Inflationary Tensor Fossils in CMB and LSS

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mostly based on:

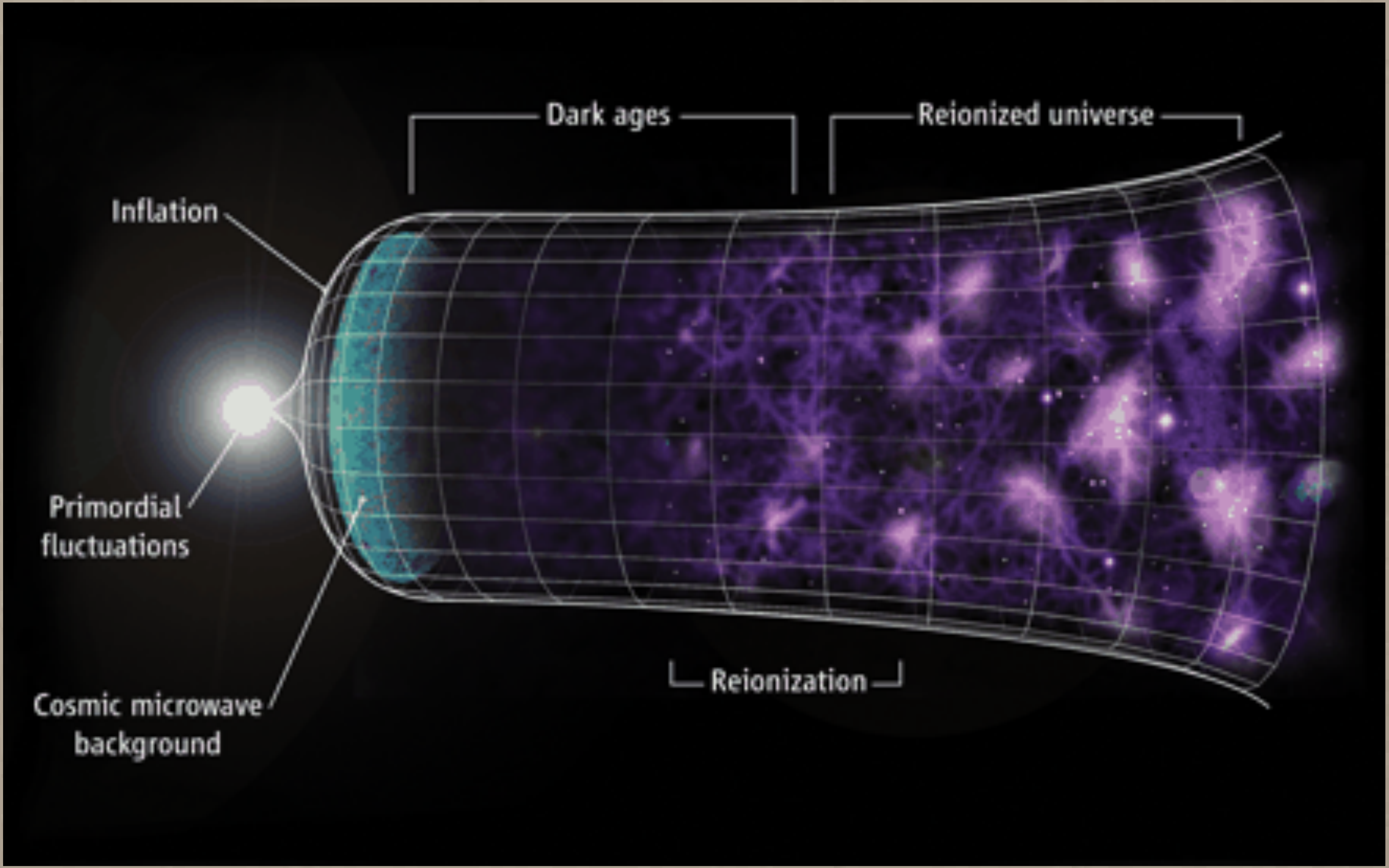
ED, M. Fasiello, M. Kamionkowski
arXiv:1504.05993

ED, M. Fasiello, D. Jeong, M. Kamionkowski
JCAP 1412 (2014) 12, 050

Outline

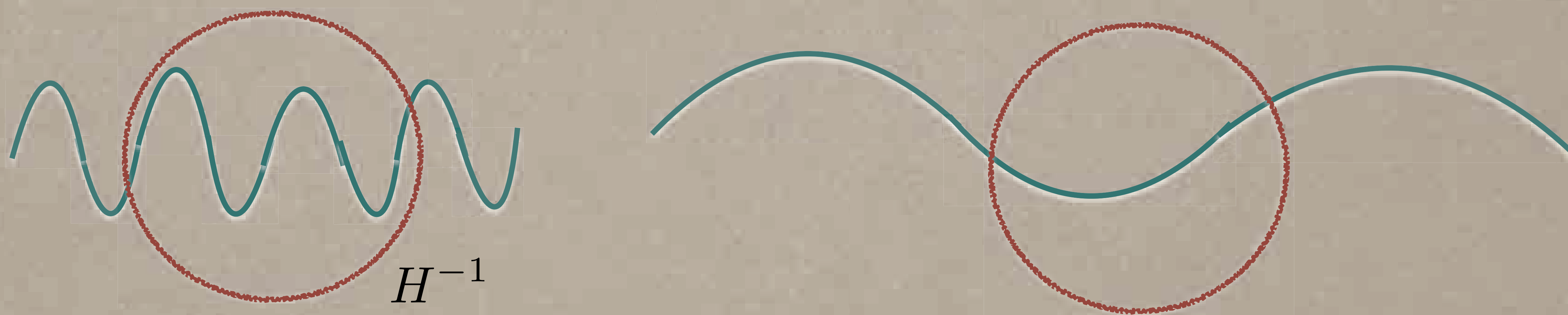
- Inflation and all that
- Gravitational fossils in CMB and LSS
- Tss predictions from some inflationary models

a little about inflation …

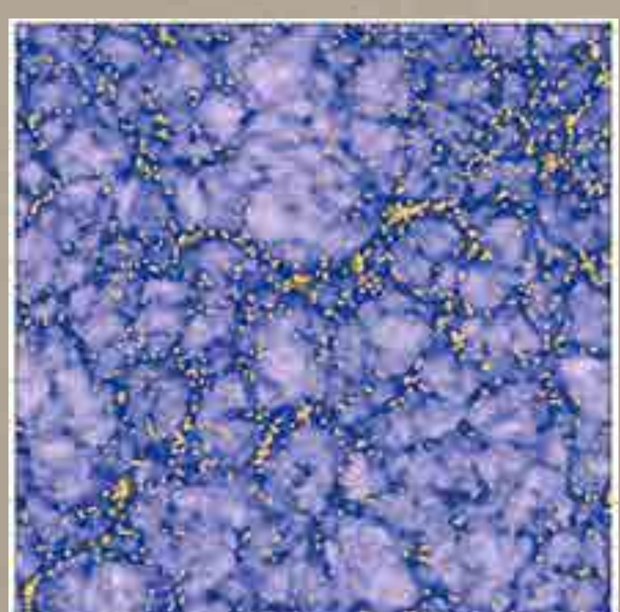


Inflation

- Era of accelerated expansion in the primordial Universe
- Mechanism for the generation of cosmological fluctuations
- Simplest realization: single-scalar field in slow-roll (SFRS)



physical scales are stretched by the expansion

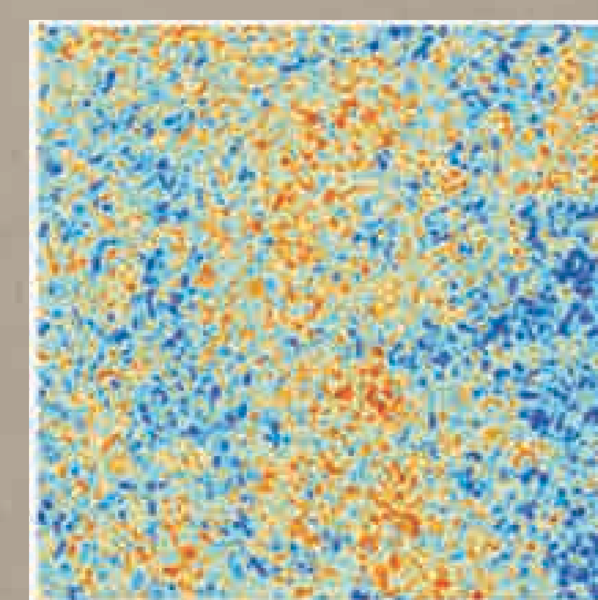


$$\phi(\vec{x},t) = \varphi(t) + \delta\phi(\vec{x},t)$$

Inflaton quantum fluctuations

↓

CMB perturbations
and LSS of the Universe



SFSR Inflation

$$\mathcal{S} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R - \int d^4x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right)$$

