On dark mesonic realization of the SIMP scheme and dark photon

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Introduction

- In this talk, we will discuss how much can we extract information on dark sector by observing communication between the dark sector and the Standard Model (SM) sector through U(1) kinetic mixing, for specific dark matter (DM) model, dark mesons in the SIMP scheme.
- This DM model makes use of Wess-Zumino-Witten (WZW) term for freeze-out process.
- Whereas the WZW term is very well known, we have mainly studied it in terms of the SM setup: SU(3) gauge group, three light quark flavors.
- When we apply the WZW term to dark sector, gauge group and flavor structure different from the SM can be chosen, which results in nontrivial DM phenomenology.
- We will focus on such interplay between theoretical concept and phenomenological application.

Contents

- Brief review on the WZW term
- SIMP scheme and its realization in the dark meson system
  : why do we need inter-sector interaction?
- U(1) kinetic mixing
  : viable parameter range of dark photon mass and kinetic mixing parameter
- The issue of chiral perturbativity
  : another 5-point self-interaction from gauged WZW term

Wess-Zumino-Witten term

- In the presence of confining force (with non-Abelian gauge symmetry) and quarks (fermions charged under it), below some scale, quark confinement occurs: quarks cannot move independently, but exist as components of 'confinement states' (mesons/baryons)
- As a phenomenological description for light mesons, composed of 'light quark' (quark mass $< \Lambda$) and its anti-quark, chiral perturbation theory is developed.

S. Weinberg, Physica 96A (1979) 327

J. Gasser, H. Leutwyler, Nucl. Phys. B250 (1985) 465; Ann. Phys. 158 (1984) 142

- Example : SM QCD

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \overline{q}_L i\not{D} q_L + \overline{q}_R i\not{D} q_R \qquad q_{L/R} = (u,d,s)^T_{L/R}$$

- Chiral symmetry

$$\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \qquad q_L \to L q_L, \quad q_R \to R q_R, \quad L, R \in \mathrm{SU}(3)$$

- As a result of quark confinement, quark bilinears break chiral symmetry:

$$\langle \overline{q}_i q_j \rangle = -\Lambda^3 \delta_{ij}, \qquad \langle \overline{q}_i \gamma_5 q_j \rangle = 0$$

- VEVs are chosen to preserve SU(3) with L=R

$$\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \xrightarrow{\text{S.S.B}} \mathrm{SU}(3)_V$$

- As a result, 8 Goldstone bosons appear (due to explicit breaking terms such as quark mass term, they become pseudo-Goldstone bosons), represented by SU(3) valued fluctuation around the vacuum:

$$U(x) = e^{2i\sum_a \lambda_a \xi_a(x)} \qquad \sum_a \lambda_a \xi^a = \frac{\sqrt{2}}{F}\begin{bmatrix} \frac{1}{\sqrt{2}}\pi_0 + \frac{1}{\sqrt{6}}\eta^0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi_0 + \frac{1}{\sqrt{6}}\eta^0 & K^0 \\ K^- & \overline{K}^0 & -\sqrt{\frac{2}{3}}\eta^0 \end{bmatrix}$$

$$\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \qquad U \to L U R^\dagger$$

- For energy scale well below Λ, where baryons are integrated out, we have an effective field theory for light mesons,

$$\mathcal{L} = \frac{F^2}{16}\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \Lambda^4 \mathcal{O}\left(\frac{p^4}{\Lambda^4}\right) + \cdots$$

- Question: How do we know the parity of mesons?

From interaction with baryons, $\mathcal{L}_{\text{int}} = \pi \overline{N} \gamma_5 N + \cdots$

we know mesons are parity odd, i.e. under $t \to t, \quad \vec{x} \to -\vec{x}, \qquad \pi \to -\pi$

Then how do we know at the energy scale where baryons are integrated out?

E. Witten, Nucl. Phys. B223 (1983) 422

The leading Lagrangian term has 3 symmetries,

1. $U \to U^T$ (charge conjugation)
2. $t \to t, \quad \vec{x} \to -\vec{x}, \quad U \to U$
3. $U \to U^\dagger$

but actual QCD symmetry is a combination of 2 and 3

- In the equations of motion,

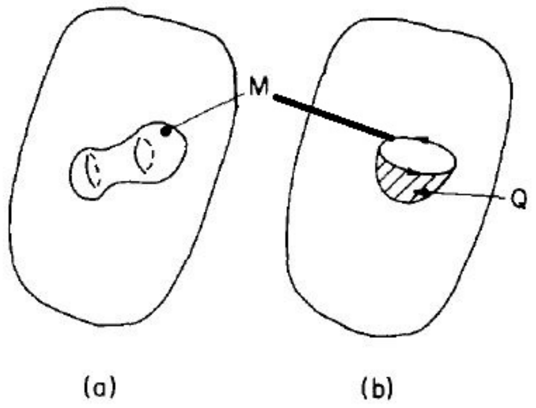
$$\partial_\mu\left(\frac{F^2}{8}U^\dagger\partial^\mu U\right) + \lambda\epsilon_{\mu\nu\rho\sigma}U^\dagger\partial^\mu UU^\dagger\partial^\nu UU^\dagger\partial^\rho UU^\dagger\partial^\sigma U = 0$$

the lowest order interaction showing P oddness of mesons is given by the second term.

This comes from the 'WZW term‘ in the Lagrangian,

$$n\Gamma_Q = n\int_Q \frac{i}{240\pi^2}d\Sigma^{ijklm}\mathrm{Tr}[U^\dagger\partial_i UU^\dagger\partial_j UU^\dagger\partial_k UU^\dagger\partial_l UU^\dagger\partial_m U]$$

where Q is the 5-dimensional 'surface' in the SU(3) group space, bounded by 4-dimensional spacetime embedding.



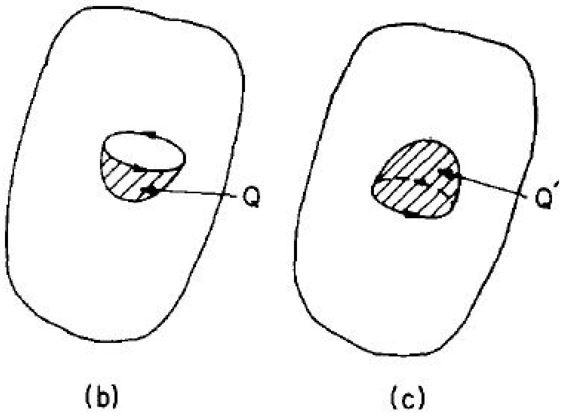
E. Witten, Nucl. Phys. B223 (1983) 422

- In the SU(3) space, the spacetime embedding bounds two different 5-surfaces, so the WZW term becomes physically meaningful only when

$$e^{in\Gamma_Q} = e^{in\Gamma_{Q'}},$$

$$\Longrightarrow \int_{Q+Q'} d\Sigma^{ijklm}\mathrm{Tr}[U^\dagger\partial_i UU^\dagger\partial_j UU^\dagger\partial_k UU^\dagger\partial_l UU^\dagger\partial_m U] = 2\pi,$$

n : integer



where $Q + Q'$ is a closed 5-surface.

E. Witten, Nucl. Phys. B223 (1983) 422

Mathematically, non-vanishing of the WZW term and existence of integer mode have a topological origin, nontrivial fifth homotopy group.

$$\pi_5(SU(3)) = \mathbb{Z}$$

- In general, given gauge group G_c, when flavor symmetry G is spontaneously broken down to its subgroup H,

H does not have an additional spontaneous breaking if it is respected by the quark mass term.

C. Vafa, E. Witten,Nucl. Phys. B234 (1984)173; Comm. Math. Phys. 95 (1984)257

For degenerate quarks,

E. Witten, Nucl. Phys. B223 (1983) 433

1) SU(N_c) gauge group : quarks are in the complex representation (distinction between quark and anti-quark). quark mass term has the form of $(i, j = 1, \cdots, N_f)$

$$m_{ij}\overline{q}_i q_j$$

For degenerate quarks which we will be interested in, $m_{ij} = m\delta_{ij}$

H= SU(N_f) is unbroken spontaneously more.

Interestingly, with G=SU(N_f)×SU(N_f), G/H= SU(N_f) and

$$\pi_5(G/H) = \pi_5(\mathrm{SU}(N_f)) = \mathbb{Z} \qquad (N_f \geq 3)$$

2) The same situation can be found for other possible gauge groups : SO(N_c) or Sp(N_c).

