Suppressing the Lorentz violations in the matter sector

– A class of extended Hořava gravity –

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Introduction

- In standard QFT, for an interaction $\mathcal{L}_{\rm int} = \lambda\,\mathcal{O}$:
  $[\lambda] \geq 0 \Rightarrow$ superficial renormalizability.
- For GR, $[\sqrt{G_N}] = -1$
  $\Rightarrow$ not renormalizable as a perturbative QFT.
- Modifying GR with high order curvature terms

$$\delta\mathcal{L} = \alpha\, R_{\mu\nu}R^{\mu\nu} + \beta R^2$$

  Modified propagator has an improved UV behavior

$$\frac{1}{k^2 - \frac{k^4}{M^2}} = \frac{1}{k^2} - \frac{1}{k^2 - M^2}$$

  Stelle 1977

  Relativistic: ∂_i^4 improves UV $\Longleftrightarrow$ ∂_t^4 compromises unitarity.
- Anisotropic scaling in UV: $t \to b^{-z}t, \quad \vec{x} \to b^{-1}\vec{x}$.
- $\partial_t^2 \leftrightarrow \partial_i^{2z}$.
- Cost: violation of local Lorentz invariance.

Lifshitz scalar analogy
Dimensional counting

- The action for a free Lifshitz scalar in $D+1$, with critical exponent z:

$$S_{\text{free}} = \int dt\, d^D x \left[\dot{\phi}^2 - \phi\,(-\triangle)^z\,\phi\right] \Leftarrow \triangle \equiv \partial_i \partial^i$$

- Field's dimension. From $[S] = 0$,

$$\underbrace{-z}_{dt}\ \underbrace{-D}_{d^D x} + \overbrace{2\,z}^{\partial} + \overbrace{2\,[\phi]}^{\phi^2} = 0 \Rightarrow [\phi] = \frac{D-z}{2}$$

- Interaction $\lambda\,\phi^n$: for $z \geq D$, we have $[\lambda] > 0,\ \forall n > 0$.
- Interaction $\lambda\,(\partial_i^{2z}, \phi^n)$: closer to gravity

$$-D - z + [\lambda] + 2\,z + n\,[\phi] = 0 \Longrightarrow [\lambda] = \frac{(n-2)(z-D)}{2}$$

Again, if $z \geq D$, we have $[\lambda] \geq 0,\ \forall n > 2$.

Generalization to gravity: Hořava's theory [Hořava '09]

Symmetry

- Momentum dimensions from scaling: $[x] = -1$, $[t] = -z$.
- A compatible symmetry: foliation-preserving diffeos (FDiff)

$$t \to t'(t) \qquad \vec{x} \to \vec{x}'(t, \vec{x})$$

- ADM decomposition provides a natural parametrization

$$ds^2 = -N^2\, c^2 dt^2 + g_{ij} \left(dx^i + N^i dt\right) \left(dx^j + N^j dt\right)$$

Building blocks ($z = D$)

$$\begin{array}{rcccl} [K_{ij}] & = & z & \longleftrightarrow & \dot{\phi} \\ [D_i] & = & 1 & \longleftrightarrow & \partial_i \\ [a_i] & = & 1 & \longleftrightarrow & \partial_i \phi \\ [R_{ij}] & = & 2 & \longleftrightarrow & \partial_i \partial_j \phi \end{array}$$

$(a_i \equiv \partial_i \log N)$

Action for Hořava gravity

- $z = D = 3$ Minimal model:

$$\mathcal{L}_{HG} = K_{ij}K^{ij} - \lambda K^2 + 2\alpha\, a_i a^i + \beta R + \frac{1}{M_*^2}\mathcal{L}_4 + \frac{1}{M_*^4}\mathcal{L}_6$$

with

$$\mathcal{L}_4 = \alpha_1 R D_i a^i + \alpha_2 D_i a_j D^i a^j + \beta_1 R_{ij} R^{ij} + \beta_2 R^2 + \dots ,$$
$$\mathcal{L}_6 = \alpha_3 D_i D^i R D_j a^j + \alpha_4 D^2 a_i D^2 a^i + \beta_3 D_i R_{jk} D^i R^{jk} + \beta_4 D_i R D^i R + \dots .$$

