HYDRODYNAMICS OF ROTATING STARS

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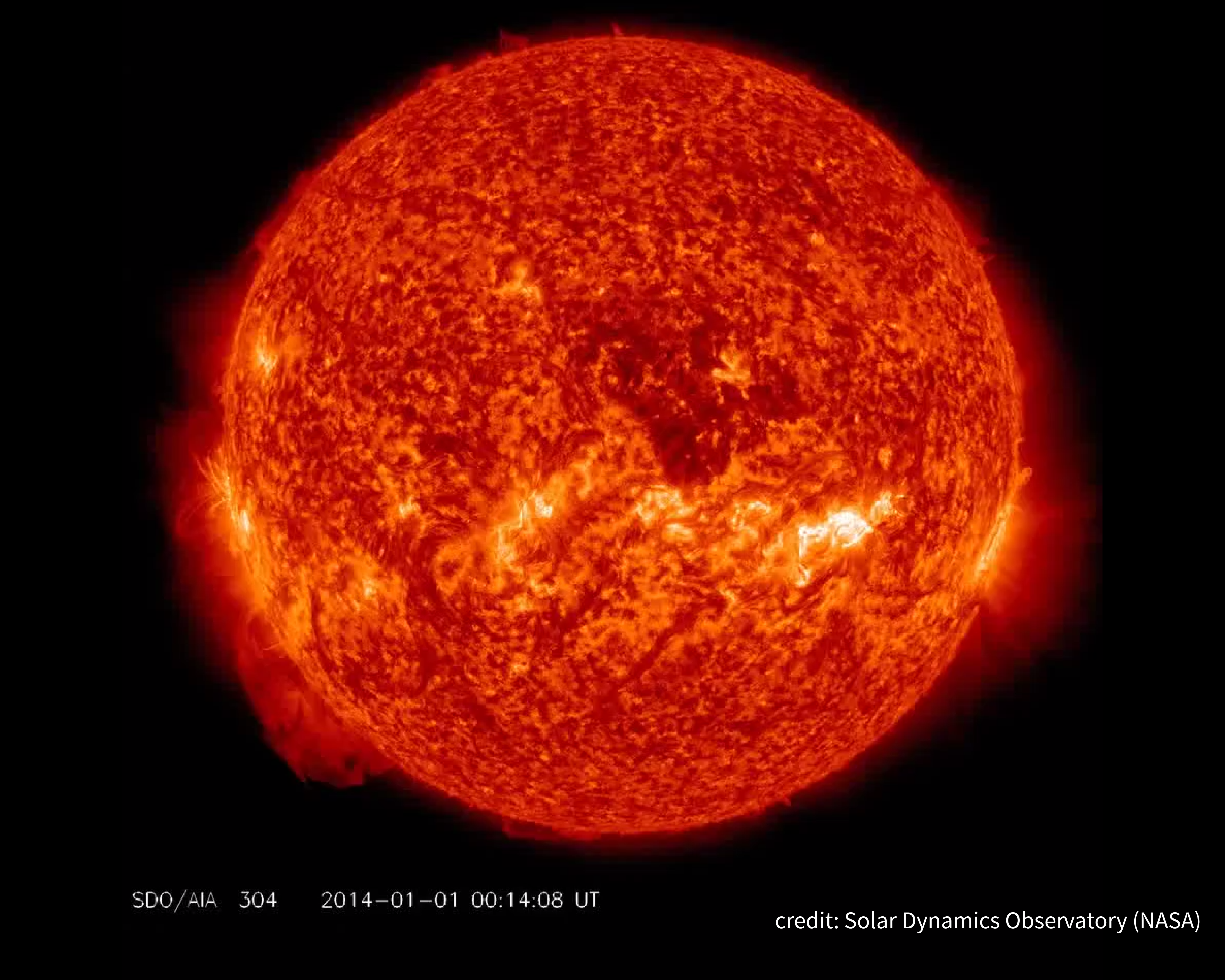
from January: Newcastle University

WHY SHOULD WE CONSIDER ROTATING STARS?



credit: Solar Dynamics Observatory (NASA)

IMPORTANCE OF ROTATION IN STARS

- change in mechanical structure
- additional instabilities and mixing
- initial rotation and magnetic field of supernova progenitors

ROTATION PROFILES IN STARS

based on A. Maeder “Physics, Formation and Evolution of Rotating Stars” (2009)

Roche approximation:
treat gravity as if mass were located at centre of star

CYLINDRICAL ROTATION

- Ω is constant on cylinders of constant $\varpi = r\sin\vartheta$.
- hydrostatic equilibrium:
  $\nabla P = -\rho\nabla\Phi + \frac{1}{2}\Omega^2\nabla\varpi^2$
  with $\varpi = r\sin\vartheta$
- can be written as
  $\nabla P = -\rho\nabla\Psi$ with $\Psi = \Phi - \frac{1}{2}\Omega^2\varpi^2$
- barotropic: equipotentials and isobars coincide

SHELLULAR ROTATION

- Ω is constant on isobars ($P = \mathrm{const}$).
- centrifugal force cannot be derived from potential
- hydrostatic equilibrium:
  $\nabla P = -\rho \nabla \Psi + \varpi^2 \Omega \nabla \Omega$
  with $\Psi = \Phi - \frac{1}{2}\Omega^2 \varpi^2$
- baroclinic: equipotentials and isobars do **not** coincide
- density and temperature vary on shell

WHICH ONE IS REALIZED?