- homogeneous field with small fluctuations: $\phi(\vec{x},t) = \varphi(t) + \delta\phi(\vec{x},t)$

$$\ddot{\varphi} + 3H\dot{\varphi} + V^{'}(\varphi) = 0$$

- the potential V dominates the energy density: expansion

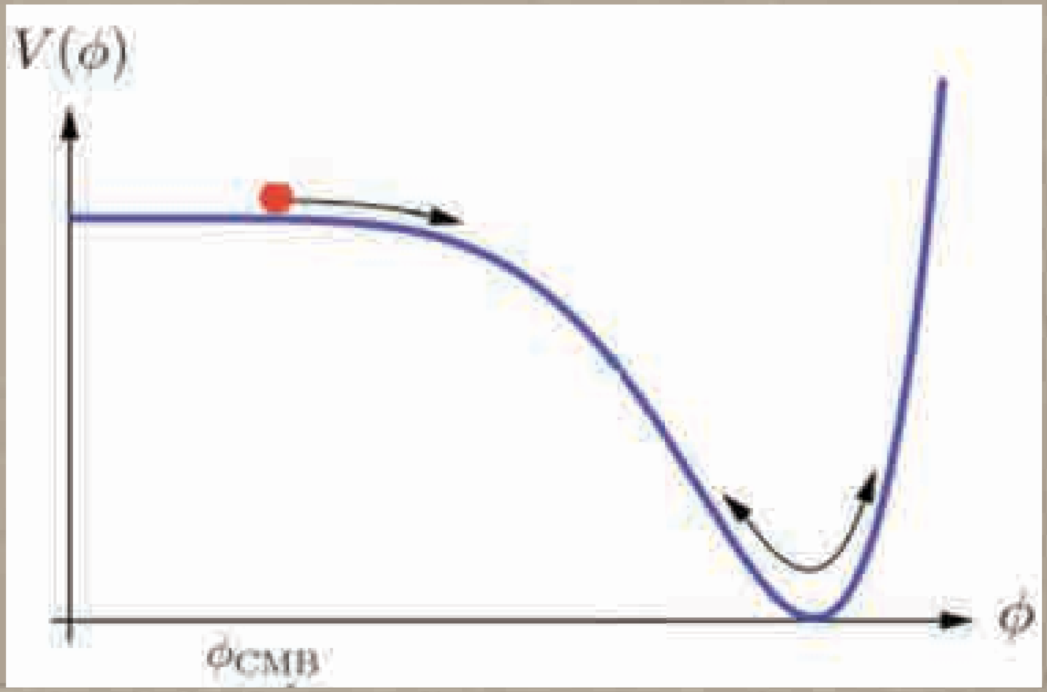
$$3M_P^2H^2 = \frac{\dot{\varphi}^2}{2} + V(\varphi) \qquad \rho = \frac{\dot{\varphi}^2}{2} + V(\varphi) \quad p = \frac{\dot{\varphi}^2}{2} - V(\varphi)$$

$$\dot{\varphi}^2 \ll V(\varphi) \longrightarrow \quad p \simeq -\rho$$

- V nearly flat : slow-roll conditions

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{1}{2}\left(\frac{M_P V^{'}}{V}\right)^2 \ll 1$$

$$\eta \equiv \frac{M_P^2 V^{''}}{V} \ll 1$$



SFSR Inflation

Scalar sector: from $\delta\phi$ to primordial curvature perturbation

$$ds^2 = dt^2 - a^2(t)e^{2\zeta(\vec{x},t)}d\vec{x}^2$$

(in uniform energy density gauge)

primordial scalar power spectrum:

$$\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\rangle = (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2)\frac{2\pi^2}{k_1^3}\mathcal{P}_\zeta(k_1)$$

nearly scale-invariant

$$\mathcal{P}_\zeta(k) \sim \frac{1}{\epsilon}\left(\frac{H}{M_P}\right)^2 k^{n_s-1}, \quad n_s = 1 - 4\epsilon - 2\delta$$

Planck $(5\,\sigma)$

$$n_s = 0.968 \pm 0.006$$

Tensor sector:

$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + \gamma_{ij}(\tau,\vec{x}))\,dx^i dx^j\right]$$

transverse
traceless

primordial tensor power spectrum:

$$\mathcal{P}_\gamma \sim \left(\frac{H}{M_P}\right)^2 k^{n_T}, \qquad n_T \sim -\epsilon$$

Amplitude of gravity waves =
energy-scale of inflation !

several directions to search for imprints of primordial GW, e.g. :

- CMB polarization (B-modes)

- Lensing effects from GW:

  galaxy distribution [Dodelson-Rozo Stebbins 2003, Schmidt-Jeong 2012, Dai-Kamionkowski-Jeong 2012, . . .]

  CMB [Cooray-Kamionkowski-Caldwell 2005, Dodelson 2010, . . .]

  21-cm fluctuations [Pen-Masui 2010, Book-Kamionkowski-Schmidt 2012]

- Anisotropic/non-Gaussian effects introduced in CMB and LSS [Dai-Jeong-Kamionkowski 2012-2013]

  ⟶ "tensor fossils"

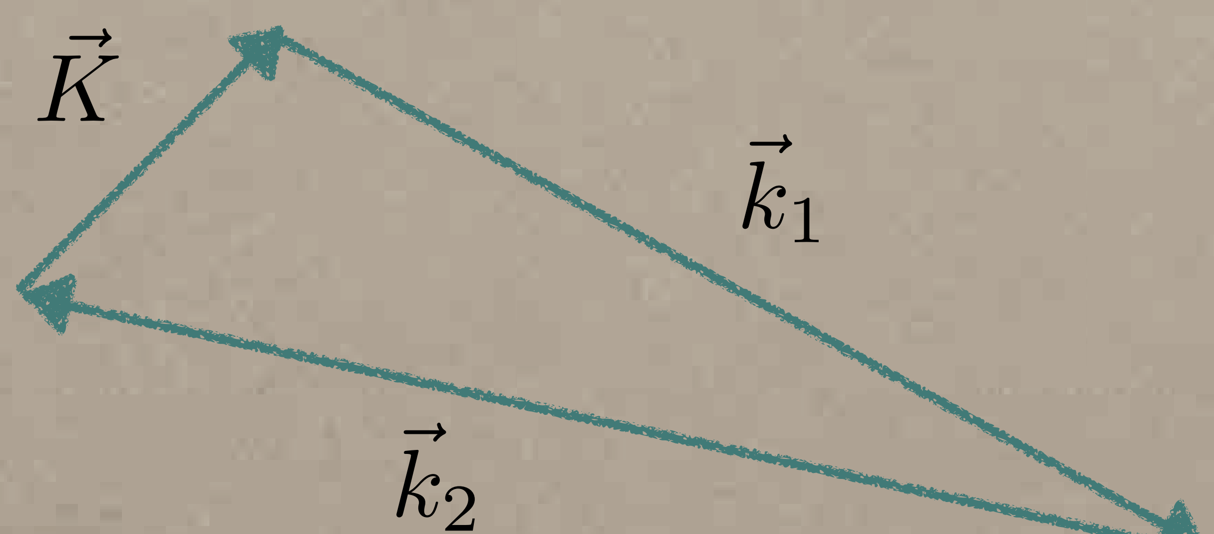
tss bispectrum :

$$\langle \gamma_p(\vec{K})\zeta(\vec{k}_1)\zeta(\vec{k}_2)\rangle = (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{K})B_p(K,k_1,k_2)$$