Blas, Pujolas, Sibiryakov 2009-2010

- $t \to t'(t)$ not enough to remove 1 dof $\Rightarrow$ scalar graviton
- Dispersion relation for tensor modes

$$\omega^2 = \beta k^2 - \beta_1 \frac{k^4}{M_*^2} - \beta_3 \frac{k^6}{M_*^4}$$

- Scalar mode is more subtle, but in the UV it goes $\omega^2 \propto \frac{k^6}{M_*^4}$.
- At low energies ($k \ll M_*$), where higher derivative terms are suppressed, ~GR is recovered for $\lambda = \beta = 1$, $\alpha = 0$.

Constraints on the IR theory

Theoretical consistency

- Scalar kinetic term $= \frac{3\lambda-1}{\lambda-1} > 0 \qquad \Rightarrow \lambda < 1/3$ or $\lambda > 1$.
- Dispersion relations in IR

$$\omega^2_{IR,t} = \beta k^2 \,, \qquad \omega^2_{IR,s} = \frac{\beta(\beta-\alpha)(\lambda-1)}{\alpha(3\,\lambda-1)} k^2$$

- To avoid gradient instability $\omega^2_{IR} > 0 \quad \Rightarrow 0 < \alpha < \beta$.
- Problem with IR theory: high order terms with ∂_t^2 violate the perturbative expansion, power-counting under suspicion. For $\alpha \sim \lambda - 1$, the non-perturbativity scale is $M_{NP} = \sqrt{\alpha} M_p$.
- However, if high D operators contribute at a lower energy, i.e. $M_* < M_{NP}$, the “strong coupling” is beyond reach. More on this later.

Blas, Pujolas, Sibiryakov 2010

Constraints on the IR theory
Observational constraints

- Stringent constraint comes from the PPN parameters α_1 and $\alpha_2 \Leftarrow$ preferred-frame effects.

$$|\alpha_1| \lesssim 10^{-4}\,, \qquad |\alpha_2| \lesssim 10^{-7}\,.$$

Will 2006

- Barring any special cancellations, the constraint gives

$$\alpha\,,\ \beta - 1\,,\ \lambda - 1 \lesssim 10^{-7} \div 10^{-6}$$

Blas, Pujolas, Sibiryakov 2010

- Non-perturbativity scale $M_{NP} = \sqrt{\alpha} M_p \lesssim 10^{16}\mathrm{GeV}$
- Requiring that the theory is perturbative at all scales imposes $M_* < M_{NP} < 10^{16}\mathrm{GeV}$.

Constraints on high dim. operators

- Theoretical considerations and solar system tests imply $M_* \lesssim 10^{16}\mathrm{GeV}$.
- Using only gravitational bound, from sub-mm tests, $M_* \gtrsim 10^{-2}\mathrm{eV}$.
- Enormous window for M_*.

LV in the matter sector

Constraints on maximum attainable velocity for different species
e.g. Coleman, Glashow 1998

- Cherenkov radiation bound: $c_p - c_\gamma < 10^{-23}$
- Frame of CMB: $|c_m - c_\gamma| < 6 \times 10^{-22}$
- Neutrino oscillations: $|c' - c|_{\nu_e \nu_\mu} < 6 \times 10^{-22}$
- Radiative muon decay: $|c' - c|_{e\mu} < 4 \times 10^{-21}$
- Neutral kaons: $|c_{K_L} - c_{K_R}| < 3 \times 10^{-21}$.

- Planck scale preferred frame $\Rightarrow$ LV at low energies $\sim 1\%$
  Collins, Perez, Sudarsky, Urrutia, Vucetich 2004
- Concrete example for multiple Lifshitz fields: 1–loop correction to δc^2. Although $\delta c^2 = 0$ can be an attractive IR fixed point, flow is too slow. Unnaturally strong fine-tuning unavoidable.
  Iengo, Russo, Serone 2009

Constraints on higher order operators

- Even if matter sector SM, graviton loops still generate LV.
- A symmetry to prevent lowest order LV terms? e.g. SUSY, but extension to Hořava gravity highly non-trivial.