In these cases, quarks are real and pseudoreal, respectively, so no dictinction between quarks and anti-quarks:

$$T_a = -T_a^T, \qquad T_a = -JT_a^T J, \quad (J = i\sigma_2 \otimes \mathbb{I})$$

quark mass term is written as

$$m^{(rs)(ij)} q^{\alpha}_{r,i} q_{\alpha s,j} + \mathrm{h.c.}$$

$\alpha, \beta, \cdots$: spinor indices

$r, s, \cdots$: gauge multiplet indices

$i, j, \cdots$: gauge multiplet indices

The degenerate mass term,

$$m^{(rs)(ij)} = m\delta^{rs}\delta^{ij}, \quad \mathrm{SO(N_c)}$$

$$m^{(rs)(ij)} = mJ^{rs}J^{ij}, \quad \mathrm{Sp(N_c)}$$

respects SO(N_f) (SO(N_c) case) / Sp($2N_f$) (Sp(N_c) case) flavor group, and also

$$\pi_5(G/H) = \pi_5(\mathrm{SU}(N_f)/\mathrm{SO}(N_f)) = \quad \mathbb{Z} \qquad (N_f \geq 3)$$

$$\pi_5(\mathrm{SU}(2N_f)/\mathrm{Sp}(2N_f)) \qquad (N_f \geq 2)$$

- Topological index n is determined by the gauged WZW term: under electromagnetic interaction, quark charges are given by Q=diag. (2/3, -1/3, -1/3) and the gauged WZW term is given by

$$\Gamma_Q - e\int d^4x A_\mu J^\mu + \frac{ie^2}{24\pi^2}\int d^4x \epsilon^{\mu\nu\rho\sigma}\partial_\mu A_\nu A_\rho \mathrm{Tr}[Q^2\partial_\sigma U U^\dagger + Q^2 U^\dagger \partial_\sigma U + QUQU^\dagger \partial_\sigma U U^\dagger]$$

$$J^\mu = \frac{1}{48\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}[Q\partial_\nu U U^\dagger \partial_\rho U U^\dagger \partial_\sigma U U^\dagger + QU^\dagger \partial_\nu U U^\dagger \partial_\rho U U^\dagger \partial_\sigma U]$$

The first term gives AAAV anomaly, ($\gamma\pi^+\pi^0\pi^-$ vertex)

The second term gives AVV anomaly ($\pi^0\gamma\gamma$ vertex)

W. A. Bardeen, Phys. Rev. 184 (1969) 1848

so we can match the coefficients : $n = N_c$

- Another approach (J.Wess, B. Zumino, Phys. Lett.B37(1971) 95)

If G=SU(3)×SU(3) were gauged, broken symmetry (G/H) is anomalous.

"The gauged effective field theory of the Goldstone bosons must have an anomaly for the fictitious local symmetries which is equal to that produced by the trapped fermions in the underlying theory" (S. Weinberg, QFT II)

$$\begin{aligned}\mathcal{T}_a(x)\Gamma[\xi, A] &= G_a[x, A] \\ &= \frac{i}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}[T^a(\partial_\mu A_\nu \partial_\rho A_\sigma - \frac{i}{2}(\partial_\mu A_\nu A_\rho A_\sigma - A_\mu \partial_\nu A_\rho A_\sigma + A_\mu A_\nu \partial_\rho A_\sigma))]\end{aligned}$$

$$A_\mu = A_\mu^a T^a$$

SIMP dark mesons

- As a phenomenological application, we consider the suggestion that the WZW term is responsible for freeze-out of dark sector meson.
- SIMP (Strongly Interacting Massive Particle) scheme :

  (pseudo-) scalar thermal DM

  $3 \rightarrow 2$ self-annihilation is used to freeze-out

E. D. Carlson, M. E. Machacek, L. J. Hall, ApJ 398 (1992) 43

Y. Hochberg, E. Kuflik, T. Volansky, J. G. Wacker, Phys.Rev. Lett. 113 (2014) 171301

DM self interaciton is motivated by small scale problems of cold DM (CDM) model

- core-cusp problem : inconsistency in DM density profile at the center of galaxy
- too-big-to-fail problem : overabundance of massive, dense subhalos in CDM prediction

but also strongly restricted to be small (bullet cluster)

$$\frac{\sigma_{\rm scattering}}{m_{\rm DM}} \leq 1{\rm cm}^2{\rm g}^{-1}$$

- At the moment of freeze-out, $\frac{m_{\rm DM}}{T_f} \sim 20$

$$n_{\rm DM}^2 \langle \sigma v^2 \rangle (T_f) = H(T_f)$$

$$n_{\rm DM} = \frac{\rho_{\rm DM}}{m_{\rm DM}} = \left(\frac{\xi}{1+\xi}\frac{3}{4}\frac{g_*(T_{\rm eq})}{g_{*S}(T_{\rm eq})}T_{\rm eq}\right)\frac{s}{m_{\rm DM}} \equiv c\frac{T_{\rm eq}s}{m_{\rm DM}}$$

$$\xi = \frac{\rho_{\rm DM}}{\rho_{\rm baryon}}, \qquad s = \frac{2\pi}{45}g_{*S}(T_{\rm eq})T^3$$

$$\rho_m = \rho_{\rm eq}\left(\frac{T}{T_{\rm eq}}\right)^3 = \left(\frac{\pi^2}{30}g_*(T_{\rm eq})T_{\rm eq}^4\right)\left(\frac{T}{T_{\rm eq}}\right)^3 = \left(\frac{3}{4}\frac{g_*(T_{\rm eq})}{g_{*S}(T_{\rm eq})}T_{\rm eq}\right)s$$

$$\langle \sigma v^2 \rangle = \Big(\prod_{i=1,2,3}\frac{1}{2E_i}\Big)\Big(\prod_{f=1,2}\frac{d^3p_f}{(2\pi)^3 2E_f}\Big)|\mathcal{M}|^2(2\pi)^4\delta^4(P_i - P_f) \sim \frac{\alpha_{\rm eff}^2}{m_{\rm DM}^5}$$

$$H = 0.33 g_*^{1/2}\frac{T^2}{m_{\rm pl}}$$

$$m_{\rm DM} \sim \alpha(T_{\rm eq}^2 m_{\rm pl})^{1/3} \sim \text{GeV}$$

- Challenge : as a result of $3 \to 2$ self annihilation, two final DMs get hotter as freeze-out proceeds.

e.g. If three initial DMs are almost non-relativistic,

$$E_{\rm CM} = 3m_{\rm DM} \qquad \Longrightarrow \qquad E_f = \frac{3}{2}m_{\rm DM} \quad \text{for each final DM}$$

This can spoil structure formation.

- Resolution:

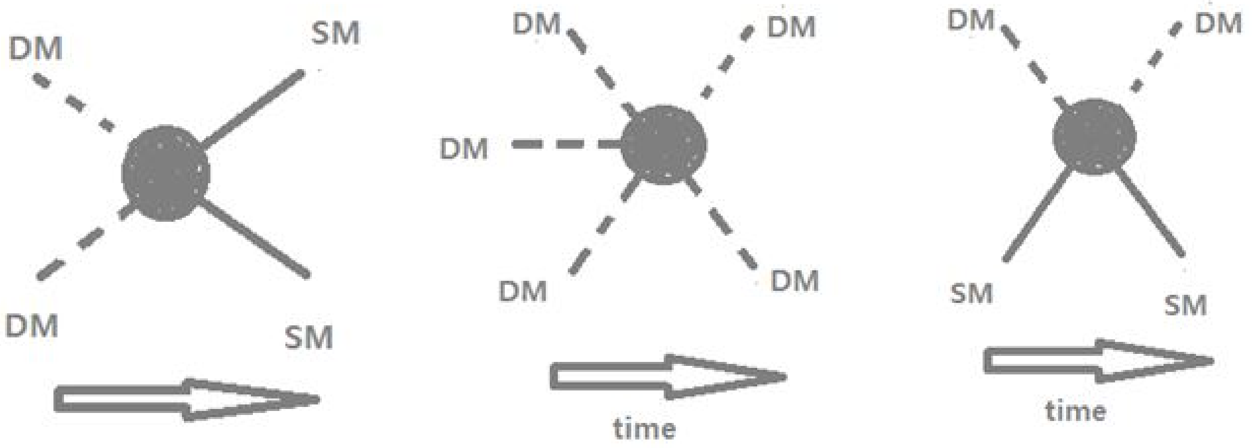
DM sector is not thermally isolated, but has a thermal contact with the SM sector :

Two sectors are in kinematic equilibrium, and DM can send its remaining thermal energy to the SM sector.

Phenomenologically interesting as it requires sizable interaction between DM and SM sectors.

