Gluino Coannihilation

Feng Luo

w/ John Ellis, Jason Evans and Keith Olive, 1503.07142, 1510.03498

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Kavli Institute for the Physics and Mathematics of the Universe
東京大学 国際高等研究所 カブリ数物連携宇宙研究機構
KAVLI IPMU

Outline

- background
  - supersymmetric dark matter
  - thermally freeze-out mechanism
  - coannihilation mechanism
- gluino coannihilation
- gluino-gluino bound state
- breakdown of gluino coannihilation by large squark masses
- results

Question to be addressed (very loosely)

How *Heavy* could Dark Matter be in Supersymmetry?

Background 1: supersymmetry – particle content

or, more precisely, the Minimal Supersymmetric extension of the Standard Model (MSSM)

$$\begin{aligned}
\textit{quarks, leptons}: &\quad u, d, \nu, e \\
\textit{squarks, sleptons}: &\quad \tilde{u}, \tilde{d}, \tilde{\nu}, \tilde{e} \\
\textit{gluons}, W\ \textit{bosons}, B\ \textit{boson}: &\quad g, W^{\pm}, W^{0}, B \\
\textit{gluinos, winos, bino}: &\quad \tilde{g}, \tilde{W}^{\pm}, \tilde{W}^{0}, \tilde{B} \\
\textit{higgs}: &\quad H_{\rm u}^{+}, H_{\rm u}^{0}, H_{\rm d}^{0}, H_{\rm d}^{-} \\
\textit{higgsinos}: &\quad \tilde{H}_{\rm u}^{+}, \tilde{H}_{\rm u}^{0}, \tilde{H}_{\rm d}^{0}, \tilde{H}_{\rm d}^{-}
\end{aligned}$$

From interaction eigenstates to mass eigenstates

- Neutralinos $\tilde{\chi}^0_{1,2,3,4}$ are linear combinations of $\tilde{B}$, $\tilde{W}^0$ and $\tilde{H}^0_{\rm u,d}$
- Charginos $\tilde{\chi}^{\pm}_{1,2}$ are linear combinations of $\tilde{W}^{\pm}$, $\tilde{H}^{+}_{\rm u}$ and $\tilde{H}^{-}_{\rm d}$

Background 1: R-parity – supersymmetric dark matter

Introduce R-parity, defined as $R \equiv (-1)^{3B+L+2s}$
(motivation: forbid interaction terms which make proton decay)

- $R = +1$ for all the Standard Model fermions and gauge bosons, as well as all the higgs bosons
- $R = -1$ for all the sparticles, i.e., the s- and -ino

Background 1: *R*-parity – supersymmetric dark matter

If *R*-parity is multiplicatively conserved, e.g., $R(A)R(B) = R(C)R(D)$ for reaction $A + B \to C + D$, or say, the number of sparticles on both sides of a reaction should be both even or odd,

$\Rightarrow$ *the Lightest Supersymmetric Particle (LSP) is stable, and furthermore, it could be a dark matter (DM) candidate if colour and electrically neutral.*

In this talk, I will consider that the LSP is the lightest neutralino $\tilde{\chi}_1^0$ $(\equiv \chi)$.

Background 2: get the DM relic abundance through thermally freeze-out mechanism

Consider the evolution of particle 1 in the expanding Universe:
w/o interaction,

$$0 = \frac{d\left(n_1 a^3\right)}{dt} = a^3 \frac{dn_1}{dt} + 3a^2 n_1 \frac{da}{dt} = a^3 \left[\frac{dn_1}{dt} + 3H(T)n_1\right].$$

w/ a reaction and its inverse reaction $1 + 2 \leftrightarrow 3 + 4$,

$$\frac{dn_1}{dt} + 3H(T)n_1 = -\alpha n_1 n_2 + \beta n_3 n_4 .$$

This is the Boltzmann equation.
$\alpha = \langle \sigma v_{12} \rangle_{1+2\to 3+4}$, i.e., the interaction rate per particle 1 is $\langle \Gamma \rangle_1 \equiv n_2 \langle \sigma v_{12} \rangle_{1+2\to 3+4}$.
chemical equilibrium $\Rightarrow \alpha n_1^{eq} n_2^{eq} = \beta n_3^{eq} n_4^{eq}$.

$$\Rightarrow \frac{dn_1}{dt} + 3H(T)n_1 = -\langle \sigma v_{12} \rangle_{1+2\to 3+4} \left[n_1 n_2 - n_1^{eq} n_2^{eq} \left(\frac{n_3 n_4}{n_3^{eq} n_4^{eq}}\right)\right]$$

