Institute for the Physics and Mathematics of the Universe, 20 October, 2009, Tokyo

Finite size effects in integrable models: planar AdS/CFT

Zoltán Bajnok,

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II_B superstring on $AdS_5 \times S^5$		$\mathcal{N}=4$ D=4 $SU(N)$ SYM
$\sum_1^5 Y_i^2 = R^2 \qquad -+++-=-R^2$	$\equiv$	$A_\mu, \Psi_i, \overline{\Psi}_i, \Phi_a$
$\frac{R^2}{\alpha'}\int \frac{d\tau d\sigma}{4\pi}\left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M\right)+\dots$		$\frac{2}{g_{YM}}\int d^4x \mathrm{Tr}\left[-\frac{1}{4}F^2 - \frac{1}{2}(D\Phi)^2 + i\overline{\Psi}\not{D}\Psi + V\right]$
		$V(\Phi,\Psi)=\frac{1}{4}[\Phi,\Phi]^2+\overline{\Psi}[\Phi,\Psi]$
		gaugeinvariant ope: $\mathcal{O}=Tr(\Phi^2)$

Couplings: $\sqrt{\lambda}=\frac{R^2}{\alpha'}$, $g_s=\frac{\lambda}{N}\to 0$	strong$\leftrightarrow$weak	$\lambda = g_{YM}^2 N$, $N\to\infty$ planar
QM for string: 2D field theory	$\Downarrow$	$\langle \mathcal{O}_n(x)\mathcal{O}_m(0)\rangle = \frac{\delta_{nm}}{\vert x\vert^{2\Delta_n(\lambda)}}$
String spectrum $E(\lambda)$		Anomalous dim $\Delta(\lambda)$
$E(\lambda)=E(\infty)+\frac{E_1}{\sqrt{\lambda}}+\frac{E_2}{\lambda}+\dots$		$\Delta(\lambda)=\Delta(0)+\lambda\Delta_1+\lambda^2\Delta_2+$

AdS $\longleftrightarrow$ integrable model $\longleftrightarrow$ CFT

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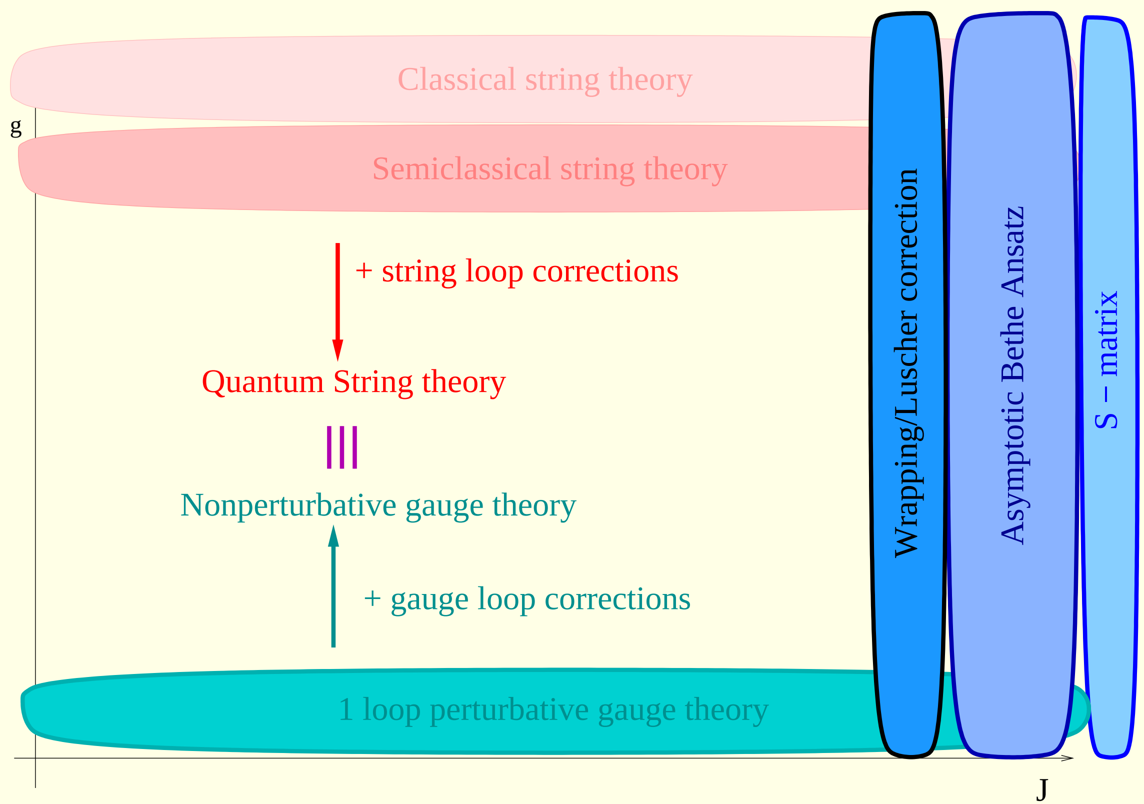

