Topological strings, black holes, and matrix models

Minxin Huang

IPMU

MH, A. Klemm, and S. Quackenbush, hep-th/0612125,
MH , A. Klemm, M. Mariño and A. Tavanfar, arXiv:0704.2440 [hep-th],
MH and A.Klemm, arXiv:0902.1325 [hep-th].

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- Introduction and motivation. Summary of Results. (30 minutes)

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- Relation to matrix models.

- Conclusion and future directions.

- Topological strings: A $N = (2,2)$ supersymmetric non-linear sigma model from world sheet Σ to target space X.

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- There are two types of topological twistings: A-model and B-model. We are interested in the topological string partition function

$$Z = \exp(\sum_{g=0}^{\infty} \lambda^{2g-2} F^{(g)}(t_i))$$

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where t_i are Kahler moduli in the case of A-model, and complex structure moduli in the case of B-model.

- Topological A-model counts holomorphic curves in target space X, and has a rigorous mathematical formulation known as Gromov-Witten theory. Topological B-model is a complex structure deformation theory known as Kodaira-Spencer theory.

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- Related to black hole physics according the OSV conjecture.

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- Many techniques have been developed to study topological string theory. For example, topological strings on a class of non-compact toric Calabi-Yaus are essentially solved to all genera by topological vertex formalism.

- A long standing problem: How to solve topological strings on compact Calabi-Yau spaces? Progress are very limited.

- A famous example: the Quintic manifold, a degree 5 hypersurface in $\mathbb{CP}^4$.
  Candelas et al solve the prepotential, i.e. the counting genus zero curve, using physical idea of mirror symmetry.
  The mirror symmetry results are later proven by mathematicians using Kontsevich's localization methods, Givental; Lian, Liu, Yau.
  At higher genus, the only available approach is the BCOV(Bershadsky, Cecotti, Ooguri, Vafa) method. One use holomorphic anomaly equation to compute $F^{(g)}$ recursively in genus g. This was done by BCOV (in 1993) up to genus 2.

- The BCOV holomorphic anomaly equation

$$\bar{\partial}_{\bar{k}} F^{(g)} = \frac{1}{2} \bar{C}_{\bar{k}}^{ij} \left(D_i D_j F^{(g-1)} + \sum_{r=1}^{g-1} D_i F^{(r)} D_j F^{(g-r)} \right)$$

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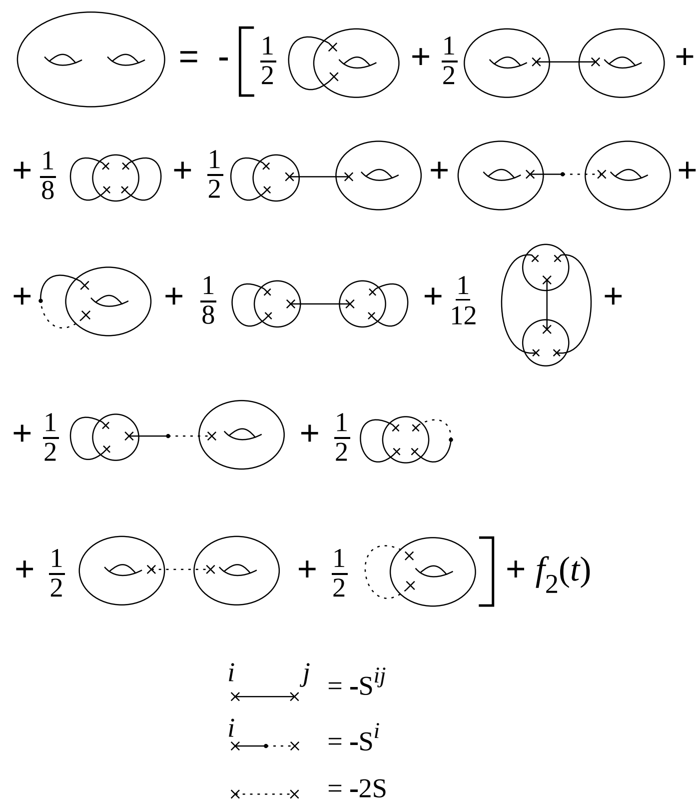
- An example of BCOV diagrams, at genus 2.