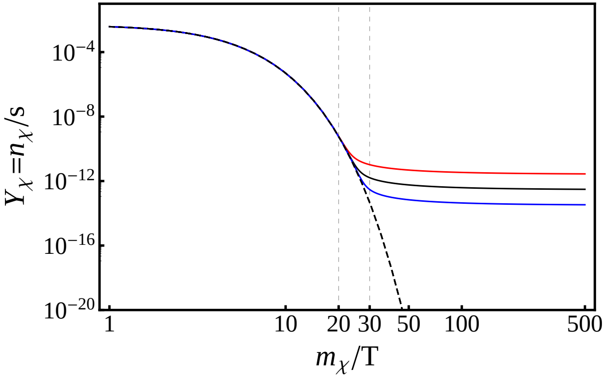
Considering DM self-annihilation and creation: $\chi\chi \leftrightarrow SM's$,

$$n_1 = n_2 \equiv n_\chi, n_3 = n_3^{eq}, n_4 = n_4^{eq}$$

$$\Rightarrow \boxed{\frac{dn_\chi}{dt} + 3H(T)n_\chi = -\langle \sigma v \rangle_{\chi\chi \to SM's} \left[n_\chi^2 - \left(n_\chi^{eq} \right)^2 \right]}$$

Introduce $Y_\chi \equiv n_\chi / s$ to factor out the dilution due to the Cosmic expansion,

$$\Rightarrow \frac{d \ln Y_\chi}{d \ln(m_\chi / T)} = -\frac{n_\chi \langle \sigma v \rangle_{\chi\chi \to SM's}}{H(T)} \left[1 - (Y_\chi^{eq} / Y_\chi)^2 \right]$$



$$\begin{aligned} m_\chi &= 1\,TeV, \langle\sigma v\rangle = (0.1, 1, 10) \times 3 \times 10^{-26} cm^3/s \\ \Rightarrow \Omega_\chi h^2 &= Y_{\chi,0} s_0 m_\chi h^2/\rho_{crit} \sim (1, 0.1, 0.01) \end{aligned}$$

$\Omega_{CDM} h^2 = 0.1193 \pm 0.0014$ (*1-σ, Planck Collaboration, 1502.01589*)

A too small $\langle\sigma v\rangle_{\chi\chi\to SM's}$ necessarily means $(\Omega_\chi)_{prediction} > (\Omega_{CDM})_{obs}$?

Background 3: coannihilation

If there is another R-odd species χ_2 almost degenerate in mass with the LSP χ_1,

and if χ_2 has a big annihilation cross section with itself and/or with χ_1,

and if χ_1 can efficiently convert to χ_2,

then χ_1 and χ_2 can freeze out together at a lower temperature resulting in a smaller dark matter abundance than if without the existence of χ_2.

(*Griest and Seckel, Phys. Rev. D 43, 3191*)

$$\begin{aligned} \chi_1\chi_1 &\leftrightarrow SM,\ \chi_1\chi_2 \leftrightarrow SM,\ \chi_2\chi_2 \leftrightarrow SM \\ \chi_1 SM &\leftrightarrow \chi_2 SM,\ \chi_2 \leftrightarrow \chi_1 SM \end{aligned}$$

(note: the 'SM' are not necessarily the same, and can be one or several)

$$\begin{aligned} \frac{dn_1}{dt} + 3Hn_1 = & -\sum_{j=1}^{2} \langle \sigma v \rangle_{1j\to SM} \left[n_1 n_j - n_1^{eq} n_j^{eq} \right] \\ & - \left[\langle \Gamma \rangle_{1SM\to 2SM} + \langle \Gamma \rangle_{1SM\to 2} \right] n_1 \\ & + \left[\langle \Gamma \rangle_{2SM\to 1SM} + \langle \Gamma \rangle_{2\to 1SM} \right] n_2 \\ \frac{dn_2}{dt} + 3Hn_2 = & -\sum_{j=1}^{2} \langle \sigma v \rangle_{2j\to SM} \left[n_2 n_j - n_2^{eq} n_j^{eq} \right] \\ & + \left[\langle \Gamma \rangle_{1SM\to 2SM} + \langle \Gamma \rangle_{1SM\to 2} \right] n_1 \\ & - \left[\langle \Gamma \rangle_{2SM\to 1SM} + \langle \Gamma \rangle_{2\to 1SM} \right] n_2 \end{aligned}$$

If all the $\langle \sigma v \rangle$'s and $\langle \Gamma \rangle$'s are known, then can solve for n_1 and n_2.
Also note that $n_i^{eq} = g_i \left(\frac{m_i T}{2\pi} \right)^{3/2} e^{-m_i/T}$ for $T \ll m_i$.

