Holographic Resolution of Cosmological Singularities

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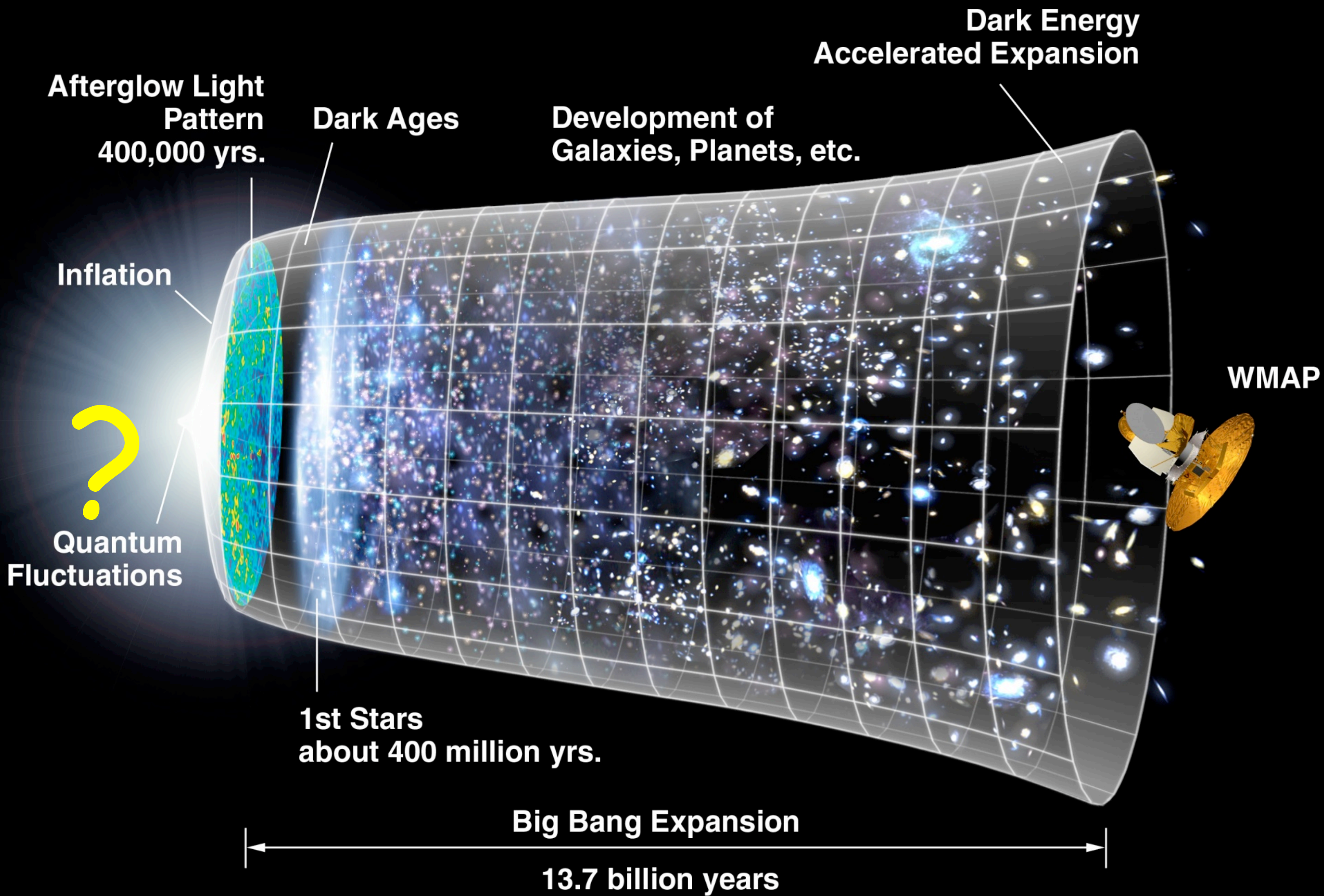
with Ben Craps, Thomas Hertog

* Motivation
* Holographic cosmology
* $AdS^5 x S^5$ and AdS^4 x S^7
* Dual CFT
* Quantum Singularity Resolution
* Bounce and backreaction
* Speculation

Caveats:
work still in progress
rests on unproven conjectures
so far, under good calculational control
only in a regime far from observation

Concordance Inflationary Model



this picture is very successful
but theoretically problematic

* the singularity
* inflation:
  ad hoc initial conditions
  fine-tuned potential
* extreme fine tuning of Λ
* other coincidence problems

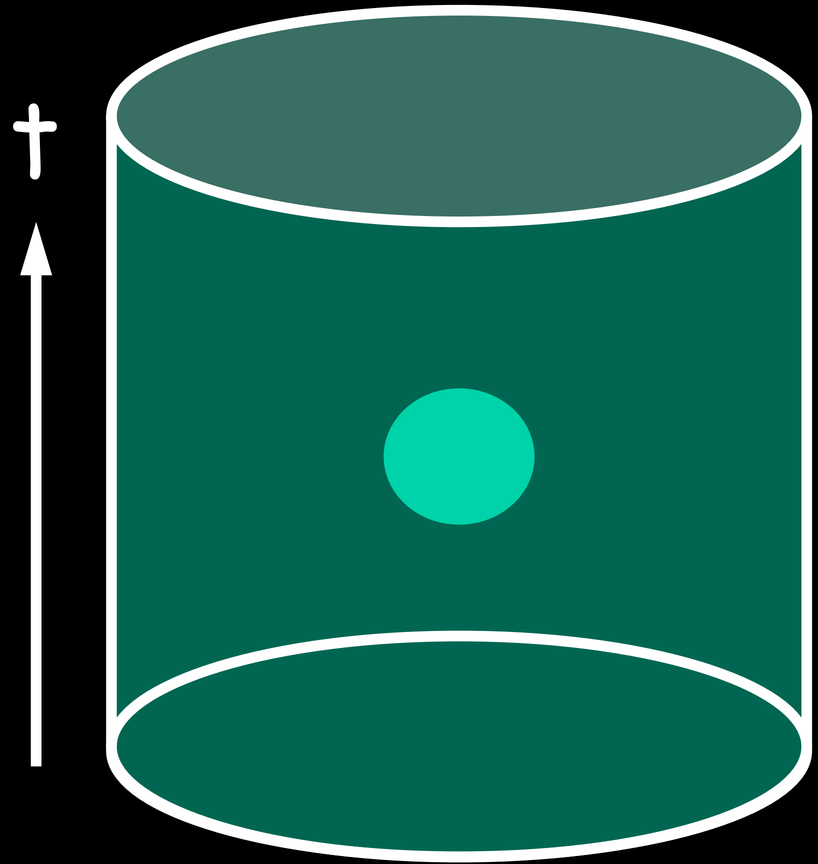
Was the singularity the beginning?

YES -> inflation needed -> tuning/IC problems
-> many resort to anthropic principle
-> "end of fundamental physics"
NO -> pre-big bang epoch can play the same roles -> cyclic universe possible
-> new solutions to Λ tuning problem
-> dynamical selection of string vacua?

