Running Non-Minimal Inflation with Stabilized Inflaton Potential

Nobuchika Okada

University of Alabama

In collaboration with Digesh Raut (U. of Alabama)
arXiv: 1509.04439

Seminar @ IPMU, Nov. 05, 2015

The Standard Big-Bang Cosmology

The success of the Standard Big-Bang Cosmology

Expansion law: $$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3M_{Pl}^2}\rho$$

Continuity equation: $$\dot{\rho} + 3H(\rho + p) = 0$$

- Hubble expansion

  Hubble’s law: expansion of the Universe

- Cosmic Microwave Background (CMB)

  2.725K radiation, Planck distribution

- Big-Bang nucleosynthesis

  Success in synthesizing light nuclei in the early Universe

Problems of Big-Bang Cosmology

Big-Bang Cosmology: $\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p) = -\frac{1}{6}(1+3w)\rho$

w=1/3 : radiation
w=0 : matter

$\ddot{a} < 0$ Decelerating expansion

``Naturalness/Initial condition problem''

Flatness problem

Fine-tuning of the spatial curvature parameter

Horizon problem

Observed CMB is isotropic
nevertheless two regions have never contacted with each other

Origin of density fluctuation

need the seed of density fluctuation for the large scale structure formation of the Universe

Basic Idea of Inflationary Universe

Suppose the existence of a stage in the early universe with $\ddot{a} > 0$

``Inflation''

Accelerating Expansion

Simple example: de Sitter space

Positive cosmological constant (vacuum energy)

ρ_Λ $\quad w = -1 \rightarrow p = -\rho_\Lambda$

Expansion law: $H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\rho$

Continuity equation: $\dot{\rho} + 3H(\rho + p) = 0$

⇒

$a \propto e^{H_I t}$

$\rho_\Lambda = \text{const.}$

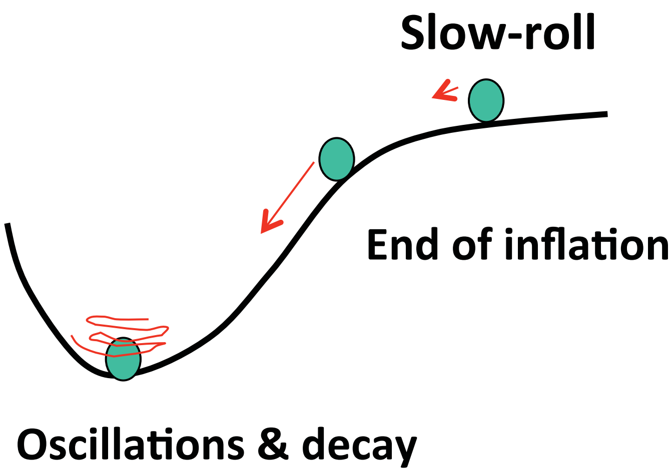
Simple inflation model

The picture we seek....

Inflation before Big-Bang → Big-bang cosmology

Slow-roll inflation

A scalar field (inflaton) slowly "rolling down" to its potential minimum



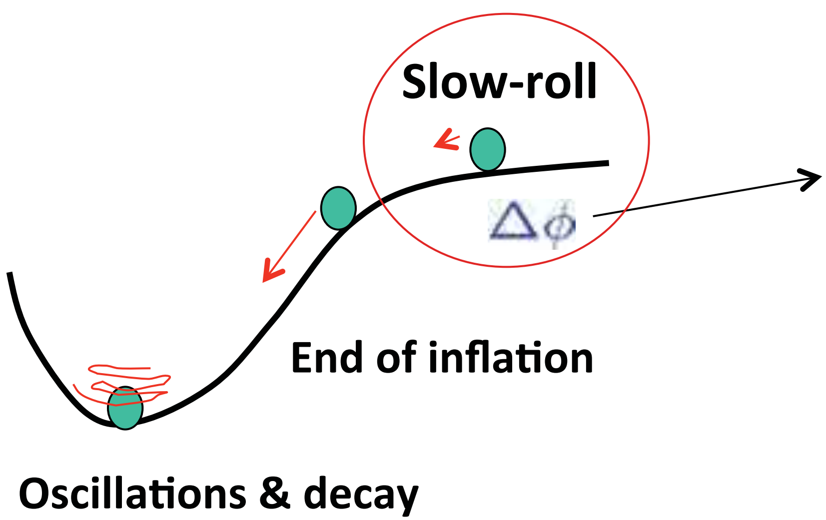
1. Inflation at slow-roll era (E =K+ V~V)
2. End of Inflation (K ~ V)
3. Coherent oscillations
4. Decays to Standard Model particles
5. Reheating → Big-Bang Cosmology

Exponential expansion solves

- flatness problem ← spatial curvature flattened
- horizon problem ← small causal region expanded

Quantum fluctuations of inflaton + inflation

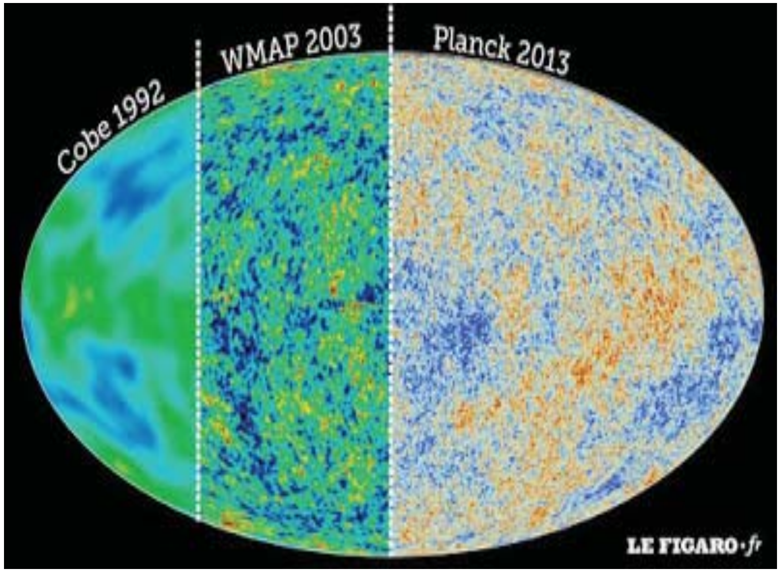
- primordial density fluctuation



quantum fluctuation is magnified to cosmic scale by inflation

$$a \propto e^{H_I t}$$

Planck 2015 results VS. Inflationary predictions

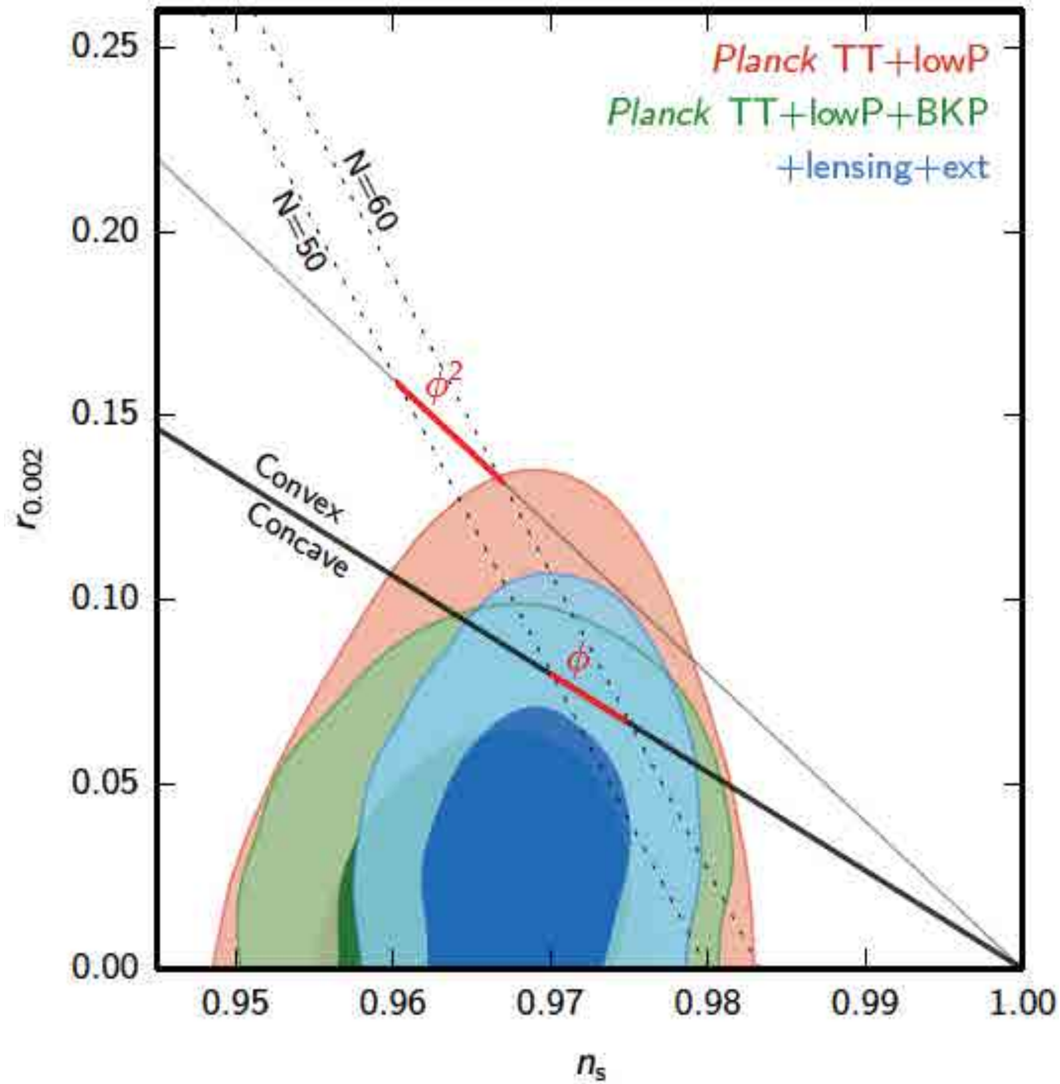


$$T = 2.725\ \mathrm{K}$$

$$\frac{\delta T}{T} \sim 10^{-5}$$

The observational cosmology is now a precision science!

