5d/6d SCFTs
and
5-brane (Tao) diagrams

Kim, Sung-Soo
Korea Institute for Advanced Study

2015-12-01

IPMU

This work is based on various collaborations with

Hirotaka Hayashi (Instituto de Fisica Teorica),
Kimyeong Lee (KIAS),
Masato Taki (RIKEN),
Futoshi Yagi (KIAS)

arXiv:1504.03672

arXiv:1505.04439

arXiv:1509.03300

We report
a new understanding of 5d SCFTs
in connection with 6d SCFTs
through "Tao web diagrams"

6d SCFTs

(2,0) theory: worldvolume theory of M5 branes but mysterious…

(1,0) theory: F-theory classification.

[Heckman-Morrison-Vafa ’13]
[Del Zotto-Heckman-Tomasiello-Vafa ’14]
[many more …]

5d SCFTs (simple example)

SU(2) gauge theory with **N_f = 0,1,…,7** flavors has non-trivial UV fixed point (Superconformal theory)

[Seiberg '96]

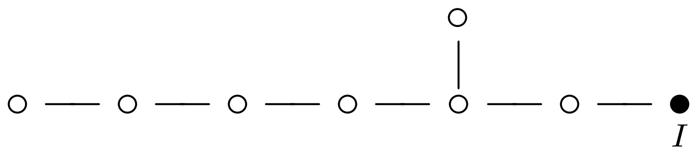
At UV fixed point, the global symmetry is enhanced

$$SO(2N_f) \times U(1)_I \subset E_{N_f+1}$$

N_f flavors

Topological symmetry from instanton particle

En Flavor Symmetry



$$SO(2N_f) \times U(1)$$

$$N_f = 7 \quad E_8$$

$$N_f = 6 \quad E_7$$

$$N_f = 5 \quad E_6$$

$$N_f = 4 \quad E_5 = SO(10)$$

$$N_f = 3 \quad E_4 = SU(5)$$

$$N_f = 2 \quad E_3 = SU(3) \times SU(2)$$

$$N_f = 1 \quad E_2 = SU(2) \times U(1)$$

$$N_f = 0 \quad E_1 = SU(2)$$

5d and 6d relations

6d N=(2,0)

A circle compactification of **6d** (2,0) theory
= 5d **Maximally** **supersymmetric Yang-Mills** theory

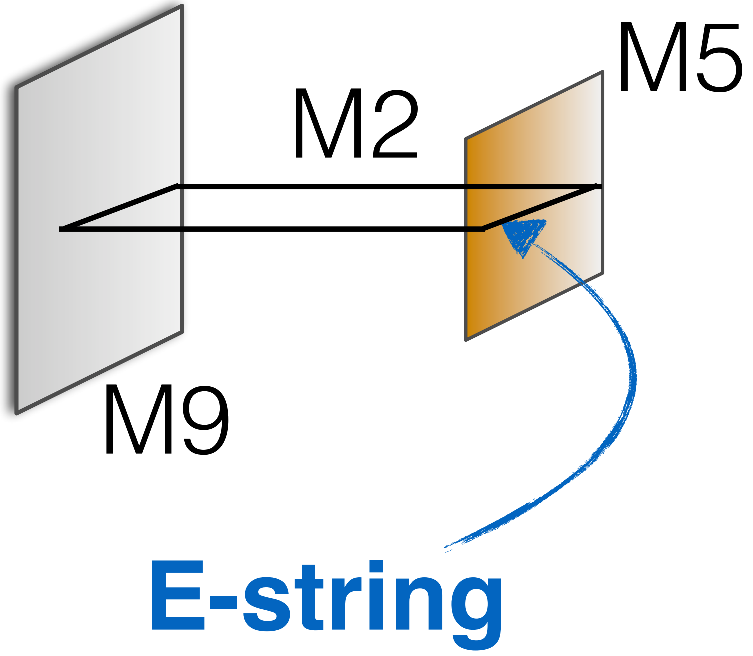
$$\boxed{g^2_{YM} = R_6}$$

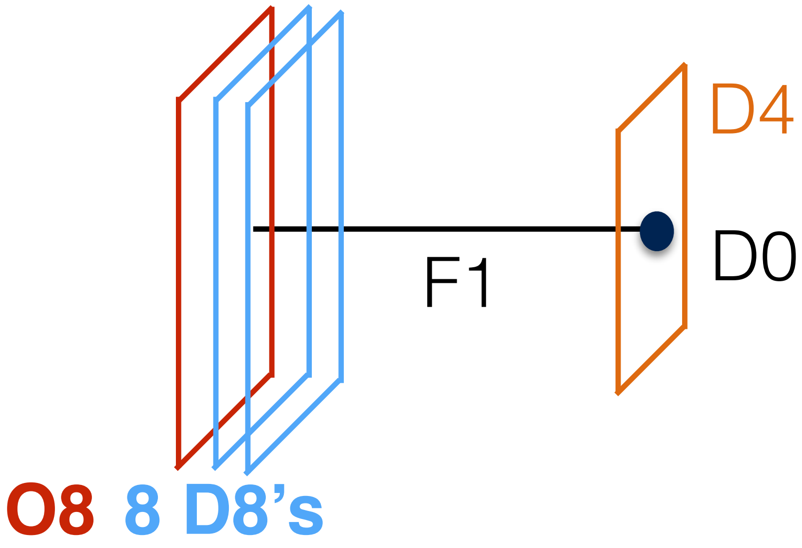
circle radius

6d N=(1,0)

One of the famous examples of (1,0) theory:

E-string theory





E-string theory on a circle = **5d SU(2) theory with Nf=8**

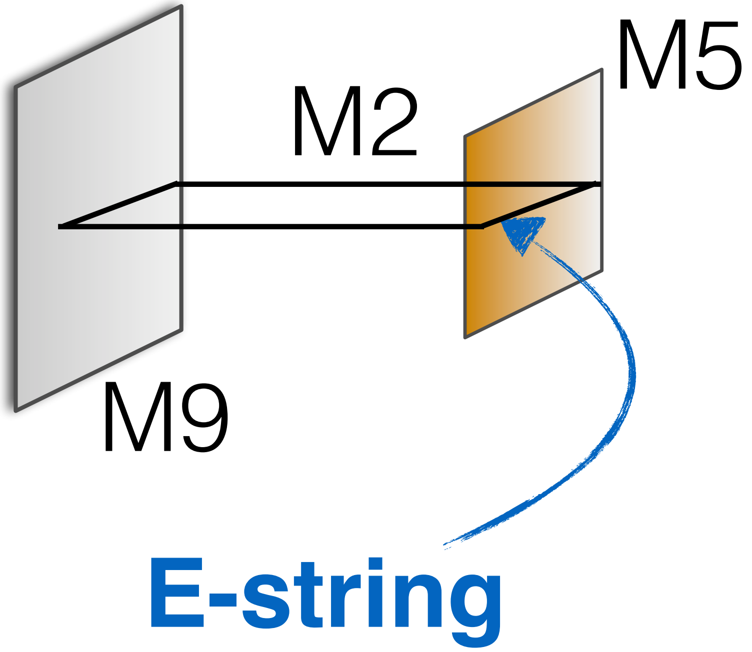
KK modes = **Instantons**

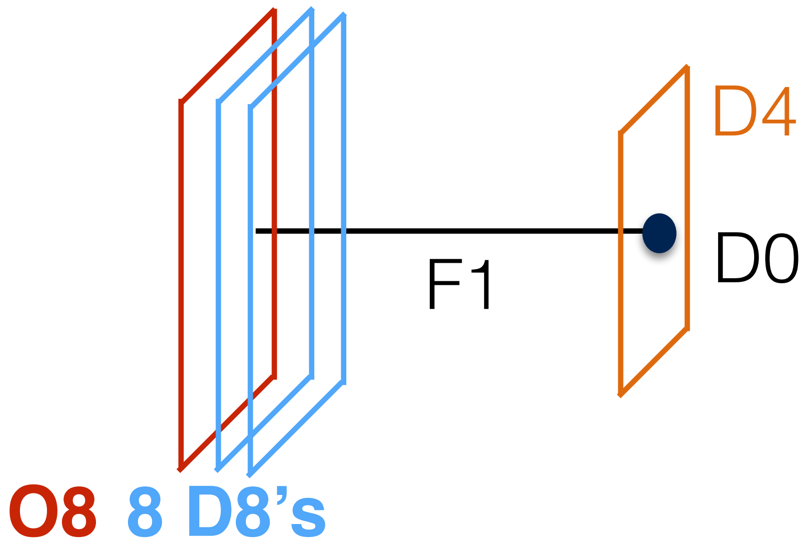
E-string partition function
(elliptic genus)

['14 Seok Kim, Joonho Kim, Kimyeong Lee, Jaemo Park, Vafa]

5d SU(2) w/ Nf=8

E-string theory





E-string theory on a circle = **5d SU(2) theory with Nf=8**

KK modes = **Instantons**

UV completion is a 6d SCFT !

Q1: How do we determine what 5d theories have UV completion as 6d SCFTs?

Q1: How do we determine what 5d theories have UV completion as 6d SCFTs?

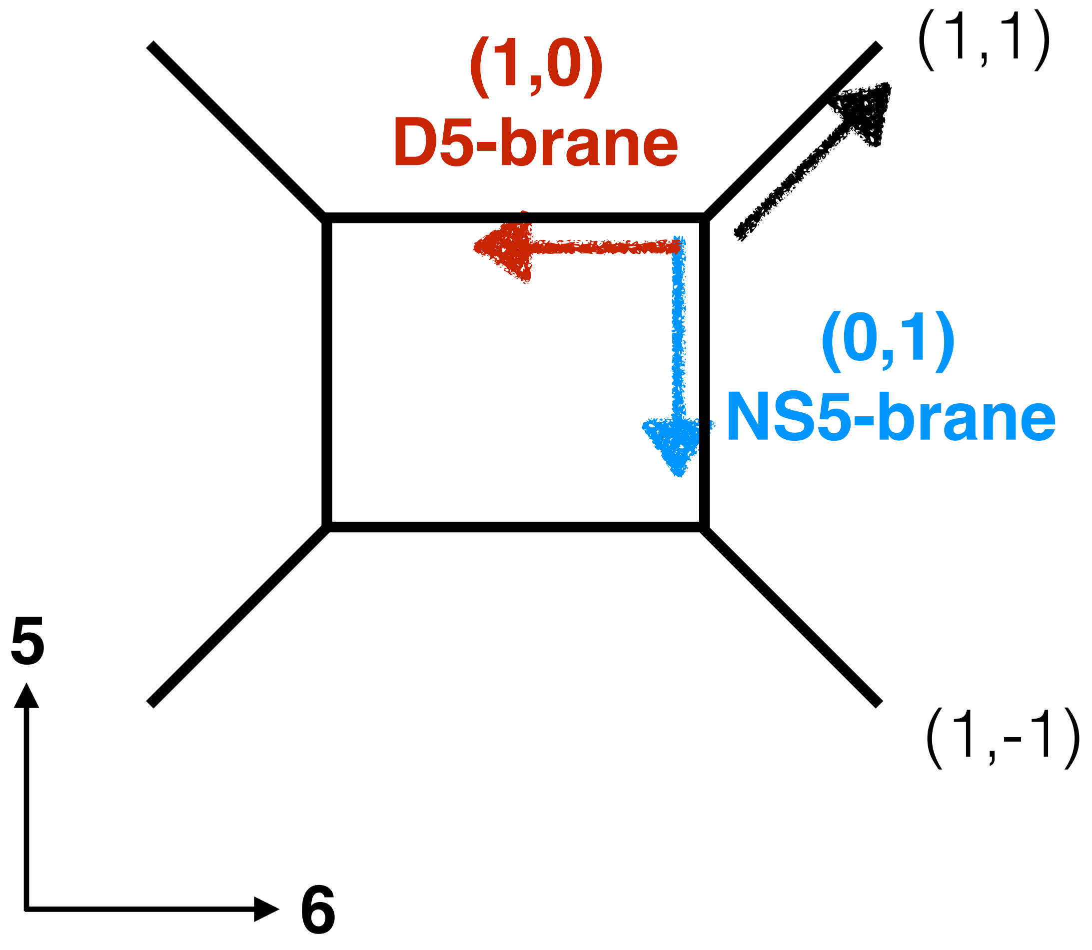
A1: We developed a diagrammatic way that distinguishes 6d UV completions from 5d UV completions —> Tao diagram

(p,q) web diagram and 5d SU(2) theory

[Aharony-Hanany, '97]

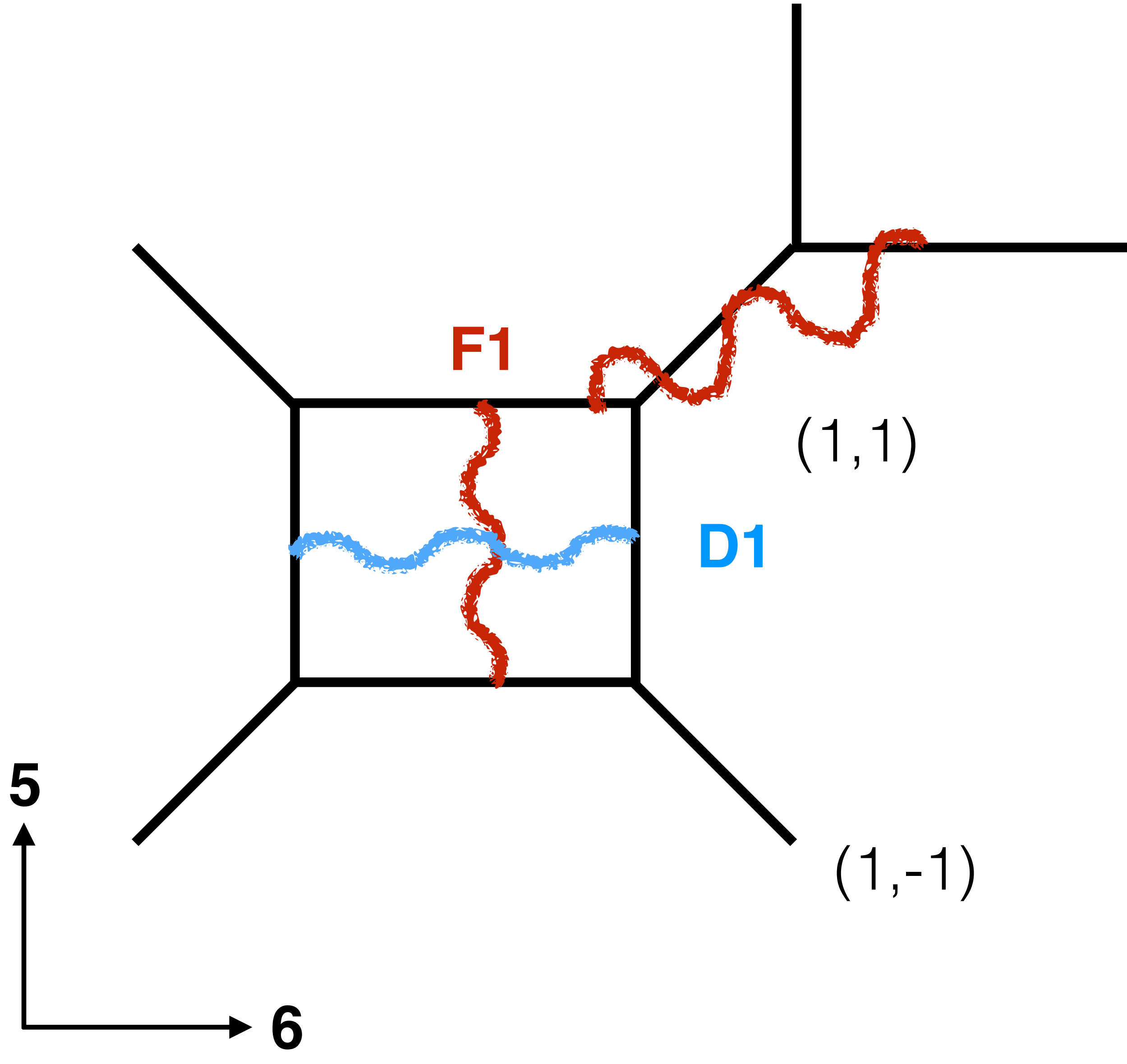
Type IIB configuration with **charge conservation**, **tension balance**: ***(p,q) web diagram***

