Thermalization after Inflation

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Based on 1312.3097, 1402.2846 and 1506.xxxxx
In collaboration with K.Harigaya, M.Kawasaki, M.Yamada

Introduction

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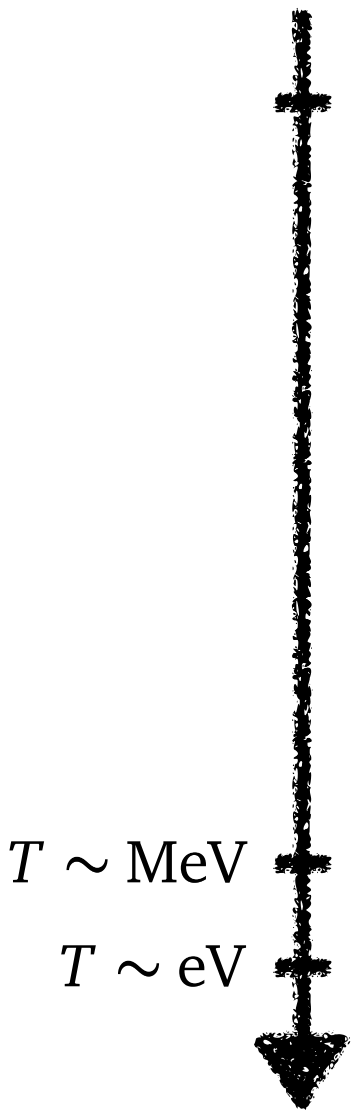
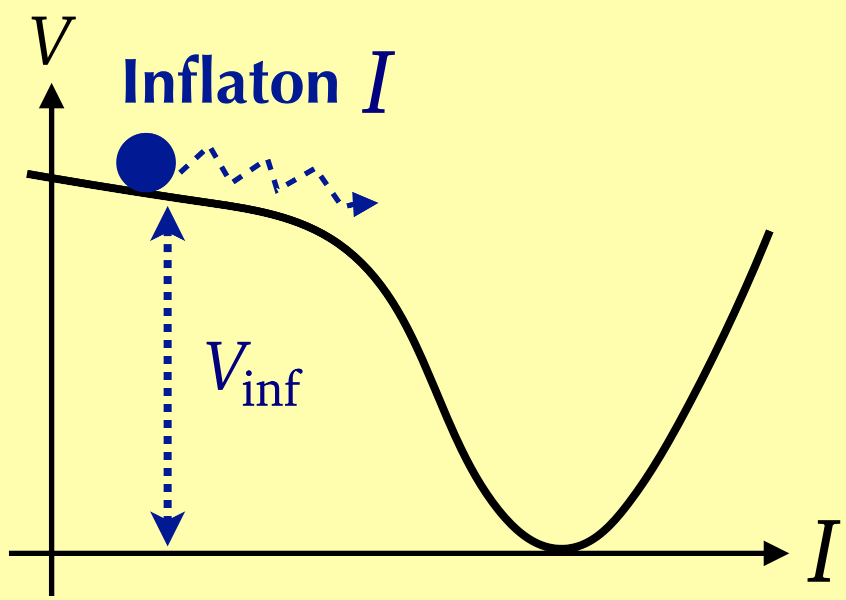
■ Inflationary Cosmology

- Inflation is caused by a potential energy of a scalar field, *i.e.* **inflaton**.
- Inflaton should convert its energy into radiation:





H: Hubble parameter
a: scale factor

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 \simeq \frac{V_{\rm inf}}{3M_{\rm pl}^2}$$

$$\Rightarrow a \propto e^{Ht}$$

Accelerated Expansion !

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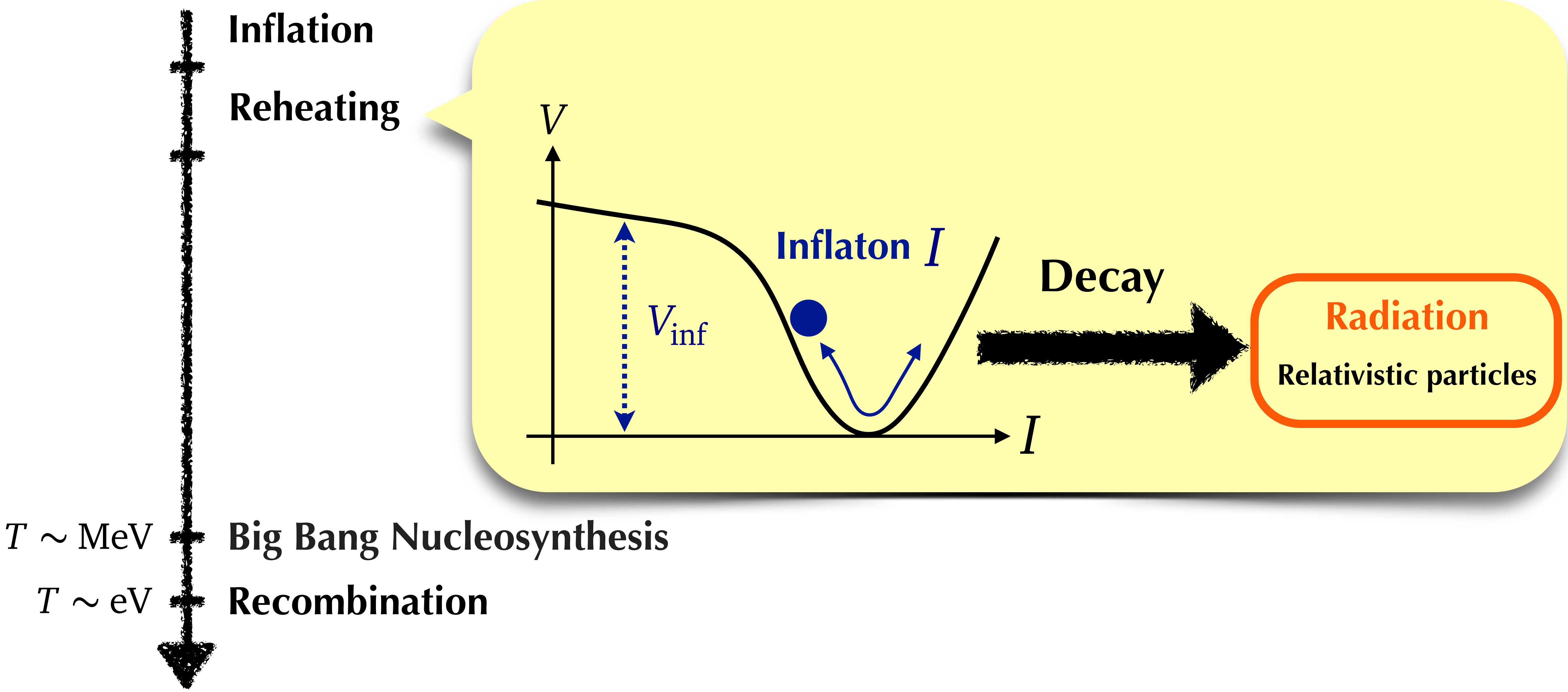

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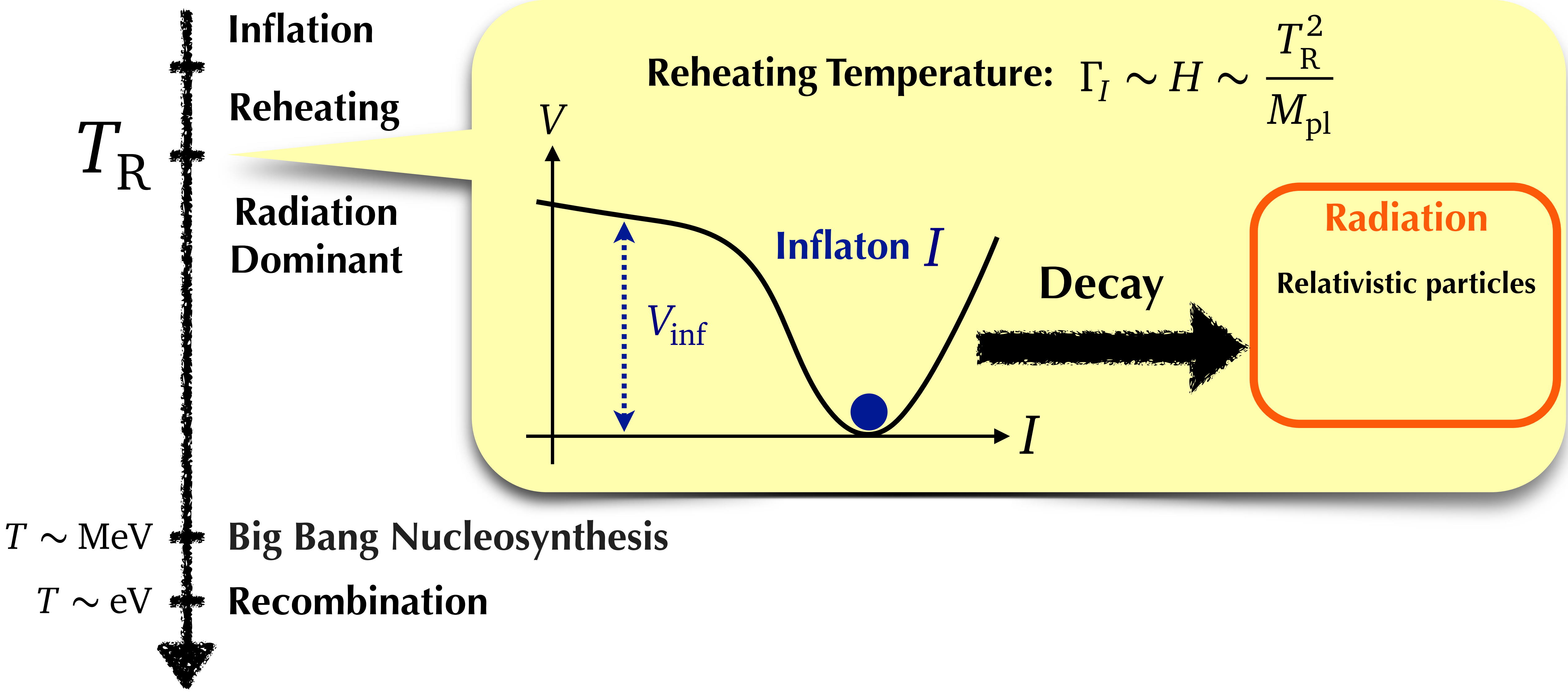

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Inflation

Reheating

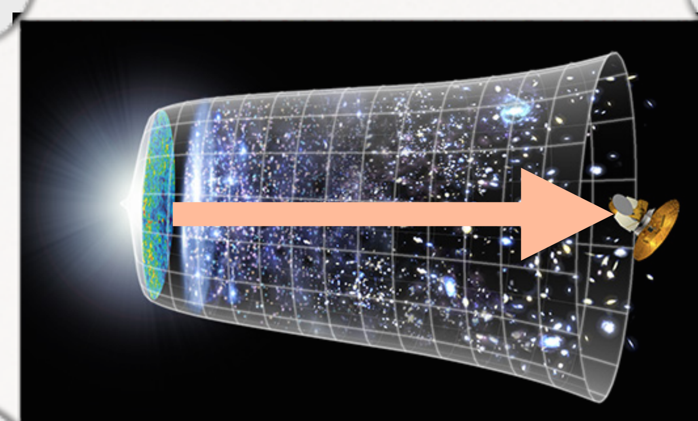
T_{R}

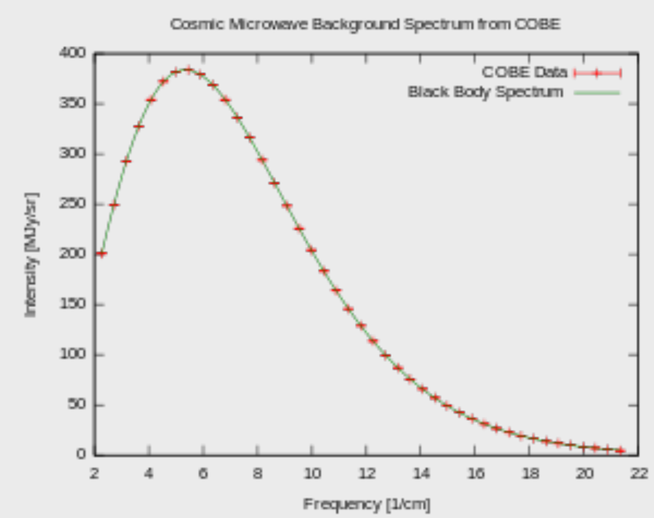
Radiation Dominant

$T \sim \mathrm{MeV}$ **Big Bang Nucle synthesis**

$T \sim \mathrm{eV}$ **Recombination**

Our Universe was in thermal equilibrium





Planck Distribution w/ T = 2.725 K

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Inflation

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Our Universe was in thermal equilibrium

- proton-neutron conversion

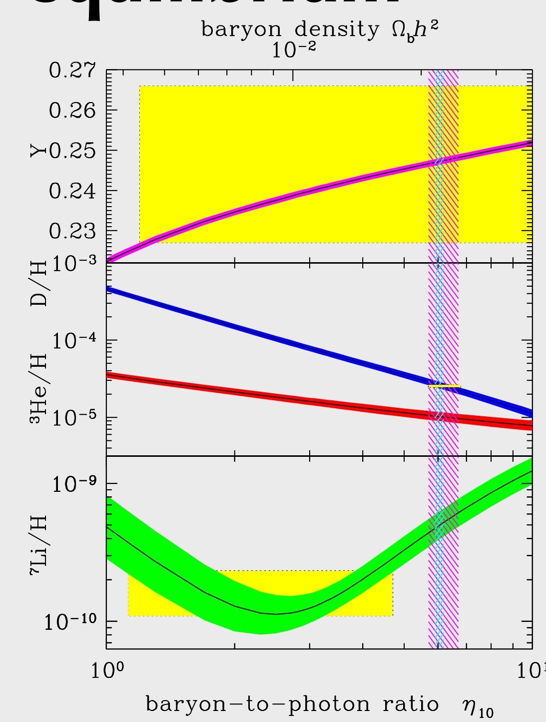
$$\begin{aligned} &p + e^- \longleftrightarrow n + \nu_e \\ &p + \bar{\nu}_e \longleftrightarrow n + e^+ \\ &p + e^- + \bar{\nu}_e \longleftrightarrow n \end{aligned} \quad \longrightarrow \quad \Gamma \sim \sigma n_{\rm th} \sim G_{\rm F}^2 T^5$$

- Freeze-out of (n/p): $\Gamma \sim H \longleftrightarrow T_{\rm F} \sim 0.8\,\mathrm{MeV}$

$$(n_n/n_p) = \exp(-\Delta m/T_{\rm F}) \simeq 1/6 \rightarrow 1/7$$

- Abundance of ^{4}He

$$Y = \frac{M(^4H_{\rm e})}{M(p)+M(n)} \simeq \frac{4m_p n_n/2}{m_p n_p + m_n n_n} \simeq \frac{2n_n/n_p}{1+n_n/n_p}$$



[PDG `14]