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  2. Computational complexity in BCOV method: the number of diagrams grows exponentially with genus. A normal laptop can handle the diagrams only up to about genus 6, even for the simplest one parameter models such as the quintic.

- The calculation was pushed up to genus 3 for the quintic, using further information from the counting of BPS states known as Gopakumar-Vafa invariants. Katz, Klemm, Vafa, hep-th/9910181.

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  1. We solve the holomorphic anomaly equation directly without the BCOV Feynman diagrams, by using the idea of formulating topological strings as polynomials Yamaguchi, Yau, hep-th/0406078. The computational complexity of the method grows only polynomially in genus.

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  2. We discover novel boundary conditions at the conifold point of the moduli space, i.e. the "gap" condition c.f. Huang, Klemm, hep-th/0605195, which fix the holomorphic ambiguity to a large extend.

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  2. We discover novel boundary conditions at the conifold point of the moduli space, i.e. the "gap" condition c.f. Huang, Klemm, hep-th/0605195, which fix the holomorphic ambiguity to a large extend.

- We are able to solve a class of one-parameter Calabi-Yau models to very high genus, e.g. genus ~ 26 (up to genus 51 in principle) for the quintic.

5 minutes break

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- Picard-Fuchs equation, periods, and mirror map.

$$\{(\psi\partial_\psi)^4 - \psi^{-1}(\psi\partial_\psi - \frac{1}{5})(\psi\partial_\psi - \frac{2}{5})(\psi\partial_\psi - \frac{3}{5})(\psi\partial_\psi - \frac{4}{5})\}\omega = 0$$

The equation can be solved by asymptotic series at $\psi = \infty$,

$$\vec{\Pi} = \begin{pmatrix} \int_{B_1}\Omega \\ \int_{B_2}\Omega \\ \int_{A^1}\Omega \\ \int_{A^2}\Omega \end{pmatrix} = \begin{pmatrix} F_0 \\ F_1 \\ X_0 \\ X_1 \end{pmatrix} = \omega_0 \begin{pmatrix} 2F^{(0)} - t\partial_t F^{(0)} \\ \partial_t F^{(0)} \\ 1 \\ t \end{pmatrix}$$

The mirror map has a logarithmic behavior

$$2\pi i t(\psi) = -\log(5^5\psi) + \frac{154}{625\psi} + \frac{28713}{390625\psi^2} + \cdots$$

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- The Kahler potential and metric

$$K := -\log i(\bar{X}^i F_i - X^i \bar{F}_i), \qquad G_{\psi\bar{\psi}} := \partial_\psi \partial_{\bar{\psi}} K$$

Topological strings as polynomials

Yamaguchi and Yau, hep-th/0406078

- Define the following generators

$$A_p := \frac{(\psi\partial_\psi)^p G_{\psi\bar\psi}}{G_{\psi\bar\psi}}, \quad B_p := \frac{(\psi\partial_\psi)^p e^{-K}}{e^{-K}}, \quad (p = 1, 2, 3, \cdots)$$

$$C := C_{\psi\psi\psi}\psi^3, \quad X := \frac{1}{1-\psi}$$

These generators satisfy the derivative relations

$$\psi\partial_\psi A_p = A_{p+1} - A_1 A_p, \quad \psi\partial_\psi B_p = B_{p+1} - B_1 B_p, \quad \psi\partial_\psi X = X(X-1)$$

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- The independent generators are (A_1, B_1, B_2, B_3, X). One can use the Picard-Fuchs equation and special geometry relation to show B_4 and A_2 are polynomials of $(A_1 \equiv A, B_1 \equiv B, B_2, B_3, X)$.