	0	1	2	3	4	5	6	7	8	9
NS5	–	–	–	–	–	–				
D5	–	–	–	–	–		–			
(1,1)	–	–	–	–	–	-	-			

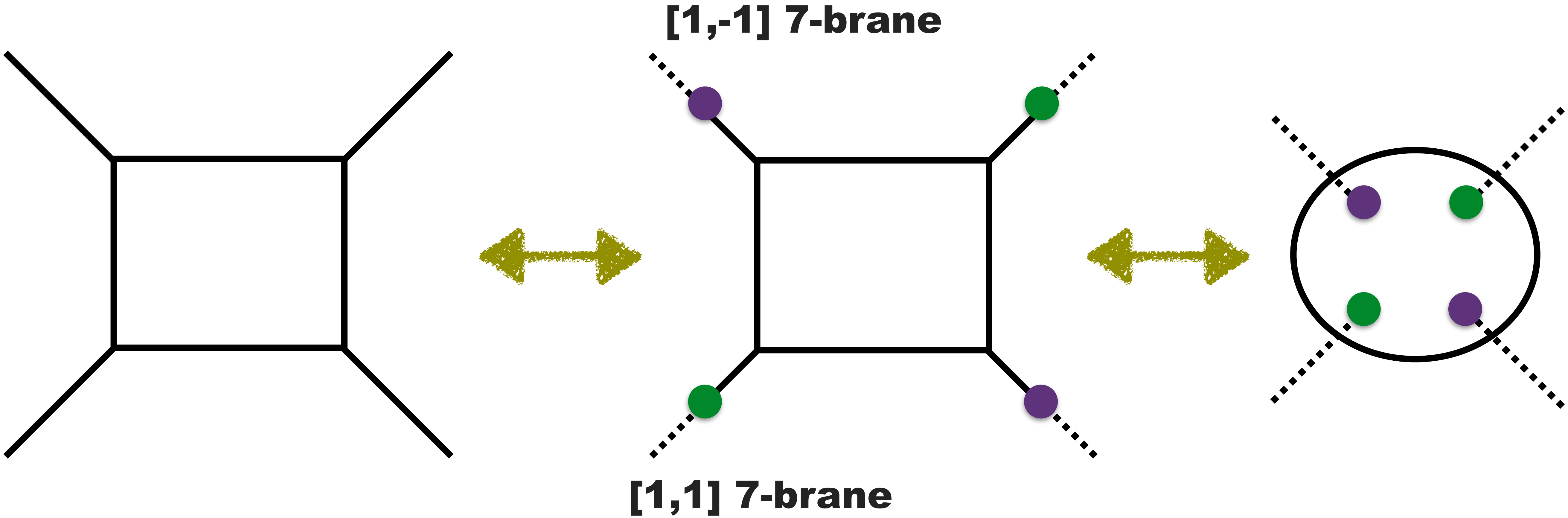


Flavors = semi infinite D5

	0	1	2	3	4	5	6	7	8	9
NS5	–	–	–	–	–	–				
D5	–	–	–	–	–		–			
(1,1)	–	–	–	–	–	-	-			

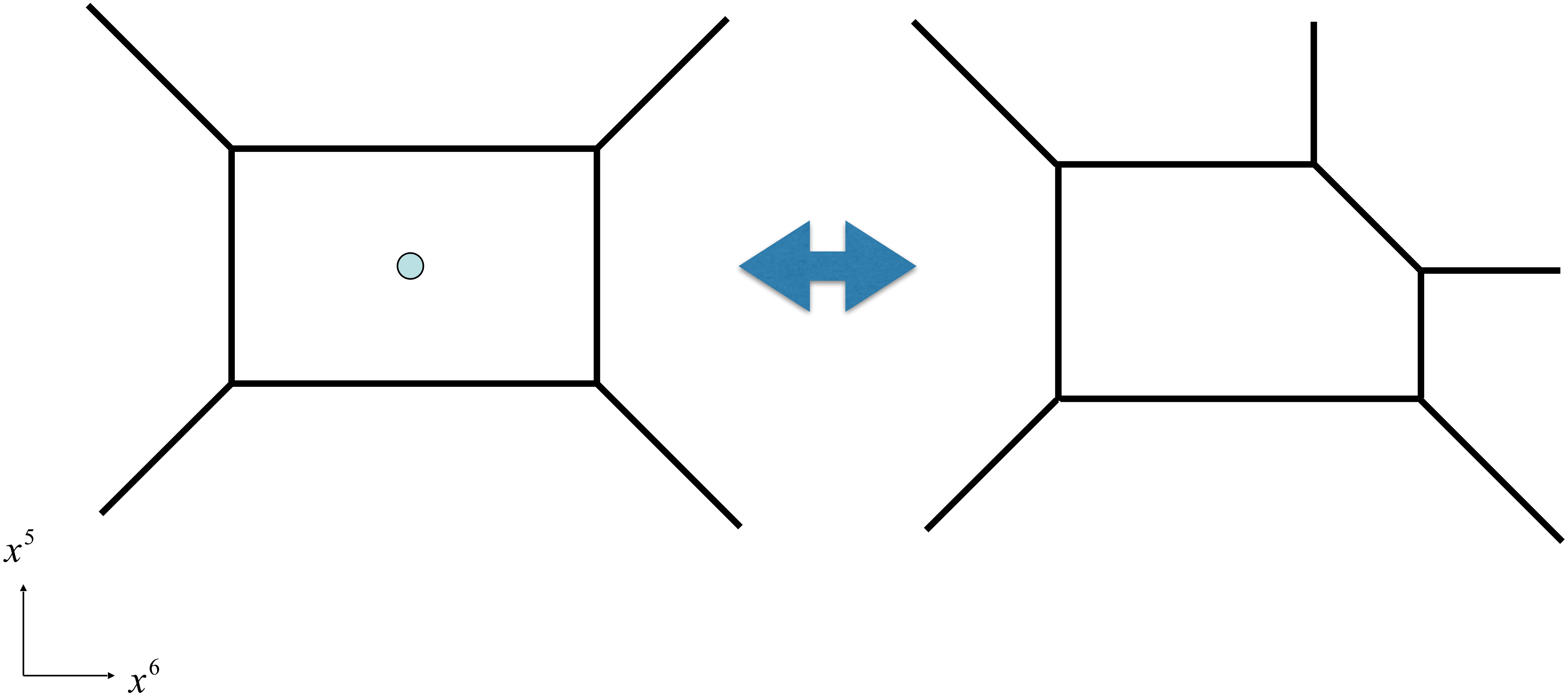


5d SU(2) theory via 5-branes and 7-branes



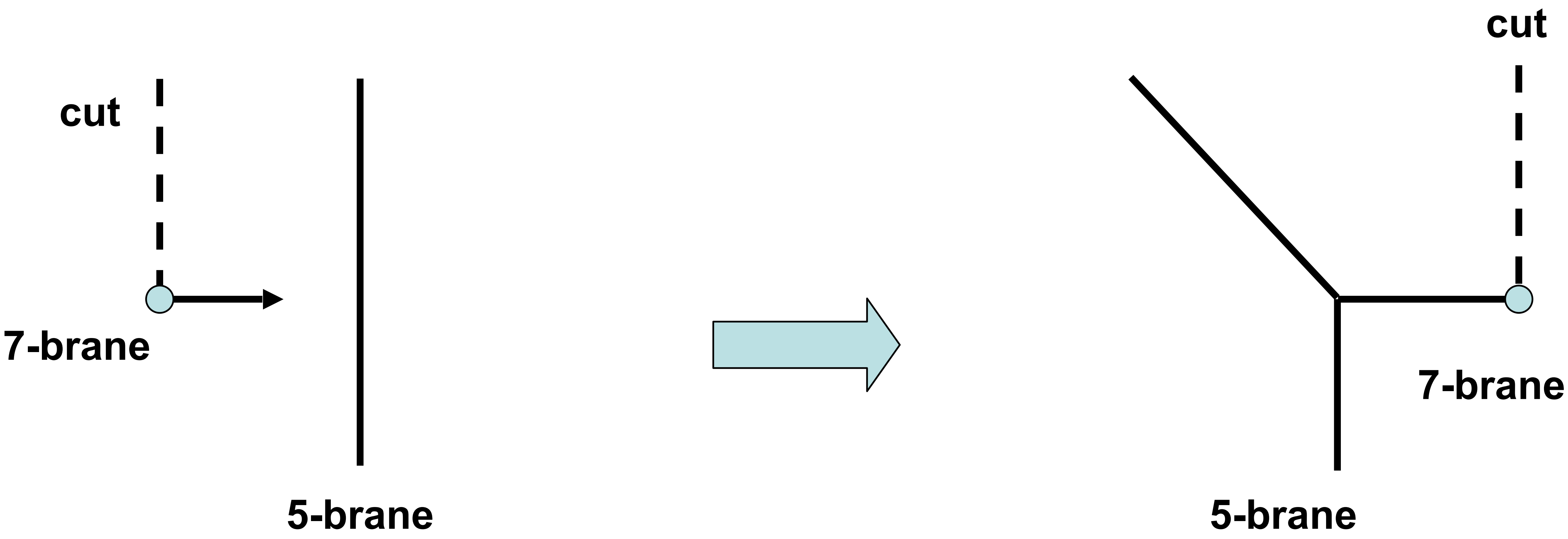
	0	1	2	3	4	5	6	7	8	9
NS5	—	—	—	—	—	—	.			
D5	—	—	—	—	—	.	—			
D7	—	—	—	—	—	.	.	—	—	—

5D N=1 SU(2) SYM with N_f=1 flavor

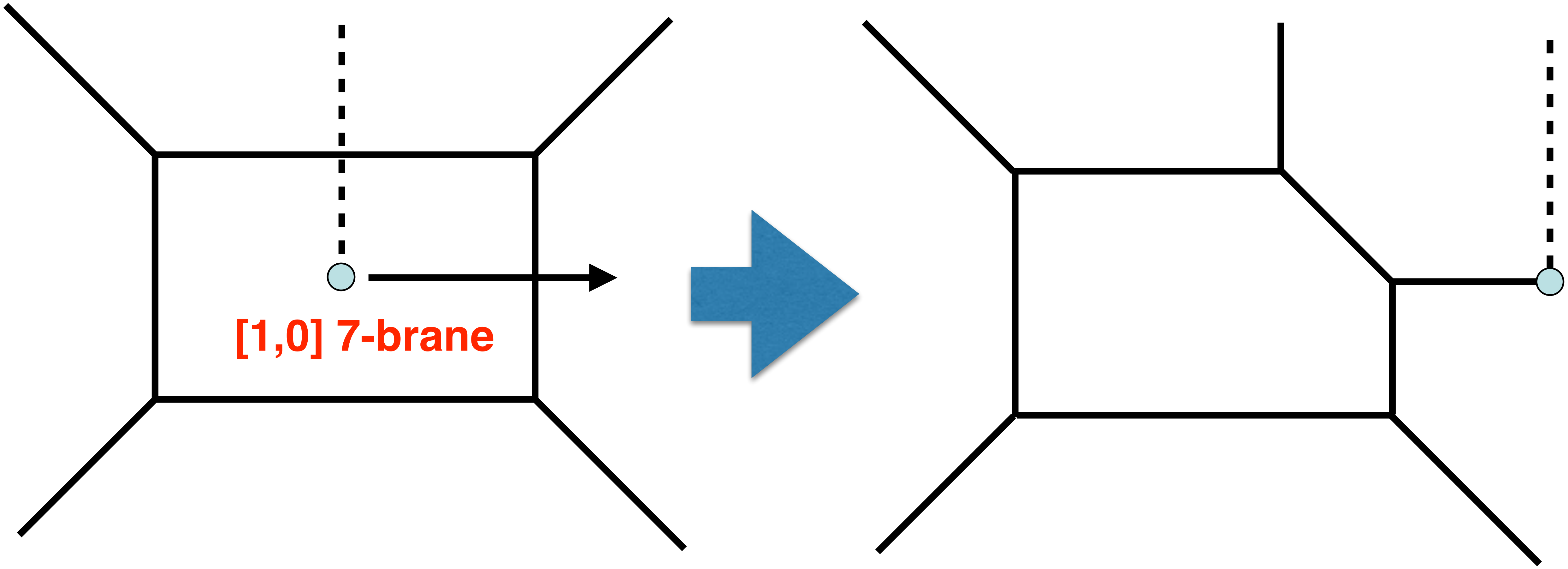


	0	1	2	3	4	5	6	7	8	9
5-brane	–	–	–	–	–	web		.	.	.
7-brane	–	–	–	–	–	.	.	–	–	–

Hanany-Witten transition

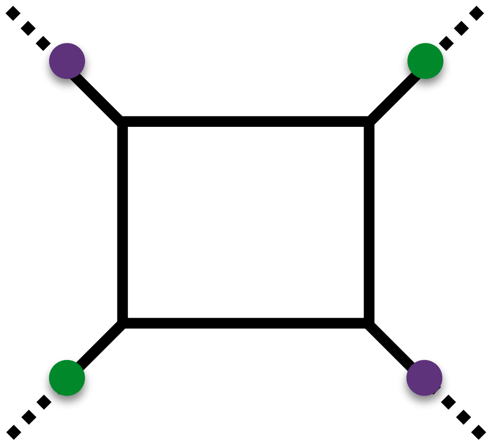


5D N=1 SU(2) SYM with N_f=1 flavor

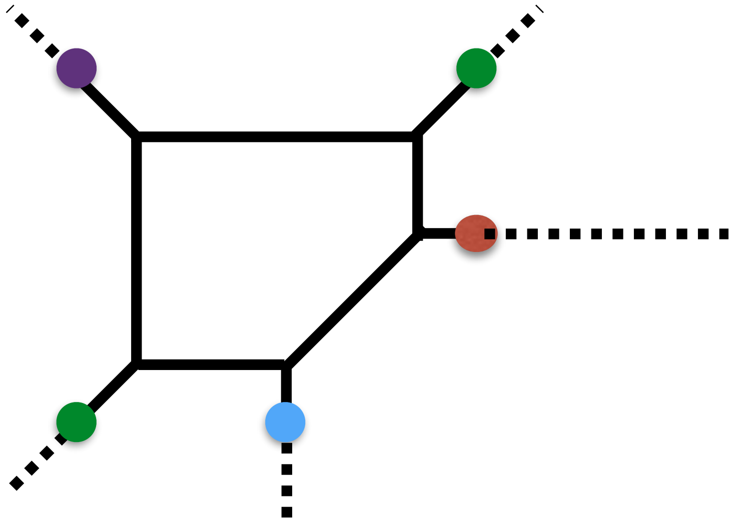


without changing the masses of the theory

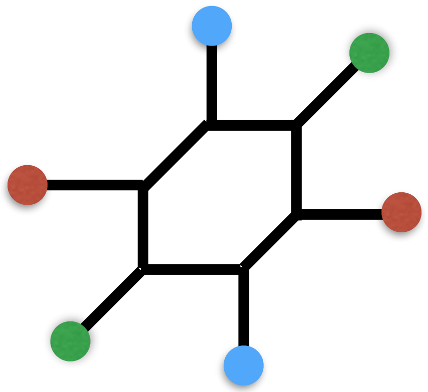
Flavors = D7 brane ●



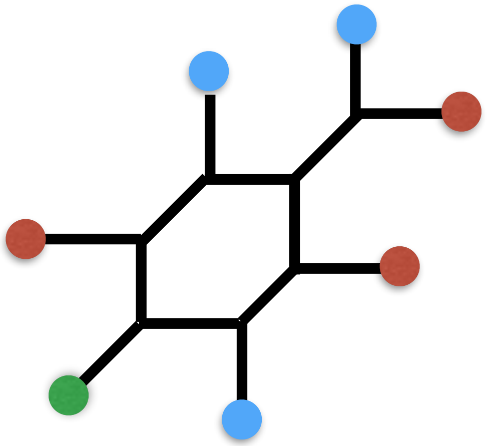
Nf= 0



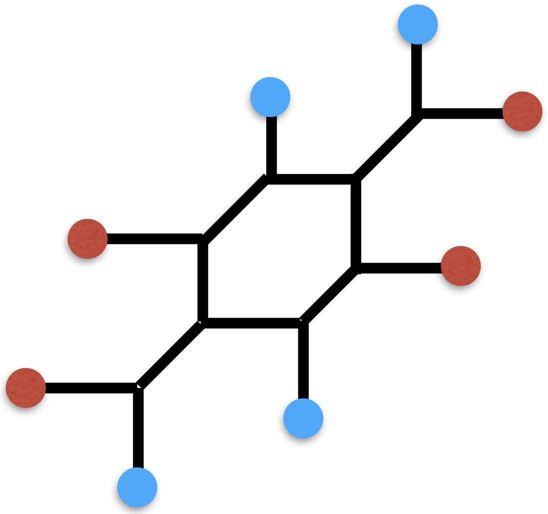
Nf= 1



Nf= 2

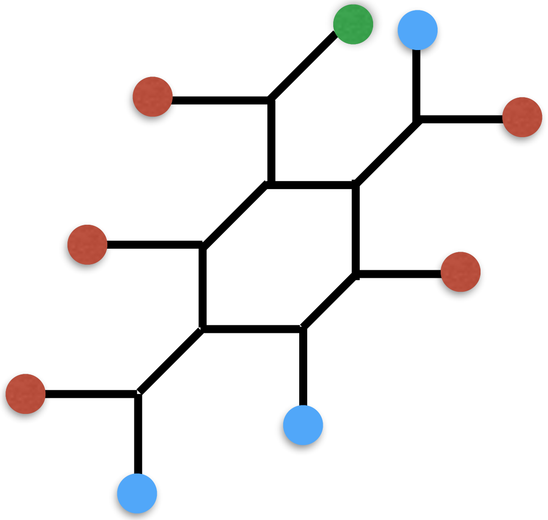


Nf= 3

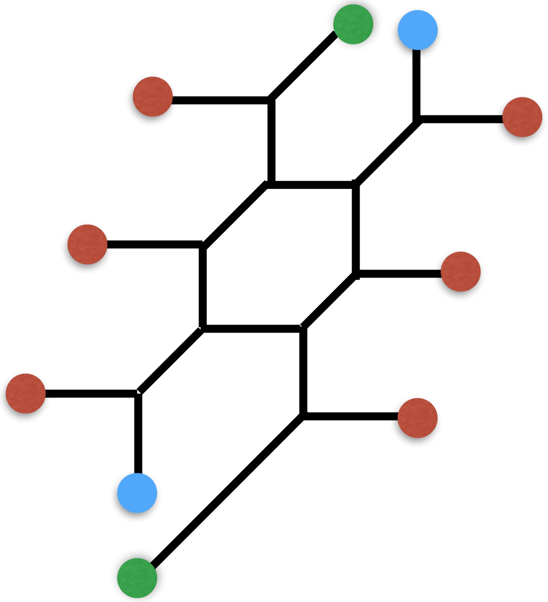


Nf= 4

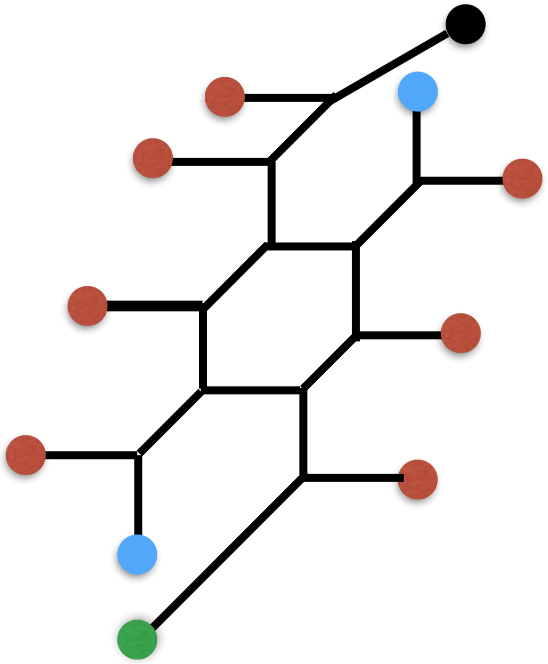
Five, six, seven flavors seem problematic,



$N_f = 5$



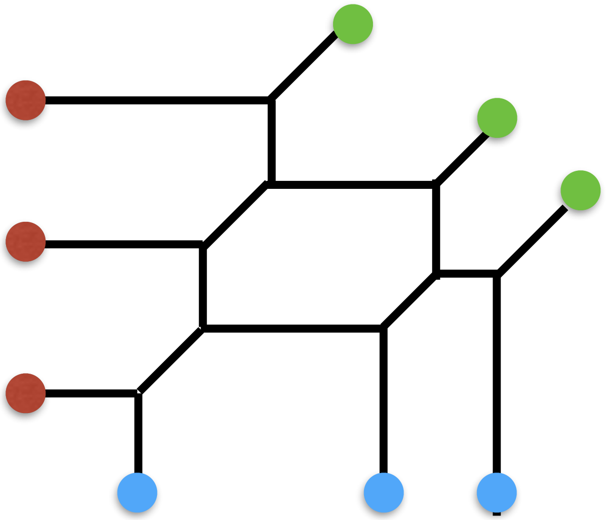
$N_f = 6$



$N_f = 7$

but possible to make sense using 7-brane monodromies

e.g., Nf=5



[Benini-Benvenuti-Tachikawa, '09]

finite configuration diagram

5d SCFT

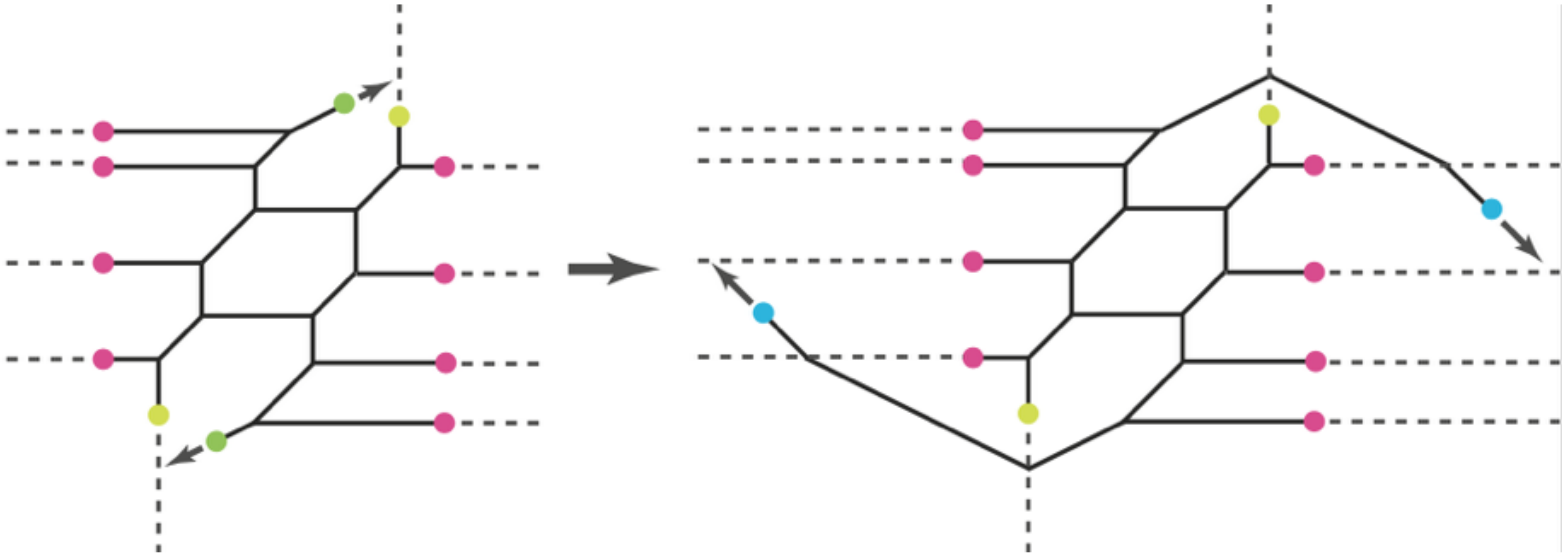
Flavor decoupling of 5d SU(2) theory of **$N_f=8$**

↓

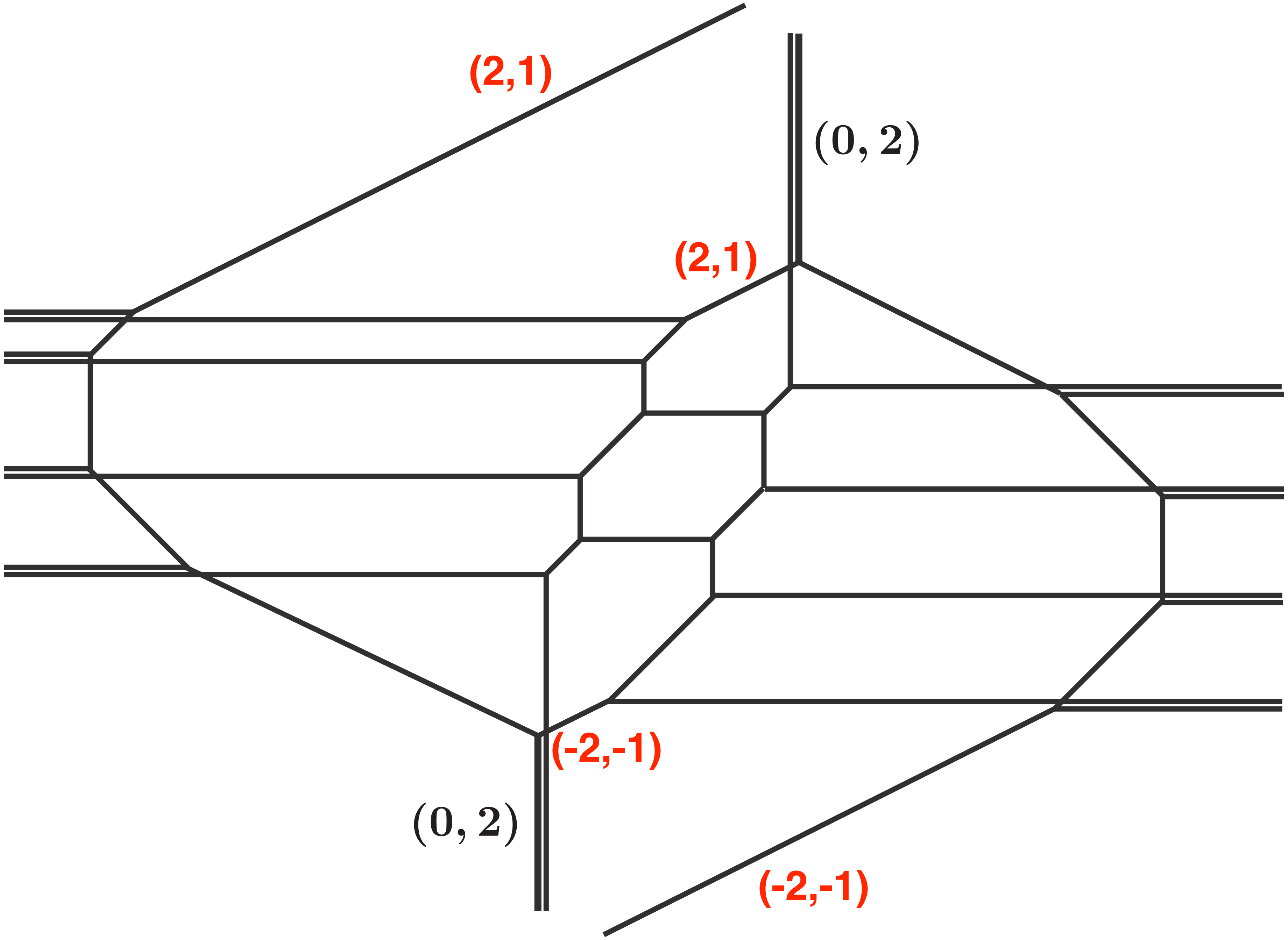
SU(2) gauge theory with **$N_f = 0,1,\ldots,7$** flavors
of 5d UV fixed point (Superconformal theory)

