Towards a complete $\Delta(27) \times SO(10)$ SUSY GUT

Fredrik Björkeroth[1]

in collaboration with: Francisco José de Anda[2], Ivo de Medeiros Varzielas[1], Steve King[1]

[1] *University of Southampton, UK*
[2] *CUCEI, Universidad de Guadalajara, México*

IPMU seminar, 24.05.16

UNIVERSITY OF Southampton

invisibles
neutrinos, dark matter & dark energy physics

Outline

1. Motivation
   - Unanswered questions
   - Grand unification
   - Non-Abelian flavour symmetry
2. The model
   - Yukawa structure
   - Mass matrices
   - Proton decay
   - Solving doublet-triplet splitting
3. Summary & Outlook

Based on work in
1512.00850
(hep-ph)

Unanswered questions in model-building

Why are there three generations of fermions?

Are neutrinos Majorana or Dirac fermions?

Why is there such a strong hierarchy in particle masses?

What is the origin of large lepton mixing?

How large is leptonic CP violation?

Why is there a Baryon Asymmetry of the Universe (BAU)?

Why do the gauge couplings appear to converge at $\sim 10^{15-16}$ GeV?

Unanswered questions in model-building

Why are there three generations of fermions?

Are neutrinos Majorana or Dirac fermions?

Why is there such a strong hierarchy in particle masses?

What is the origin of large lepton mixing?

How large is leptonic CP violation?

Why is there a Baryon Asymmetry of the Universe (BAU)?

Why do the gauge couplings appear to converge at $\sim 10^{15-16}$ GeV?

GUTs

Discrete flavour symmetry

Leptogenesis

SUSY

Unanswered questions in model-building

Why are there three generations of fermions?

Are neutrinos Majorana or Dirac fermions?

Why is there such a strong hierarchy in particle masses?

What is the origin of large lepton mixing?

How large is leptonic CP violation?

Why is there a Baryon Asymmetry of the Universe (BAU)?

Why do the gauge couplings appear to converge at $\sim 10^{15-16}$ GeV?

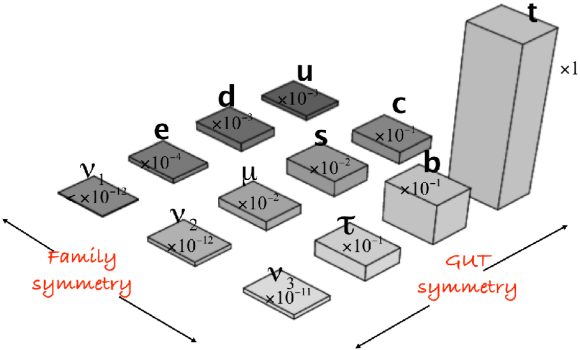
+

What is the lifetime of the proton?

How is doublet-triplet splitting achieved?

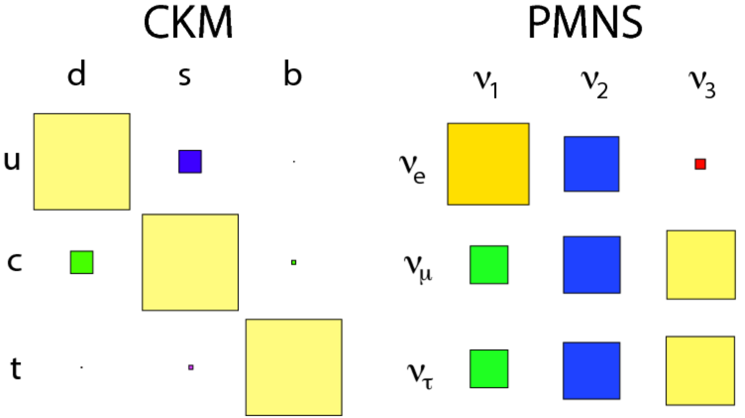
What is the scale of the MSSM μ-term?

