Perturbative gauge theory results from strings in $AdS_5 \times S^5$

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work with Z. Bajnok (also with T. Łukowski, A. Hegedus)

Outline

1. **Motivation**
   - $\mathcal{N} = 4$ Super Yang-Mills and the AdS/CFT correspondence
2. **How to describe strings in $AdS_5 \times S^5$**
3. **Anomalous dimensions from strings in $AdS_5 \times S^5$**
4. **Anomalous dimensions in gauge theory**
5. **Basic example: the Konishi operator**
   - Definition
   - Direct 4-loop perturbative computation
6. **Computation of Konishi anomalous dimension from strings in $AdS_5 \times S^5$**
7. **Further developments**
8. **Conclusions**

What is $\mathcal{N} = 4$ Super Yang-Mills and why bother about it?

- $\mathcal{N} = 4$ Super Yang Mills ($\equiv$ ordinary Yang-Mills+4 adjoint fermions +6 adjoint scalars+ appropriate self-interactions)
- This theory is
  1. supersymmetric
  2. conformal (scale-invariant even at the quantum level)
- $\mathcal{N} = 4$ *SYM may be the 'harmonic oscillator' of four dimensional gauge theories* – D. Gross
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The AdS/CFT correspondence

$\boxed{\mathcal{N} = 4 \text{ Super Yang-Mills theory}} \quad \equiv \quad \boxed{\text{Superstrings on } AdS_5 \times S^5}$

strong coupling	(semi-)classical strings
nonperturbative physics	or supergravity
very difficult	'easy'
weak coupling	highly quantum regime
'easy'	very difficult

New ways of looking at nonperturbative gauge theory physics
Translates various gauge theory properties into *geometrical* and/or *gravitational* problems
but very difficult to test...

Goal: Interpolate from strong to weak coupling to reach perturbative results staying on the string theory side of the correspondence

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Integrability program:

Goal: Solve exactly $\mathcal{N} = 4$ SYM or superstrings in $AdS_5 \times S^5$ in the large N_c limit for any value of the coupling $\lambda = g_{YM}^2 N_c$

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Basic questions

Gauge theory side:

- $\mathcal{N} = 4$ is an exact CFT
- There is a basis of (local) operators such that correlation functions have the form

$$\langle O(x)O(y)\rangle = \frac{const}{|x-y|^{2\Delta}}$$

- The (anomalous) dimension Δ will be a complicated function of the coupling constant $g^2 = \lambda/16\pi^2$
- **Basic question:** Find the anomalous dimensions of all operators in $\mathcal{N} = 4$ SYM as a function of the coupling constant g^2
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AdS/CFT dictionary

Operators in $\mathcal{N} = 4$ SYM	$\longleftrightarrow$	(quantized) string states in $AdS_5 \times S^5$
Single trace operators	$\longleftrightarrow$	single string states
Multitrace operators	$\longleftrightarrow$	multistring states
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How to describe strings in $AdS_5 \times S^5$?

- Consider a closed string in $AdS_5 \times S^5$:

- The embedding coordinates of the point (τ, σ) are *quantum fields* $X^\mu(\tau, \sigma)$ on the worldsheet which has the geometry of a cylinder
- String theory in $AdS_5 \times S^5 \equiv$ a specific two dimensional quantum field theory defined on a cylinder (worldsheet QFT)
- It turns out that this worldsheet QFT is *integrable*

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Anomalous dimensions from strings in $AdS_5 \times S^5$

- In $AdS_5 \times S^5$ this worldsheet QFT is *integrable* — if we put this theory on an infinite plane, it has an infinite number of (nonlocal) conserved quantities
- This 2D QFT has massive particles whose scattering is described by so-called S-matrices
- S-matrices reduce to products of $2 \to 2$ scattering S-matrices which have to satisfy the Yang-Baxter Equation

$$S_{12}S_{23}S_{13} = S_{13}S_{23}S_{12}$$

- One can use this to find the S-matrix exactly as a function of the coupling constant $g^2 = \lambda/16\pi^2$
- The worldsheet QFT is solved exactly in infinite volume!

> Caveat: We still have to use this information to find the energy levels ($\equiv$ anomalous dimensions) on a cylinder (*closed string!*) of finite size

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Spectrum on a cylinder

- In general this is a very complicated problem even for integrable QFT's
- Something may be said about large volume behaviour
- The spectrum on a cylinder of large size is given by a Bethe ansatz

$$\text{``}e^{ip_i L} = \prod_{k \neq i} S(p_i, p_k)\text{''}$$

- In fact, it coincides exactly with the Asymptotic Bethe Ansatz of [Beisert, Staudacher] proposed earlier for gauge theory
- But on top of this there are virtual corrections — Lüscher corrections

- Lead to violations of the Bethe Asnatz
- All this has a direct counterpart in gauge theory!