[Seiberg '96]

Brane configuration for 5d SU(2) theory with Nf=8 flavors



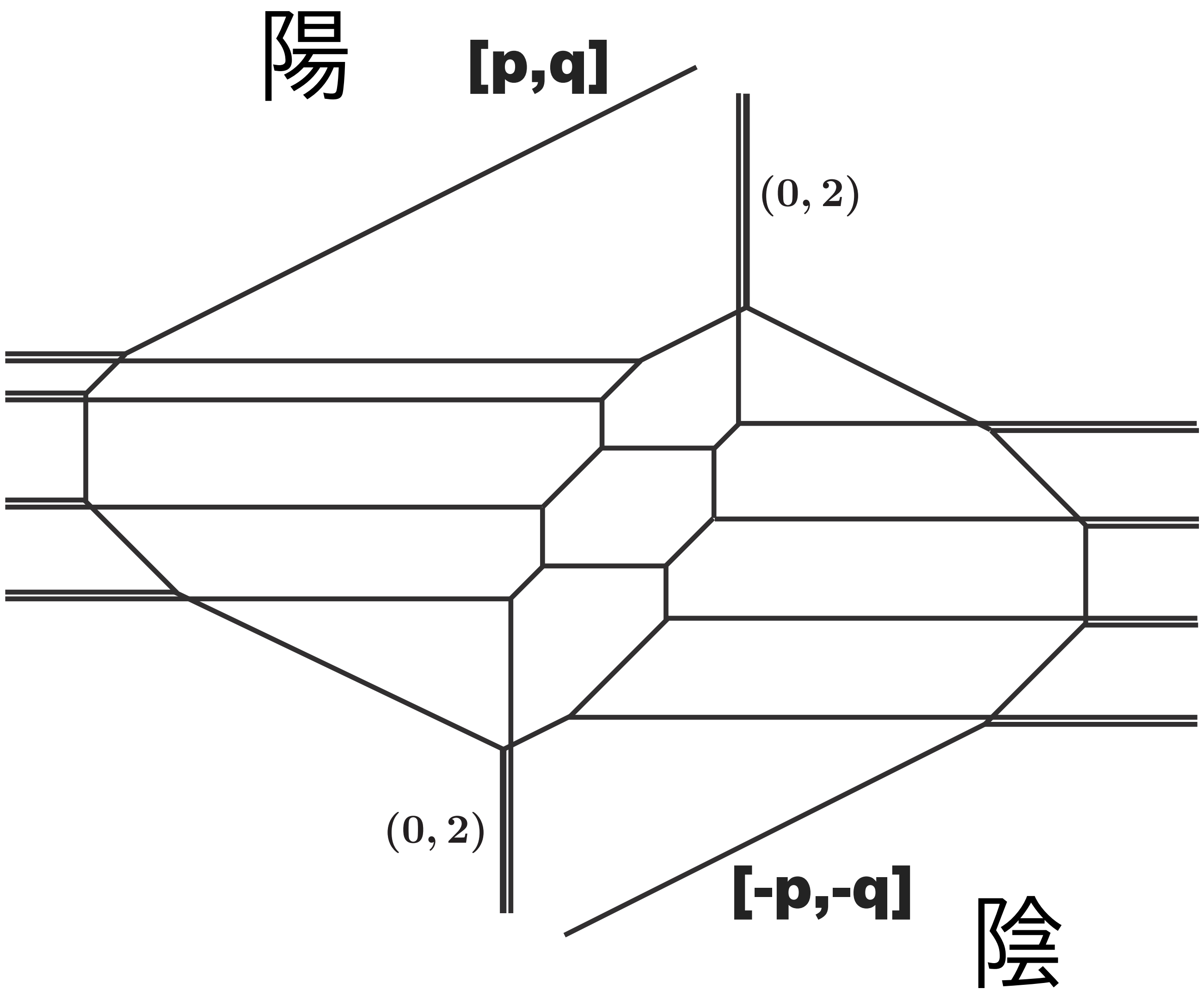
By pulling out 7-branes to infinity



Spirally rotating!   One revolution: charges remain the same.

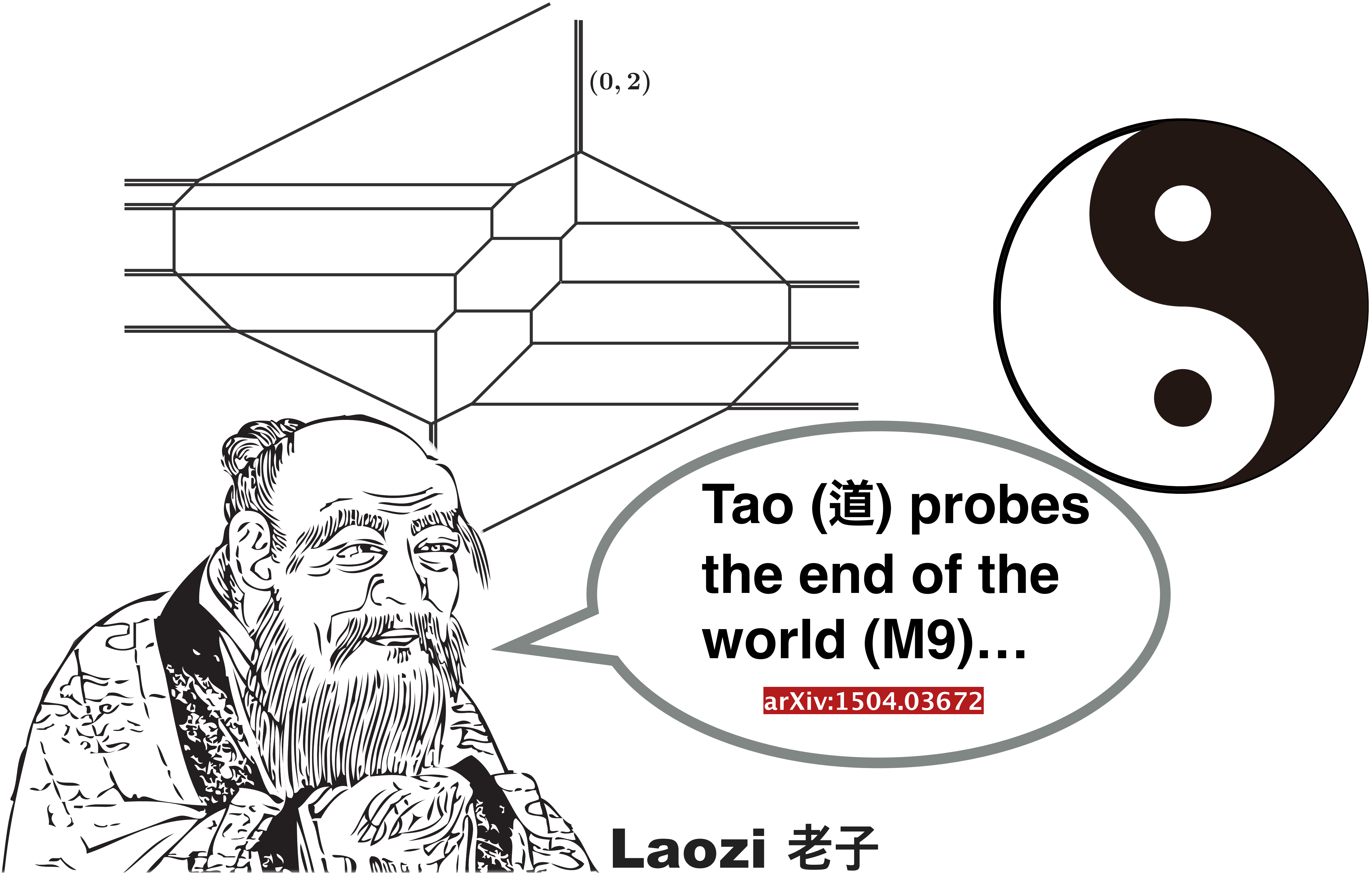
Infinitely rotating spiral diagram

The shape looks like



We call it **Tao diagram**…

道



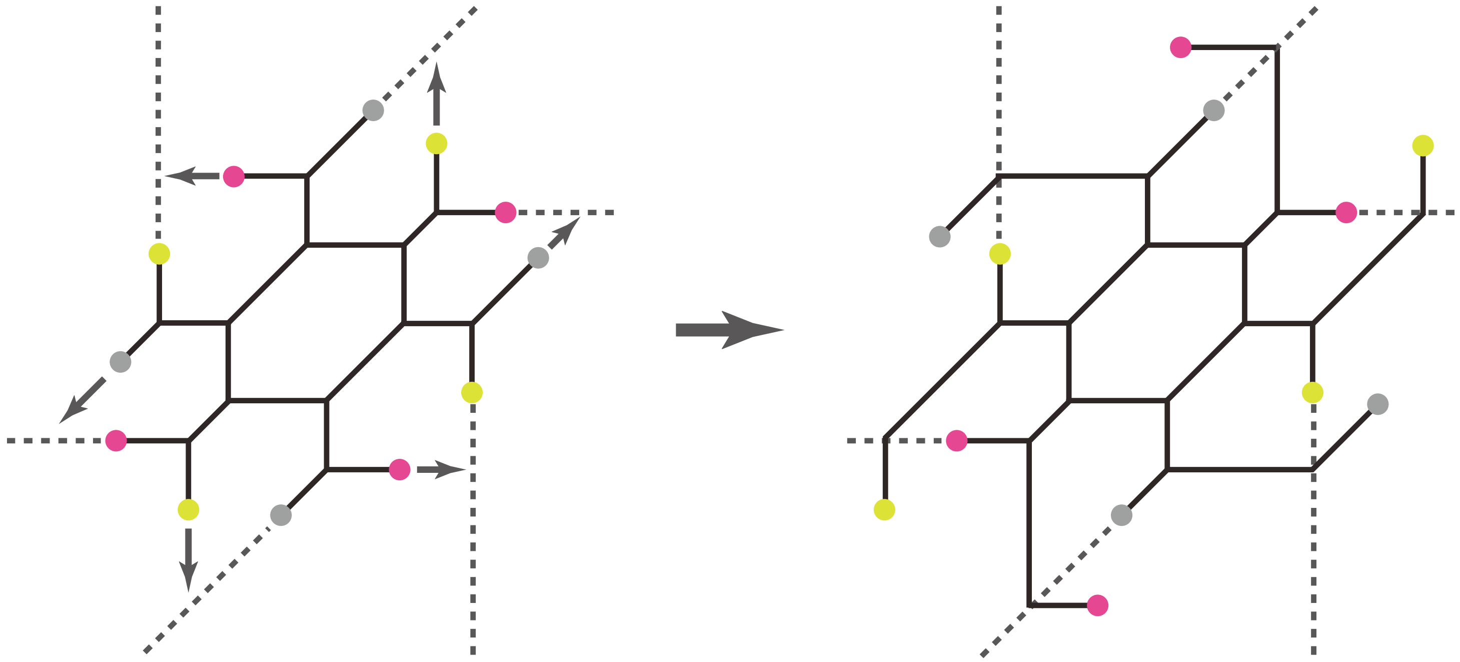
Tao (道) probes the end of the world (M9)…

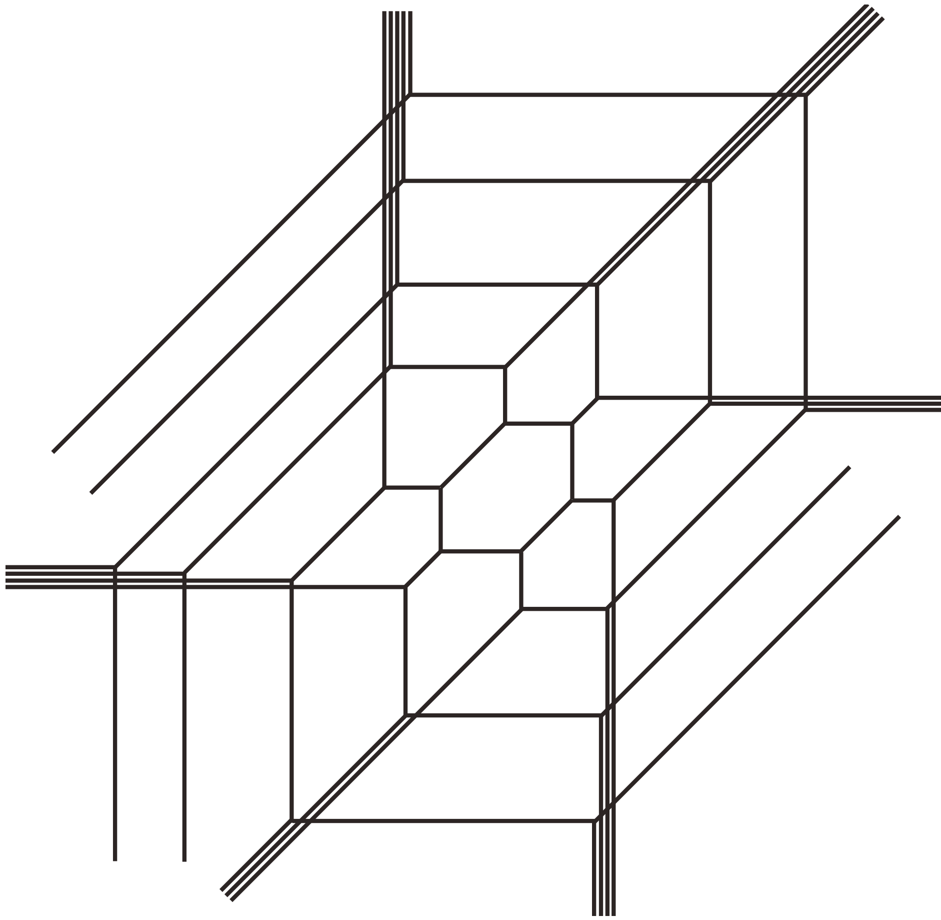
arXiv:1504.03672

Laozi 老子

There are **various equivalent** forms of Tao diagram:

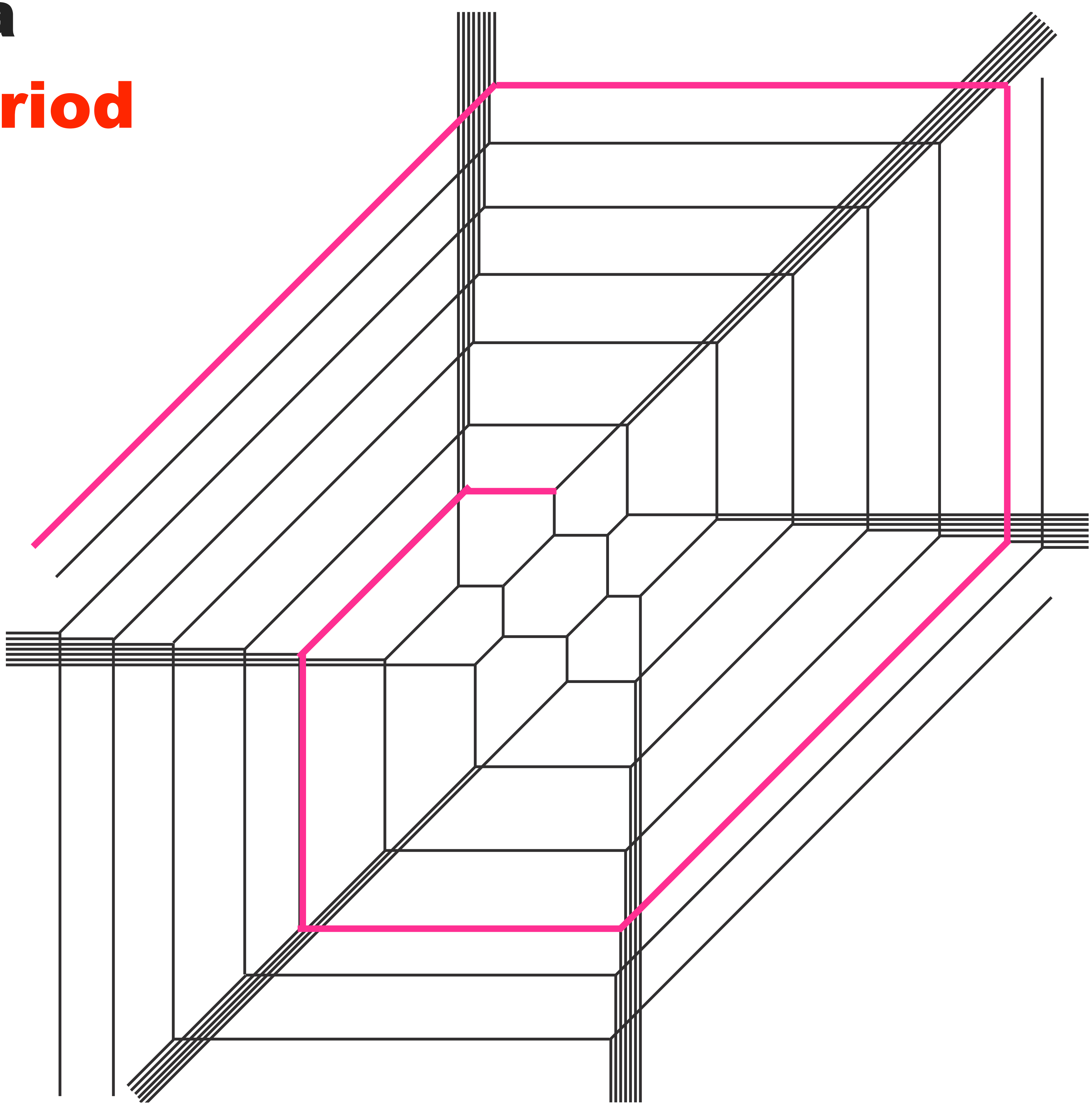
Another **Tao diagram for SU(2) gauge theory with 8 flavors**