- Such an inter-sector interaction includes another annihilation channel, so it is required to be smaller than 3 → 2 annihilation rate.
- Inter-sector interaction for thermal contact is dominated by scattering of DM off the SM particles in thermal bath.

$$n_{\rm DM}\langle\sigma v\rangle_{\rm ann} < n_{\rm DM}^2\langle\sigma v^2\rangle_{3\to2} < n_{\rm SM}\langle\sigma v\rangle_{\rm scatt},$$



- Dark mesons in SIMP scheme Y. Hochberg, E. Kuflik, H. Murayama, T. Volansky, J. G. Wacker, Phys. Rev. Lett. 115 (2015) 021301

(Degenerate) dark meson is a good example of self-interacting DM.

Since leading term in the WZW term is a 5-point interaction, it can be used to 3 → 2 interaction in the SIMP scheme.

From

$$S_{\mathrm{WZW}} = -\frac{iN_c}{240\pi^2}\int d\Sigma^{ijklm}\mathrm{Tr}[U^{-1}\partial_i U U^{-1}\partial_j U U^{-1}\partial_k U U^{-1}\partial_l U U^{-1}\partial_m U]$$
$$= \frac{N_c}{240\pi^2}\int d^4x \epsilon^{\mu\nu\rho\sigma}\pi^a\partial_\mu\pi^b\partial_\nu\pi^c\partial_\rho\pi^d\partial_\sigma\pi^e \mathrm{Tr}(T_aT_bT_cT_dT_e) + \cdots .$$

we have

$$\langle\sigma v^2\rangle_{3\to 2} = \frac{5\sqrt{5}N_c^2 m_\pi^5}{2\pi^5 F^{10}}\frac{t^2}{N_\pi^3}\left(\frac{T_F}{m_\pi}\right)^2$$

- On the other hand, leading 4-point interaction

$$-\frac{1}{6F^2}\mathrm{Tr}(\xi^2\partial\xi_\mu\partial^\mu\xi - \xi\partial_\mu\xi\xi\partial^\mu\xi) \quad \in \quad \frac{F^2}{16}\mathrm{Tr}(\partial_\mu U\partial^\mu U^\dagger)$$

gives 2 → 2 scattering rate,

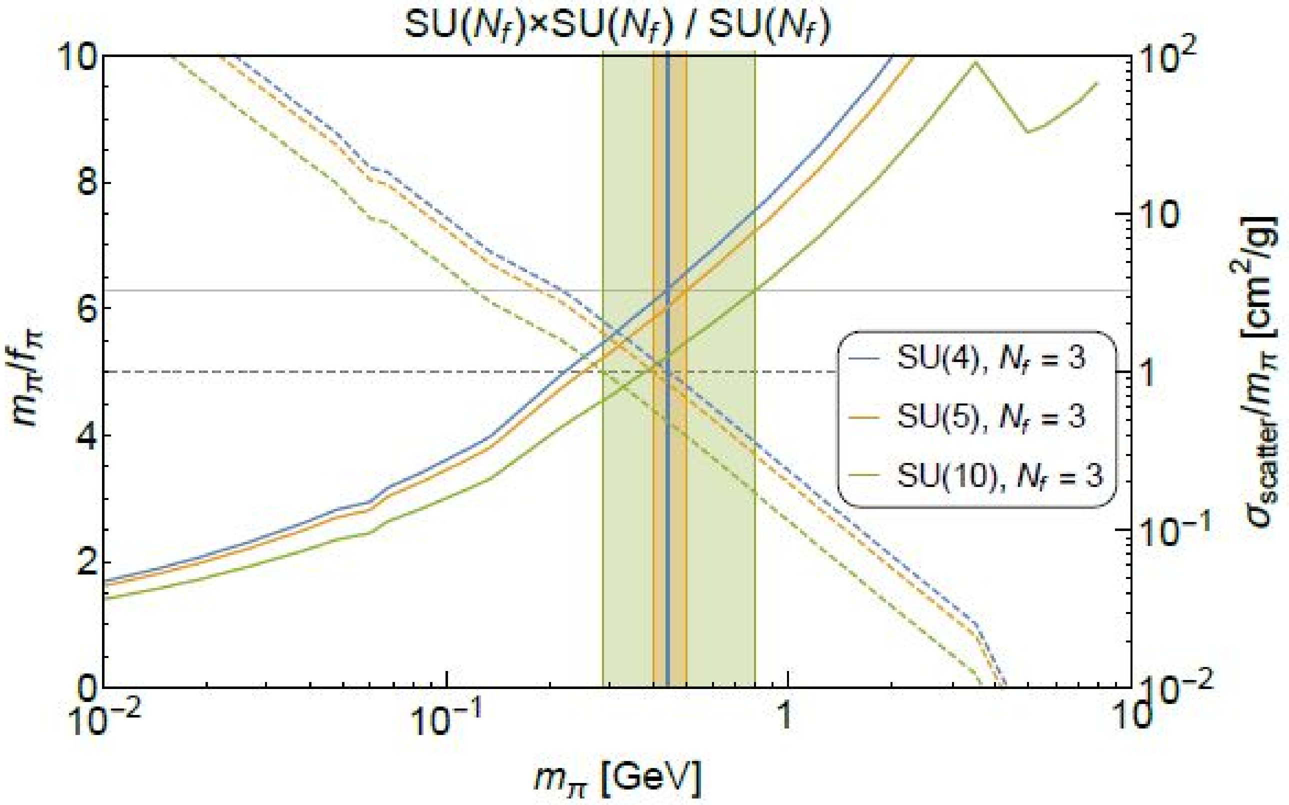
$$\sigma_{\mathrm{self}} = \frac{m_\pi^2}{32\pi F^4}\frac{a^2}{N_\pi^2},$$

which will be restricted to be

$$\frac{\sigma_{\mathrm{scattering}}}{m_{\mathrm{DM}}} \leq 1\mathrm{cm}^2\mathrm{g}^{-1}$$

G_c	G_f/H	N_π	t^2	$N_f^2a^2$
$\mathrm{SU}(N_c)$	$\frac{\mathrm{SU}(N_f)\times\mathrm{SU}(N_f)}{\mathrm{SU}(N_f)}$ $(N_f \geq 3)$	N_f^2-1	$\frac{4}{3}N_f(N_f^2-1)(N_f^2-4)$	$8(N_f-1)(N_f+1)(3N_f^4-2N_f^2+6)$
$\mathrm{SO}(N_c)$	$\mathrm{SU}(N_f)/\mathrm{SO}(N_f)$ $(N_f \geq 3)$	$\frac{1}{2}(N_f+2)(N_f-1)$	$\frac{1}{12}N_f(N_f^2-1)(N_f^2-4)$	$(N_f-1)(N_f+2)(3N_f^4+7N_f^3-2N_f^2-12N_f+24)$
$\mathrm{Sp}(N_c)$	$\mathrm{SU}(2N_f)/\mathrm{Sp}(2N_f)$ $(N_f \geq 2)$	$(2N_f+1)(N_f-1)$	$\frac{2}{3}N_f(N_f^2-1)(4N_f^2-1)$	$4(N_f-1)(2N_f+1)(6N_f^4-7N_f^3-N_f^2+3N_f+3)$

- Typically, (Y. Hochberg, E. Kuflik, H. Murayama, T. Volansky, J. G. Wacker, Phys. Rev. Lett. 115 (2015) 021301)



$$n_{\rm DM}^2 \langle \sigma v^2 \rangle (T_f) = H(T_f)$$

$$\text{gives} \quad N_c^2 \left(\frac{m_\pi}{F} \right)^{10} \frac{1}{m_\pi^3} = \text{const.}$$

$$\frac{\sigma_{\rm scattering}}{m_{\rm DM}} \leq 1 {\rm cm}^2 {\rm g}^{-1}$$

Completion of the model:
Putting interaction between dark- and SM sector

- In order to avoid the DM overheating, inter-sector interaction between DM- and SM sector is required.
- As a renormalizable interaction, one can think of Higgs portal and U(1) gauge kinetic mixing.
- Higgs portal is not satisfactory: At freeze-out temperature (10~ 100MeV), only light particles (electron, neutrino, photon) are relativistic in thermal bath : The electron Yukawa coupling is too small for enough inter-sector scattering rate.
- This motivates considering U(1) kinetic mixing between two sectors.

H. M. Lee, **MSS**, Phys. Lett. B748 (2015) 316

Completion of the model: Putting U(1) kinetic mixing between dark- and SM sector

- First step: introducing U(1) gauge interaction in the dark sector.

For SU(N_c) gauge group, Dirac mass term is allowed, so dark quarks can be vector-like under dark U(1)$_D$.

Question: Can U(1)$_D$ unbroken?