To answer the question, must go beyond GR -> quantum gravity: string theory, AdS/CFT

holographic gravity

Maldacena,
Witten ...



theory with gravity
in spacetime bulk

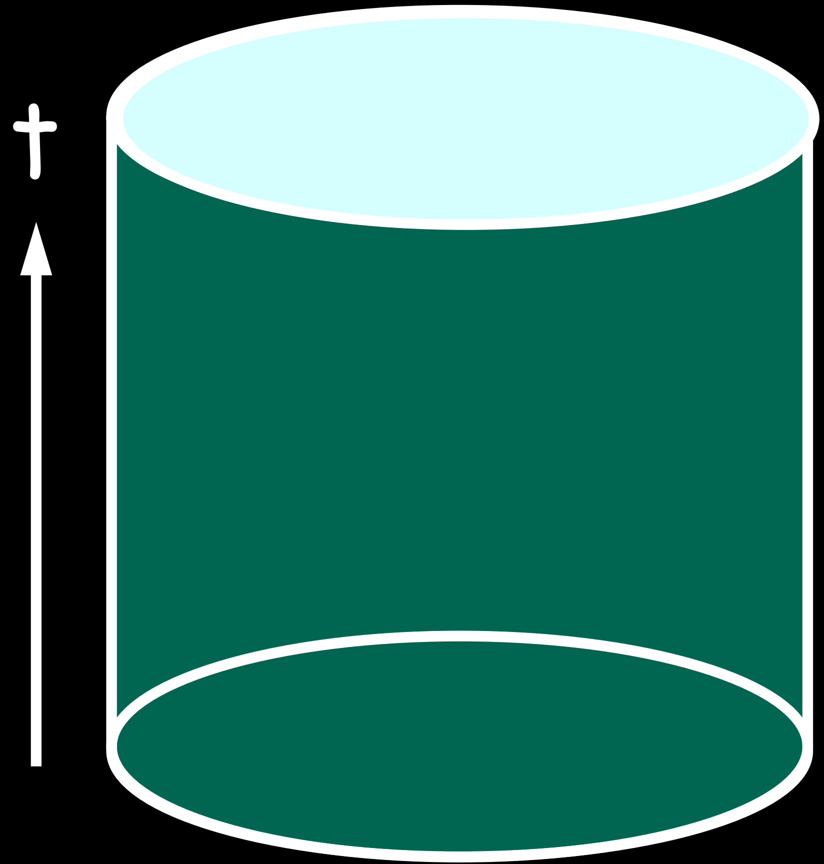
dual to

a theory with no
gravity on boundary!

(a CFT)

holographic cosmology

Hertog+Horowitz



generalized BCs
(still AdS-invariant,
but SUSY-breaking)
allow the bulk to evolve
to a singularity

what happens in the
dual theory?

holographic cosmology

D=11 Supergravity on $AdS^4 \times S^7$: 4d truncation (units $8\pi G = 1$)

$$\int d^4x \left(\frac{1}{2}R - \frac{1}{2}(\partial\varphi)^2 + R_{AdS}^{-2}(2 + \cosh(\sqrt{2}\varphi) \right)$$

Background

$$ds^2 = R_{AdS}^{-2} \left[-(1+r^2)dt^2 + (1+r^2)^{-1}dr^2 + r^2 d\Omega_2^2 \right]$$

General solution for φ :

$$\varphi \sim \frac{\alpha(t,\Omega)}{r} + \frac{\beta(t,\Omega)}{r^2}, \qquad r \to \infty$$

Boundary conditions:

SUSY: α or $\beta = 0$ $\rightarrow \varphi = 0$ stable, no cosmological dynamics

Generalised: $\beta = -h\alpha^2$ $\rightarrow$ unstable, evolves to a big crunch

Generalised BCs correspond to adding a SUSY-breaking deformation to the CFT

$$\alpha(t,\Omega) \quad \sim \quad \mathcal{O} = \mathrm{Tr}(\Phi^2) \equiv \phi^2$$

$$\beta(t,\Omega) \quad \sim \quad J(\mathcal{O})$$

$$\beta = -h\,\alpha^2 \quad \rightarrow \quad \mathcal{S} \rightarrow \mathcal{S} + \int h\,\mathcal{O}^3$$

Cosmological BCs are dual to a deformation which is unbounded below: schematically,

$$AdS^4 \times S^7 \quad \rightarrow V_{def} \sim -\phi^6 \text{ in D=2+1}$$

$$AdS^5 \times S^5 \quad \rightarrow V_{def} \sim -\phi^4 \text{ in D=3+1}$$

The cosmological singularity develops when ϕ runs to infinity on the boundary

Does QFT with an unbounded negative potential make sense?

* no ground state or equilibrium

* but this may be just what we need for cosmology

$$\mathcal{S}_{grav} \sim - \int dt\, a\, \dot{a}^2 \qquad \leftrightarrow \qquad \mathcal{S}_{dual} \sim Vol \int dt \left((\dot{\bar{\phi}})^2 + \bar{\phi}^4 \right)$$

* Boundary sphere finite -> homogeneous mode is quantum mechanical, not classical

* Can define unitary boundary conditions at infinite ϕ , so-called "self-adjoint extensions"

* WKB becomes exact at large ϕ, so can use the semiclassical approximation, described by complex classical solutions

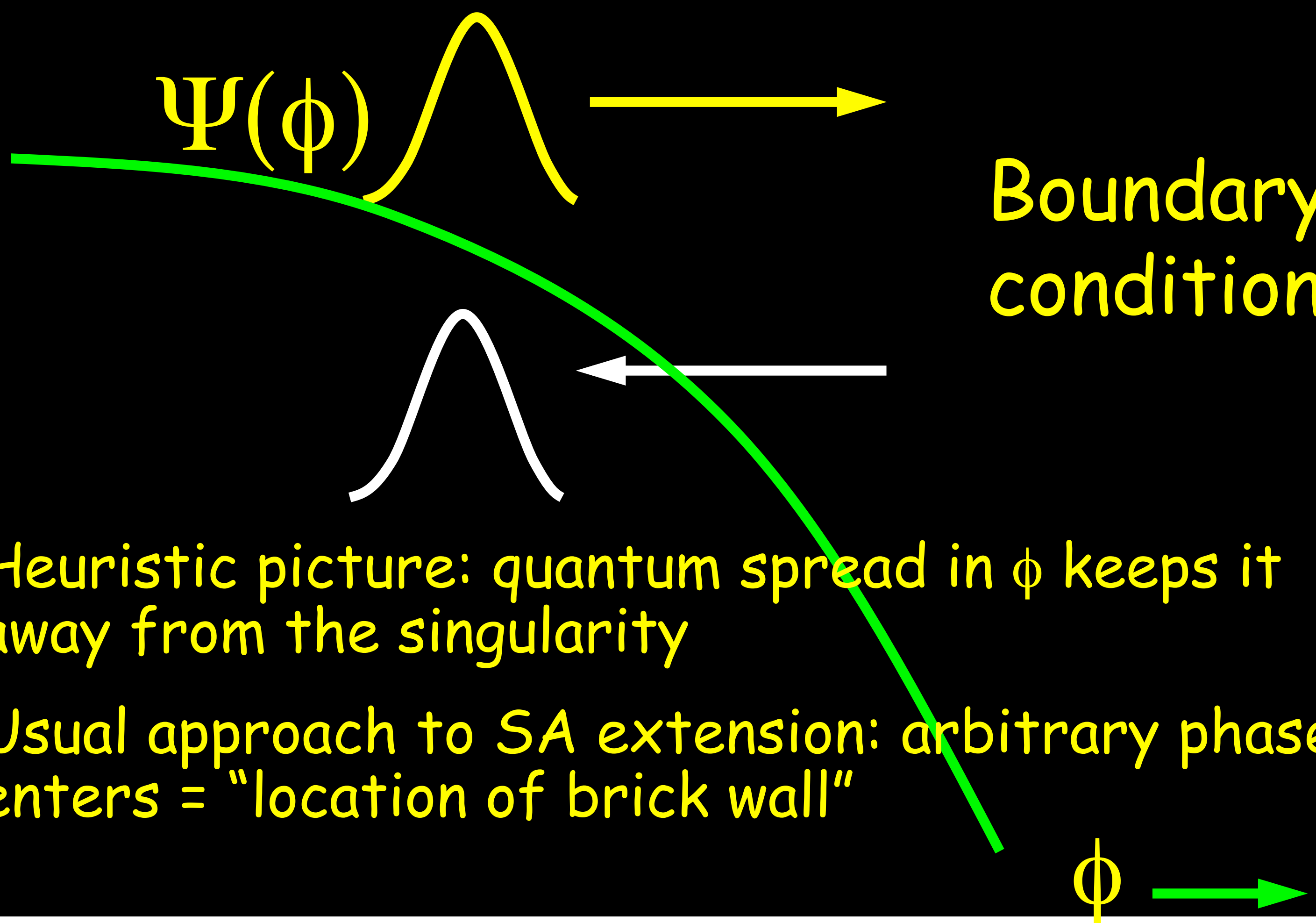
Our first focus was on $AdS^5 x S^5$

Not realistic : a 5d cosmology but

Dual theory is N=4 SYM, SU(N) *gauge group*

Deformed potential is renormalizable and asymptotically free -> perturbation theory becomes reliable as singularity is approached

Quantum mechanics of dual theory



1 Complex Classical Solutions and Quantum Mechanics

Consider a quantum mechanical particle with $H(x,p) = \frac{1}{2}p^2 + V(x)$. At time t_i we prepare the system in a Gaussian wavepacket centred on $x = x_c$ and with momentum $p = p_c$:

$$\frac{x}{2L} + i\frac{pL}{\hbar} = \frac{x_c}{2L} + i\frac{p_c L}{\hbar}. \tag{1.1}$$

View this as an eigenvalue equation, to obtain

$$\Psi^L_{x_c,p_c}(x) \sim e^{i\frac{p_c x}{\hbar}} e^{-\frac{(x-x_c)^2}{4L^2}}. \tag{1.2}$$

For given L, we have $\Delta x = L$, $\Delta p = \hbar/(2L)$, saturating the uncertainty relation.