Fermion masses



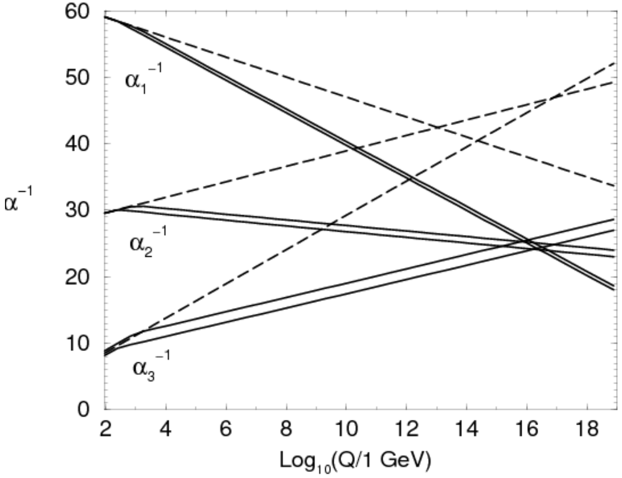
[King, 1301.1340]

Mixing matrices



[Stone, 1212.6374]

Grand unification



[Bhattacharyya, 0807.3883]

Grand unification

In $SO(10)$, MSSM Higgs doublets are contained in larger reps, such as a $\mathbf{10}$. When gauge symmetry is broken via SU(5),

$$\mathbf{10} \rightarrow \mathbf{5} + \bar{\mathbf{5}} \rightarrow \mathbf{3} + \mathbf{2} + \bar{\mathbf{3}} + \mathbf{2}.$$

Explaining why doublets are light, while triplets are heavy, is the doublet-triplet splitting problem.

Grand unification

In $SO(10)$, MSSM Higgs doublets are contained in larger reps, such as a **10**. When gauge symmetry is broken via SU(5),

$$\mathbf{10} \to \mathbf{5} + \overline{\mathbf{5}} \to \mathbf{3} + \mathbf{2} + \overline{\mathbf{3}} + \mathbf{2}.$$

Explaining why doublets are light, while triplets are heavy, is the doublet-triplet splitting problem.

Furthermore, we need at least two **10**s, H^u_{10} and H^d_{10} (otherwise no mixing). This means we have (at least) four doublets in the theory, when we only want two $\to$ **doublet-doublet splitting**.

Analogous scale splitting problems are ubiquitous: any good GUT should resolve them.

Naturalness problem

Non-abelian discrete flavour symmetry

Aim: explain the existence of 3 families of fermions and describe the internal Yukawa structure

Proposal: introduce discrete global symmetry G_F that has triplet representations.

History:

- (Constrained) sequential dominance [King 1999]
- A_4 symmetry to explain large mixing angles [Ma, Rajasekaran 2001]
- A_4 flavon model giving tribimaximal (TBM) mixing [Altarelli, Feruglio 2005]

Sequential dominance (SD)

Sequential dominance conditions:

1. First RH neutrino (often lightest) primarily responsible for $m_3 \sim 50$ meV
2. Second RH neutrino responsible for $m_2 \sim 9$ meV
3. Last RH nearly decoupled, gives $m_1 \lesssim 1$ meV

Predictions:

- Normal Ordering + mass hierarchy
- Naturally large mixing angles: $\theta_{13} \gtrsim \left| \frac{m_2}{m_3} \right| \sim 0.1 \ (\approx 6^\circ)$

  [King, hep-ph/0204360]

Constrained sequential dominance (CSD)

SD yields neutrino parameters in terms of Yukawa + RH Majorana matrices. Define

$$M_R = \begin{pmatrix} X & 0 & 0 \\ 0 & Y & 0 \\ 0 & 0 & Y' \end{pmatrix} \quad Y^\nu = \begin{pmatrix} a & b & b' \\ c & d & d' \\ e & f & f' \end{pmatrix}$$

$$\text{SD condition: } \frac{\{a, c, e\}^2}{X} \gg \frac{\{b, d, f\}^2}{Y} \gg \frac{\{b', d', f'\}^2}{Y'}$$

CSD proposes relationships between elements of Y^ν, increasing predictivity. Original CSD(n) in flavour basis:

$$Y^\nu = \begin{pmatrix} 0 & b & * \\ a & nb & * \\ a & (n-2)b & * \end{pmatrix}, \quad a, b \in \mathbb{C},\ n \in \mathbb{Z}_+$$