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$$\begin{aligned} P_{g=0}^{(3)} &= 1 \\ P_{g=1}^{(1)} &= -\frac{31}{3} B + \frac{1}{12}(X-1) - \frac{1}{2} A + \frac{5}{3} \\ P_g^{(n+1)} &= \psi \partial_\psi P_g^{(n)} - [n(A+1) + (2-2g)(B - \frac{1}{2} X)] P_g^{(n)} \end{aligned}$$

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- Define a change of variable

$$(A, B, B_2, B_3, X) \to (u, v_1, v_2, v_3, X)$$

by the followings

$$B = u, \quad A = v_1 - 1 - 2u, \quad B_2 = v_2 + uv_1,$$
$$B_3 = v_3 - uv_2 + uv_1X - \frac{2}{5}uX$$

- The anti-holomorphic derivative of the generators can be related to each other. Only $\partial_{\bar{\psi}}A_1$ and $\partial_{\bar{\psi}}B_1$ are independent. The BCOV holomorphic anomaly equations are

$$\frac{\partial P_g}{\partial u} = 0$$

$$(\frac{\partial}{\partial v_1} - u\frac{\partial}{\partial v_2} - u(u+X)\frac{\partial}{\partial v_3})P_g = -\frac{1}{2}(P_{g-1}^{(2)} + \sum_{r=1}^{g-1} P_r^{(1)} P_{g-r}^{(1)})$$

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- The Main Proposition: Each P_g, $(g \geq 2)$ is a degree $3g-3$ inhomogeneous polynomial of v_1, v_2, v_3, X, where one assigns the degree $1, 2, 3, 1$ for v_1, v_2, v_3, X, respectively. Yamaguchi and Yau.

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- The number of terms n_g in P_g grows polynomially with genus g.

$$n_g \preceq (3g-3)^4$$

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- There are $3g - 2$ unknown constants at each genus g.

Boundary conditions

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So in the holomorphic limit, the generators A_p and B_p are

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- The period ω_0 and mirror map t can be solved asymptotically at each singular point of the moduli space by the Picard-Fuchs equation.

- Boundary condition at the orbifold point $\psi = 0$. The Picard-Fuchs equation has 4 power series solutions that go like $\omega_0 \sim \psi^{\frac{1}{5}}$, $\omega_1 \sim \psi^{\frac{2}{5}}$, $\omega_2 \sim \psi^{\frac{3}{5}}$, $\omega_3 \sim \psi^{\frac{4}{5}}$.

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- The topological string amplitudes are

$$F^{(g)}_{\text{orbifold}} = \lim_{\bar{\psi}\to 0} \omega_0^{2(g-1)} \left(\frac{1-\psi}{\psi}\right)^{g-1} P_g \sim \frac{P_g}{\psi^{\frac{3}{5}(g-1)}}$$

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- P_g is a power series of ψ, starting from a constant. This imposes

$$\left\lceil \frac{3}{5}(g-1) \right\rceil$$

number of conditions on the holomorphic ambiguity in P_g.

- Boundary condition at the conifold point $\psi = 1$. Picard-Fuchs equation around $z = \psi - 1$ have four solutions that go like

$$\vec{\Pi} = \begin{pmatrix} \omega_0 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \begin{pmatrix} 1 + \mathcal{O}(z) \\ z + \mathcal{O}(z^2) \\ z^2 + \mathcal{O}(z^3) \\ \omega_1 \log(z) + \mathcal{O}(z^4) \end{pmatrix}$$

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- We define a dual mirror map $t_D = \frac{\omega_1}{\omega_0}$. We find the topological strings around the conifold point has a "gap" structure in the t_D coordinate

$$\begin{aligned} F^{(g)}_{\text{conifold}} &= \lim_{\bar{z}\to 0} \omega_0^{2(g-1)} \left(\frac{1-\psi}{\psi}\right)^{g-1} P_g \\ &= \frac{(-1)^{g-1} B_{2g}}{2g(2g-2) t_D^{2g-2}} + \mathcal{O}(t_D^0), \end{aligned}$$

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  This fixes $2g-2$ coefficients in the holomorphic ambiguity.

- An arbitrary change of the basis $\omega_0 \to \omega_0 + b_1 \omega_1 + b_2 \omega_2$ does not affect this gap like structure.