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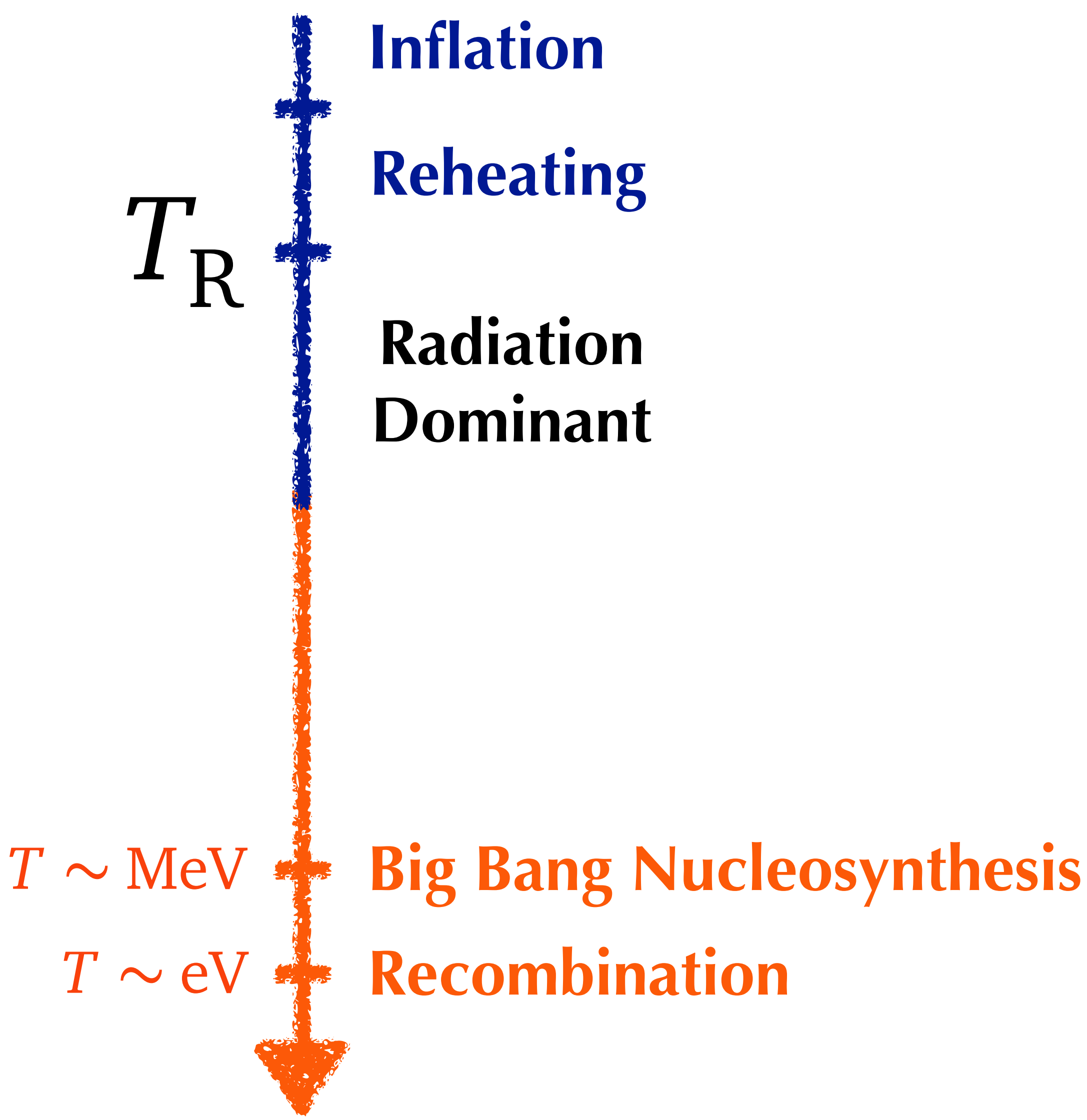

Far From Thermal Equilibrium

- May strongly depend on details of reheating dynamics.

Thermal Equilibrium

- Does not depend on details of reheating.
- Simply characterized by the temperature.
- More predictable.

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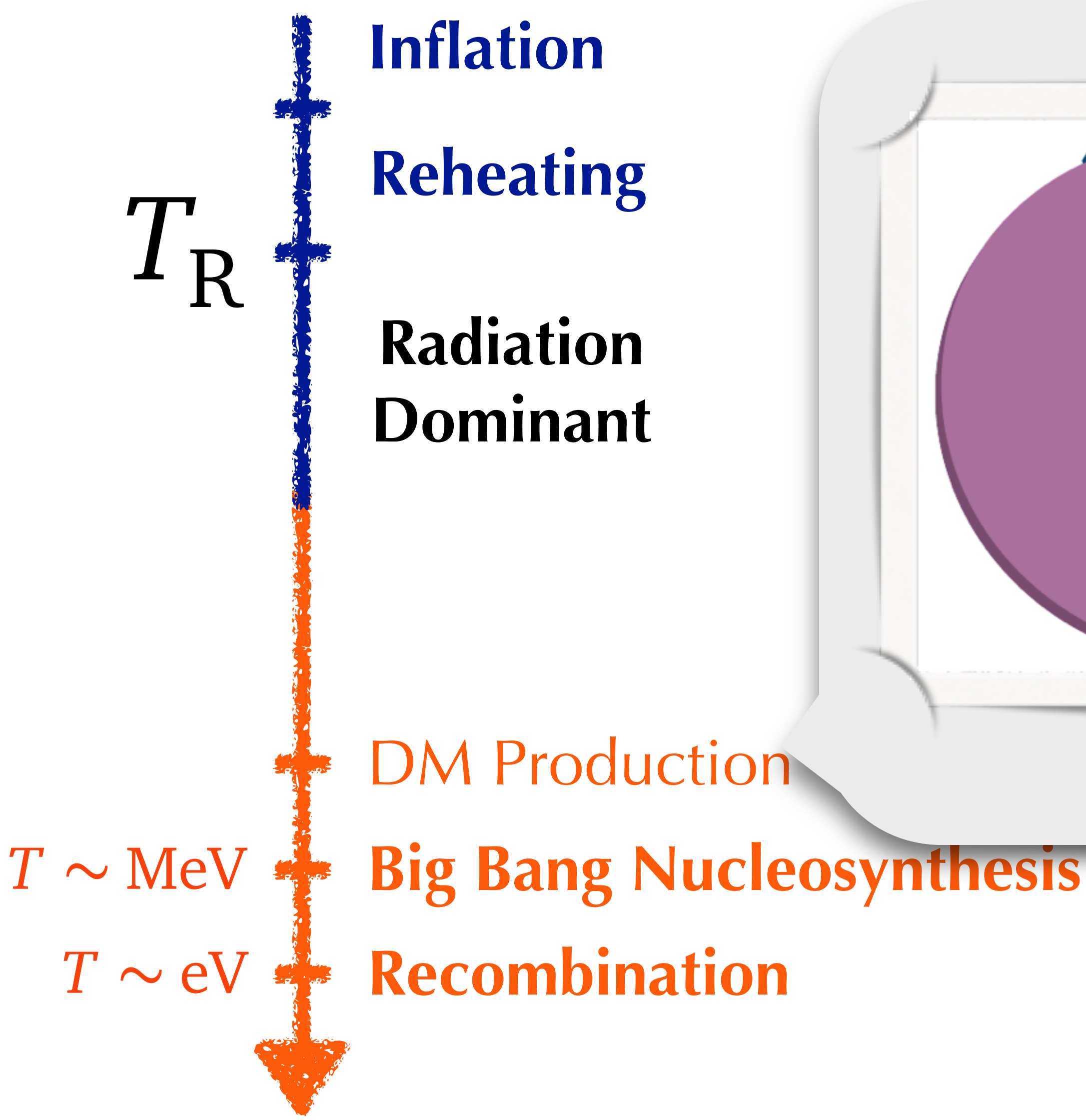
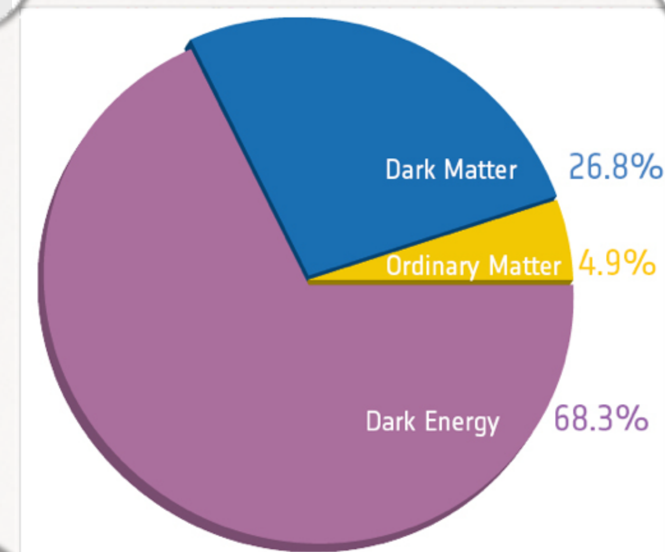
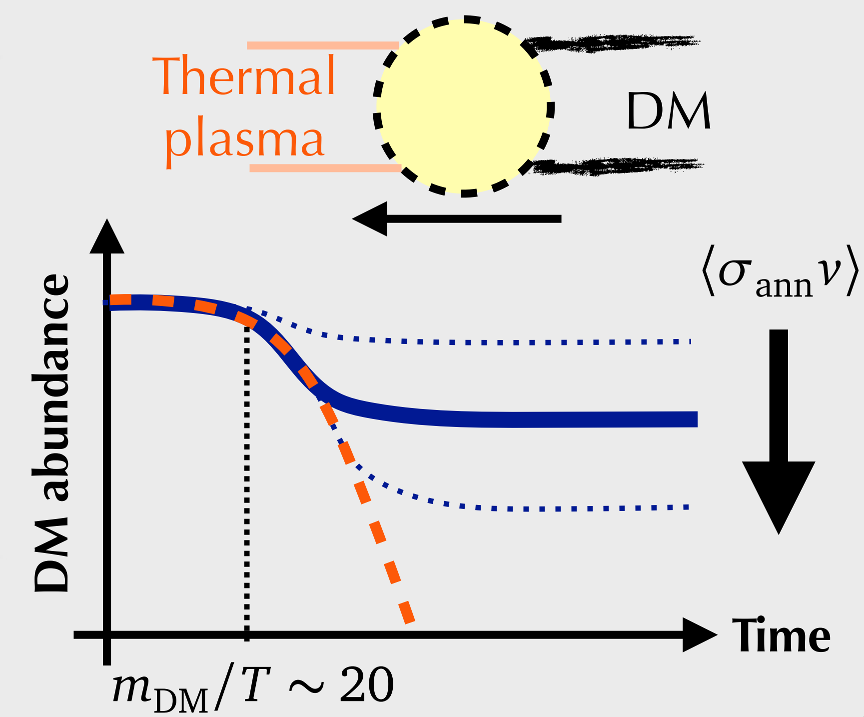




[Planck 2013]

Freeze-out DM production



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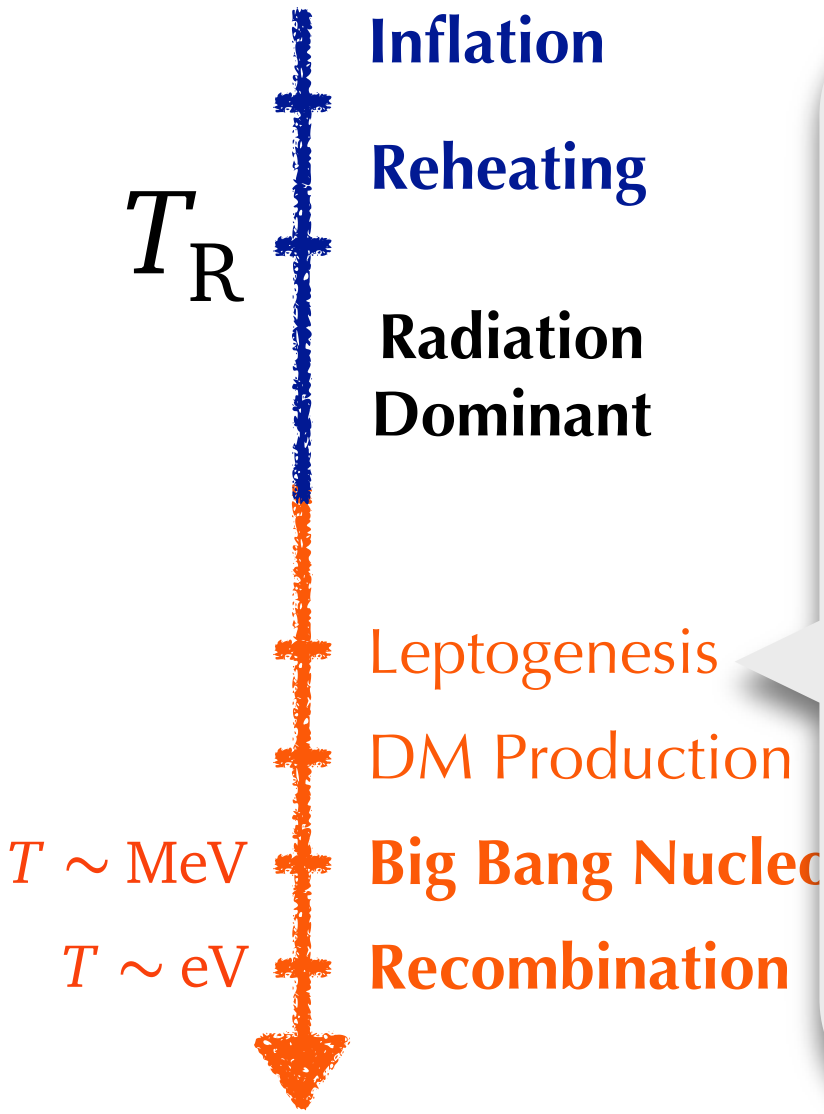


Baryon Asymmetry of Universe

- Baryon-to-photon ratio: $\eta \equiv n_B / n_\gamma$

$$\eta(\text{CMB}) \simeq 6.0 \times 10^{-10}, \quad 5.7 \times 10^{-10} < \eta(\text{BBN}) < 6.7 \times 10^{-10}$$

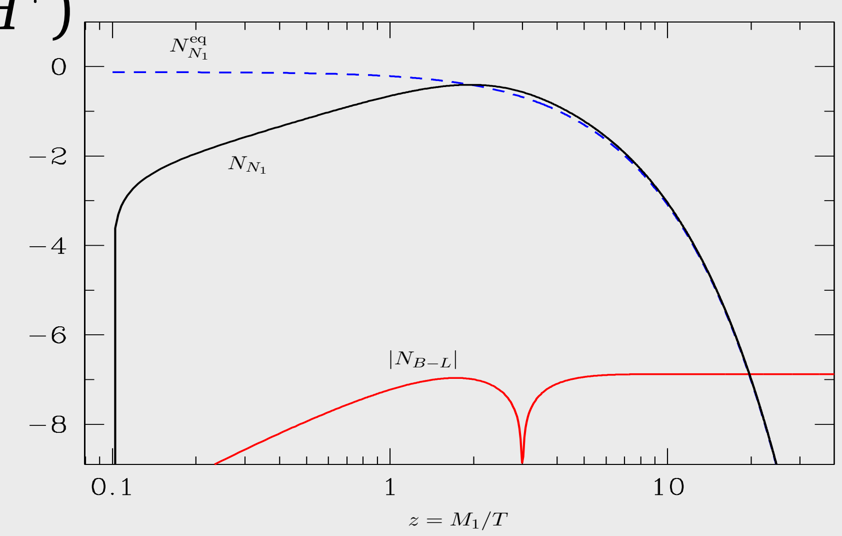
- **Leptogenesis** via asymmetric decay of right-handed neutrino

[Fukugita, Yanagida]

$$\Gamma(N \to l + H) \neq \Gamma(N \to \bar{l} + H^*)$$

- Sphaleron breaks **B+L**

$$\Delta L \to \Delta B$$



[Buchmuller, Peccei, Yanagida]