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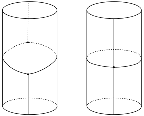

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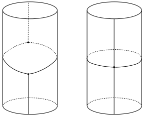

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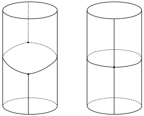

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"S-matrix" in $\mathcal{N}=4$ SYM

- Consider long operators made up of two complex scalar fields of $\mathcal{N}=4$ SYM

$$\mathrm{tr}\ \underbrace{ZZZZZZ}_{\text{'vacuum'}} XZXXZZXZZZZXZZ \underbrace{XXXXXXX}_{\text{excitations}} ZZZZ$$

or better

$$\mathrm{tr}\ ZZZZZZZZZZZZZZZZZXZZZZZZZZZZZZXZZZZZZZZZXZZZ$$

- X's are excitations. . .
- Fourier transform in position space $\longrightarrow$ associate a momentum p_i to each excitation.
- String side: excitations of string worldsheet with definite worldsheet momentum ($\equiv$ magnons)
- Implement the dilatation operator in this language... Staudacher
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Anomalous dimensions in gauge theory

- Since $\mathcal{N} = 4$ SYM is exactly conformal anomalous dimensions may be defined simply through two-point correlation functions

$$\langle O(x)O(y) \rangle = \frac{const}{|x-y|^{2\Delta}}$$

- The dimension Δ depends in a nontrivial way on the coupling $g^2 = \lambda/16\pi^2$ where $\lambda \equiv g_{YM}^2 N_c$ is kept fixed in the limit $N_c \to \infty$
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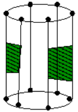

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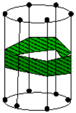

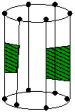

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$$\text{“}e^{ip_i L} = \underbrace{\prod_{k \neq i} S(p_i, p_k; g^2)\text{”}}_{\text{(Beisert-Staudacher Asymptotic Bethe Ansatz)}}$$

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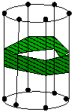
- Expand in g. The result can be trusted only up to terms of order g^{2L}



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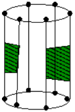

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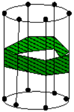

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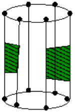

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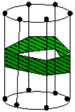

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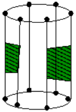

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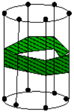

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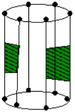

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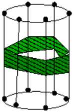

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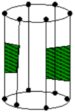

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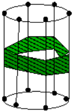

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Basic example: the Konishi operator

- For an operator composed of L fields in SYM we expect the Bethe Ansatz to give exact answers up to order g^{2L}.
- (Conceptually) simplest example: the Konishi operator

$$\mathrm{tr}\,\Phi_i^2 \quad \longleftrightarrow \quad \mathrm{tr}\,Z^2X^2 + \ldots \quad \longleftrightarrow \quad \mathrm{tr}\,ZD^2Z + \ldots$$

- Its anomalous dimension should be given by the ABA exactly up to 3 loops:

$$E_{Bethe} = 4 + 12g^2 - 48g^4 + 336g^6 - (2820 + 288\zeta(3))g^8 + \ldots$$

- The true result is

$$E = E_{Bethe} + \Delta_{wrapping}E$$

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$$\mathrm{tr}\,\Phi_i^2 \quad \longleftrightarrow \quad \mathrm{tr}\,Z^2X^2 + \ldots \quad \longleftrightarrow \quad \mathrm{tr}\,ZD^2Z + \ldots$$

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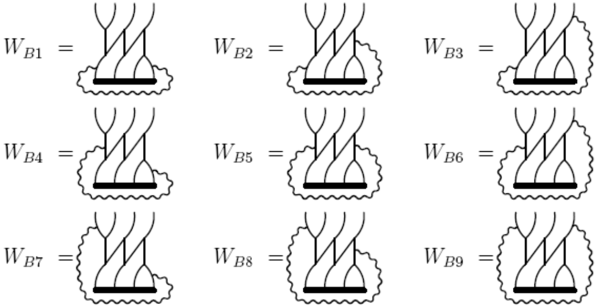