This is more practical and **useful for computations**

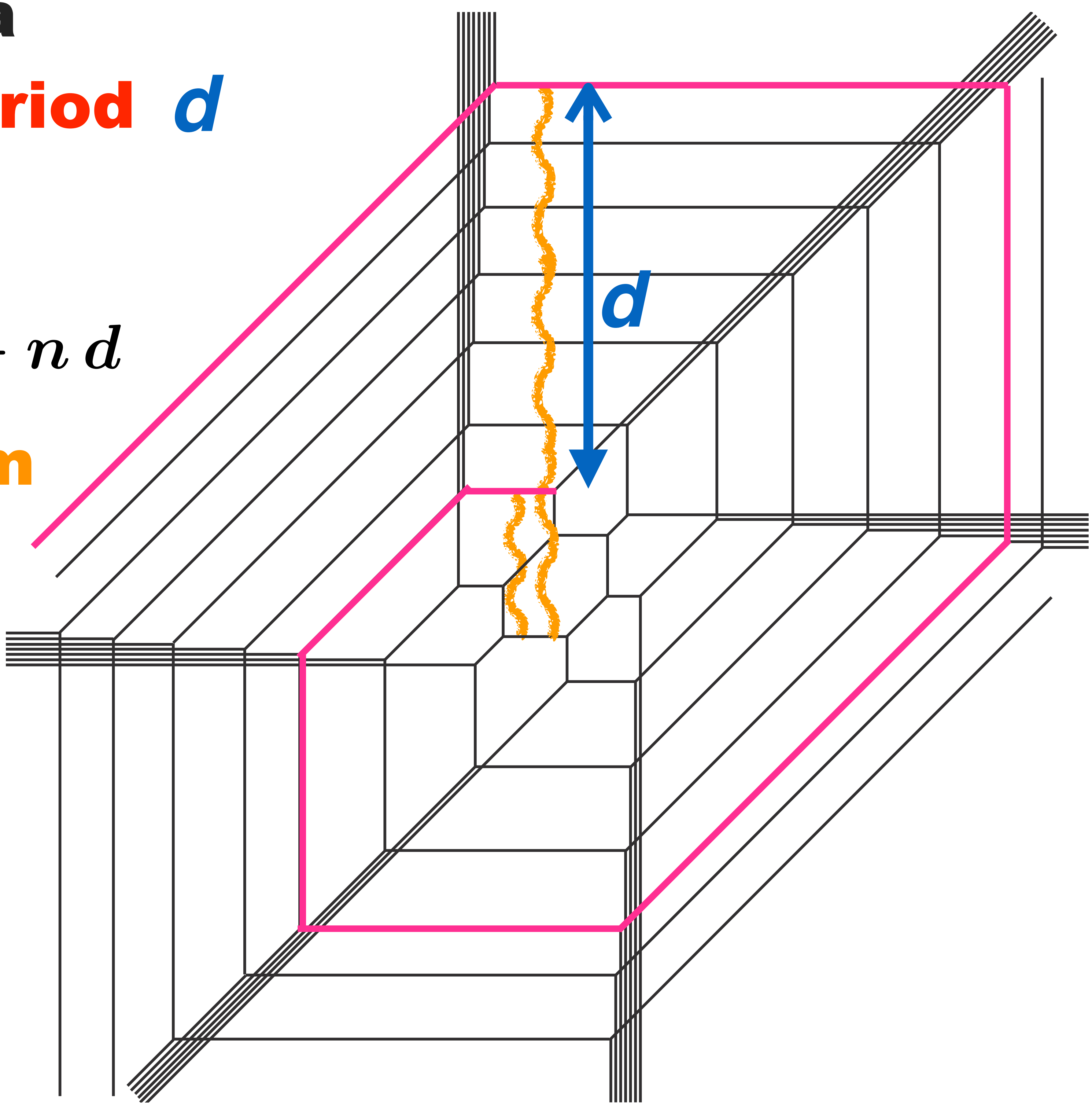
spiral with a constant period



spiral with a
constant period d

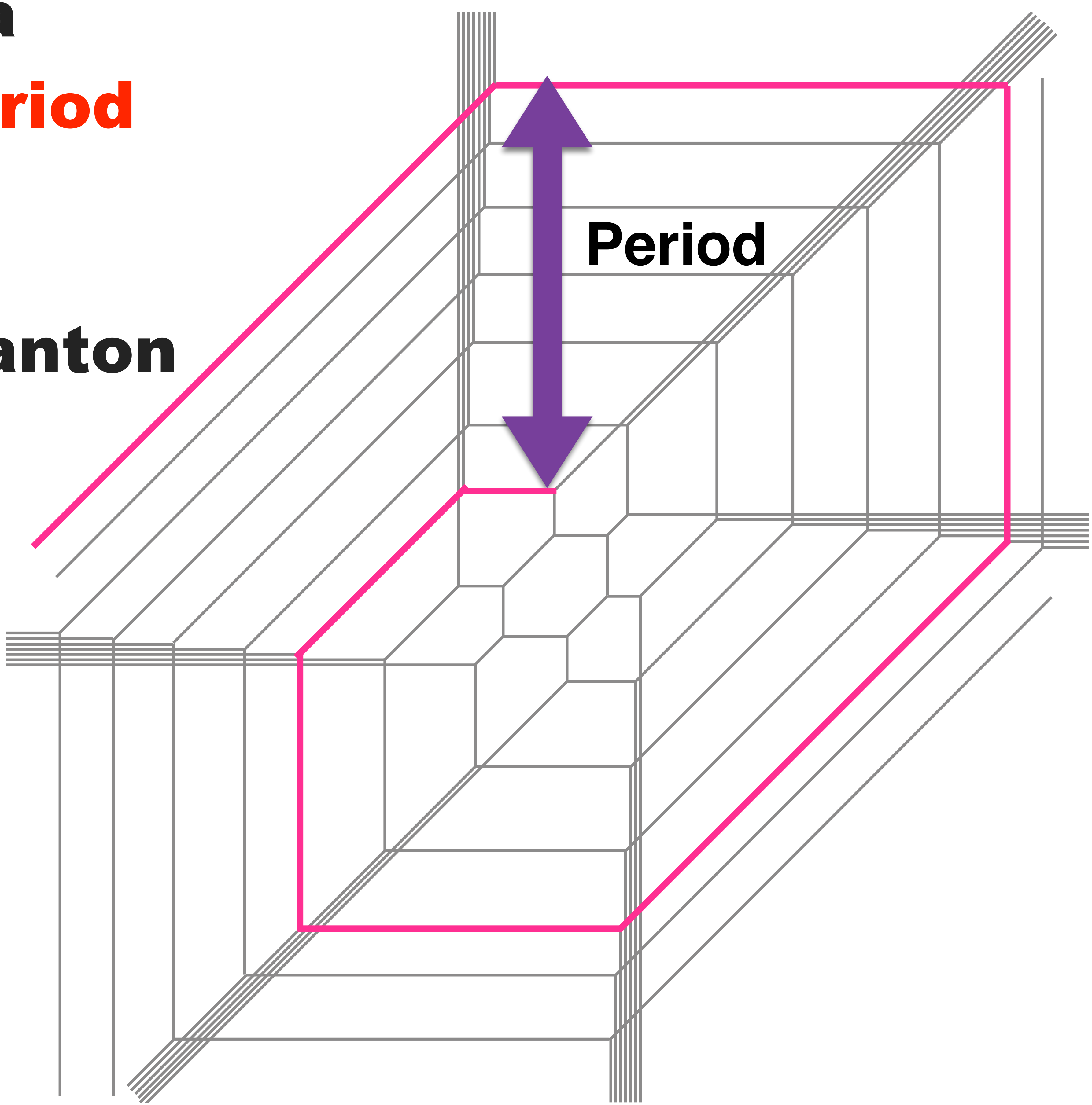
$$m_{(n)} = m_{(0)} + n\, d$$

KK spectrum



spiral with a
constant period

period = instanton
$\sim R^{-1}$



Tao diagrams give new perspective on 6d SCFTs

Tao diagram: infinite spirals (KK spectrum)
constant period (compactified radius)

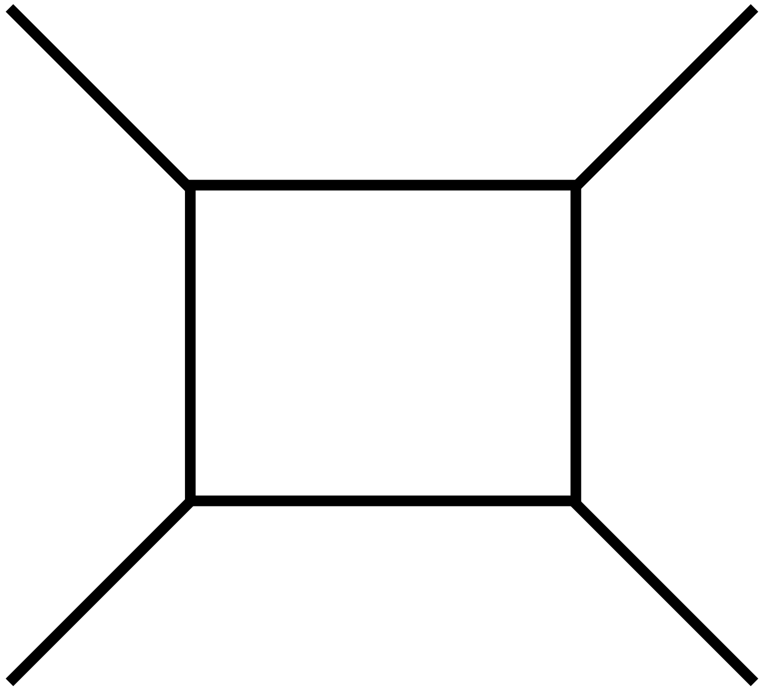
- Naturally identified as **a 6d theory on a circle** (compactification radius emerges…)

- Computational tool:
  Partition function

Topological Vertex formalism

[Vafa et al.]

web diagram

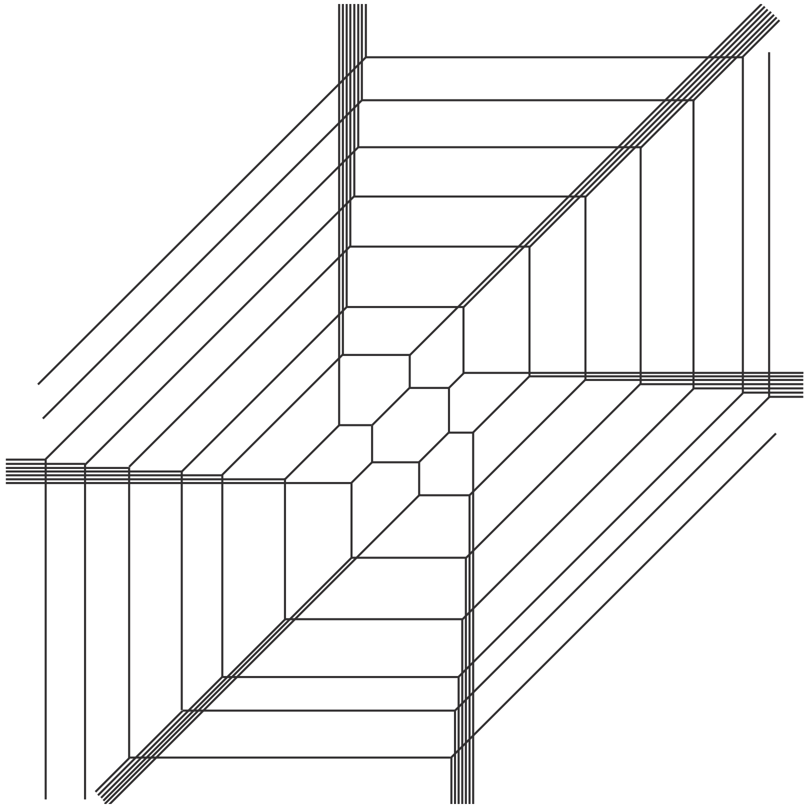


BPS partition function

$$Z = \cdots$$

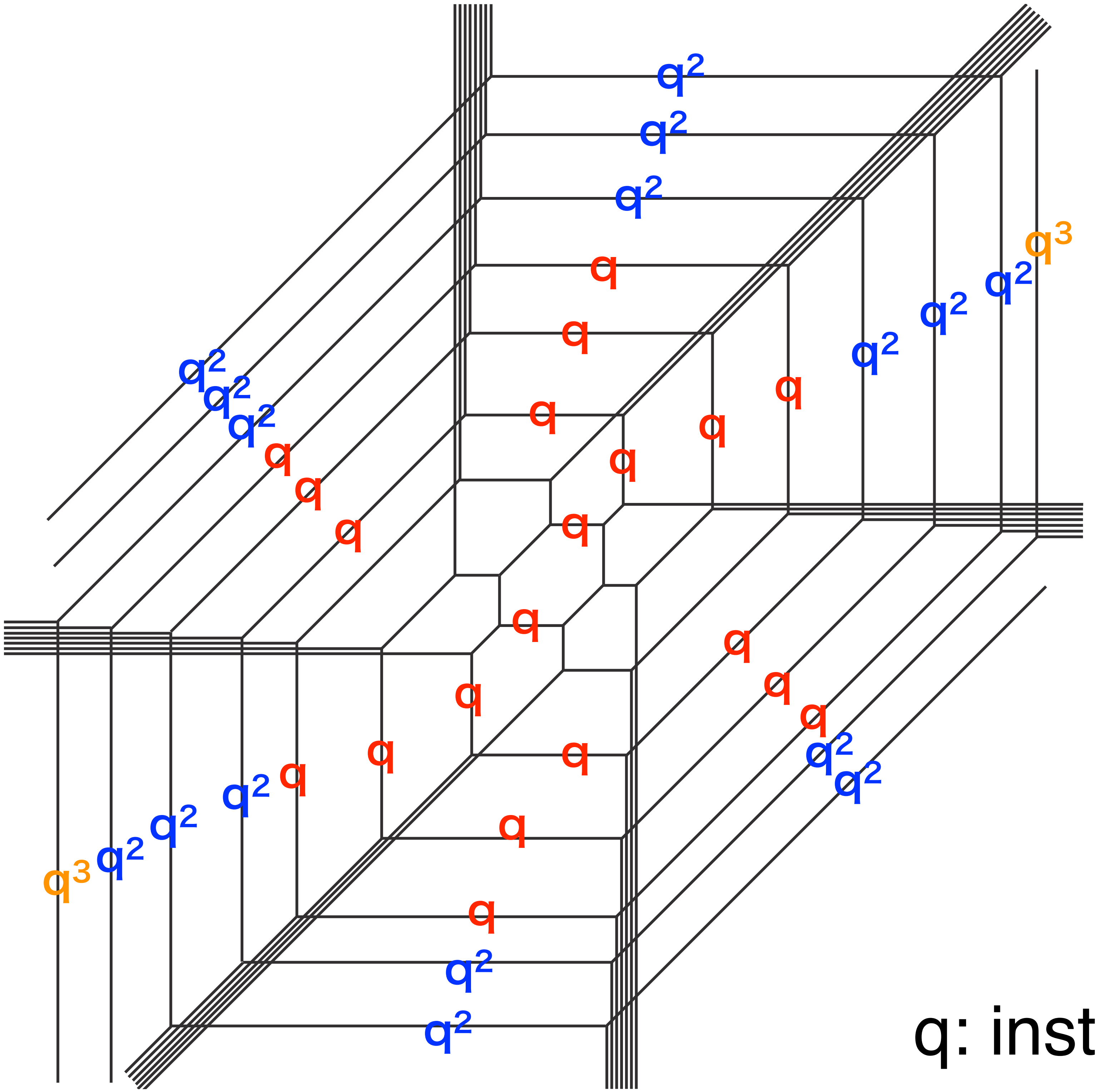
Topological Vertex formalism

web diagram



BPS partition function

$$Z \overset{?}{=} \cdots$$



q: instanton factor

Compute order-by-order in **q**

1-instanton, **2-instanton**, … , up to $\mathbf{q}^k$ order

Partition function from Tao diagram

$$Z_{E\text{-}string} = \mathrm{PE}\Big[\sum_{m=0}^{\infty} \mathcal{F}_m(y, A, q)\mathfrak{q}^m\Big] = \mathrm{PE}\Big[\frac{1}{(1-q)(1-q^{-1})}\sum_{n=1}^{\infty}\tilde{f}_n A^n\Big]$$

$$\begin{aligned}
\tilde{f}_1 &= \chi^{(1)} + \chi_{\mathrm{c}}\,\mathfrak{q} + \Big(2\chi_{\mathbf{2}}(q)\chi^{(1)} + \chi^{(3)} + \chi^{(1)}\Big)\mathfrak{q}^2 + \Big(\chi^{(1)}\chi_{\mathrm{s}} + 2\chi_{\mathbf{2}}(q)\chi_{\mathrm{c}}\Big)\mathfrak{q}^3 \\
&\quad + \Big(3\chi_3(q) + 4\chi_2(q) + 2)\chi^{(1)} + 2\chi_2(q)\chi^{(3)} + \chi^{(5)} + \chi^{(1)}\chi^{(2)}\Big)\mathfrak{q}^4 + \mathcal{O}(\mathfrak{q}^5), \qquad (4.54) \\
\tilde{f}_2 &= -2 - 2\chi_{\mathrm{s}}\,\mathfrak{q} - \Big(2\chi^{(4)} + (3\chi_{\mathbf{2}}(q) + 2)\chi^{(2)} + 4(\chi_{\mathbf{3}}(q) + \chi_{\mathbf{2}}(q) + 1)\Big)\mathfrak{q}^2 \\
&\quad - \Big(2\chi^{(2)}\chi_{\mathrm{s}} + 3\chi_{\mathbf{2}}(q)\chi^{(1)}\chi_{\mathrm{c}} + 4(\chi_{\mathbf{3}}(q) + \chi_{\mathbf{2}}(q) + 1)\chi_{\mathrm{s}}\Big)\mathfrak{q}^3 \\
&\quad + \Big((5\chi_4(q) + 6\chi_3(q) + 11\chi_2(q) + 8)\chi^{(2)} + (4\chi_3(q) + 4\chi_2(q))\chi^{(4)} + (3\chi_2(q) - 2)\chi^{(6)} \\
&\qquad + (4\chi_3(q) + 3\chi_2(q) + 2)(\chi^{(1)})^2 + 3\chi_2(q)\chi^{(1)}\chi^{(3)} + 2\chi^{(1)}\chi^{(5)} + 2(\chi^{(2)})^2 + 2(\chi_s)^2 \\
&\qquad + (6\chi_5(q) + 8\chi_4(q) + 16\chi_3(q) + 20\chi_2(q) + 10)\Big)\mathfrak{q}^4 + \mathcal{O}(\mathfrak{q}^5). \qquad (4.55)
\end{aligned}$$

arXiv:1504.03672

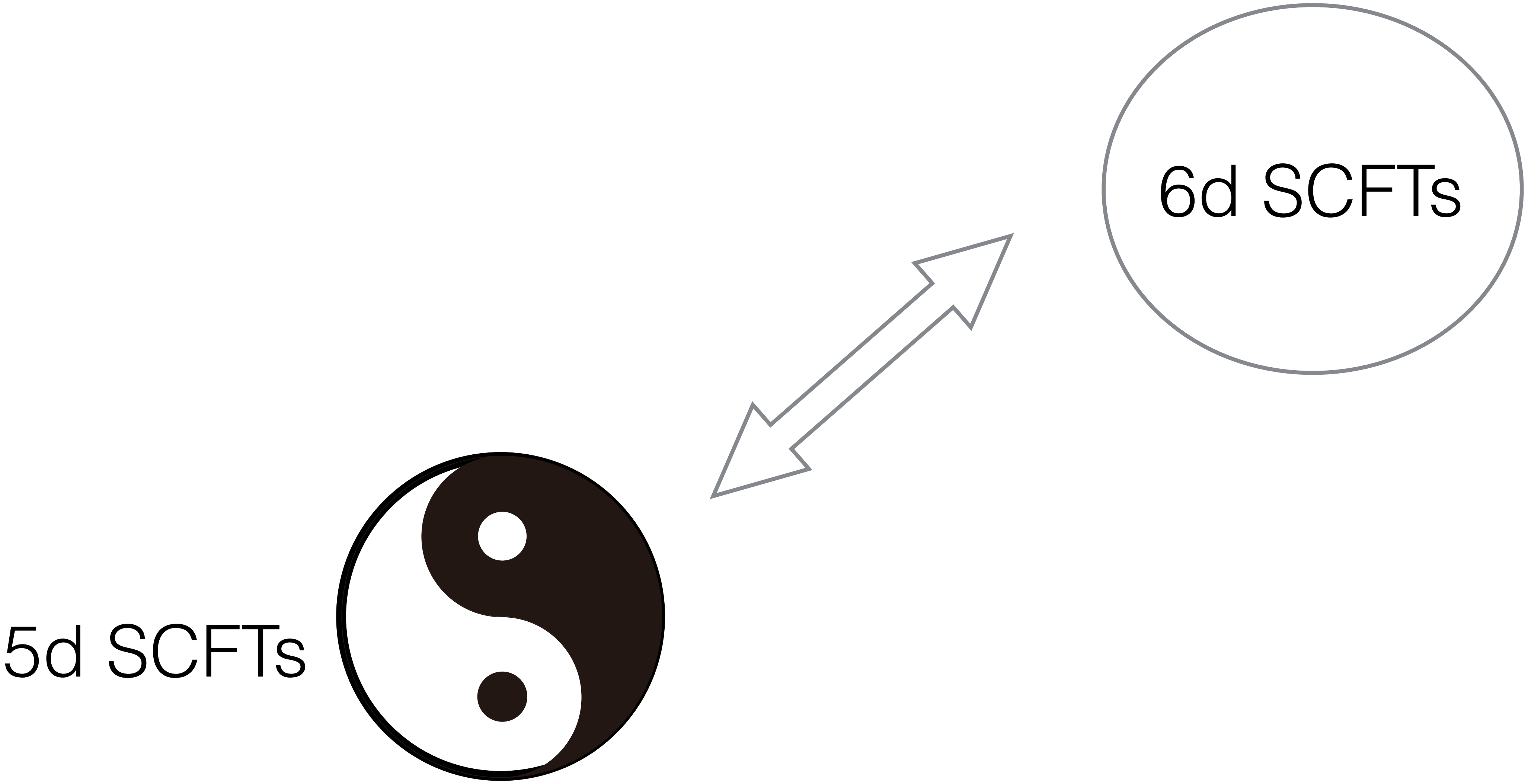
reproduces the E-string partition function (elliptic genus)

by (up to 4 instantons)

['14 Kim, Kim, Lee, Park, Vafa]

Tao diagram indeed sees the E-string theory on a circle

New understandings of 5d/6d SCFTs : Summary (Part I)



