2HDM scalar potential and the role of symmetries

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Motivation

- Supersymmetry relies on two-Higgs-doublet structure.
- PQ symmetry can be imposed to rotate away the CPV term from QCD Lagrangian when there are two scalar doublets. This leads to axions. Simplest versions are ruled out.
- With one Higgs doublet it is not possible to generate BAU of sufficient size. 2HDM provides additional room.
- Natural extension keeping $\rho = 1$ at tree level.

$$\rho^{\rm tree} = \frac{\sum_{i=1}^{N} \left\{ T_i(T_i+1) - \frac{Y_i^2}{4} \right\} v_i}{\frac{1}{2} \sum_{i=1}^{N} Y_i^2 v_i}$$

The scalar potential

Parametrization 1 :

$$V = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right) + \frac{\beta_1}{2} \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{\beta_2}{2} \left(\Phi_2^\dagger \Phi_2 \right)^2$$

$$+ \beta_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) + \beta_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) + \left\{ \frac{\beta_5}{2} \left(\Phi_1^\dagger \Phi_2 \right)^2 + \text{h.c.} \right\} .$$

Parametrization 2 :

$$V = \lambda_1 \left(\Phi_1^\dagger \Phi_1 - \frac{v_1^2}{2} \right)^2 + \lambda_2 \left(\Phi_2^\dagger \Phi_2 - \frac{v_2^2}{2} \right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 - \frac{v_1^2 + v_2^2}{2} \right)^2$$

$$+ \lambda_4 \left((\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) - (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \right) + \lambda_5 \left(\text{Re}\, \Phi_1^\dagger \Phi_2 - \frac{v_1 v_2}{2} \right)^2 + \lambda_6 \left(\text{Im}\, \Phi_1^\dagger \Phi_2 \right)^2$$

- All potential parameters are real.
- There is a Z_2 symmetry in the potential ($\Phi_1 \to \Phi_1, \Phi_2 \to -\Phi_2$).
- m_{12}^2 and λ_5 break the Z_2 symmetry softly.

The two parametrizations

- Parametrization 1 is ‘more general’ than 2. The second parametrization assumes that both scalars receive vevs.
- The ‘inert doublet’ limit can be achieved in the first one, not in second. Putting $\beta_2 = \beta_3 = \beta_4 = \beta_5 = m_{12}^2 = 0$, $m_{22}^2 > 0$, and $v^2 = v_1^2 = -m_{11}^2/\beta_1$.
- SM potential: $V \sim \mu^2|\phi|^2 + \lambda|\phi|^4$, and $V' \sim \lambda\left(|\phi|^2 - v^2/2\right)^2$. They are not always equivalent.

If we assume that both scalars receive vevs, then

$$m_{11}^2 = -(\lambda_1 v_1^2 + \lambda_3 v^2)\ ;\ m_{22}^2 = -(\lambda_2 v_2^2 + \lambda_3 v^2)\ ;\ m_{12}^2 = \frac{\lambda_5}{2} v_1 v_2\ ;\ \beta_1 = 2(\lambda_1 + \lambda_3)\ ;$$

$$\beta_2 = 2(\lambda_2 + \lambda_3)\ ;\ \beta_3 = 2\lambda_3 + \lambda_4\ ;\ \beta_4 = \frac{\lambda_5 + \lambda_6}{2} - \lambda_4\ ;\ \beta_5 = \frac{\lambda_5 - \lambda_6}{2}\ .$$

Physical Eigenstates

We express the scalar doublets as

$$\Phi_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} w_i^+ \\ (h_i + v_i) + i z_i \end{pmatrix} .$$

$$\begin{pmatrix} \omega^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} w_1^\pm \\ w_2^\pm \end{pmatrix} , \quad m_{H+}^2 = \frac{\lambda_4}{2} v^2 .$$

$$\begin{pmatrix} \zeta \\ A \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} , \quad m_A^2 = \frac{\lambda_6}{2} v^2 .$$