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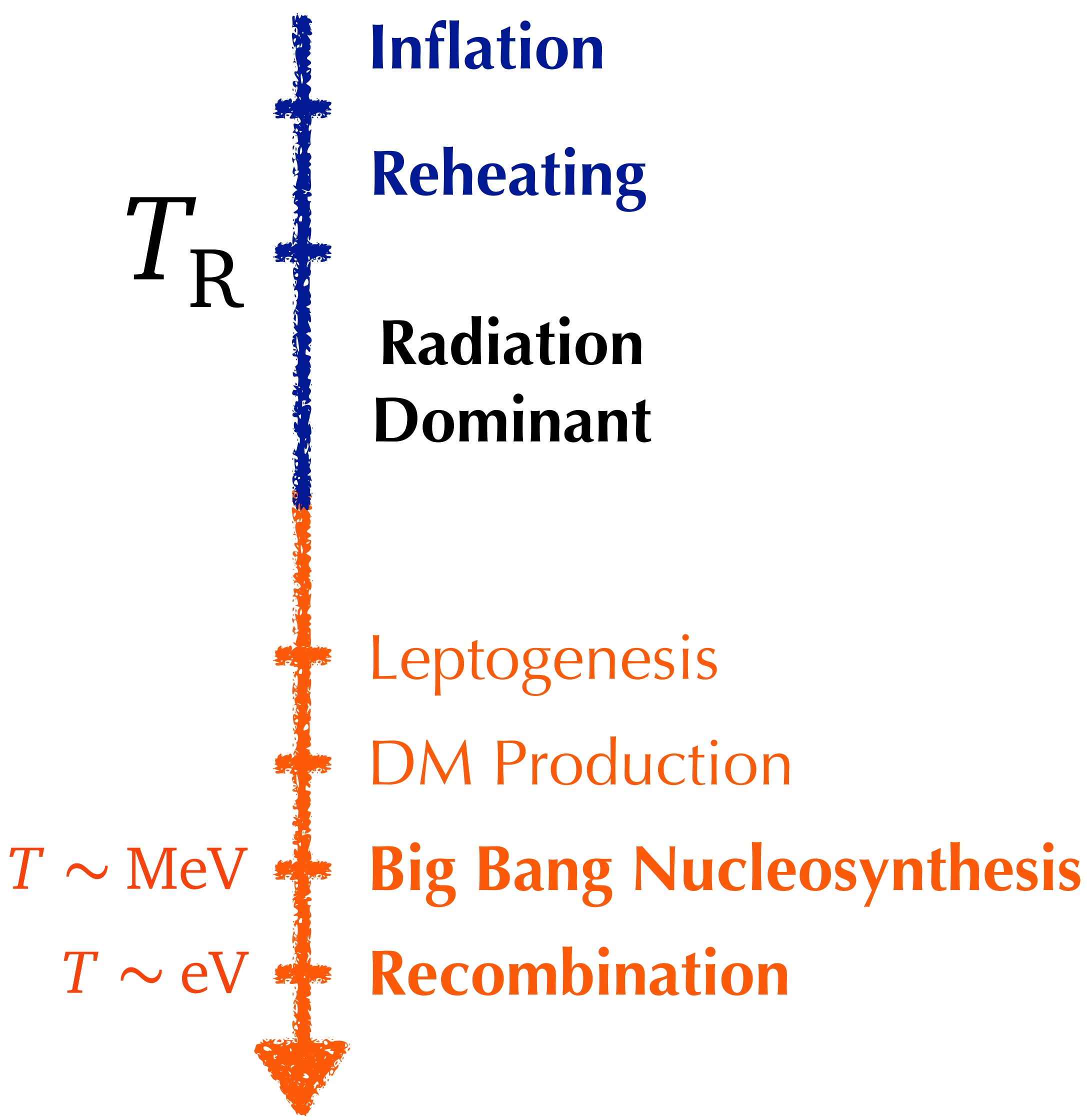

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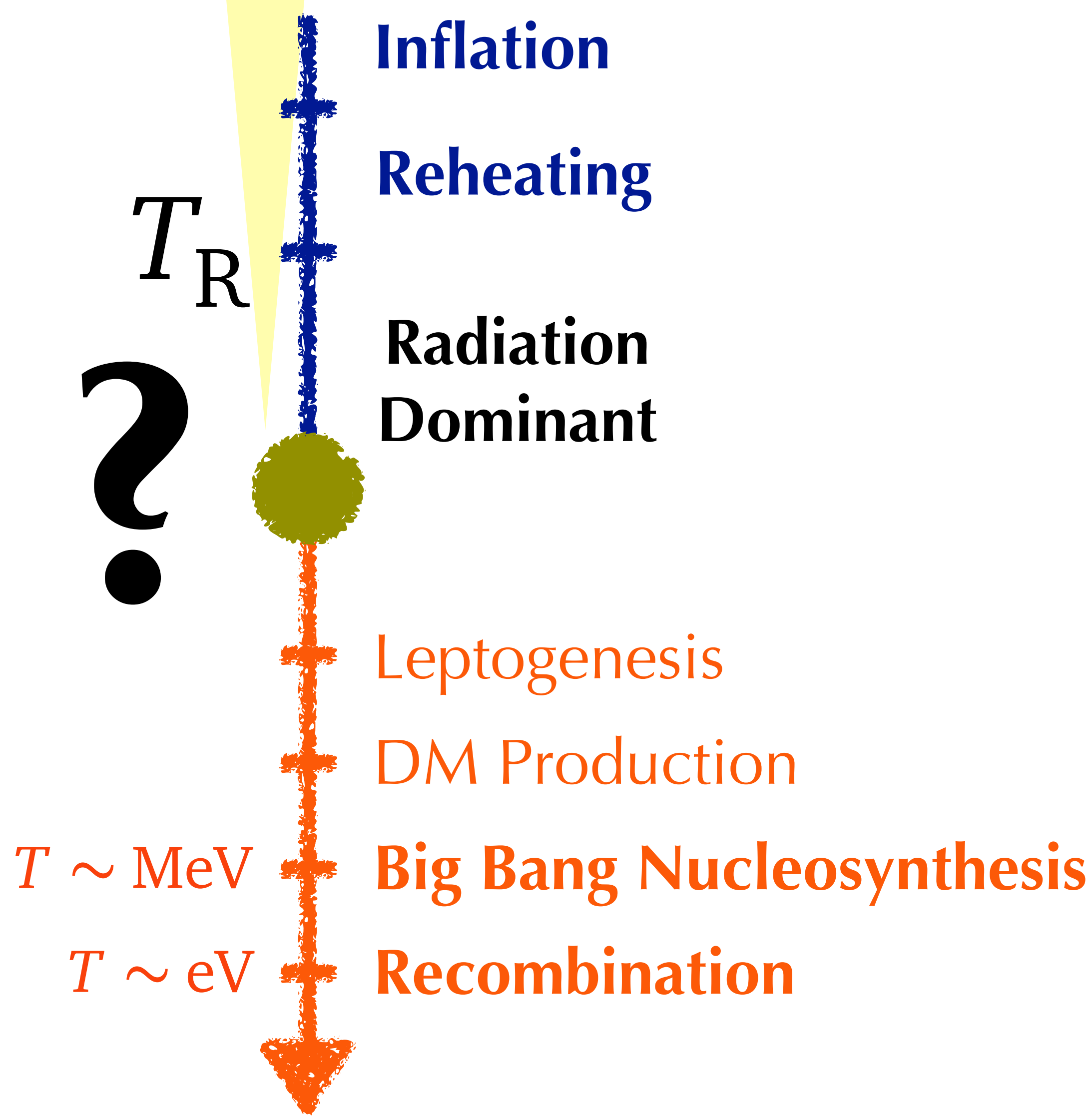
Introduction

■ Main Theme of This Talk

- **Thermalization of radiation: When and How?**
- Implications on Heavy particle production and Symmetry restoration.



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Outline

- Introduction
- Naive Estimation
- Bottom-up Thermalization
- Implications: Heavy particle production and Symmetry restoration
- Summary

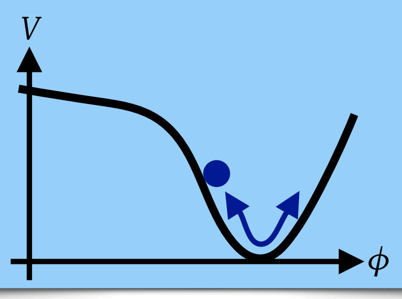
Naive Estimation

Reheating

- There are several paths to produce radiation.
- Depend on **interactions** between inflaton and radiation.

Far From Thermal Equilibrium

- Homogeneous condensation of inflaton



Large

$\lambda^2 I^2 |\chi|^2, \lambda I \bar{\psi}\psi, \cdots$

Non-perturbative Production

Instant Preheating + Thermal Dissipation

[KM, Nakayama;...]

Non-thermal Fixed-point

- Turbulence

[Micha, Tkachev; Berges, Rothkopf, Schmidt; ...]

Small

Planck-suppressed operator

$c \frac{I}{M_{\rm pl}} F^a_{\mu\nu} F^{a\mu\nu}, \cdots$

Perturbative Production

Thermal Equilibrium

- Fluid of thermal plasma

Reheating

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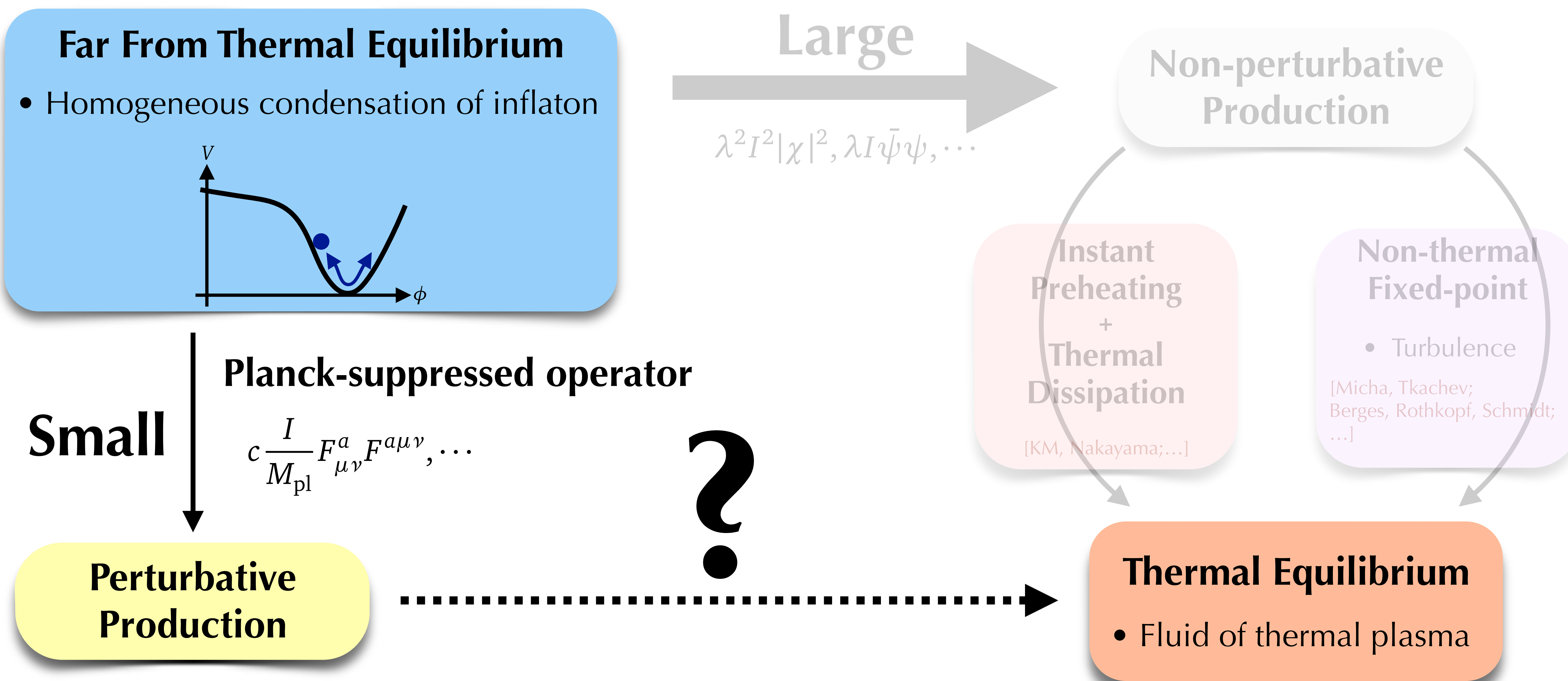

Reheating

- Necessary ingredients to study thermalization after inflation.
- Three parameters:
  - Mass of inflaton: $\mathbf{m_I}$
  - Decay rate of inflaton: $\mathbf{\Gamma_I}$
  - Dominant coupling of decay products: $\boldsymbol{\alpha}$

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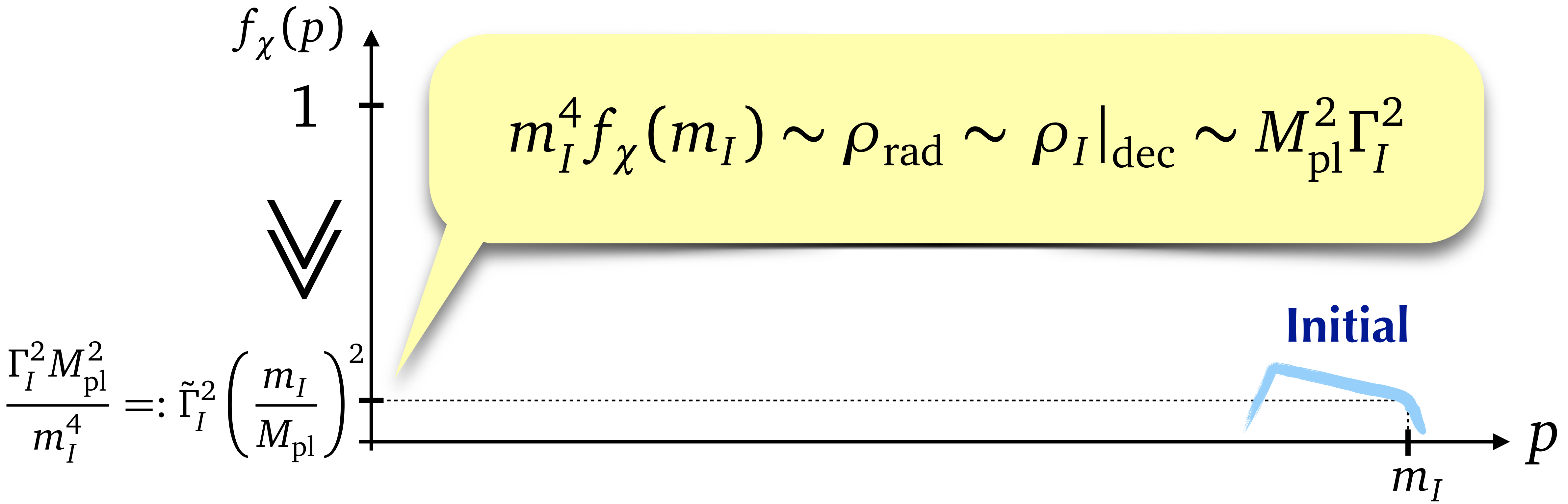
Rescaled prm:

$$\tilde{\Gamma}_I \equiv \frac{\Gamma_I}{m_I^3/M_{\rm pl}^2}$$

- See what happens at $\Gamma_I \sim H$ as an illustration.