For a SHO, take $L = \sqrt{\hbar/(2\omega)}$. Then (1.1) is $a = \sqrt{\omega/(2\hbar)}\,(x + ip/\omega)$, obeying $[a, a^\dagger] = 1$. The initial condition that the SHO is in its ground state is just $a = 0$. More generally, the rhs of (1.1) is called the "coherent state parameter".

[redacted]

2 The Semiclassical Expansion

Substitute

$$\Psi(x,t) = A(x,t)e^{iS(x,t)/\hbar} \tag{2.1}$$

into the time-dependent Schrödinger equation

$$i\hbar\frac{\partial\Psi}{\partial t} = \left(-\frac{\hbar^2}{2}\frac{\partial^2}{\partial x^2} + V(x)\right)\Psi. \tag{2.2}$$

and equate powers of $\hbar$. At leading order we obtain the Hamilton-Jacobi equation, solved by

$$S_{Cl}(x,t) = \int_{t_i}^{t} dt'\,(p_{Cl}\dot{x}_{Cl} - H(x_{Cl},p_{Cl})) + B(x_i,p_i) \tag{2.3}$$

where $x(t)$ satisfies (1.1) at time t_i and $x_{Cl} = x$ at time t. The boundary term is needed to ensure $\delta S = 0$ is equivalent to the classical equations of motion:

$$B(x_i,p_i) = \frac{p_i^2 L^2}{i\hbar}. \tag{2.4}$$

3 Example: Free Particle

Complex classical solution:

$$x(t) = x_f + (t - t_f)\frac{p_c + \frac{i\hbar}{2L^2}(x_f - x_c)}{1 + \frac{i\hbar}{2L^2}(t_f - t_i)}, \tag{3.1}$$

$$S_{Cl}(x_f, t_f) = \frac{L^2}{i\hbar}\frac{\left(p_c + \frac{i\hbar}{2L^2}(x_f - x_c)\right)^2}{1 + \frac{i\hbar}{2L^2}(t_f - t_i)}. \tag{3.2}$$

At next order in $\hbar$, we find the amplitude

$$A(x,t) = \frac{N}{\sqrt{1 + i\frac{\hbar t}{2L^2}}} \tag{3.3}$$

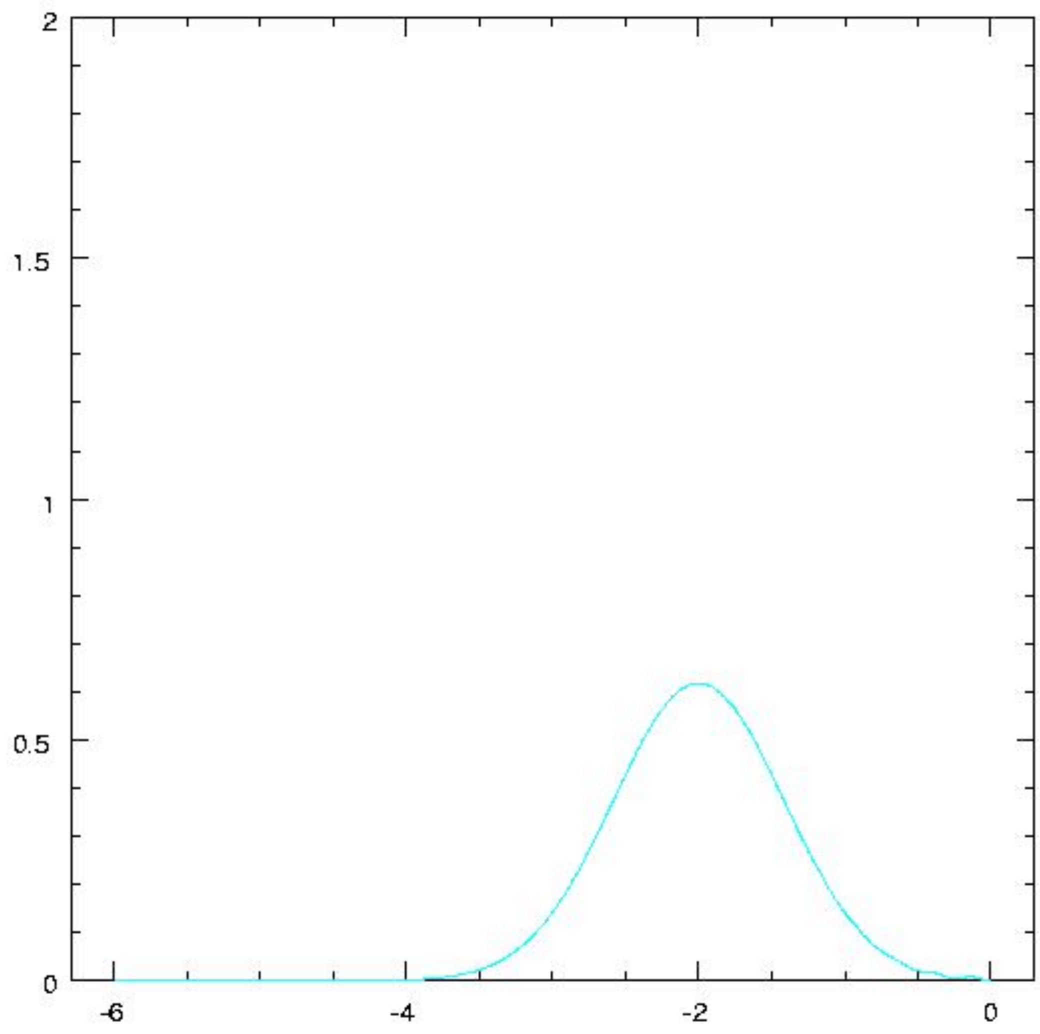
with N a constant determined by normalizing the wavefunction. For a free particle, semi-classical approximation is exact.

Can generalize to include a brick wall: use method of images,

$$\Psi(x,t) = \Psi_1(x,t) - \Psi_2(x,t), \tag{3.4}$$

where Ψ_2 obtained by reflecting ICs through $x = 0$ in complex x-plane.