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} .$$

Counting of parameters

$$
\begin{aligned}
\lambda_1 &= \frac{1}{2v^2\cos^2\beta}\left[m_H^2\cos^2\alpha + m_h^2\sin^2\alpha - \frac{\sin\alpha\cos\alpha}{\tan\beta}\left(m_H^2 - m_h^2\right)\right] - \frac{\lambda_5}{4}\left(\tan^2\beta - 1\right)\,,\\
\lambda_2 &= \frac{1}{2v^2\sin^2\beta}\left[m_h^2\cos^2\alpha + m_H^2\sin^2\alpha - \sin\alpha\cos\alpha\tan\beta\left(m_H^2 - m_h^2\right)\right] - \frac{\lambda_5}{4}\left(\cot^2\beta - 1\right)\\
\lambda_3 &= \frac{1}{2v^2}\frac{\sin\alpha\cos\alpha}{\sin\beta\cos\beta}\left(m_H^2 - m_h^2\right) - \frac{\lambda_5}{4}\,,\\
\lambda_4 &= \frac{2}{v^2}m_{H+}^2\,,\\
\lambda_6 &= \frac{2}{v^2}m_A^2\,.
\end{aligned}
$$

8 parameters: 6 lambdas, v_1, v_2 (or, $v = 246\ \mathrm{GeV}$, $\tan\beta$). All lambdas, except λ_5, can be traded for $m_h (= 125\ \mathrm{GeV})$, m_{H+}, m_H, m_A and α.

The Alignment limit

$$H^0 = \frac{1}{v}(v_1 h_1 + v_2 h_2)$$

has gauge couplings exactly as the SM Higgs boson and its orthogonal combination (R) does not have any RVV trilinear couplings. H^0 also mimics the SM Higgs in Yukawa sector.

$$\begin{pmatrix} H^0 \\ R \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} .$$

$$\begin{aligned} H &= \cos(\beta-\alpha) H^0 - \sin(\beta-\alpha) R\,, \\ h &= \sin(\beta-\alpha) H^0 + \cos(\beta-\alpha) R\,. \end{aligned}$$

If we want the lightest CP-even physical scalar h to posses SM-like couplings, we must set $\sin(\beta-\alpha) = 1$, which is the definition of the alignment limit.
Number of free parameters is then reduced by one.

Global U(1) symmetry

$$\Phi_1 \to \Phi_1 \,, \qquad \Phi_2 \to e^{i\theta}\Phi_2 \,.$$

On the quartic terms, this symmetry is realized by putting

$$\lambda_5 = \lambda_6 \,,$$

which means that the potential now reads

$$\begin{aligned} V \quad = \quad & \lambda_1 \left(\Phi_1^\dagger \Phi_1 - \frac{v_1^2}{2} \right)^2 + \lambda_2 \left(\Phi_2^\dagger \Phi_2 - \frac{v_2^2}{2} \right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 - \frac{v_1^2 + v_2^2}{2} \right)^2 \\ & + \lambda_4 \left((\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) - (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \right) + \lambda_5 \left| \Phi_1^\dagger \Phi_2 - \frac{v_1 v_2}{2} \right|^2 \,. \end{aligned}$$

$v_2 \neq 0$ spontaneously breaks the global symmetry. The λ_5 terms avoids the appearance of massless pseudoscalar by explicitly breaking the U(1) symmetry. The psedo-scalar can be light.

Light A or $H, H^\pm$ can be perfectly accommodated in ‘alignment limit’! This is often called ‘decoupling limit’ - be alert!!