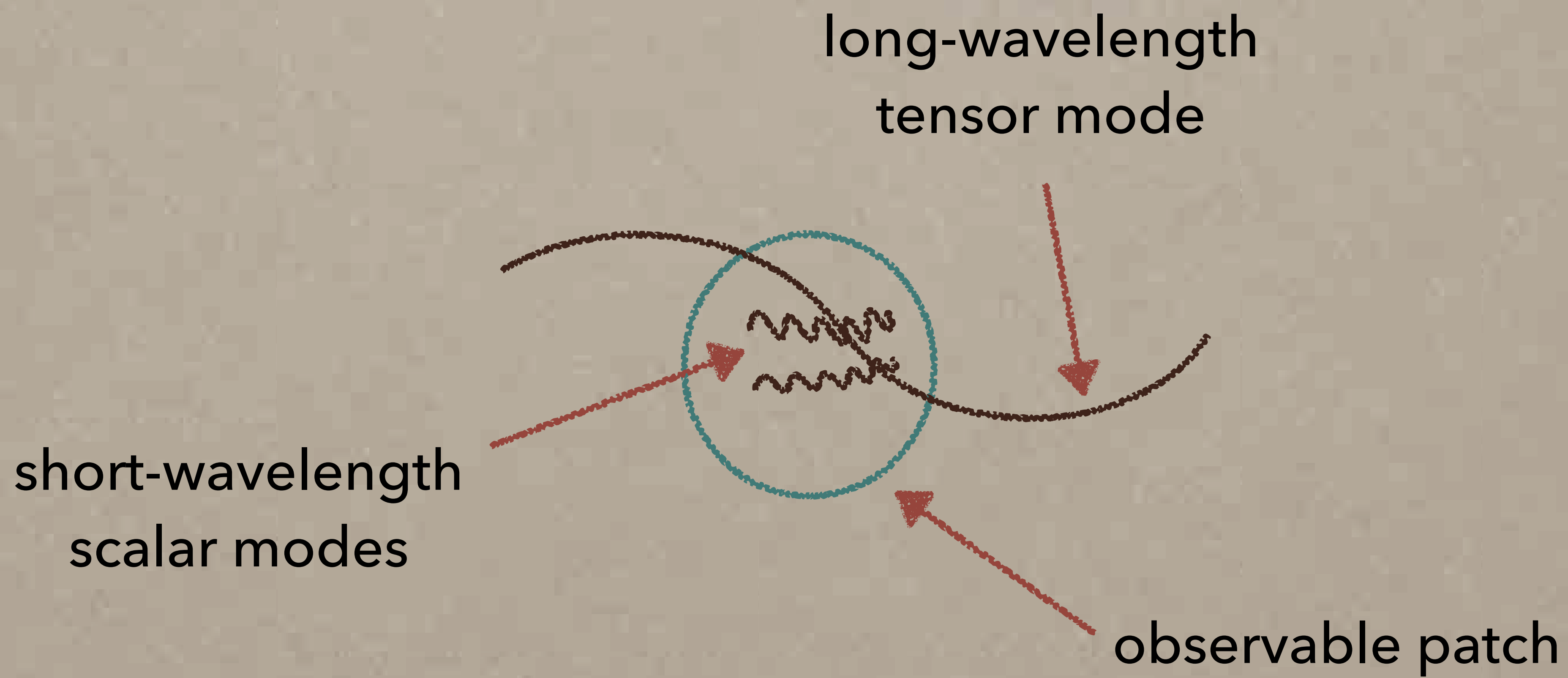
$$\equiv$$

$$\mathcal{B}_p(K,k_1,k_2)\epsilon^p_{ij}(\hat{K})\hat{k}^i_1\hat{k}^j_2$$

squeezed limit : $K \ll k_1 \simeq k_2$



A correlation (tss) between a long wavelength tensor mode with two short wavelength scalar modes affects the scalar power spectrum



fossil equation :

diagonal part

tensor mode induced

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\rangle_\gamma = (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2)P_\zeta(k_1)$$
$$+ \sum_p \int \frac{d^3K}{(2\pi)^3}(2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{K})$$
$$\times \gamma_p^*(\vec{K})\frac{B_p(K,k_1,k_2)}{P_\gamma^p(K)}$$

tss bispectrum

tensor mode

tensor power spectrum

[Jeong-Kamionkowski 2012-2013]

(1) super-Hubble K :

cannot resolve $|\vec{k}_1 + \vec{k}_2| = K < a_0 H_0$

$$\begin{aligned}\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\rangle_\gamma &= (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2)P_\zeta(k_1) \\ &+ \sum_p \int \frac{d^3K}{(2\pi)^3}(2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{K}) \\ &\times \gamma_p^*(\vec{K})\frac{B_p(K,k_1,k_2)}{P_\gamma^p(K)}\end{aligned}$$

(small K)

quadrupolar correction

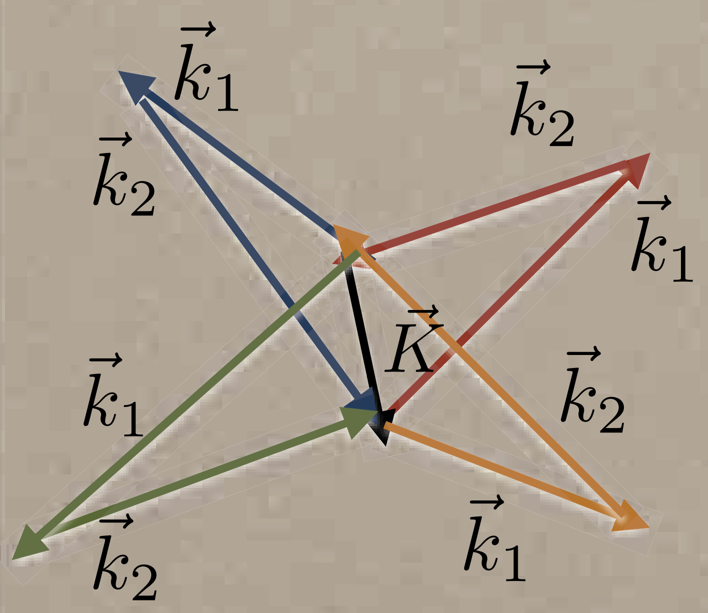
$$P(\vec{k}) \simeq P(k)\left[1 + \alpha\,\gamma_{ij}\hat{k}_i\hat{k}_j\right]$$

$$\alpha \propto \frac{B_p}{P_\zeta P_\gamma^p}$$

[Giddings-Sloth 2011, Jeong-Kamionkowski 2012]

(2) sub-Hubble : $K > aH$

$$\langle\zeta(\vec{k}_1)\zeta(\vec{k}_2)\rangle_\gamma = (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2)P_\zeta(k_1)$$
$$+ \sum_p \int \frac{d^3K}{(2\pi)^3}(2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{K})$$
$$\times \gamma_p^*(\vec{K})\frac{B_p(K,k_1,k_2)}{P_\gamma^p(K)}$$

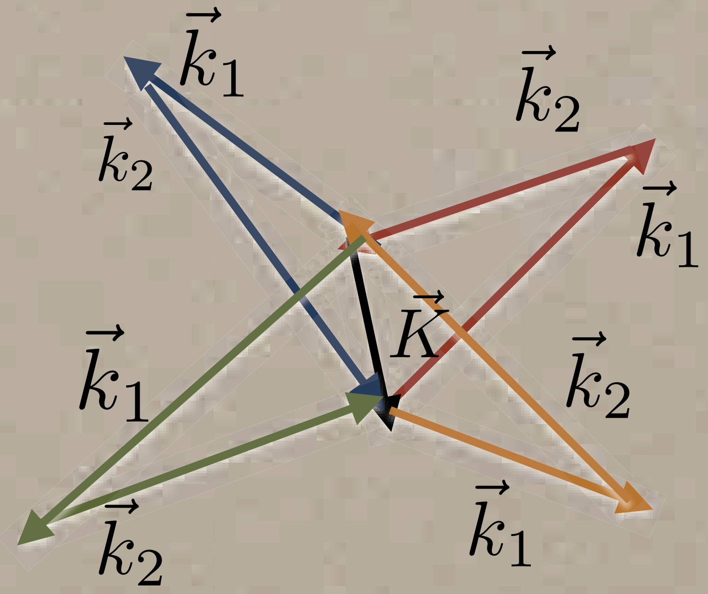


can use pairs $\vec{k}_1+\vec{k}_2=-\vec{K}$ to estimate $\gamma_p(k)$

[Jeong-Kamionkowski 2012]

estimating tensors from primordial density fluctuations

$$\langle \delta(\vec{k}_1)\delta(\vec{k}_2)\rangle_{\gamma_p(\vec{K})} \sim (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{K})\gamma_p^*(\vec{K})\,\frac{B_p(\vec{K},\vec{k}_1,\vec{k}_2)}{P_\gamma^p(K)}$$



- naive estimator :

$$\widehat{\gamma_p(\vec{K})} = \sum_{\vec{k}_1+\vec{k}_2=-\vec{K}} \frac{\delta(\vec{k}_1)\delta(\vec{k}_2)}{B_p(\vec{K},\vec{k}_1,\vec{k}_2)/P_\gamma^p(K)}$$

[Jeong-Kamionkowski 2012]