$$m_I^4 f_\chi(m_I) \sim \rho_{\rm rad} \sim \rho_I|_{\rm dec} \sim M_{\rm pl}^2 \Gamma_I^2$$



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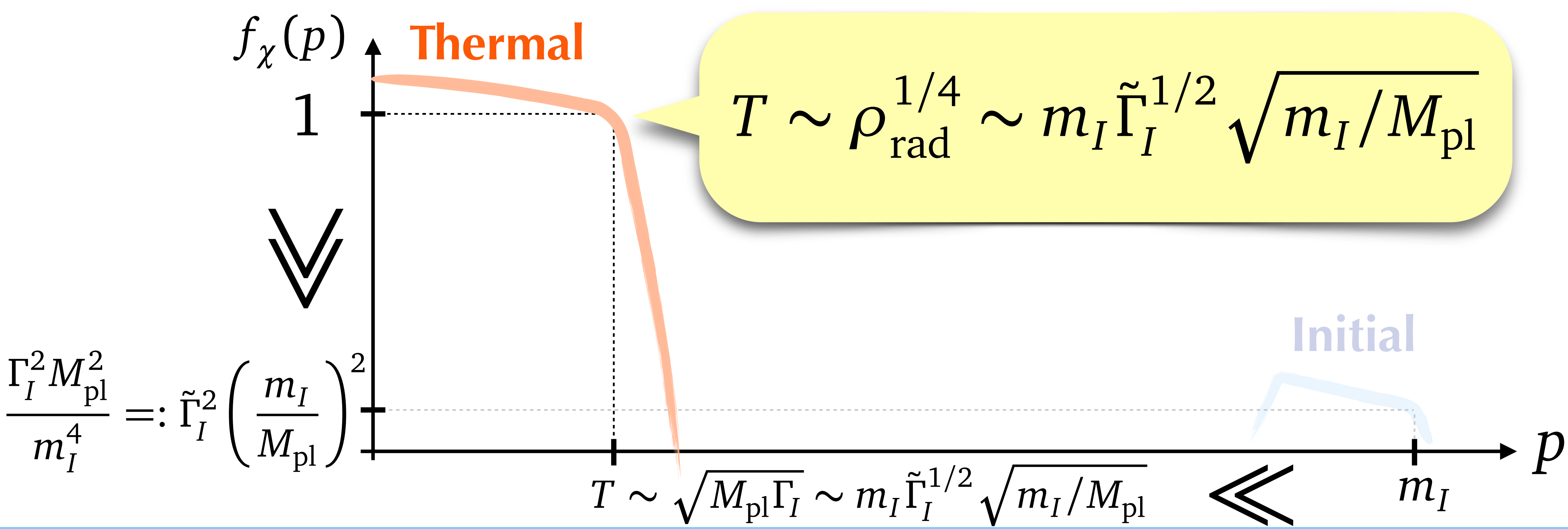

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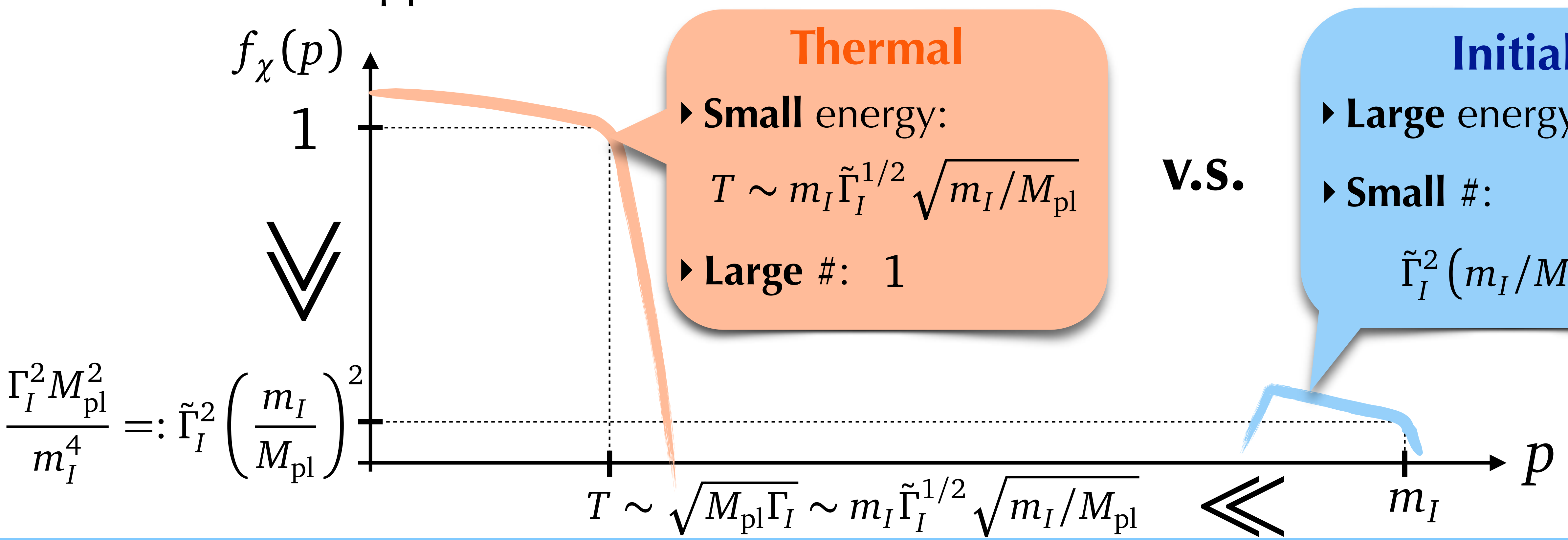
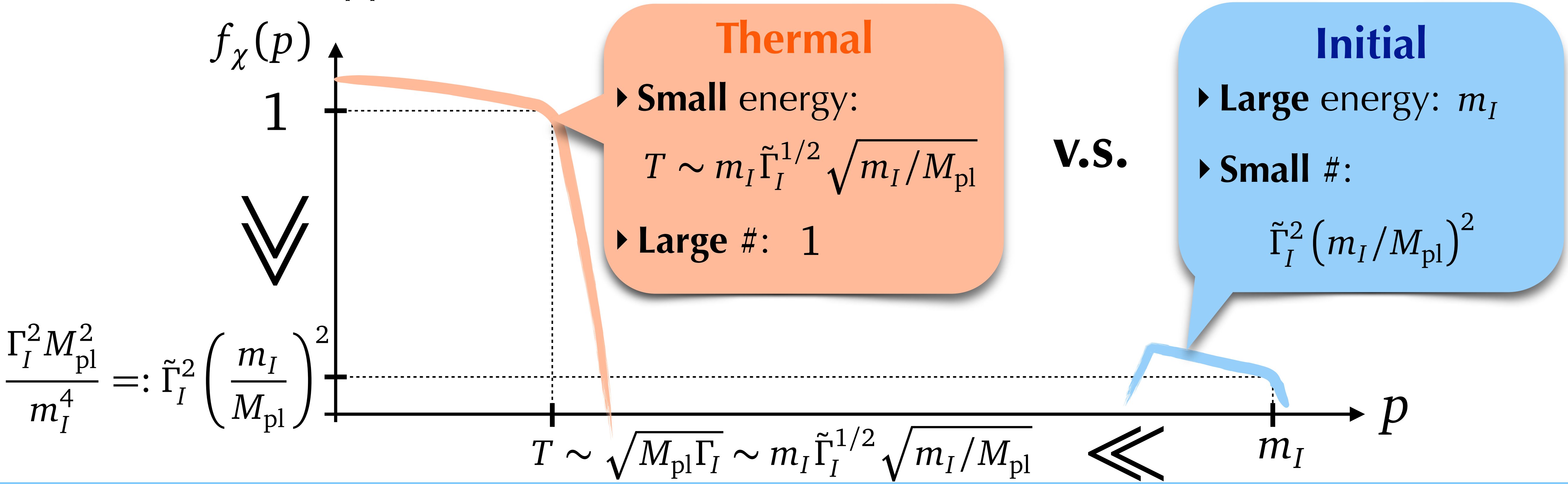


Reheating

■ **Thermalization after Reheating via Small Decay Rate**

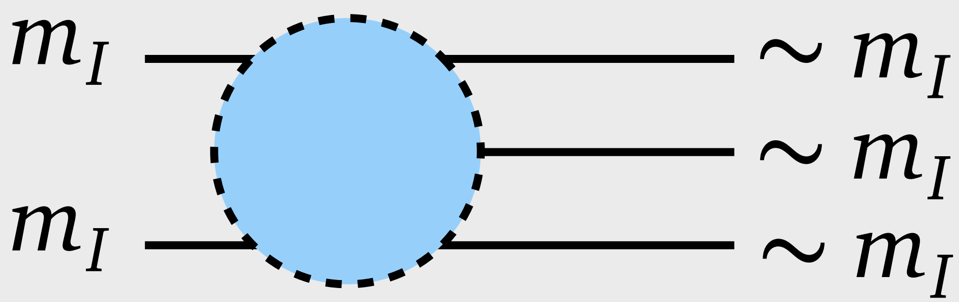
- **Number violating process plays crucial roles!**
  - ➡ Time scale of #-violating process v.s. Hubble parameter, H
    - Depends on **α**
- See what happens at $\Gamma_I \sim H$ as an illustration.



Naive Estimation

■ **Number Violating Processes** (*naive estimation*)

- Apparently, #-violating "hard" process seems to efficiently increase # and reduce energy per one-particle…



$$\Gamma \sim \frac{\alpha^3}{m_I^2} \times f_\chi(m_I) m_I^3$$

$$\sim \alpha^3 m_I \tilde{\Gamma}_I^2 \left(\frac{m_I}{M_{\rm pl}} \right)^2$$

➡ **Delayed thermalization ???** [Ellis et al.,1987; McDonald, '00; Allahverdi, '00; …]

$$\frac{\Gamma}{H} \sim \frac{\Gamma}{\Gamma_I} \sim \alpha^3 \tilde{\Gamma}_I \ll 1$$

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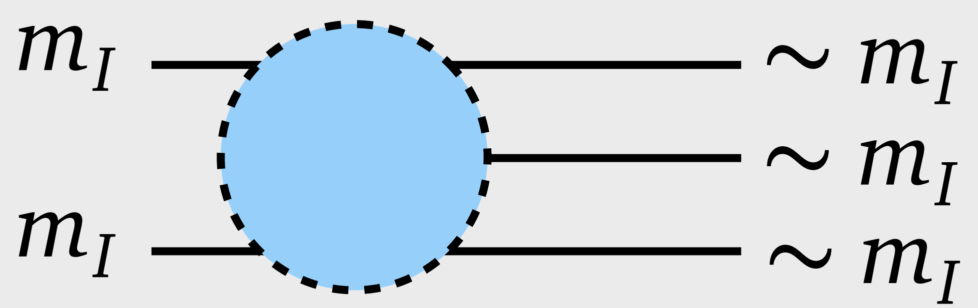

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[redacted] $\sim m_I$ [redacted] m_I

m_I [redacted] $\sim \alpha^3 m_I \tilde{\Gamma}_I^2 \left(\frac{m_I}{M_{\rm pl}}\right)^2$

WRONG…

➡ **De** [redacted] **…!?** [Ellis et al.,1987; McD [redacted]

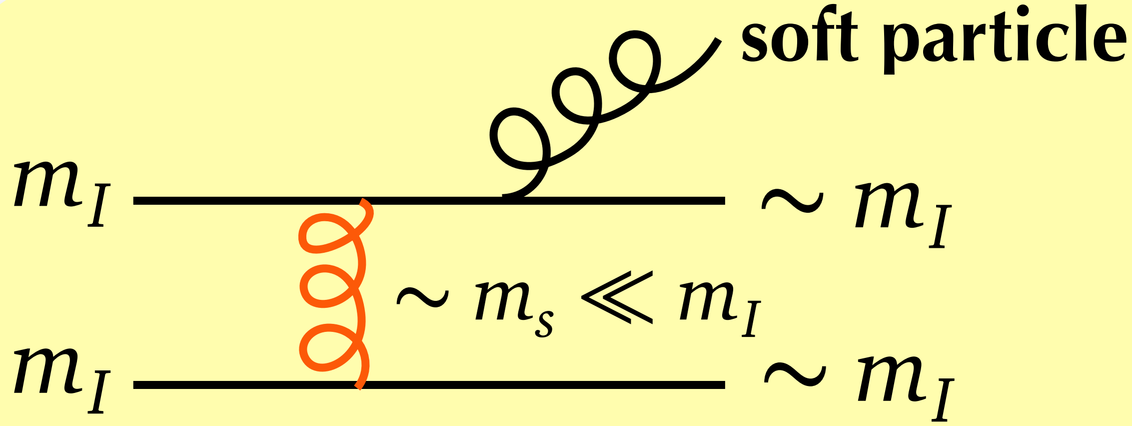
$$\frac{\Gamma}{H} \sim \frac{\Gamma}{\Gamma_I} \sim \alpha^3 \tilde{\Gamma}_I \ll 1$$

Bottom-up Thermalization

Thermalization

■ "Soft" Number Violating Processes

- t-channel enhancement of "soft" processes



w/ $m_s^2 \sim \alpha \int_p \frac{f_\chi(p)}{p}$

$$\frac{\Gamma}{H} \sim \frac{1}{\Gamma_I}\frac{\alpha^3}{m_s^2} f(m_I) m_I^3 \sim \alpha^2 \frac{m_I}{\Gamma_I} \gg 1$$

$$\sim \tilde{\Gamma}_I^{-1}\left(\alpha \frac{M_{\rm pl}}{m_I}\right)^2$$

[Davidson, Sarkar, '00]

➡ Rapid production of soft particles

- This process **alone cannot** efficiently reduce the energy of hard primaries.
- This is because the energy loss per event is too small.

Thermalization

■ Thermalization of under-occupied primaries

- Isotropic and under-occupied distribution of hard primaries: f(p) ≪ 1.
- Hard primaries are charged under non-Abelian (SM) gauge group.