Stability and Unitarity limits

Conditions for the potential to be bounded from below:

$$\lambda_1 + \lambda_3 > 0\,, \quad \lambda_2 + \lambda_3 > 0\,,$$
$$2\lambda_3 + \lambda_4 + 2\sqrt{(\lambda_1 + \lambda_3)(\lambda_2 + \lambda_3)} > 0\,,$$
$$2\lambda_3 + \lambda_5 + 2\sqrt{(\lambda_1 + \lambda_3)(\lambda_2 + \lambda_3)} > 0\,.$$

Upper bounds from perturbative unitarity: Scattering amplitudes involving longitudinal gauge bosons and Higgs bosons comprise the elements of an S-matrix, having 2-particle states as rows and columns. The eigenvalues are restricted by $|a_0| < 1$.

$$\Big|2\lambda_3 - \lambda_4 + 2\lambda_5\Big| \leq 16\pi\,, \quad \Big|2\lambda_3 + \lambda_4\Big| \leq 16\pi\,,$$
$$\Big|2\lambda_3 + \lambda_5\Big| \leq 16\pi\,, \quad \Big|2\lambda_3 + 2\lambda_4 - \lambda_5\Big| \leq 16\pi\,,$$
$$\Big|3(\lambda_1 + \lambda_2 + 2\lambda_3) \pm \sqrt{9(\lambda_1 - \lambda_2)^2 + (4\lambda_3 + \lambda_4 + \lambda_5)^2}\Big| \leq 16\pi\,,$$
$$\Big|(\lambda_1 + \lambda_2 + 2\lambda_3) \pm \sqrt{(\lambda_1 - \lambda_2)^2 + (\lambda_4 - \lambda_5)^2}\Big| \leq 16\pi\,,$$
$$\Big|(\lambda_1 + \lambda_2 + 2\lambda_3) \pm (\lambda_1 - \lambda_2)\Big| \leq 16\pi\,.$$

Constraints

Put the following constraints:

- $\beta - \alpha = \pi/2$ and $\lambda_5 = \lambda_6$
- $m_h = 125$ GeV
- $m_{H+} > 100$ GeV, which is a rough lower bound from direct searches

On top of it put the constraints from oblique electroweak T parameter:

$$T = \frac{1}{16\pi \sin^2\theta_w M_W^2}\left[F(m_{H+}^2, m_H^2) + F(m_{H+}^2, m_A^2) - F(m_H^2, m_A^2)\right],$$