$$\frac{dn}{dt} + 3Hn = -\sum_{i,j=1}^{2} \langle \sigma v \rangle_{ij\to SM} \left[n_i n_j - n_i^{eq} n_j^{eq} \right],$$

where $n = n_1 + n_2$.

efficient conversion: $\langle \Gamma \rangle_{1SM\to 2SM} + \langle \Gamma \rangle_{1SM\to 2} \gg H$
$\Rightarrow n_1/n_2 \approx n_1^{eq}/n_2^{eq}$ (this can be checked by explicitly solving for n_1 and n_2)

$$\frac{dn}{dt} + 3Hn = -\sum_{i,j=1}^{2} \langle \sigma v \rangle_{ij\to SM} \frac{n_i^{eq} n_j^{eq}}{n_{eq}^2} \left[n^2 - n_{eq}^2 \right]$$

- if $m_2 \gg m_1$, $\Rightarrow n_{eq} \approx n_1^{eq}$, ●● $\approx \langle \sigma v \rangle_{11\to SM}$
- if $m_2 = m_1$, $\Rightarrow$ ●● $= \frac{g_1^2 \langle \sigma v \rangle_{11\to SM} + g_2^2 \langle \sigma v \rangle_{22\to SM} + 2 g_1 g_2 \langle \sigma v \rangle_{12\to SM}}{(g_1+g_2)^2}$

(Recall w/o coannihilation: $\boxed{\frac{dn_\chi}{dt} + 3H(T) n_\chi = -\langle \sigma v \rangle_{\chi\chi\to SM's} \left[n_\chi^2 - \left(n_\chi^{eq}\right)^2 \right]}$)

End of background: gluino coannihilation

Also, considering that typically $\langle \sigma v \rangle_{ann}$ decreases with the increase of the annihilating particle masses,

$\Rightarrow$ *there could be a largest possible LSP mass which gives* $(\Omega_\chi)_{prediction} = (\Omega_{CDM})_{obs}$*, and it is possible to be achieved in the gluino coannihilation scenario at* $m_{\tilde{g}} = m_\chi$*, because* $\langle \sigma v \rangle_{\tilde{g}\tilde{g} \to SM}$ *is big (strong interaction) and* $g_{\tilde{g}}$ *is also big* $(= 16)$.

Question to be addressed (more accurately)

How *Heavy* could Dark Matter be in the gluino-neutralino coannihilation scenario in the MSSM?

In this talk:

- Gluino bound-state effect
  (how does this effect help to achieve the largest DM mass?)
- Breakdown of coannihilation by large squark masses
  (how do large squark masses prevent from achieving the largest DM mass?)
- Results based on simplified supersymmetric spectra defined at the weak scale and from more complete CMSSM-like models

(CMSSM = Constrained MSSM, with the soft supersymmetry-breaking parameters constrained to be universal at the input GUT scale)

Gluino Coannihilation

$$\chi\chi \leftrightarrow SM,\ \chi\tilde{g} \leftrightarrow q\bar{q},\ \tilde{g}\tilde{g} \leftrightarrow q\bar{q} \text{ or } gg$$
$$\chi q \leftrightarrow \tilde{g} q,\ \tilde{g} \leftrightarrow \chi q\bar{q}$$

Key elements in the calculations:

1. *Sommerfeld enhancement* (and suppression) for $\tilde{g}\tilde{g} \to q\bar{q}$ or gg

Explanation:
depends on the colour configuration of the initial $\tilde{g}\tilde{g}$, the long range Coulomb like potential between $\tilde{g}\tilde{g}$ can be attractive (or repulsive)

$\Rightarrow$ modify the otherwise free initial particle wave function

$\Rightarrow$ enhance (or suppress) the $\tilde{g}\tilde{g}$ annihilation cross sections

(*De Simone, Giudice and Strumia, 1402.6287*
Harigaya, Kaneta and Matsumoto, 1403.0715)

Gluino bound-state effect

$$\chi\chi \leftrightarrow SM,\ \chi\tilde{g} \leftrightarrow q\bar{q},\ \tilde{g}\tilde{g} \leftrightarrow q\bar{q} \text{ or } gg,$$
$$\tilde{g}\tilde{g} \leftrightarrow \tilde{R}g, \tilde{R} \leftrightarrow gg$$
$$\chi q \leftrightarrow \tilde{g}q,\ \tilde{g} \leftrightarrow \chi q\bar{q}$$

Key elements in the calculations:

2. *Bound-state effects:* $\tilde{g}\tilde{g} \leftrightarrow \tilde{R}g$, $\tilde{R} \leftrightarrow gg$

Explanation:
(1) Similar to $e^-p \leftrightarrow H\gamma$, the attractive Coulomb like potential between the $\tilde{g}'s$ can make the formation of gluino-gluino bound state $\tilde{R}$ possible.