NB: solution for localized wavepackets generically complex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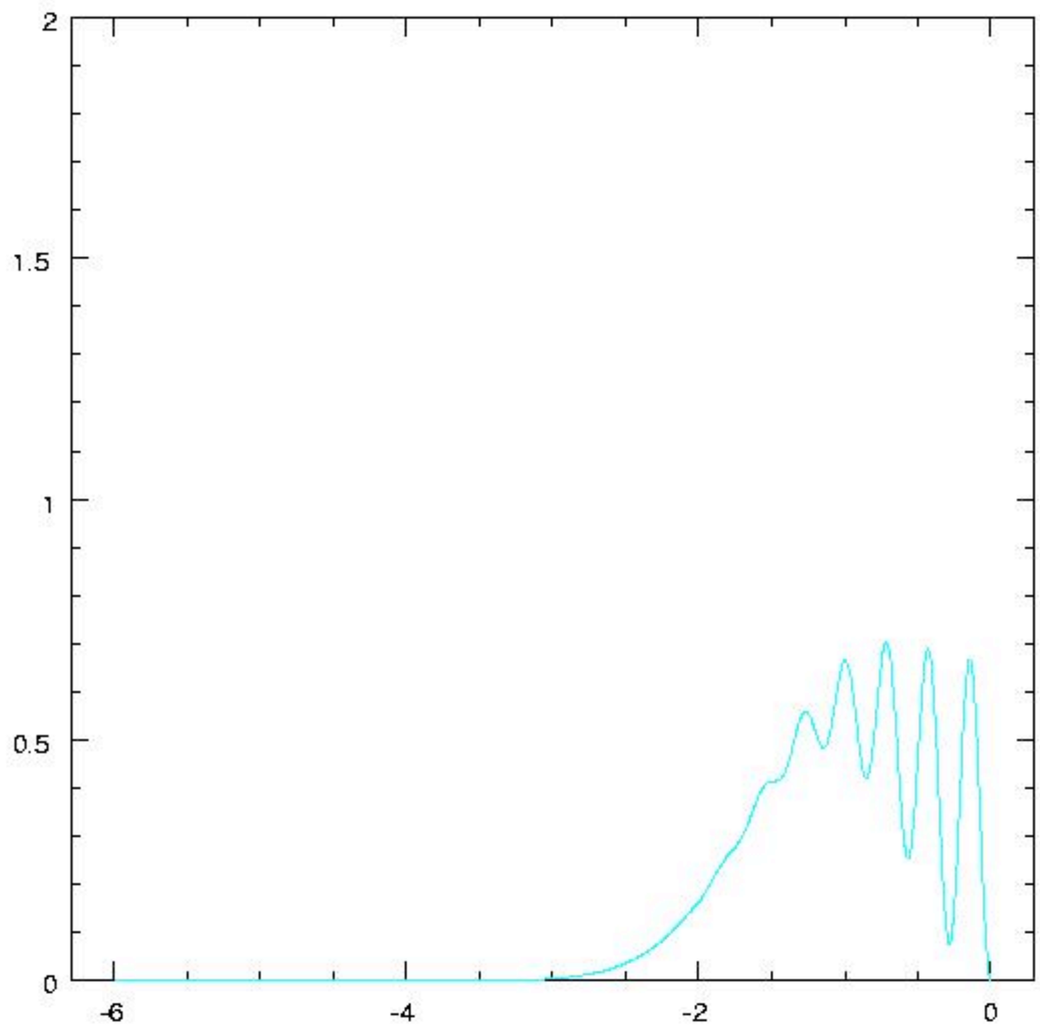

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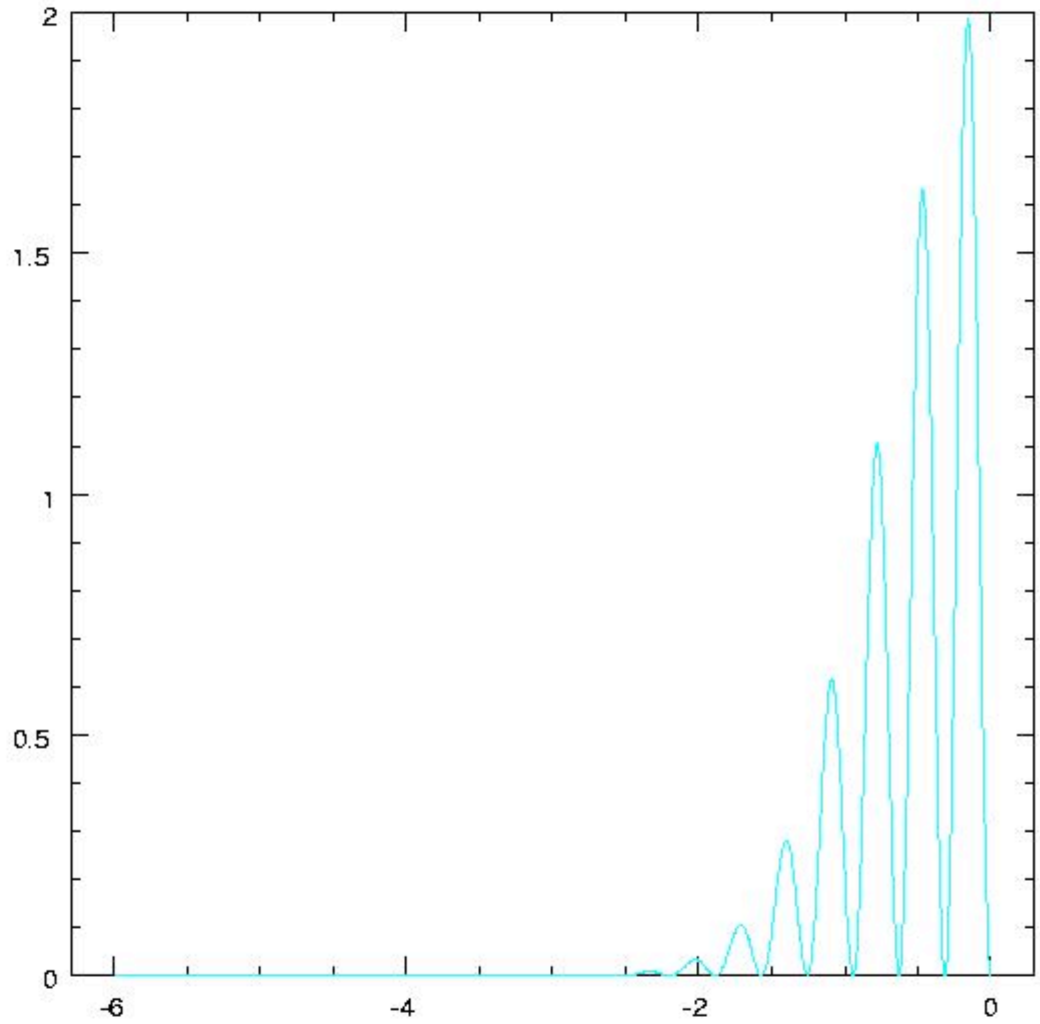

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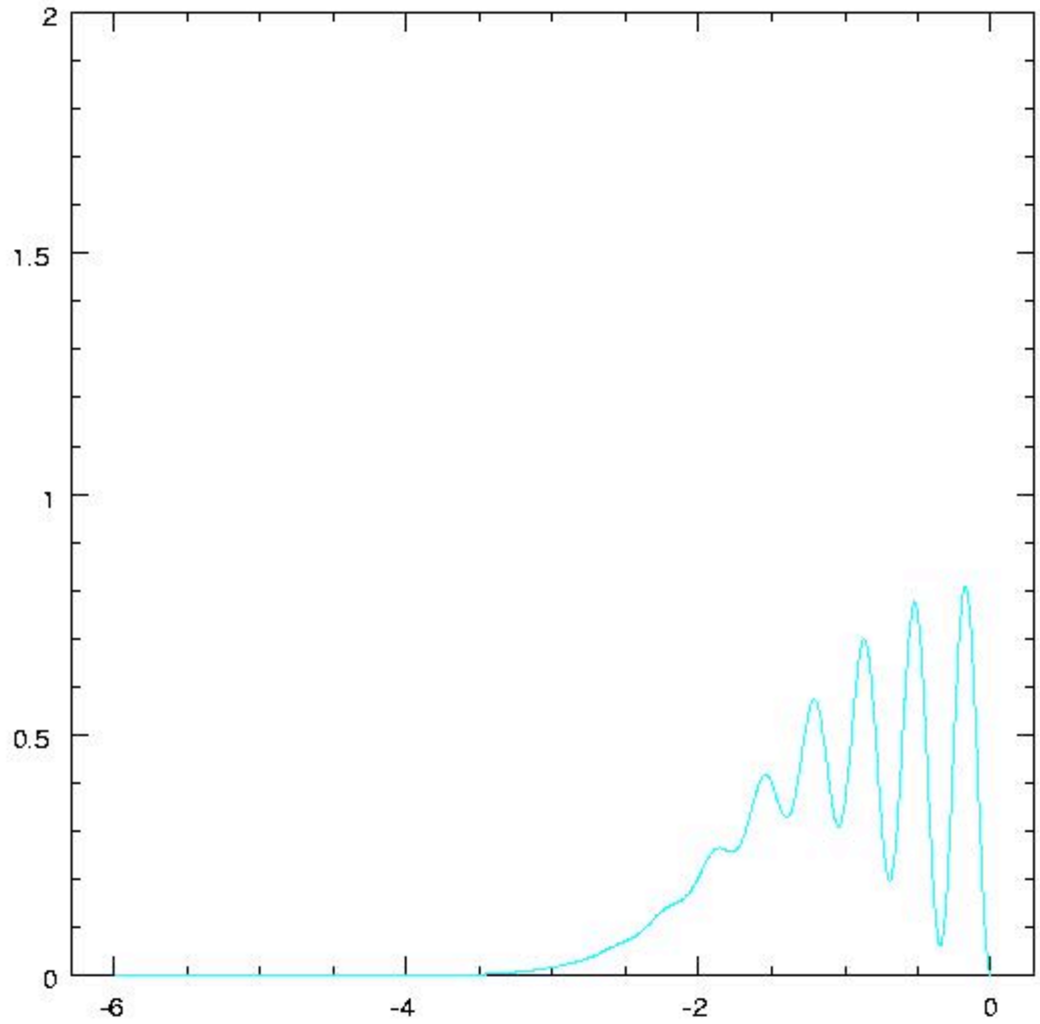