- turbulent mixing is more efficient in horizontal direction (i.e. on isobars) Zahn (1992)
- magnetic fields might produce models closer to $\Omega = \mathrm{const.}$.
- convection zones might break shellular rotation
- strong rotation can lead to Rayleigh–Taylor unstable models

ROTATION IN STELLAR MODELS

TIMESCALES

- free-fall timescale: $\tau_{\mathrm{ff},\odot} = 27\,\mathrm{min}$
- Kelvin–Helmholtz timescale: $\tau_{\mathrm{KH},\odot} = 2 \times 10^7\ \mathrm{yr}$
- nuclear timescale: $\tau_{\mathrm{nuc},\odot} = 10^{11}\ \mathrm{yr}$

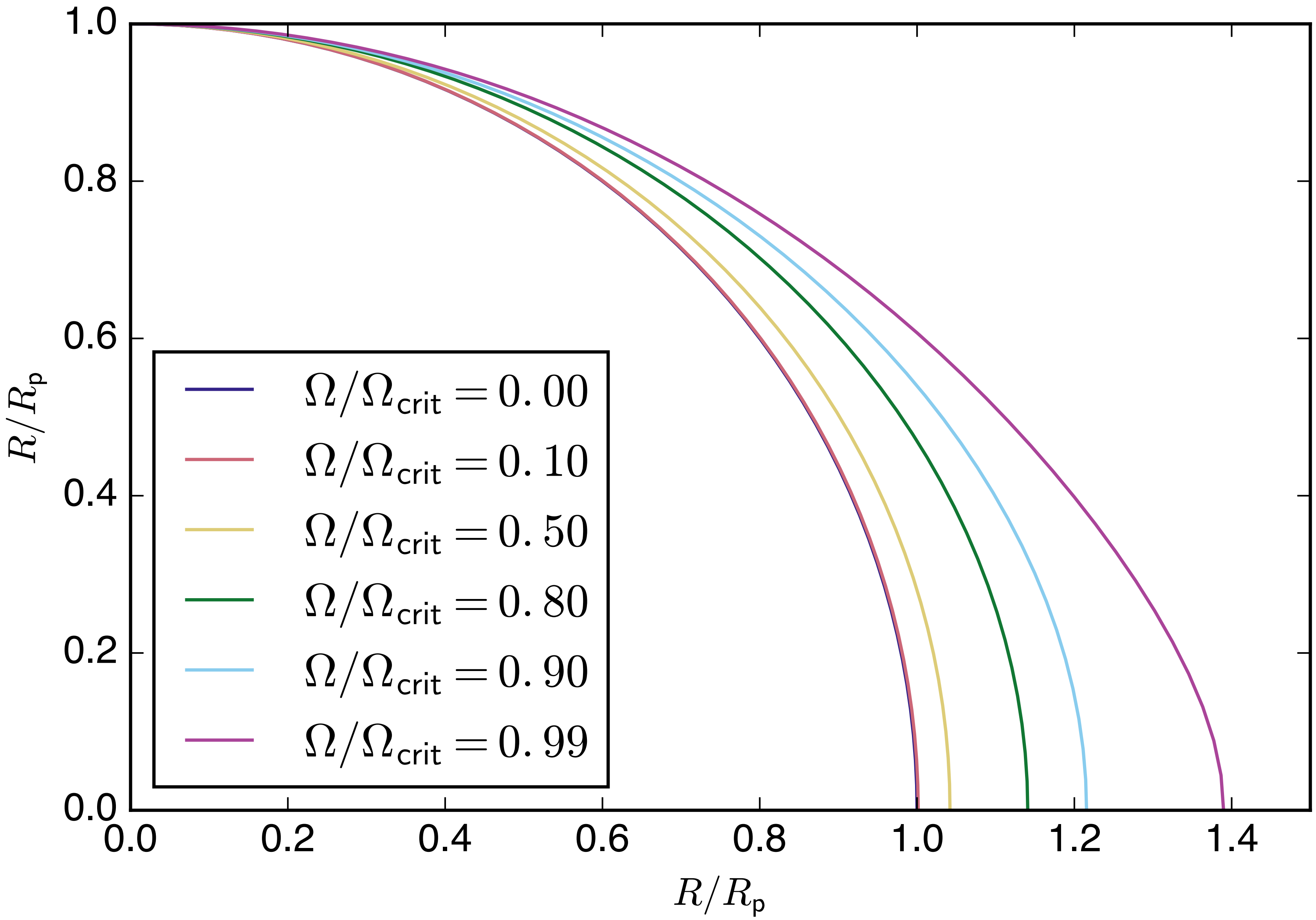
$$\tau_{\mathrm{nuc},\odot} = 10^3\ \tau_{\mathrm{KH},\odot} = 10^{15}\ \tau_{\mathrm{ff},\odot}$$

ONE-DIMENSIONAL MODELS

- purely hydrostatic models needed to bridge vast timescales
- non-rotating hydrostatic stars are spherical
- hydrodynamic phenomena (e.g. convection) are treated using prescriptions (MLT, etc.)