Constrained sequential dominance (CSD)

$$Y^\nu = \begin{pmatrix} 0 & b & * \\ a & nb & * \\ a & (n-2)b & * \end{pmatrix}, \quad a, b \in \mathbb{C}, \; n \in \mathbb{Z}_+$$

This arrangement can be produced by coupling fermions to triplet flavons ϕ, which get VEVs like

$$\phi_{\mathrm{atm}} \propto \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \phi_{\mathrm{sol}} \propto \begin{pmatrix} 1 \\ n \\ n-2 \end{pmatrix}$$

Successful model based on SU(5) with CSD(3) has been built [FB, de Anda, de Medeiros Varzielas, King, 1503.03306]

In our $SO(10)$ model, Y^ν does *not* look like this (is symmetric), but flavons with these alignments ($n = 3$) will be used again.

The model

Symmetries of the model

$$SO(10) \times \Delta(27) \times \mathbb{Z}_9 \times \mathbb{Z}_{12} \times \mathbb{Z}_4^R \times \mathrm{CP}$$
$$||$$
$$SU(5)$$
$$\Downarrow$$
$$SU(3) \times SU(2) \times U(1) \times \mathbb{Z}_2^R$$

MSSM fields

$$\begin{aligned} \Psi &= (\mathbf{16}, 3) && \rightarrow \text{fermions} \\ H_{10}^u, H_{10}^d &= (\mathbf{10}, 1) && \\ H_{16}, H_{\overline{16}} &= (\mathbf{16}, 1), (\overline{\mathbf{16}}, 1) && \rightarrow H_u, H_d \end{aligned}$$

Field content

Field	Representation			
	$\Delta(27)$	$SO(10)$	$\mathbb{Z}_4^R$	
Ψ	3	16	1	Contains SM fermions
$H_{10}^{u,d}$	1	10	0	Break electroweak symmetry
$H_{16,\overline{16}}$	1	$16, \overline{16}$	0	Break $SO(10)$
H_{45}	1	45	0	Break SU(5)
H_{DW}	1	45	2	Gives DT splitting via DW mechanism
$\bar{\phi}_i$	$\bar{3}$	1	0	Produces CSD(n) mass matrices
ξ	1	1	0	Gives mass hierarchies, μ term
Z, Z''	1	1	2	Break $\mathbb{Z}_4^R \to \mathbb{Z}_2^R$ R-parity
A_i	3	1	2	} Aligns triplet flavons $\bar{\phi}_i$
O_{ij}	1_{ij}	1	2	