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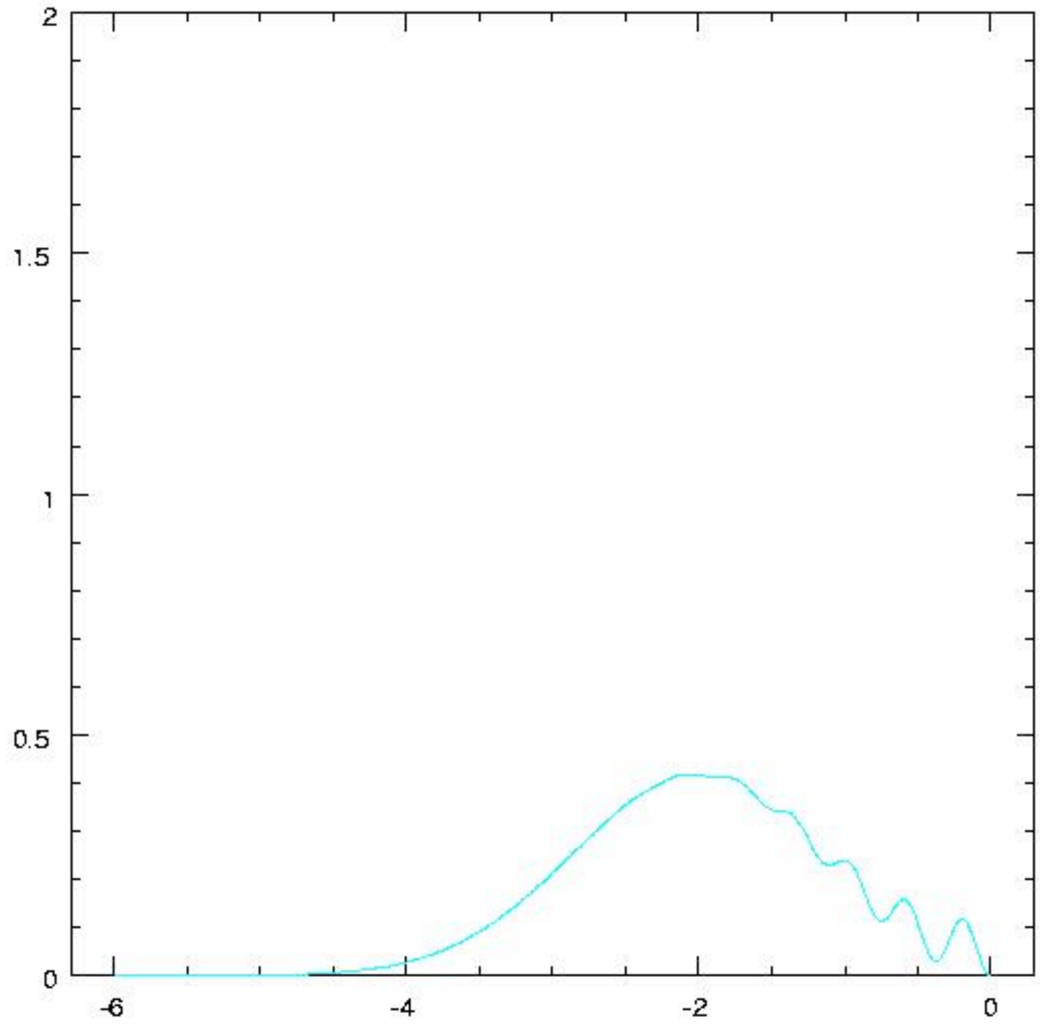
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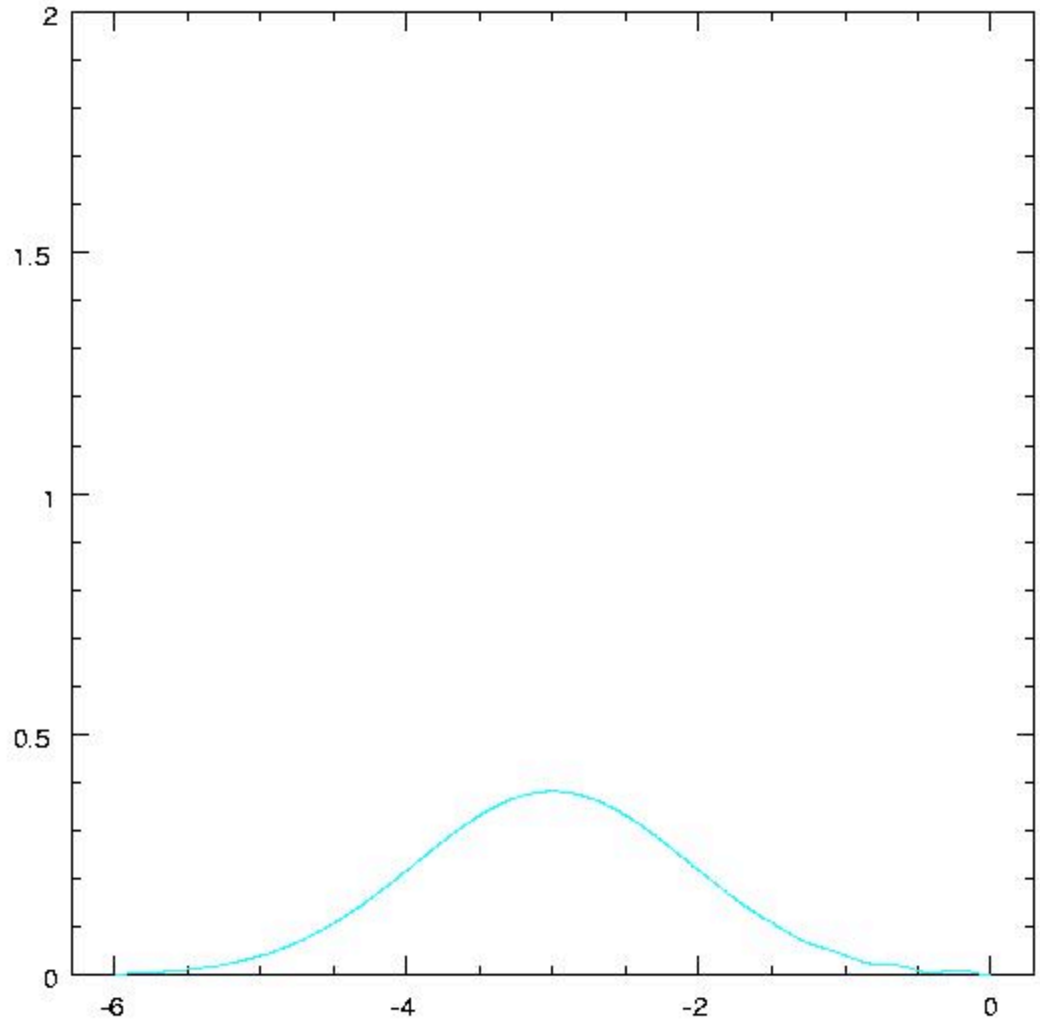

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Inhomogeneous modes

$$\delta\ddot{\phi}_{\mathbf{k}} = -k^2 \delta\phi_{\mathbf{k}} + \overline{\phi}^2 \delta\phi_{\mathbf{k}}$$

quantum spreading keeps $\overline{\phi}$ finite:
complex solution generically avoids infinity

-> for generic values of final $\overline{\phi}$, have a UV cutoff in particle creation

5d AdS cosmology

Craps, Hertog, NT 2008, and in preparation

Dual theory is SYM with $-f\ \phi^4$ deformation. The associated coupling f is asymptotically free, but it runs at one loop, and leading order in large N, breaking scale invariance.