Planck 2015, arXiv: 1502.01589

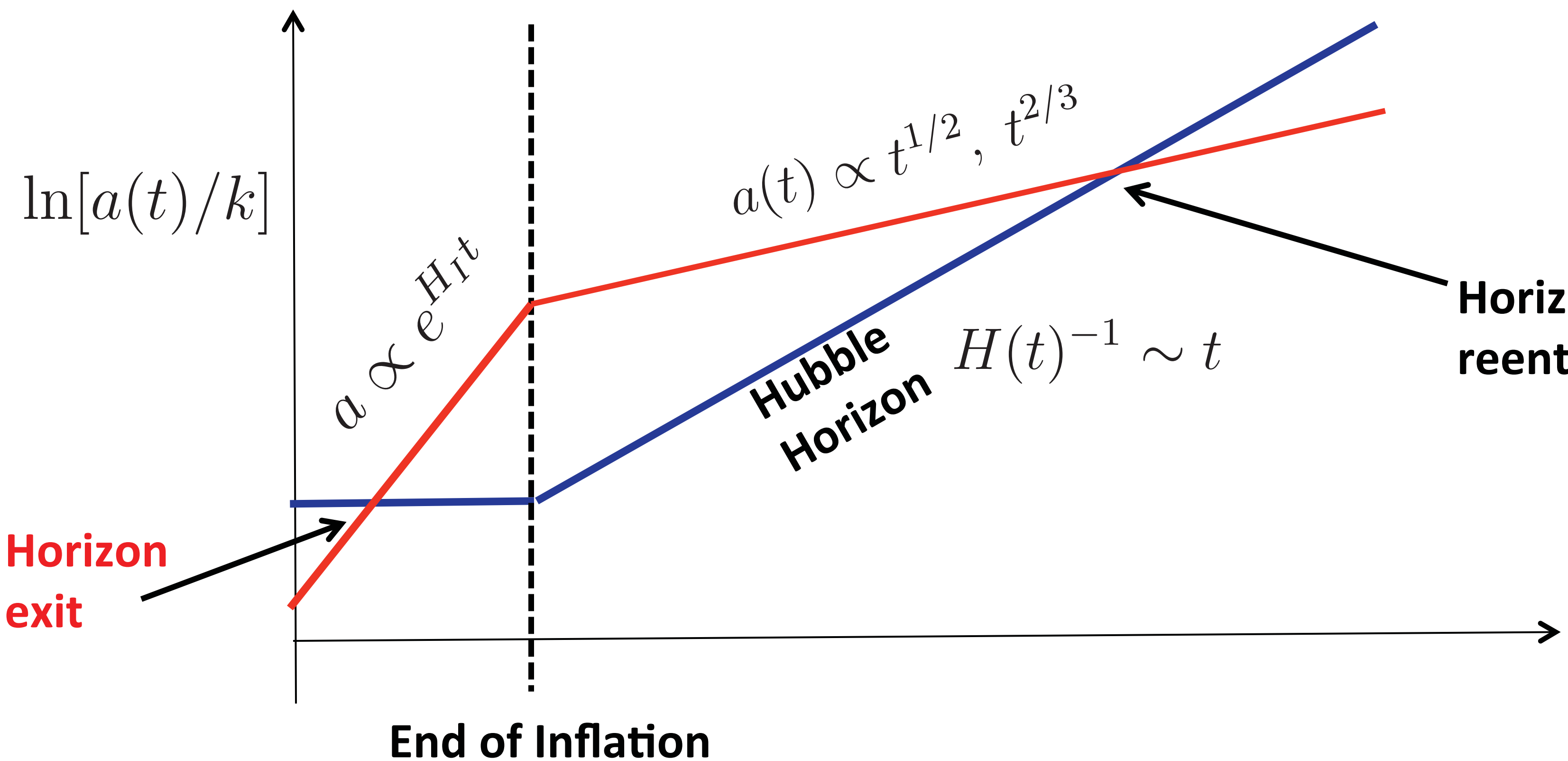


Inflationary predictions

Power spectrum of curvature perturbation: $\mathcal{P}_{\mathcal{S}} = \frac{9}{4\pi^2}\frac{H^6}{(V')^2}$

Power spectrum of tensor perturbation: $\mathcal{P}_{\mathcal{T}} = 8\left(\frac{H}{2\pi}\right)^2$

Evolution of density fluctuation



Constraints

- Planck 2015 results: $\mathcal{P}_{\mathcal{S}} = 2.2 \times 10^{-9}$
  with a pivot scale $k_0 = 0.002\ \mathrm{Mpc}^{-1}$

- e-folding number $N = \int_{\phi_e}^{\phi_0} \frac{V \mathrm{d}\phi}{V'}$ ***= 50-60***

Inflationary predictions (all evaluated at the pivot scale)

Spectral index: $$n_s - 1 = \frac{d \ln \mathcal{P}_{\mathcal{S}}}{d \ln k} = -6\epsilon + 2\eta$$

Tensor-to-scalar ratio: $$r = \mathcal{P}_{\mathcal{T}} / \mathcal{P}_{\mathcal{S}} \rightarrow r = 16\epsilon$$

Running of spectral index: $$\alpha = 16\epsilon\eta - 24\epsilon^2 - 2\zeta^2$$

in terms of "slow-roll parameters" $$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2, \quad \eta = \frac{V''}{V}, \quad \zeta^2 = \frac{V'V'''}{V^2}$$

Simple Inflationary Models

(1) Quadratic potential

$$V = \frac{1}{2}m^2\phi^2$$

(2) Quartic potential

$$V = \frac{\lambda}{4}\phi^4$$

(3) Quartic potential with non-minimal gravitational coupling

Jordan frame:

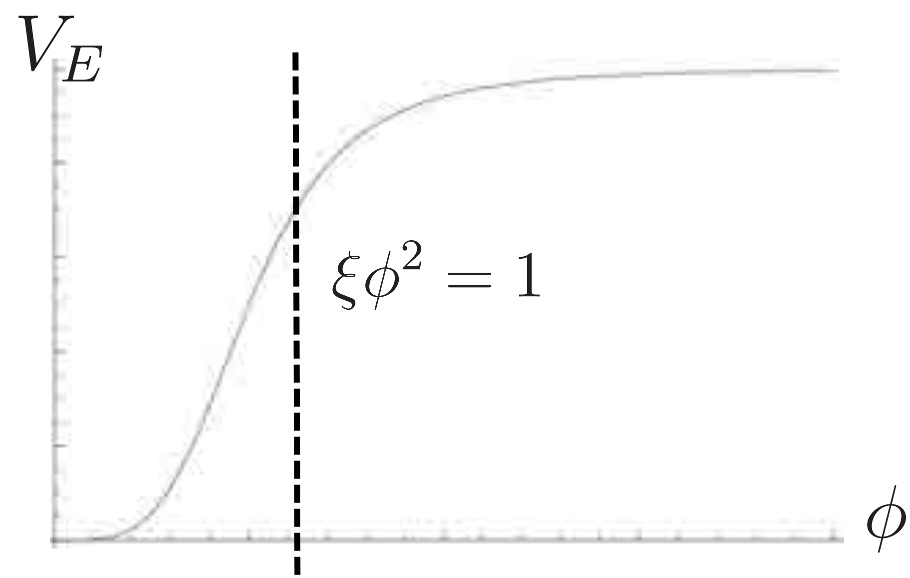
$$S_J^{\text{tree}} = \int d^4x \sqrt{-g} \left[-\left(\frac{1+\xi\phi^2}{2} \right) \mathcal{R} + \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}\phi^4 \right]$$

Einstein frame: $g_{E\mu\nu} = (1+\xi\phi^2) g_{\mu\nu}$

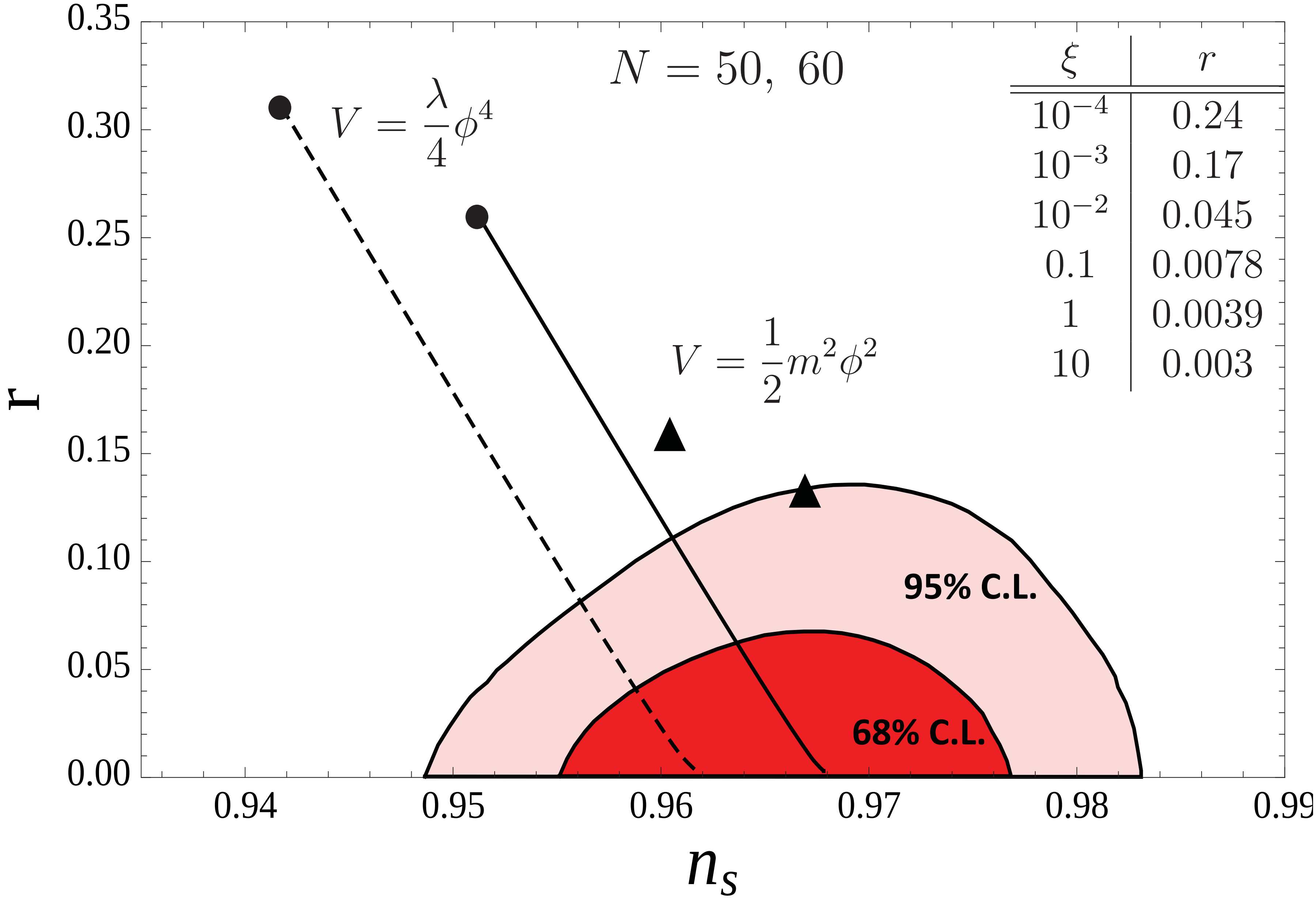
$$S_E = \int d^4x \sqrt{-g_E} \left[-\frac{1}{2}\mathcal{R}_E + \frac{1}{2}(\partial_E \sigma_E)^2 - V_E(\sigma_E(\phi)) \right]$$

$$\left(\frac{d\sigma}{d\phi} \right)^{-2} = \frac{\left(1+\xi\phi^2\right)^2}{1+(6\xi+1)\xi\phi^2}$$