Answer: No. Unbroken U(1)$_D$ gives too many phenomenological problems.

$$\Gamma_Q - e\int d^4x A_\mu J^\mu + \frac{ie^2}{24\pi^2}\int d^4x \epsilon^{\mu\nu\rho\sigma}\partial_\mu A_\nu A_\rho \mathrm{Tr}[Q^2\partial_\sigma U U^\dagger + Q^2 U^\dagger \partial_\sigma U + QUQU^\dagger \partial_\sigma U U^\dagger]$$

$$J^\mu = \frac{1}{48\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}[Q\partial_\nu U U^\dagger \partial_\rho U U^\dagger \partial_\sigma U U^\dagger + QU^\dagger \partial_\nu U U^\dagger \partial_\rho U U^\dagger \partial_\sigma U]$$

1. AVV anomalies: dark meson decay (similar to pion (or η meson) decay into two photons) : forbidden by an appropriate charge assignment

For SU(3) flavor group case, Q=diag.(q_1, q_2, q_3)

$$\mathrm{Tr}(T_3 Q^2) = 0 \quad \Longrightarrow \quad q_1^2 - q_2^2 = 0$$

$$\mathrm{Tr}(T_8 Q^2) = 0 \quad \Longrightarrow \quad q_1^2 + q_2^2 - q_3^2 = 0$$

2. AAAV anomalies: another self-annihilation channel (similar to two pion annihilation into a pion and a photon) : cannot be forbidden by a charge assignment.

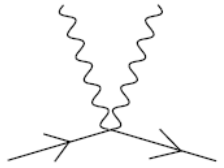
$$\mathrm{Tr}(T_{3,8}\frac{1}{2}[T_1 + iT_2, T_1 - iT_2]Q) = 0 \quad \Longrightarrow \quad q_1 \pm q_2 = 0$$

$$\mathrm{Tr}(T_{3,8}\frac{1}{2}[T_4 + iT_5, T_4 - iT_5]Q) = 0 \quad \Longrightarrow \quad q_1 = q_3 = 0$$

$$\mathrm{Tr}(T_{3,8}\frac{1}{2}[T_6 + iT_7, T_6 - iT_7]Q) = 0 \quad \Longrightarrow \quad q_2 = q_3 = 0$$

gauge coupling cannot be made arbitrarily small for kinetic equilibrium with SM sector.

Conclusion: dark photon mass is larger than dark meson mass. (It also forbids two dark mesons decay into two dark photon through the tree level seagull diagram.)



- For SO(N_c) or Sp(N_c) gauge group, dark U(1) is compatible with dark quark mass term only if it is spontaneously broken so that dark quark mass is provided by $U(1)_D$ Higgs mechanism.

Challenges in charge assignment:

1) Free of gauge anomalies(G_c-G_c-$U(1)_D$, $U(1)_D$-$U(1)_D$-$U(1)_D$)

2) absence of AVV anomaly (Even kinematically forbidden, dark meson still can decay into SM particles through kinetic mixing)

3) absence of linear coupling between dark mesons and dark photon through PCAC (also induce dark meson decay)

$$F\partial_\mu \pi V^\mu$$

we may introduce another gauge interaction.

- As the simplest case for phenomenology, we consider SU(N_c) with flavor symmetry

$$\mathrm{SU}(3)\times\mathrm{SU}(3)/\mathrm{SU}(3)$$

with U(1)$_D$ under which dark quarks are charged in the vector-like way.

-Anomaly free so we don't need to introduce additional gauge symmetry or heavy quarks

- U(1)$_D$ is broken, but it is not relevant to light dark quark mass.

AVV anomalies are absent by the charge assignment Q=diag. (1, -1, -1)

$$\mathcal{L}_{D\mathrm{int}} = - i2g_D(\partial_\mu\tilde{K}^+\tilde{K}^- - \tilde{K}^+\partial_\mu\tilde{K}^- + \partial_\mu\tilde{\pi}^+\overline{\tilde{\pi}}^- - \tilde{\pi}^+\partial_\mu\overline{\tilde{\pi}}^-)V^\mu$$
$$+ 4g_D^2(\tilde{K}^+\tilde{K}^- + \tilde{\pi}^+\tilde{\pi}^-)V_\mu V^\mu.$$

$$D_\mu U = \partial_\mu U + ig_D[Q_D, U]V_\mu.$$

- Gauge kinetic mixing between dark U(1)D and SM hypercharge U(1)

$$\mathcal{L}_{\mathrm{U(1)_D}} = -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{\sin\chi}{2}V_{\mu\nu}B^{\mu\nu} + \frac{1}{2}m_V^2 V_\mu V^\mu.$$

After diagonalizing gauge kinetic and mass terms by

$$\begin{bmatrix} B_\mu \\ W_\mu^3 \\ V_\mu \end{bmatrix} = \begin{bmatrix} c_W & -s_W c_\zeta + t_\chi s_\zeta & -s_W s_\zeta - t_\chi c_\zeta \\ s_W & c_W c_\zeta & c_W s_\zeta \\ 0 & -\frac{s_\zeta}{c_\chi} & \frac{c_\zeta}{c_\chi} \end{bmatrix} \begin{bmatrix} A_\mu \\ Z_\mu \\ A'_\mu \end{bmatrix},$$

where

$$\tan 2\zeta = \frac{m_Z^2 s_W \sin 2\chi}{m_V^2 - m_Z^2(c_\chi^2 - s_W^2 s_\chi^2)},$$

- Mass eigenvalues are given by 0 (for photon) and

$$m_\pm^2 = \frac{1}{2}\left[m_Z^2(1 + s_W^2 t_\chi^2) + \frac{m_V^2}{c_\chi^2} \pm \sqrt{\left(m_Z^2(1 + s_W^2 t_\chi^2) + \frac{m_V^2}{c_\chi^2}\right)^2 - \frac{4}{c_\chi^2} m_Z^2 m_V^2}\right].$$

$m_+^2 \simeq m_Z^2$ and $m_-^2 \simeq m_V^2$ in the $\chi \to 0$ limit.

- In terms of diagonalized basis,

$$\mathcal{L}_{\rm int} = A_\mu J^\mu_{\rm EM} + Z_\mu \left[(c_W s_\zeta t_\chi) J^\mu_{\rm EM} + (c_\zeta - s_W t_\chi s_\zeta) J^\mu_Z - \frac{s_\zeta}{c_\chi} J^\mu_D \right]$$
$$+ A'_\mu \left[(-c_W c_\zeta t_\chi) J^\mu_{\rm EM} + (s_\zeta + s_W t_\chi c_\zeta) J^\mu_Z + \frac{c_\zeta}{c_\chi} J^\mu_D \right].$$

$$\epsilon_\gamma \equiv c_W c_\zeta t_\chi, \qquad \epsilon_Z \equiv s_\zeta + s_W t_\chi c_\zeta.$$

$$\tan 2\zeta = \frac{m_Z^2 s_W \sin 2\chi}{m_V^2 - m_Z^2 (c_\chi^2 - s_W^2 s_\chi^2)},$$

For $m_V \ll m_Z, \quad \zeta \simeq -s_W \chi$

no leading Z-mixing

$m_V \simeq m_Z \quad \zeta \simeq (m_Z^2 t_W \epsilon_\gamma)/(m_V^2 - m_Z^2)$

Z-mixing enhanced

- Paramerters m_V and ϵ_γ

are restricted from above by ground-based experiments:

$$\text{For } 0.02\text{GeV} < m_V < 10.2\text{GeV}, \quad e^+e^- \to \gamma\gamma_D \to \gamma\ell^+\ell^- \quad (\text{BaBar}) \quad \epsilon_\gamma < 6\times10^{-4}$$

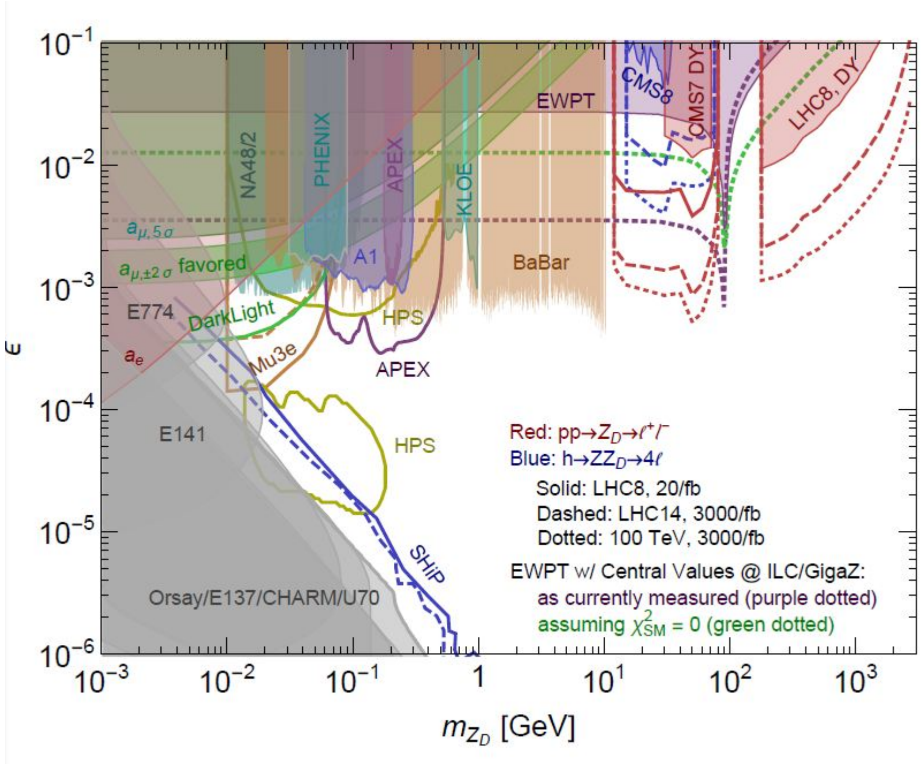
$$\text{For } 20\text{GeV} < m_V < 30\text{GeV}, \quad h \to Z\gamma_D \quad (\text{CMS8}) \quad \epsilon_\gamma < 5\times10^{-2}$$