Groot-Nibbelink, Pospelov 2005; Xue 1010; Redigolo 2012; Pujolas, Sibiryakov 2012

Constraints from higher order operators

- Assume lowest order LV operators absent (e.g. fine tuned).
- Matter dispersion relation will still get high order modification above some scale $M_{*,m}$, e.g.

$$E^2 = m^2 + p^2 + \frac{p^4}{M_{*,m}^2}$$

- Synchrotron radiation constraints from the Crab nebula $M_{*,m} \gtrsim 2 \times 10^{16}\mathrm{GeV}$

Liberati, Maccione, Sotiriou 2012

- For a universal LV scale $M_{*,m} \sim M_*$, the bound is in conflict with the allowed region for M_*.

Scale separation mechanism [Pospelov, Shang '10]

- Gravity: HG, $z = D = 3$. Matter: No LV.
- Feedback of LV from graviton loops $f(M_*/M_p)$. LV in the matter sector under control if $M_* \ll M_p$. [Reminder: M_* has a vast allowed range.]
- 1-loop corrections to matter propagator:

$$c_v^2 - c_s^2 = (\dots)\frac{M_*^2}{M_p^2}\log\frac{\Lambda_{\mathrm{UV}}^2}{M_*^2} + (\dots)\frac{\Lambda_{UV}^2}{M_p^2}$$

- 2nd term diverges⇒Naturalness problem. From vector graviton loops.
- Vector part of HG = Vector part of GR. Propagator $\sim \frac{1}{\vec{k}^2}$.

Resolution, also from [Pospelov, Shang '10]

- New term $\frac{1}{M_*^2} D^i K_{ik} D_j K^{jk} \Longrightarrow$ 2 time, 2 space derivatives.
- Scalar & Tensor still $\omega_{UV}^2 \sim k^6$. Vector propagator $\frac{1}{\vec{k}^4}$, which is sufficient.
- The degree of non-universality of speeds:

$$c_v^2 - c_s^2 = (\dots)\frac{M_*^2}{M_p^2}\log\frac{\Lambda_{\mathrm{UV}}^2}{M_*^2}$$

- Other *DKDK* terms? Generically S&T have $\omega^2 \sim k^4$. Colombo, AEG, Sotiriou '14
- Does this imply non-renormalizable? If tuned to have $\omega^2 \sim k^6$, is tuning stable? **Is it even necessary?**

Using Lifshitz scalar as a proxy
[Colombo, AEG, Sotiriou '15]

- The Lifshitz analogue with mixed derivatives

$$S = \int dt\, d^D x \left[\dot\phi(-\triangle)^y \dot\phi - \phi(-\triangle)^z \phi + \lambda(\partial_t^{p_t}, \partial_i^{p_x}, \phi^n)\right]$$

- Scaling: $t \to b^{-m} t$, $\vec{x} \to b^{-1}\vec{x}$.
- $\partial_i^{2y}\partial_t^2 \leftrightarrow \partial_i^{2z} \implies m + y = z$.
- We consider a diagram with L loops, V vertices, I/E internal/external lines.
- Λ_{UV} contributions:

$$\begin{aligned} \text{Loops}: &\quad \int d\omega\, d^D k &\longrightarrow&\quad \Lambda_{UV}^{m+D} \\ \text{Internal}: &\quad \frac{1}{k^2\omega^2 - k^{2z}} &\longrightarrow&\quad \Lambda_{UV}^{-2z} \\ \text{Vertices}: &\quad \partial_i^p &\longrightarrow&\quad \Lambda_{UV}^{p} \end{aligned}$$

where $p \equiv m\, p_t + p_x$.