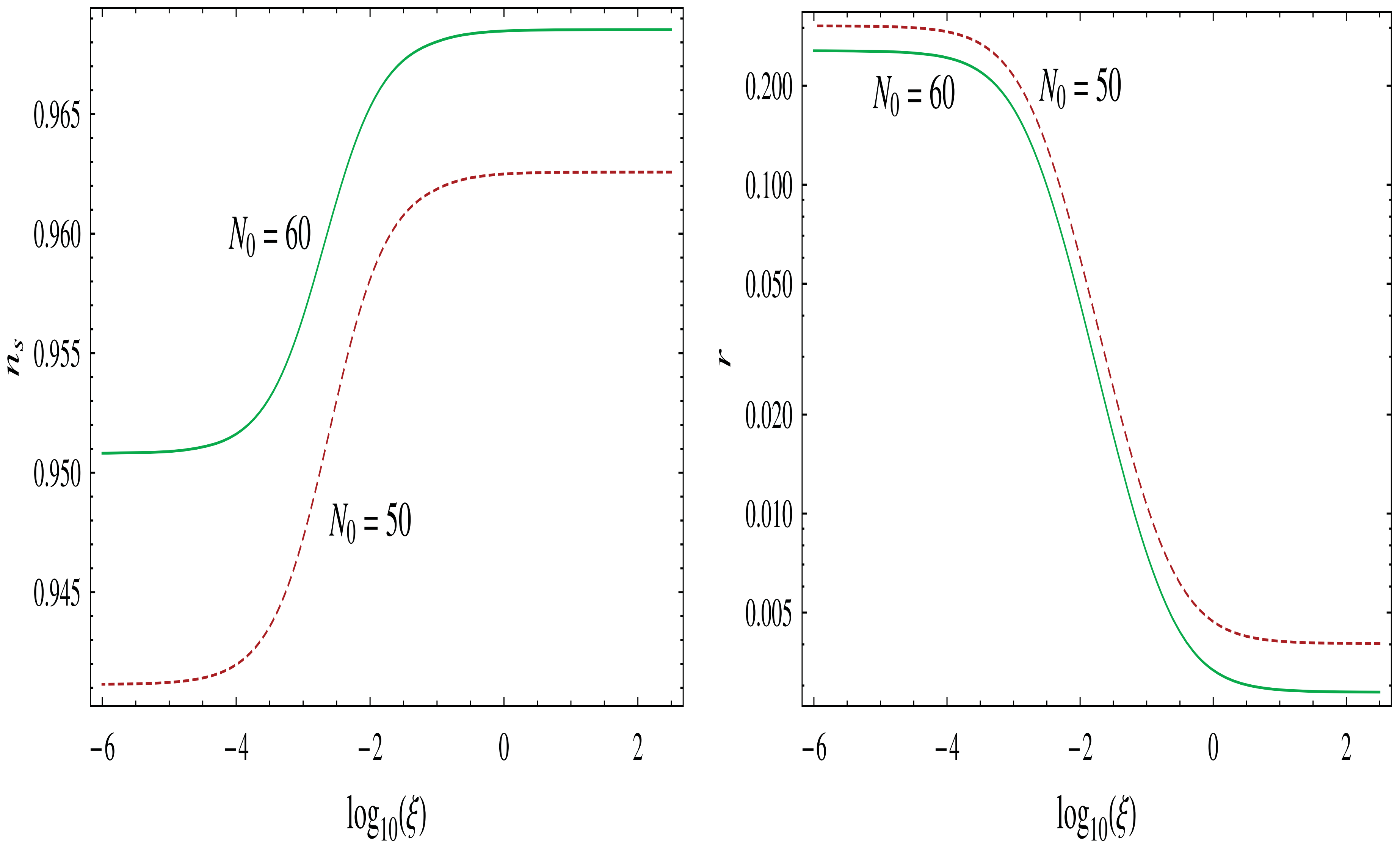
$$V_E = \frac{\frac{\lambda}{4}\phi^4}{(1+\xi\phi^2)^2}$$

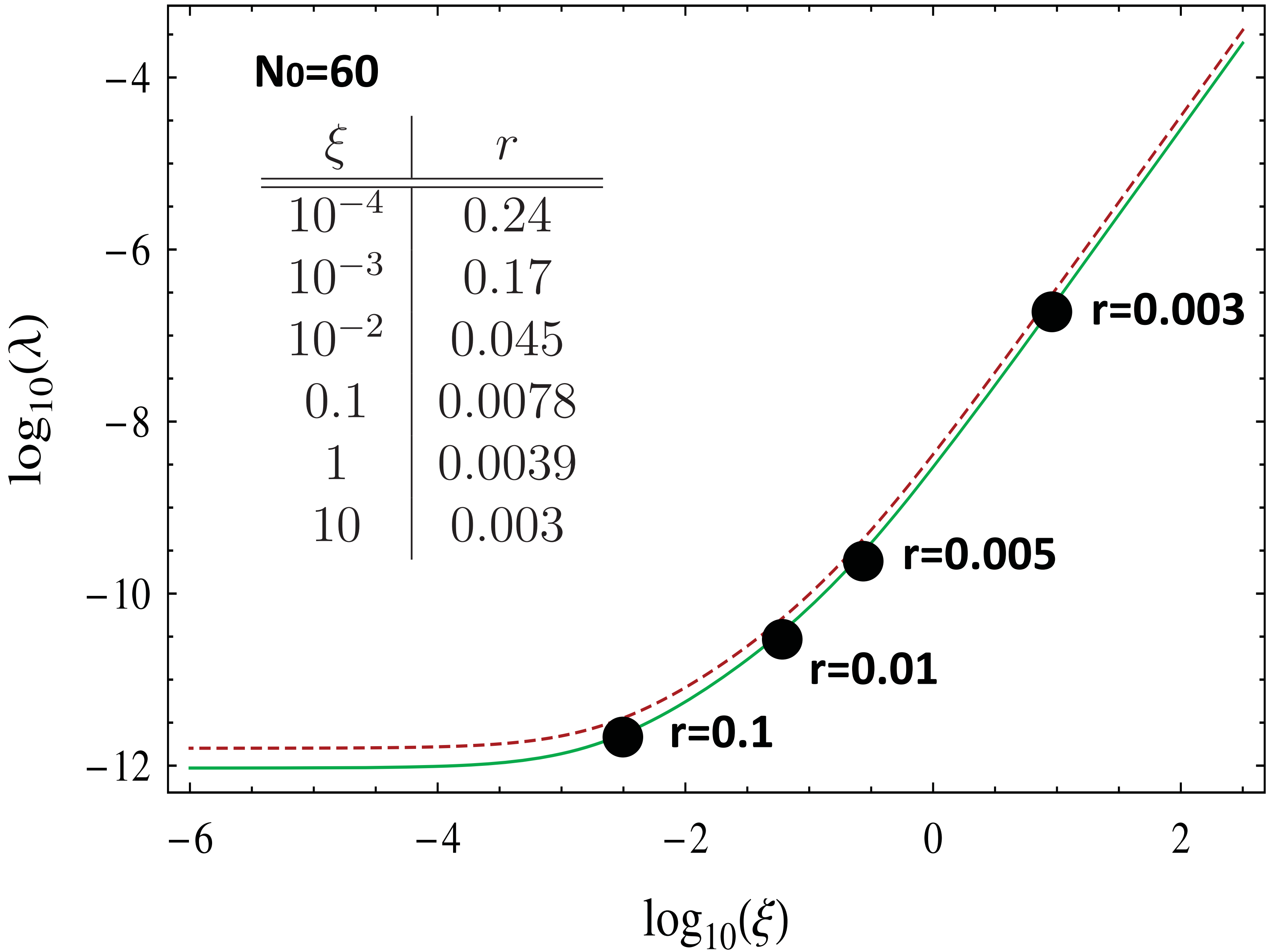


Non-minimal model nicely fit the data with a suitable xi



NO, Rehman & Shafi, Phys. Rev. D 82, 043502 (2010)




N0=60
ξ | r
10^-4 | 0.24
10^-3 | 0.17
10^-2 | 0.045
0.1 | 0.0078
1 | 0.0039
10 | 0.003
r=0.003
r=0.005
r=0.01
r=0.1
−4
−6
−8
−10
−12
log10(λ)
−6
−4
−2
0
2
log10(ξ)

Non-minimal model nicely fits the Planck 2015 data

What we need?

Quartic coupling & non-minimal gravitational coupling

It seems easy to realize the inflationary universe compatible to the Planck 2015 results

Questions

What is inflaton?

Is the scalar only for inflation?

A more compelling inflationary scenario would be where the inflaton plays another important role in particle physics

Most interesting example would be

inflaton=Higgs field in spontaneously broken gauge theories

Inflartionary Universe with Inflaton=Higgs field

Non-minimal model nicely fits the Planck 2015 data, but tree-level analysis is sufficient?