$$\text{For } 30\text{GeV} < m_V < 70\text{GeV}, \ m_V > 10^2\text{GeV}, \quad \text{Drell}-\text{Yan } \gamma_D(\text{LHC8}) \quad \epsilon_\gamma < 10^{-2}$$

+ Electroweak precision test

$$\epsilon_\gamma \lesssim 2\times10^{-2} \quad \text{for} \quad 10\text{GeV} < m_V < 80\text{GeV}$$

$$\epsilon_\gamma \lesssim 2.5\times10^{-3} \quad \text{for} \quad m_V \simeq m_Z$$



D. Curtin, R.Essig, S.Gori, J. Shelton, JHEP1502 (2015)157

- Caution: bounds above are assuming dark photon decays into SM particles only.

(B. Batell, M. Pospelov, A. Ritz, Phys. Rev. D79 (2009) 115008)

But when $m_V > 2m_\pi$

dark photon decay channel into dark mesons opens : rescaling is required.

$$\mathrm{BR}(\gamma_D \to \mathrm{SM}_i) = \frac{\Gamma(\gamma_D \to \mathrm{SM}_i)}{\sum_i \Gamma(\gamma_D \to \mathrm{SM}_i) + \Gamma(\gamma_D \to \pi\pi)} \simeq \left(\frac{\epsilon_\gamma^2 \alpha}{\alpha_D}\right) \frac{\Gamma(\gamma_D \to \mathrm{SM}_i)}{\sum_i \Gamma(\gamma_D \to \mathrm{SM}_i)},$$

Since the experimental limits on the visible modes depend on

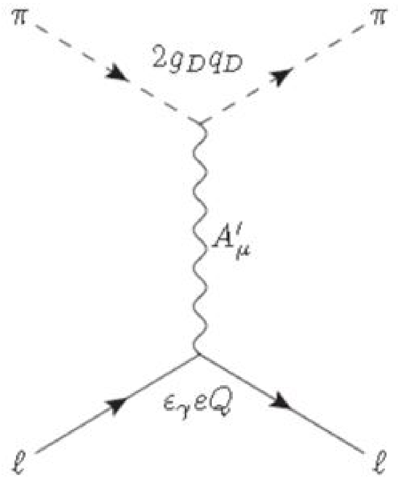
$$\epsilon_\gamma^2 \mathrm{Br}(\gamma_D \to e^+ e^-)$$

the bound on ϵ_γ gets weaker by a factor $[\alpha_D/(\epsilon_\gamma^2 \alpha)]^{1/2}$.

- On the other hand, requirement of consistent SIMP scheme,

$$n_{\rm DM}\langle\sigma v\rangle_{\rm ann} < n_{\rm DM}^2\langle\sigma v^2\rangle_{3\to 2} < n_{\rm SM}\langle\sigma v\rangle_{\rm scatt},$$

implements a lower bound as well as upper bound.



annihilation rate: $n_{\rm DM}\times[\mathcal{O}(10^2)\alpha\alpha_D\epsilon_\gamma^2 m_\pi^2/(N_\pi m_V^4)]$

$$\epsilon_\gamma \lesssim 0.05\left(\frac{N_c}{10}\right)\left(\frac{m_V}{10\,\text{GeV}}\right)\left(\frac{0.5\,\text{GeV}}{m_\pi}\right)^{5/2}.$$

- Scattering rate:

$$n_{\rm DM}^2\langle\sigma v^2\rangle_{3\to2} < n_{\rm SM}\langle\sigma v\rangle_{\rm scatt} = \sum_{\ell=e,\mu} n_\ell\langle\sigma v\rangle_{{\rm scatt},\ell} + n_\nu\langle\sigma v\rangle_{{\rm scatt},\nu} + n_{\pi_{\rm SM}}\langle\sigma v\rangle_{{\rm scatt},\pi},$$

$\pi+\ell\to\pi+\ell$

$$\langle\sigma v\rangle_{{\rm scatt},\ell} = \frac{4}{N_\pi}\cdot\left[192\epsilon_\gamma^2 + \frac{24(8s_W^4-4s_W^2+1)\epsilon_Z^2}{c_W^2 s_W^2}\right]\pi\alpha\alpha_D\frac{m_\pi^2}{m_V^4}\left(\frac{T_F}{m_\pi}\right),$$

$\pi+\nu\to\pi+\nu$

$$\langle\sigma v\rangle_{{\rm scatt},\nu} = \frac{4}{N_\pi}\cdot\frac{24\pi\alpha\alpha_D\epsilon_Z^2 m_\pi^2}{c_W^2 s_W^2 m_V^4}\left(\frac{T_F}{m_\pi}\right),$$

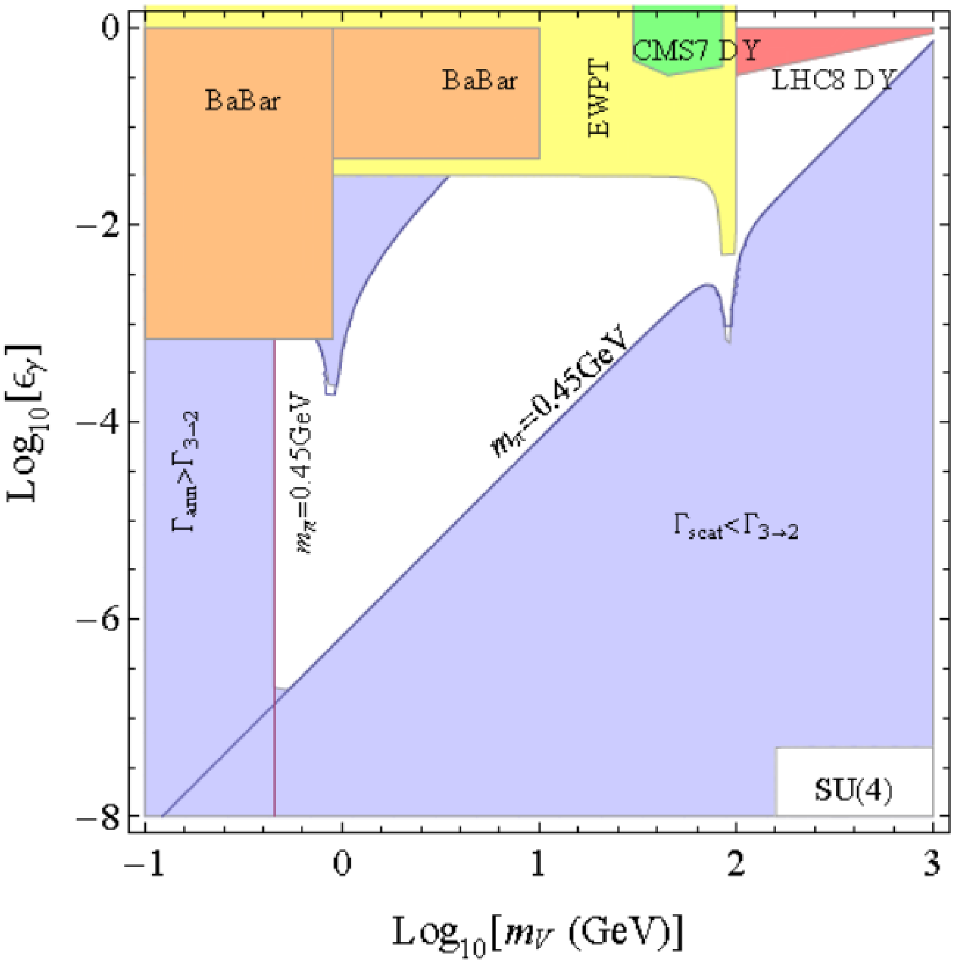
$\pi+\pi_{\rm SM}^\pm\to\pi+\pi_{\rm SM}^\pm$