The differences come from

- colour wave functions of $\tilde{g}\tilde{g}$ and $\tilde{R}$
- the two gluinos are two identical Majorana fermions

Gluino bound-state effect

(2) $\tilde{R}$ becomes a favourable state when the temperature of the Universe drops below the binding energy of $\tilde{R}$.

(3) The $\tilde{R}$ *annihilation decay* $\tilde{R} \to gg$ effectively enhances the $\tilde{g}\tilde{g}$ annihilation cross section.
(also note that the $\tilde{R}$ annihilation decay rate is much larger than the gluino decay rate for $m_{\tilde{q}} > m_{\tilde{g}}$)

Result:

$$\frac{dn}{dt} + 3Hn \approx -\sum_{i,j=\chi,\tilde{g}} \langle \sigma v \rangle_{ij \to SM} \left[n_i n_j - n_i^{eq} n_j^{eq} \right] - \langle \sigma v \rangle_{\tilde{g}\tilde{g} \to \tilde{R}g} \frac{\langle \Gamma \rangle_{\tilde{R} \to gg}}{\langle \Gamma \rangle_{\tilde{R} \to gg} + \langle \Gamma \rangle_{\tilde{R}g \to \tilde{g}\tilde{g}}} \left[n_{\tilde{g}} n_{\tilde{g}} - n_{\tilde{g}}^{eq} n_{\tilde{g}}^{eq} \right]$$

In the $v \to 0$ limit,

$$\frac{(\sigma v)_{\tilde{g}\tilde{g} \to \tilde{R}g}}{\text{Sommerfeld enhanced } (\sigma v)_{\tilde{g}\tilde{g} \to gg}} \approx 1.44$$

Breakdown of coannihilation by a large squark mass

$$\chi\chi \leftrightarrow SM,\ \chi\tilde{g} \leftrightarrow q\bar{q},\ \tilde{g}\tilde{g} \leftrightarrow q\bar{q} \text{ or } gg,$$
$$\tilde{g}\tilde{g} \leftrightarrow \tilde{R}g, \tilde{R} \leftrightarrow gg$$
$$\chi q \leftrightarrow \tilde{g}q,\ \tilde{g} \leftrightarrow \chi q\bar{q}$$

Key elements in the calculations:

3. *Breakdown of coannihilation by a large squark mass*

Explanation:
the gluino ONLY has a colour charge, while the neutralino DOES NOT have a colour charge, in the above processes neutralino can only interact with gluino through vertices involving a squark in the propagator: $\chi - q - \tilde{q}$ and $\tilde{q} - \tilde{g} - q$

$\Rightarrow$ when $m_{\tilde{q}}$ is very large, $\chi q \leftrightarrow \tilde{g}q$ and $\tilde{g} \leftrightarrow \chi q\bar{q}$ are ineffective

$\Rightarrow$ coannihilation mechanism breaks down, gluino annihilations and bound-state effects *cannot* reduce the neutralino density *even if* they are large and *even if* $\tilde{g}$ and χ are degenerate in mass

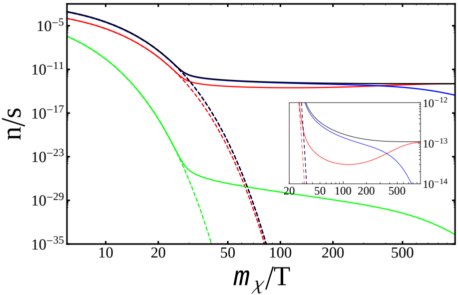
Results based on simplified supersymmetric spectra

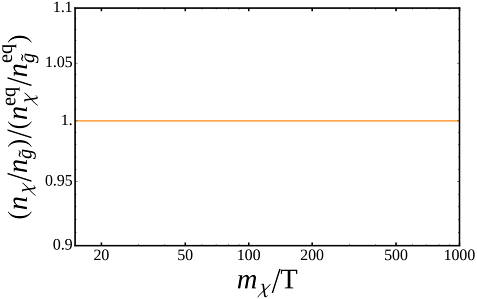
To illustrate the physics, let's first see results based on simplified supersymmetric spectra, assuming degenerate squark masses, and that the neutralino is a pure state of either a Bino, Wino, or Higgsino.

Therefore, the free parameters are simply the neutralino mass, m_χ, the gluino mass, $m_{\tilde{g}}$ and the squark masses, $m_{\tilde{q}}$.