- During inflation, the quartic coupling is very small
- Quantum corrections may change inflaton potential drastically

Inflaton=Higgs field in a spontaneously broken gauge field theory

Variety of interactions like in the Standard Model

- ✓ Quartic inflaton coupling λ
- ✓ Gauge coupling g
- ✓ Yukawa coupling Y

$$V(\phi) = \lambda\left(\phi^\dagger\phi - v^2\right)^2 \simeq \lambda\left(\phi^\dagger\phi\right)^2 \qquad \phi \gg v$$

RGE for the inflaton quartic coupling

$$16\pi^2\mu\frac{d\lambda}{d\mu} = C_1\,\lambda^2 + \lambda(-C_2\,g^2 + C_3\,Y^2) + C_4\,g^4 - C_5\,Y^4$$

(C_i > 0 are constants)

Gauge & Yukawa couplings are independent of the quartic coupling

If $\lambda \ll g^2,\ Y^2$

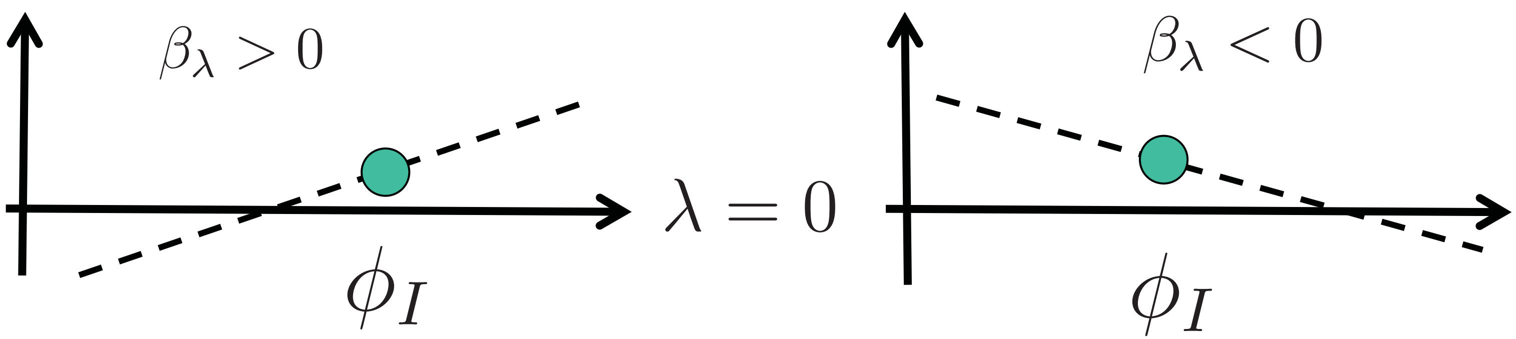
$$16\pi^2\mu\frac{d\lambda}{d\mu} \simeq C_4\,g^4 - C_5\,Y^4$$

The gauge & Yukawa couplings can drastically change the shape of infalton potential

What happens?

Running quartic coupling is very close to 0

$$0 < \lambda(\phi_I) \ll 1$$



Effective inflaton potenial is likely to be drastically changed

→ not suitable for inflation any more

$$V_{\text{eff}} \simeq \frac{1}{4}\lambda(\mu = \phi)\phi^4$$

To resolve this problem, we may impose $\beta_\lambda(\mu = \phi_I) = 0$

(i) $16\pi^2 \mu \frac{d\lambda}{d\mu} \simeq C_4\, g^4 - C_5\, Y^4 = 0$

→ relation between gauge coupling and Yukawa coupling

(ii) Inflationary predictions are altered from those in tree-level

$$\frac{dV}{d\phi}\Big|_{\phi=\phi_I} = \frac{1}{4}\frac{d\lambda}{d\phi}\Big|_{\phi=\phi_I}\phi_I^4 + \lambda(\phi_I)\phi_I^3 = \left(\frac{1}{4}\beta_\lambda(\phi_I) + \lambda(\phi_I)\right)\phi_I^3 = \lambda(\phi_I)\phi_I^3$$

→ same as tree-level one

$\frac{d^2V}{d\phi^2}\Big|_{\phi=\phi_I}$, $\frac{d^3V}{d\phi^3}\Big|_{\phi=\phi_I}$ are different from tree-level ones

Therefore, $n_s \neq n_s^{\rm tree}$, $r = r^{\rm tree}$, $\alpha \neq \alpha^{\rm tree}$

Sample Model: Minimal B-L extension of the SM @ TeV

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$
q_L^i	$\mathbf{3}$	$\mathbf{2}$	$+1/6$	$+1/3$
u_R^i	$\mathbf{3}$	$\mathbf{1}$	$+2/3$	$+1/3$
d_R^i	$\mathbf{3}$	$\mathbf{1}$	$-1/3$	$+1/3$
ℓ_L^i	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	-1
N^i	$\mathbf{1}$	$\mathbf{1}$	0	-1
e_R^i	$\mathbf{1}$	$\mathbf{1}$	-1	-1
H	$\mathbf{1}$	$\mathbf{2}$	$-1/2$	0
Φ	$\mathbf{1}$	$\mathbf{1}$	0	$+2$

$$\mathcal{L} \supset -Y_D^{ij}\overline{N^i}H^\dagger \ell_L^j - \frac{1}{2}Y_N^i \Phi \overline{N^{ic}} N^i + \text{h.c.}$$

- 3 right-handed neutrinos for anomaly cancellation
- B-L symmetry breaking → Z' boson mass, N_R mass
- See-saw mechanism is automatically implemented
- B-L breaking at TeV → LHC signature for Z' & N_R

Inflaton = B-L Higgs field

Non-minimal B-L inflation

NO, Rehman & Shafi, PLB 701 (2011) 520

$$S_J^{tree} = \int d^4x\sqrt{-g}\left[-\left(\frac{m_P^2}{2}+\xi\Phi^\dagger\Phi\right)\mathcal{R} + (D_\mu\Phi)^\dagger g^{\mu\nu}(D_\nu\Phi) - \lambda\left(\Phi^\dagger\Phi - \frac{v_{B-L}^2}{2}\right)^2\right]$$

Tree-Level Analysis

(i) Fix xi
(ii) Fix Ne=60
(iii) Planck constraint: $\mathcal{P}_{\mathcal{S}} = 2.2\times 10^{-9}$
(iv) Inflationary predictions

Running Non-minimal B-L Inflation with stabilized inflaton potential

NO & Raut,
arXiv: 1509.04439

$$V_E^{\rm tree} \to \; V_E(\phi) = \frac{1}{4}\lambda(\Phi)\;\Phi^4, \quad \text{where } \Phi \equiv \phi/\sqrt{1+\xi\phi^2},$$

George, Mooij & Postma,
JCAP 1402 (2014) 024

$$\left\{\begin{aligned} (4\pi)^2\frac{dg}{d\ln\Phi} &= 12g^3 \\ (4\pi)^2\frac{dY}{d\ln\Phi} &= \frac{5}{2}Y^3 - 6Yg^2 \\ (4\pi)^2\frac{d\lambda}{d\ln\Phi} &= 20\lambda^2 + \lambda\left(-48g^2 + 2Y^2\right) + 96g^4 - 3Y^4 \end{aligned}\right.$$

For simplicity, we consider $Y = Y_N^i$

Inflation analysis

Free parameters: $\lambda, \ g, \ Y$

- Fix ξ and N=60
- From analysis with the tree-level potential, we fix

  $\Phi_I \ \ \& \ \ \lambda(\Phi_I)$

- Stability condition at Φ_I

$$(4\pi)^2 \frac{d\lambda}{d\ln\Phi} = 20\lambda^2 + \lambda\left(-48g^2 + 2Y^2\right) + 96g^4 - 3Y^4 = 0$$

→ *g* is a unique free parameter

Inflationary predictions as a function of g

$$n_s(g) \neq n_s^{\rm tree}, \ \ r(g) = r^{\rm tree}, \ \ \alpha(g) \neq \alpha^{\rm tree}$$

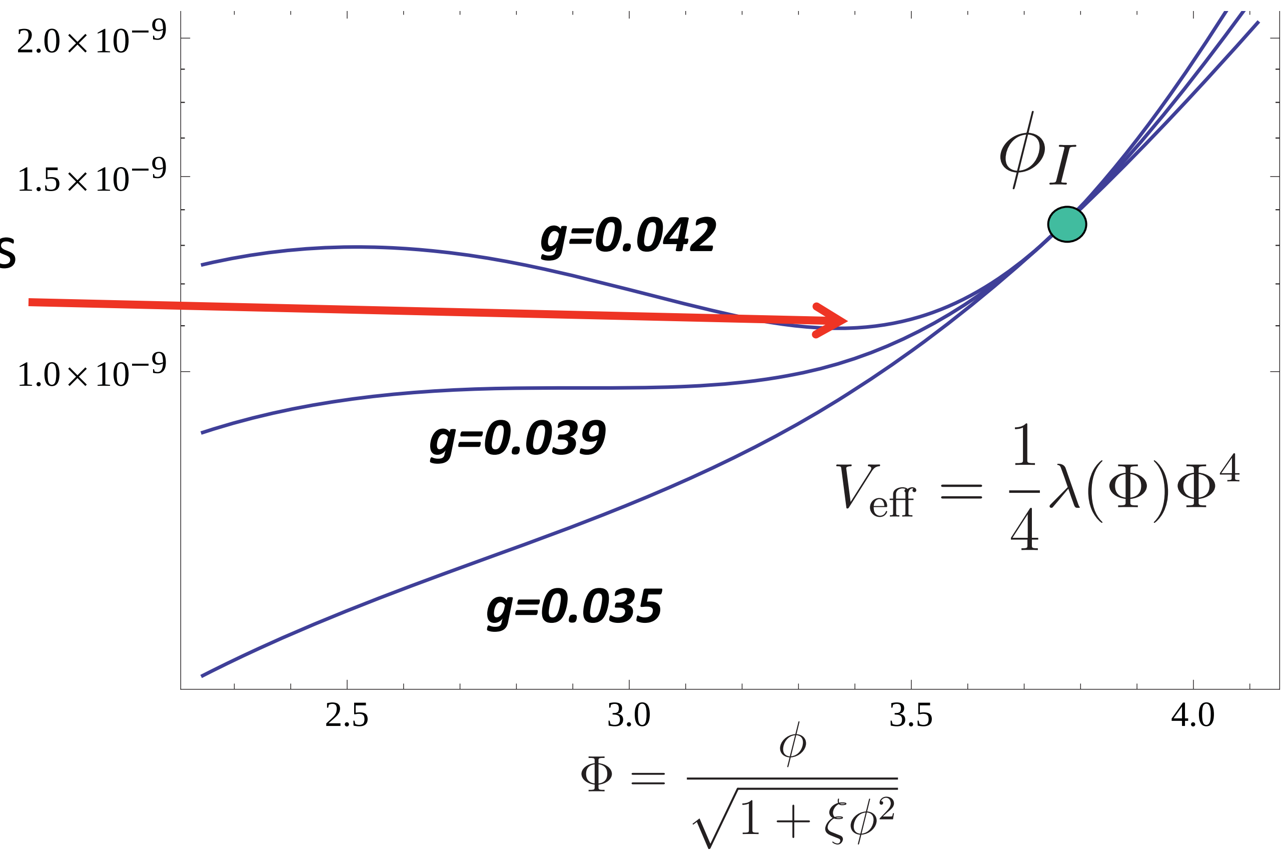
Results for a fixed xi

Example: $\xi = 0.0687 \rightarrow \Phi_I = 18.9, \quad \lambda(\Phi_I) = 6.71 \times 10^{-12}$