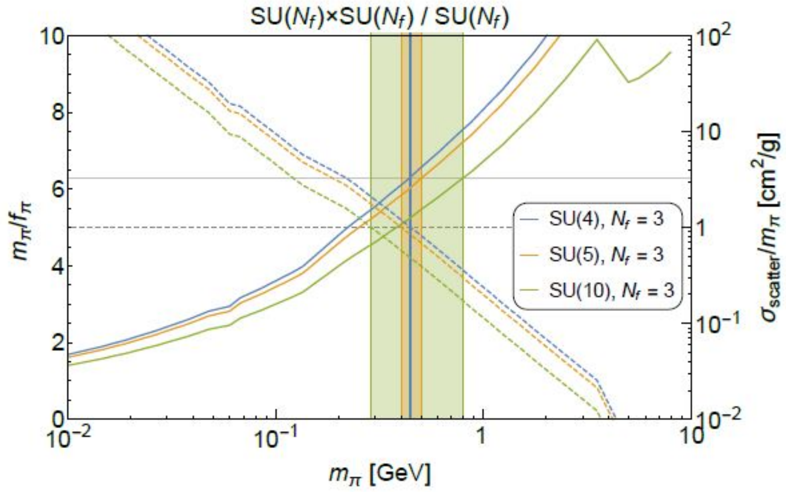
$$\langle\sigma v\rangle_{{\rm scatt},\pi} = \frac{4}{N_\pi}\cdot\left[192\epsilon_\gamma^2 + \frac{192\epsilon_Z^2}{c_W^2 s_W^2}\frac{1}{4}(1-2s_W^2)^2\right]\pi\alpha\alpha_D\frac{m_\pi^2}{m_V^4}\left(\frac{T_F}{m_\pi}\right).$$

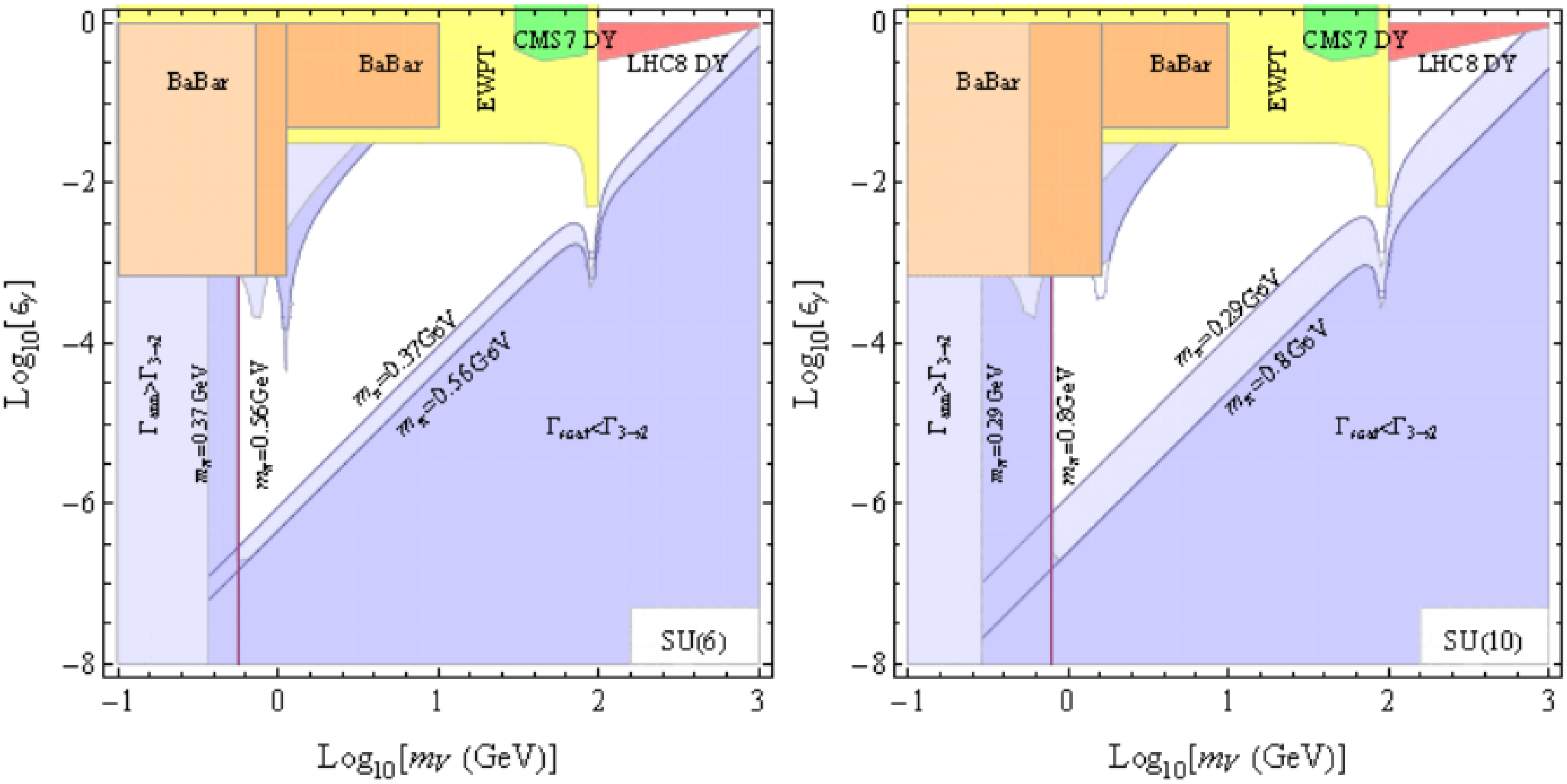
$$\alpha_D\epsilon_\gamma\left(\frac{m_\pi}{m_V}\right)^2 \gtrsim 10^{-8}.$$

H. M. Lee, M. –S. Seo, Phys. Lett. B748 (2015) 316

$\alpha_D=1/4\pi$







BaBar
BaBar
EWPT
CMS7 DY
LHC8 DY
$\Gamma_{ann} > \Gamma_{3\to2}$
m_π=0.37 GeV
m_π=0.56GeV
m_π=0.37GeV
m_π=0.56GeV
$\Gamma_{scat} < \Gamma_{3\to2}$
SU(6)
m_π=0.29 GeV
m_π=0.8GeV
m_π=0.29GeV
m_π=0.8GeV
SU(10)
$\mathrm{Log}_{10}[\epsilon_\gamma]$
$\mathrm{Log}_{10}[m_V\ (\mathrm{GeV})]$

Perturbativity issue (working on progress)

- Discussions so far has been made based on leading order calculations.
- However, in order that three particles meet together at the same space-time point, (= very small phase space) large coupling is required: As a result, perturbation parameter in chiral perturbation theory, m/F turns out to be quite large.
- Then, what happens if we take next-to-leading order calculations into account?