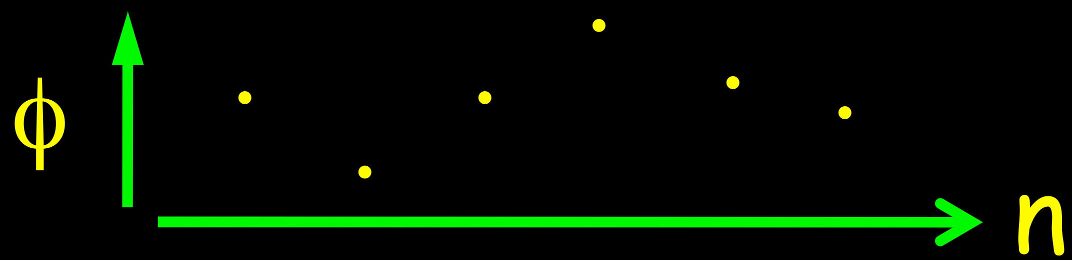
This results in particle production across the bounce.

Backreaction is not parametrically small and the calculation is not therefore under control

Cannot conclude that 5d cosmology "bounces"

Second problem: "brick wall" does not work nonlinearly in QFT

latticize:



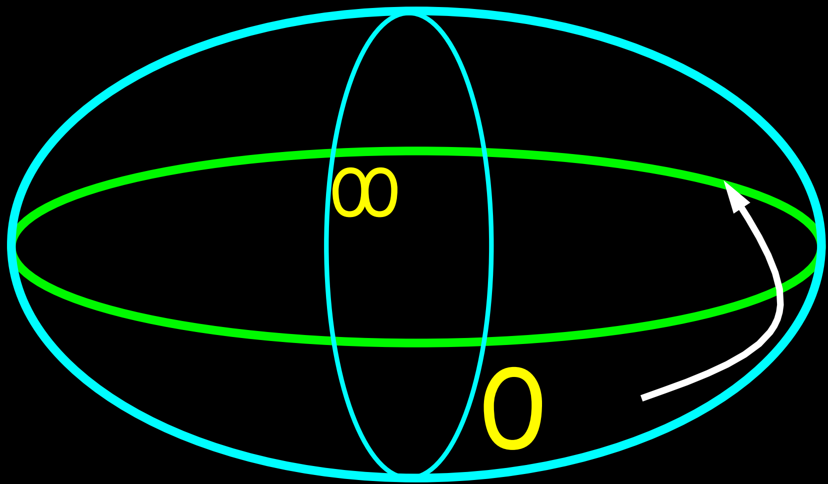
Action contains $(\phi_n - \phi_{n+1})^2/dx^2 \to -2\phi_n\phi_{n+1}/dx^2$

Equation of motion $\ddot{\phi}_{n+1} \sim \phi_n/dx^2$

If $\phi_n \sim 1/|t|$, then $[\dot{\phi}_{n+1}]_-^+ \sim \int_-^+ dt/|t| \to \infty$

Analytic self-adjoint extension

Wavefunction calculated with complex classical solutions
-> trajectories on the Riemann sphere, naturally
compactified at infinite ϕ

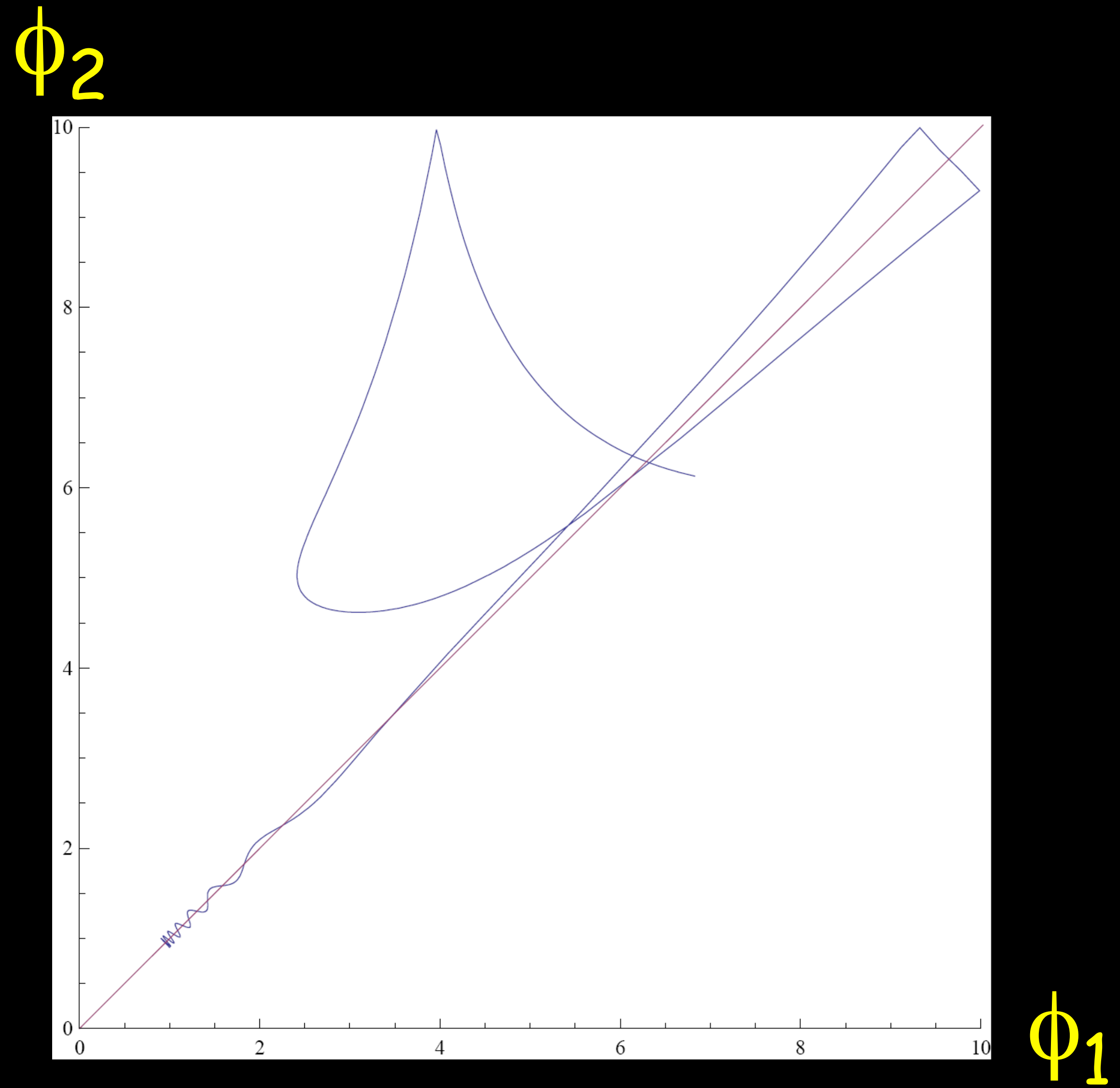


eg ϕ^4 : $\phi_{classical} \sim 1/(i\,\varepsilon - t\,)$

Wavefunction leaves at + infinity, re-enters at – infinity
Unitary, no arbitrary phase enters