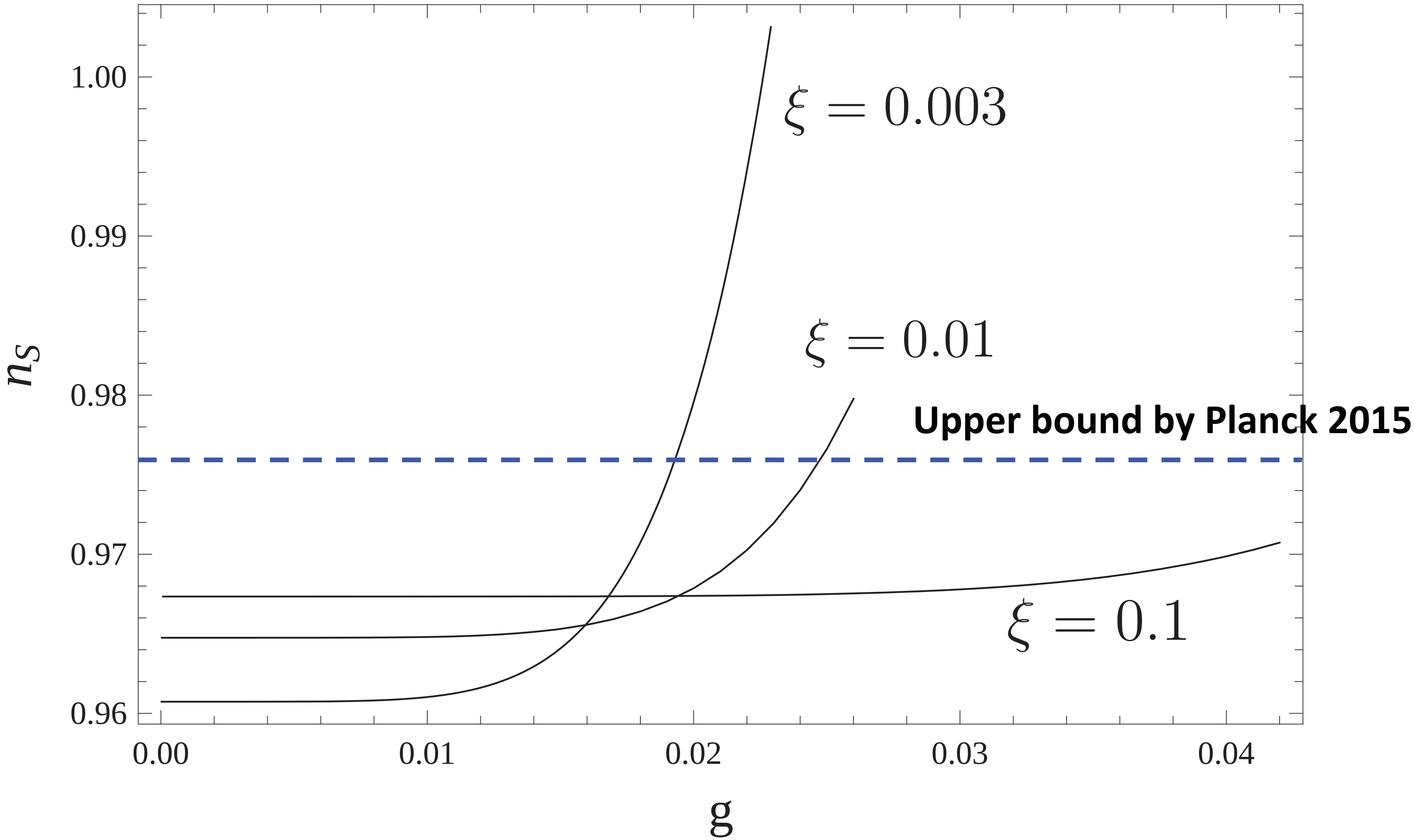
$$r = 0.1$$

There is a theoretical upper bound on *g*

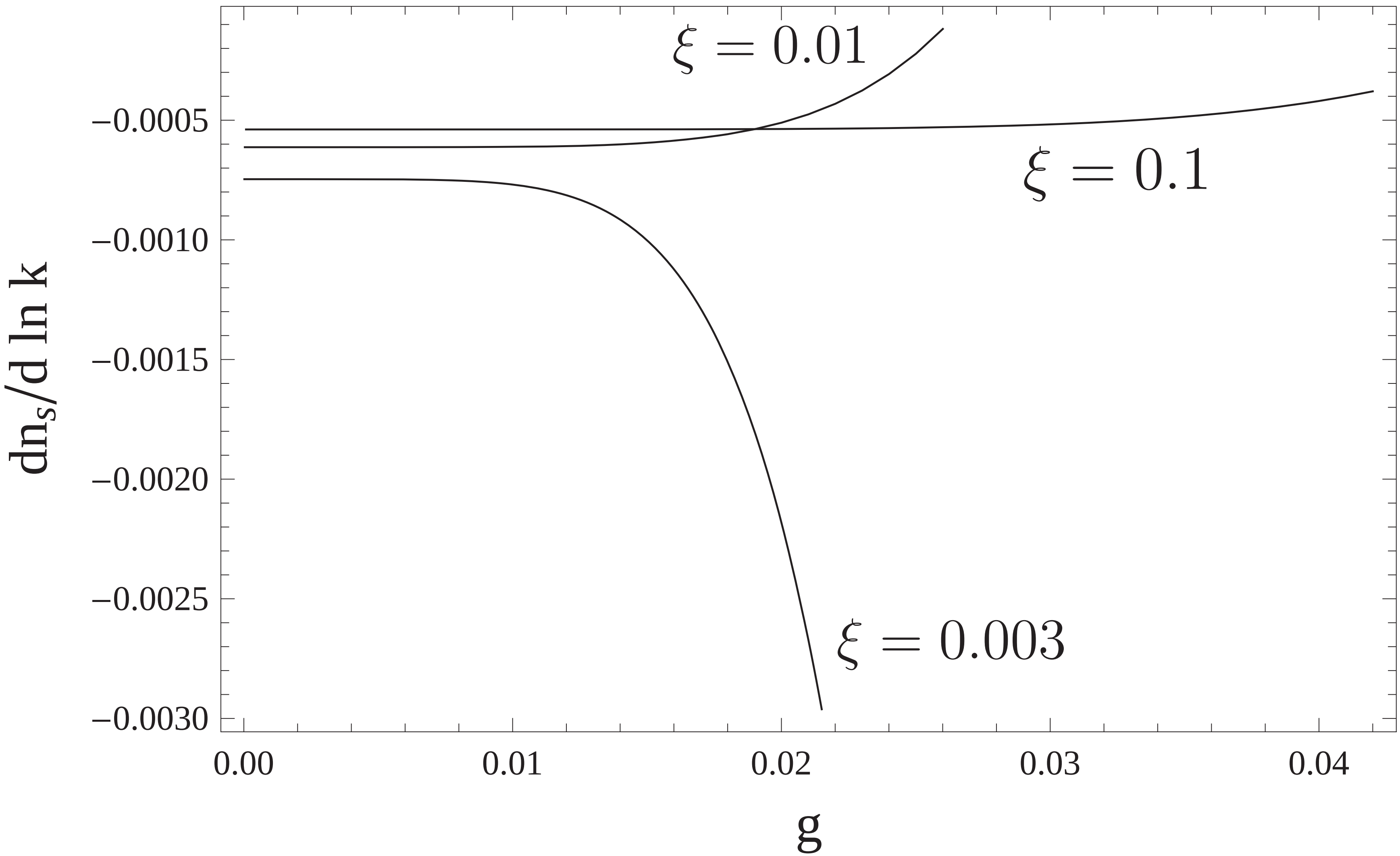
Too large *g* creates a minimum where inflaton is trapped (eternal inflation)



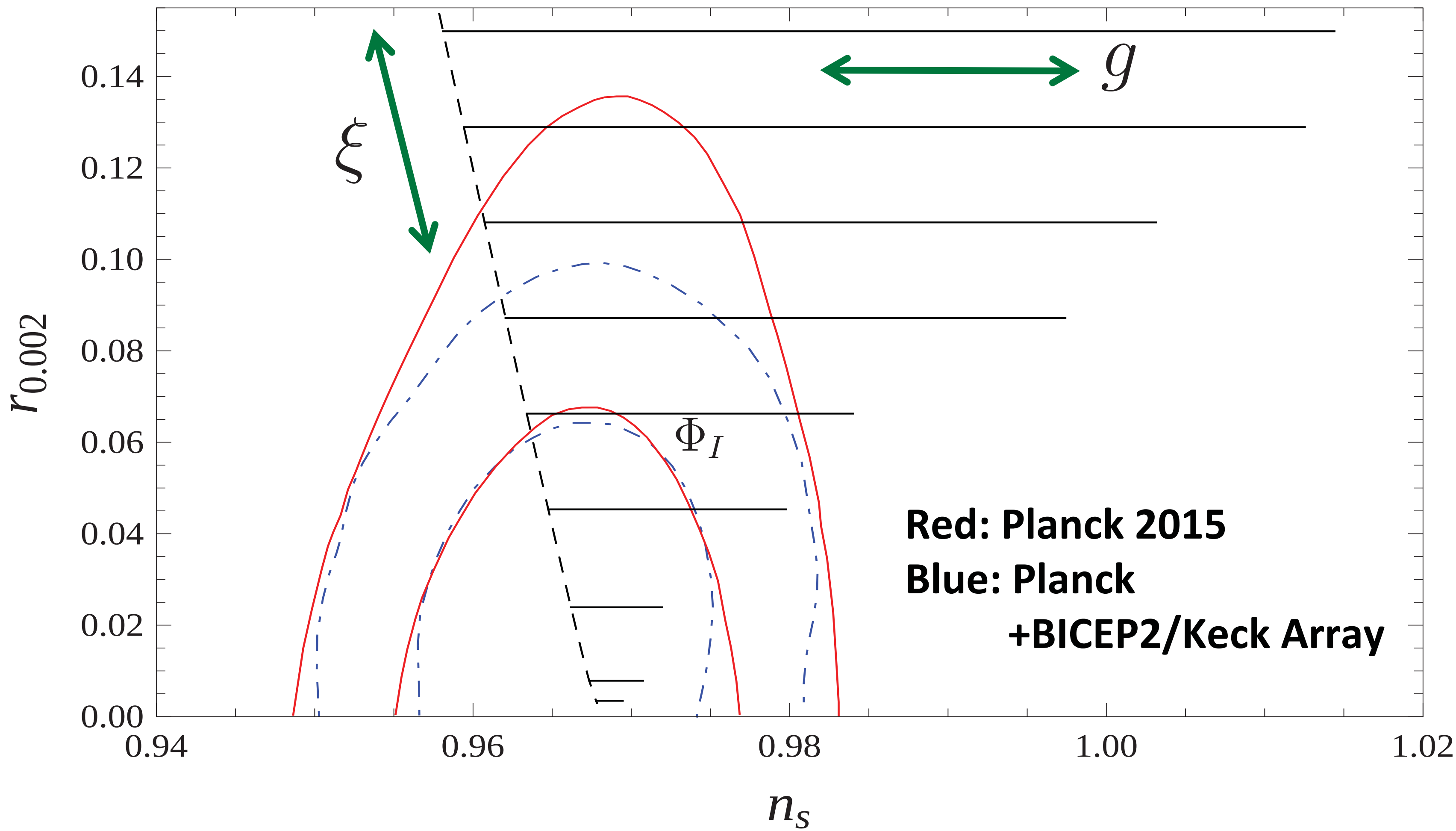
Inflationary Predictions



Inflationary Predictions



Results for variations of xi and g (N=60)



