Entanglement entropy and higher genus partition function in AdS_3/CFT_2

Jie-qiang Wu (Peking University)

Based on [arXiv: 1405.6254, 1506.03206, 1509.02062] with Bin Chen

Kavli Institute for the Physics and Mathematics of the Universe

April 26th, 2016

Outline

1. Introduction
2. RE
3. HRE
4. Partition function
5. Conclusion

- Recently, entanglement entropy draw a lot of attention from different regions
- In this work, we will use entanglement entropy as a measurement to check AdS_3/CFT_2

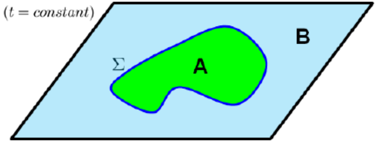
AdS_3/CFT_2

- AdS/CFT claim that a $d+1$ dimensional quantum gravity is dual to a d dimensional field theory, and $Z_{\text{gravity}} = Z_{\text{field theory}}$ Maldacena(1996)
- AdS_3/CFT_2 is a good arena to check AdS/CFT

- There are a lot of check for AdS_3/CFT_2 including: the asymptotic symmetry group, the thermal-dynamics, and conformal block Brown, Henneaux (1986) Strominger, Vafa (1998)
- It is believed that weakly coupled three dimensional gravity should correspond to a large central charge CFT with sparse light spectrum; $c = \frac{3l}{2G}$
  Hartman arXiv:1303.6955, 1405.5137
- Furthermore for pure gravity, we expect the corresponding field theory only has vacuum module states in light spectrum

Entanglement entropy

- Dividing a quantum system into two part $A \bigcup B$, the reduced density matrix for subsystem A is



$$\rho_A = \mathrm{tr}_B \rho \qquad (1.1)$$

  - All physics in sub-system A is described by ρ_A
  - The entanglement entropy can be defined as

$$S_{EE}(A) = -\mathrm{tr}_A \rho_A \log \rho_A \qquad (1.2)$$

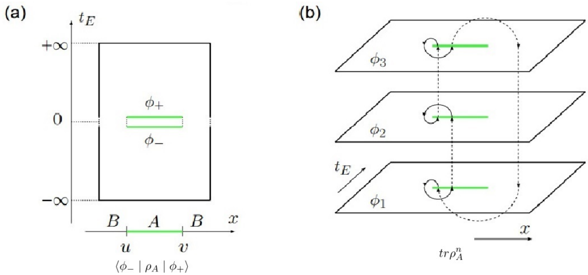
  - For pure state, $\rho = \mid \psi \rangle \langle \psi \mid$, $S_{EE}(A) = S_{EE}(A^c)$
  - For thermal state, where $\rho = e^{-\beta H}$, $S_{EE}(A) \neq S_{EE}(A^c)$

- In field theory, we usually calculate the entanglement entropy by replica trick

$$S_{EE}(A) = \lim_{n\to 1} S_n(A) \tag{1.3}$$

where

$$S_n = -\frac{1}{n-1}\log \mathrm{tr}\rho_A^n = -\frac{1}{n-1}\log\frac{Z_n}{Z_1^n} \tag{1.4}$$



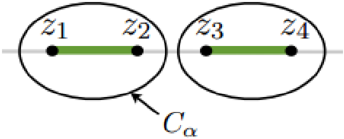
- The n-sheeted surface can be written as a n-copied field theory on one surface and the twist boundary condition can be replaced by some twist operators localted at the branch point

$$\mathrm{tr}\rho_A^n = \frac{Z_n}{Z_1^n} = \langle \mathcal{T}(z_1)\mathcal{T}(z_2)...\rangle \tag{1.5}$$

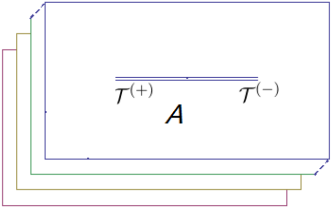
- Because of the infinite degree of freedom, the entanglement entropy in field theory is quite hard to calculate
- In 2d CFT, the conformal symmetry give more constrain for the entanglement entropy
- When the Riemann surface is genus-0 there is universal result
- For example, single interval Rényi entropy in infinite space at zero temperature

$$S_{EE} = \frac{c}{6}(1 + \frac{1}{n}) \log \frac{l}{\epsilon} \tag{1.6}$$

- When the Riemann surface has higher genus, for example the Rényi entropy for two interval case or one interval on finite space at finite temperature, the result depend on spectrum and OPE in CFT