- rotating stars are not spherical
- use isobars as coordinate instead

DEFORMATION OF A ROTATING STAR



STELLAR STRUCTURE

$$\frac{dr}{dM} = \frac{1}{4\pi r^2 \rho}$$
$$\frac{dP}{dM} = -\frac{GM}{4\pi r^4}$$
$$\frac{dL}{dM} = \epsilon_{\rm nuc} - \epsilon_\nu + \epsilon_{\rm grav}$$
$$\frac{d\ln T}{dM} = -\frac{GM}{4\pi r^4} \min\left[\nabla_{\rm ad}, \nabla_{\rm rad}\right]$$

$$\frac{dr_P}{dM_P} = \frac{1}{4\pi r_P^2 \bar{\rho}}$$
$$\frac{dP}{dM_P} = -\frac{GM_P}{4\pi r_P^4} f_P$$
$$\frac{dL_P}{dM_P} = \epsilon_{\rm nuc} - \epsilon_\nu + \epsilon_{\rm grav}$$
$$\frac{d\ln T}{dM_P} = -\frac{GM_P}{4\pi r_P^4} f_P \min\left[\nabla_{\rm ad}, \nabla_{\rm rad}\frac{f_T}{f_P}\right]$$

INSTABILITIES

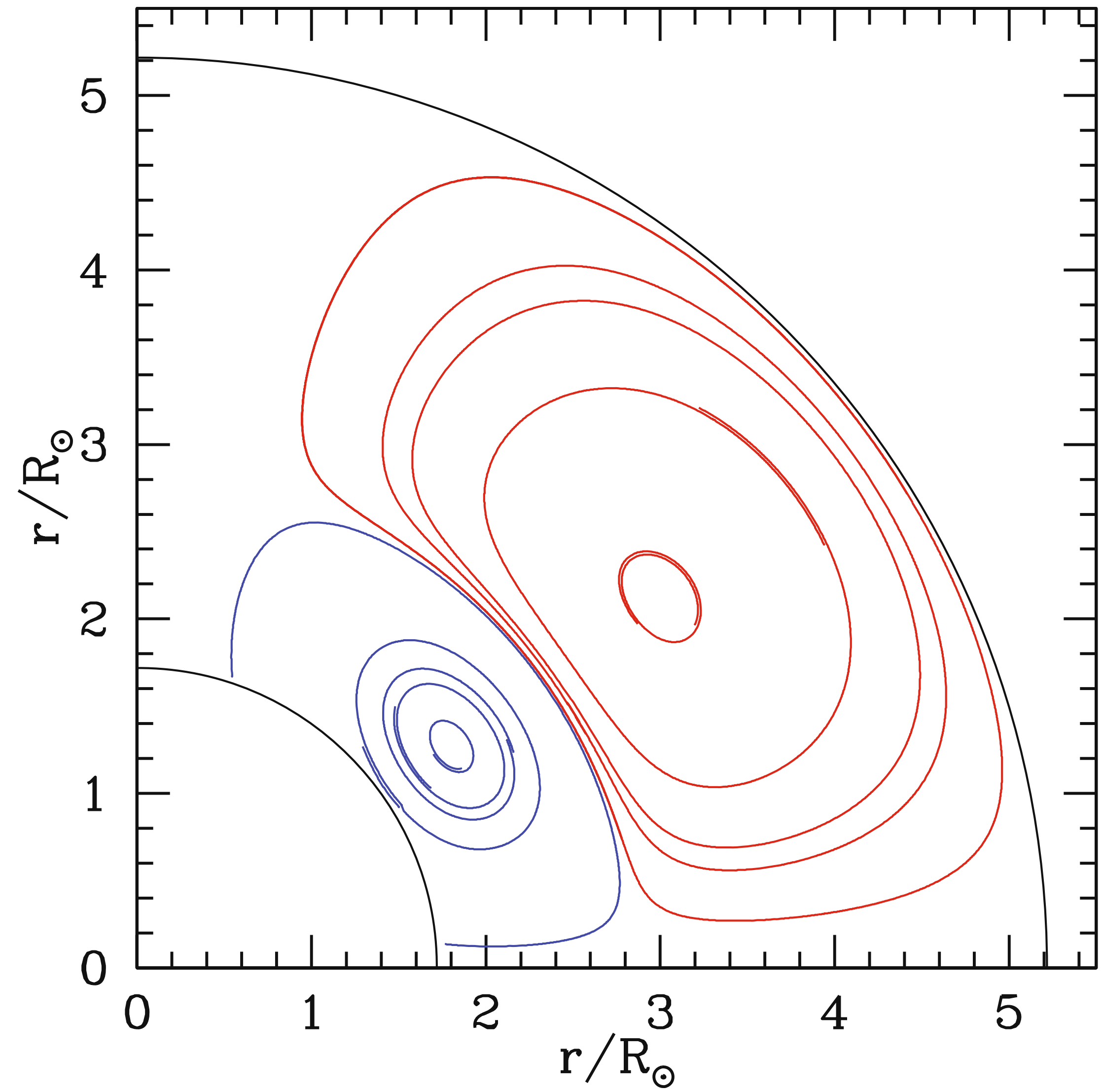
- rotation can cause many new types of instabilities that cause additional mixing
  e.g. dynamical shear, secular shear, GSF, ABCD,…
- hydrostatic 1D models cannot treat these self-consistently
- usually solved by prescribing a diffusion coefficient