For $\xi \lesssim 0.01$, experimental constraints on the spectral index is more severe than the theoretical one

Low energy observables

New particle mass spectrum: $m_{Z'} = 2 g v_{BL}, \ m_N = \frac{Y}{\sqrt{2}} v_{BL}, \ m_\phi = \sqrt{2\lambda} v_{BL}$

$$m_{Z'} : m_N : m_\phi = 2g : \frac{Y}{\sqrt{2}} : \sqrt{2\lambda}$$

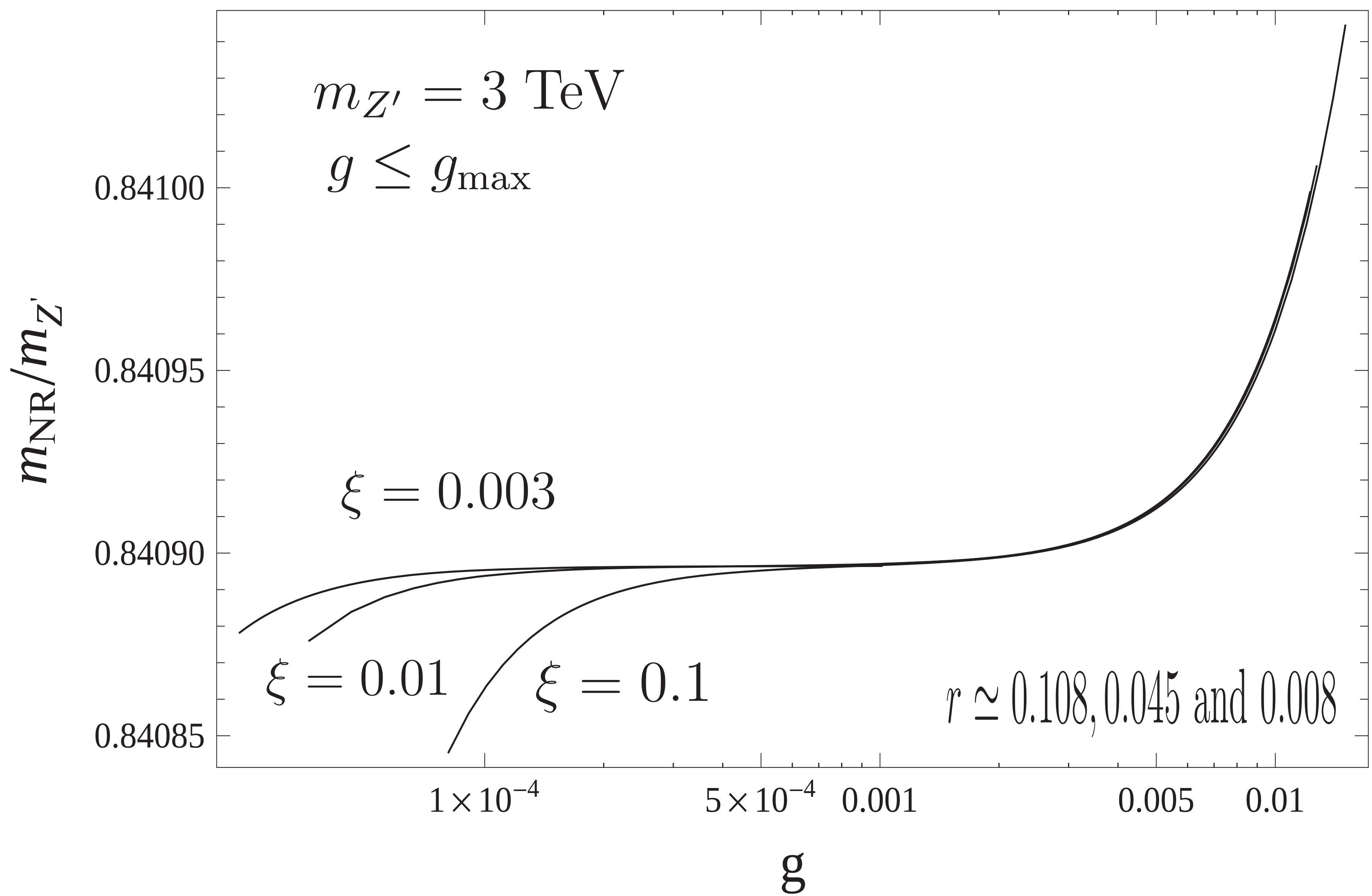
- At $\mu = \phi_I, \ \lambda$ is fixed
- Stability condition → *Y=Y(g)*
- Inflationary predictions as a function of *g*

Therefore, the mass ratio has a correlation with the inflationary predictions

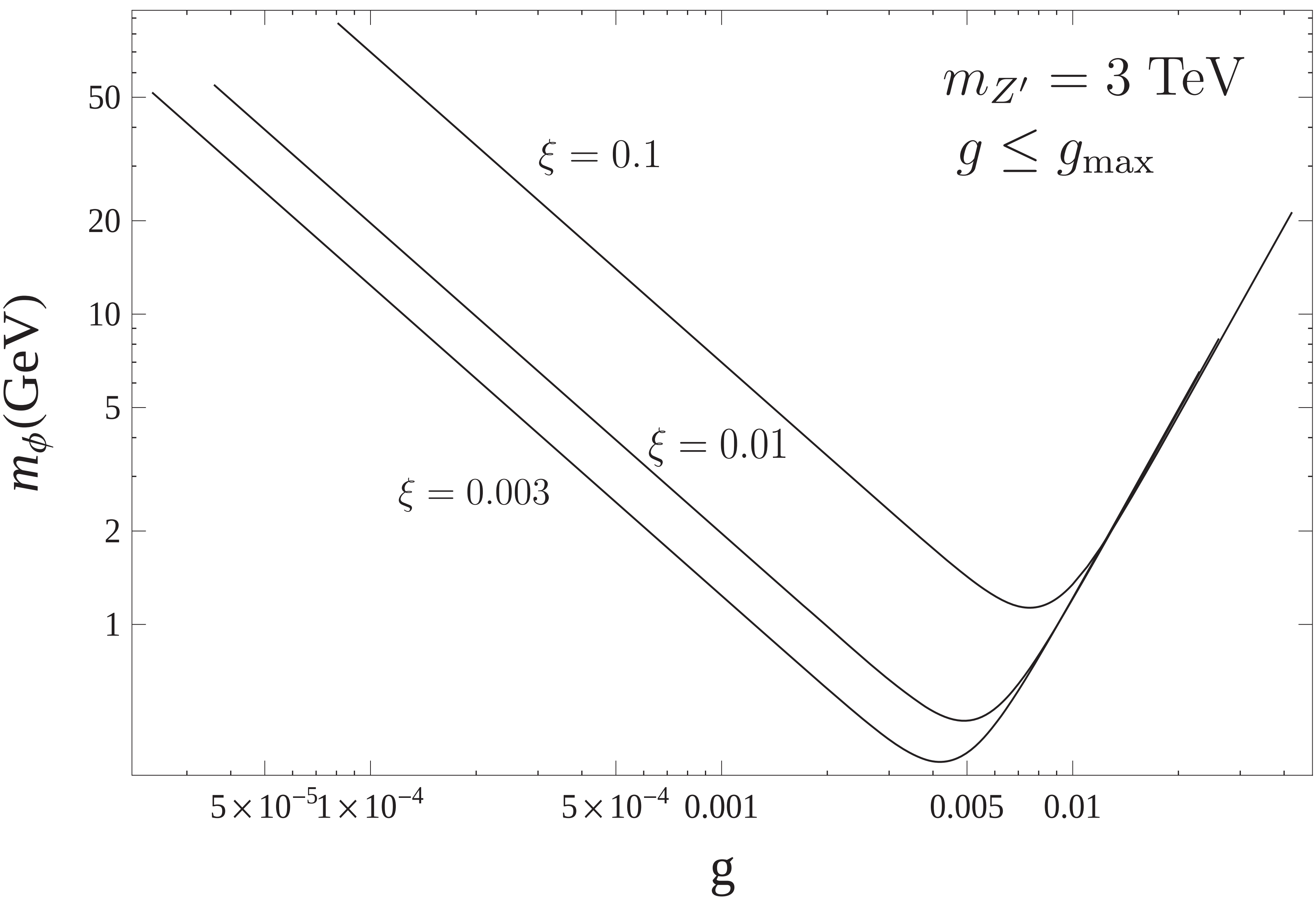
Example) $g_{\rm Max} = 0.0392, \ \ m_{Z'} = 3 \ {\rm TeV}$