Finite J (volume) integrable models: (Lee-Yang, sinh-Gordon, sine-Gordon)$\longleftrightarrow$AdS/CFT

Motivation: AdS/CFT

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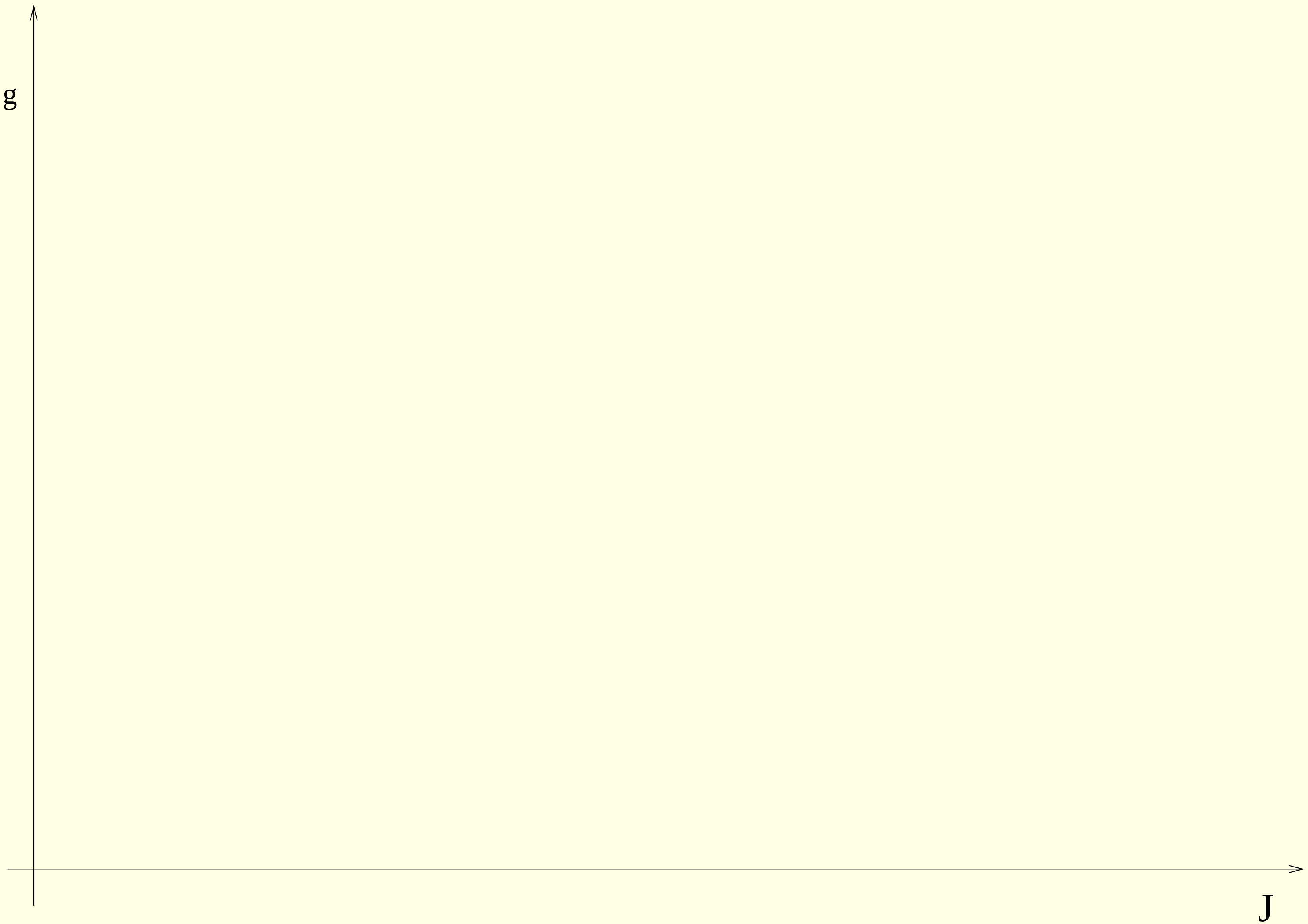