Yukawa superpotential

$$\begin{aligned}
\mathcal{W}_Y = \boldsymbol{\Psi}_i \boldsymbol{\Psi}_j H^u_{10} \Bigg[& \bar{\phi}^i_{\mathrm{dec}} \bar{\phi}^j_{\mathrm{dec}} \sum_{n=0}^{2} \frac{\lambda^{(u)}_{\mathrm{dec},n}}{\langle H_{45} \rangle^n M_\chi^{2-n}} + \bar{\phi}^i_{\mathrm{atm}} \bar{\phi}^j_{\mathrm{atm}} \xi \sum_{n=0}^{3} \frac{\lambda^{(u)}_{\mathrm{atm},n}}{\langle H_{45} \rangle^n M_\chi^{3-n}} \\
& + \bar{\phi}^i_{\mathrm{sol}} \bar{\phi}^j_{\mathrm{sol}} \xi^2 \sum_{n=0}^{4} \frac{\lambda^{(u)}_{\mathrm{sol},n}}{\langle H_{45} \rangle^n M_\chi^{4-n}} + \bar{\phi}^i_{\mathrm{sol}} \bar{\phi}^j_{\mathrm{dec}} \xi \left(\frac{\lambda^{(u)}_{\mathrm{sd},1}}{\langle H'_{45} \rangle^2 M_\chi} + \frac{\lambda^{(u)}_{\mathrm{sd},2}}{\langle H'_{45} \rangle^2 \langle H_{45} \rangle} \right) \Bigg] \\
+ \boldsymbol{\Psi}_i \boldsymbol{\Psi}_j H^d_{10} \Bigg[& \bar{\phi}^i_{\mathrm{dec}} \bar{\phi}^j_{\mathrm{dec}} \xi \sum_{n=0}^{3} \frac{\lambda^{(d)}_{\mathrm{dec},n}}{\langle H_{45} \rangle^n M_\chi^{3-n}} + \bar{\phi}^i_{\mathrm{atm}} \bar{\phi}^j_{\mathrm{atm}} \xi^2 \sum_{n=0}^{4} \frac{\lambda^{(d)}_{\mathrm{atm},n}}{\langle H_{45} \rangle^n M_\chi^{4-n}} \\
& + \bar{\phi}^i_{\mathrm{sol}} \bar{\phi}^j_{\mathrm{sol}} \xi^3 \sum_{n=0}^{5} \frac{\lambda^{(d)}_{\mathrm{sol},n}}{\langle H_{45} \rangle^n M_\chi^{5-n}} \Bigg] \\
+ \boldsymbol{\Psi}_i \boldsymbol{\Psi}_j H_{\overline{16}} H_{\overline{16}} \Bigg[& \bar{\phi}^i_{\mathrm{dec}} \bar{\phi}^j_{\mathrm{dec}} \xi^3 \frac{\lambda^{(M)}_{\mathrm{dec}}}{M_\chi^2 M^4_{\Omega_{\mathrm{dec}}}} + \bar{\phi}^i_{\mathrm{atm}} \bar{\phi}^j_{\mathrm{atm}} \xi^4 \frac{\lambda^{(M)}_{\mathrm{atm}}}{M_\chi^3 M^4_{\Omega_{\mathrm{atm}}}} \\
& + \bar{\phi}^i_{\mathrm{sol}} \bar{\phi}^j_{\mathrm{sol}} \xi^5 \frac{\lambda^{(M)}_{\mathrm{sol}}}{M_\chi^4 M^4_{\Omega_{\mathrm{sol}}}} \Bigg]
\end{aligned}$$

Yukawa superpotential

$$
\begin{aligned}
\mathcal{W}_Y = \; & \Psi_i \Psi_j H_{10}^d \xi \left[\bar{\phi}^i_{\mathrm{dec}} \bar{\phi}^j_{\mathrm{dec}} C^{(d)}_{\mathrm{dec}}(3) + \bar{\phi}^i_{\mathrm{atm}} \bar{\phi}^j_{\mathrm{atm}} \xi C^{(d)}_{\mathrm{atm}}(4) + \bar{\phi}^i_{\mathrm{sol}} \bar{\phi}^j_{\mathrm{sol}} \xi^2 C^{(d)}_{\mathrm{sol}}(5) \right] \\
& + \Psi_i \Psi_j H_{10}^u \left[\bar{\phi}^i_{\mathrm{dec}} \bar{\phi}^j_{\mathrm{dec}} C^{(u)}_{\mathrm{dec}}(2) + \bar{\phi}^i_{\mathrm{atm}} \bar{\phi}^j_{\mathrm{atm}} \xi C^{(u)}_{\mathrm{atm}}(3) + \bar{\phi}^i_{\mathrm{sol}} \bar{\phi}^j_{\mathrm{sol}} \xi^2 C^{(u)}_{\mathrm{sol}}(4) \right. \\
& \qquad \left. + \bar{\phi}^i_{\mathrm{sol}} \bar{\phi}^j_{\mathrm{dec}} \xi C'^{(u)}_{\mathrm{sd}}(3) \right] \\
& + \Psi_i \Psi_j H_{\overline{16}} H_{\overline{16}} \xi^3 \\
& \qquad \times \left[\bar{\phi}^i_{\mathrm{dec}} \bar{\phi}^j_{\mathrm{dec}} D^{(M)}_{\mathrm{dec}}(2) + \bar{\phi}^i_{\mathrm{atm}} \bar{\phi}^j_{\mathrm{atm}} \xi D^{(M)}_{\mathrm{atm}}(3) + \bar{\phi}^i_{\mathrm{sol}} \bar{\phi}^j_{\mathrm{sol}} \xi^2 D^{(M)}_{\mathrm{sol}}(4) \right]
\end{aligned}
$$