Figure C.1: Wrapping diagrams with chiral structure $\chi(1,2,3)$

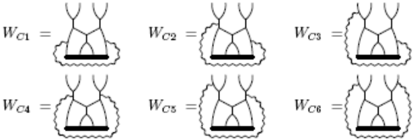


Figure C.2: Wrapping diagrams with chiral structure $\chi(1,3,2)$

$W_{C1} \rightarrow *$	1	$W_{C4} \rightarrow$ finite	
$W_{C2} \rightarrow *$	2	$W_{C5} \rightarrow -W_{C3}$	
$W_{C3} \rightarrow -W_{C5}$		$W_{C6} \rightarrow$ finite	

Table C.2: Results of D-algebra for diagrams with structure $\chi(1,3,2)$

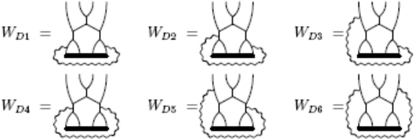
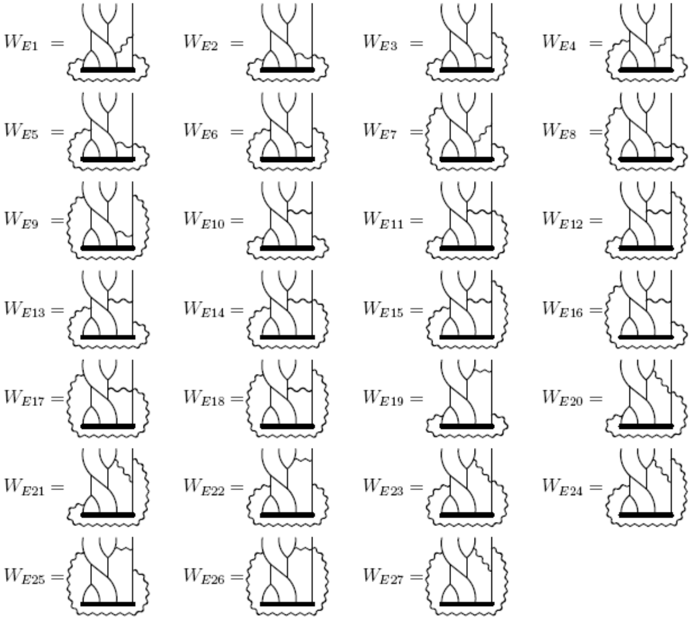
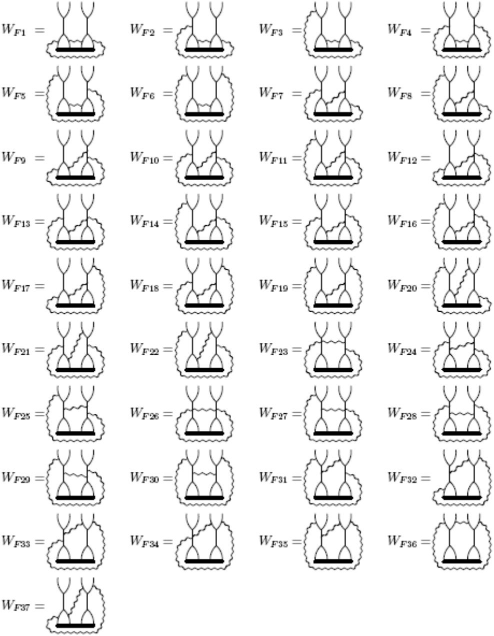
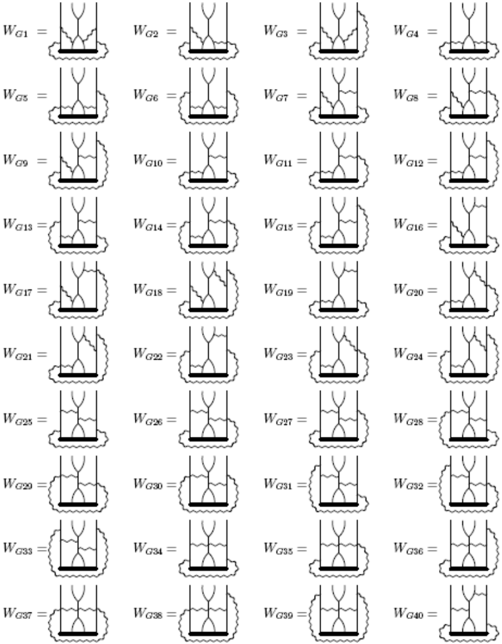