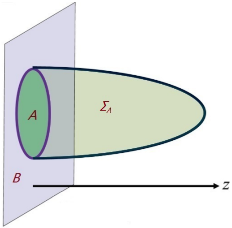
(a) double interval: genus n-1



(b) finite temperature: genus n

- For a CFT with holographic correspondence, it is suggested the entanglement entropy can be evaluated by Ryu-Takayanagi formula

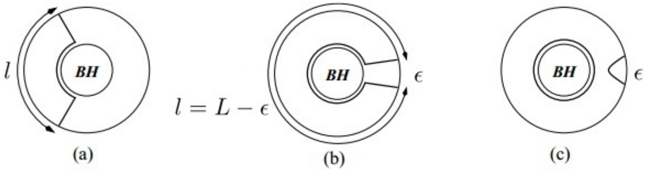
$$S_{EE} = \frac{\text{Area}(\Sigma_A)}{4G}, \text{ with } \partial(\Sigma_A) = \partial(A) \quad (1.7)$$



Shinsei Ryu and Tadashi Takayanagi Phys.Rev.Lett.96:181602,2006

- In AdS_3/CFT_2 the minimal surface is a geodesic

- An interesting case is in black hole background



Tatsuo Azeyanagi, Tatsuma Nishioka, Tadashi Takayanagi

Phys.Rev.D77:064005,2008

- There is a phase transition when the interval is large enough

$$S_{EE}(L-\epsilon) = S_{th} + S_{EE}(\epsilon) \tag{1.8}$$

- By using replica trick in bulk, the Ryu-Takayanagi formula can be derived for single connected regain based on some assumptions A.Lewkowycz, J.Maldacena (2013)
- and more rigorously in AdS_3/CFT_2 for vacuum state with multi-interval

  Hartman, arXiv: 1303.6955

  Faulkner, arXiv: 1303.7221

- In this work, we study one interval entanglement entropy in finite space at finite temperature
- We consider all different cases: low/high temperature, small/large interval
- In gravity side, we derive the Ryu-Takayanagi formula in black hole background and also study the phase transition for small/large interval
- In field theory side, we consider a large c CFT with sparse light spectrum where $c = \frac{3l}{2G}$
- The result in field theory and gravity match with each other up to some level and imply a correspondence for AdS_3/CFT_2
- In another work, we study the partition function on any higher genus Riemann surface and keep c^0 order of $\frac{1}{c}$ expansion. We find agreement with holographic calculation

5 Minutes Break

AdS_3/CFT_2

- The AdS_3/CFT_2 correspondence for pure gravity is not fully understood Alexander Maloney, Edward Witten JHEP 1002:029,2010
- For torus partition function:
- In field theory $Z = \sum_i e^{-\beta E_i}$, we don't know the full spectrum
- In gravity

$$Z = \sum_{\text{saddle point}} e^{-\frac{1}{4G} S_E} Z_{\text{quantum}} \tag{1.9}$$

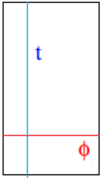
- We don't know which saddle point should include in the summation

- In semi-classical limit, both the gravity and CFT are simplified
- In field theory, the contribution from heavy operators ($h \sim c$) is non-perturbatively depressed in partition function as $e^{-\beta h_i}$.
- If we expand the partition function **properly**, such that the expansion converge fast enough, it is believed that only light representation have perturbative contribution

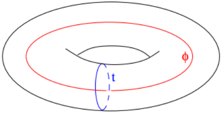
$$\begin{aligned} &\text{Light operator} \qquad h \sim \mathcal{O}(1) \\ &\text{Heavy operator} \qquad h \sim \mathcal{O}(c) \end{aligned} \tag{1.10}$$

- In gravity side, because of $e^{-\frac{1}{4G}S_E}$, only the saddle point with smallest action dominate in the partition function and other saddle points are non-perturbatively depressed as $O(e^{-c})$