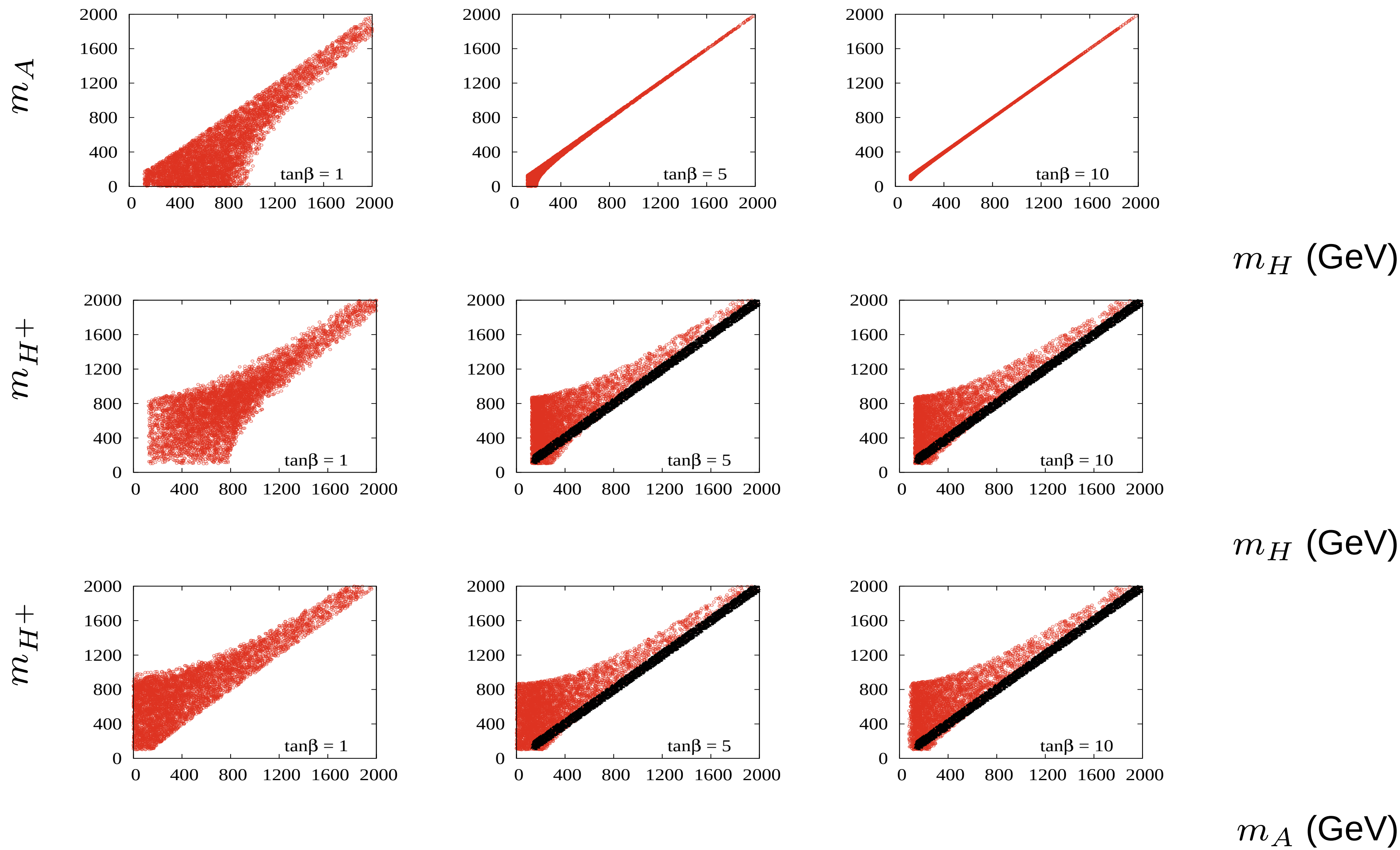
with

$$F(x,y) = \frac{x+y}{2} - \frac{xy}{x-y}\, \ln(x/y)\,.$$

The new physics contribution to the T-parameter as

$$T = 0.05 \pm 0.12\,.$$

Plots of mass spectra



Salient features

- It follows from unitarity and stability

$$0 \le (m_H^2 - m_A^2)(\tan^2\beta + \cot^2\beta) + 2m_h^2 \le \frac{32\pi v^2}{3} \,.$$

For $\tan\beta$ away from unity, H and A are almost degenerate.

- There is a similar correlation between m_H and m_{H+}, but this time without any dependence on $\tan\beta$.

$$\left| 2m_{H+}^2 - m_H^2 - m_A^2 + m_h^2 \right| \le 16\pi v^2 \,.$$

- The unitarity conditions apply on the difference of their squared masses. Any individual mass can be arbitrarily large. This conclusion crucially depends on the existence of a U(1) symmetry of the potential. When the symmetry of the potential is only a discrete Z_2, considerations of unitarity do restrict the individual non-standard masses.
- The constraints from the T-parameter are stronger than that from unitarity and stability.

Diphoton decay width

Only charged scalars provide additional contributions.

$$\mu_{\gamma\gamma} \equiv \frac{\sigma(pp \to h)}{\sigma^{\rm SM}(pp \to h)} \cdot \frac{\text{BR}(h \to \gamma\gamma)}{\text{BR}^{\rm SM}(h \to \gamma\gamma)} = \frac{\Gamma(h \to \gamma\gamma)}{\Gamma^{\rm SM}(h \to \gamma\gamma)} .$$

Parametrize the coupling of h to the charged scalars as

$$g_{hH^+H^-} \equiv \kappa \frac{g m_{H+}^2}{M_W} ,$$

Then

$$\mu_{\gamma\gamma} = \frac{\left|\mathcal{A}_W + \frac{4}{3}\mathcal{A}_t + \kappa \mathcal{A}_{H+}\right|^2}{\left|\mathcal{A}_W + \frac{4}{3}\mathcal{A}_t\right|^2} .$$

If κ saturates to some finite value in the limit when the charged scalar is too heavy, the effect will not decouple (as $\mathcal{A}_{H+}$ saturates to $\frac{1}{3}$).