EXAMPLE FOR DYNAMICAL SHEAR

- Richardson number $Ri = \frac{N^2}{(\partial v / \partial r)^2}$
- unstable if $Ri < Ri_c = \frac{1}{4}$
- apply $D = \frac{1}{3} r \Delta\Omega \Delta r$ in unstable zones

MERIDIONAL CIRCULATION

- driven by thermal imbalance on isobars
- studied since Eddington (1928)
- self-consistent theory (in framework of shellular rotation) by Zahn (1992)
- leads to transport of composition and angular momentum



credit G. Meynet

2D STELLAR MODELS

- need at least two spatial dimensions to describe structure of rotating star
- ESTER code built to produce self-consistent 2D steady-state solutions of rotating stars
  (Espinosa Lara & Rieutord, 2013)
- convection zones can be confined to certain latitudes
- meridional circulation
- no time evolution done so far
- hydrodynamical instabilities still need to be prescribed

HYDRODYNAMICS

- hydrodynamics is a first-principle verification of stellar models
- long timescales involved make it difficult to approach

SEVEN-LEAGUE HYDRO CODE

- solves the compressible Euler equations in 1-, 2-, 3-D
- explicit and implicit time integration
- flux preconditioning to ensure correct behaviour at low Mach numbers
- other low Mach number schemes (e.g. AUSM^{+}-up)
- works for low and high Mach numbers on the same grid
- hybrid (MPI, OpenMP) parallelization (works up to 458 752 cores)
- several solvers for the linear system:
  BiCGSTAB, GMRES, Multigrid, (direct)
- arbitrary curvilinear meshes
  using a rectangular computational mesh
- gravity solver (monopole, Multigrid)
- radiation in the diffusion limit
- general equation of state
- general nuclear reaction network

DYNAMICAL SHEAR INSTABILITIES

collaborators:
Raphael Hirschi (Keele) and Cyril Georgy (Geneva)
Friedrich Röpke (Heidelberg), Leonhard Horst (Heidelberg)

ROTATIONAL INSTABILITIES AND THEIR TREATMENT IN 1D CODES

- instabilities: dynamical shear, secular shear, GSF, …
- usually treated as diffusion in SE codes

E.G. DYNAMICAL SHEAR

- Richardson number $Ri = \frac{N^2}{(\partial v / \partial r)^2}$
- unstable if $Ri < Ri_c = \frac{1}{4}$
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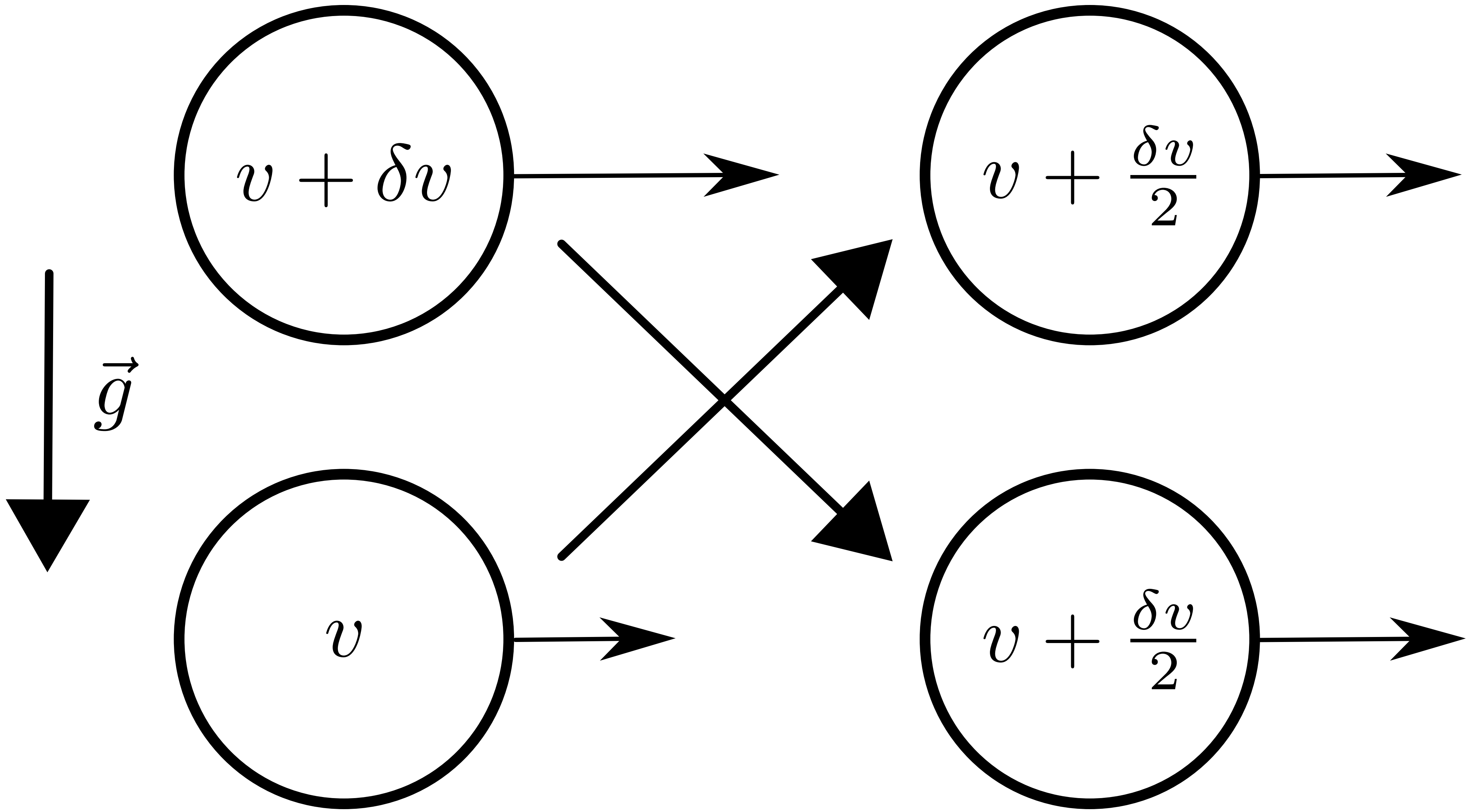