- optimal estimator for a single mode : inverse variance weighting

$$\widehat{\gamma_p(\vec{K})} = \sigma^2 \sum_{\vec{k}} \frac{B_p(K, k_1, k_2)/P^p_\gamma(K)}{2V P^{tot}(k) P^{tot}(|\vec{K} - \vec{k}|)} \delta(\vec{k}) \delta(\vec{K} - \vec{k})$$

$$\sigma^2 = \left[\sum_{\vec{k}} \frac{|B_p(\vec{K}, \vec{k}, \vec{K} - \vec{k})/P_\gamma(K)|^2}{2V P^{tot}(k) P^{tot}(|\vec{K} - \vec{k}|)} \right]^{-1}$$

random variable — its variance

$$\{y_i, \sigma_i^2\}$$

$$\hat{y} = \frac{\sum_i (y_i/\sigma_i^2)}{\sum_j (1/\sigma_j^2)}$$

$$\sigma^2 = \left(\sum_i \frac{1}{\sigma_i^2} \right)^{-1}$$

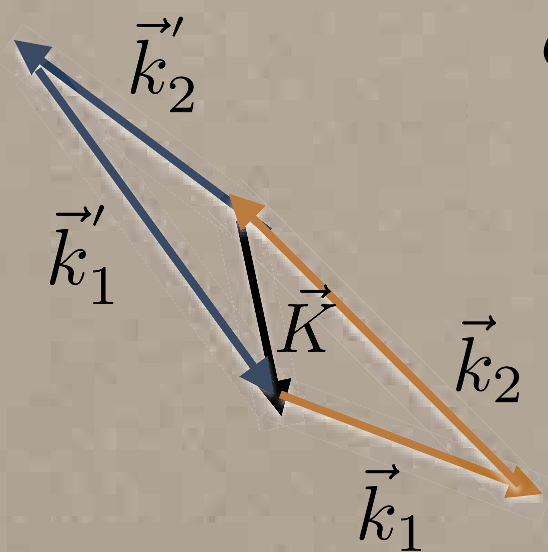
[Jeong-Kamionkowski 2012]

- optimal estimator for the power amplitude :
  stochastic GW background with $P_p(K) = A_\gamma P^f_\gamma(K)$

$$\widehat{A_\gamma} = \sigma_\gamma^2 \sum_{\vec{K},p} \frac{\left(P^f_\gamma(K)\right)^2}{\left(P^n_p(K)\right)^2} \left(\frac{\widehat{|\gamma_p(\vec{K})|^2}}{V} - P^n_p(K) \right)$$

optimal sum over different K-modes

$$\sigma_\gamma^{-2} = \sum_{\vec{K},p} \frac{\left(P^f_\gamma(K)\right)^2}{2\left(P^n_p(K)\right)^2}$$



$$P^n_p \equiv \left[\sum_{\vec{k}} \frac{|B_p(\vec{K},\vec{k},\vec{K}-\vec{k})/P_\gamma(K)|^2}{2V P^{tot}(k) P^{tot}(|\vec{K}-\vec{k}|)} \right]^{-1}$$

[Jeong-Kamionkowski 2012]

in fact, it is more general than that …

For any primordial field F with non-zero F-s-s correlation, i.e. :

$$\langle F_p(\vec{K})\delta(\vec{k}_1)\delta(\vec{k}_2)\rangle = (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{K})P_p(K)f_p(\vec{k}_1,\vec{k}_2)\epsilon^p_{ij}(\hat{K})k_1^i k_2^j$$

the new field induces a non-zero off-diagonal component in the local density power spectrum :

$$\langle\delta(\vec{k}_1)\delta(\vec{k}_2)\rangle_{F_p(\vec{K})} = (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{K})F_p^*(\vec{K})f_p(\vec{k}_1,\vec{k}_2)\epsilon^p_{ij}(\hat{K})k_1^i k_2^j$$

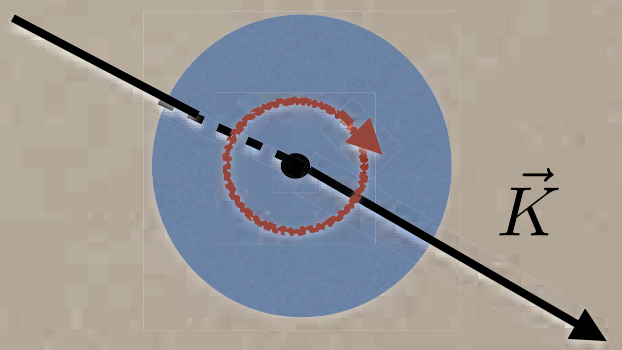
<u>bottom line</u>:
detection of primordial fields through off-diagonal correlation

[Jeong-Kamionkowski 2012]

field F can be anything … e.g. :

scalar : $\epsilon^0_{ij} \propto \delta_{ij}$ ⟶ isotropic modulation

vector (transverse)
tensor (transverse/traceless) ⟶ azimuthal dependence w.r.t. direction of $\vec{K}$



dependence from the azimuthal about $\vec{K}$ distinguishes scalars from vectors/tensors

[Jeong-Kamionkowski 2012]

from theory to observations : late time effects

$$B_p^{obs}(k_L, k_S, k_S) = B_p(k_L, k_S, k_S) + \frac{1}{2} P_\gamma(k_L) P_\zeta(k_S) \epsilon^p_{ij} \hat{k}^i_S \hat{k}^j_S \frac{\partial \ln P_\zeta(k_s)}{\partial \ln k_s}$$

observed Bispectrum

primordial Bispectrum

$= -B_{cc}(k_L, k_S, k_S)$

up to $\mathcal{O}\left(\frac{k_L}{k_S}\right)^2$ corrections

(from late time effects)

models of inflation that preserve consistency conditions predict a very small (unobservable) tensor-scalar-scalar correlation

[Tanaka-Urakawa 2011, Pajer-Schmidt-Zaldarriaga 2013, Dai-Jeong Kamionkowski 2013, Dai-Pajer-Schmidt 2015, ...]

Consistency conditions in inflation

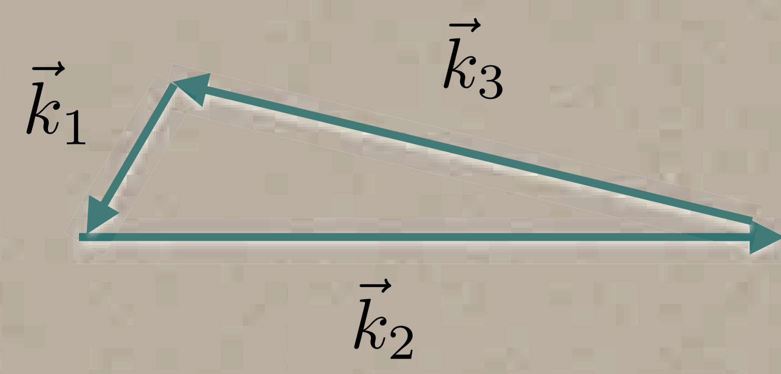
- form of n-point function is fixed in terms of (n-1)-point functions, in the soft limit for one of the modes

- apply to single-clock models, with perturbations that become constant at late times : long wavelength mode only rescales background for the short wavelengths

- can be derived from symmetries of the action (invariance under space diffs)

[Maldacena 2003, Creminelli-Zaldarriaga 2004, Goldberger-Hui-Nicolis 2013, Berezhiani and J. Khoury 2014, etc. . . .]

Examples of ccs : 3-point ⟷ 2-point function



$$k_L \equiv k_1 \ll k_2 \simeq k_3 \equiv k_S$$

- scalar correlator :

$$B_{cc}^{\zeta\zeta\zeta}(k_L, k_S, k_S) \simeq -P_\zeta(k_L) P_\zeta(k_S) \frac{\partial \ln \left[k_S^3 P_\zeta(k_S)\right]}{\partial \ln k_S}$$

$$= -(n_s - 1) P_\zeta(k_L) P_\zeta(k_S)$$

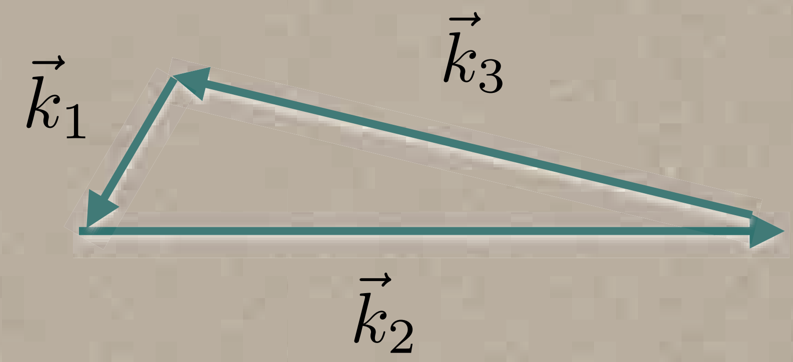
in SFSR :

$$\mathcal{L}_{\zeta\zeta\zeta} \sim \epsilon^2 \left[a^3 \dot{\zeta}^2 \zeta + ...\right]$$

$$\longrightarrow \quad f_{NL}^{local} \sim 1 - n_s$$

[Maldacena 2003]

Examples of ccs : 3-point ⟷ 2-point function



$$k_L \equiv k_1 \ll k_2 \simeq k_3 \equiv k_S$$

- *tss* correlator :

$$B_{cc}^{\gamma\zeta\zeta}(k_L, k_S, k_S) \simeq -\frac{1}{2} P_\gamma(k_L) P_\zeta(k_S) \epsilon_{ij} \hat{k}_S^i \hat{k}_S^j \frac{\partial \ln P_\zeta(k_S)}{\partial \ln k_S}$$

in SFSR :

$$= \frac{3}{2} P_\gamma(k_L) P_\zeta(k_S) \epsilon_{ij} \hat{k}_S^i \hat{k}_S^j$$