Decoupling vs nondecoupling

When the symmetry is Z_2

$$\kappa = -\frac{1}{m_{H+}^2}\left(m_{H+}^2 + \frac{m_h^2}{2} - \frac{\lambda_5 v^2}{2}\right)$$

Decoupling can be achieved by tuning $m_{H+}^2 \simeq \lambda_5 v^2/2$.
When the symmetry is U(1)

$$\kappa = -\frac{1}{m_{H+}^2}\left(m_{H+}^2 - m_A^2 + \frac{m_h^2}{2}\right)$$

Unitarity and T-parameter together restrict the numerator ensuring decoupling.
The key point is that the soft global symmetry breaking parameter λ_5 is now related to a physical scalar mass, and mass square differences are constrained. Thus, no tuning is involved.
For decoupling, there must be a non-SSB component in the mass term of the heavy particle.
When $\lambda_5 = 0$, nondecoupling would restrict the number of additional scalars, strictly when both scalars receive vevs.

3HDM scalar potential

S_3 or A_4 symmetric flavor models are typical examples which employ three Higgs doublets.

$$\begin{aligned}
V_{\rm 3HDM}^{S_3} \quad = \quad & -\mu_1^2(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2) - \mu_3^2\phi_3^\dagger\phi_3 \\
& +\lambda_1(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2)^2 + \lambda_2(\phi_1^\dagger\phi_2 - \phi_2^\dagger\phi_1)^2 \\
& +\lambda_3\left\{(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1)^2 + (\phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2)^2\right\} \\
& +\lambda_4\left\{(\phi_3^\dagger\phi_1)(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1) + (\phi_3^\dagger\phi_2)(\phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2) + \text{h.c.}\right\} \\
& +\lambda_5(\phi_3^\dagger\phi_3)(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2) + \lambda_6\left\{(\phi_3^\dagger\phi_1)(\phi_1^\dagger\phi_3) + (\phi_3^\dagger\phi_2)(\phi_2^\dagger\phi_3)\right\} \\
& +\lambda_7\left\{(\phi_3^\dagger\phi_1)(\phi_3^\dagger\phi_1) + (\phi_3^\dagger\phi_2)(\phi_3^\dagger\phi_2) + \text{h.c.}\right\} + \lambda_8(\phi_3^\dagger\phi_3)^2 \,.
\end{aligned}$$

There will be two pairs of charged scalars:

$$\kappa_i = -\left(1 + \frac{m_h^2}{2m_{i+}^2}\right) \;\; \text{for } i = 1, 2\,.$$

Global symmetry for S_3 potential

Assume a global SO(2) symmetry, then $\lambda_4 = 0$, and introduce a soft breaking term $(-\mu_{12}^2 \phi_1^\dagger \phi_2)$. Then

$$m_{h'}^2 = 2\mu_{12}^2 \,,$$

where h', H and $h(= 125)$ are the three CP even Higgses. h' coupling is peculiar as it does not have any $h'VV$ triliear coupling.

$$\begin{aligned}\kappa_1 &= -\frac{1}{m_{1+}^2}\left(m_{1+}^2 - m_{h'}^2 + \frac{m_h^2}{2}\right) \,, \\ \kappa_2 &= -\left(1 + \frac{m_h^2}{2m_{2+}^2}\right) \,.\end{aligned}$$

Note that $(|m_{1+}^2 - m_{h'}^2|)$ is constrained from unitarity.

With an extended global symmetry SO(2)$\times$U(1), together with an extra soft breaking parameter which is related to m_{A2}, decoupling in κ_2 can be ensured.

Outlook

- With increasing LHC Higgs data we are gradually pushed to Alignment limit. But this can still accommodate light (rather not so heavy) additional scalars.
- Symmetries of the scalar potential and their soft breaking terms play crucial role, especially in ensuring decoupling.
- Flavor symmetries also decide the scalar structure. New scalars with exotic behavior are present in 3HDM (S_3, S_4, A_4, $\Delta(27)$, $\cdots$).
- Validity up to high scale puts a constraint on $\tan\beta$.