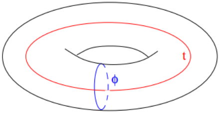
- There is a phase transition between low and high temperature
- In field theory, because we only insert vacuum Virasoro representation, we need the expansion converge fast enough, such that the heavy operators have no perturbative contributions
- In low temperature we expand in imaginary time direction; in high temperature we expand in spacial direction
- In gravity side, the on-shell action will change with temperature. For high temperature BTZ black hole's action is smaller, while for low temperature thermal AdS is smaller



(c) torus



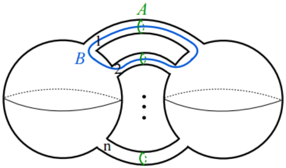
(d) BTZ black hole



(e) Thermal AdS

- For higher genus Riemann surface, the discussion is similar

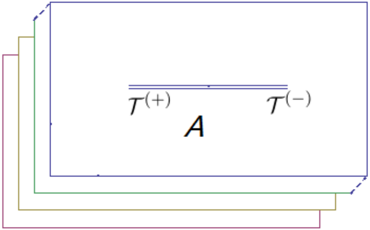
- In field theory we use sewing construction: cutting the Riemann surface and inserting a complete bases



  - If the expansion converge fast enough, only vacuum Virasoro representation state give perturbative contribution and others' are depressed non-perturbatively
  - In gravity side we find gravity solution whose boundary is the higher genus Riemann surface
  - The solution with smallest on-shell action dominate
  - We expect the classical, 1-loop, 2-loop, ... orders in gravity match with c, c^0, c^{-1}... orders in CFT
  - There is also phases transition:
    in field theory, we insert state in different cycle
    in gravity, different saddle point dominate

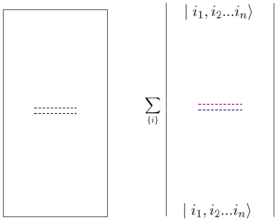
Rényi entropy in CFT

- We study the Rényi entropy S_n in finite space at finite temperature
- By replica trick the Rényi entropy transform to partition function on a genus-n Riemann surface
- We will to cut and insert Vacuum Virasoro representation at some cycles
- For different parameter condition we should cut and insert state in different cycles



Low temperature

- We insert a complete bases at each sheet of the torus



- After cutting the surface at n cycles it transform to a summation of multi-point correlation function on genus zero surface

- We can calculate S_n as an expansion with respect to $e^{-\frac{\beta}{L}}$
- We also take an expansion with $\frac{1}{c}$, the c c^0 c^{-1} order should match with classical, 1-loop, 2-loop order in gravity

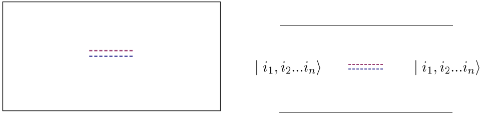
$$\begin{aligned} & S_n^{\text{classical}} \\ = \quad & c\{\frac{1}{6n}(1+n)\log \ \sin\frac{\pi l}{L} - \frac{1}{9}\frac{1}{n^3}(n^2-1)(n+1)\sin^4(\frac{\pi l}{L})e^{-\frac{4\pi\beta}{L}} \\ & -\frac{4}{9}\frac{1}{n^3}(n^2-1)(n+1)\sin^4(\frac{\pi l}{L})\cos^2(\frac{\pi l}{L})e^{-\frac{6\pi\beta}{L}} + O(e^{-\frac{8\pi\beta}{L}}) \end{aligned}$$

$$S_n^{\text{1-loop}} = \{-\frac{2n}{n-1}(\frac{1}{n^4}\frac{\sin^4(\frac{\pi l}{L})}{\sin^4\frac{\pi l}{nL}} - 1)e^{-\frac{4\pi\beta}{L}} + O(e^{-\frac{6\pi\beta}{L}})$$

- When $n \to 1$ we can read the entanglement entropy
- The classical entanglement entropy gives geodesic length in global AdS space which support RT formula
- From the $S^{\text{1-loop}}$ we see $S_{EE}(l) \neq S_{EE}(L-l)$