energy conservation: $Ri = \left(\frac{N^2}{\varpi \frac{\partial \Omega}{\partial r}} \right) < \frac{1}{4}$ for instability

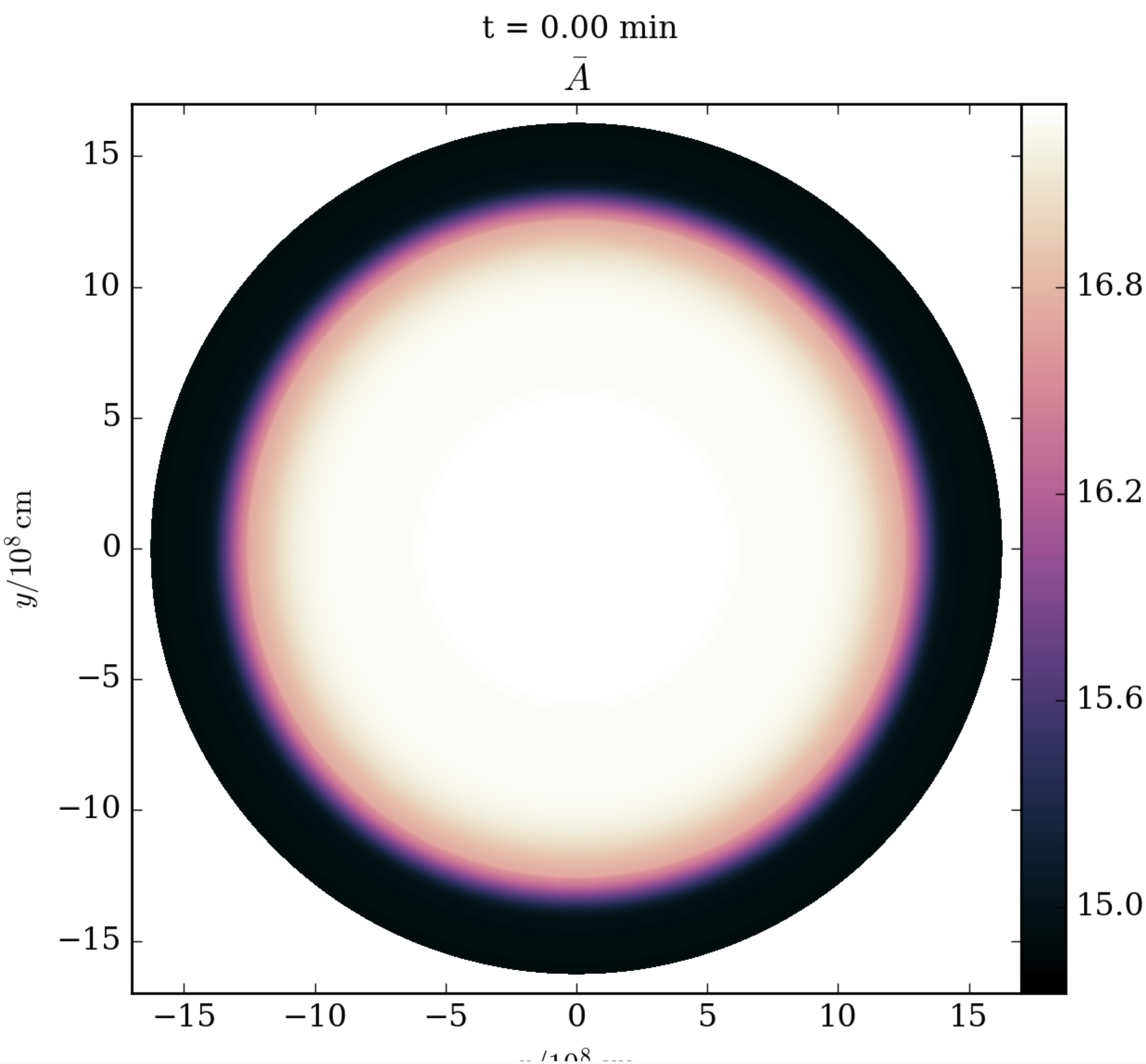
THE STELLAR MODEL

- should be shear unstable in stellar evolution code
- should not show other instabilities at the same time
- should be on similar time scale in stellar evolution and hydro code

- Geneva stellar evolution code
- $20\ M_\odot$ ZAMS star, 40% crit. rotation
- post core O burning phase
- C/Ne shell interface
- convectively stable
- Ri unstable

SIMULATIONS WITH SLH

- 2D equatorial plane
- more than 6 hours of physical time
- special mapping of GENEC data to keep convective stability