Result: neutralino is a Bino

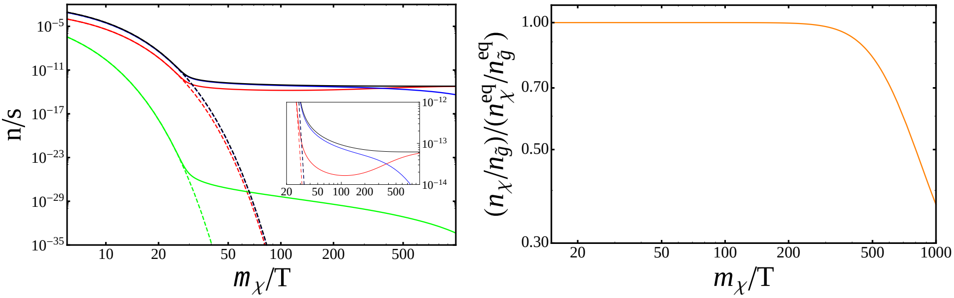
Results obtained by solving the coupled Boltzmann equations for χ, $\tilde{g}$ and $\tilde{R}$.





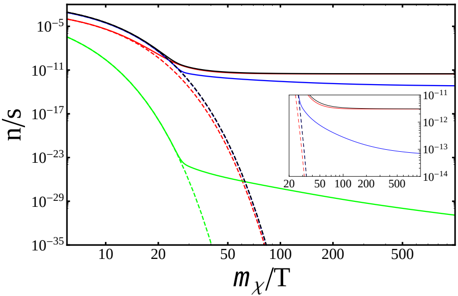
$m_\chi = 7$ TeV, $m_{\tilde{g}} - m_\chi = 40$ GeV, $m_{\tilde{q}}/m_{\tilde{g}} = 1.1$.
(red: χ, blue: $\tilde{g}$, green: $\tilde{R}$, black: $\chi + \tilde{g}$)

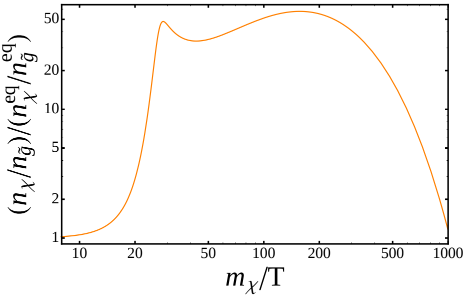
Result: Bino



$m_\chi = 7$ TeV, $m_{\tilde{g}} - m_\chi = 40$ GeV, $m_{\tilde{q}}/m_{\tilde{g}} = 10$.

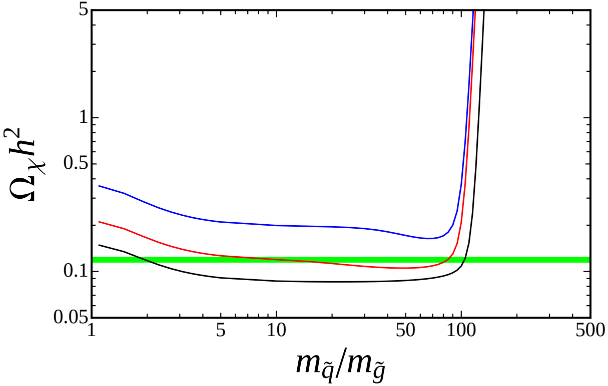
Result: Bino





$m_\chi = 7$ TeV, $m_{\tilde{g}} - m_\chi = 40$ GeV, $m_{\tilde{q}}/m_{\tilde{g}} = 120$.

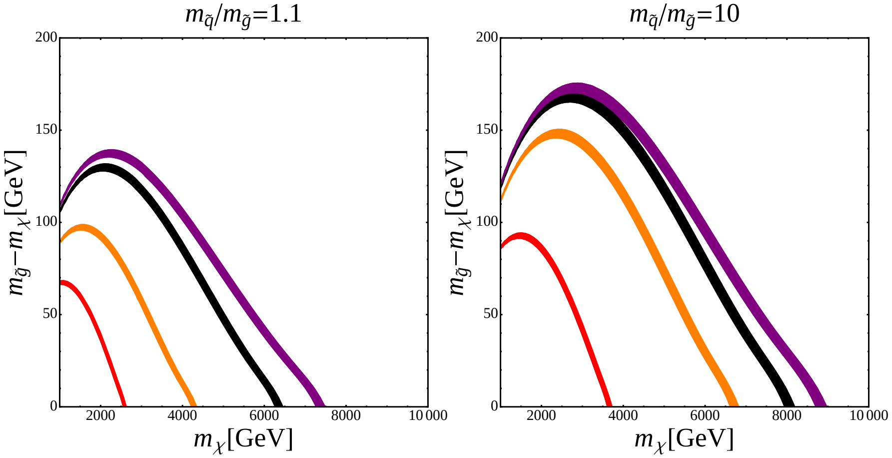
Result: Bino



$m_\chi = 7$ TeV, $m_{\tilde{g}} - m_\chi = 0$ (black), 40 GeV (red), 120 GeV (blue).