High temperature small interval

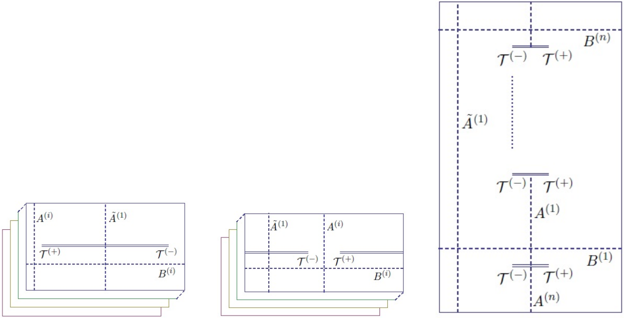
- For high temperature with small interval, we just need to take a modular transformation
- We insert a complete bases at each imaginary time circle; the result is an expansion with respect to $e^{-\frac{L}{\beta}}$



$$S_n \,|_{classical}$$
$$= \frac{c}{6}\frac{1+n}{n}\log\sinh(2\pi Tl) - \frac{c}{9}\frac{(n+1)(n^2-1)}{n^3}\left\{\sinh^4(\pi Tl)e^{-4\pi TL}\right.$$
$$\left. +4\sinh^4(\pi Tl)\cosh^2(\pi Tl)e^{-6\pi TL}\right\} + O(e^{-8\pi TL}),$$

- This result is only for small interval case; for large interval case $l \sim L$, the expansion converge slowly
- It implies for large interval case this expansion cannot evaluate the perturbative result

- For small interval case, we insert a complete bases at $A^{(i)}$ cycle; however in large interval case, the geometry near $A^{(i)}$ change fast,
- We insert a complete basis at $\tilde{A}^{(1)}$ cycle



- Cutting along $\tilde{A}^{(1)}$ cycle the Riemann surface still have higher-genus, we need to take an OPE expansion for the two twist operators
- For classical level we can read out the entanglement entropy by taking $n \to 1$

$$S_{EE}(L-l) = S_{th} + S_{EE}(l) \tag{2.1}$$

- For quantum level, the Rényi entropy is an expansion with $e^{-\frac{2\pi L}{n\beta}}$ and l,

Holographic Rényi entropy

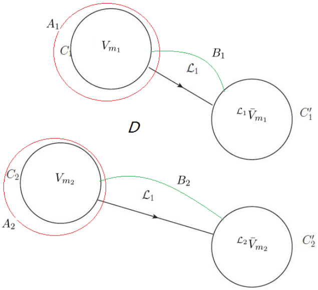
- For holographic Rényi entropy, we still use replica trick
- We calculate the gravity partition function whose boundary is the n-sheeted Riemann surface

$$Z = \sum e^{-S_E} Z \,|_{\text{quantum}} \tag{3.1}$$

- In semi-classical limit, only the saddle point with smallest action dominate; while other saddle points are depressed non-perturbatively

Schottky Uniformization

- Every genus-g Riemann surface can be described by Schottky Uniformization as $\mathcal{M} = \Omega/\Gamma$, where Γ is sub-group of global conformal transformation generated by g element $\mathcal{L}_j$



- We choose $2g$ circles C_j $C_j^{'}$ and global conformal transformation $\mathcal{L}_j$
- $\mathcal{L}_j$ identify C_j with $C_j^{'}$
- The regain outside the circles C_j and $C_j^{'}$ is a genus-g Riemann surface

$$\frac{\mathcal{L}(z) - a_j}{\mathcal{L}(z) - r_j} = q\frac{z - a_j}{z - r_j} \tag{3.2}$$

where q is called the multiplier of $\mathcal{L}$

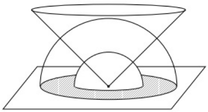
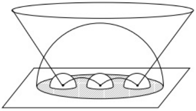
- The Schottky uniformization can be extended into the bulk: Riemann surface Ω/Γ, the handle-body solutions AdS_3/Γ

$$AdS_3: \;\; ds^2 = \frac{dzd\bar{z} + dy^2}{y^2} \tag{3.3}$$

- Using quaternion $u = z + jy$

$$\mathcal{L}: z = \frac{az+b}{cz+d} \quad \mathcal{L}: u = (au+b)(cu+d)^{-1} \tag{3.4}$$

where a b c d are complex numbers

K. Krasnov, Adv. Theor. Math. Phys. 4, 929 (2000)

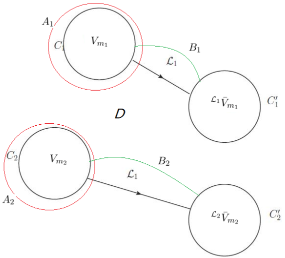
Conformal transformation $\Sigma_n \to \Omega/\Gamma$