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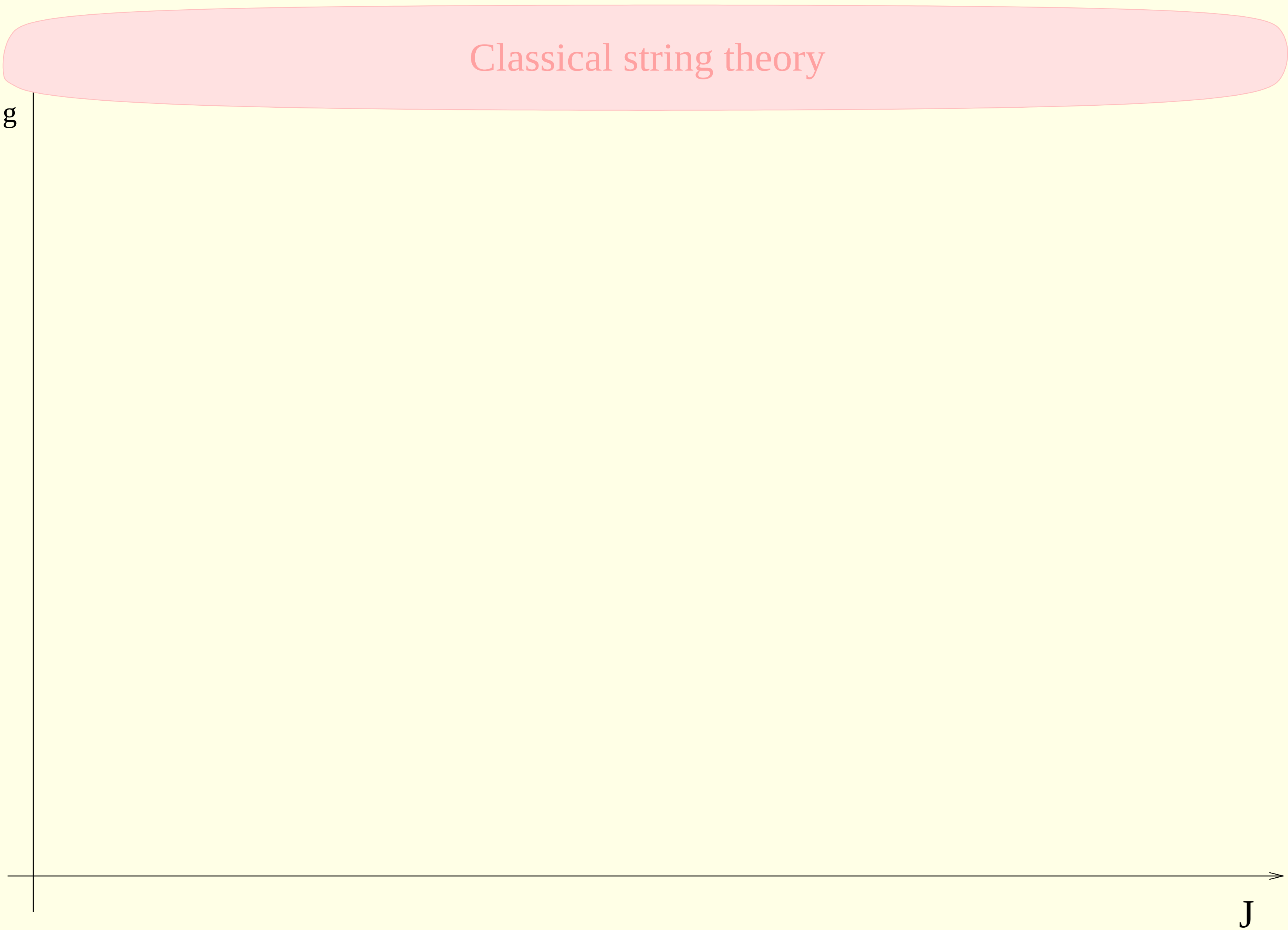