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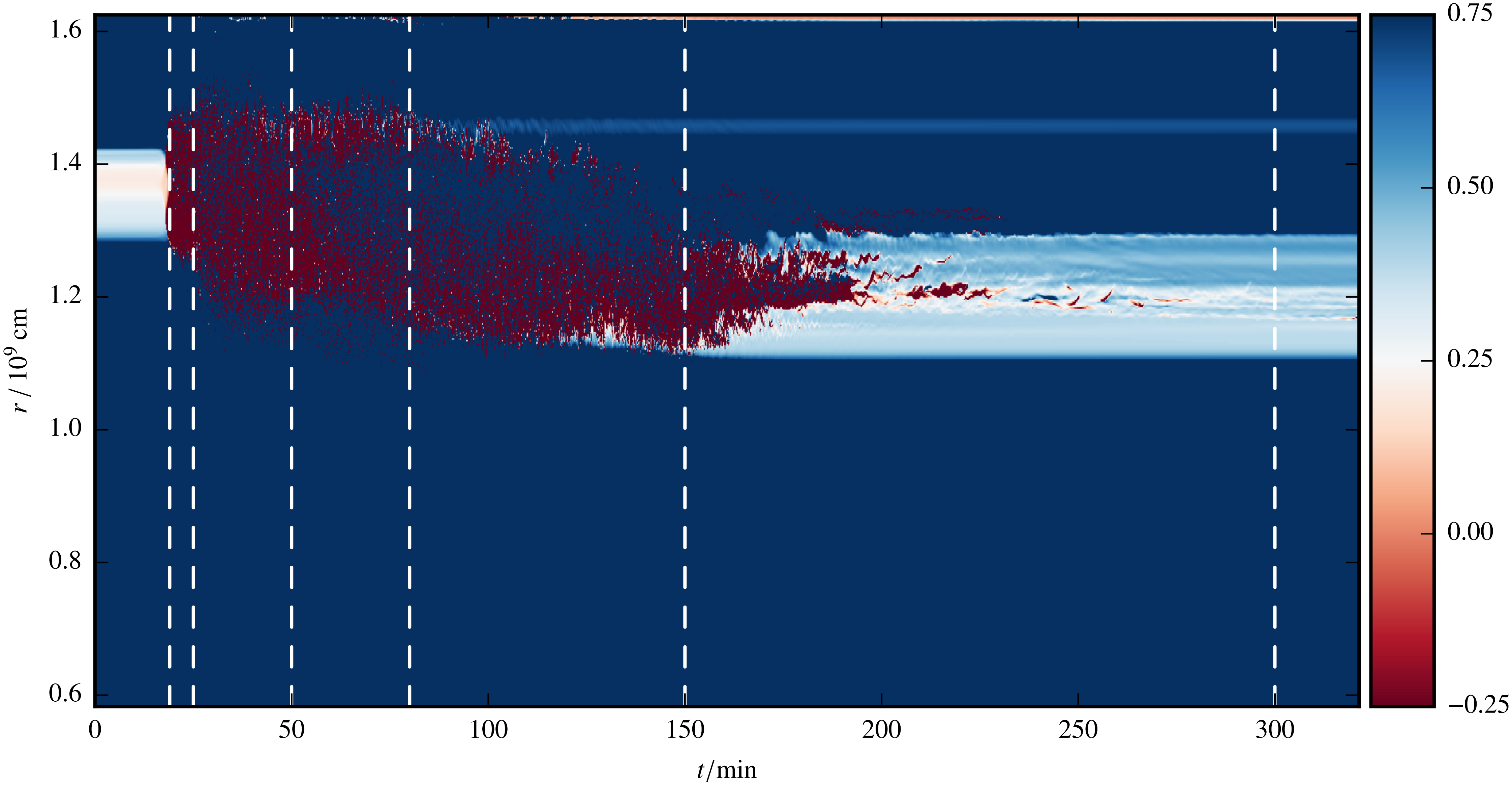