- We can solve a differential equation on Σ_n

$$\partial^2\psi(z) + \frac{6}{c}T(z)\psi(z) = 0 \tag{3.5}$$

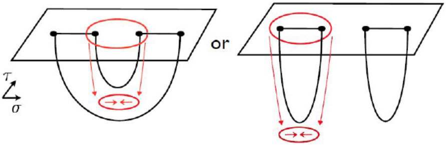
- If we choose $T(z)$ properly, such that $\psi(z)$ is single valued along $A^{(i)}$ cycle then $w(z) = \frac{\psi_1(z)}{\psi_2(z)}$ gives a conformal transformation to universal covering
- For each choice of trivial cycles, we can find a Schottky uniformization
- Most of time, we cannot solve the differential equation analytically
- We can take an expansion with respect to some parameter on the Riemann surface and solve it order by order, for example $e^{-\frac{\beta}{L}}$ (in low temperature), $e^{-\frac{L}{\beta}}$ (in large temperature), $L-l$ (large interval)

- For genus-g Riemann surface we can find the gravity solution systematically:



- Choose 2g canonical cycles A_i B_i
- Transform to the coordinate for Schottky Uniformization
- Extend the identification to the bulk
- A_i cycles are contractible in bulk

- Different choice of A_i B_i cycles give different gravity solutions

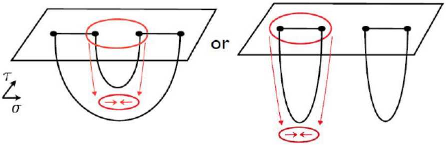


- We can evaluate the classical action and quantum correction for the handle-body solution
- The classical action equals to an on-shell Liouville action in the boundary
- The 1-loop partition function in handle body solution was derived by heat kernel method

$$Z_{\text{1-loop}} = \prod_{\gamma} \prod_{m=2}^{\infty} \frac{1}{|1 - q_{\gamma}^{m}|}, \tag{3.6}$$

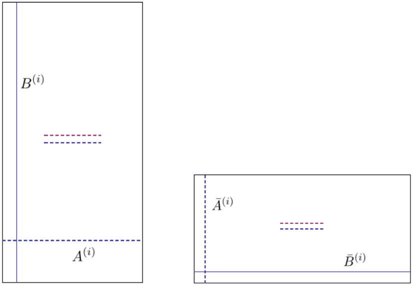
where γ is primary conjugate class in Schottky group $\gamma \neq \gamma_0^n$
Simone Giombi, Alexander Maloney, Xi Yin JHEP 0808:007,2008

- For a Riemann surface there are infinite gravity solutions with the same asymptotic boundary Maloney, Witten 2007 Faulkner, arXiv: 1303.7221
- We assume the dominate saddle point have replica symmetry
- For two interval case there are two choices for saddle point



- The holographic entanglement entropy can be computed in classical and 1-loop level; the classical result equals to RT formula Thomas Faulkner 1303.7221 Taylor Barrella, Xi Dong, Sean A. Hartnoll, Victoria L. Martin JHEP 1309:109,2013

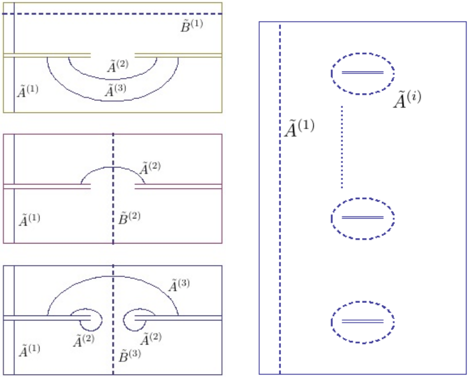
- For finite temperature case the Σ_n is a genus-n Riemann surface