Rescaled prms:

$$\tilde{t} \equiv m_I t, \quad \tilde{\Gamma}_I \equiv \frac{\Gamma_I}{m_I^3/M_{\rm pl}^2}$$

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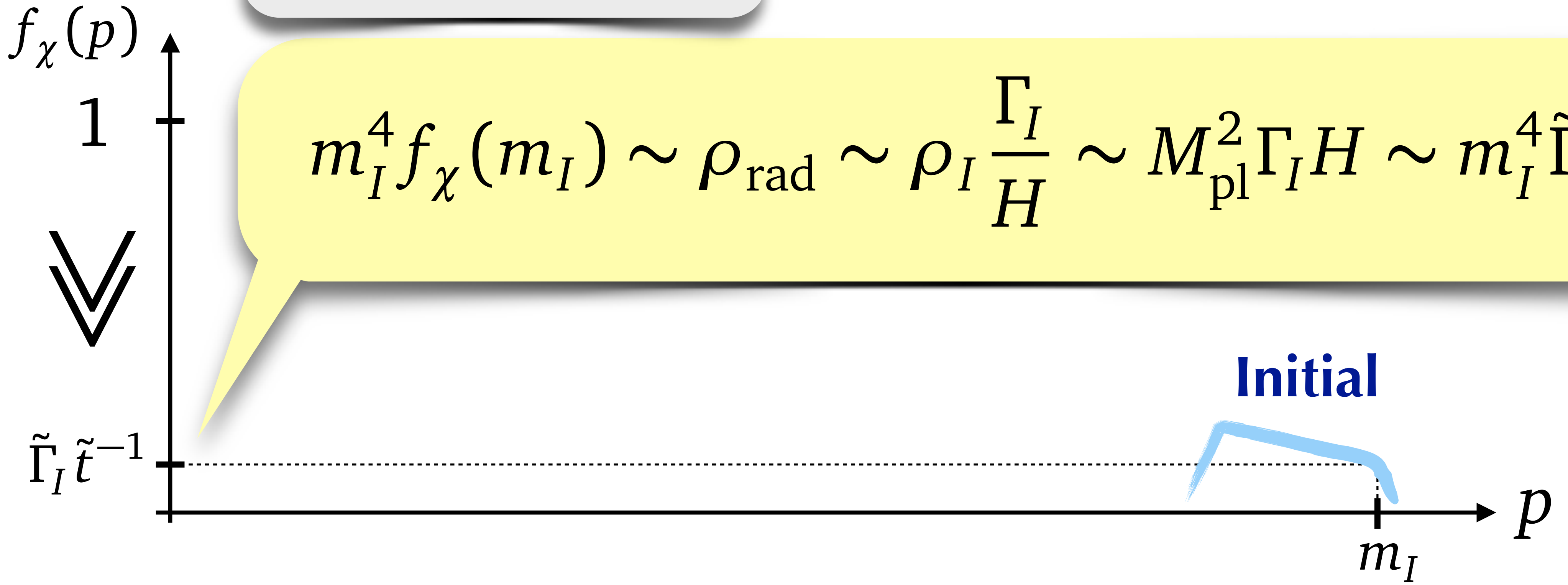

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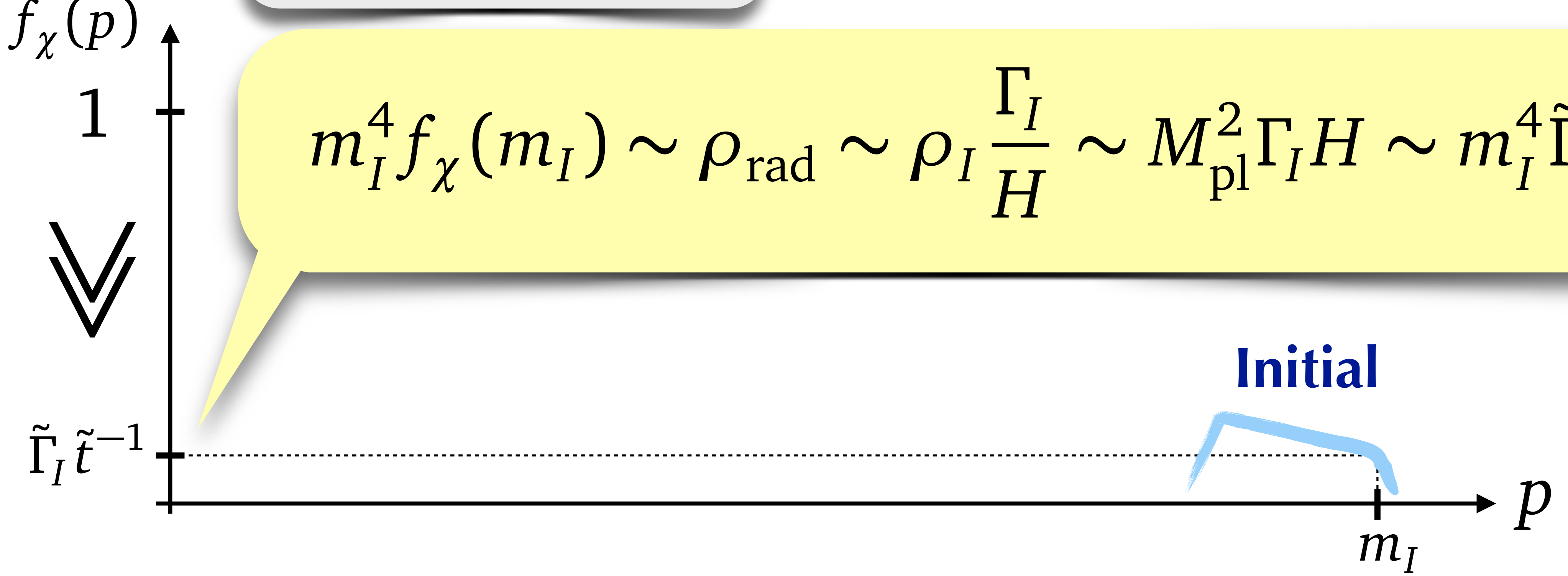

Basic Formalism

[Arnold, Moore and Yaffe, hep-ph/0209353]

■ Effective Kinetic Equations

- **Assumption**: weak coupling, perturbative occupancy and modes w/ $p \gg m_s$.

$$\alpha \ll 1 \qquad \alpha f(p) \ll 1 \qquad m_s^2 \sim \alpha \int_p \frac{f(p)}{p}$$

- **Kinetic Equations**: dynamics of quasi-particles

$$\partial_t f(p,t) = -\mathcal{C}_{2\leftrightarrow 2}[f](p) - \mathcal{C}_{1\leftrightarrow 2}[f](p)$$ at leading order in αf

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‣ Diffusion via t-channel enhancement

→ Random Walk in momentum space…

$$\Delta p^2 \sim \hat{q}_{\rm el} t$$

$$\hat{q}_{\rm el} \sim \int d^2 q_\perp \frac{d\Gamma_{\rm el}}{dq_\perp^2} q_\perp^2 \sim \alpha^2 \int_{\mathbf{p}'} f(\mathbf{p}')[1+f(\mathbf{p}')]$$ w/ $$\frac{d\Gamma_{\rm el}}{dq_\perp^2} \sim \frac{\alpha^2}{q_\perp^2 (q_\perp^2 + m_s^2)} \int_{\mathbf{p}'} f(\mathbf{p}')[1+f(\mathbf{p}')]$$

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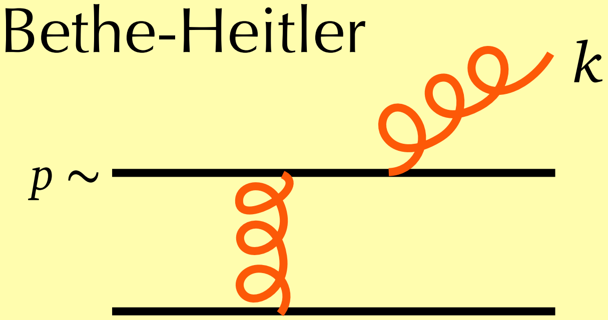
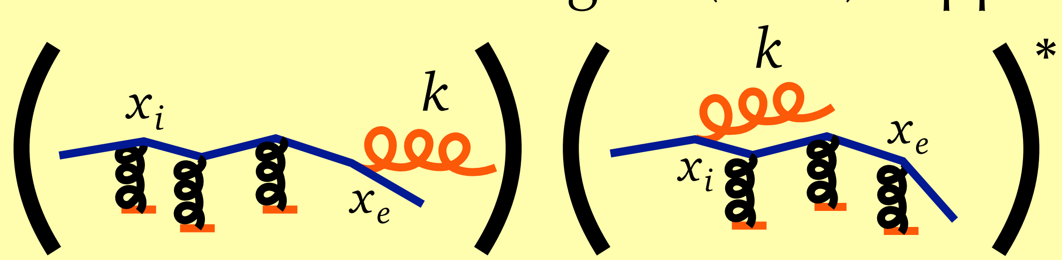
‣ Splitting via t-channel enhancement (effective "1 to 2" process)

- Bethe-Heitler



v.s.

- Landau-Pomeranchuk-Migdal (LPM) suppression

Re 

➡**Destructive interference** unless the phase varies

$1 \lesssim k\cdot(x_e - x_i) \sim tk\theta^2$ w/ $\theta \sim k_\perp/k \sim \sqrt{\hat{q}_{\rm el} t}/k$ ➡ $t > \sqrt{\frac{k}{\hat{q}_{\rm el}}} \equiv t_{\rm form}$

$$\Rightarrow \quad \Gamma_{\rm split}(k) \sim \alpha \min\left[\Gamma_{\rm el},\quad t_{\rm form}^{-1}\right]$$

Thermalization

■ Thermalization of under-occupied primaries

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- Hard primaries are charged under non-Abelian (SM) gauge group.

➡ **Thermalization proceeds from the soft sector.** [Kurkela, Moore, '11; Kurkela, Lu, '14; Baier et al., '00]

(0). Initial

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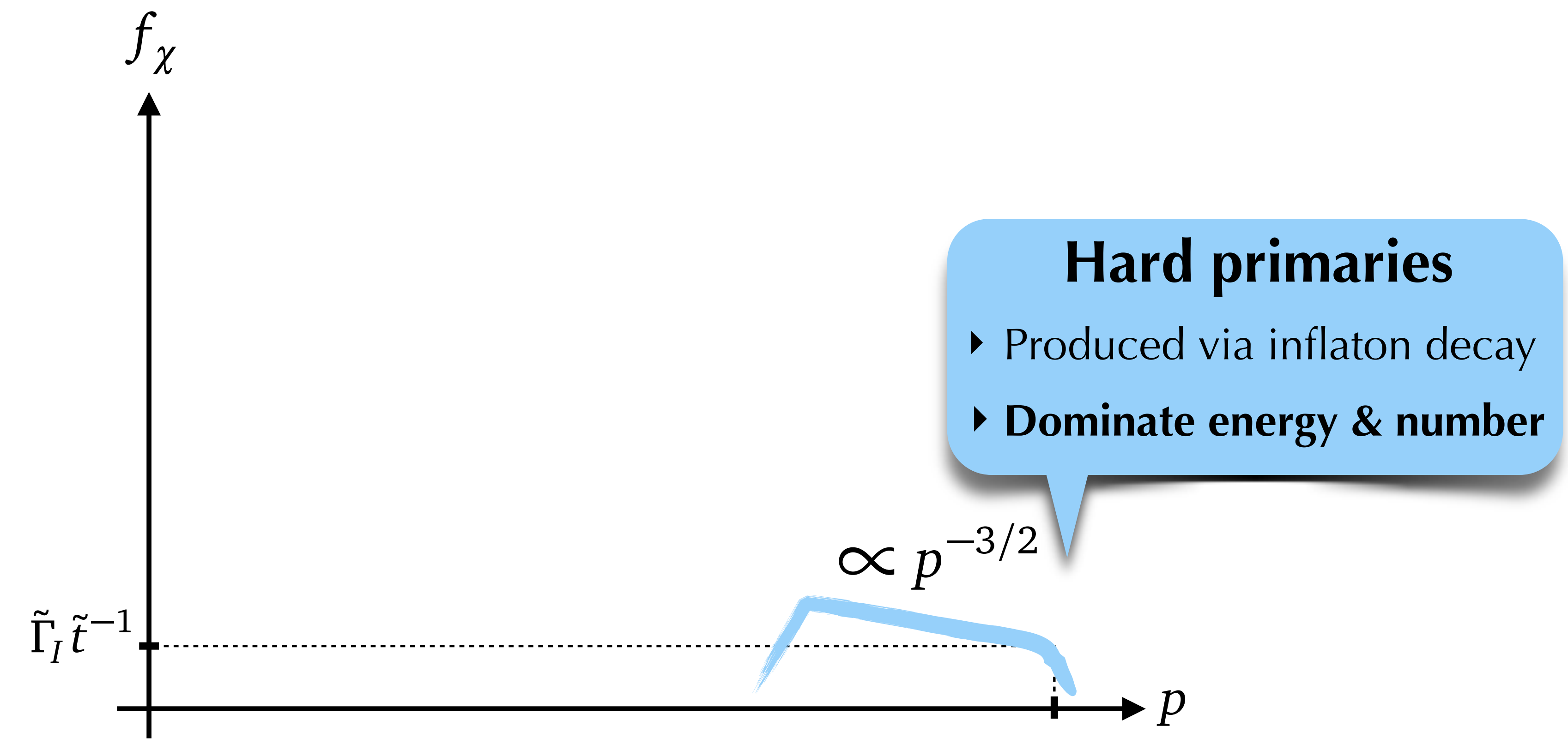

[KM, K.Harigaya]

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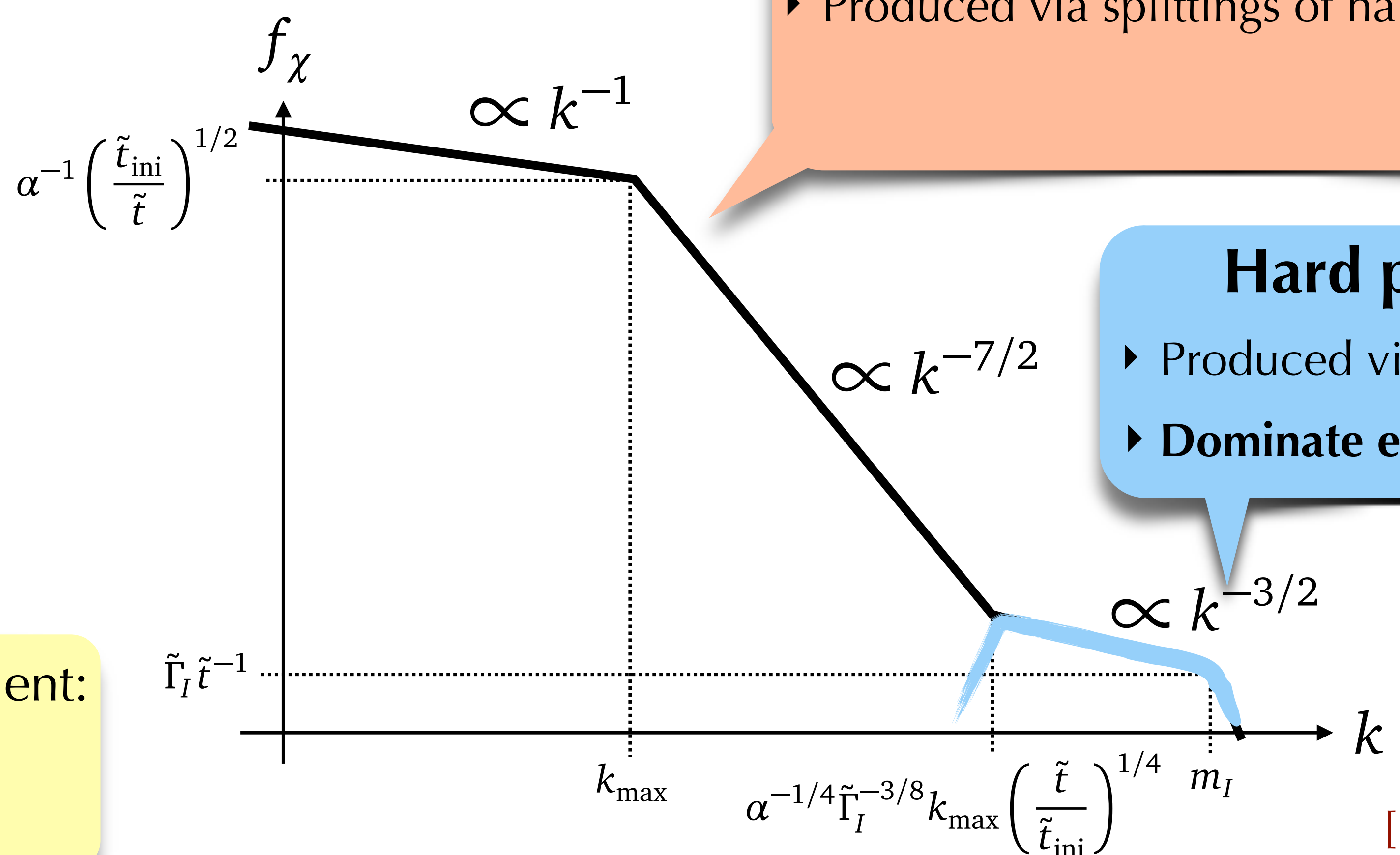
$$k_{\rm max}/m_I \sim \alpha \tilde{\Gamma}_I^{1/2}$$
$$\tilde{t}_{\rm ini} = m_I/k_{\rm max}$$