- The leading coefficients of the conifold expansion were actually pointed out long time ago, Ghoshal, Vafa, hep-th/9506122. The gap condition is first observed recently in the context of $SU(2)$ Seiberg-Witten theory, Huang, Klemm, hep-th/0605195.

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- A physical explanation of the gap condition: Integrating out the massless black hole state in a graviphoton background...

- Gopakumar-Vafa-Schwinger Computation: In $\mathcal{N}=2$ supergravity, we integrate out a charged BPS hypermultiplet of $e=m=\frac{t}{\lambda}$, and Lorentz Group $SO(4)=SU(2)_L\times SU(2)_R$ representation

$$[(\frac{1}{2},0)+2(0,0)]\bigotimes(j_L,j_R)$$

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- The Gopakumar-Vafa-Schwinger Computation generates the following term in the effective action

$$\begin{aligned} S &= \int d^4x F(t,\lambda)R_+^2, \\ \text{where} \quad F(t,\lambda) &= \int_\epsilon^\infty \frac{ds}{s}\,\frac{\mathrm{Tr}\,(-1)^F\exp(-st)\exp(-2s\lambda\sigma_L)}{(2\sin(\frac{s\lambda}{2}))^2} \end{aligned}$$

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$$\begin{aligned} S &= \int d^4x F(t,\lambda)R_+^2, \\ \text{where} \quad F(t,\lambda) &= \int_\epsilon^\infty \frac{ds}{s}\,\frac{\mathrm{Tr}\,(-1)^F \exp(-st)\exp(-2s\lambda\sigma_L)}{(2\sin(\frac{s\lambda}{2}))^2} \end{aligned}$$

- In type IIB compactification near the conifold, there is only one light particle: the massless black hole.

- The topological string near the conifold should be, (up to regular terms of the period t),

$$F(\lambda,t)=\int_\epsilon^\infty \frac{ds}{s}\,\frac{\mathrm{exp}(-st)}{(2\,\mathrm{sin}(\frac{s\lambda}{2}))^2}=\sum(\frac{\lambda}{t})^{2g-2}\frac{(-1)^{g-1}B_{2g}}{2g(2g-2)}+\mathcal{O}(t^0)$$

This is precisely the gap condition.

- Boundary conditions at infinity $\psi = \infty$. The constant map contribution of manifold M, Faber, Pandharipande, math.ag/9810173,

$$\lim_{t \to \infty} F^{(g)}_{\text{A-model}} = \frac{(-1)^{g-1} B_{2g} B_{2g-2}}{4g(2g-2)(2g-2)!} \chi(M)$$

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$$F^{(g)}_{\text{instanton}} = \sum_{\beta\in H_2(M,\mathbb{Z})} r^{(g)}_\beta \exp(2\pi i t\beta)$$

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- Re-organize the world sheet instanton contributions

$$\sum_{g=0}^{\infty}\lambda^{2g-2}F^{(g)}_{\text{instanton}} = \sum_{g=0}^{\infty}\sum_{\beta}\sum_{m=1}^{\infty} n^{(g)}_{\beta}\left(\frac{e^{2\pi i t\beta m}}{m}\right)\left(2\sin\frac{m\lambda}{2}\right)^{2g-2}$$

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- The quintic example: one kahler modulus, $\beta = d$ is the degree of the holomorphic map. The GV invariants

g	d=1	d=2	d=3	d=4	d=5
0	2875	609250	317206375	242467530000	229305888887625
1	0	0	609250	3721431625	12129909700200
2	0	0	0	534750	75478987900
3	0	0	0	8625	-15663750
4	0	0	0	0	49250
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- Boundary condition: at each genus, the Gopakumar-Vafa invariants vanish $n_{d}^{(g)} = 0$ for low degree d holomorphic maps.