where

$$
\begin{aligned}
C^{(f)}_{\mathrm{flavon}}(N) &= \sum_{n=0}^{N} \frac{\lambda^{(f)}_{\mathrm{flavon},n}}{\langle H_{45} \rangle^n M_\chi^{N-n}} \sim \frac{1}{M^N_{\mathrm{GUT}}}, \\
D^{(M)}_{\mathrm{flavon}}(N) &= \frac{\lambda^{(M)}_{\mathrm{flavon}}}{M_\chi^N M^4_{\Omega_{\mathrm{flavon}}}} \sim \frac{1}{M^{N+4}_{\mathrm{GUT}}}
\end{aligned}
$$

Mass matrices

Schematically, Yukawa superpotential looks like

$$\mathcal{W} \sim \Psi\Psi H \left(\bar{\phi}_{\mathrm{dec}}\bar{\phi}_{\mathrm{dec}}\xi^{n} + \bar{\phi}_{\mathrm{atm}}\bar{\phi}_{\mathrm{atm}}\xi^{n+1} + \bar{\phi}_{\mathrm{sol}}\bar{\phi}_{\mathrm{sol}}\xi^{n+2}\right) + \ldots$$

- ξ gets a VEV below the GUT scale, i.e. $\langle\xi\rangle \sim 0.1 M_{\mathrm{GUT}}$. [Froggatt, Nielsen 1979]
- In our model, flavon VEVs $\langle\bar{\phi}\rangle$ also have scale differences: $\langle\bar{\phi}_{\mathrm{dec}}\rangle \gg \langle\bar{\phi}_{\mathrm{atm}}\rangle \gtrsim \langle\bar{\phi}_{\mathrm{sol}}\rangle$.

Coupling of flavons $\bar{\phi}$ to ξ^{n} explains the existence of mass hierarchies.

Mass matrices

Schematically, Yukawa superpotential looks like

$$\mathcal{W} \sim \Psi\Psi H \left(\bar{\phi}_{\mathrm{dec}}\bar{\phi}_{\mathrm{dec}}\xi^{n} + \bar{\phi}_{\mathrm{atm}}\bar{\phi}_{\mathrm{atm}}\xi^{n+1} + \bar{\phi}_{\mathrm{sol}}\bar{\phi}_{\mathrm{sol}}\xi^{n+2}\right) + \ldots$$

- ξ gets a VEV below the GUT scale, i.e. $\langle\xi\rangle \sim 0.1 M_{\mathrm{GUT}}$. [Froggatt, Nielsen 1979]
- In our model, flavon VEVs $\langle\bar{\phi}\rangle$ also have scale differences: $\langle\bar{\phi}_{\mathrm{dec}}\rangle \gg \langle\bar{\phi}_{\mathrm{atm}}\rangle \gtrsim \langle\bar{\phi}_{\mathrm{sol}}\rangle$.

Coupling of flavons $\bar{\phi}$ to ξ^n explains the existence of mass hierarchies.

Flavons gain vacuum alignments

$$\bar{\phi}_{\mathrm{atm}} = v_{\mathrm{atm}}\begin{pmatrix}0\\1\\1\end{pmatrix}, \quad \bar{\phi}_{\mathrm{sol}} = v_{\mathrm{sol}}\begin{pmatrix}1\\3\\1\end{pmatrix}, \quad \bar{\phi}_{\mathrm{dec}} = v_{\mathrm{dec}}\begin{pmatrix}0\\0\\1\end{pmatrix}$$

Mass matrix

$SO(10)$ unification $\Rightarrow$ all* Yukawa matrices have the **same structure**:

$$m \sim m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix} + m_c \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Exception: additional terms that couple to H_{10}^u involving new field H'_{45}:

$$\overline{\phi}^i_{\mathrm{sol}}\overline{\phi}^j_{\mathrm{dec}}\xi\left(\frac{\lambda^{(u)}_{\mathrm{sd},1}}{\langle H'_{45}\rangle^2 M_\chi} + \frac{\lambda^{(u)}_{\mathrm{sd},2}}{\langle H'_{45}\rangle^2 \langle H_{45}\rangle}\right)$$

Gives additional contribution to up-quark matrix:

$$m_{sd} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

Neutrino mass matrix

Neutrino mass matrix also has this structure after seesaw!