Elastic scattering should be efficient:

$$\Gamma_{\rm el} > H \leftrightarrow \tilde{t} > \tilde{t}_{\rm ini}$$



Soft daughters
- Produced via splittings of hard primaries

Hard primaries
- Produced via inflaton decay
- **Dominate energy & number**

[KM, K.Harigaya]

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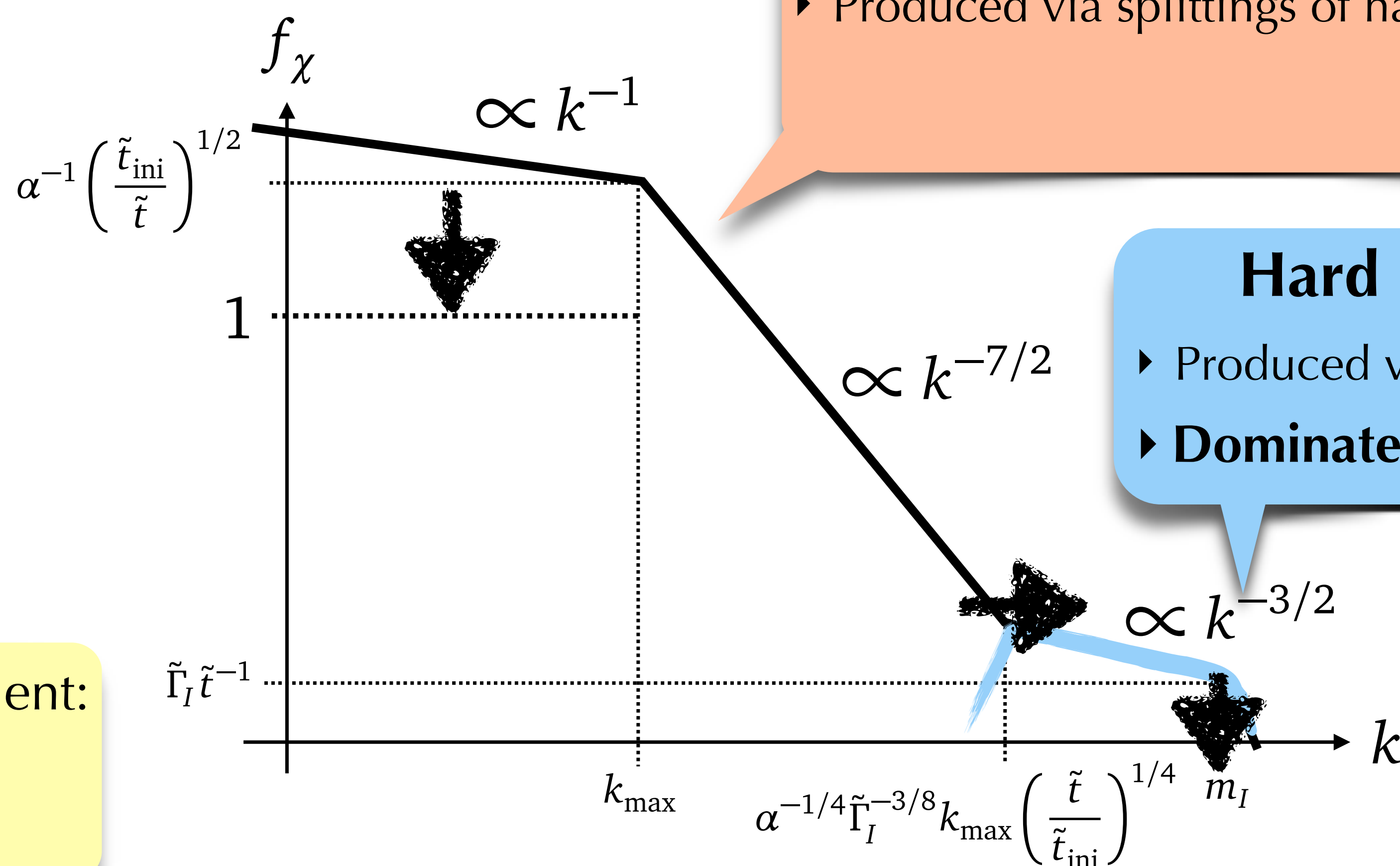
$$\tilde{t}_{\rm ini} = m_I/k_{\rm max}$$

Elastic scattering should be efficient:

$$\Gamma_{\rm el} > H \leftrightarrow \tilde{t} > \tilde{t}_{\rm ini}$$

Soft daughters
- Produced via splittings of hard primaries

Hard primaries
- Produced via inflaton decay
- **Dominate energy/number**



[KM, K.Harigaya]

Thermalization

■ Thermalization of under-occupied primaries

- Isotropic and under-occupied distribution of hard primaries: f(p) ≪ 1.
- Hard primaries are charged under non-Abelian (SM) gauge group.

➡ **Thermalization proceeds from the soft sector.** [Kurkela, Moore, '11; Kurkela, Lu, '14; Baier et al., '00]

$(ii).\ \tilde{t}_{\rm soft} \lesssim \tilde{t} \lesssim \tilde{t}_{\rm max}$

Rescaled prms:

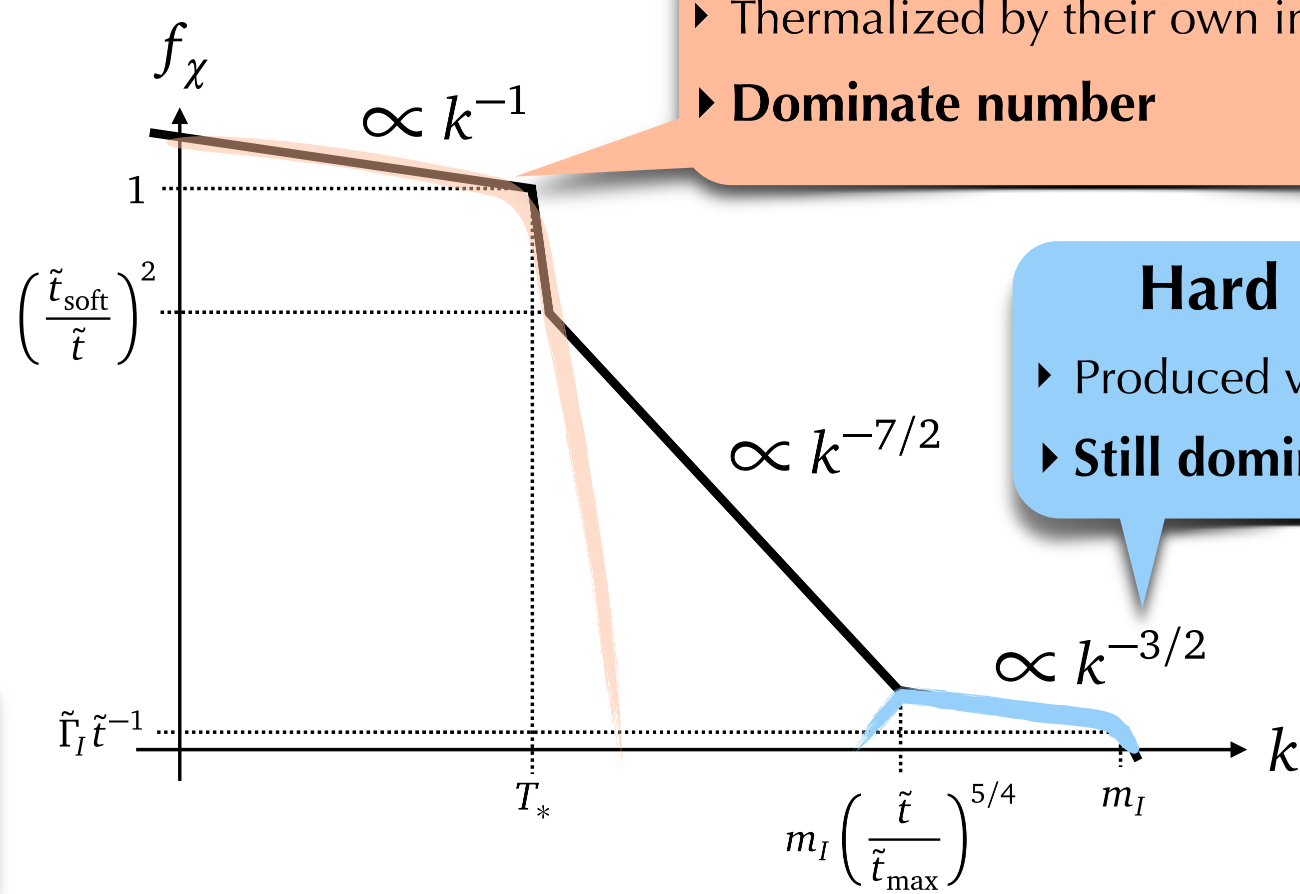
$$\tilde{t} \equiv m_I t, \quad \tilde{\Gamma}_I \equiv \frac{\Gamma_I}{m_I^3/M_{\rm pl}^2}$$

$$T_*/m_I \sim \alpha^4 \tilde{\Gamma}_I \tilde{t}$$

$$\tilde{t}_{\rm soft} = \alpha^{-2} \tilde{t}_{\rm ini}$$

Soft daughters are thermalized:

$$\alpha^2 T_* > H \leftrightarrow \tilde{t} > \tilde{t}_{\rm soft}$$



Soft daughters
- Thermalized by their own interactions
- **Dominate number**

Hard primaries
- Produced via inflaton decay
- **Still dominate energy**

[KM, K.Harigaya]

Thermalization

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$(iii).\ \tilde{t}_{\rm max} \lesssim \tilde{t} \lesssim \tilde{t}_{\rm rh}$

Rescaled prms:

$$\tilde{t} \equiv m_I t,\ \ \tilde{\Gamma}_I \equiv \frac{\Gamma_I}{m_I^3/M_{\rm pl}^2}$$

$$T_* \sim \rho_{\rm rad}^{1/4} \sim \left(\tilde{\Gamma}_I \tilde{t}^{-1}\right)^{1/4} m_I$$

$$\tilde{t}_{\rm max} \sim \alpha^{-16/5} \tilde{\Gamma}_I^{-3/5}$$

Hard primaries soon breaks up:

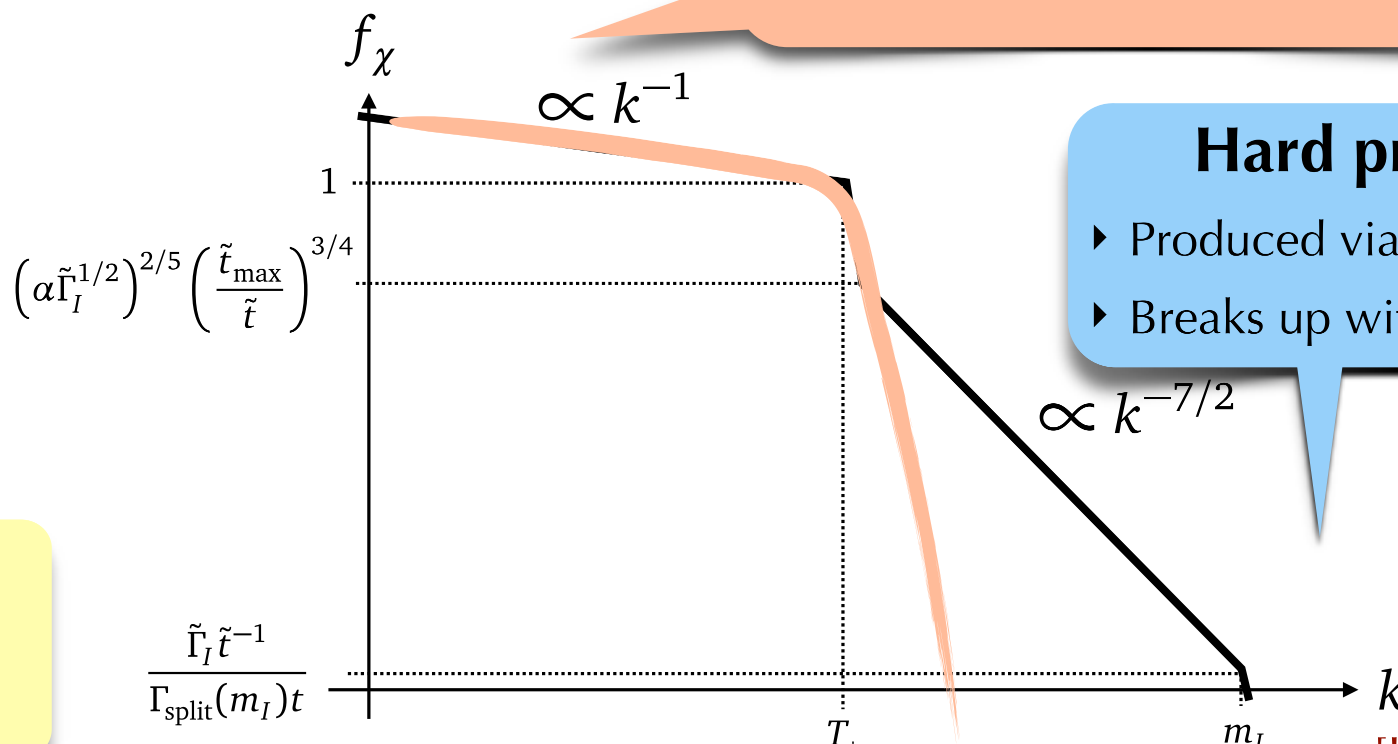
$$\Gamma_{\rm split}(m_I) > H \leftrightarrow \tilde{t} > \tilde{t}_{\rm max}$$

Soft daughters

- Thermalized by their own interactions
- **Dominate number/energy**

Hard primaries

- Produced via inflaton decay
- Breaks up within $\Gamma_{\rm split}(m_I)$



[KM, K.Harigaya]