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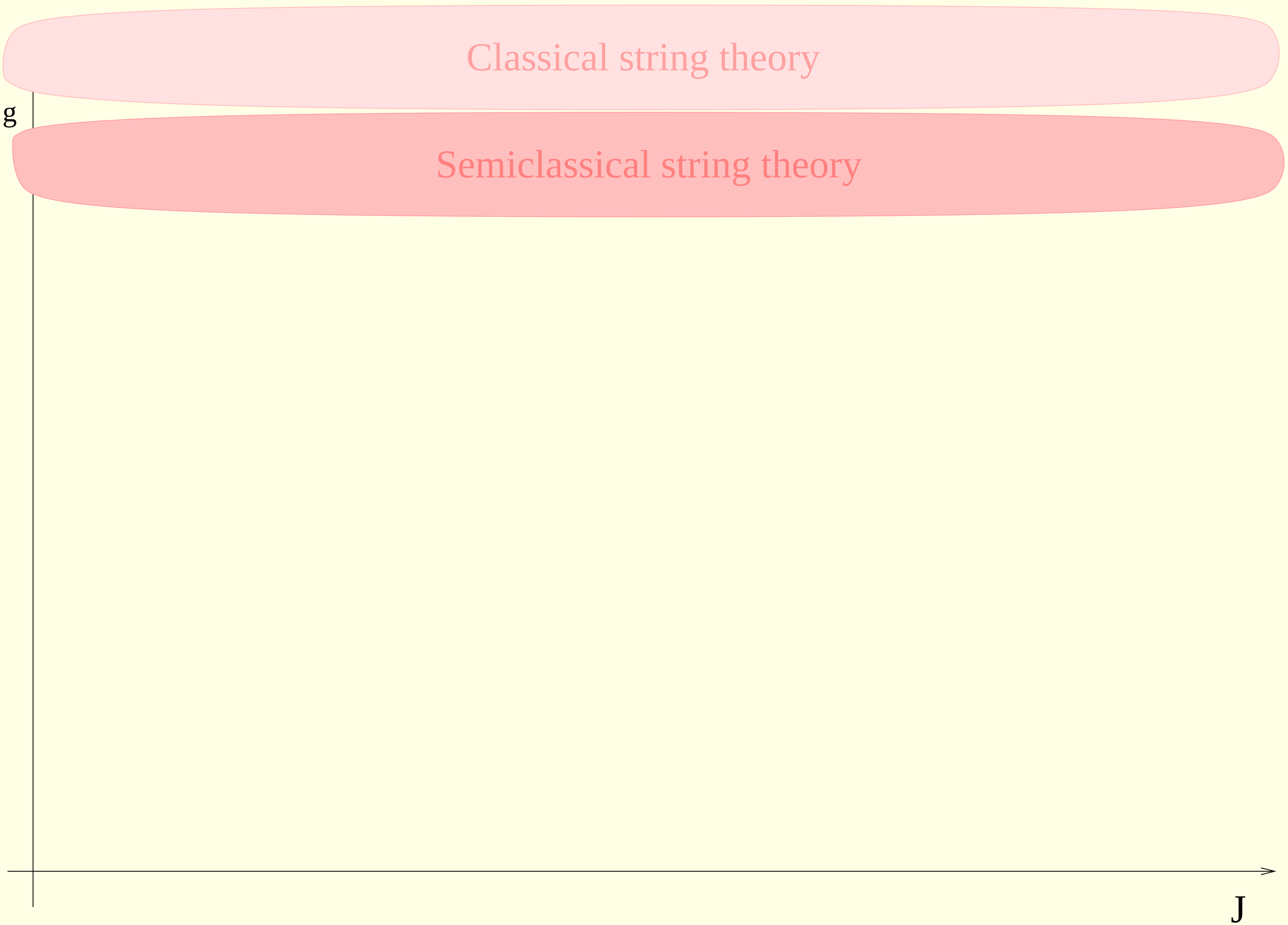