$\mathcal{L}_{\gamma\zeta\zeta} \sim a\, \epsilon\, \gamma_{ij} \partial_i \zeta \partial_j \zeta$

[Maldacena 2003]

intuitive understanding of ccs :

$k_3 \ll k_1,\, k_2$

k_3 crosses the horizon much earlier than $k_1,\, k_2$, it is frozen by the time the other modes cross

$$a \;\rightarrow\; a\, e^{\zeta_3} \simeq a(1+\zeta_3)$$ ⟵ background rescaling

the effect of the soft mode is that of anticipating the horizon exit of the hard modes by an amount $\delta t \simeq -\zeta_3/H$

$$\langle \zeta_1 \zeta_2 \rangle_{\zeta_3} \sim \frac{d}{dt} \langle \zeta_1 \zeta_2 \rangle \delta t \sim \frac{1}{H} \frac{d}{dt} \langle \zeta_1 \zeta_2 \rangle \zeta_3$$

$$\longrightarrow \langle \zeta_1 \zeta_2 \zeta_3 \rangle \sim (1-n_s) P_\zeta(k_1) P_\zeta(k_3)$$

[Maldacena 2003]

to summarize:

observed *tss* correlator :

(primordial) (from projection effects)

$$B_{obs}^{\gamma\zeta\zeta}(k_L, k_S, k_S) = B^{\gamma\zeta\zeta}(k_L, k_S, k_S) + \frac{1}{2} P_\gamma(k_L) P_\zeta(k_S) \epsilon_{ij} \hat{k}_S^i \hat{k}_S^j \frac{\partial \ln P_\zeta(k_S)}{\partial \ln k_S}$$

ccs imply $B^{\gamma\zeta\zeta}(k_L, k_S, k_S) \simeq -\frac{1}{2} P_\gamma(k_L) P_\zeta(k_S) \epsilon_{ij} \hat{k}_S^i \hat{k}_S^j \frac{\partial \ln P_\zeta(k_S)}{\partial \ln k_S} + \mathcal{O}\left(\frac{k_L}{k_S}\right)^2$

→ very small $B_{obs}^{\gamma\zeta\zeta}$

similar condition applies to the scalar bispectrum

a detection of a 3-point correlation in the squeezed limit would rule out a very large class of inflationary models!

take home message so far :

the squeezed limit is a powerful discriminant for inflationary models

only models violating consistency conditions for single-clock inflation may predict observably large fossils signals

violation of ccs

~~single-clock dynamics~~

- isocurvature modes (multi-field models)

- non-Bunch Davies initial states

- system has not reached attractor (e.g. during non-attractor phase preceding inflation)

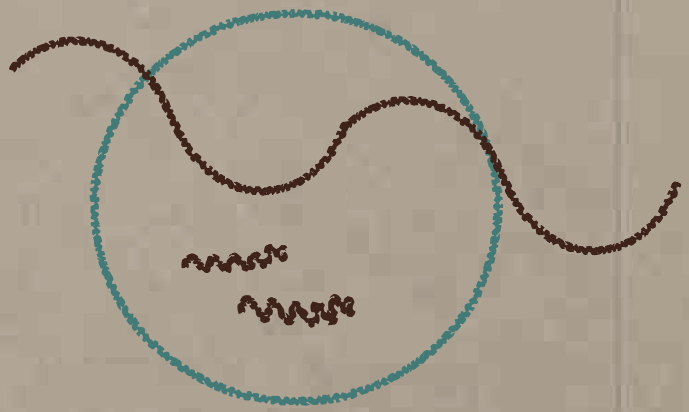
- models with broken space diffs

Fossils from Inflation with non-Bunch Davies initial states

BD: usual initial state for cosmological fluctuations
= vacuum state in Minkowski

non BD : may arise from some pre-inflationary era (e.g. anisotropic) or from a number of other scenarios (e.g. tunneling from a false vacuum+ inflation within the bubble, ...)
[Dei-Paban 2012, Lello-Boyanovsky-Holman 2013, Sugimura-Komatsu 2013, Yamamoto-Sasaki-Tanaka 1996, Agullo-Ashtekar-Nelson 2013, ...]

excited initial state → soft-hard momenta correlation can arise before horizon exit



fossil-type signatures can be at reach for observations

[Brahma-Nelson-Shandera 2013]

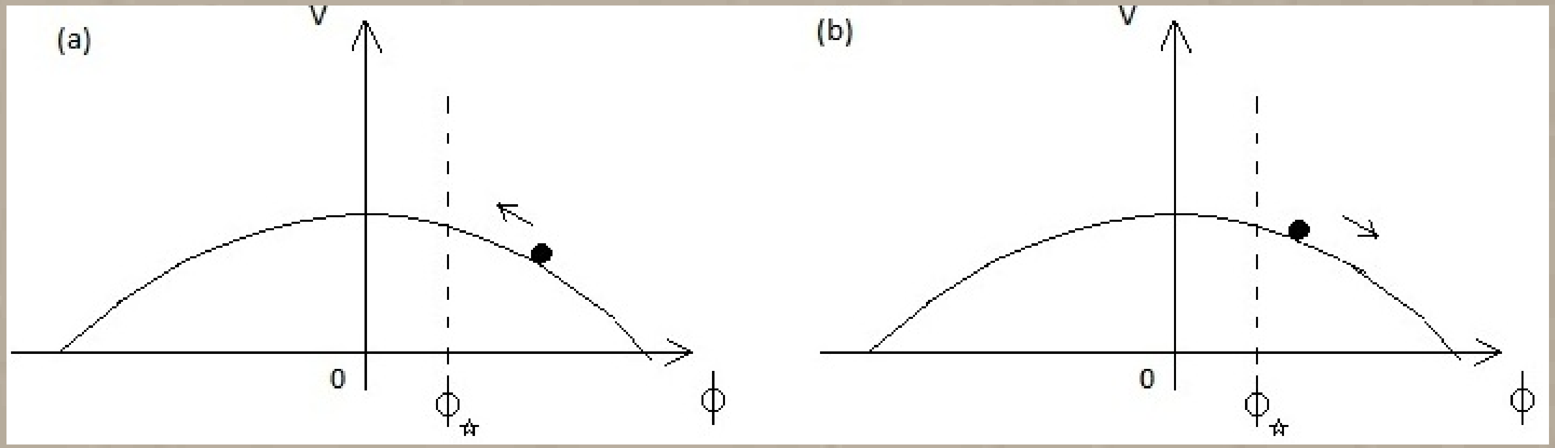
Non-attractor single-field inflation

- example of single-field model which can violate ccs for scalar correlators
- non-attractor phase at the initial stage of inflation (t<t*)
- during the non-attractor phase curvature fluctuation is not conserved
- larger value of fnl in the local limit for modes exiting horizon before t*

**[Namjoo-Firouzjahi-Sasaki 2012, Kinney 2005,
Chen-Firouzjahi-Komatsu-Namjoo-Sasaki 2013]**

Non-attractor single-field inflation

$$P = X + \frac{X^n}{M^{4\,(n-1)}} - V(\phi) \qquad n > 1$$



t<t*: the field climbs up the potential until it stops

$$\epsilon \sim a^{-6} \qquad \rightarrow \qquad \zeta_{[k \ll aH]} \sim a^3$$

t>t*: field rolls back down

$\epsilon \sim$ constant, ζ conserved super-horizon

[Chen-Firouzjahi-Komatsu-Namjoo-Sasaki 2013]

TSS from Inflation with a non-attractor phase

curvature fluctuation not conserved during non-attractor phase

$\langle\zeta\zeta\zeta\rangle$ violates ccs (order k_L^0)

tensor modes are conserved outside the horizon

$\langle\gamma\zeta\zeta\rangle$ obeys ccs (order k_L^0)

interesting dependence from the transition time (t*) at order k_L^2

Solid Inflation

Very different symmetry-breaking pattern than standard inflationary models

e.g. EFTI of **[C. Cheung et al, 2008]**

$$\bar{\phi} = \bar{\phi}(t)$$

time translation are broken, Goldstone mode π ⟶ adiabatic perturbations

Now background quantities are space-dependent

$$\langle \phi^I \rangle = x^I$$

key fact nevertheless, can recover homogeneity and isotropy by employing internal symmetries of the fields

[S.Endlich, A.Nicolis, J. Wang, 2013]

Internal Symmetries

$$\begin{cases} \phi^I \to \phi^I + a^I \\ \phi^I \to O^I_J \phi^J \end{cases} \longleftarrow \quad \text{hence the "Solid" nomenclature}$$