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Rescaled prms:

$$\tilde{t} \equiv m_I t, \ \ \tilde{\Gamma}_I \equiv \frac{\Gamma_I}{m_I^3/M_{\rm pl}^2}$$

$$T \sim T_{\rm R}(\tilde{t}_{\rm rh}/\tilde{t})^{1/2}$$

$$\tilde{t}_{\rm rh} \sim \tilde{\Gamma}_I^{-1}\frac{M_{\rm pl}^2}{m_I^2}$$

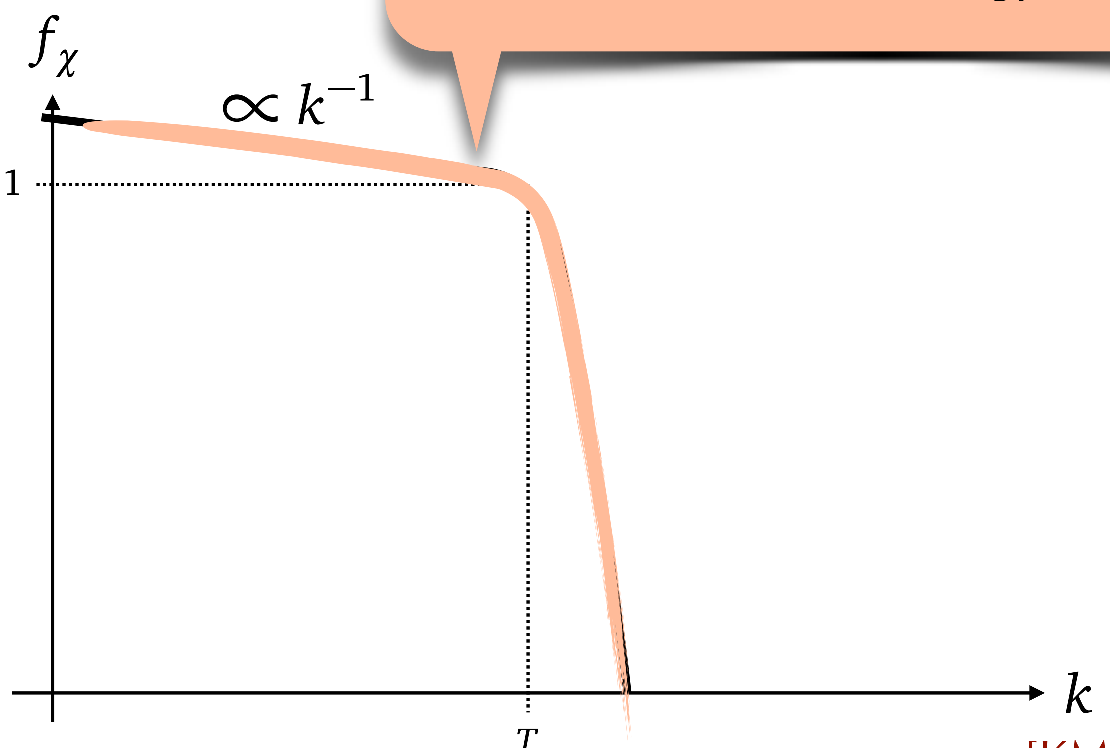
$$T_{\rm R} \sim \sqrt{\Gamma_I M_{\rm pl}}$$

Inflaton decays completely:

$$\Gamma_I > H \leftrightarrow \tilde{t} > \tilde{t}_{\rm rh}$$

Thermal Plasma

- Thermalized by their own interactions
- **Dominate number/energy**



[KM, K.Harigaya]

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$(i).\ \tilde{t}_{\rm ini} \lesssim \tilde{t} \lesssim \tilde{t}_{\rm soft}$	Production of soft population and its thermalization	$\tilde{t}_{\rm ini} \sim \alpha^{-1}\tilde{\Gamma}_I^{-1/2}$
$(ii).\ \tilde{t}_{\rm soft} \lesssim \tilde{t} \lesssim \tilde{t}_{\rm max}$	Splittings of hard primaries in thermalized soft sector	$\tilde{t}_{\rm soft} \sim \alpha^{-3}\tilde{\Gamma}_I^{-1/2}$
$(iii).\ \tilde{t}_{\rm max} \lesssim \tilde{t} \lesssim \tilde{t}_{\rm rh}$	Thermalized radiation (with a small tail)	$\tilde{t}_{\rm max} \sim \alpha^{-16/5}\tilde{\Gamma}_I^{-3/5}$
$(iv).\ \tilde{t}_{\rm rh} \lesssim \tilde{t}$	Completion of reheating	$\tilde{t}_{\rm rh} \sim \tilde{\Gamma}_I^{-1} M_{\rm pl}^2/m_I^2$

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➡ **One can rely on** $T \sim \rho_{\rm rad}^{1/4}$ **after** $\tilde{t}_{\rm max} \sim \alpha^{-16/5}\tilde{\Gamma}_I^{-3/5}$

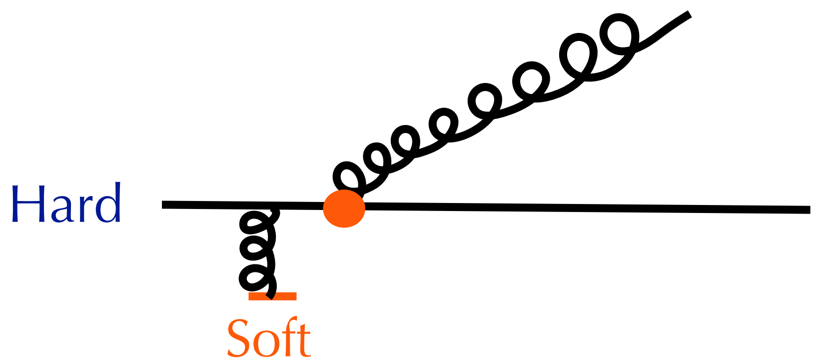
[KM, K.Harigaya]

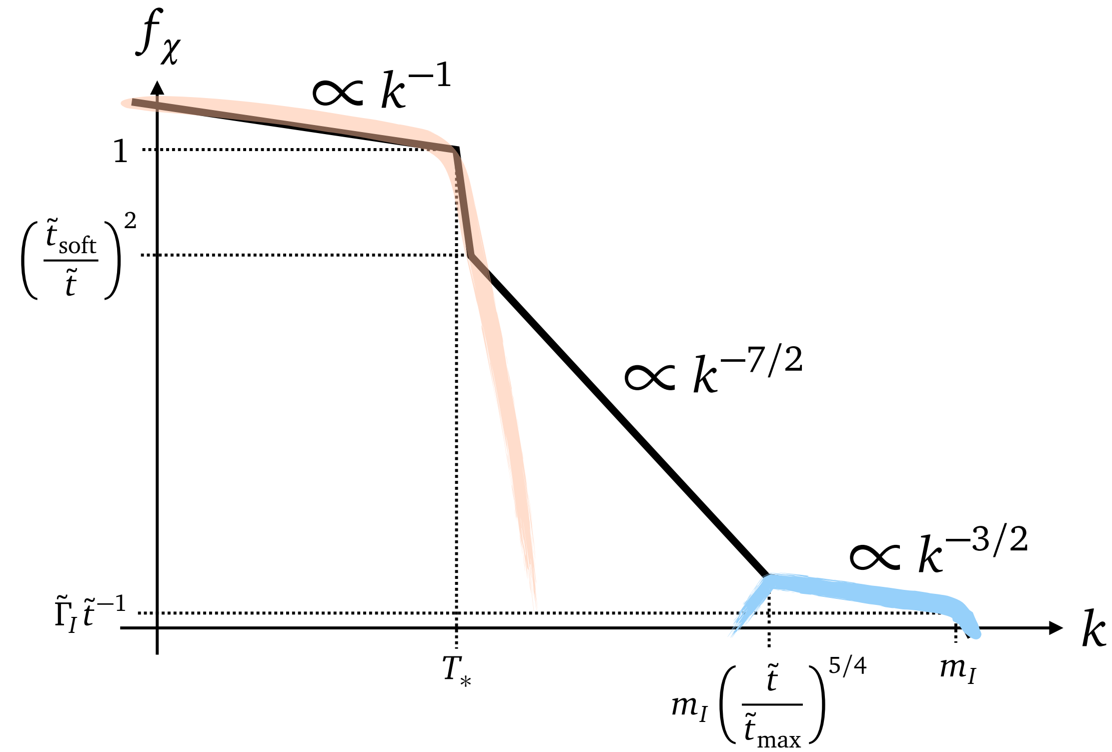
Thermalization

[Kurkela, Moore, ’11; Kurkela, Lu, ’14; Baier et al., ‘00]

■ Bottleneck Process determines t_{max}

- Splitting of remaining hard primaries





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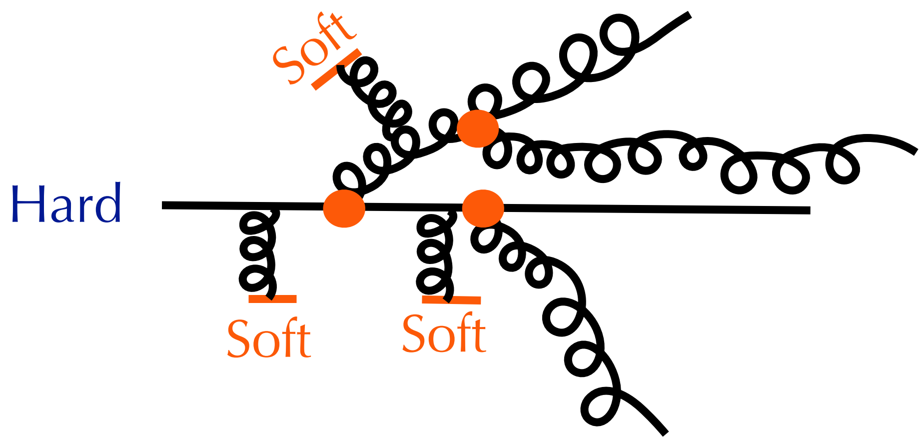

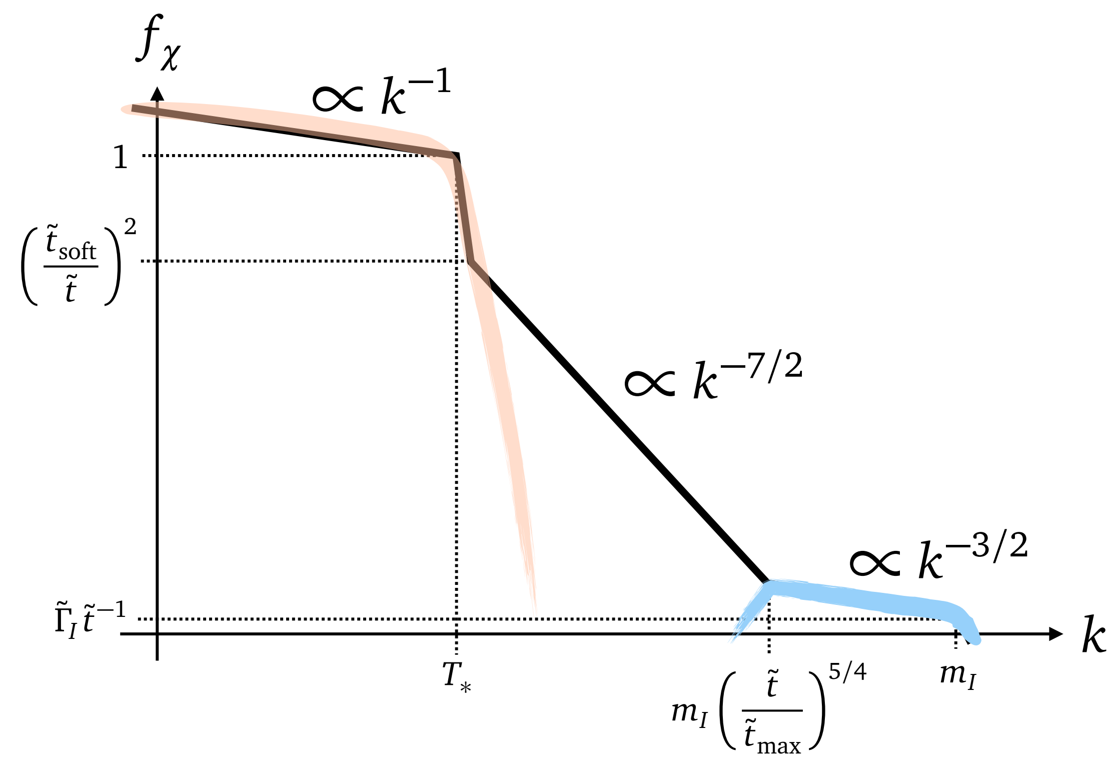

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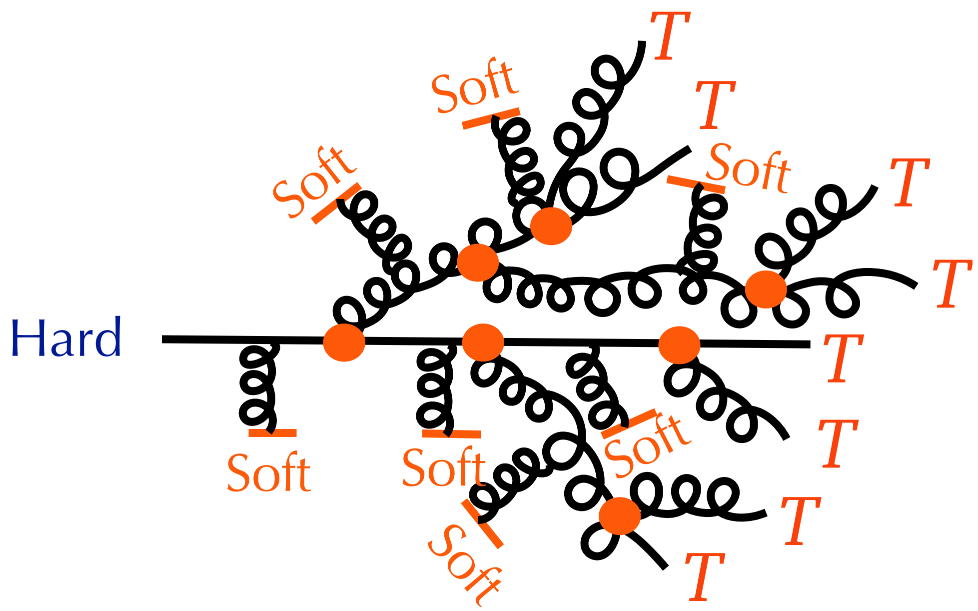

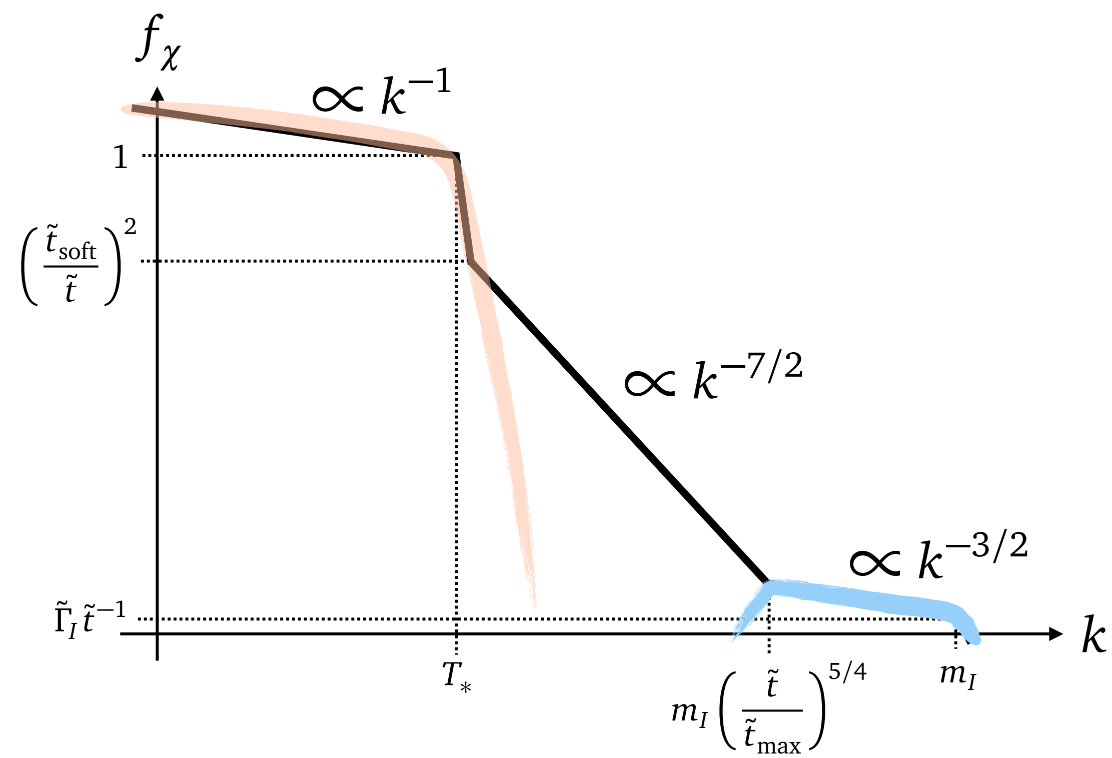

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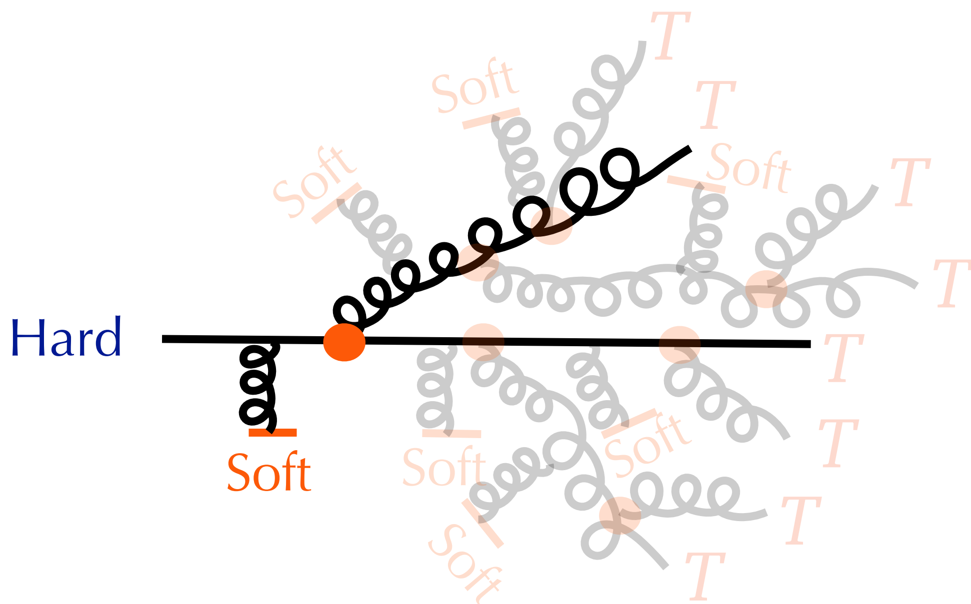