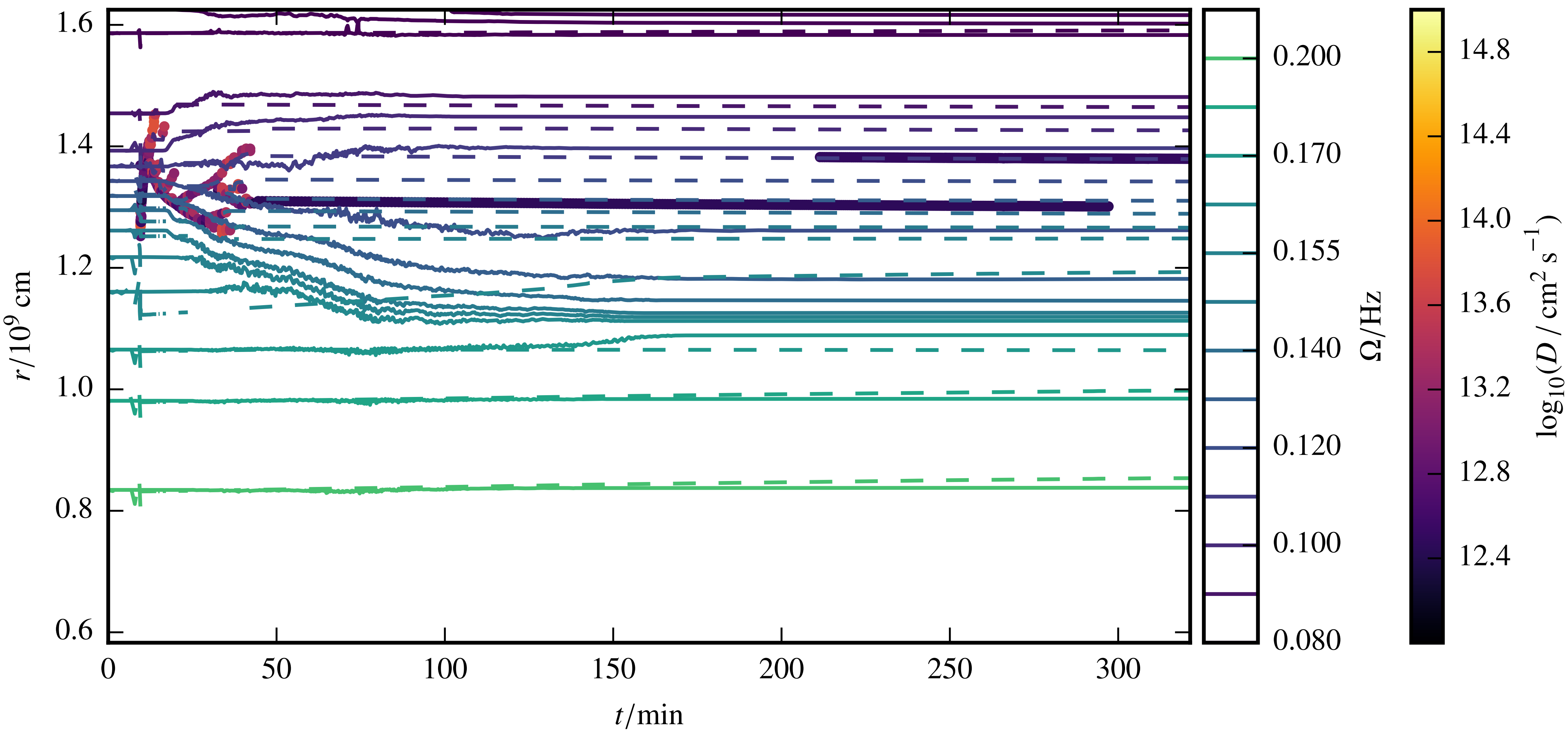
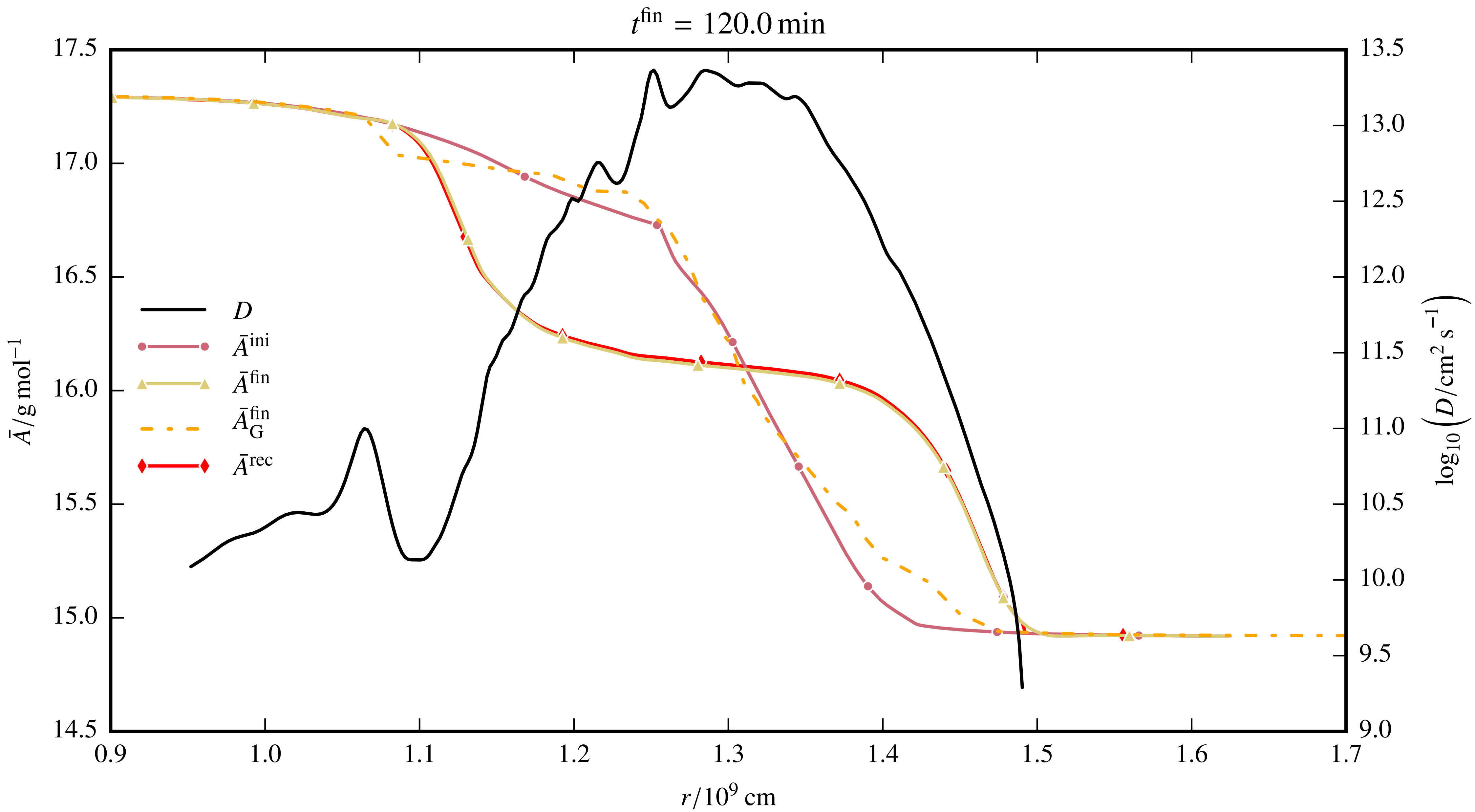
RICHARDSON NUMBER