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- Count the number of unknown constants

$$3g-2-(\lceil\frac{3}{5}(g-1)\rceil+2g-2+1+a_g)=[\frac{2}{5}(g-1)]-a_g$$

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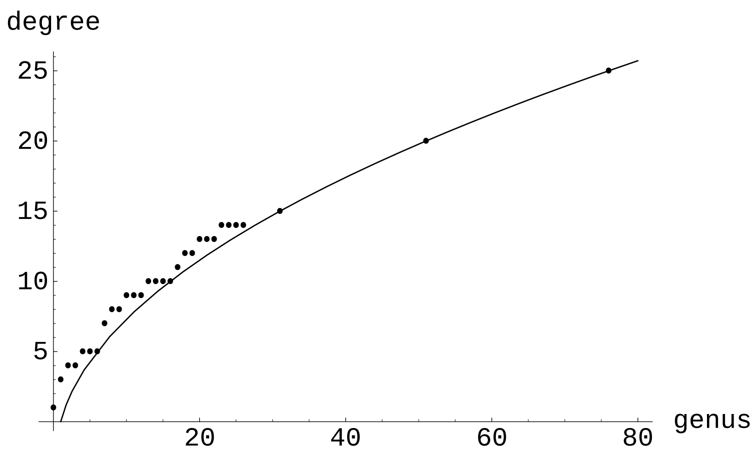
- This is true for low genus, (up to $g \sim 51$ for the quintic) . However, asymptotically

$$a_g \sim \sqrt{g}, \quad \text{when} \quad g \longrightarrow \infty$$

So far our calculation is limited only by the power of our computational facilities.



- The analysis can be straightforwardly generalized to one-parameter Calabi-Yau models, realized as hypersurfaces or complete intersections in weight projective spaces.

$$X_5(1^5)\quad X_6(1^4,2),\quad X_8(1^4,4),\quad X_{10}(1^3,2,5),\quad X_{3,3}(1^6),$$
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- On the other hand, we discover a rich variety of singularity structures around the orbifold point. The 13 models fall into 4 classes.

Four cases

(1). No massless charged state. The F^g are regular at the orbifold point $\psi = 0$, imposing boundary conditions. This includes models $X_5(1^5)$, $X_6(1^4, 2)$, $X_8(1^4, 4)$, $X_{10}(1^3, 2, 5)$,$X_{3,3}(1^6)$, $X_{2,2,2,2}(1^8)$, $X_{4,4}(1^4, 2^2)$,$X_{6,6}(1^2, 2^2, 3^2)$.

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(4). Multiple massless charged states. The F^g are singular with no obvious structures at the orbifold point. However the scaling of masses of these light states imposes some boundary conditions. This includes the model $X_{4,3}(1^5,2)$, $X_{6,4}(1^3,2^2,3)$.

Castelnuovo's theory

- We make many predictions for the Gopakumar-Vafa invariants. The counting of BPS states of a degree d can be calculated from the cohomology of the moduli space $\mathcal{M}$ of the D0-D2 brane bound states. This algebraic geometric counting is known as Castelnuovo's theory. (Katz, Klemm, Vafa, 1999)

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- Examples from the quintic:

  1. Genus $g = 6$, degree $d = 5$: $n_5^6 = 10$, $n_5^g = 0$ $(g \geq 7)$.

  2. Genus $g = 16$, degree $d = 10$: $n_{10}^{16} = -50$, $n_{10}^g = 0$ $(g \geq 17)$.

- Some basics: The moduli space of $\mathbb{P}^k$ moving in $\mathbb{P}^n$ is the Grassmannian $\mathbb{G}(k,n)$. The complex dimension and Euler number are

$$\dim(\mathbb{P}^k) = k, \quad \chi(\mathbb{P}^k) = k+1,$$
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- Similarly, consider a complete intersection of degree $(1,2,5)$ in $\mathbb{P}^4$. This is a curve of genus 16, degree 10. The moduli space of curves in the quintic is $\mathbb{P}^4 \times \mathbb{P}^9$, so we recover the BPS number

$$n_{10}^{16} = (-1)^{4+9} 5 \cdot 10 = -50$$

Applications for black hole physics

- Compactify M-theory on a compact Calabi-Yau 3-fold. The 5-D supergravity has a BPS black hole solution (BMPV black hole) with graviphoton charge Q, angular momentum J of the $SU(2)_L \subset SO(4)$. The classical entropy of the black hole is one quarter of the horizon area

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- An open problem: How to count the black hole microstates? Much more difficult than the Strominger-Vafa black hole.