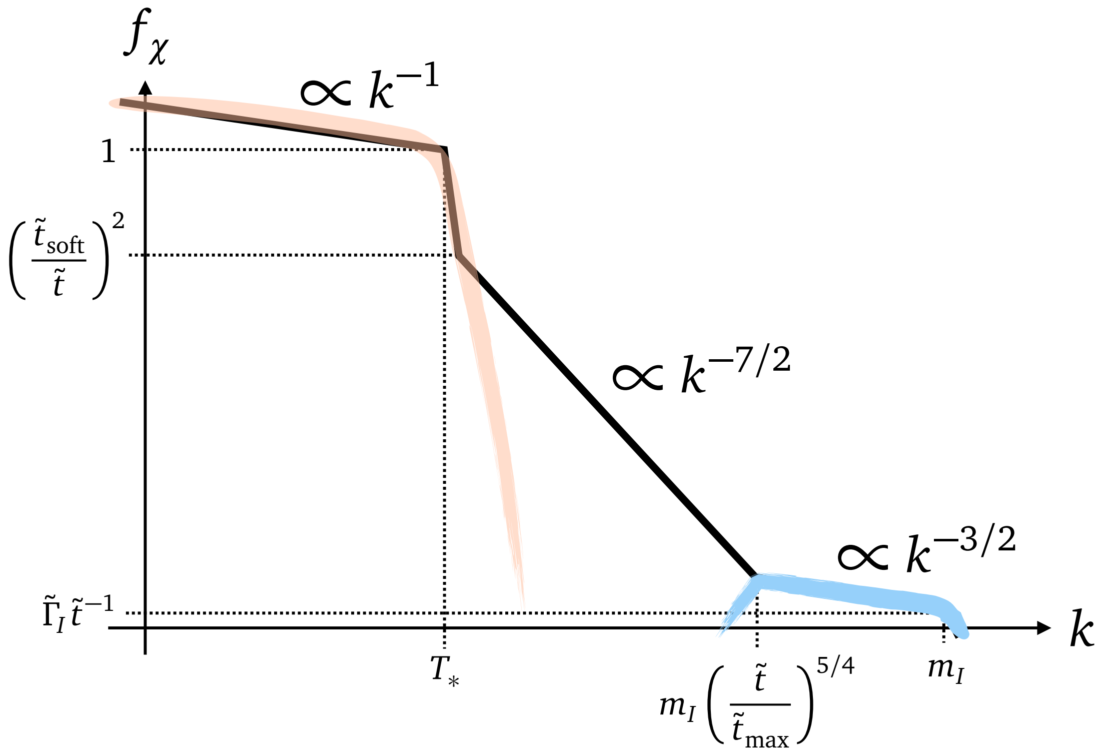

- Time scale of bottleneck process:

$$1 \sim \Gamma_{\rm split}(m_I)t \leftrightarrow t \sim (\alpha^2 T)^{-1}\sqrt{m_I/T}$$

➡ Plugging in $T_*/m_I \sim \alpha^4 \tilde{\Gamma}_I \tilde{t}, \ldots$

$$\tilde{t}_{\rm max} \sim \alpha^{-16/5}\tilde{\Gamma}_I^{-3/5}$$

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$(iv).\ \tilde{t}_{\rm rh} \lesssim \tilde{t}$ Completion of reheating

$$\tilde{t}_{\rm ini} \sim \alpha^{-1}\tilde{\Gamma}_I^{-1/2}$$
$$\wedge$$
$$\tilde{t}_{\rm soft} \sim \alpha^{-3}\tilde{\Gamma}_I^{-1/2}$$
$$\wedge$$
$$\tilde{t}_{\rm max} \sim \alpha^{-16/5}\tilde{\Gamma}_I^{-3/5}$$
?
$$\tilde{t}_{\rm rh} \sim \tilde{\Gamma}_I^{-1} M_{\rm pl}^2/m_I^2$$

➡ **Thermalized before the complete decay of inflaton**

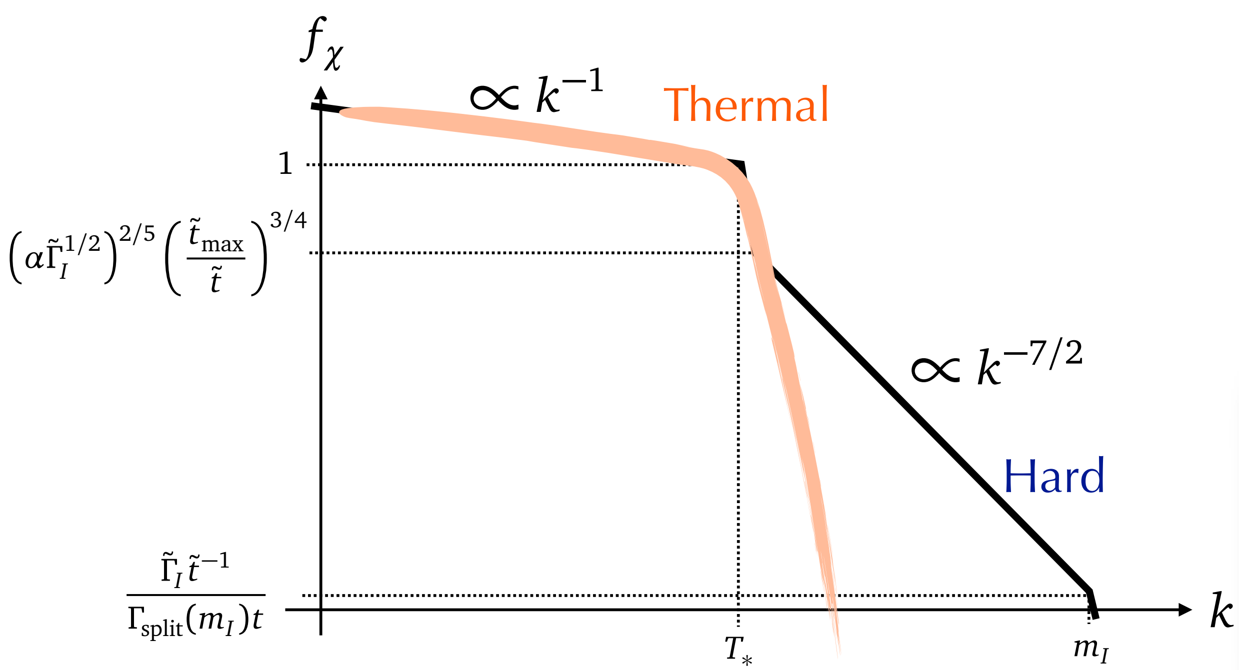
$$\tilde{t}_{\rm rh} > \tilde{t}_{\rm max} \leftrightarrow \alpha > 10^{-3}\left(\frac{m_I}{10^{13}\,{\rm GeV}}\right)^{5/8}\tilde{\Gamma}_I^{1/8}$$

Implications

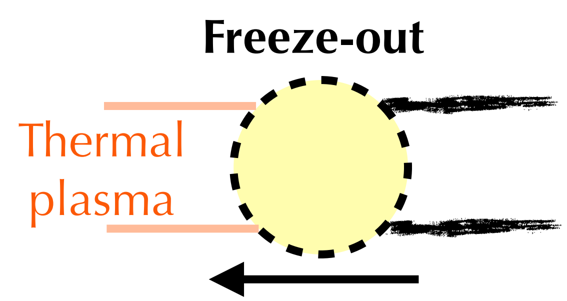
Heavy Particle

■ Particle Production Processes w/ $m \gg T_\mathrm{R}$

- Production from direct inflaton decay $\left.\frac{n_\mathrm{dir}}{s}\right|_\mathrm{now} \simeq \frac{3T_\mathrm{R}}{4m_I}\mathrm{Br}\,(I \to \text{Heavy particle})$
- Production from background plasma

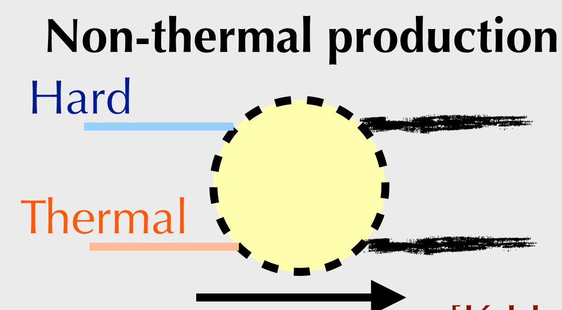


‣ Thermal Freeze-out ($T_* > T_\mathrm{FO} \gg T_\mathrm{R}$)



$$\left.\frac{n_\mathrm{th}}{s}\right|_\mathrm{now} \simeq \left.\frac{n_\mathrm{th}}{s}\right|_\mathrm{FO}\left(\frac{T_\mathrm{R}}{T_\mathrm{FO}}\right)^5$$

‣ Non-thermal production via scatterings between thermal plasma and hard primaries for $T m_I > m^2$



$$\left.\frac{n_\text{n-th}}{s}\right|_\mathrm{now} \simeq \left(\frac{\alpha_\mathrm{new}}{\alpha}\right)^2\left(\frac{T_\mathrm{R}}{m}\right)^3$$

for $m_I T_\mathrm{R} \gtrsim m^2$

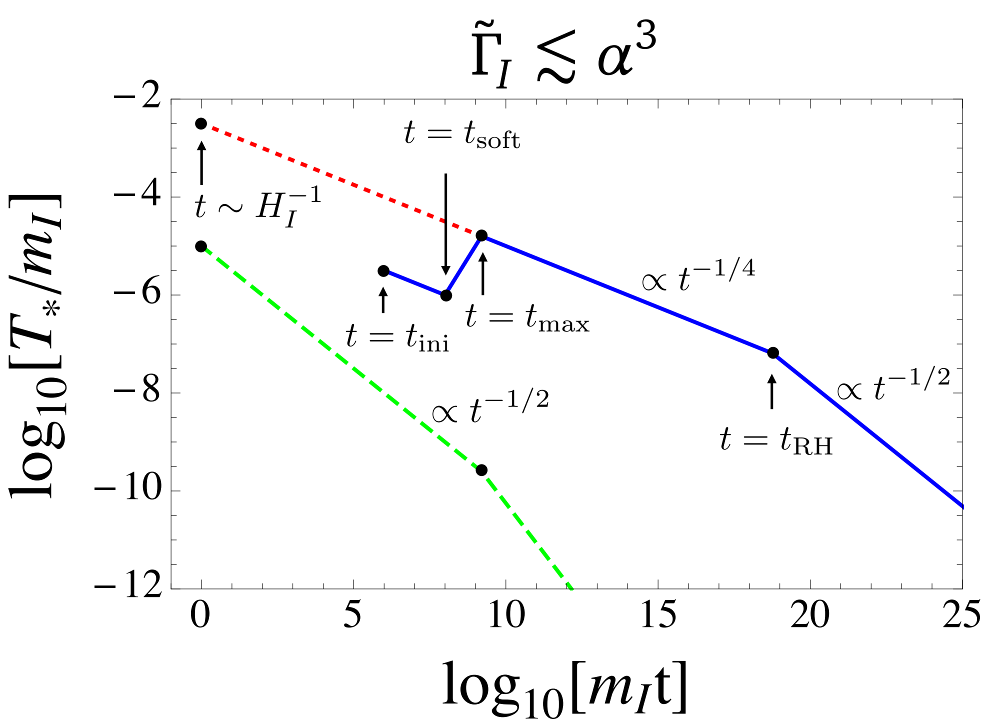
[K.Harigaya, KM, M.Kawasaki, M.Yamada]

Symmetry Restoration

■ Effective potential of scalar field

- Consider a scalar Φ, which is an order parameter and couples with radiation.
- Φ receives finite density corrections from background plasma.
  - Near Φ ~ 0: $m^2_{\phi,\mathrm{eff}} \sim \alpha_\phi \int_p \frac{f_\chi(p)}{p} = \alpha_\phi T_*^2$ *e.g.*, $g_\phi^2 \phi^2 |\chi|^2, g_\phi \phi \bar{\chi} \chi, \cdots$

- **Evolution of T***



- BBN sets lower bound to Γ_I.

$$\tilde{\Gamma}_I \sim 10^{-24} \left(\frac{T_R}{1\,\mathrm{MeV}}\right)^2 \left(\frac{m_I}{10^{13}\,\mathrm{GeV}}\right)^{-3}$$

- Maximum T* for the smallest Γ_I.

$$T_*|_{\max} \sim 5 \times 10^2\,\mathrm{GeV} \times \left(\frac{\alpha}{0.1}\right)^{4/5} \left(\frac{\tilde{\Gamma}_I}{10^{-24}}\right)^{2/5} \left(\frac{m_I}{10^{-13}\,\mathrm{GeV}}\right)$$

[KM, M.Yamada]

Summary

Summary

- A small decay rate of inflaton (e.g., Planck-suppressed one) results in a small number density of decay products, which apparently delays the thermalization after reheating.
- Contrary to the naive expectation, we found the **condition for instantaneous thermalization after T_R**, which is satisfied in most cases: $\tilde{t}_{\rm rh} > \tilde{t}_{\rm max} \leftrightarrow \alpha > 10^{-3} \left(\frac{m_I}{10^{13}\,\text{GeV}} \right)^{5/8} \tilde{\Gamma}_I^{1/8}$
- Heavy particles with $m \gg T_R$ can be produced scatterings between the hard primaries and the thermal plasma.
- The evolution of effective temperature is different from that obtained under instantaneous thermalization assumption.