Figure C.3: Wrapping diagrams with chiral structure $\chi(2,1,3)$


$W_{E1} =$
$W_{E2} =$
$W_{E3} =$
$W_{E4} =$
$W_{E5} =$
$W_{E6} =$
$W_{E7} =$
$W_{E8} =$
$W_{E9} =$
$W_{E10} =$
$W_{E11} =$
$W_{E12} =$
$W_{E13} =$
$W_{E14} =$
$W_{E15} =$
$W_{E16} =$
$W_{E17} =$
$W_{E18} =$
$W_{E19} =$
$W_{E20} =$
$W_{E21} =$
$W_{E22} =$
$W_{E23} =$
$W_{E24} =$
$W_{E25} =$
$W_{E26} =$
$W_{E27} =$


$W_{F1} =$
$W_{F2} =$
$W_{F3} =$
$W_{F4} =$
$W_{F5} =$
$W_{F6} =$
$W_{F7} =$
$W_{F8} =$
$W_{F9} =$
$W_{F10} =$
$W_{F11} =$
$W_{F12} =$
$W_{F13} =$
$W_{F14} =$
$W_{F15} =$
$W_{F16} =$
$W_{F17} =$
$W_{F18} =$
$W_{F19} =$
$W_{F20} =$
$W_{F21} =$
$W_{F22} =$
$W_{F23} =$
$W_{F24} =$
$W_{F25} =$
$W_{F26} =$
$W_{F27} =$
$W_{F28} =$
$W_{F29} =$
$W_{F30} =$
$W_{F31} =$
$W_{F32} =$
$W_{F33} =$
$W_{F34} =$
$W_{F35} =$
$W_{F36} =$
$W_{F37} =$


W_{G1} =
W_{G2} =
W_{G3} =
W_{G4} =
W_{G5} =
W_{G6} =
W_{G7} =
W_{G8} =
W_{G9} =
W_{G10} =
W_{G11} =
W_{G12} =
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W_{G14} =
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W_{G16} =
W_{G17} =
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W_{G31} =
W_{G32} =
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W_{G39} =
W_{G40} =

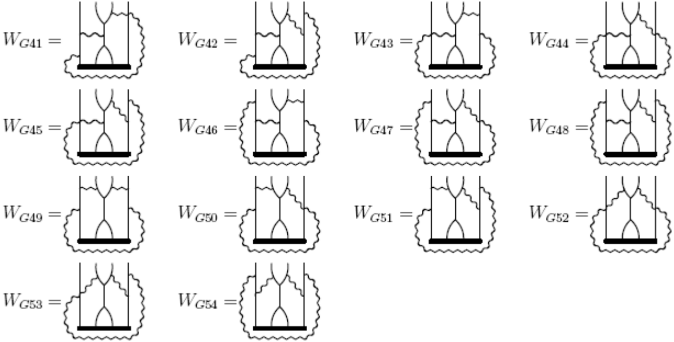
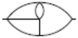


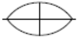
Figure C.6: Wrapping diagrams with chiral structure $\chi(1)$ (continued)

$I_1 = J_1 =$  $= \frac{1}{(4\pi)^8}\Big(-\frac{1}{24\varepsilon^4}+\frac{1}{4\varepsilon^3}-\frac{19}{24\varepsilon^2}+\frac{5}{4\varepsilon}\Big)$

$I_2 =$  $= \frac{1}{(4\pi)^8}\Big(-\frac{1}{24\varepsilon^4}+\frac{1}{4\varepsilon^3}-\frac{19}{24\varepsilon^2}+\frac{1}{\varepsilon}\Big(\frac{5}{4}-\zeta(3)\Big)\Big)$

$I_3 = J_5 =$  $= \frac{1}{(4\pi)^8}\Big(-\frac{1}{12\varepsilon^4}+\frac{1}{3\varepsilon^3}-\frac{5}{12\varepsilon^2}-\frac{1}{\varepsilon}\Big(\frac{1}{2}-\zeta(3)\Big)\Big)$

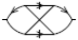
$I_4 =$  $= \frac{1}{(4\pi)^8}\Big(-\frac{1}{6\varepsilon^4}+\frac{1}{3\varepsilon^3}+\frac{1}{3\varepsilon^2}-\frac{1}{\varepsilon}(1-\zeta(3))\Big)$

$I_5 =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}5\zeta(5)$

$I_6 =$  $= \frac{1}{(4\pi)^8}\Big(\frac{1}{12\varepsilon^2}-\frac{7}{12\varepsilon}\Big)$ $I_7 =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}(-\zeta(3))$