(o) low temperature

(p) high temperature small interval

- For high temperature large interval we suggest a new monodromy condition



- The trivial cycles $A^{(i)}$ in holograpic calculation are the same cycles to cut and insert bases in field theory

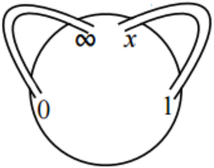
- In each case we can calculate the holographic Rényi entropy in classical and 1-loop level
- The result is an expansion with respect to some parameter in Riemann surface, and we can calculate it order by order
- By brute force, we can calculate the Rényi entropy to some finite orders and find agreement with the field theory
- Is it possible to prove this correspondence to all orders?

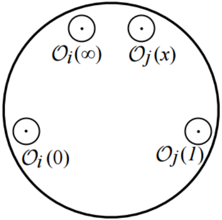
Higher genus partition function

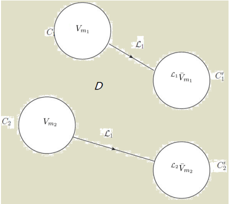
- We study the partition function on any higher genus Riemann surface
- The c^1 order contribution in partition is captured by the conformal anomaly, which is a Liouville action and match with the on-shell action in gravity K. Krasnov, Adv. Theor. Math. Phys. 4, 929 (2000)
- **We will calculate the c^0 order partition function in field theory and match with the 1-loop partition function in gravity handle body solution**

$$Z\mid_{1-loop}=\prod_{\gamma}\prod_{m=2}^{\infty}\frac{1}{|1-q_{\gamma}^{m}|} \tag{4.1}$$

- The partition function can be calculated by sewing construction







$$Z_g = \sum_{m_1,m_2,\ldots m_g} \langle {}^{\mathcal{L}_1}\bar{O}^{(1)}_{m_1} O^{(1)}_{m_1} \, {}^{\mathcal{L}_2}\bar{O}^{(2)}_{m_2} O^{(2)}_{m_2} \ldots {}^{\mathcal{L}_g}\bar{O}^{(g)}_{m_g} O^{(g)}_{m_g} \rangle \quad (4.2)$$

where $O^{(j)}_{m_j}$ is inserted in C_j and ${}^{\mathcal{L}_j}\bar{O}^{(j)}_{m_j}$ is inserted in C_j'

- As previous discussion, only vacuum Virasoro representation states give perturbative contributions

- If we only keep leading order of $\frac{1}{c}$ expansion, there are some simplifications in calculation :
- 1) The Virasoro algebra is decoupled to a series of creation and annihilation operators

$$[L_m, L_n] \sim \frac{c}{12} m(m^2 - 1)\delta_{m+n} \tag{4.3}$$

- 2) The vacuum Virasoro representation states are orthogonal each other

$$\prod_{m=2}^{\infty} L_{-m}^{r_m} \mid 0\rangle \ : \prod_{m=2}^{\infty} \partial^{m-2} T(z) : \tag{4.4}$$

- 3) The multi-stress tensor correlation function is captured by a summation of two point functions' product

$$\langle T(z_1)T(z_2)T(z_3)T(z_4)...\rangle = \sum_P \langle T(z_{P1})T(z_{P2})\rangle\langle T(z_{P3})T(z_{P4})\rangle... \tag{4.5}$$

- For state $\prod_{m=2}^{\infty} L_{-m}^{r_m} \mid 0\rangle$, we define the particle number as $r = \sum_{m=2}^{\infty} r_m$
- We can classify the vacuum Virasoro representation by particle number
- Single particle operator: $V_m \sim \partial^{m-2} T(z)$
  r-particle operator: $:\prod_{j=1}^{r} V_{m_j}:$

Genus-1 as a warm-up

- The result is well know $Z_1 = \prod_{m=2}^{\infty} \frac{1}{1-q^m}$
- We try to derive it in another way

$$Z_1 = \sum_{r=0}^{\infty} \frac{1}{r!} \sum_{\{m_j\}} \langle : (\prod_{j=1}^{r} {}^{\mathcal{L}}\bar{V}_m(r_1)) : \quad : (\prod_{j=1}^{r} V_m(a_1)) : \rangle + O(\frac{1}{c})$$