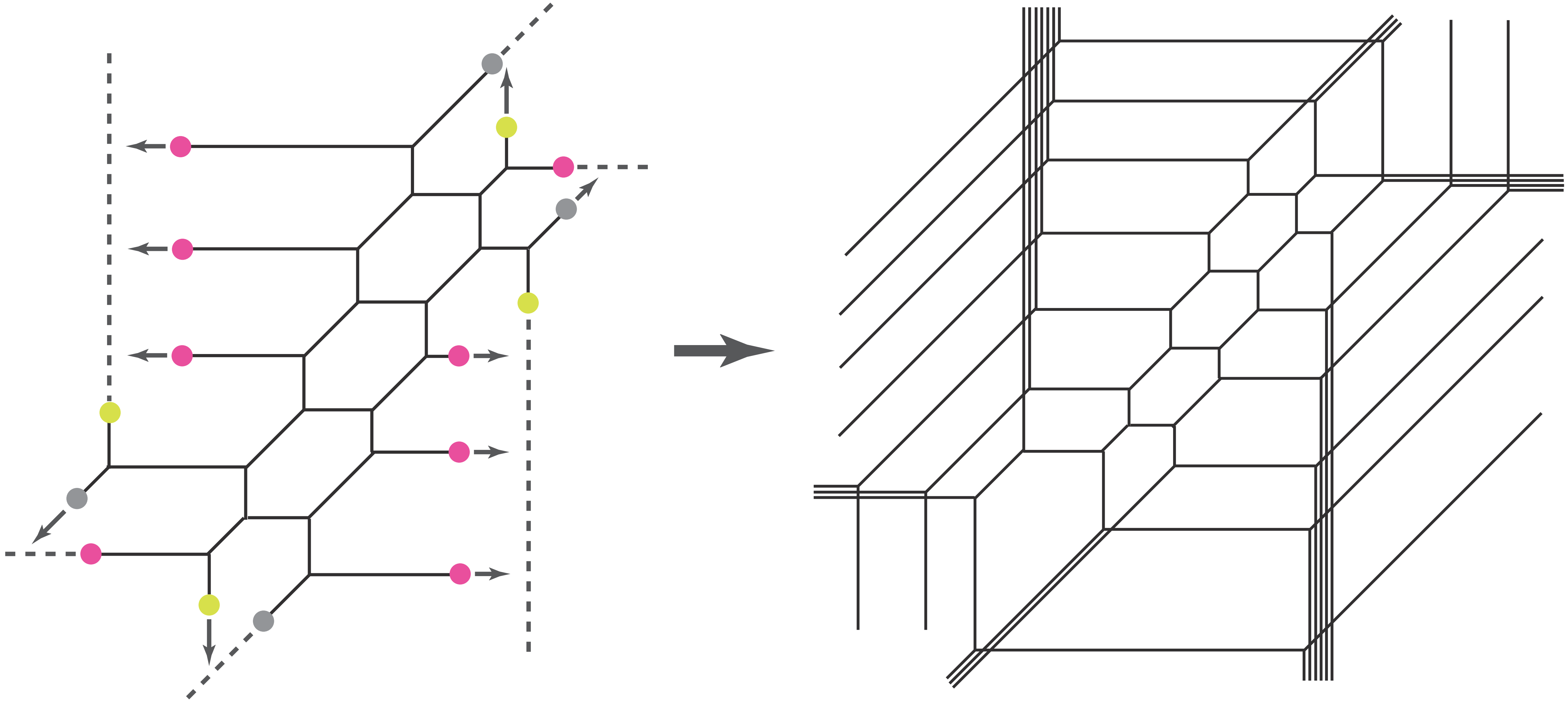
6d SCFTs

5d SCFTs

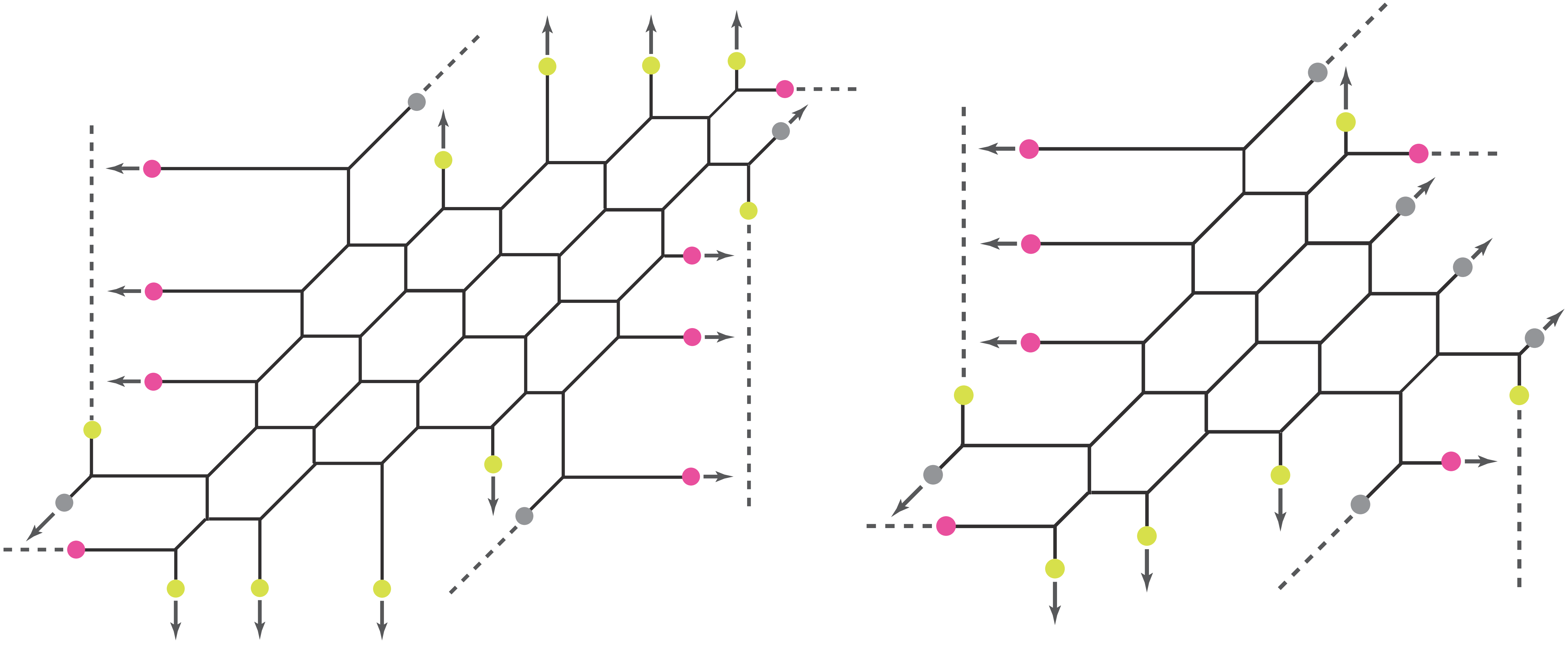
Tao diagram

Many more Tao web diagrams

5d SU(N) **$N_f=2N+4$**



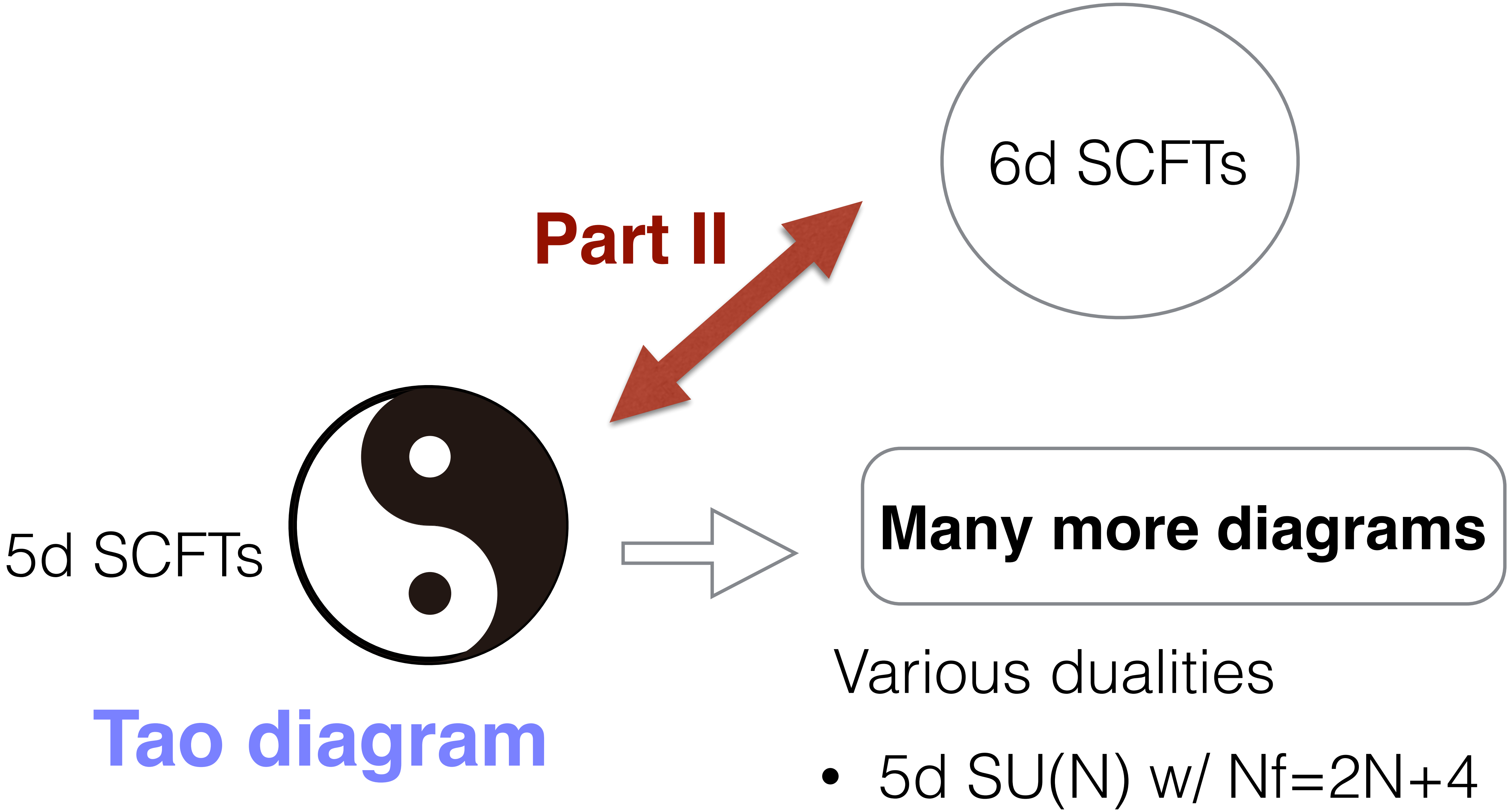
Quiver type



Claim:

Tao web diagrams imply that a 5d theory has UV completion as a 6d SCFT

Preview (Part II)



6d SCFTs

Part II

5d SCFTs

Tao diagram

Many more diagrams

Various dualities

- 5d SU(N) w/ Nf=2N+4
- 5d UV dualities

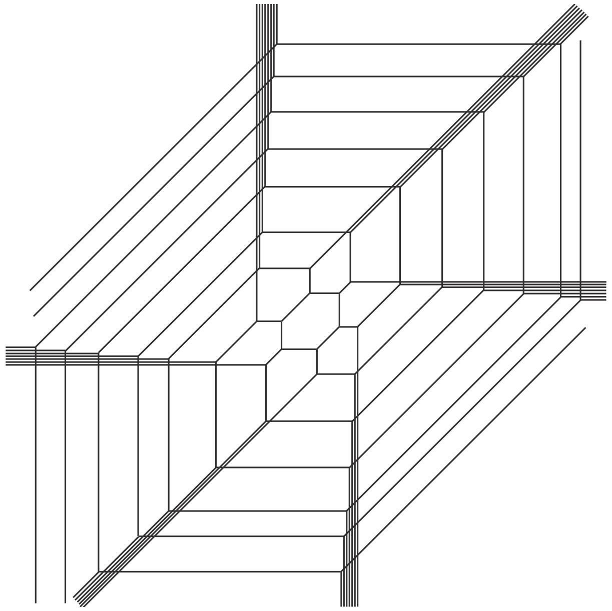
Tao diagrams connecting 5d and 6d SCFTs

Tao diagram: infinite spirals (KK spectrum)
constant period (compactified radius)

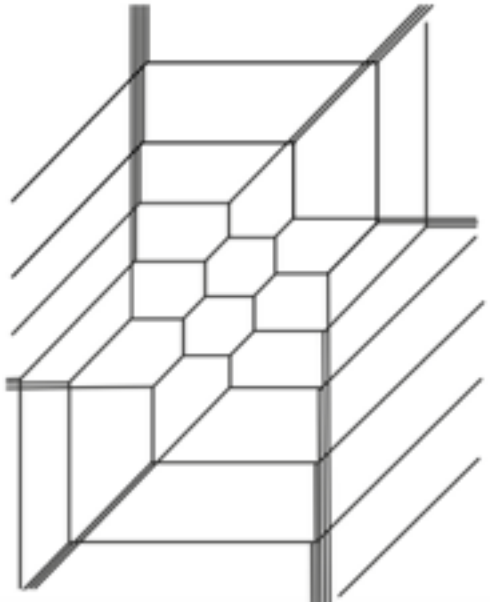
- Naturally identified as **a 6d theory on a circle** (compactification radius emerges…)

- Computational tool:
  Partition function

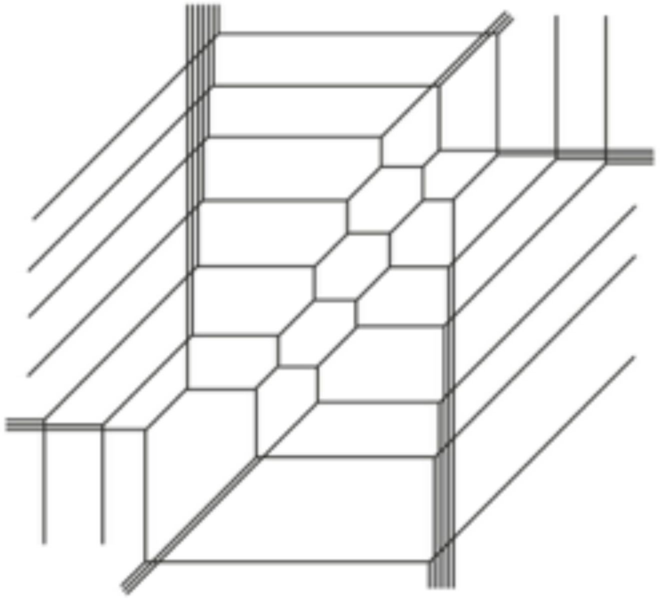
Many more Tao web diagrams



$N = 2$

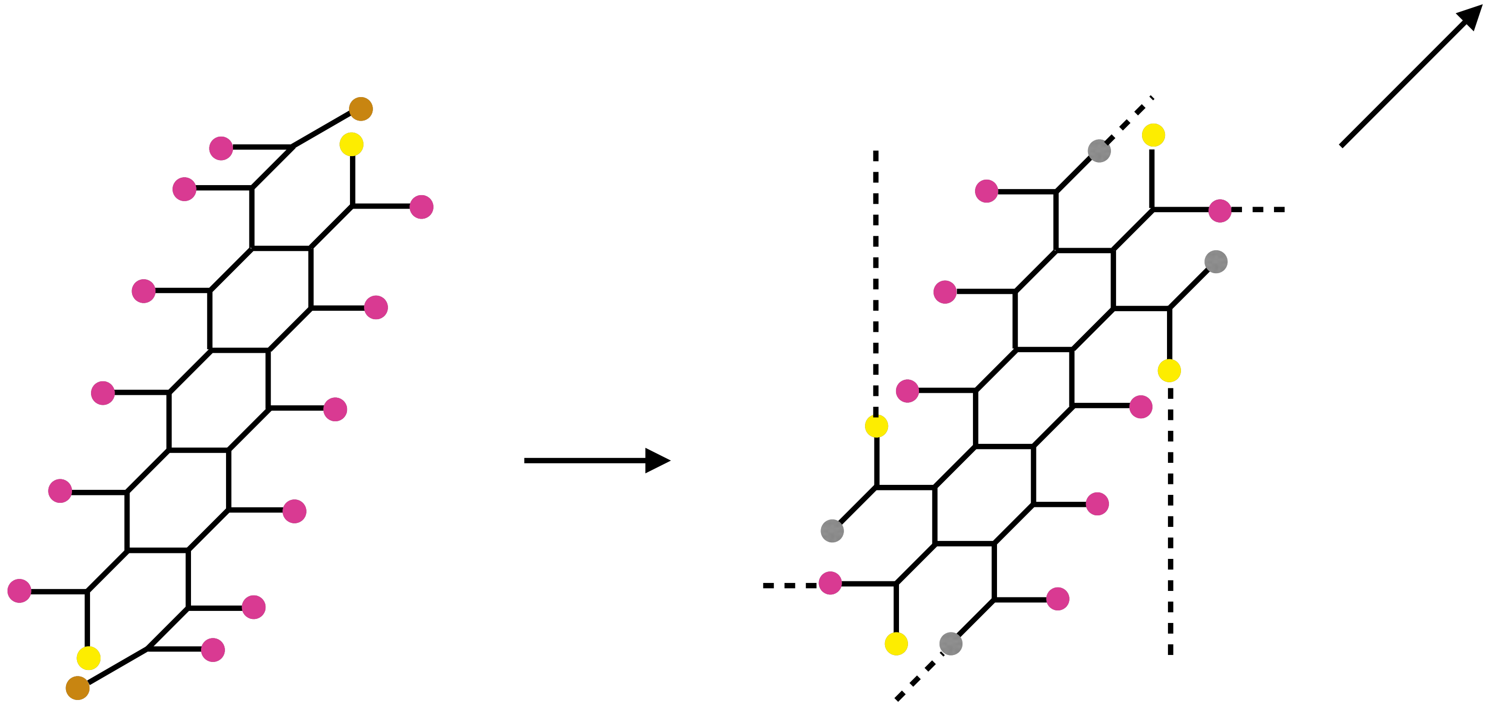


$N = 3$



$N = 4$

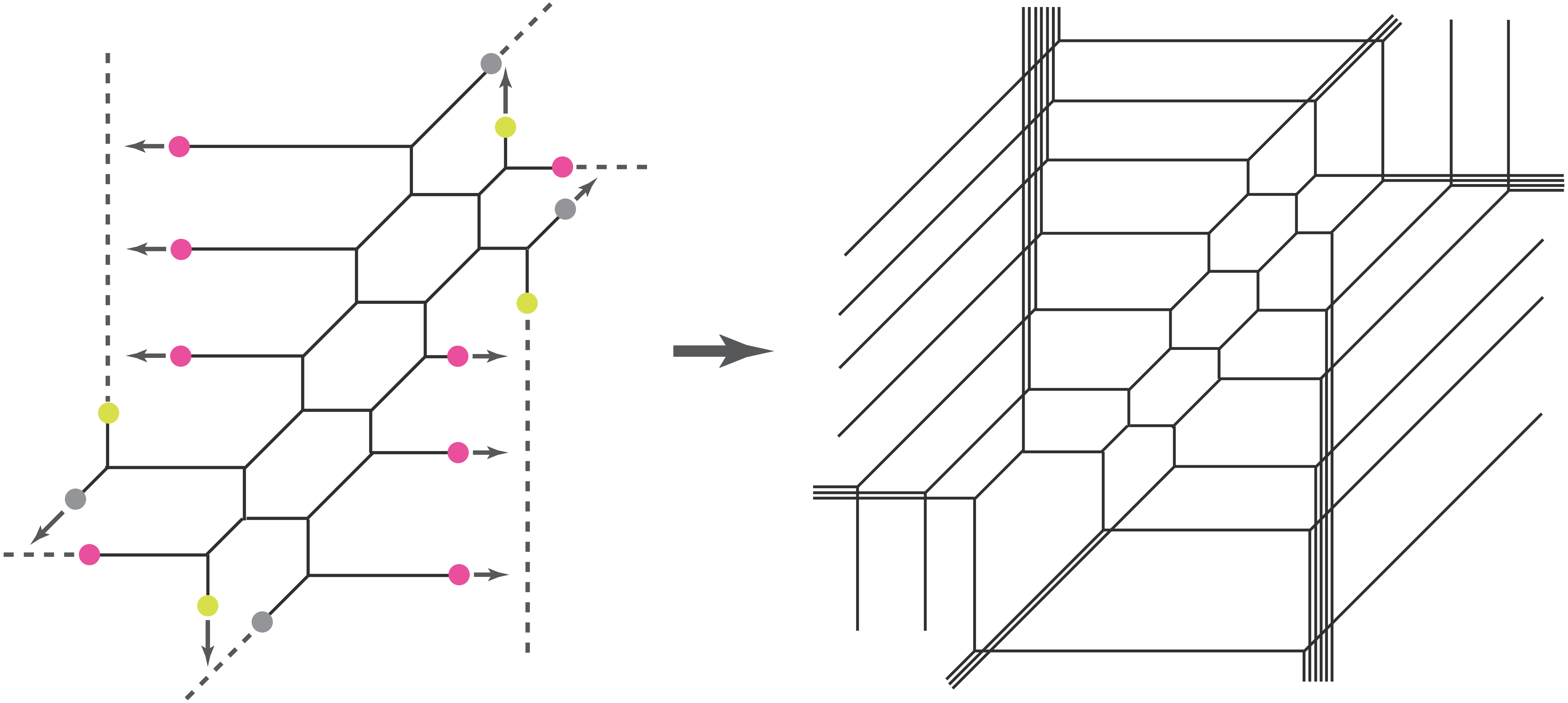
…



$SU(4), N_f = 12$

Many more Tao web diagrams

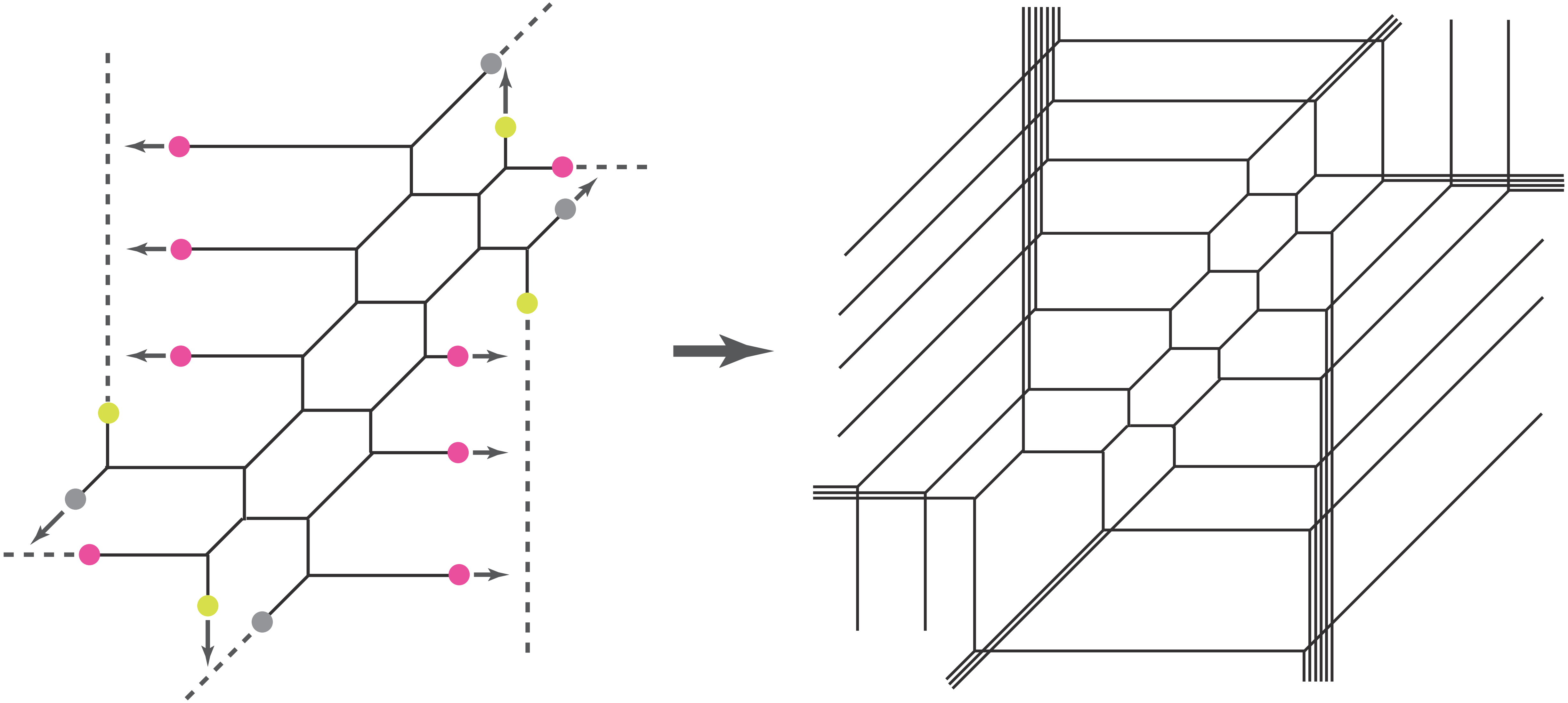
5d SU(N) **$N_f=2N+4$**



What is 6d SCFT for this Tao?