Using Lifshitz scalar as a proxy

Superficial degree of divergence–Limitations of dimensional counting

- Collecting these, degree of divergence $(\Lambda_{UV})^{\delta}$:

$$\delta \leq (D + m)L - 2\,z\,I + p\,V$$

- Identities: $L - I + V = 1\,, \qquad n\,V = E + 2\,I$
- Eliminating L and I, we get the familiar relation:

$$\delta \leq D + m - [\phi]\,E - [\lambda]\,V$$

- Dimensional counting works if $[\phi] \geq 0$, so that $[\lambda] > 0$ is sufficient for p-c renormalizability.
- However, for $[\phi] < 0$, it is not enough to have $[\lambda] > 0$, the relation is opaque. However, it demonstrates that dimensional counting assumes positive dimensional fields!
- Also relevant for standard Hořava theory (no mixed derivatives $y = 0$). Only the minimal theory has $[\phi] = 0$.

Using Lifshitz scalar as a proxy

Superficial degree of divergence–# of derivatives as a renormalizability criterion

- A more useful relation can be found by reusing the identities to eliminate E and I

$$\delta \leq 2\,z + 2\,[\phi]\,L - (2z - p)\,V$$

- For $[\phi] \leq 0$, vertex term determines degree of divergence.
- For interaction terms with weighted derivatives $p \leq 2z$, $\Longrightarrow \delta$ bounded from above by $2\,z$. $\Rightarrow$ p-c renormalizable.
- For $p > 2\,z$, for a given loop order, there are always diagrams with large enough vertices.
- Therefore, renormalizability condition is $[\phi] \leq 0$, or

$$\boxed{z + y \geq D}\,, \text{or using } m + y = z, \quad \boxed{m + 2\,y \geq D}$$

provided that derivatives in the interaction term satisfy

$$\boxed{p = m\,p_t + p_x \leq 2z}$$

Using Lifshitz scalar as a proxy

Constraints from unitarity

- Condition on the number of derivatives in an interaction:

$$p = m\,p_t + p_x \leq 2z$$

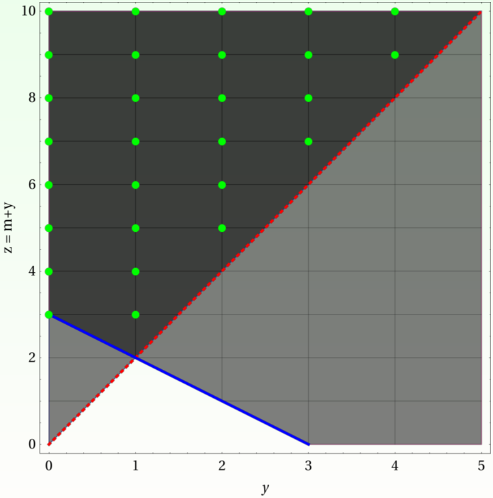
- p_t and p_x have maximum values

$$p_x \leq 2z\,, \qquad p_t \leq \frac{2z}{m} = 2\left(1 + \frac{y}{m}\right)$$

- If $p_t \geq 4$, unitarity compromised! Therefore, the only sensible theories are those which satisfy $\boxed{y < m}$
- In standard Hořava, $y = 0$ and $m = z$, so this is never an issue. But applies to mixed derivative cases.
- For the $y = 1$ case in $D = 3$ (analogue of the Pospelov-Shang term), renormalizability: $m \geq 1$; unitarity $m > 1$. Minimal version is not unitary! The first unitary theory has $m = 2$, $z = 3$. Pospelov-Shang's action is a tuned version of this theory! Generically, one also needs other $\nabla K \nabla K$ terms as well as $K^2 a^2$, $K^2 R$, $a\,K\,\nabla K$ type terms. Fourth order dispersion relations, power-counting renormalizable and unitary.

Allowed region in D=3

Power-counting renormalizable and unitary Hořava–like theories in D=3



Conclusions

- Preservation of LI in matter sector is the biggest challenge for LV gravity theories.
- A promising mechanism relies on a hierarchy between M_* and M_p. Naturalness problem from vector loops.
- Resolution: extend HG by new terms that can modify the vector sector. Pospelov-Shang used a mixed derivative term, as deformation to HG. However, these generically modify scaling anisotropy.
- Specifically, the generalization of Pospelov-Shang terms lead to $\omega^2 \sim k^4$ without undermining p-c renormalizability.
- Power-counting uncovers a new class of Hořava–like theories that are p-c renormalizable and unitary.
- Mixed derivatives: any improvements to original theory?
- Other ways of modifying vector part?