At $SU(5)$ level:

$$\begin{array}{ccccccc} \Psi & \rightarrow & F & + & T & + & N^c \\ \mathbf{16} & \rightarrow & \bar{\mathbf{5}} & + & \mathbf{10} & + & \mathbf{1} \end{array}$$

Relevant superpotential:

$$\kappa^\nu_{\rm atm}(\bar\phi_{\rm atm}F)(\bar\phi_{\rm atm}N^c) + \kappa^\nu_{\rm sol}(\bar\phi_{\rm sol}F)(\bar\phi_{\rm sol}N^c) + \kappa^\nu_{\rm dec}(\bar\phi_{\rm dec}F)(\bar\phi_{\rm dec}N^c)$$
$$+\, \kappa^M_{\rm atm}(\bar\phi_{\rm atm}N^c)(\bar\phi_{\rm atm}N^c) + \kappa^M_{\rm sol}(\bar\phi_{\rm sol}N^c)(\bar\phi_{\rm sol}N^c) + \kappa^M_{\rm dec}(\bar\phi_{\rm dec}N^c)(\bar\phi_{\rm dec}N^c)$$

$$\Downarrow$$

$$\begin{array}{c} \\ \bar\phi_{\rm atm}F \\ \bar\phi_{\rm sol}F \\ \bar\phi_{\rm dec}F \\ \bar\phi_{\rm atm}N^c \\ \bar\phi_{\rm sol}N^c \\ \bar\phi_{\rm dec}N^c \end{array}
\begin{array}{c} \begin{array}{cccccc} \bar\phi_{\rm atm}F & \bar\phi_{\rm sol}F & \bar\phi_{\rm dec}F & \bar\phi_{\rm atm}N^c & \bar\phi_{\rm sol}N^c & \bar\phi_{\rm dec}N^c \end{array} \\
\begin{pmatrix} 0 & 0 & 0 & \kappa^\nu_{\rm atm} & 0 & 0 \\ 0 & 0 & 0 & 0 & \kappa^\nu_{\rm sol} & 0 \\ 0 & 0 & 0 & 0 & 0 & \kappa^\nu_{\rm dec} \\ \kappa^\nu_{\rm atm} & 0 & 0 & \kappa^M_{\rm atm} & 0 & 0 \\ 0 & \kappa^\nu_{\rm sol} & 0 & 0 & \kappa^M_{\rm sol} & 0 \\ 0 & 0 & \kappa^\nu_{\rm dec} & 0 & 0 & \kappa^M_{\rm dec} \end{pmatrix} \end{array}$$

Neutrino mass matrix

Diagonalisation gives effective terms

$$-\frac{(\kappa^\nu_{\mathrm{atm}})^2}{\kappa^M_{\mathrm{atm}}}(\bar{\phi}_{\mathrm{atm}}F)(\bar{\phi}_{\mathrm{atm}}F) - \frac{(\kappa^\nu_{\mathrm{sol}})^2}{\kappa^M_{\mathrm{sol}}}(\bar{\phi}_{\mathrm{sol}}F)(\bar{\phi}_{\mathrm{sol}}F) - \frac{(\kappa^\nu_{\mathrm{dec}})^2}{\kappa^M_{\mathrm{dec}}}(\bar{\phi}_{\mathrm{dec}}F)(\bar{\phi}_{\mathrm{dec}}F)$$

This produces the effective light neutrino mass matrix

$$m^\nu = \mu_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \mu_b e^{i\eta} \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix} + \mu_c e^{i\eta'} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(as before)