Most general low-energy theory for three scalar fields obeying Poincare' + the internal symmetries

Building blocks $\quad B^{IJ} = \partial_\mu \phi^I \partial^\mu \phi^J$

$$\mathcal{L} = F(\frac{[B]}{1}, \frac{[B^2]}{[B]^2}, \frac{[B^3]}{[B]^3}) + \ldots \qquad [..] = \text{Trace}$$

Excitation about the background $\longrightarrow$ curvature fluctuations, observables

[S.Endlich, A.Nicolis, J. Wang, 2013]

Checks

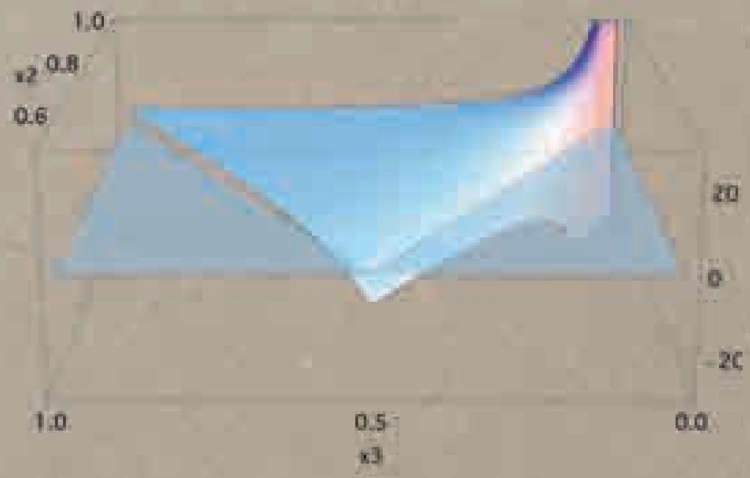
Inflating background ✓

(sub)Luminal propagation of fluctuations ✓

Strong coupling scale above H ✓

Observables

Larger non-Gaussianity and specific, very distinct shape-function



Most relevant in our context

The non-standard nature of these model manifests itself also in the form of non-conserved both ζ , γ outside the horizon, i.e. violation of consistency conditions

[S.Endlich, A.Nicolis, J. Wang, 2013]

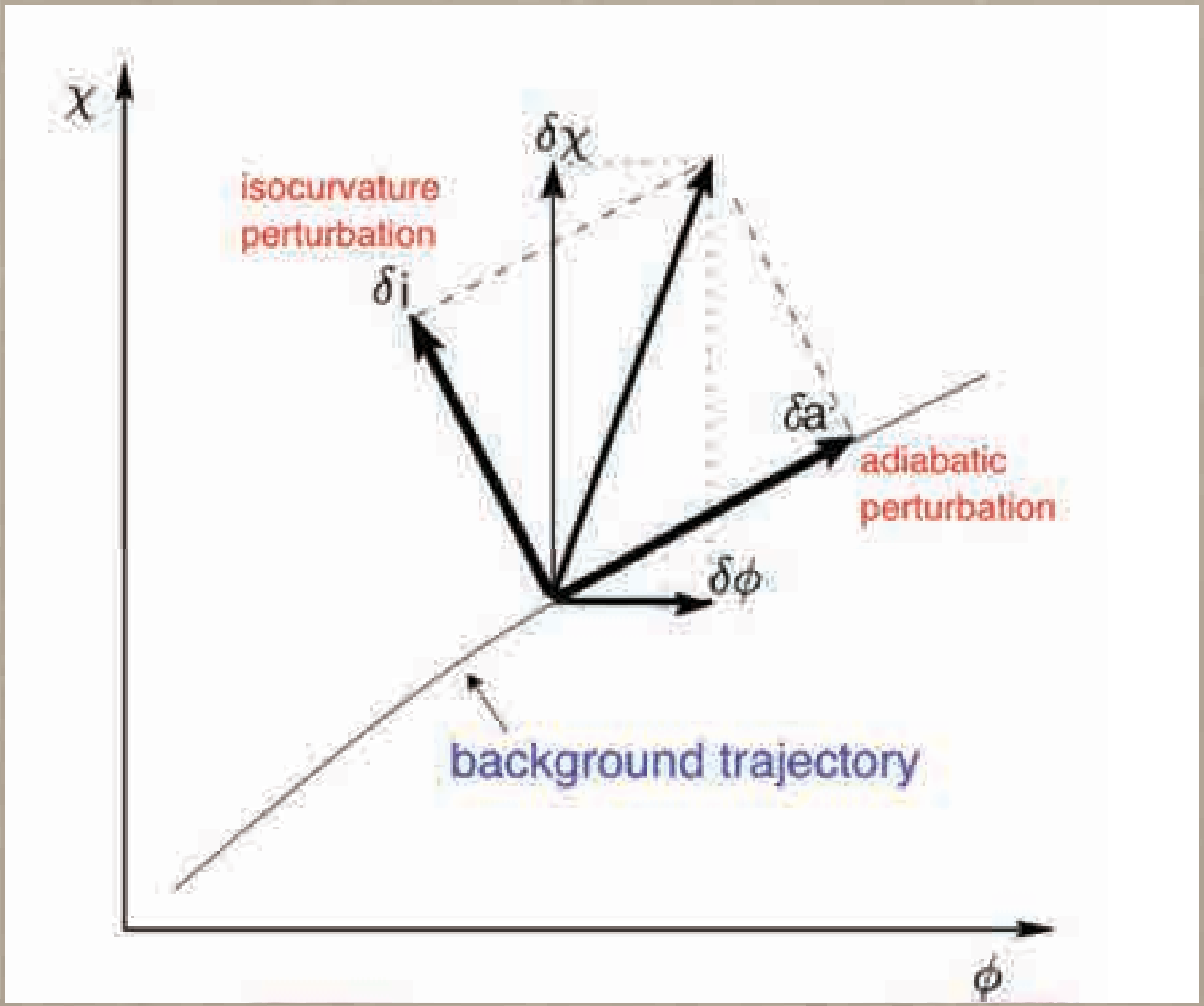
inflationary models with isocurvature modes

a natural set-up for ccs to be violated:

isocurvature perturbations + curved trajectory

↓

adiabatic fluctuations can evolve outside the horizon

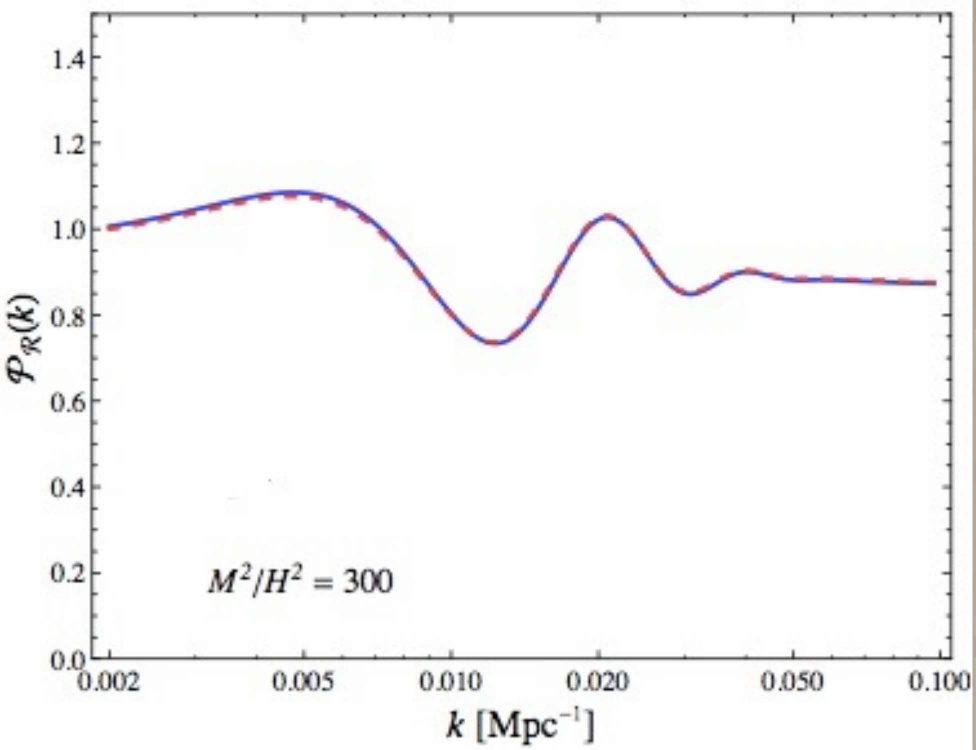


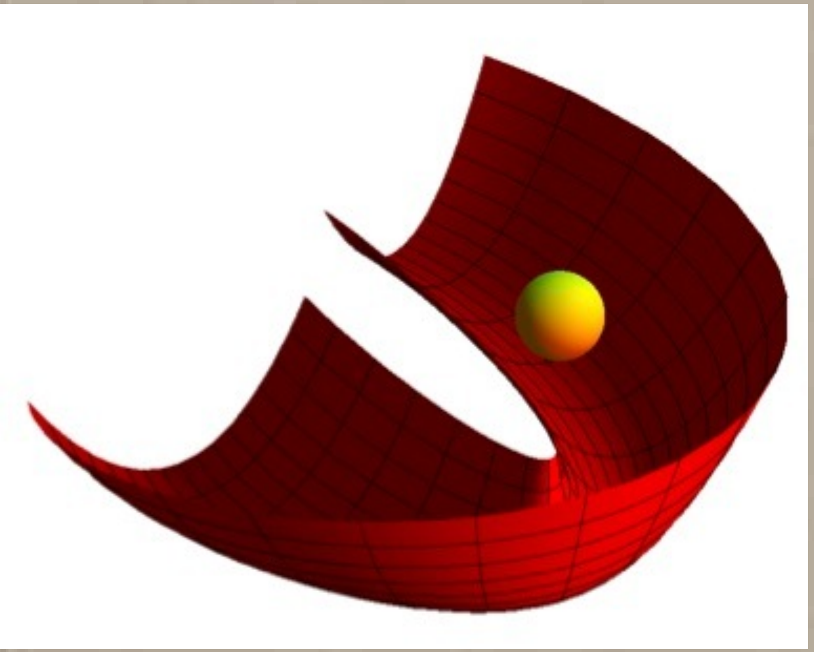
clearly not a single-clock dynamics!

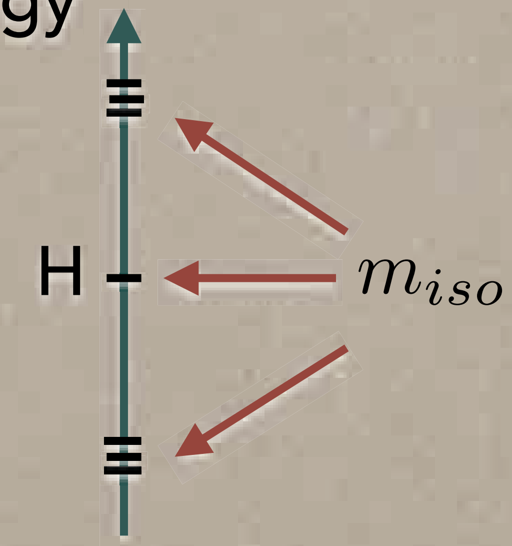
[Gordon, Wands, Bassett, Maartens, 2000] curved trajectory in field space :

conversion isocurvature → adiabatic

[Achucarro et al,Tolley-Wyman,Pi-Sasaki, Chen-Wang,Noumi-Yamaguchi, Battefeld-Freitas, Kehagias-Riotto, Maldacena-ArkaniHamed, ...]







Quasi-single-field inflation:

$$\mathcal{S}_m = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(\sigma+r)^2 g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V_{sr}(\theta) - V(\sigma)\right]$$

adiabatic mode (tangential)

isocurvature mode (radial)

assumptions :

- constant turn
- massive isocurvature: m ~ H (e.g. SUSY motivated)

[Chen-Wang 2010,Baumann-Green 2011,Pi-Sasaki 2012 ...]

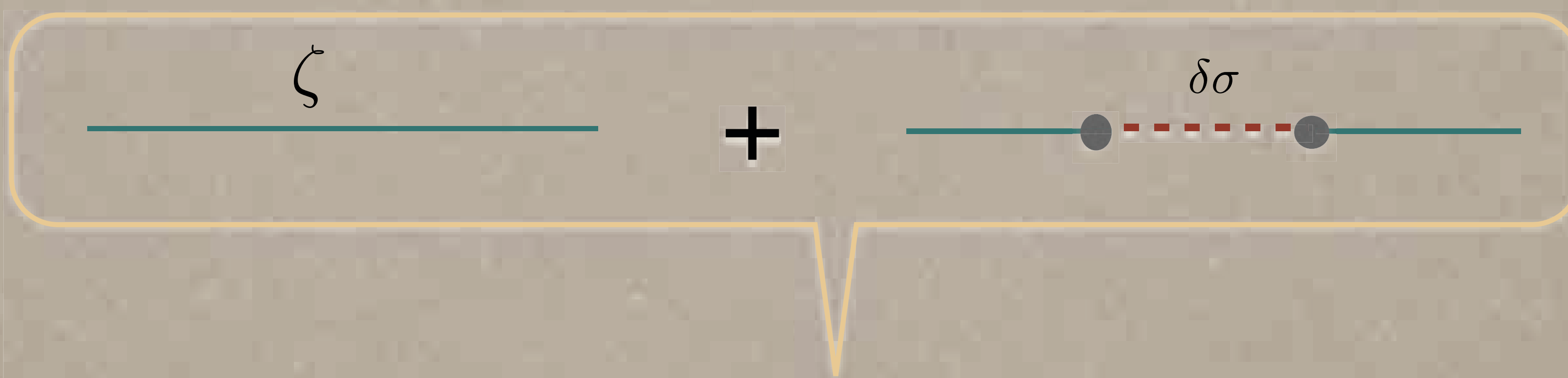
linear perturbations

simplifying assumption :

- mixing term is a small correction to quadratic Lagrangian

$$\delta\mathcal{L}_2 \propto \dot{\theta}_0 \delta\sigma \delta\dot{\theta} \qquad \left(\frac{\dot{\theta}_0}{H}\right)^2 \ll 1$$

$\langle \zeta^2 \rangle \sim$



$+ \; \ldots$

$$\mathcal{P}_\zeta = \frac{H^4}{4\pi^2 R^2 \dot{\theta}_0^2}\left[1 + 8\,\mathcal{C}(\nu)\frac{\dot{\theta}_0^2}{H^2}\right]$$

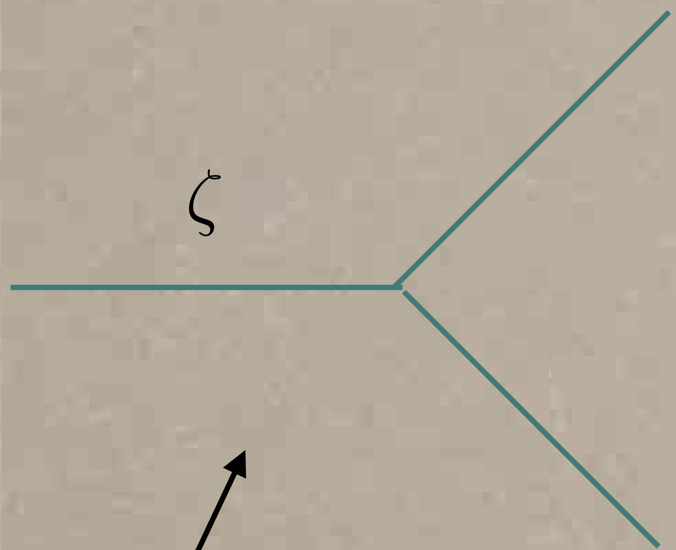
$$\left[\zeta \simeq -\frac{1}{\sqrt{2\epsilon}M_P}\delta\theta\right]$$

$$\delta\theta_k(\tau) \propto \frac{H}{R\sqrt{2k^3}}(1 + ik\tau)e^{-ik\tau} \qquad \left(\nu \equiv \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}\right)$$

$\delta\sigma_k(\tau) \propto (-\tau)^{3/2-\nu}$ ← (decays at late times)

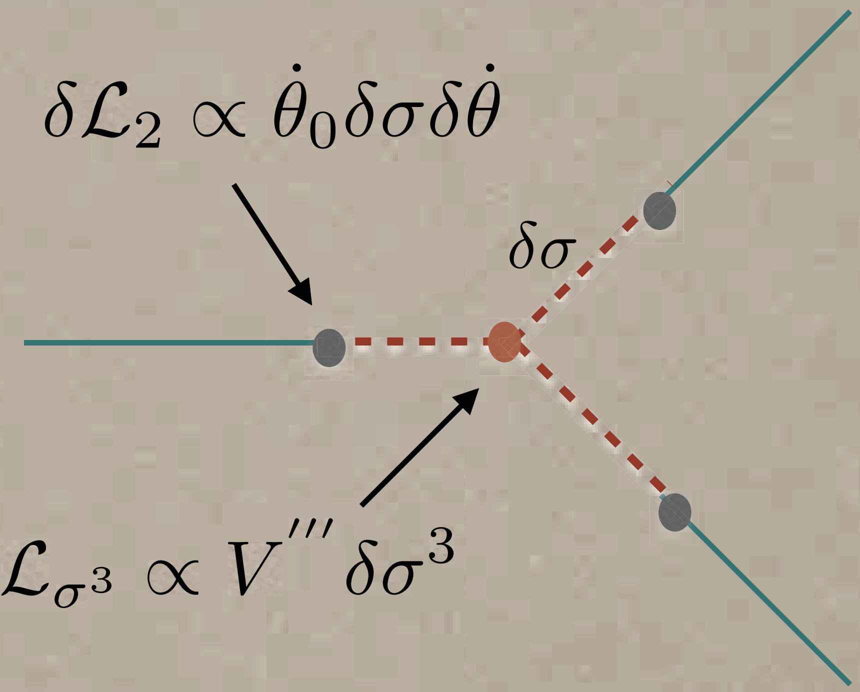
[Chen-Wang,2010]

scalar bispectrum



(as in standard SFSR)

+



$\delta\mathcal{L}_2 \propto \dot{\theta}_0 \delta\sigma \delta\dot{\theta}$

$\mathcal{L}_{\sigma^3} \propto V''' \delta\sigma^3$

+ . . .