- There are contributions from different particle number states :

$$Z^{(0)} = 1 \tag{4.6}$$

$$Z^{(1)} = \sum_{m=2}^{\infty} \langle {}^{\mathcal{L}}\bar{V}_m(r_1) V_m(a_1) \rangle = Tr_{\mathcal{H}_1} q^{L_0} = \sum_{m=2}^{\infty} q^m \tag{4.7}$$

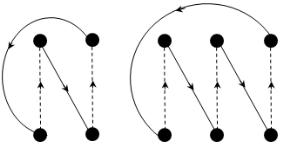
where q is the multiplier of conformal transformation $\mathcal{L}$

$$Z^{(2)} = \frac{1}{2} \sum_{m_1=2}^{\infty} \sum_{m_2=2}^{\infty} \langle : {}^{\mathcal{L}}\bar{V}_{m_1}(r_1) {}^{\mathcal{L}}\bar{V}_{m_2}(r_1) :: V_{m_1}(a_1) V_{m_2}(a_1) : \rangle \tag{4.8}$$

- For multi-particle states, the c^0 order is captured by a summation of two point functions product

$$\begin{aligned}&\langle :{}^{\mathcal{L}}\bar{V}_{m_1}(r_1){}^{\mathcal{L}}\bar{V}_{m_2}(r_1)\ldots{}^{\mathcal{L}}\bar{V}_{m_r}(r_1) :: V_{m_1}(a_1)V_{m_2}(a_1)\ldots V_{m_r}(a_1) :\rangle\\ &=\sum_{\{P\}}\langle{}^{\mathcal{L}}\bar{V}_{m_{P_1}}(r_1)V_{m_1}(a_1)\rangle\langle{}^{\mathcal{L}}\bar{V}_{m_{P_2}}(r_1)V_{m_2}(a_1)\rangle\ldots\langle{}^{\mathcal{L}}\bar{V}_{m_{P_r}}(r_1)V_{m_r}(a_1)\rangle\\ &+O(\frac{1}{c})\end{aligned}$$

- We develop a diagram language to calculate the partition functions



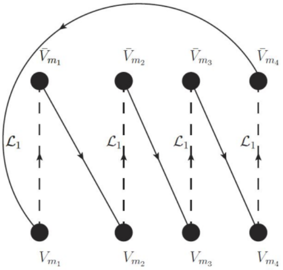


Figure: Diagram Language

- The lower and upper dots denote single vertex operators V_m inserting at the cycle
- The number of dots denote particle number of the state
- The dashed lines denote summation over m
- The solid line denotes the contraction of two single-particle operator.
- A closed contour with dashed and solid lines is called a link
- The number of dashed line is called the length of link

- A link is a product of two point functions, such that the previous ket state equals the next bra state

$$\sum_{\{m_t\}} \langle^{\mathcal{L}} \bar{V}_{m_{t_2}} V_{m_{t_1}} \rangle \langle^{\mathcal{L}} \bar{V}_{m_{t_3}} V_{m_{t_2}} \rangle \ \dots \ \langle^{\mathcal{L}} \bar{V}_{m_{t_1}} V_{m_{t_s}} \rangle = \sum_{m=2}^{\infty} q^{ms} \tag{4.10}$$

- In the expansion for partition function, each term is a product of links

$$Z_1 \sim (1+(1)+(1)^2+\dots)(1+(2)+(2)^2+\dots)(1+(3)+(3)^2+\dots)\dots \tag{4.11}$$

(i) denote a length-i link, and $(1)^2(2)$ denote product of two length-1 link and a length-2 link

- With coefficients, we can calculate the partition function exactly

$$Z_1 = \prod_{s=1}^{\infty} \sum_{t=0}^{\infty} \frac{1}{s^t} \frac{1}{t!} (\sum_{r=2}^{\infty} q^{sr})^t = \prod_{r=2}^{\infty} \frac{1}{1-q^r}. \tag{4.12}$$

Genus-g

- For higher genus cases we need to insert vacuum Virasoro representation in each handle(cycle), for example

$$Z_2 = \sum_{m_1,m_2} \langle^{\mathcal{L}_1} \bar{O}^{(1)}_{m_1} O^{(1)}_{m_1} {}^{\mathcal{L}_2} \bar{O}^{(2)}_{m_2} O^{(2)}_{m_2} \rangle \tag{4.13}$$