The phases η, η' are determined by the VEVs of $\bar{\phi}_{\mathrm{atm}}$, $\bar{\phi}_{\mathrm{sol}}$ and $\bar{\phi}_{\mathrm{dec}}$, and are fixed by the model:

$$\eta = 2\pi/3, \qquad \eta' = 0$$

Fit

Observables	Model	Data fit 1σ range
θ^q_{12} $/^\circ$	13.024	$12.985 \to 13.067$
θ^q_{13} $/^\circ$	0.1984	$0.1866 \to 0.2005$
θ^q_{23} $/^\circ$	2.238	$2.202 \to 2.273$
δ^q $/^\circ$	69.32	$66.12 \to 72.31$
m_u /MeV	0.575	$0.351 \to 0.666$
m_c /MeV	248.4	$240.1 \to 257.5$
m_t /GeV	92.79	$89.84 \to 95.77$
m_d /MeV	0.824	$0.744 \to 0.929$
m_s /MeV	15.55	$15.66 \to 17.47$
m_b /GeV	0.939	$0.925 \to 0.948$
m_e /MeV	0.342	$0.340 \to 0.344$
m_μ /MeV	72.25	$71.81 \to 72.68$
m_τ /GeV	1.229	$1.223 \to 1.236$

Fit

Observables	Model	Data fit 1σ range
θ^l_{12} $/^\circ$	33.13	$32.83 \to 34.27$
θ^l_{13} $/^\circ$	8.59	$8.29 \to 8.68$
θ^l_{23} $/^\circ$	40.81	$40.63 \to 43.85$
δ^l $/^\circ$	280	$192 \to 318$
Δm^2_{21} $/\text{eV}^2$	7.58×10^{-5}	$(7.33 \to 7.69) \times 10^{-5}$
Δm^2_{31} $/\text{eV}^2$	2.44×10^{-3}	$(2.41 \to 2.50) \times 10^{-3}$
m_1 /meV	0.32	–
m_2 /meV	8.64	–
m_3 /meV	49.7	–
$\sum m_i$ /meV	58.7	< 230
α_{21} $/^\circ$	264	–
α_{31} $/^\circ$	323	–
$\lvert m_{ee}\rvert$ /meV	2.46	–

Flavon VEVs

The model

1. Aligns triplet flavons $\bar{\phi}_{\mathrm{atm,sol,dec}}$ in the CSD3 directions
2. Drives their VEVs and fixes the relative phases between them

1. **Alignment**
   Flavons $\bar{\phi}$ couple to driving fields $\bar{A}_i$ whose **F-term conditions** force $\bar{\phi}$ VEVs to be aligned along symmetry-preserving directions in flavour space.
   O-fields force orthogonality between different flavons, which completely breaks $\Delta(27)$.
2. **Driving**
   Additional interactions with driving fields fix the VEVs of $\bar{\phi}_{\mathrm{atm,sol,dec}}$, with sol phase equal to $\omega = 2\pi/3$ (from $\Delta(27)$)

Proton decay

Proton decay can be mediated by (SUSY) **dim-5 operators** like $\Psi\Psi\Psi\Psi$.

- **Forbidden at the GUT scale** by the symmetries, messenger sector.
- May be produced by operators suppressed by the Planck mass M_P.

Lowest-order non-zero term:

$$g\Psi\Psi\Psi\Psi\frac{Z\bar{\phi}_{\mathrm{dec}}\xi^3}{M_P^6} \to g\Psi\Psi\Psi\Psi\frac{\langle X\rangle}{M_P^2}$$

To obey limits for proton lifetime $\tau_p > 10^{32}$ yrs, we require

$g\langle X\rangle < 3 \times 10^9\ \mathrm{GeV}$ [Kaplan, Murayama, hep-ph/9406423]

Our model gives

$\langle X\rangle \sim 150$ GeV $\Rightarrow$ Proton decay is highly suppressed

Doublet-triplet splitting

In $SO(10)$, DT splitting may be achieved by the Dimoupoulos-Wilczek mechanism [Dimopoulos, Wilczek 1981, Srednicki 1982]:

- Introduce a field H_{DW} (a **45** of $SO(10)$), with VEV

$$\langle H_{DW} \rangle = \begin{pmatrix} 0 & \langle H_{U(5)} \rangle \\ -\langle H_{U(5)} \rangle & 0 \end{pmatrix}.$$

- Take $\langle H_{U(5)} \rangle \propto \mathrm{diag}(1,1,1,0,0)$
  $\Rightarrow$ only terms coupling triplets survive.