- The rise in $\Omega_\chi h^2$ at small $m_{\tilde{q}}/m_{\tilde{g}}$ is due to the s-, t- and u-channel cancellation in $\tilde{g}\tilde{g} \to q\bar{q}$ annihilation cross section.
- The very rapid rise in $\Omega_\chi h^2$ at high $m_{\tilde{q}}/m_{\tilde{g}} \gtrsim 100$ is due to the breakdown of $\tilde{g} \leftrightarrow \chi$ conversion.

Result: Bino



$\Omega_\chi h^2 = 0.1193 \pm 0.0042$ bands (3-σ).

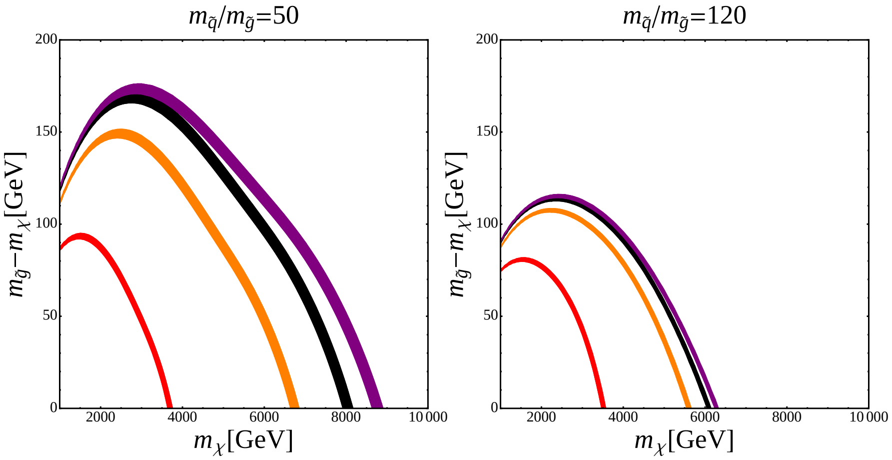
red: w/o Sommerfeld effects and w/o bound-state effects

orange: w/ Sommerfeld effects but w/o bound-state effects

black: w/ Sommerfeld effects and w/ bound-state effects

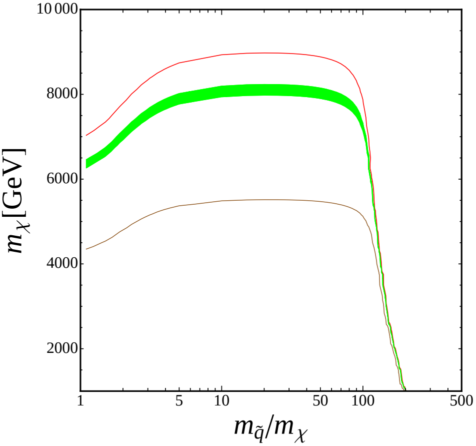
purple: w/ Sommerfeld effects and w/ 2 times bound-state effects

Result: Bino



$m_{\tilde{q}}/m_{\tilde{g}}$=50
$m_{\tilde{q}}/m_{\tilde{g}}$=120
200
150
100
50
0
2000
4000
6000
8000
10 000
$m_{\tilde{g}}-m_{\chi}$[GeV]
m_{χ}[GeV]

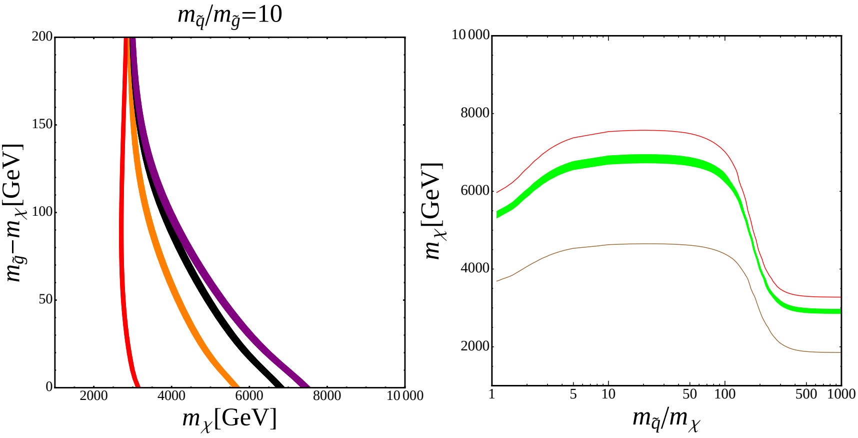
Result: Bino



For $m_{\tilde{g}} - m_\chi = 0$.