- Katz, Klemm, Vafa (KKV), 1999: The black hole microstates are counted by topological strings. For a black hole with 2-brane charge d and $SU(2)_L$ angular momentum $J = m$, the number of states are

$$N_d^m = \sum_r n_d^r \binom{2r+2}{r+1+m}$$

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- Difficulty: For non-compact Calabi-Yaus, the KKV formula can not be reliably applied to count 5D black hole microstates, since this is not really a compactification to 5D supergravity. There were not much computations of the Gopakumar-Vafa invariants for compact Calabi-Yau available (before our paper).

- We use our new results and the KKV formula the count micro-states. Consider e.g. angular momentum $m = 0$,

$$S = \log(N_d^0) = \frac{4\pi}{3\sqrt{2\kappa}} d^{\frac{3}{2}} + \mathcal{O}(d^{\frac{1}{2}})$$

Topological string data provide the values

$$f(d) = \frac{\log(N_d^0)}{d^{\frac{3}{2}}} = \frac{4\pi}{3\sqrt{2\kappa}} + \frac{b_1}{d} + \frac{b_2}{d^2} + \cdots$$

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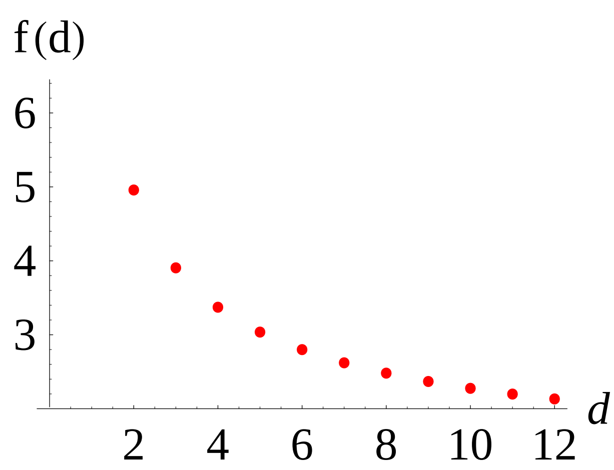

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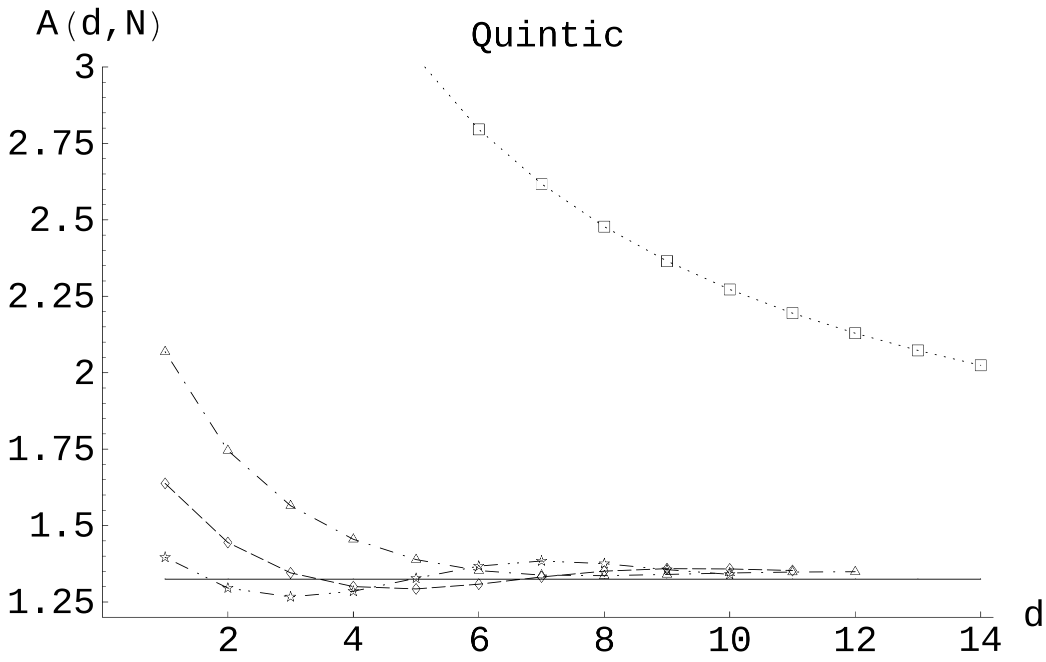