5d SU(N) N_f=2N+4

arXiv:1505.04439



Conjecture

5d N=1 SU(N) w/ Nf=2N+4 has 6d UV fixed point

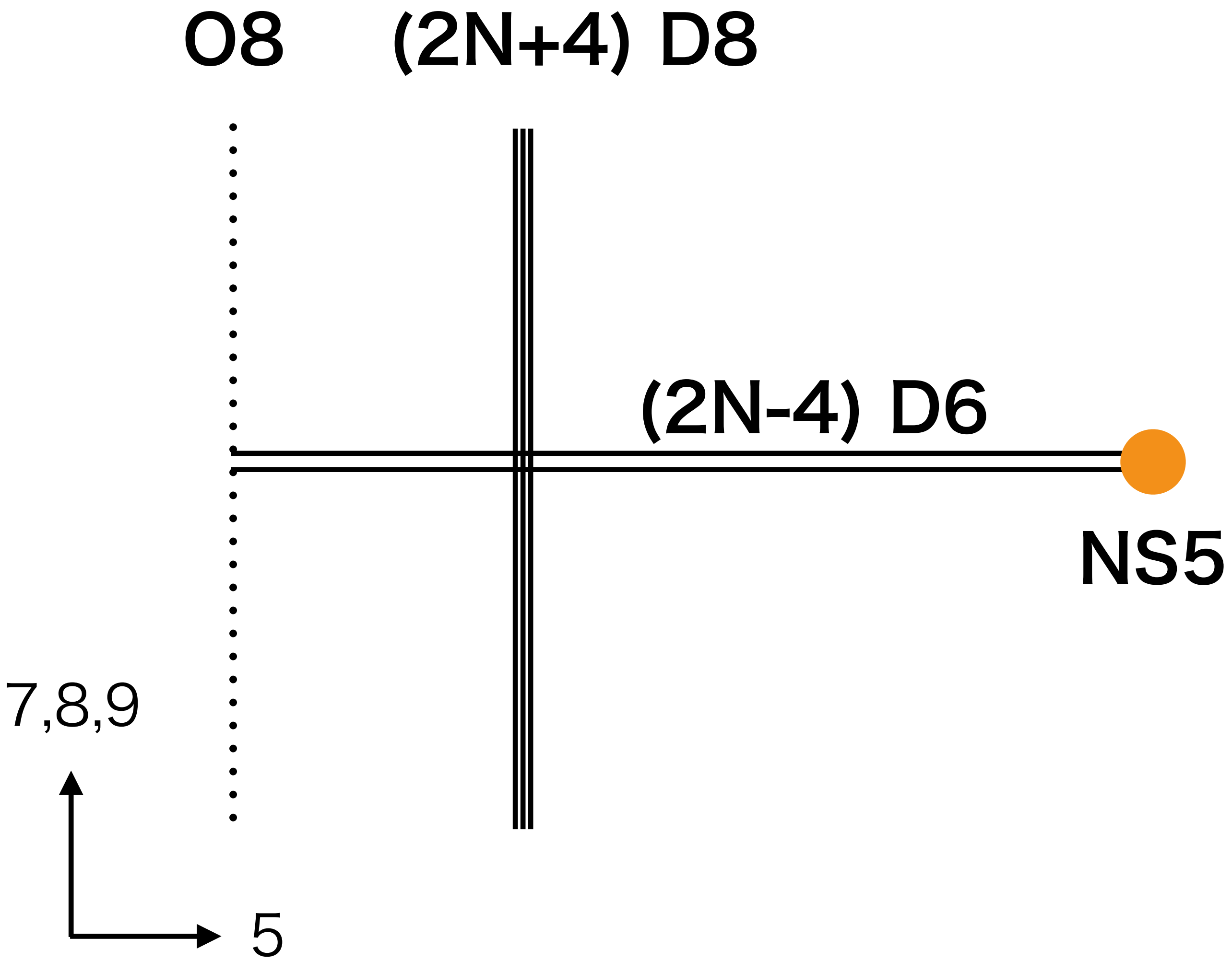
M5-brane probing DN+2 singularity
“(DN+2, DN+2) conformal matter”

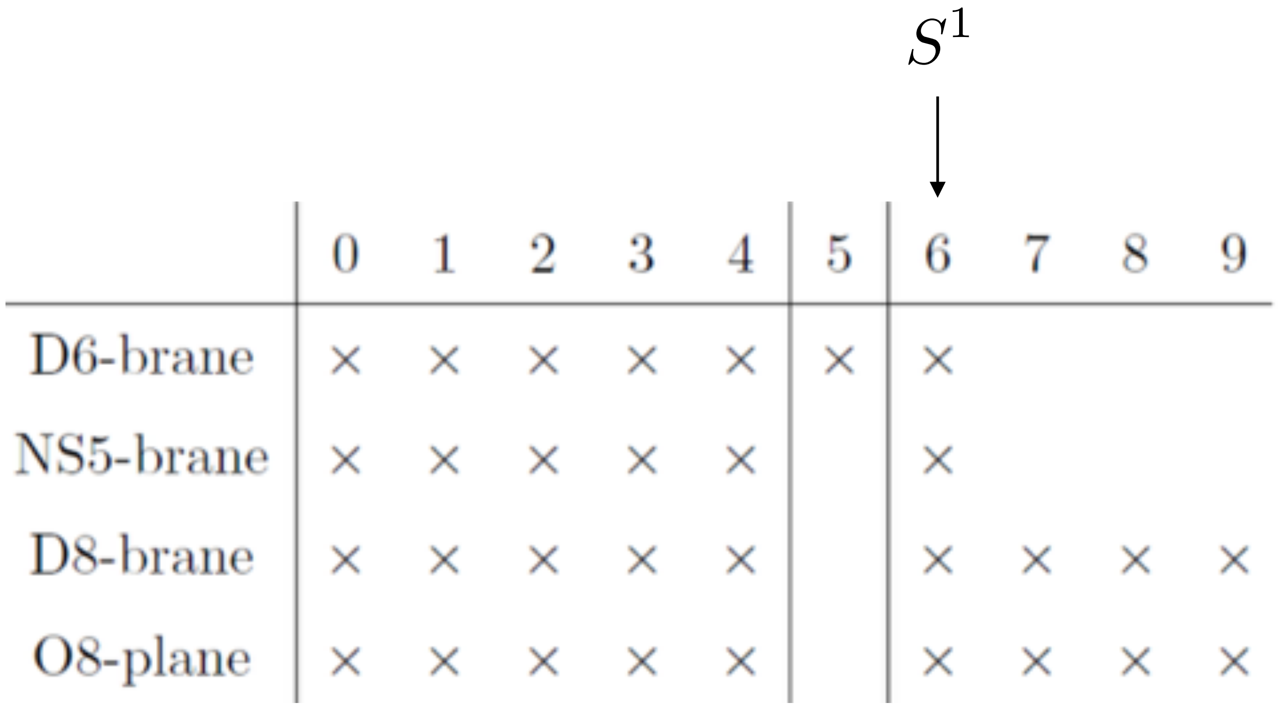
[Del Zotto - Heckman - Tomasiello - Vafa ’14]

M5-brane probing D$_{N+2}$ singularity

Tensor branch

$$6\text{d } \mathcal{N}=(1,0)\ Sp(N-2) \text{ gauge theory}$$
$$N_f = 2N+4, \text{ w/tensor multiplet}$$



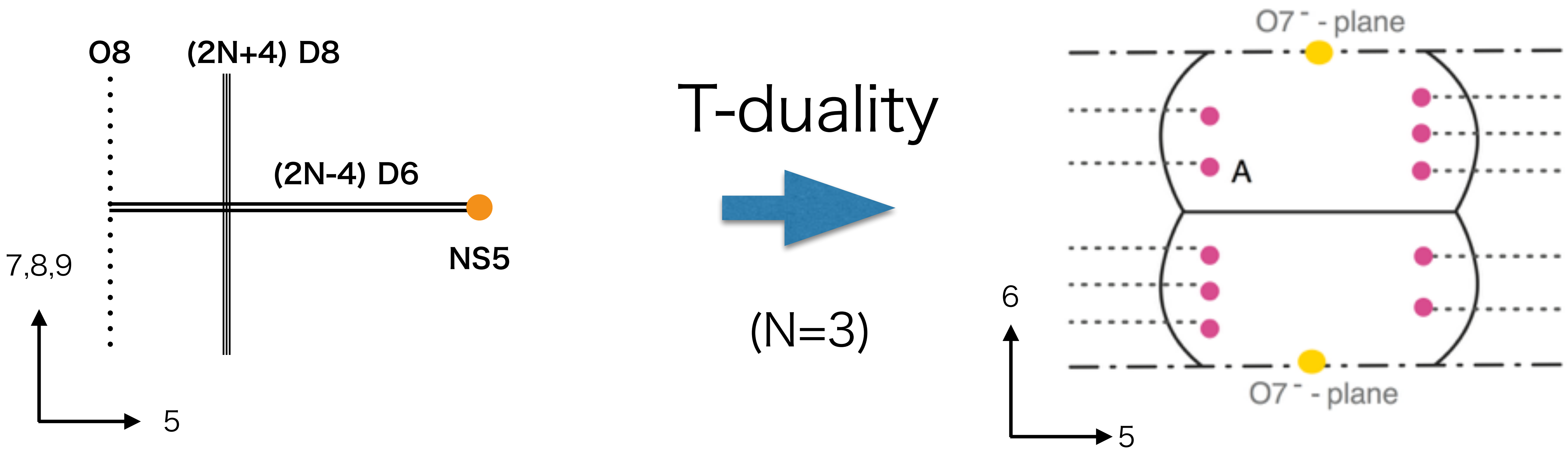


S^1

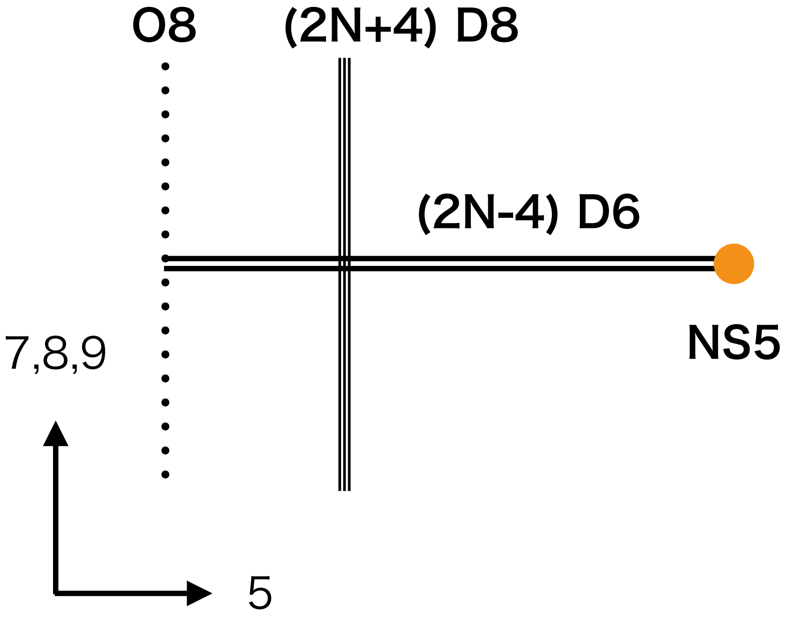
	0	1	2	3	4	5	6	7	8	9
D6-brane	×	×	×	×	×	×	×			
NS5-brane	×	×	×	×	×		×			
D8-brane	×	×	×	×	×		×	×	×	×
O8-plane	×	×	×	×	×		×	×	×	×

[Brunner, Karch '97, Hanany, Zaffaroni '97]

Diagrammatic “Derivation”

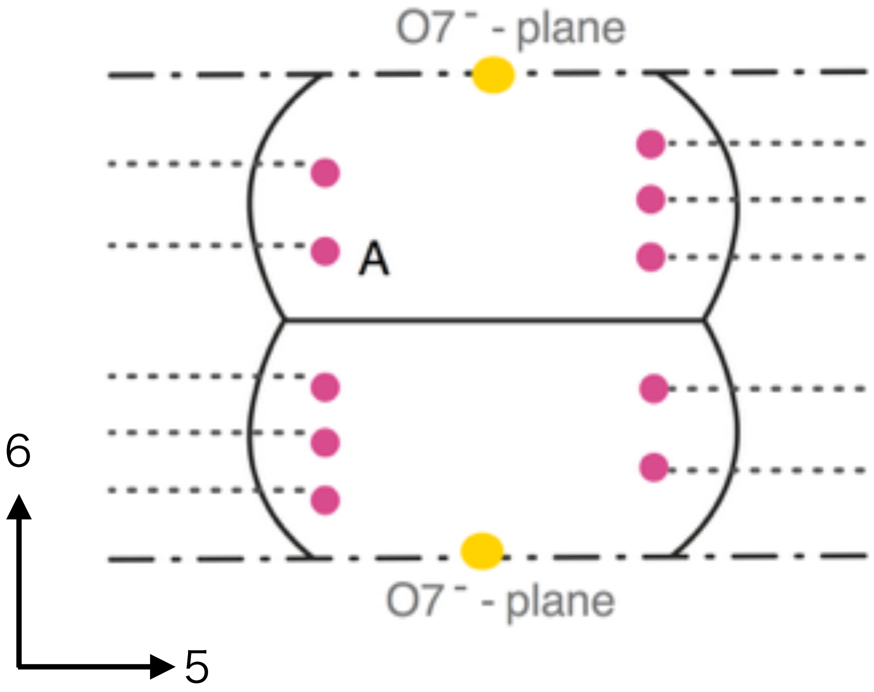


Diagrammatic “Derivation”



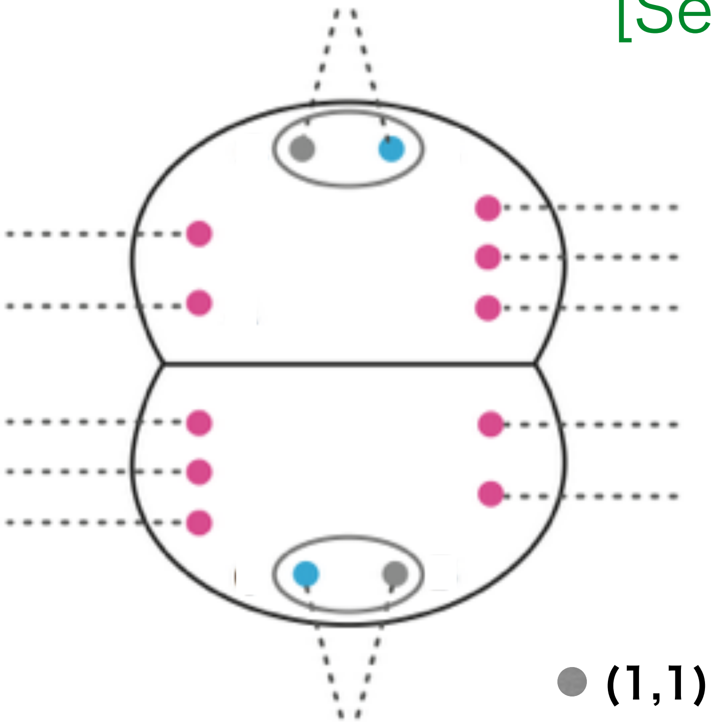
T-duality

(N=3)



O7⁻ -plane
= (1,1) 7-brane
+ (1,-1) 7-brane

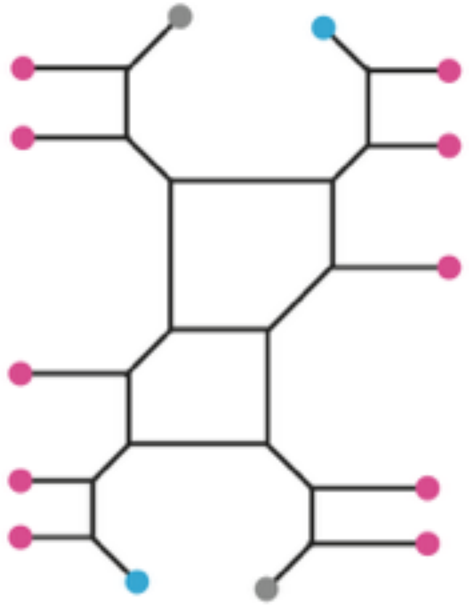
[Sen ’96]



● (1,1) 7-brane
● (1,-1) 7-brane

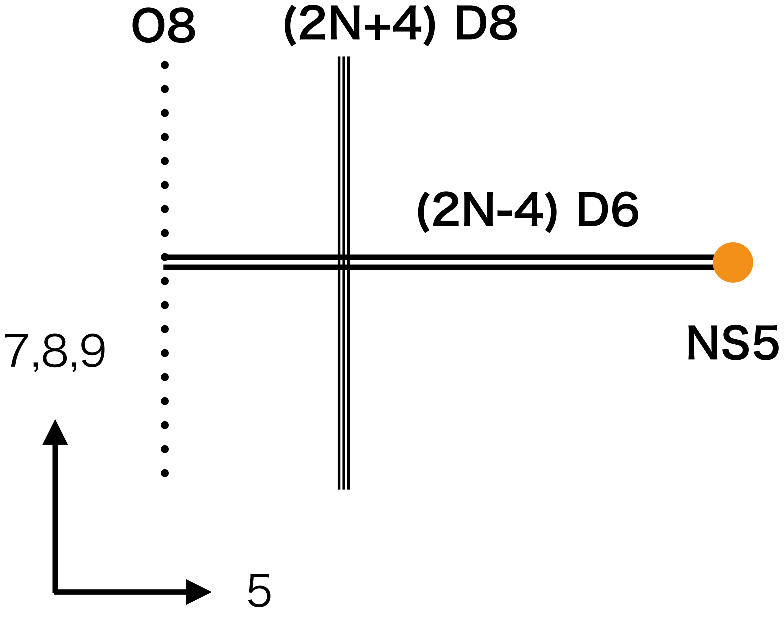
Hanany-Witten transition

$5\text{d}\ SU(N)\ N_f = 2N+4$

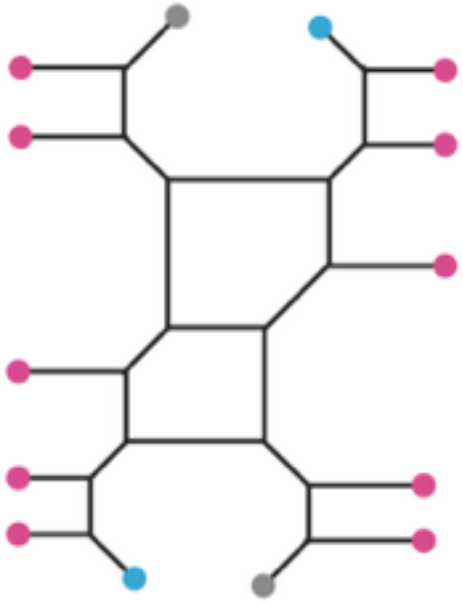


Therefore, we showed that

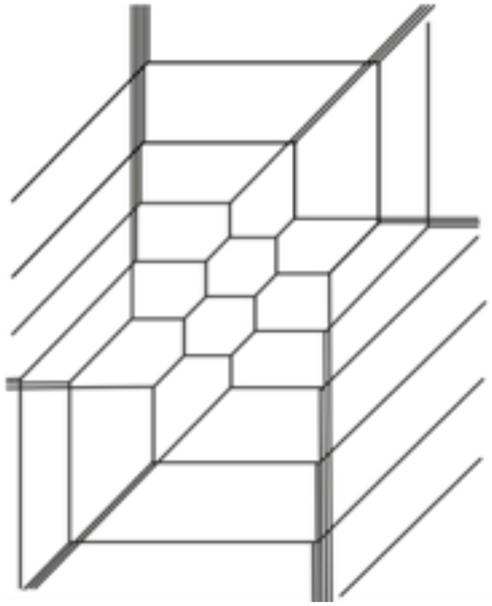
M5-brane probing
D_{N+2} singularity



$5\mathrm{d}\ SU(N)\ N_f = 2N+4$



Tao diagrams



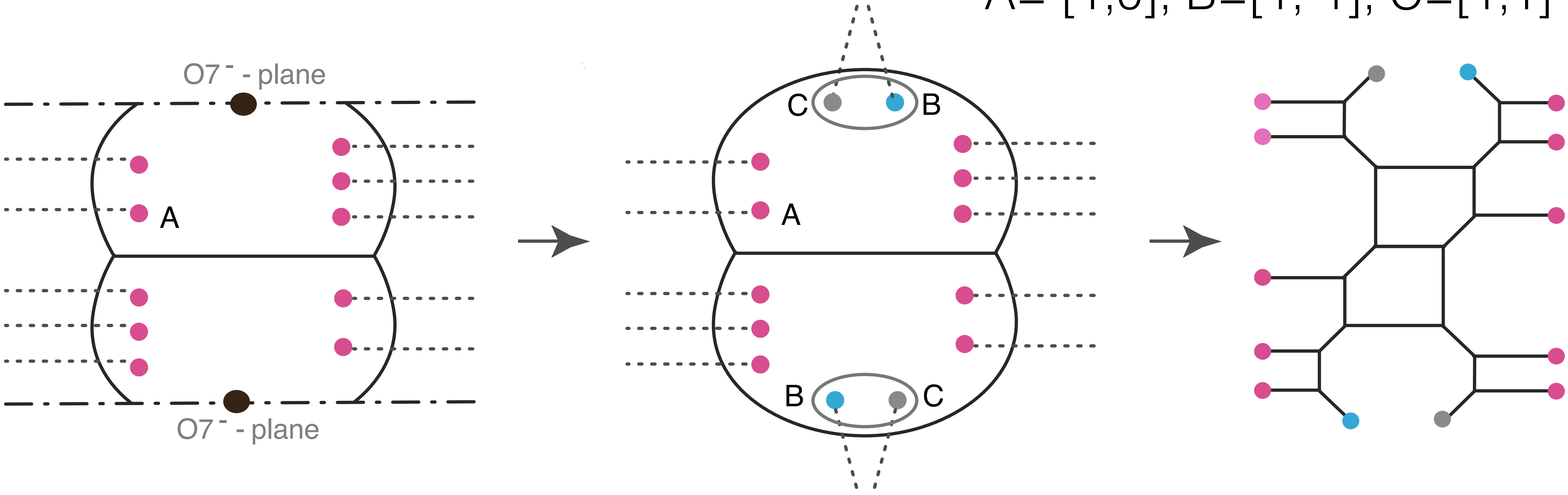
“Sp-SU duality”