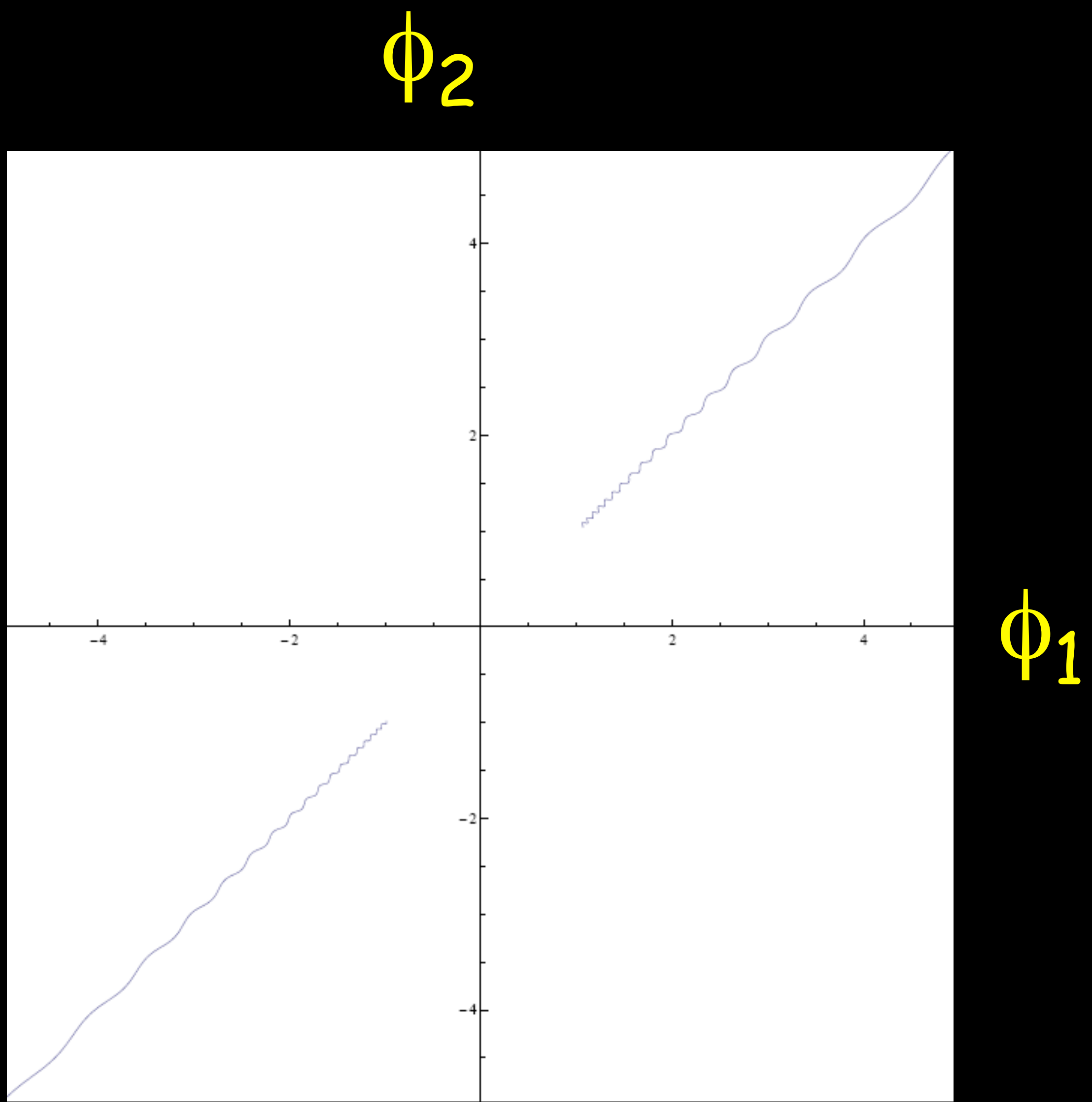
Brick wall: lattice model with 2 sites

Fully nonlinear solution: no good continuum limit



Analytic self-adjoint extension

Fully nonlinear solution: good continuum limit



Recent progress: Aharony, Bergman, Jefferis, Maldacena

M-theory on $AdS^4 \times (S^7 \sim CP^3 \times S^1)$ and S^1 orbifolded by Z_k, is dual to $U(N) \times U(N)$ SUSY-Chern-Simons

$$S = \int d^3x \left[\frac{k}{4\pi} \epsilon^{\mu\nu\lambda} \mathrm{Tr}(A_\mu \partial_\nu A_\lambda + \frac{2i}{3} A_\mu A_\nu A_\lambda - \hat{A}_\mu \partial_\nu \hat{A}_\lambda - \frac{2i}{3} \hat{A}_\mu \hat{A}_\nu \hat{A}_\lambda) - \mathrm{Tr}(D_\mu Y^A)^\dagger D^\mu Y^A + V_{\mathrm{bos}} + \text{ terms with fermions} \right],$$

$$V_{\mathrm{bos}} = -\frac{4\pi^2}{3k^2} \mathrm{Tr} \left[Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger + Y_A^\dagger Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C + 4 Y^A Y_B^\dagger Y^C Y_A^\dagger Y^B Y_C^\dagger - 6 Y^A Y_B^\dagger Y^B Y_A^\dagger Y^C Y_C^\dagger \right]$$

Large N limit: $k, N \to \infty$ with N/k ('t Hooft coupling) fixed.

AdS/CFT

$$\frac{R_{AdS}}{l_{pl}} = (kN)^{\frac{1}{6}} \qquad \text{large}$$

->

$$\frac{R_{AdS}}{l_{string}} = (N/k)^{\frac{1}{6}} \qquad \text{small at weak 'tHooft coupling}$$

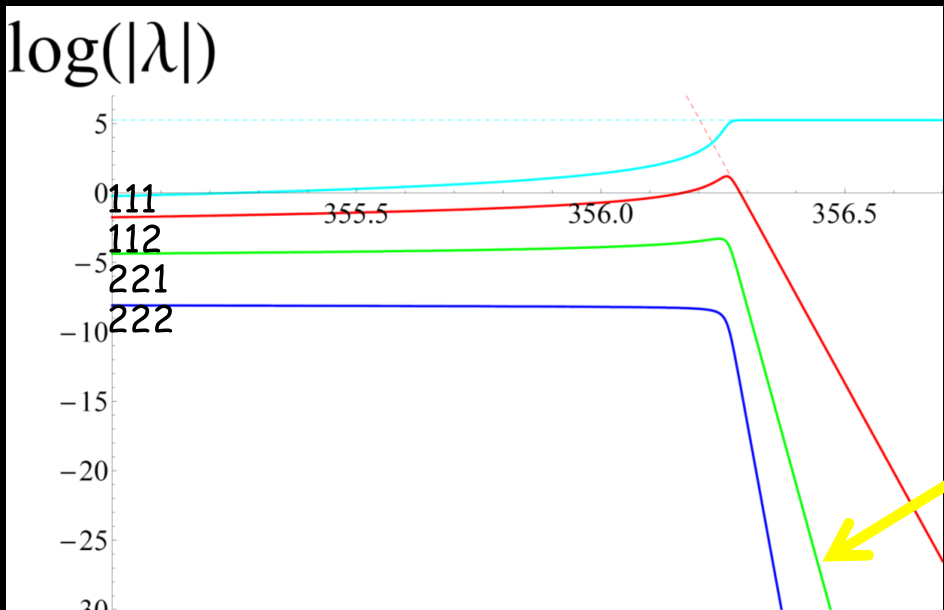
only very stringy regime is perturbative in dual CFT

Cosmological deformation of ABJM

$$\mathcal{O} = \frac{1}{N^2}\mathrm{Tr}(Y^1 Y_1^\dagger - Y^2 Y_2^\dagger)$$ -> $$V = -\frac{f}{N^4}\left[\mathrm{Tr}(Y^1 Y_1^\dagger - Y^2 Y_2^\dagger)\right]^3$$

Renormalization: analog O(N) vector model

$$V = \frac{\lambda}{N^2}\left(\vec{\phi}_1 \cdot \vec{\phi}_1 - \vec{\phi}_2 \cdot \vec{\phi}_2\right)^3$$



UV fixed point
(Bardeen, Moshe, Bander)

$\log(\phi_2)$

$\beta \sim 1/N^2$

Example of particle non-production:

Fermion (non-)creation

$\Psi \sim e^{ip^i x^i} u, \qquad i\gamma^0 \partial_0 u = (\gamma^i p^i + m(t))u \quad \rightarrow \quad \ddot{u} = -(k^2 + m^2 + i\dot{m}\gamma^0)u$

$m \propto 1/t \rightarrow$ Hankel function: positive frequency outgoing mode is $\propto H_\nu^{(2)}(t)$.

But $H_\nu^{(2)}(e^{-i\pi} z) = -e^{i\pi\nu} H_\nu^{(1)}(z)$. So positive frequency out $=$ positive frequency in.