V''' need not to be tiny

different shapes depending on
the mass of the isocurvature mode
interpolating between local and equal

. . .

$m \to 0$ $\qquad$ $m \to \frac{3H}{2}$

$k_3 \ll k_1 \simeq k_2$:

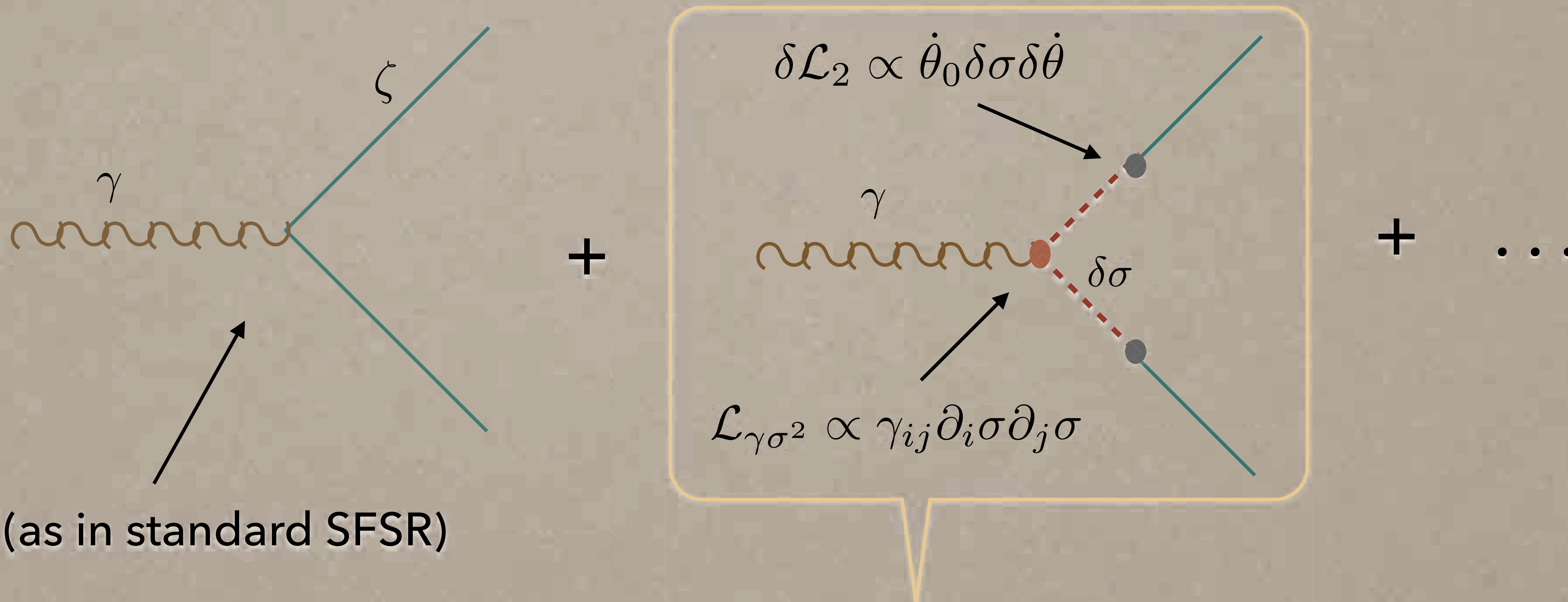
$\delta\sigma_k(\tau) \propto (-\tau)^{3/2-\nu}$

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle \sim \frac{1}{k_1^3 k_3^3} \left(\frac{k_3}{k_1} \right)^{\frac{3}{2}-\nu}$$

$\left(\nu \equiv \sqrt{\frac{9}{4} - \frac{m^2}{H^2}} \right)$

tss correlation

NEW !



$$k_3 \ll k_1 \simeq k_2 : \qquad \langle \gamma_{\vec{k}_3} \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle \propto \beta(\nu) \frac{1}{k_1^3 k_3^3}$$

(amplitude is a function of the isocurvature mass)

[ED-Fasiello-Kamionkowski 2015]

Quadrupolar correction

$$P_\zeta(\vec{k},\vec{x}) = P(k) + \Delta P(\vec{k},\vec{x})$$

tss correlation after removing projection effects

$$(|\vec{x}| \ll 1/K) \qquad \propto \beta \gamma_{ij}(\vec{x}) \hat{k}_i \hat{k}_j \qquad \beta \propto \frac{\mathcal{B}^{\gamma\zeta\zeta}_{\cancel{cc}}}{P_\gamma P_\zeta}$$

observational bounds :

(null tests) conducted for galaxy surveys **[Pullen-Hirata]**
and CMB **[Groeneboom-Eriksen, Bennett et al., Ade et al.]**

bound on the amplitude of the quadrupole : $\lesssim 0.01$

Quadrupolar anisotropy

- non-attractor Inflation:
  the non-attractor phase must have ended no later than the time our observable Universe exited horizon during inflation

- solid and quasi-single–field:
  the parameter space easily accommodates the quadrupole bound in the region where they also predict an observable off-diagonal correlation (next slide)

[ED-Fasiello-Jeong-Kamionkowski,2014]
[ED-Fasiello-Kamionkowski, 2015]

estimating GW amplitude from fossils :

$$\sigma_\gamma \propto \frac{1}{\beta^2}\left(\frac{k_{max}}{k_{min}}\right)^{-3}$$

(variance of the optimal estimator)

- $$\sigma_\gamma^{-2} = \frac{1}{2}\sum_{\vec{K},p}\left(K^3 P_p^n(K)\right)^2$$
- $$P_p^n(k_L) = \left[\sum_{\vec{k}_S} \frac{\left|B_{\not{c}c}^{\gamma\zeta\zeta}(\vec{k}_L,\vec{k}_S,\vec{k}_L-\vec{k}_S)\right|^2}{2V P_\gamma^2 P_\zeta^{tot}(k_S) P_\zeta^{tot}(|\vec{k}_L-\vec{k}_s|)}\right]^{-1}$$
- $$\sum_{\vec{k}} \to V\int \frac{d^3k}{(2\pi)^3}$$
- $$\frac{P_\zeta(k)}{P_\zeta^{tot}(k)} = \begin{cases} 1 & k < k_{max} \\ 0 & k > k_{max} \end{cases}$$

$$\mathcal{B}_{\not{c}c}^{\gamma\zeta\zeta} \simeq \beta P_\gamma P_\zeta$$

$$P_\gamma = \mathcal{A}_\gamma / k^{3+\kappa} \quad (\kappa \ll 1)$$

estimating GW amplitude from fossils : $P_\gamma = \mathcal{A}_\gamma / k^{3+\kappa} \quad (\kappa \ll 1)$

$$\mathcal{A}_\gamma \sim 3\sigma_\gamma \propto \frac{1}{\beta^2}\left(\frac{k_{max}}{k_{min}}\right)^{-3}$$

forecast for a given survey size (*s.i.*/q.s.f.) :

for A_γ near the current bound, detection possible in forthcoming surveys

$\frac{k_{max}}{k_{min}} \simeq$ 750 for Euclid; 5000 for 21-cm

[ED-Fasiello-Jeong-Kamionkowski,2014]
[ED-Fasiello-Kamionkowski, 2015]

Conclusions

- *tss* correlation in the squeezed limit affects the primordial density power spectrum (fossils: quadrupole anisotropy/off-diagonal)

- these signatures can be observable if arising from models evading ccs : another test for single-clock inflation besides the scalar local bispectrum

- the amplitude of GW may be estimated from such off-diagonal correlation (new direction for tensor mode searches)

- yet another indication that testing statistical isotropy for cosmological correlators can help constraining inflation

- single-field inflation + non standard initial conditions, or models that breaks space diffs or with multiple fields can predict observably large fossil signatures