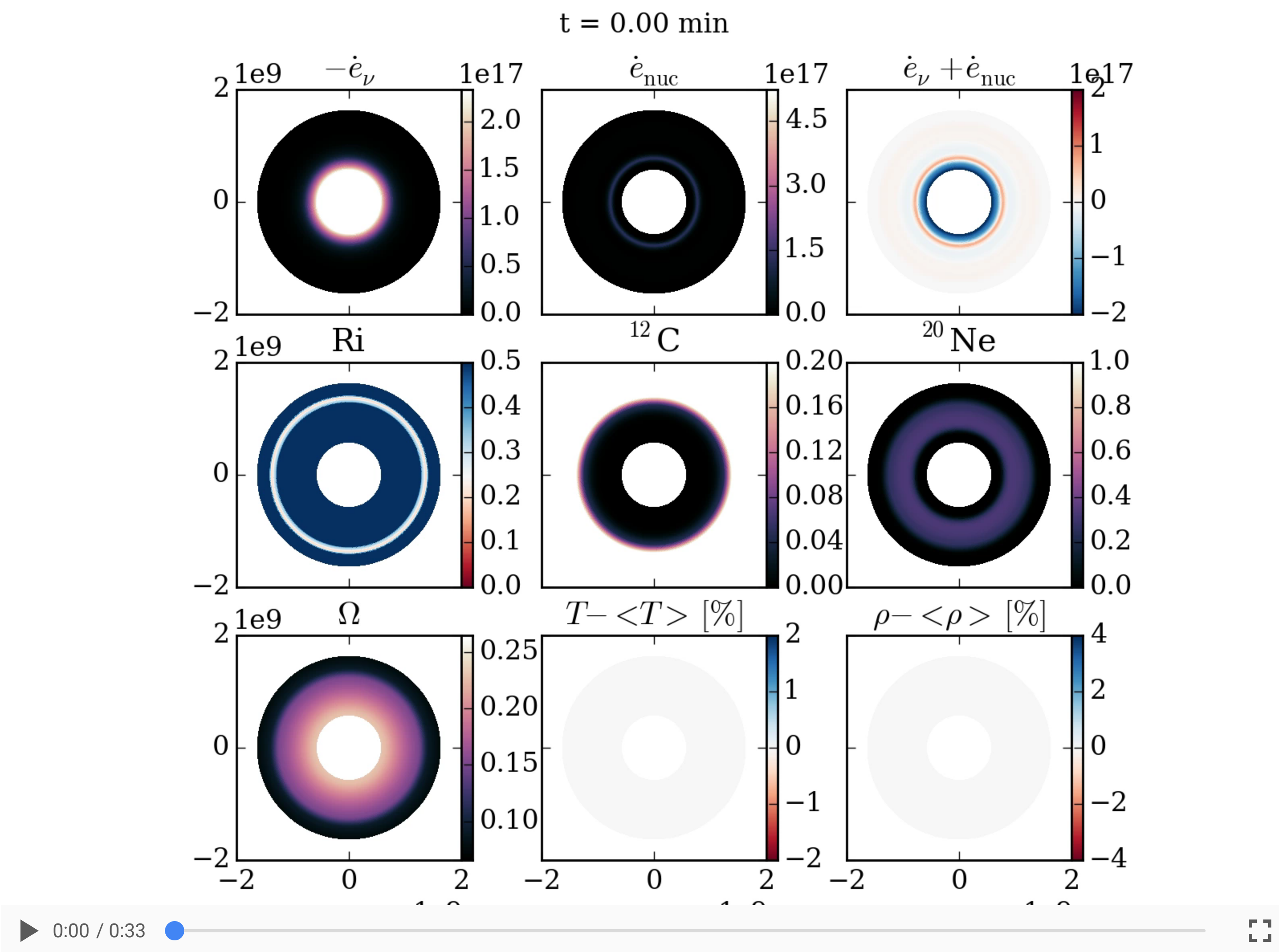
ANGULAR VELOCITY



EFFECTIVE DIFFUSION COEFFICIENT



SOURCE TERMS



CONCLUSIONS

- rotation is an important effect in stellar models
- many simplifications are needed to include it in 1D models
- 2D models are a significant improvement but far from being ready to replace 1D models
- hydrodynamics can be used to study instabilities and improve prescriptions
- dynamical shear mixing might be stronger than predicted by 1D prescription