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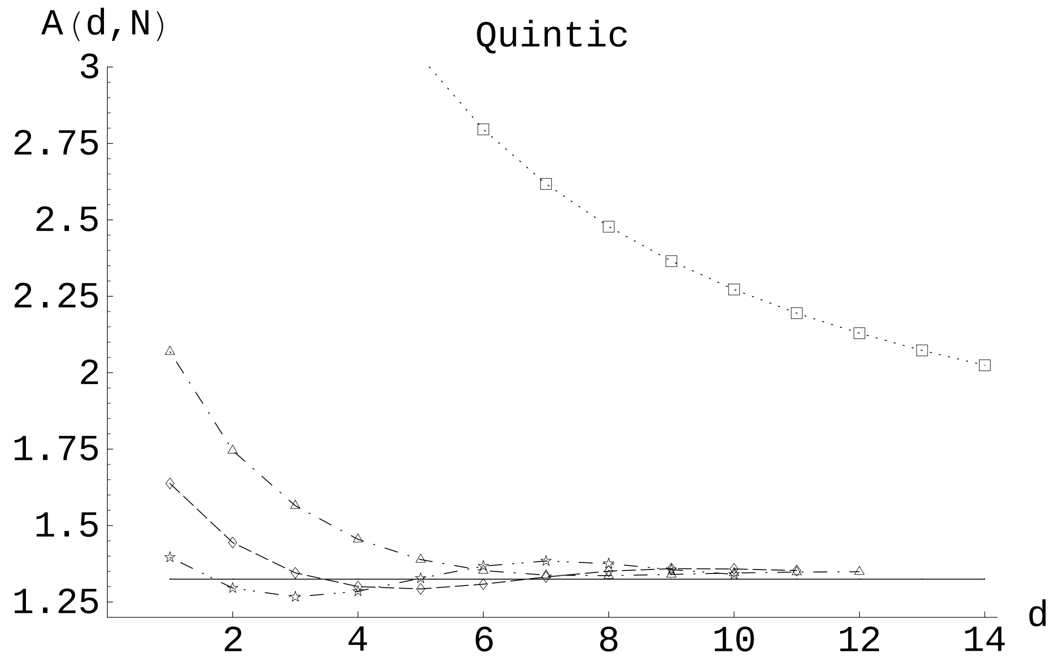

• For all 13 models, the KKV formula for counting micro-states confirms the macroscopic black hole prediction of leading coefficient with impressively small error of 1$\sim$ 3 % .

Scaling behavior of Donaldson-Thomas invariants

- The scaling behavior of Donaldson-Thomas invariants $D_{(\lambda^2 d, \lambda^3 n)}$ has important implication for OSV conjecture. Study the scaling behavior

$$\log(D_{\lambda^2 d, \lambda^3 n}) \sim \lambda^k, \qquad \lambda \to \infty.$$

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- Our analysis indicates that the value of k is indeed universal and close to $k = 2$. This strongly suggests that the "mysterious cancellations" that eventually make possible to extend the OSV conjecture to small coupling, actually take place.

A matrix model description?

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- We use this method to study gravitational couplings in the $N = 2$ $SU(2)$ Seiberg-Witten theory, and compare the results with Nekrasov's formulae. The matrix model method is more complicated than the BCOV method, but without ambiguity in the recursion relations. It also provides additional insight into open topological string amplitudes.

Conclusion and Future Directions

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- It would be interesting to develop algebraic geometric theory to systematically verify or prove our predictions.

- Continue to explore the fascinating implications for the OSV conjecture and black hole physics, higher curvature corrections to black hole entropy.

Thank You