6d Sp(N-1) theory with Nf= 2N+4, a tensor

5d SU(N) theory with Nf= 2N+4

ex: N=1-> 6d Sp(1) with Nf=10

A= [1,0], B=[1,-1], C=[1,1]



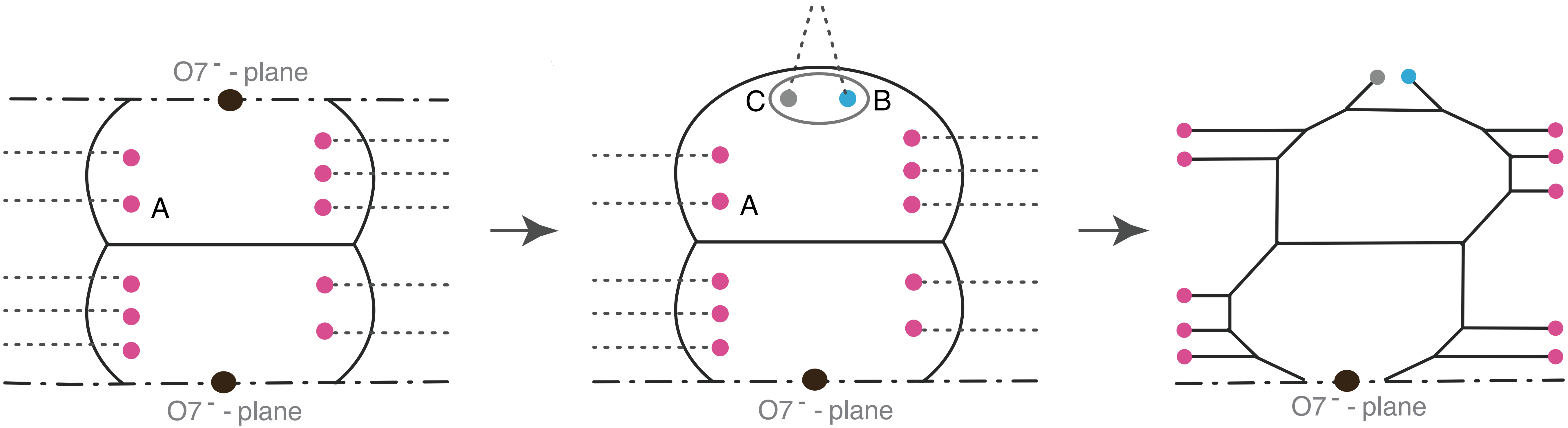
5d SU(3) with Nf=10, CS=0

[Hayashi-**SSK**-Lee-Taki-Yagi ’15]
[Yonekura ’15]

5d Sp(N-1) theory with Nf= 2N+4

Resolving **only one O7**:

[Hayashi-SSK-Lee-Yagi ’15]



5d Sp(2) with Nf=10

We thus have

5d SU(N) theory with Nf= 2N+4

↕

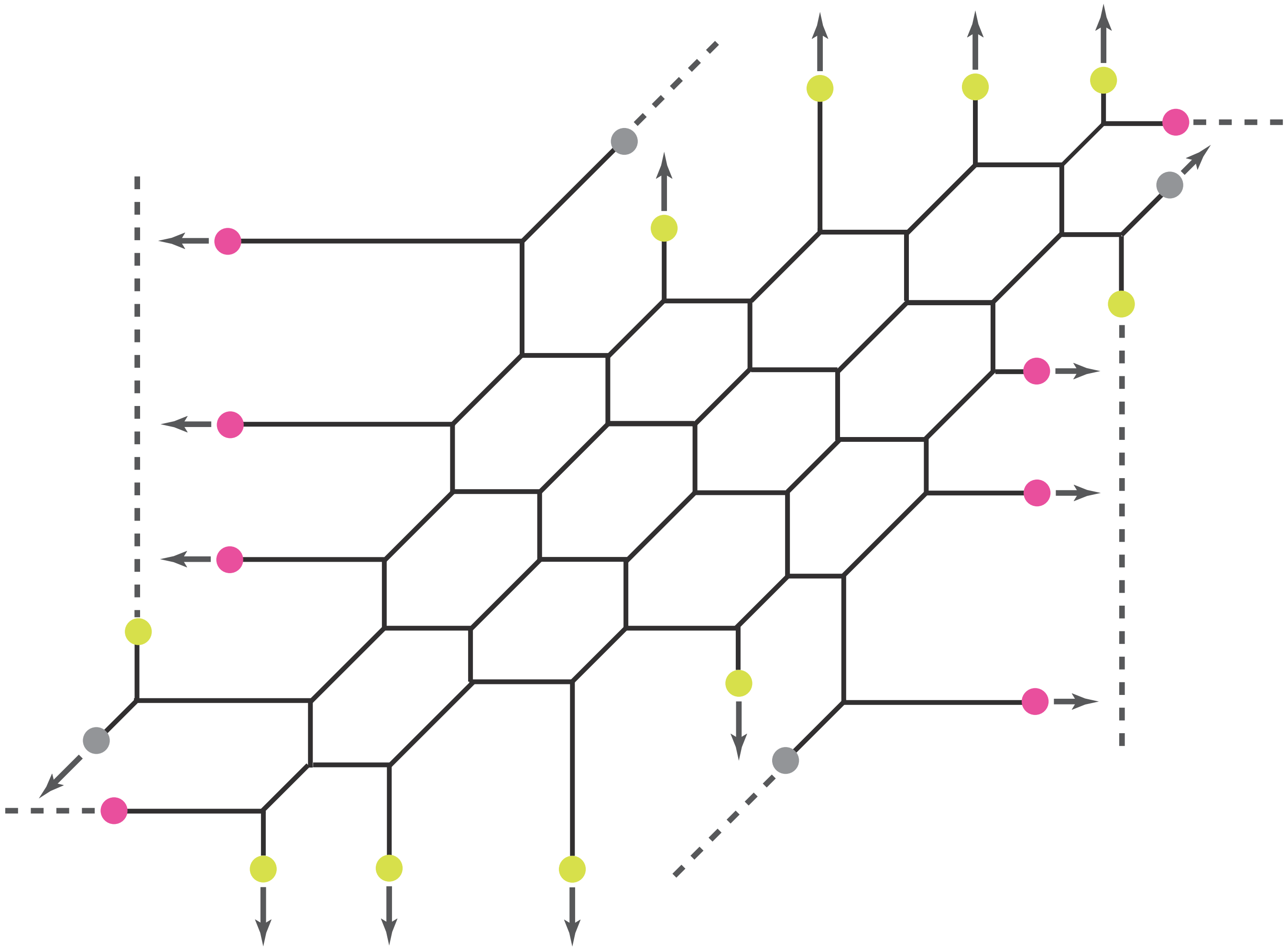
5d Sp(N-1) theory with Nf= 2N+4

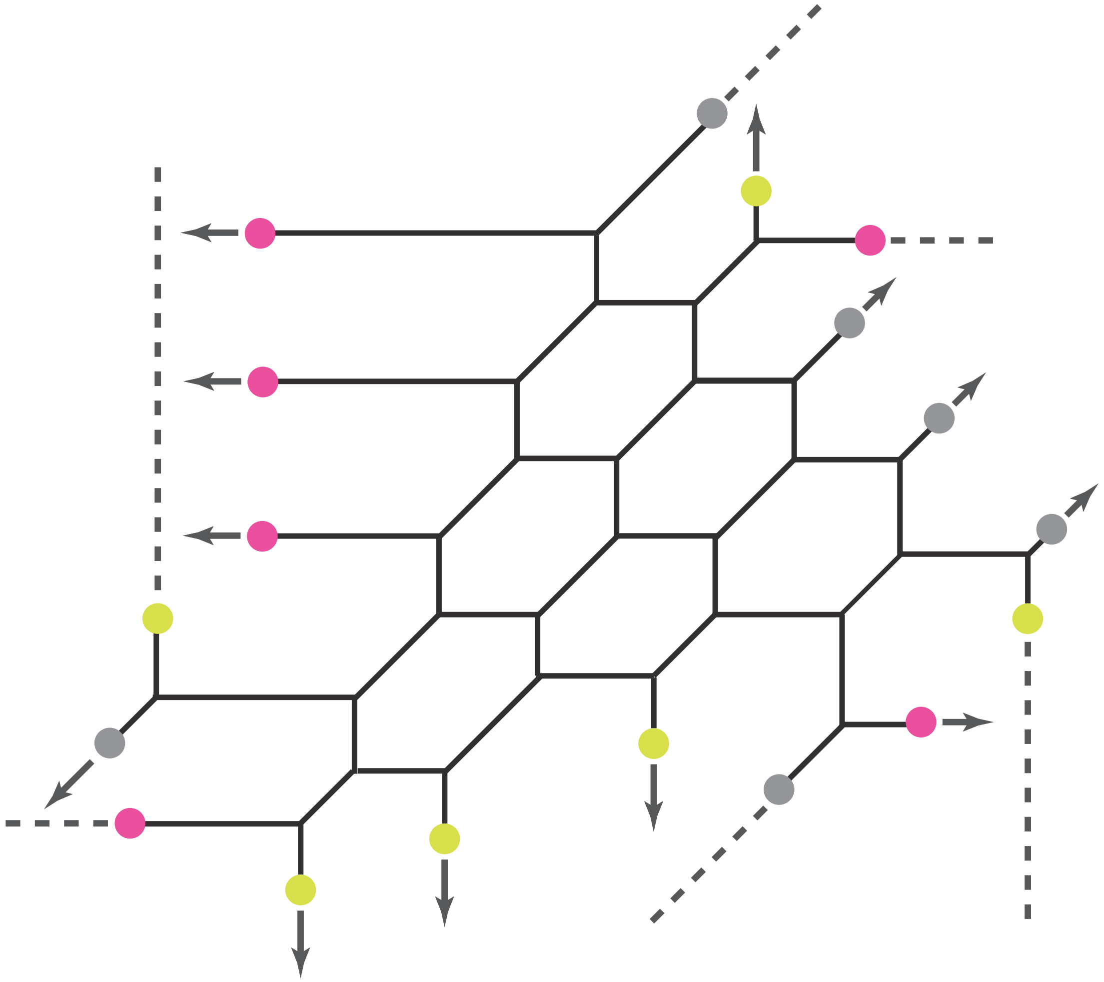
[Hee-Cheol Kim’s talk]

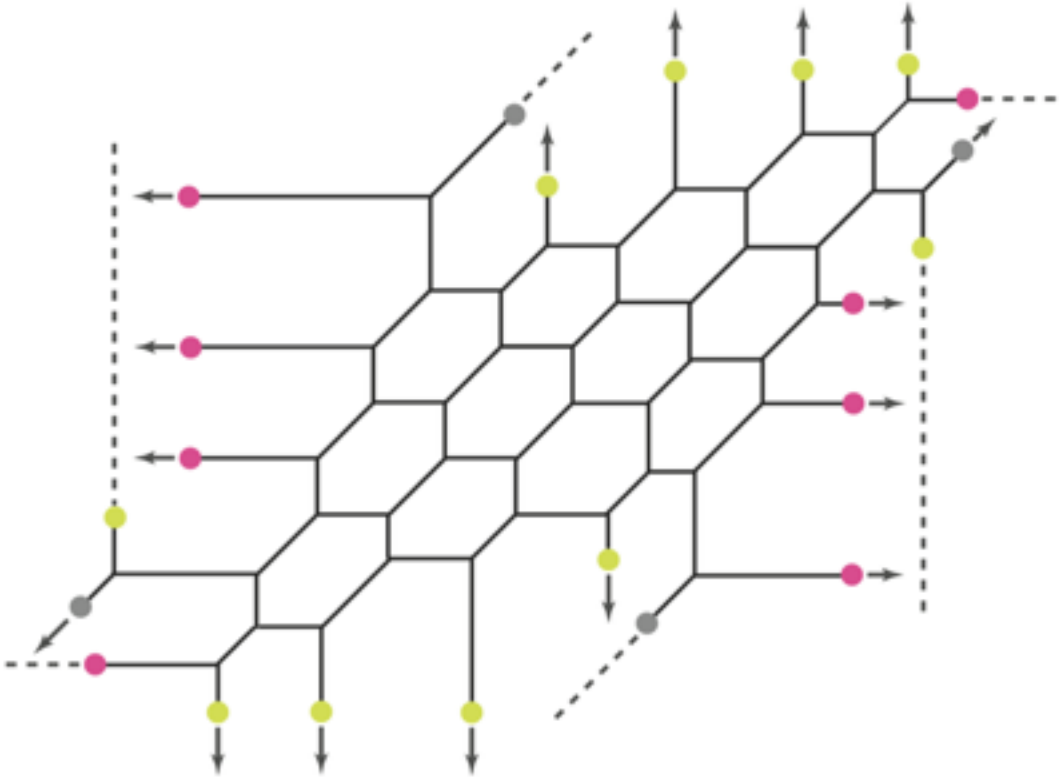
Flavor decoupling -> 5d dualities

[Gaiotto-Kim ’15]

Quiver type?







$$5\text{d}\ [N+2] - \overbrace{SU(N) - \cdots - SU(N)}^{k} - [N+2]$$

⬆

'15 Yonekura

$k = 2n+1$

$$6\text{d}\ Sp(N') - SU(2N'+8) - SU(2N'+16) - \cdots - SU(2N'+8(n-1)) - [2N'+8n]$$

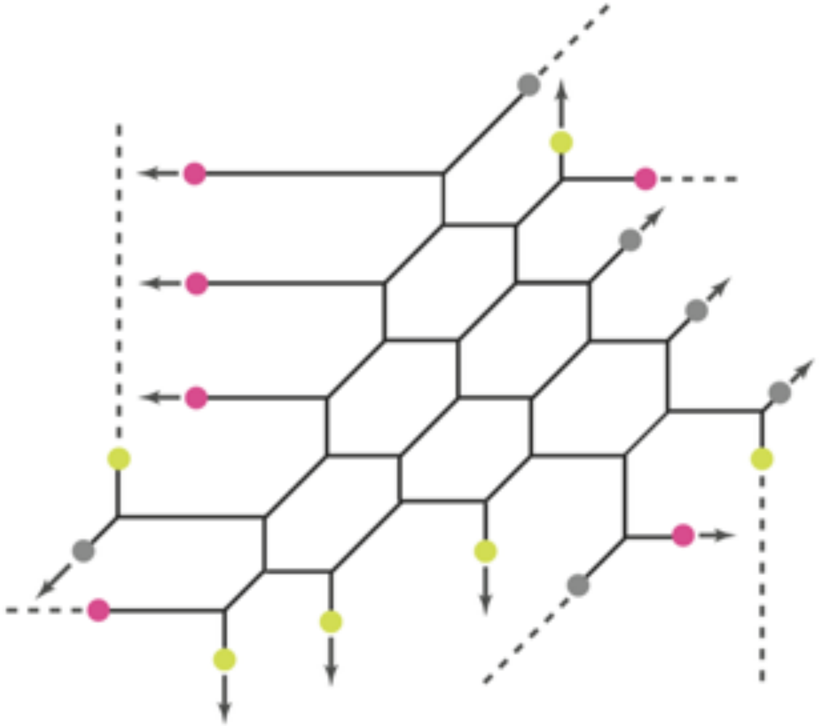
$$N' = N - 2n$$

$k = 2n$

$$6\text{d}\ [A] - SU(N') - SU(N'+8) - SU(N'+16) - \cdots SU(N'+8(n-1) - [N'+8n+8]$$

$$N' = 2(N-2n+1)$$

↖ hypermultiplet in antisymmetric representation



$$\text{5d } [N+3] - SU(N) - SU(N-1) - SU(N-2) - \cdots - SU(3) - SU(2) - [3]$$

$$(\text{“Tao-nization” of 5d } T_N)$$

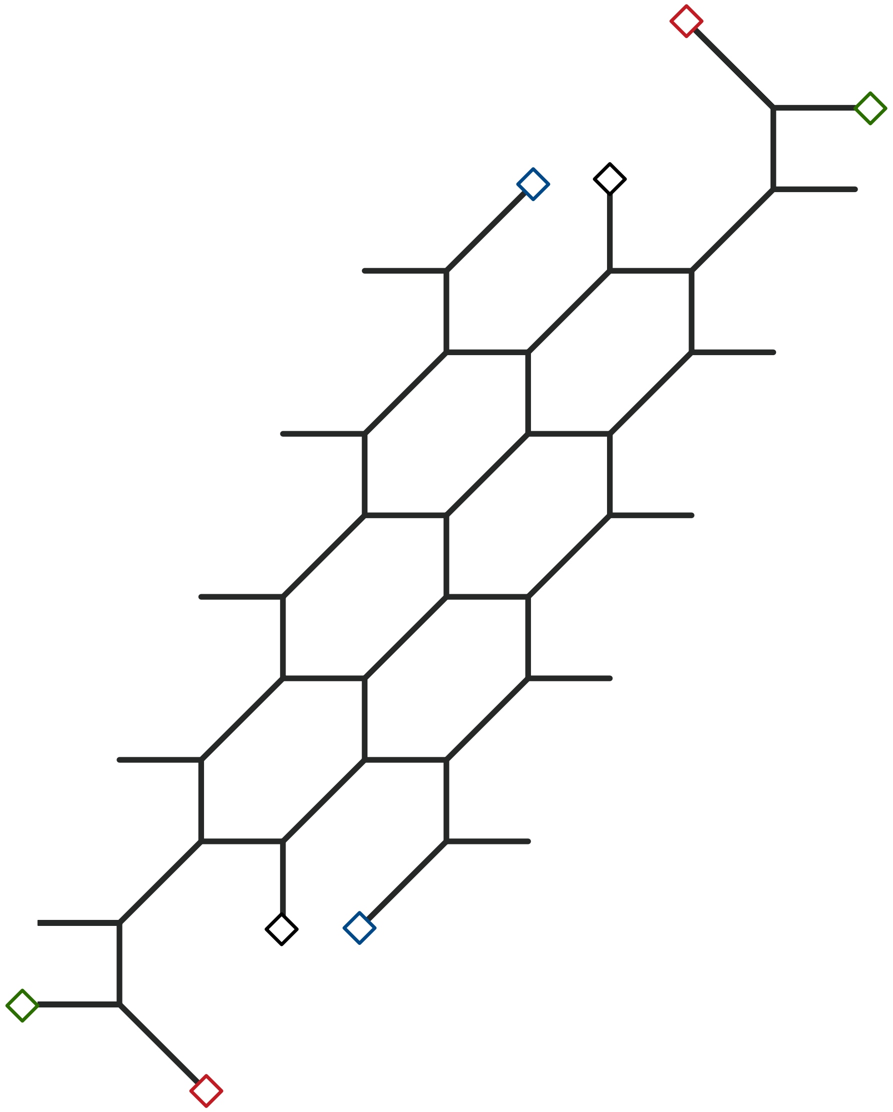
’15 Zafrir
’15 Ohmori, Shimizu

$$N = 3n: \quad \text{6d } SU(0) - SU(9) - \cdots - SU(9n) - [9n+9]$$

$$N = 3n+1: \quad \text{6d } SU(3) - SU(12) - \cdots - SU(3+9(n-1)) - [3+9n]$$

$$N = 3n+2: \quad \text{6d } \left[\tfrac{1}{2}\right]_{\Lambda^3} - SU(6) - SU(15) - \cdots - SU(6+9(n-1)) - [6+9n]$$

“UV dualities”