$I_8 =$  $= \frac{1}{(4\pi)^8}\Big(\frac{1}{4\varepsilon^2}-\frac{11}{12\varepsilon}\Big)$ $I_9 =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\Big(\frac{1}{2}\zeta(3)-\frac{5}{2}\zeta(5)\Big)$

$I_{10} =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\Big(-\frac{1}{2}-\frac{1}{2}\zeta(3)+\frac{5}{2}\zeta(5)\Big)$

$I_{11} =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\Big(-\frac{1}{4}-\frac{3}{2}\zeta(3)+\frac{5}{2}\zeta(5)\Big)$

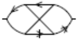
$I_{12} =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\Big(-\frac{1}{8}-\frac{1}{4}\zeta(3)+\frac{5}{4}\zeta(5)\Big)$

Table C.8: Loop integrals for 4-loop wrapping diagrams. The arrows of the same type indicate contracted spacetime derivatives

Perturbative 4-loop result for the Konishi

- The final result for the anomalous dimension of the Konishi operator is

$$\Delta = 4 + 12g^2 - 48g^4 + 336g^6 + \underbrace{\left(-2496 + 576\zeta(3) - 1440\zeta(5)\right) g^8}_{[F.Fiamberti, A.Santambrogio, C.Sieg, D.Zanon]} + \ldots$$

- The wrapping part is thus

$$\Delta_{wrapping} E = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

- Later this result was confirmed by an independent perturbative gauge theory computation using ordinary Feynman graphs by V. Velizhanin (total number of four loop diagrams: 131015)

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Our goal:

Compute the same 4-loop anomalous dimension from string theory

More generally:

How to compute these wrapping effects?

Why is this interesting?

- Want to know the complete spectrum of SYM/string theory
- For a given operator we want to eventually know the answer for any coupling
- Most interesting 'natural' operators in gauge theory are 'short'
- Finite size effects are very sensitive to the fine details of the theory
  – use them as a test of our understanding of the complete quantum worldsheet QFT
  – especially nontrivial at weak coupling
- Access information on the spectrum of short strings at strong coupling

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The Konishi computation

- We have to identify the string state corresponding to the Konishi operator...
- The Konishi operator has the same anomalous dimension as $\mathrm{tr}\, ZXZX - \mathrm{tr}\, Z^2X^2$
- number of X's $\equiv$ number of particles on the string worldsheet
- number of Z's $\equiv$ size of the cylinder
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$$\Delta E = \frac{-1}{2\pi} \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} dq \left(\frac{z^-}{z^+} \right)^2 \sum_b (-1)^{F_b} \left[S_{Q-1}(z^\pm, x_i^\pm) S_{Q-1}(z^\pm, x_{ii}^\pm) \right]_{b(11)}^{b(11)}$$

- What particles should circulate in the loop?
  - fundamental magnons ($Q = 1$)
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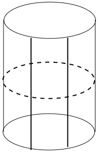

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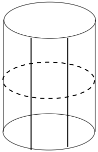

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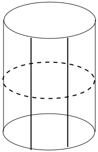

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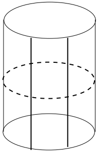

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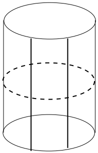

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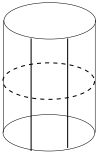

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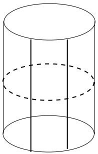

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$$\left(\frac{z^-}{z^+}\right)^2 = \frac{16 g^4}{(Q^2+q^2)^2} + \dots$$

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where

$$num(Q) = 7776\,Q(19683\,Q^{18} - 78732\,Q^{16} + 150903\,Q^{14} - 134865\,Q^{12} + \\ + 1458\,Q^{10} + 48357\,Q^8 - 13311\,Q^6 - 1053\,Q^4 + 369\,Q^2 - 10)$$

- Two last terms give at once $864\,\zeta(3) - 1440\,\zeta(5)$
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- Several new features appear...
- Due to the special form of kinematics already an *infinite* set of coefficients of the BES dressing phase starts to contribute
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- The agreement of the Konishi computation with the 4-loop weak coupling perturbative gauge theory result is an extremely nontrivial test of AdS/CFT!
- The computation of the finite size effects through Lüscher corrections is of a distinctly (2D) quantum field theoretical nature – so string theory is essential here
- The AdS/CFT correspondence allows to use the methods of integrable *two-dimensional* quantum field theories for the study of a *four-dimensional* gauge theory!!
- The result came from a single diagram – in contrast to direct perturbative computations in gauge theory which are much more complex
- This suggests that one can use string theory methods of AdS/CFT as an efficient calculational tool also at *weak coupling*
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