After RGE run: $m_{Z'} : m_N : m_\phi = 1 : 0.84 : 0.0062$

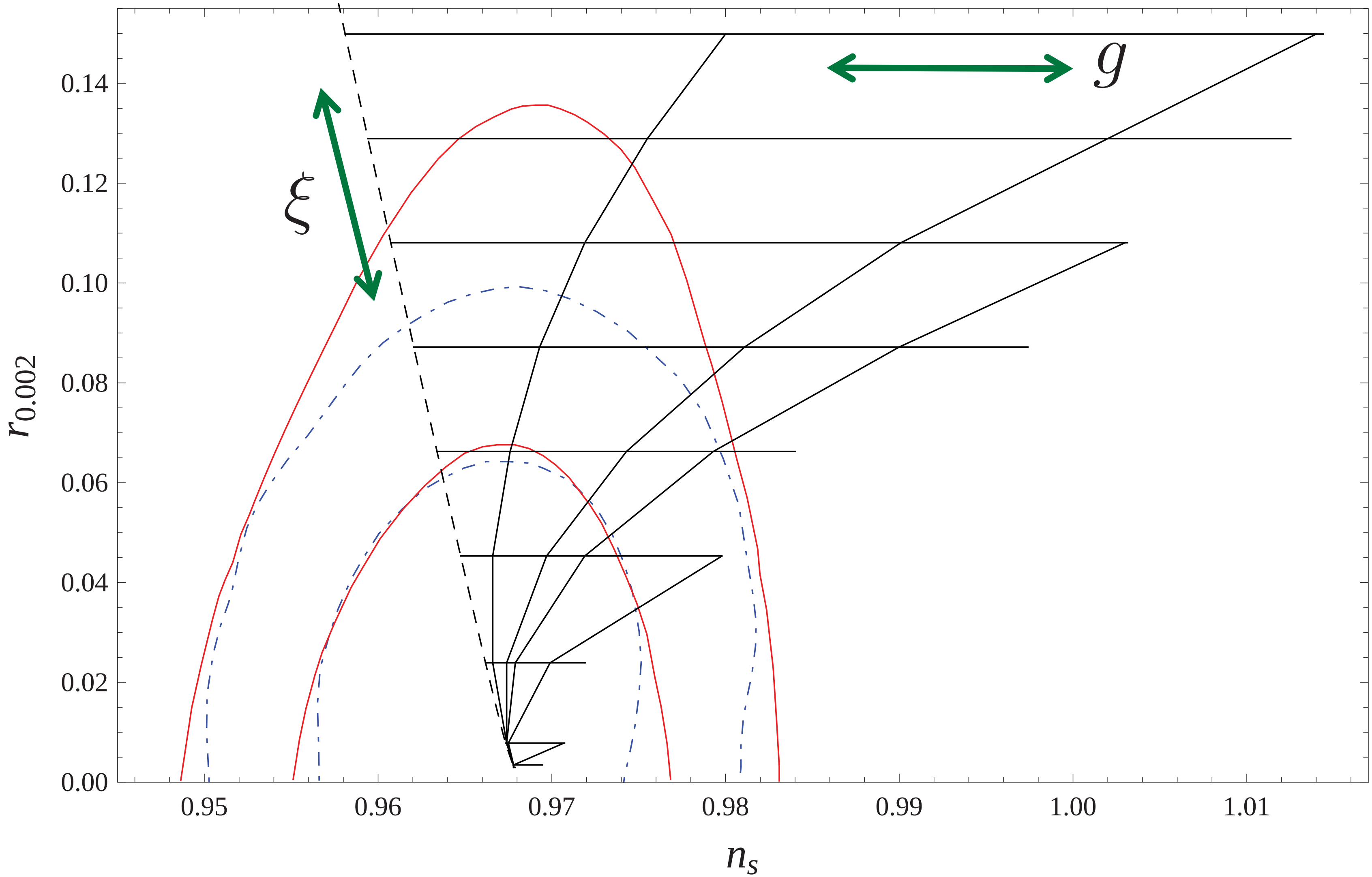
Low energy mass spectrum (right-handed neutrino)



Low energy mass spectrum (inflaton=B-L Higgs boson)



Results for variations of xi and fixed g values (N=60)



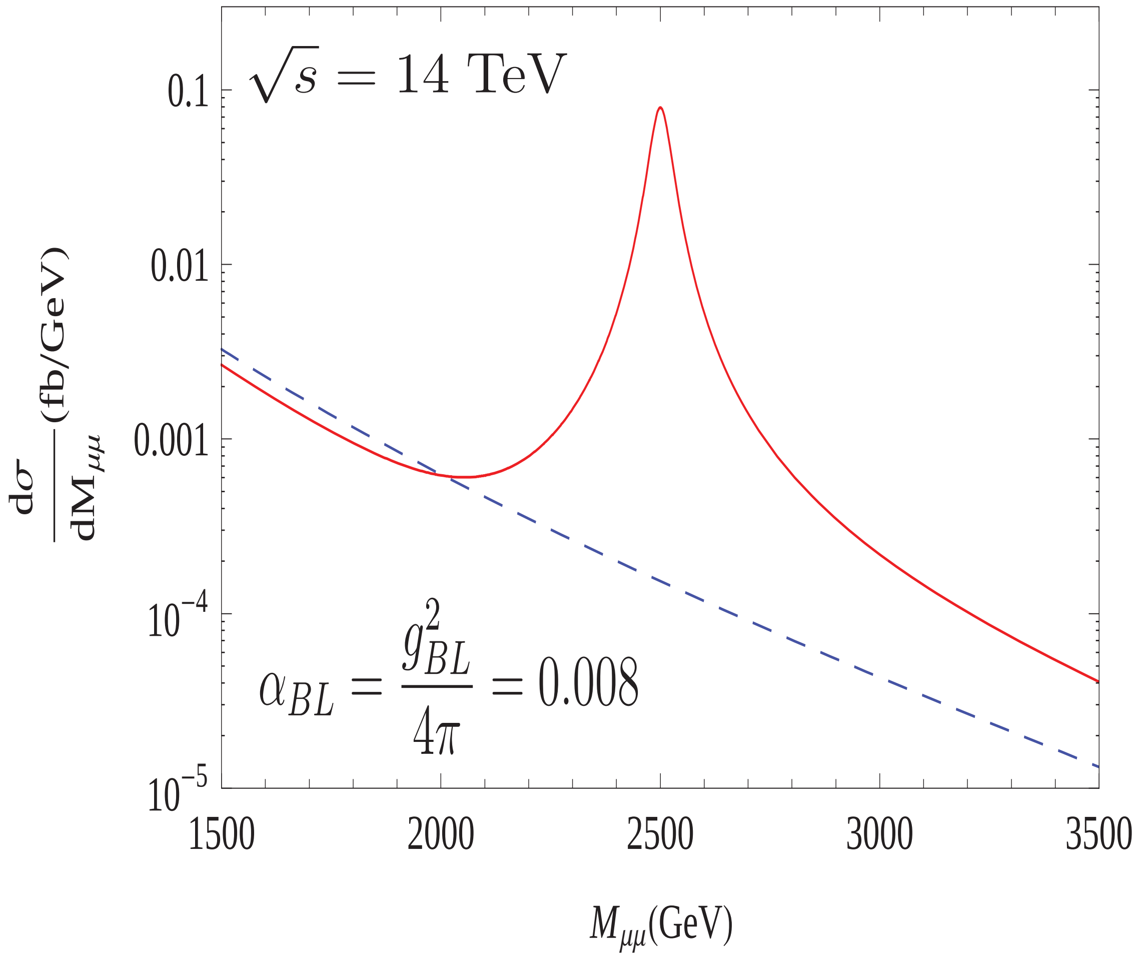
$g(v_{BL}) = 0.0184,\ 0.0216,\ 0.023,\ 0.026,$ and 0.0425 from left to right

