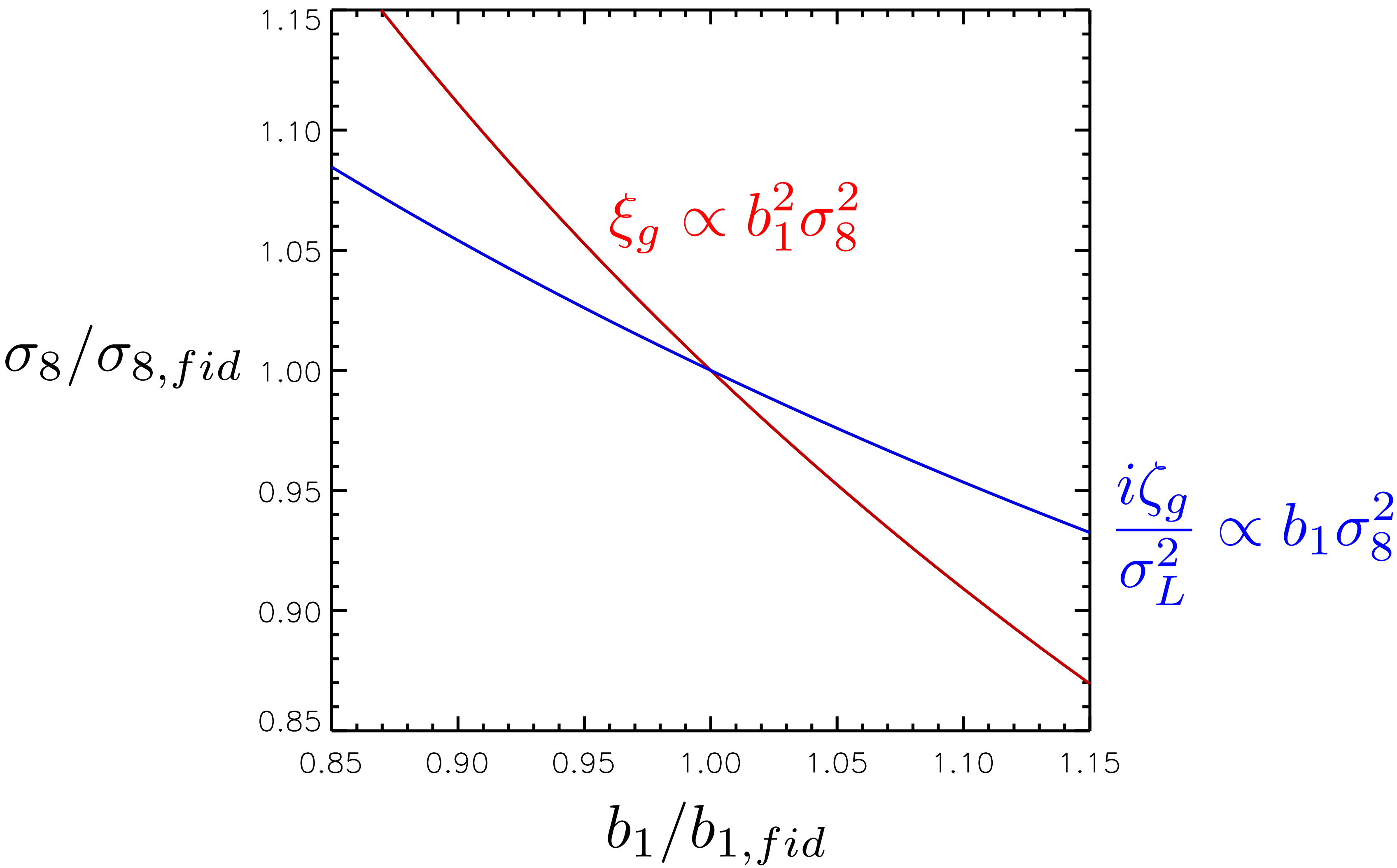
The fitted amplitudes

A1: $i\zeta(r)/\sigma_L^2$
A2: $\xi(r)$
A3: σ_L^2

var[A1]=0.108
var[A2]=0.030
var[A3]=0.038
corr[A1,A2]=0.199
corr[A1,A3]=0.021
corr[A2,A3]=0.631
data[A1]=0.812
data[A2]=1.008
data[A3]=1.092



What can we learn from the amplitudes?



Conclusion

- 3-point function contains extra information (on top of 2-point function) and so can be used to constrain the cosmological parameters.
- Position-dependent correlation function (or power spectrum) is a useful method to extract the squeezed-limit information.
- We are currently working on converting the measurements to the constrains on b_1, σ_8 , and f.