- $O^{(1)}_{m_1}$ $O^{(2)}_{m_2}$ can be any multi-particle states

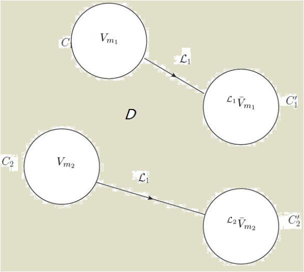
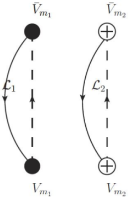


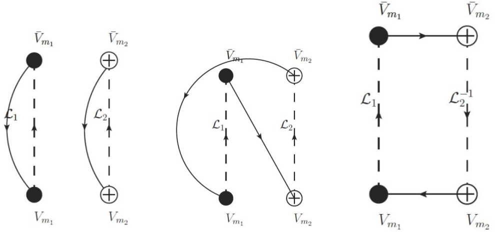
Figure: Higher genus

- In c^0 order the partition function is still an expansion of two point functions' product
- We can use diagram language to describe them
- In diagram language we use different kinds of dots to denote states inserting at different cycles



Links

- We still define a closed contour with solid and dashed line as a link
- There can be contraction between operators in different cycle



(a) link(1)(2) (b) link(12) (c) link(12^{-1})

- Another example

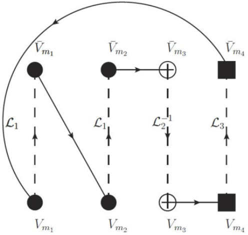


Figure: Link($112^{-1}3$)

- A link is a product of two point functions, such that the previous ket state is the same as the next bra state

$$\sum_{\{m_t\}} \langle^{\mathcal{L}} \bar{V}_{m_{t_2}}(r_2) V_{m_{t_1}}(a_1)\rangle \langle^{\mathcal{L}} \bar{V}_{m_{t_3}}(r_3) V_{m_{t_2}}(a_2)\rangle \ \dots \ \langle^{\mathcal{L}} \bar{V}_{m_{t_1}}(r_1) V_{m_{t_s}}(a_s)\rangle$$

- For a link with diagram $\gamma = (j_1 j_2 ... j_s)$ it equals to

$$\sum_{m=2}^{\infty} q^m \tag{4.15}$$

where q is the multiplier of $\mathcal{L}_{j_1}\mathcal{L}_{j_2}...\mathcal{L}_{j_s}$.

- There is a one-to-one correspondence between a link and an element in Schottky group
- We can define a primitive link γ, such that $\gamma \neq \gamma_{(0)}\gamma_{(0)}\cdots\gamma_{(0)}$, for example $(1), (2), (12), (12^{-1})$ are primary links while $(11), (1212)$ are not
- Assuming we we have all of primitive links as $\{\gamma_{(1)}\ \gamma_{(2)}\ \ldots\}$, we can build all the links as

$\gamma_{(1)},\ \gamma_{(1)}\gamma_{(1)},\ \gamma_{(1)}\gamma_{(1)}\gamma_{(1)}, \cdots, \gamma_{(2)},\ \gamma_{(2)}\gamma_{(2)}, \cdots, \gamma_{(3)}\gamma_{(3)}, \cdots$

- Each terms in the partition function is a product of these links
- With proper coefficient, we can calculate the partition function

$$Z = \prod_{\gamma}\left(\prod_{m=2}^{\infty} \frac{1}{1-q_{\gamma}^{m}}\right) \tag{4.16}$$

where γ is primary conjugate in Schottky group

Conclusion

- We study the Rényi and entanglement entropy in finite space at finite temperature
- We study different cases low/high temperature, small/large interval
- We do calculation in field theory and in gravity and find agreement in classical and 1-loop level
- In high temperature we find the phase transition when the interval is close to the whole space
- We also calculate the c^0 partition function on a general higher genus Riemann surface and the result match with gravity 1-loop partition function in handle-body solution

Outlook

- Higher order of $\frac{1}{c}$ expansion, related to gravity interaction
- Rényi entropy in higher spin black hole

Thanks

Thanks for your attention!