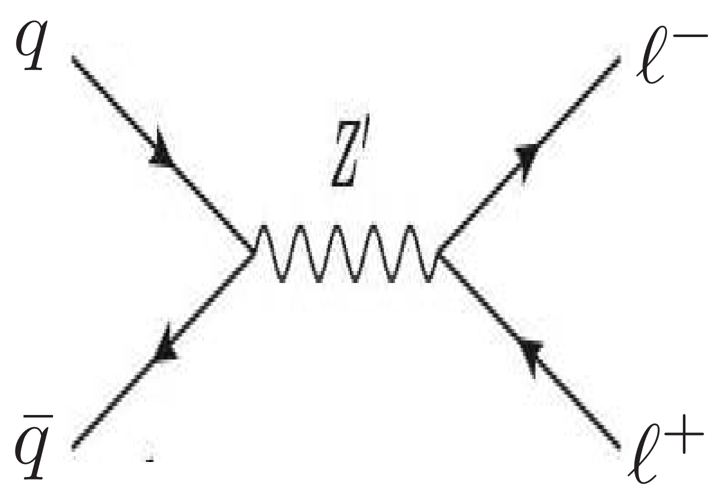
Here we have fixed $m_{Z'} = 3$ TeV

Search for Z' boson at LHC

Search for a resonance peak in the final state di-lepton invariant mass

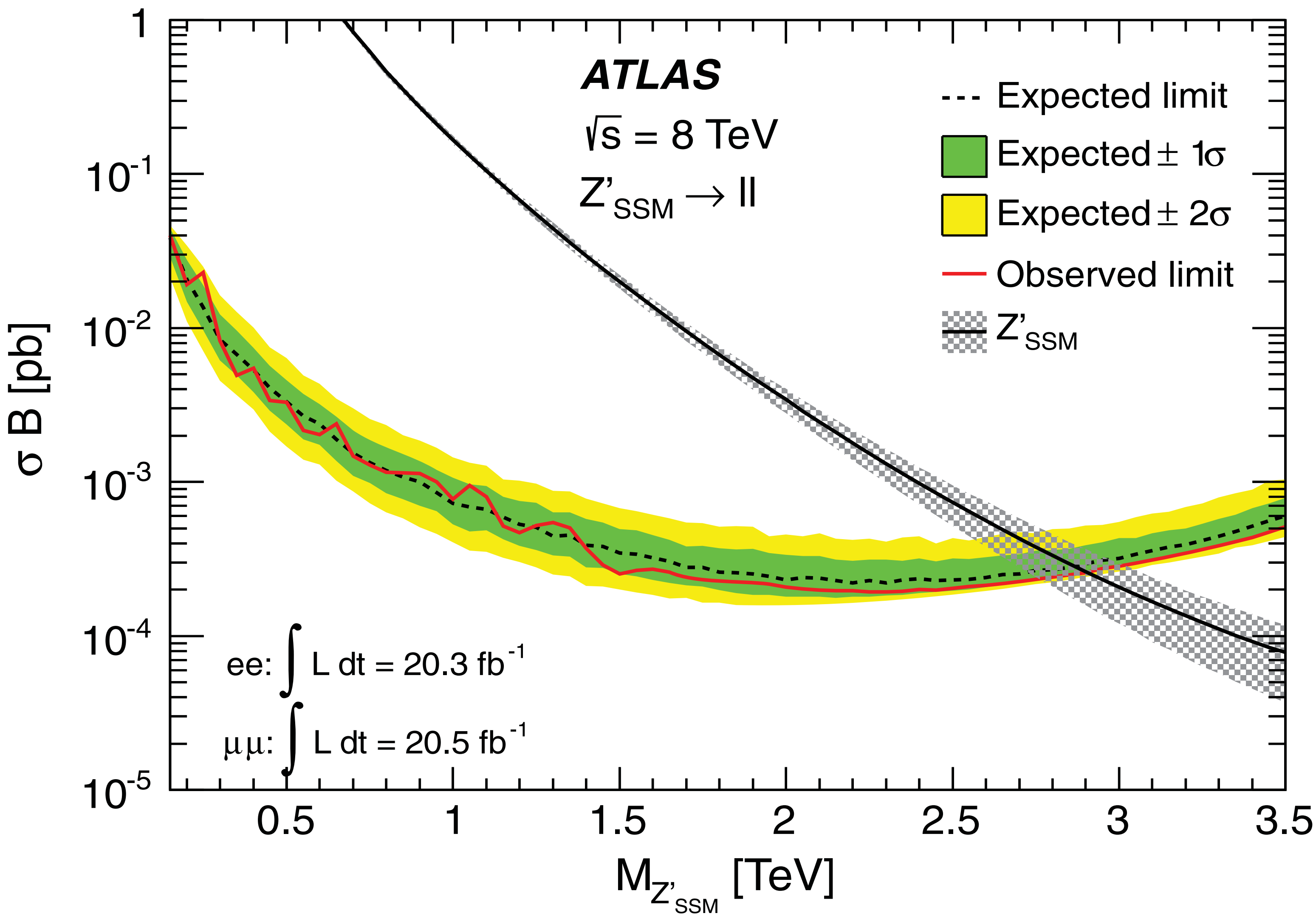
Iso, NO & Orikasa
PRD 80 (2009) 115007





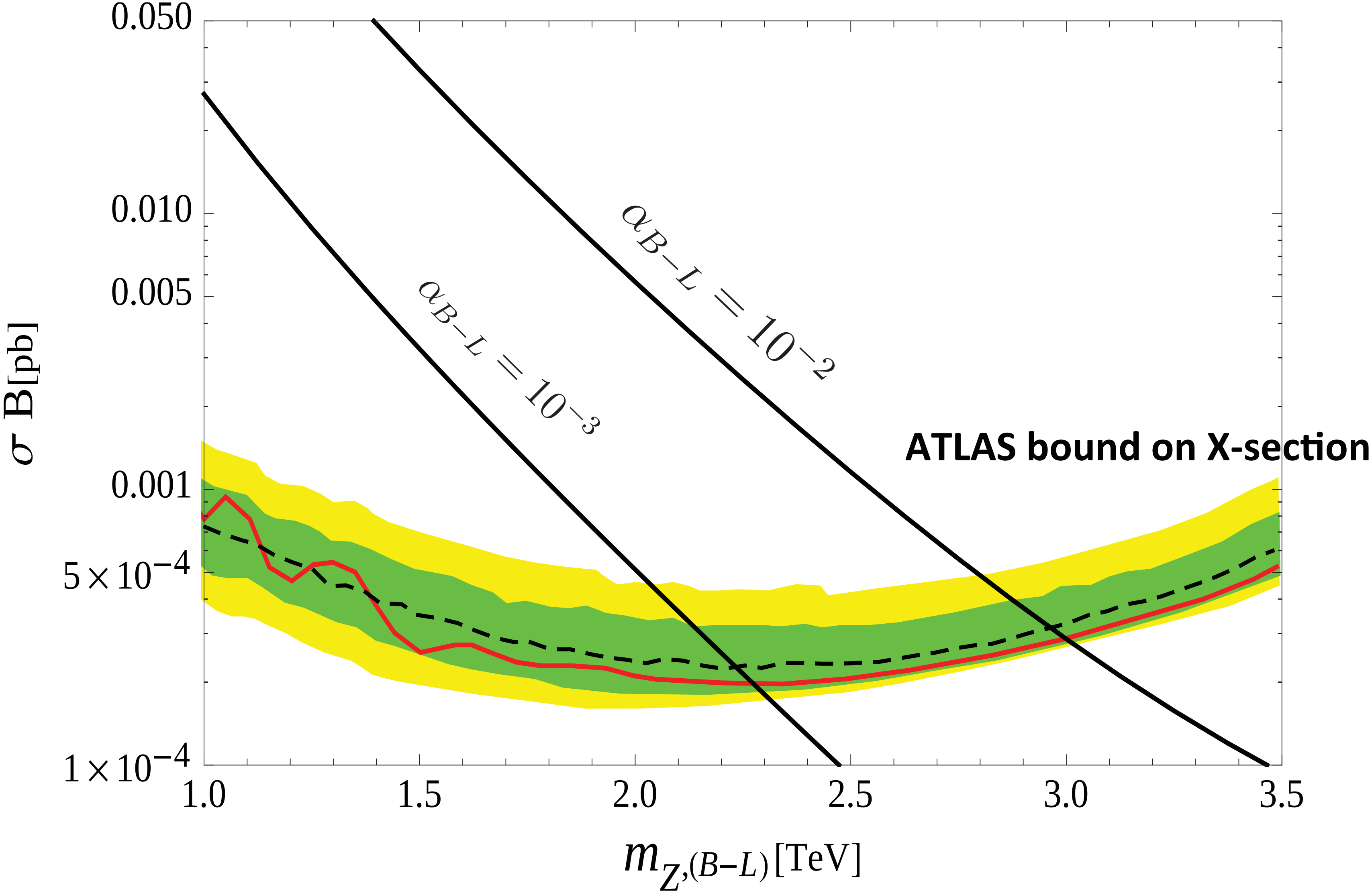
Current 8 TeV LHC results for sequential Z' model

ATLAS experiment (CMS has a similar result)

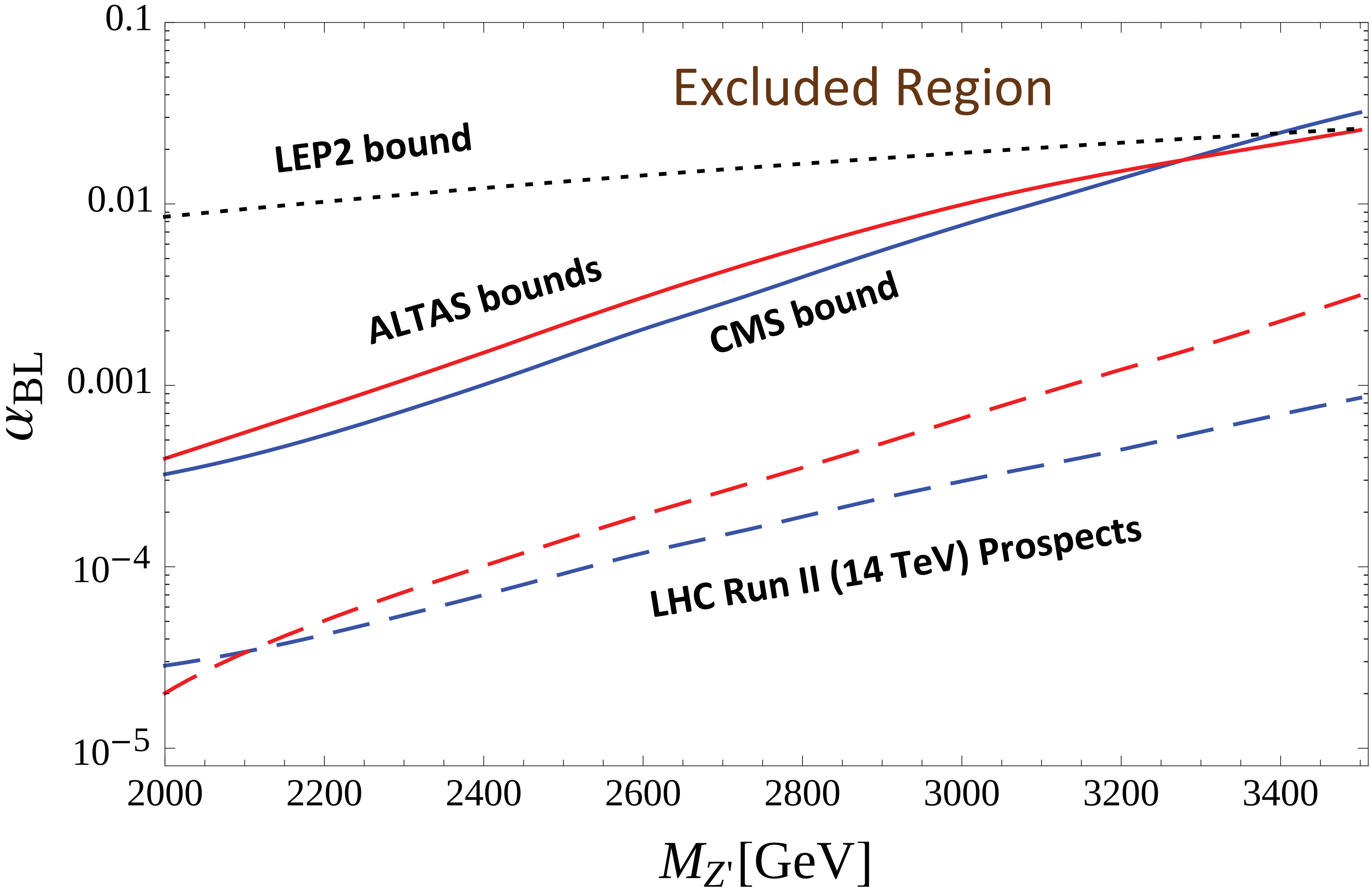


Interpretation of the current LHC results to the B-L model

NO, in preparation



Interpretation of the current LHC results to the B-L model



Summary

- We have considered a simple inflation model, non-minimal lambda phi^4 inflation
- Inflationary predictions with tree-level potential nicely fit the Planck 2015 results
- More compelling scenario → Inflaton=Higgs field
- Once quantum corrections have been taken into account, the effective inflaton potential is likely to become unstable
- In order to avoid the instability, we have imposed a vanishing beta function condition
- The condition leads to a relation among model parameters
- Inflationary predictions are altered from tree-level results

Summary (cont'd)

- As a simple example, we have considered the minimal B-L model at TeV scale, where the B-L Higgs field plays a role of inflaton
- The stability condition for the inflaton potential leads to a mass relation among Z' boson, right-handed neutrinos and the inflaton (B-L Higgs boson)
- Quantum corrections alter the inflationary predictions from those obtained from tree-level analysis
- In the system, the inflationary predictions correlate with the new particle mass spectrum
- LHC search for new particles in the B-L model is complementary to the cosmological observations