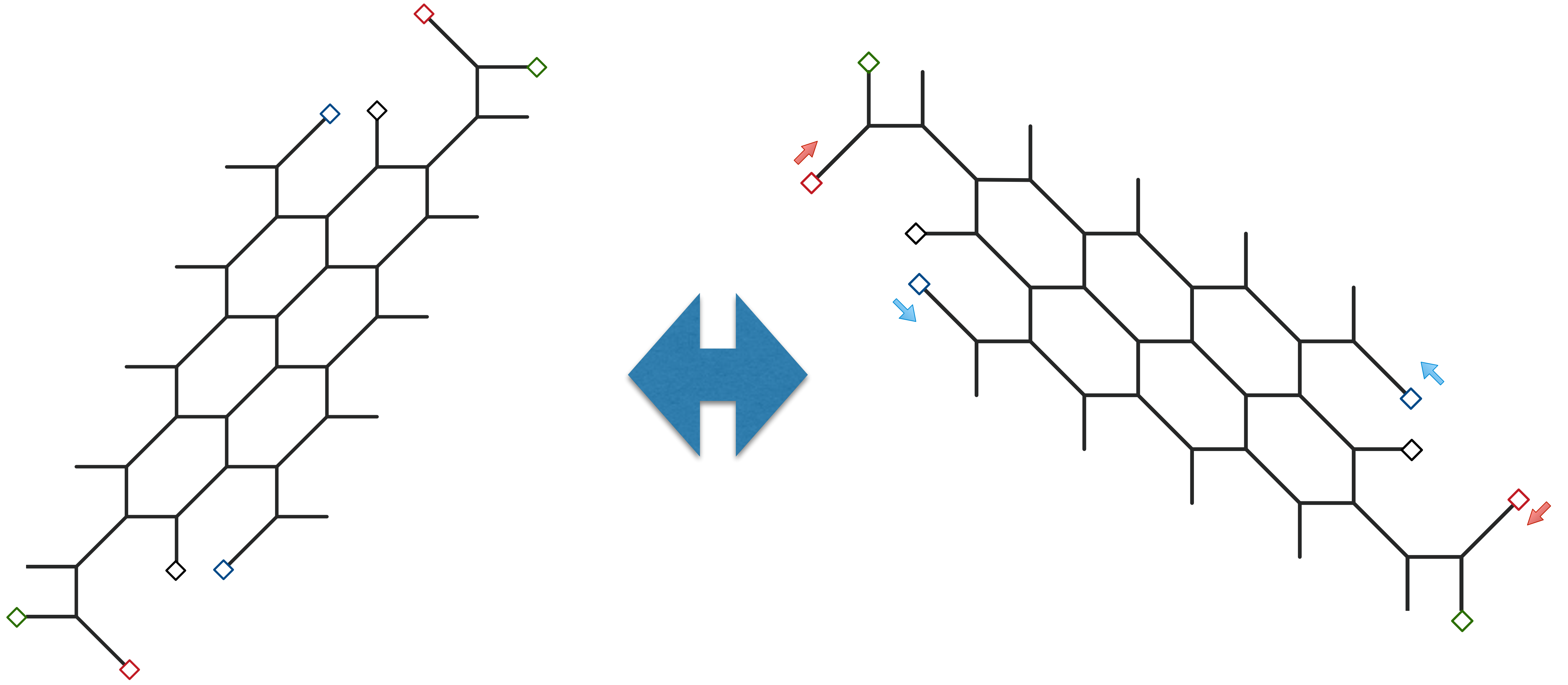


5d [6]-SU(4)-SU(4)-[6]

↑

6d [A]-SU(6)-[14]

S-duality



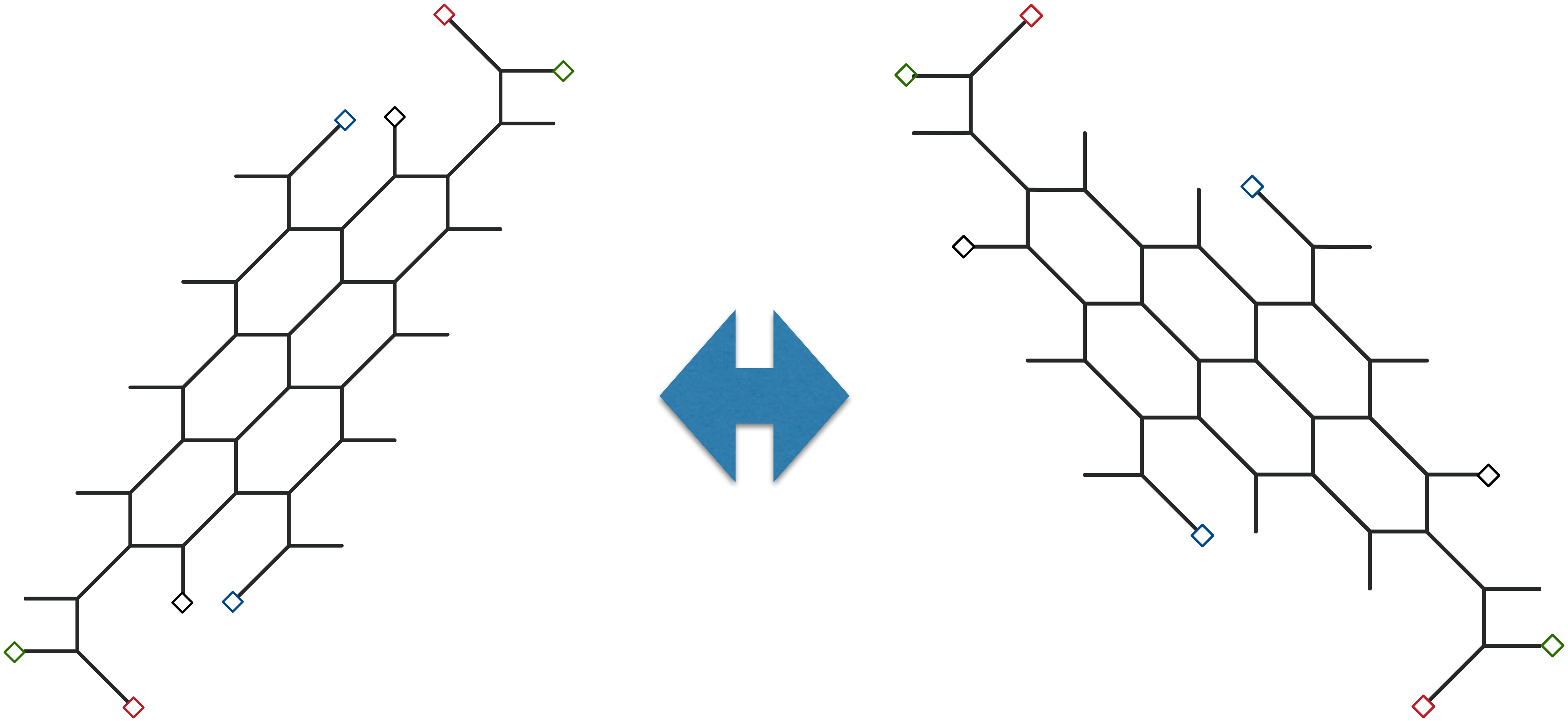
5d [6]-SU(4)-SU(4)-[6]

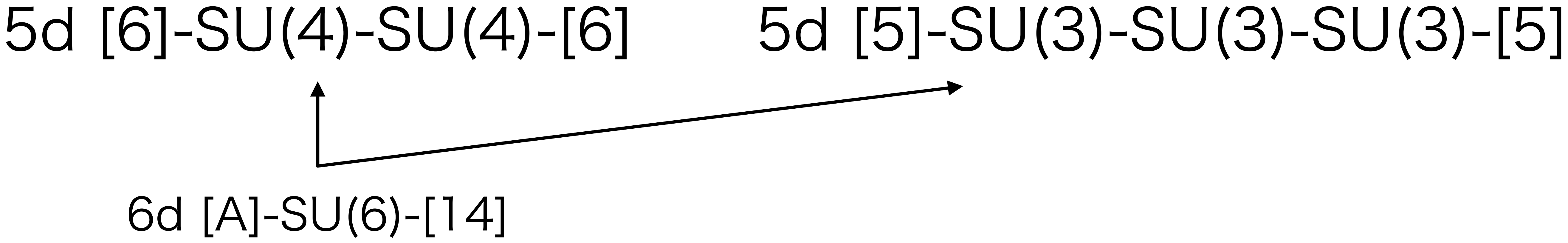
5d (?)-SU(3)-SU(3)-SU(3)-(?)

↑

6d [A]-SU(6)-[14]

S-duality



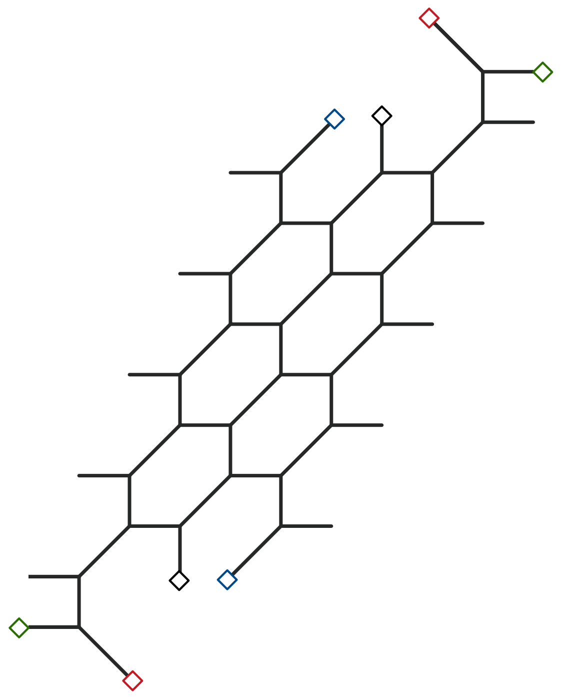


5d [6]-SU(4)-SU(4)-[6]

5d [5]-SU(3)-SU(3)-SU(3)-[5]

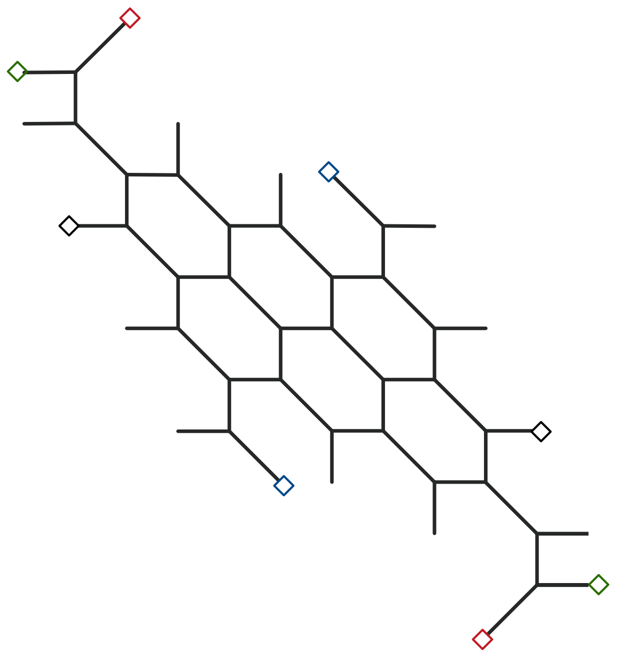
6d [A]-SU(6)-[14]

[6]-SU(4)-SU(4)-[6]



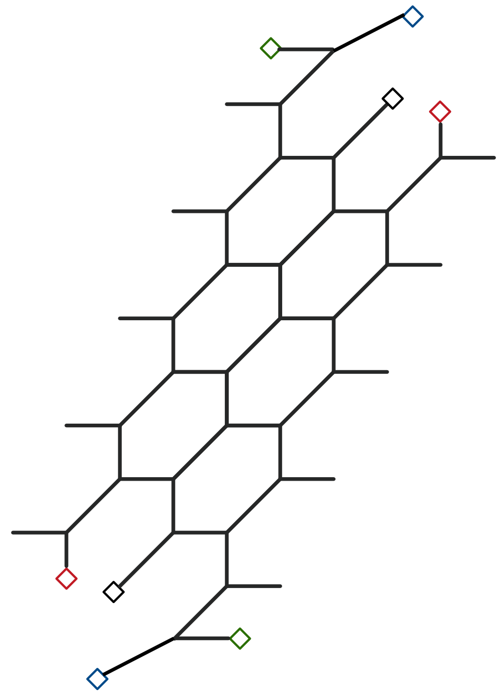
S-duality

[5]-SU(3)-SU(3)-SU(3)-[5]



Mass deformation

↓

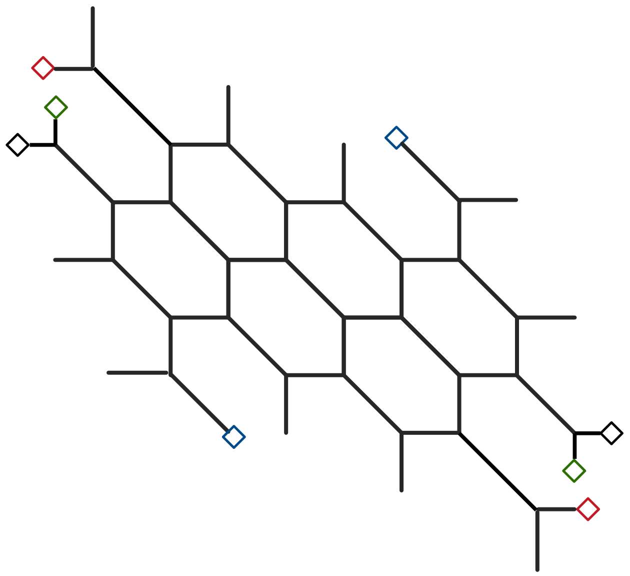


S-duality

[1] [1]

| |

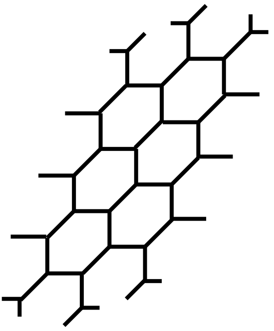
[3] - SU(2) - SU(3) - SU(3) - SU(2) - [3]



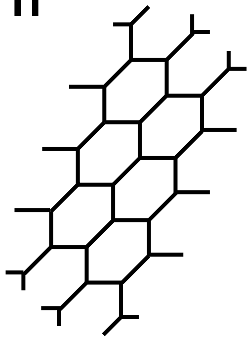
6d SU(6) theory
with Nf=14, Na=1

various 5d quivers
(depending on
D5, D7 distributions)

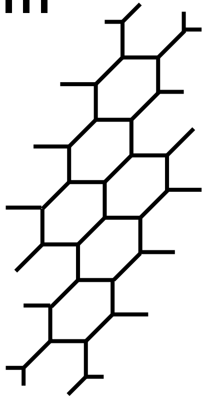
I



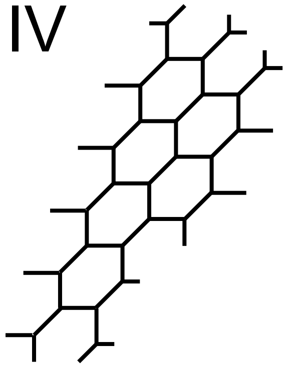
II



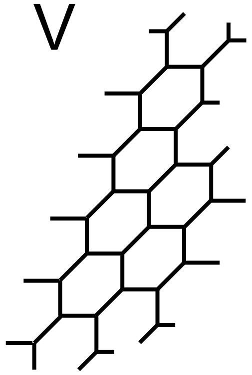
III



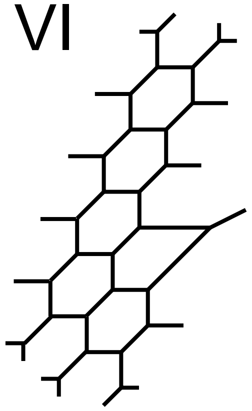
IV



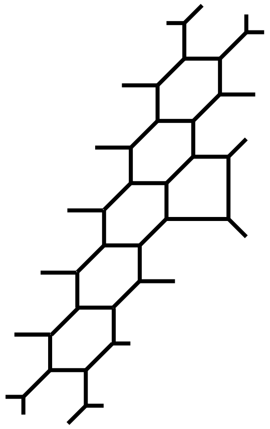
V



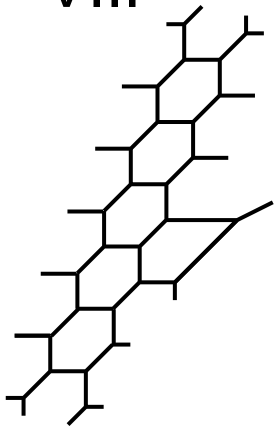
VI



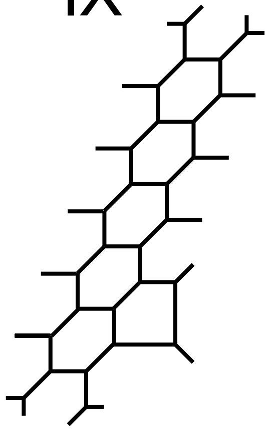
VII



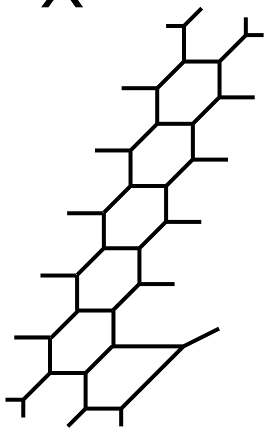
VIII



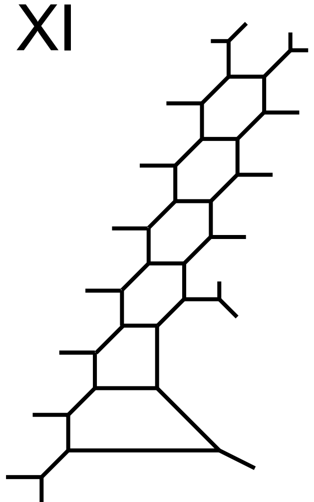
IX



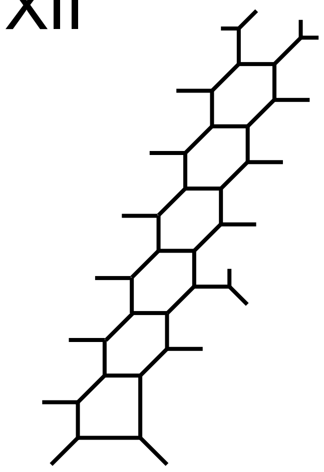
X



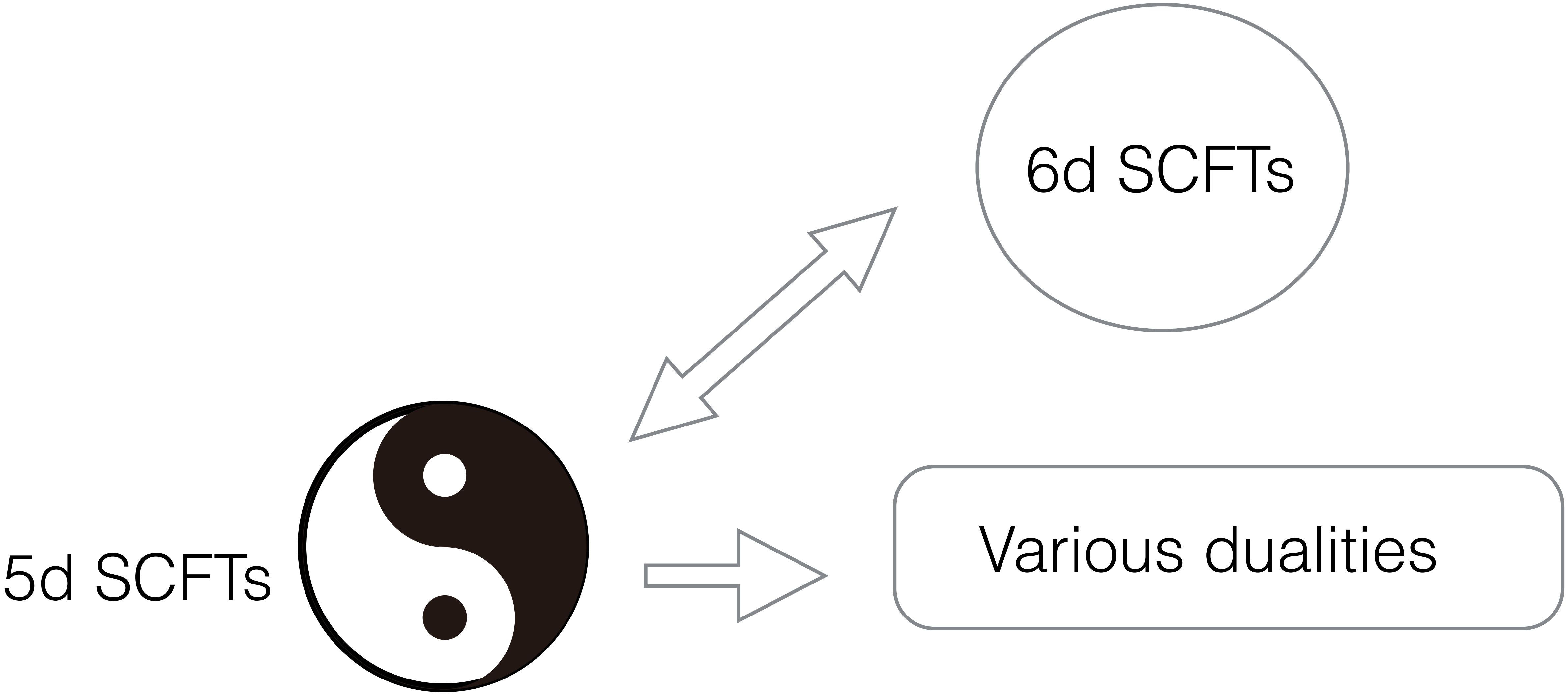
XI



XII



New understandings of 5d/6d SCFTs : Summary



6d SCFTs

5d SCFTs

Various dualities

Tao diagram

- 5d SU(N) w/ Nf=2N+4
- 5d UV dualities