- M. Hensen, K. Lanhaeble, F. Sannino, arXiv:1507.01590

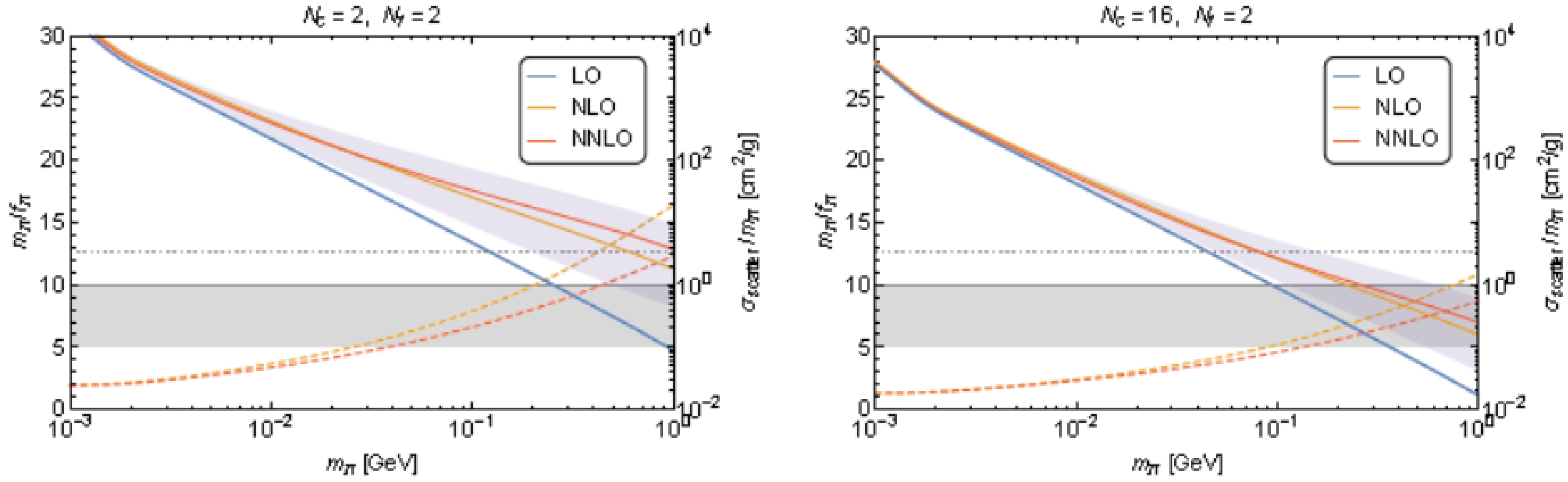
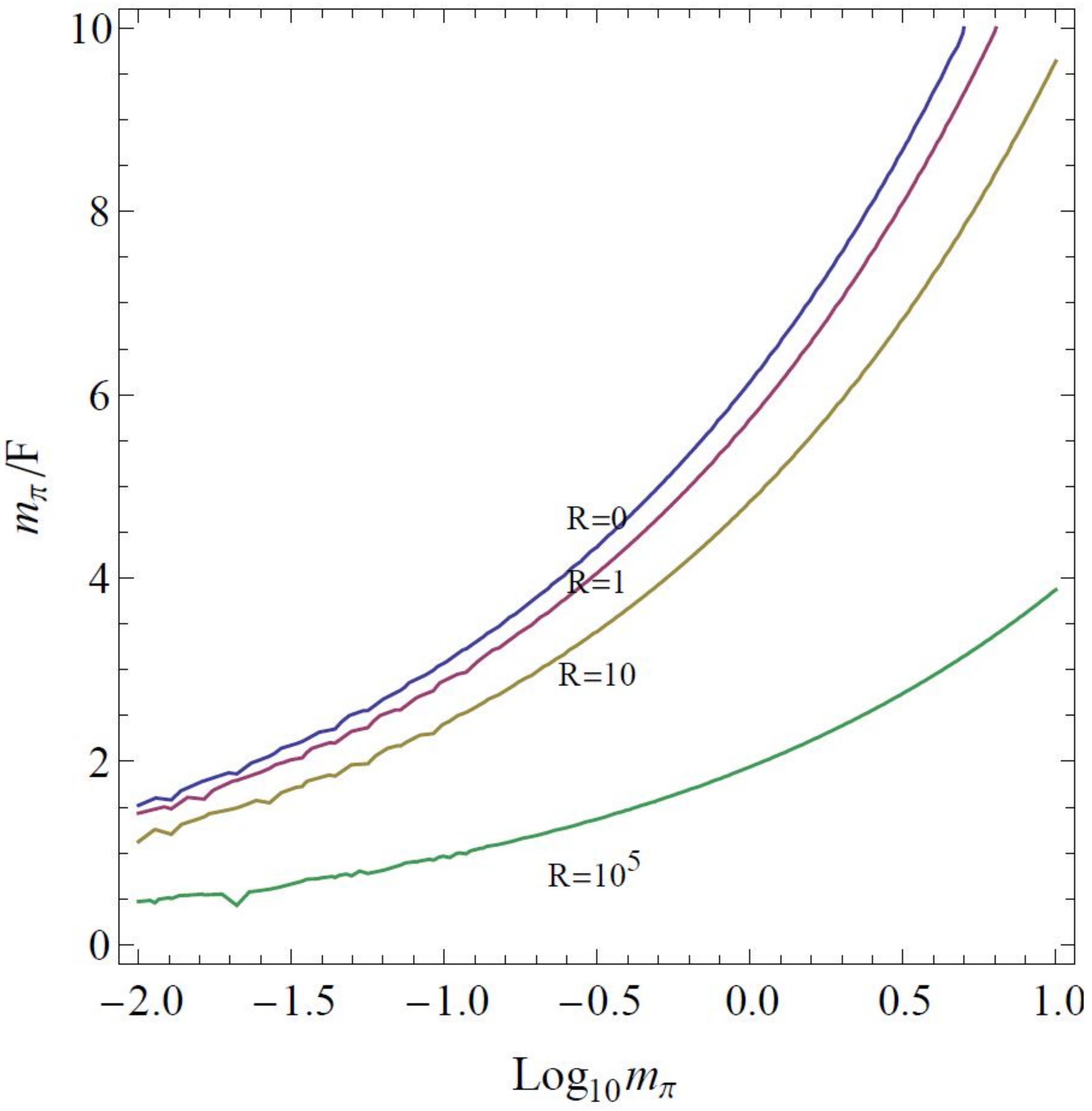


FIG. 2: *Dashed* lines belong to the left axis and *solid* lines to the right axis. The red dashed line is the NNLO solution m_π/f_π to the Boltzmann equation, the orange dashed is the NLO, and the dashed (grey) horizontal line is the upper perturbative limit $m_\pi/f_\pi = 4\pi$. The three solid lines are the cross section for the $2 \to 2$ self-interactions at LO (blue), NLO (orange) and NNLO (red). The purple band is the uncertainty from the low-energy constants (N-LECs). The solid grey band is the upper limit on the self-interactions.



R=0
R=1
R=10
R=10^5
m_π/F
$\mathrm{Log}_{10} m_\pi$

- Interestingly, the gauged WZW term provides another 5-point self interaction of dark mesons.

$$\Gamma_Q - e\int d^4x A_\mu J^\mu + \frac{ie^2}{24\pi^2}\int d^4x \epsilon^{\mu\nu\rho\sigma}\partial_\mu A_\nu A_\rho \mathrm{Tr}[Q^2\partial_\sigma U U^\dagger + Q^2 U^\dagger \partial_\sigma U + QUQU^\dagger \partial_\sigma U U^\dagger]$$

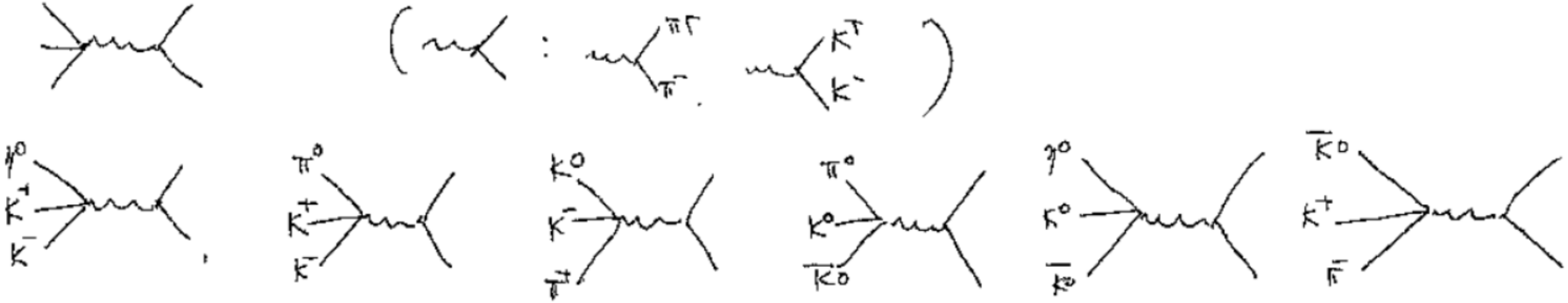
$$J^\mu = \frac{1}{48\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}[Q\partial_\nu U U^\dagger \partial_\rho U U^\dagger \partial_\sigma U U^\dagger + QU^\dagger \partial_\nu U U^\dagger \partial_\rho U U^\dagger \partial_\sigma U]$$

Second term provides 3 mesons-one dark photon interaction (AAAV)