Particle creation (in all modes) only occurs due to running of couplings,
Bogoliubov coeffts ~ β ~ f/N^6

Parametrically small:
4d cosmology bounces whereas 5d doesnt!

Model is UV-complete

* a laboratory for studying a bouncing cosmology

* particle production suppressed due to conformal invariance near singularity

Speculation: is this an alternative to inflation?

Need to study strong coupling regime, but with NO arbitrary potential:

nearly scale-invariant perturbations are automatically generated on the boundary

nearly Gaussian

small-amplitude ($1/N^6$)

adiabatic

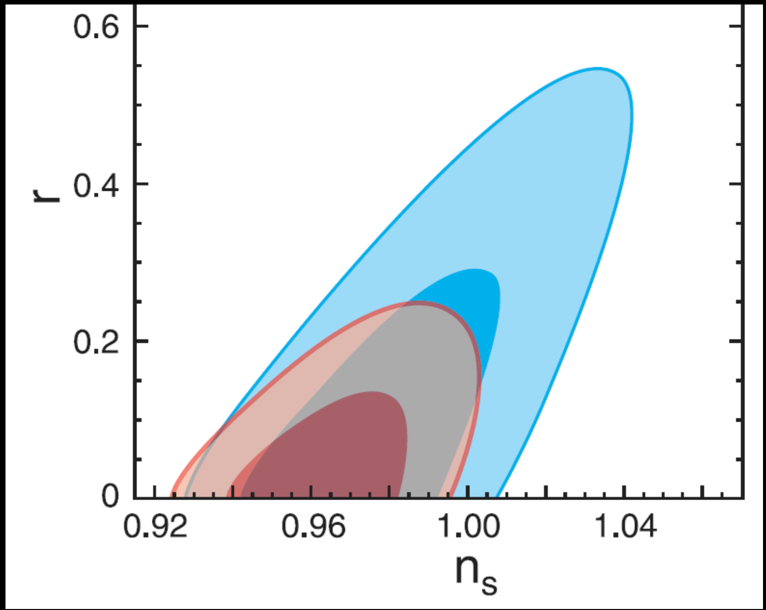
scalar

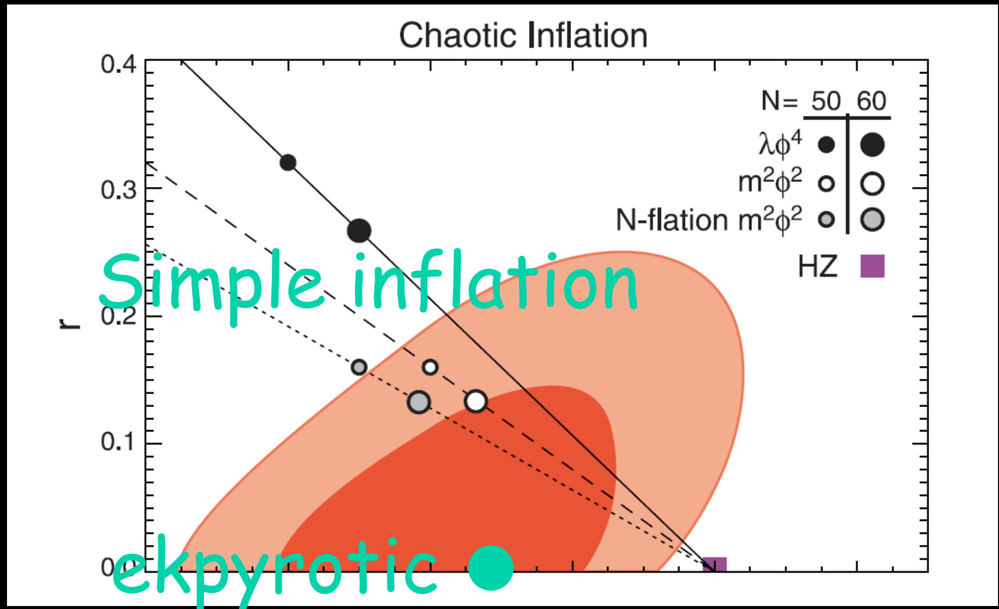
slightly red (asymptotic freedom)

need to translate these into the bulk, but expect these properties to survive

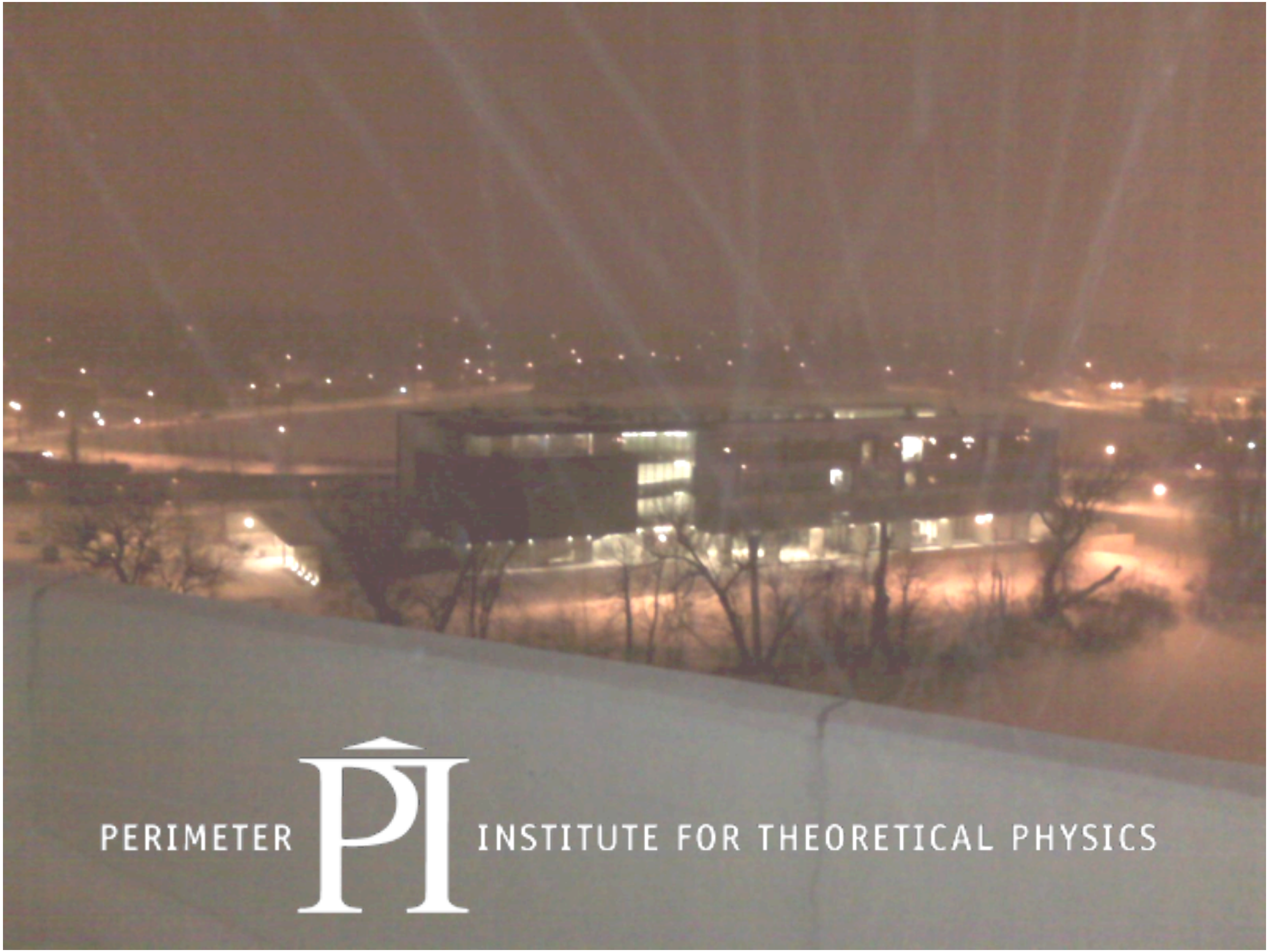
Main prediction:

no tensors, red tilt





Current limits on grav. waves, n_s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