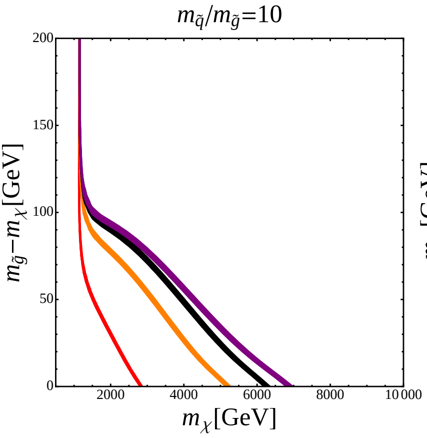
(green: $\Omega_\chi h^2 = 0.1193 \pm 0.0042$, brown and red: $\Omega_\chi h^2 = 0.05$ and 0.15)

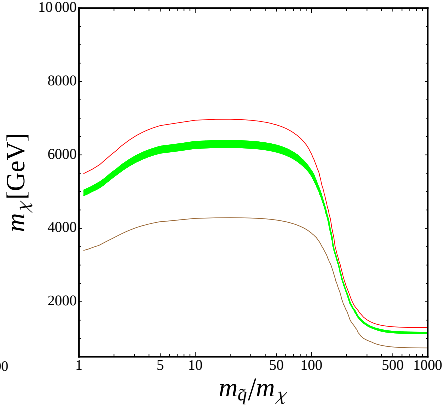
Result: Wino



$m_{\tilde{q}}/m_{\tilde{g}}$=10
200
150
100
50
0
2000
4000
6000
8000
10 000
$m_{\tilde{g}}-m_\chi$[GeV]
m_χ[GeV]
10 000
8000
6000
4000
2000
1
5
10
50
100
500
1000
m_χ[GeV]
$m_{\tilde{q}}/m_\chi$

Result: Higgsino





A remark

Why the maximum LSP mass is smaller for a Wino (~ 7 TeV) or a Higgsino (~ 6 TeV) compared to a Bino (~ 8 TeV)?

Because there are more *inert* degrees of freedom for Wino (=6) or Higgsino (=8) compared to Bino (=2) at large mass when $\chi\chi$ annihilation cross section is negligible compared to $\tilde{g}\tilde{g}$ annihilation cross section.

Recall

$$\frac{dn}{dt} + 3Hn = -\sum_{i,j=1}^{2} \langle \sigma v \rangle_{ij \to SM} \frac{n_i^{eq} n_j^{eq}}{n_{eq}^2} \left[n^2 - n_{eq}^2 \right]$$

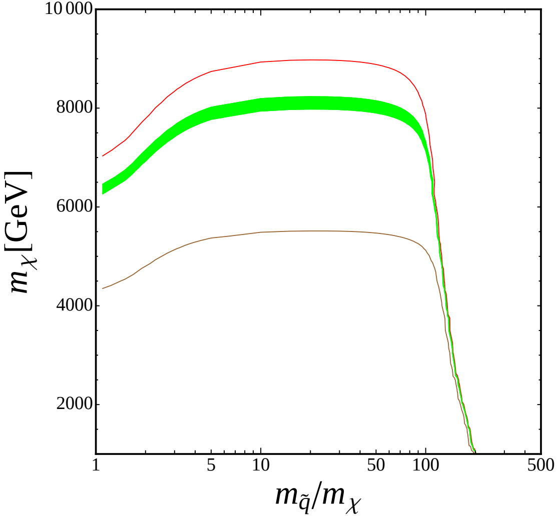
- if $m_2 = m_1$, $\Rightarrow$ ●● $= \frac{g_1^2 \langle \sigma v \rangle_{11 \to SM} + g_2^2 \langle \sigma v \rangle_{22 \to SM} + 2 g_1 g_2 \langle \sigma v \rangle_{12 \to SM}}{(g_1 + g_2)^2}$

Results from CMSSM-like models

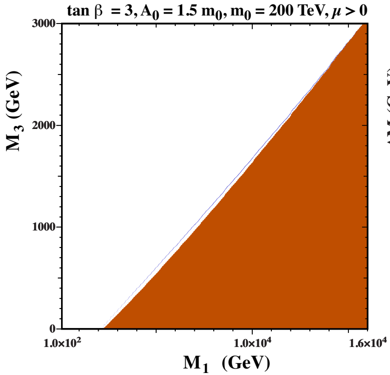
The possibility of gluino coannihilation does not arise in the CMSSM. However, gluino coannihilation can become important in variants of the MSSM such as a one-parameter extension of the CMSSM by allowing a restricted form of non-universality in the gaugino sector with $M_1 = M_2 \neq M_3$ at the input GUT scale.