H_{10}^{u}, H_{10}^{d} and $H_{16,\overline{16}}$ all contain $SU(3)$ triplets. After GUT breaking, we find that all Higgs triplets have GUT scale masses.

Doublet-doublet splitting and the μ term

H_{10}^u, H_{10}^d and $H_{16,\overline{16}}$ all contain $SU(2)$ doublets.

We only expect **two at the MSSM level**.

All others should be at least unification scale.

Introducing specific **messenger fields** Z_i, Σ_i that couple pairs of H fields to powers of ξ, we arrive at a superpotential

$$\mathcal{W}_\mu \sim Z H_{10}^u H_{10}^u \frac{\xi^6}{M_Z^6} + Z H_{10}^u H_{10}^d \frac{\xi^7}{M_Z^7} + Z H_{10}^d H_{10}^d \frac{\xi^8}{M_Z^8} + \xi H_{16} H_{\overline{16}} + \frac{Z}{M_\Sigma}\left(H_{16} H_{16} H_{10}^d + \frac{\xi^8}{M_\Sigma^8} H_{16} H_{16} H_{10}^u + H_{\overline{16}} H_{\overline{16}} H_{10}^u + \frac{\xi}{M_\Sigma} H_{\overline{16}} H_{\overline{16}} H_{10}^d \right)$$

Doublet-doublet splitting and the μ term

From that superpotential, may write the $SU(2)$ doublet mass matrix as:

$$M_D \sim \begin{array}{c} \\ H_d^u \\ H_d^d \\ H_d^{16} \end{array} \begin{array}{c} \begin{array}{ccc} H_u^u & H_u^d & H_u^{\overline{16}} \end{array} \\ \begin{pmatrix} \tilde{\xi}^6 & \tilde{\xi}^7 & \tilde{H}_{\overline{16}} \\ \tilde{\xi}^7 & \tilde{\xi}^8 & \tilde{\xi}\tilde{H}_{\overline{16}} \\ \tilde{H}_{16}\tilde{\xi}^8 & \tilde{H}_{16} & \xi/M_{\mathrm{GUT}} \end{pmatrix} \end{array} M_{\mathrm{GUT}}$$

where $\tilde{\xi} \equiv \dfrac{\langle \xi \rangle}{M_{\mathrm{GUT}}} \sim 0.1.$

Eigenvalues: $m_D \sim \tilde{\xi} M_{\mathrm{GUT}}, \quad \tilde{\xi} M_{\mathrm{GUT}}, \quad \tilde{\xi}^8 M_{\mathrm{GUT}}.$

MSSM μ term: $\dfrac{\langle \xi \rangle^8}{M_{\mathrm{GUT}}^7} H_d^d H_u^u \ll M_{\mathrm{GUT}}$

$\Rightarrow$ explains the smallness of the μ term.

Notes

Many open questions in HEP and model-building.

- Flavour GUTs can answer many of these questions!

The two models presented here are among the most **complete** and **realistic** models:

- Renormalisable!
- Good fits to data, with some tension that may allow for future tests of the models.
- But: they require a large GUT-scale field content, as well as SUSY (which has not yet been found!)

Conclusion

Why are there three generations of fermions?	✓
Are neutrinos Majorana or Dirac fermions?	✓
Why is there such a strong mass hierarchy?	✓
What is the origin of large lepton mixing?	✓
How large is leptonic CP violation?	✓
Why is there a BAU?	?
Why do the gauge couplings appear to converge?	✓
+	
What is the lifetime of the proton?	✓
How is doublet-triplet splitting achieved?	✓
What is the scale of the MSSM μ-term?	✓

Thank you!