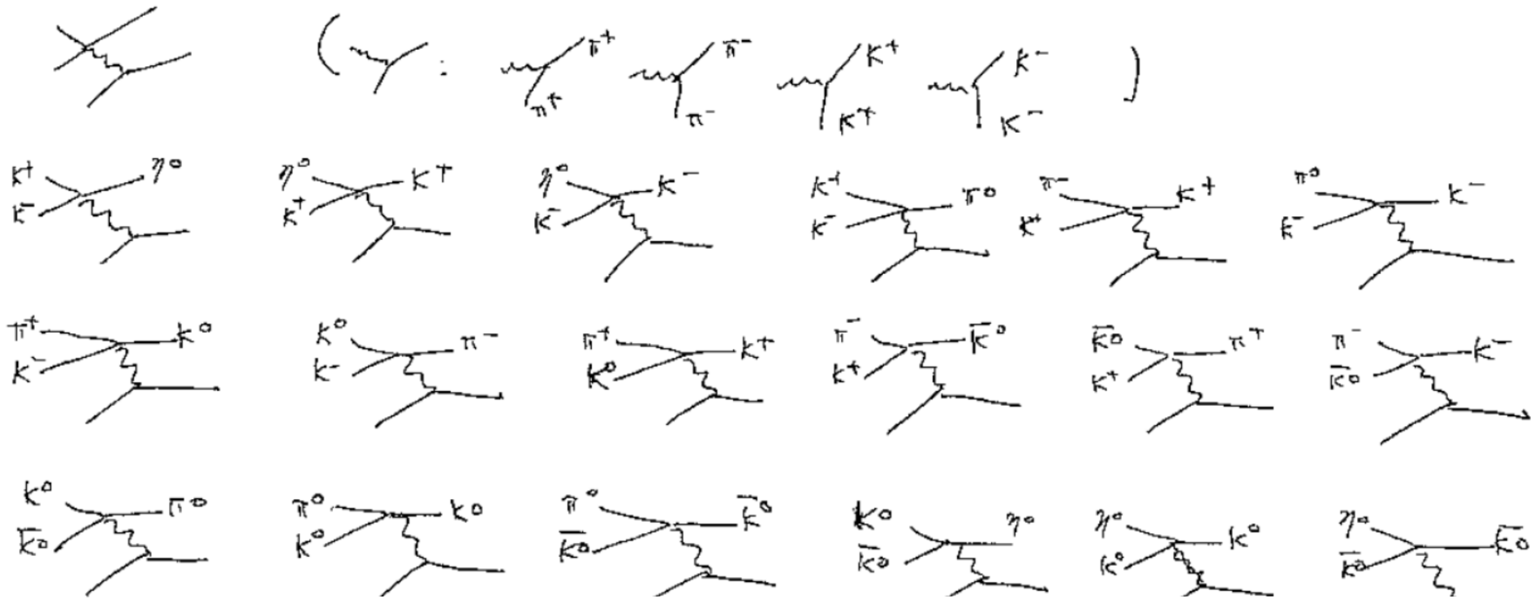
$$\begin{aligned}\mathcal{L}_{D\mathrm{int}} =& - i2g_D(\partial_\mu \tilde{K}^+\tilde{K}^- - \tilde{K}^+\partial_\mu \tilde{K}^- + \partial_\mu \tilde{\pi}^+\overline{\tilde{\pi}}^- - \tilde{\pi}^+\partial_\mu \overline{\tilde{\pi}}^-)V^\mu \\ &+ 4g_D^2(\tilde{K}^+\tilde{K}^- + \tilde{\pi}^+\tilde{\pi}^-)V_\mu V^\mu .\end{aligned}$$

- Minimal coupling provides 2-dark meson-one dark photon interaction

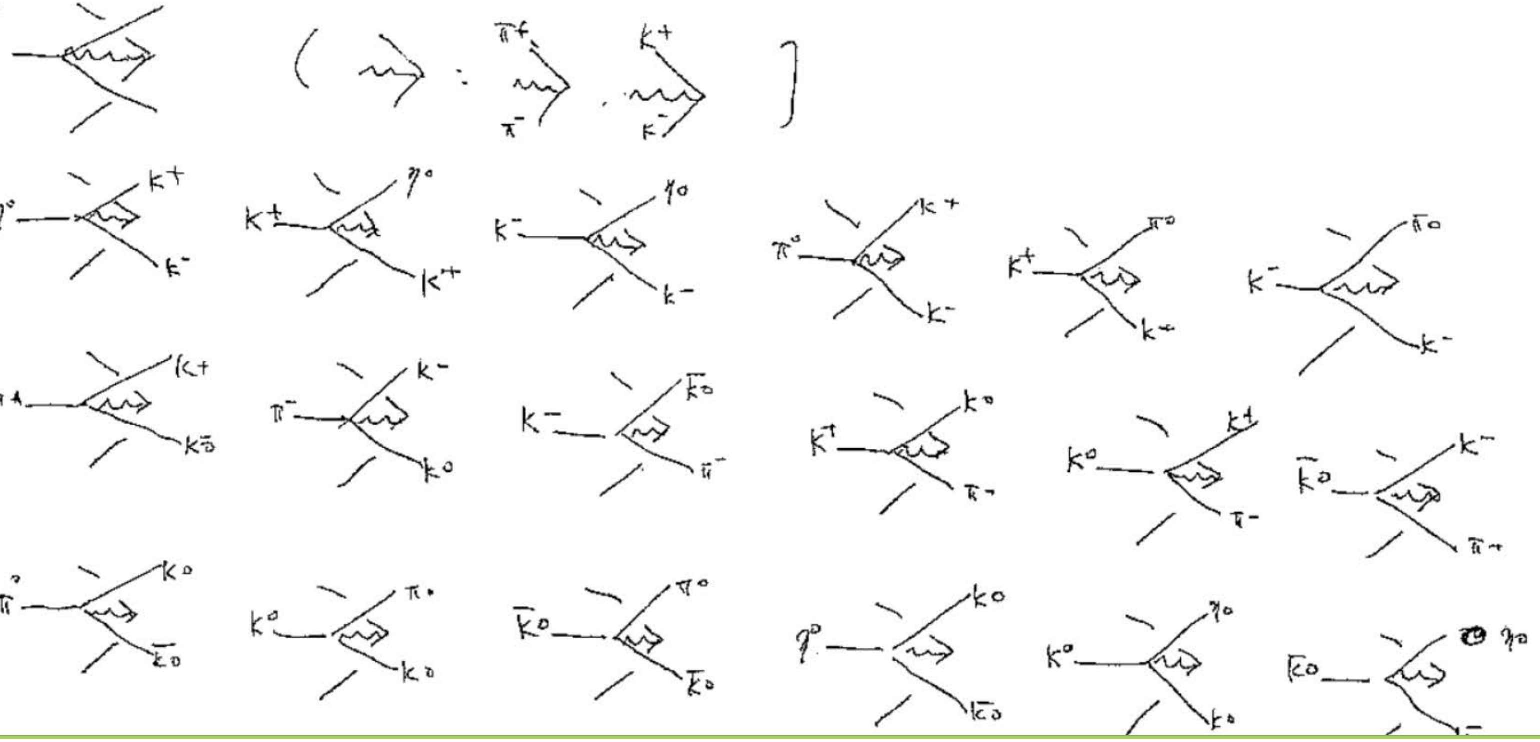
A



B



C



- For large dark photon gauge coupling, dark mesons do not degenerate any longer, so the physics becomes somewhat complicated.
- However, even dark photon gauge coupling is small, cross section can be enhanced through the resonance.
- For instance, for the process of type A, resonance occur when dark photon mass is about three times of dark meson mass.

For the freeze-out of dark matter in SIMP scheme, we start from Bolzmann equation,

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v^2 \rangle (n^3 - n_{\text{eq}}^3). \tag{60}$$

In terms of yield $Y = m/s$ and $x = m/T$, Boltzmann equation is rewritten as

$$\frac{dY}{dx} = -\langle \sigma v^2 \rangle \left[\frac{s(x)^2}{H(x)} \right]_{x=1} x^{-5} (Y^3 - Y_{\text{eq}}^3). \tag{61}$$

$$Y_\infty^2 = \left[\frac{H(x)}{s(x)^2}\right]_{x=1}\frac{1}{2J_f}.$$

$$\begin{aligned}
J_f &= \int_{x_f}^{\infty} x^{-5}\langle\sigma v^2\rangle \\
&= \frac{5\sqrt{5}}{\pi^3}c_1^2 N_c^2 \alpha_D^2 \frac{m^5}{F^6 m_V^4}\int_0^\infty dv_1 \int_0^\infty dv_2 \int_{-1}^{1} d\cos\theta_{12}\frac{v_1^4 v_2^4(1-\cos\theta_{12})}{(\delta - \frac{2}{3}(v_1^2+v_2^2+v_1 v_2\cos\theta_{12}))^2+\gamma^2} \\
&\quad \times \int_{x_f}^{\infty} dx x^{-5}\frac{3\sqrt{3}}{\pi}x^3 e^{-x(v_1^2+v_2^2+v_1 v_2\cos\theta_{12})}.
\end{aligned}$$

Summary

- SIMP scheme is interesting as it uses self interaction in its freeze-out, and explores GeV scale DM, which has not been studied too much.
- SIMP scheme is naturally realized by dark mesons, with WZW term.
- By considering existence of WZW term, together with Z' portal for consistency, we can extract much information on dark sector.
- Moreover, it provides lower bounds on parameters.
- Gauged WZW term provides another 5-point interaction, which might resolve the preturbativity issue.