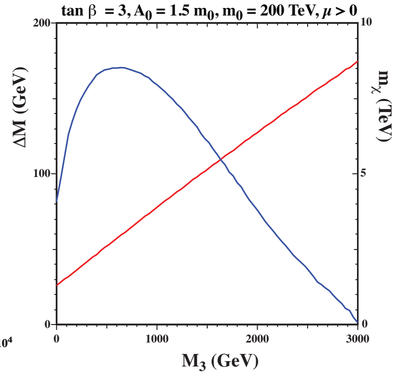
Therefore, the results depend on M_1 and M_3 as well as the usual CMSSM parameters m_0, A_0, $\tan\beta$ and the sign of μ.

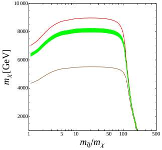
Gluino coannihilation can become important in other models as well, and in 1510.03498 we show results in models with pure gravity mediation of supersymmetry breaking with additional vector multiplets.



The following sample results correspond to choices of parameters of the CMSSM-like models featuring various parts of the green band obtained in the above simplified supersymmetric spectra.



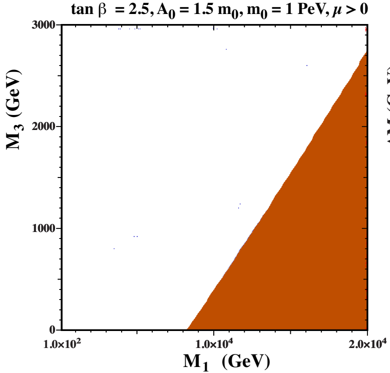


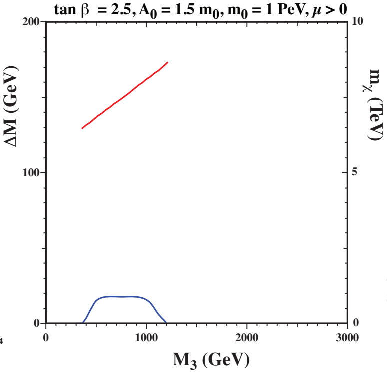


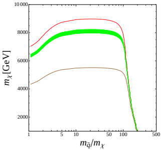
This choice of m_0 corresponds to values of $m_{\tilde{q}}/m_\chi$ along the plateau.

In the left panel, the dark blue strip shows where $\Omega_\chi h^2 = 0.1193 \pm 0.0042$, and gluino is the LSP in the brick-red shaded region.

In the right panel, the blue line shows the gluino-neutralino mass difference and the red line shows the neutralino mass, both along the dark blue strip in the left panel and as functions of M_3.

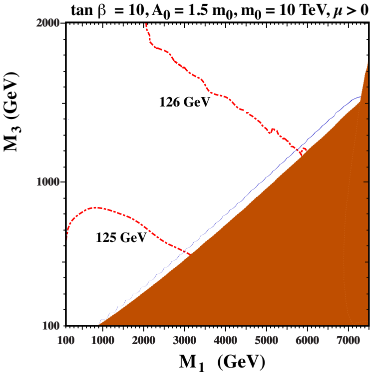


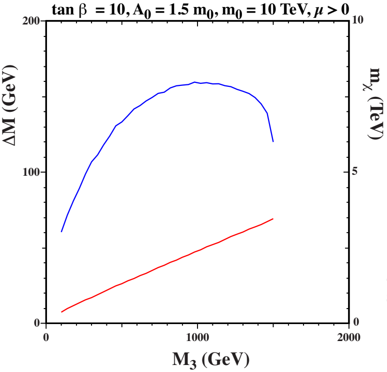


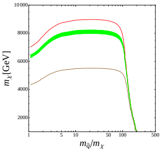


This choice of m_0 corresponds to values of $m_{\tilde{q}}/m_\chi$ extending from beyond the right-end of the plateau at small M_3 to values along the plateau at large M_3.

The gluino coannihilation strip therefore has two end-points where $\Delta M \to 0$.







This choice of m_0 corresponds to values of $m_{\tilde{q}}/m_{\chi}$ beyond the left-end of the plateau.

In the left panel, the lighter stop is the LSP in the brick-red shaded region in the upper right corner.

Summary

- There could be a largest possible LSP mass achievable in the neutralino-gluino coannihilation scenario.
- Bound states effectively enhance the gluino annihilation cross section, and they help to achieve the largest DM mass.
- The neutralino-gluino coannihilation mechanism can be broken by large squark masses.
- Gluino coannihilation can become important in variants of the MSSM such as CMSSM-like models with non-universality in the gaugino sector.

Finally, for collider probe of the gluino coannihilation scenario, see, e.g., Nagata, Otono and Shirai, 1504.00504.