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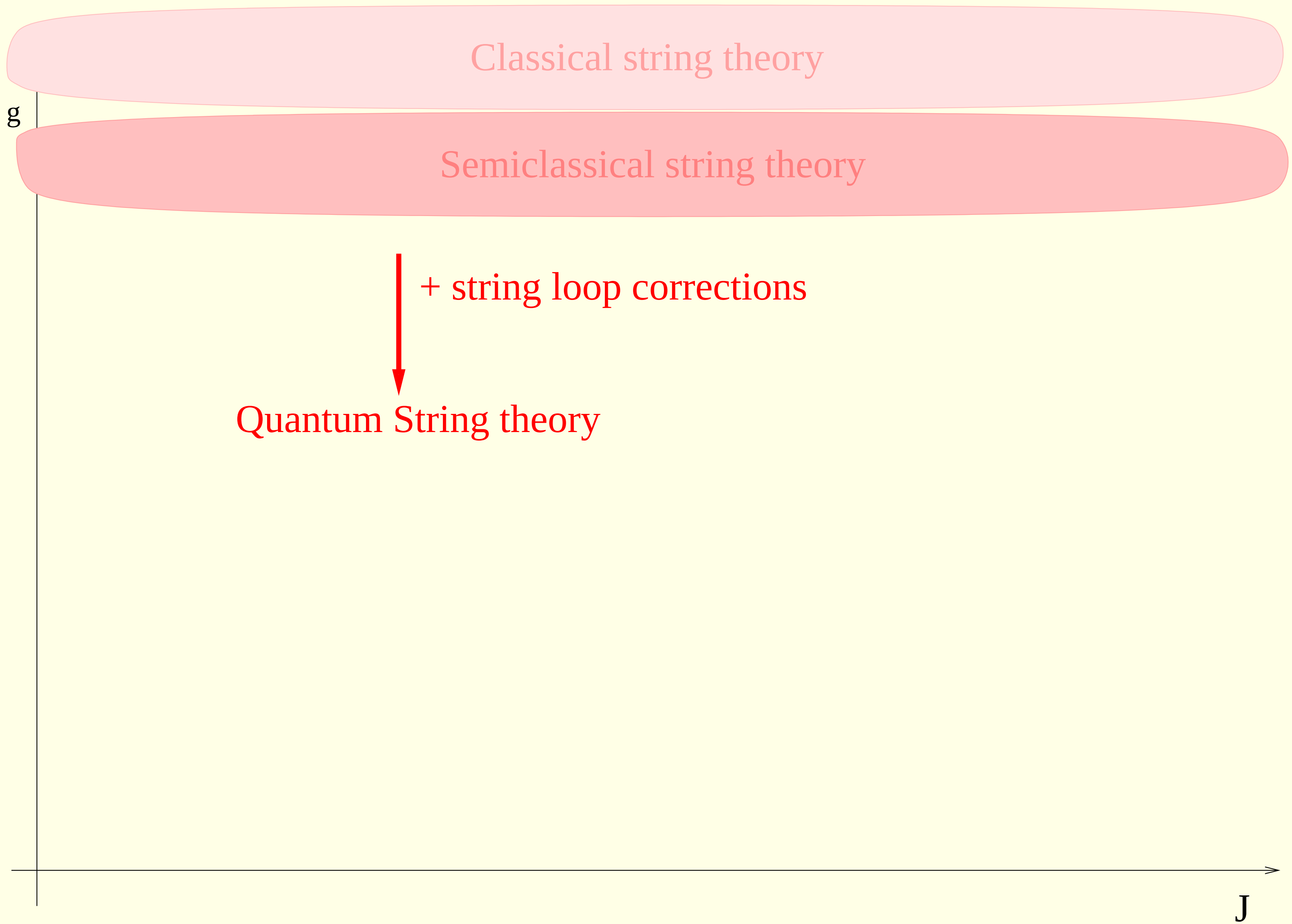

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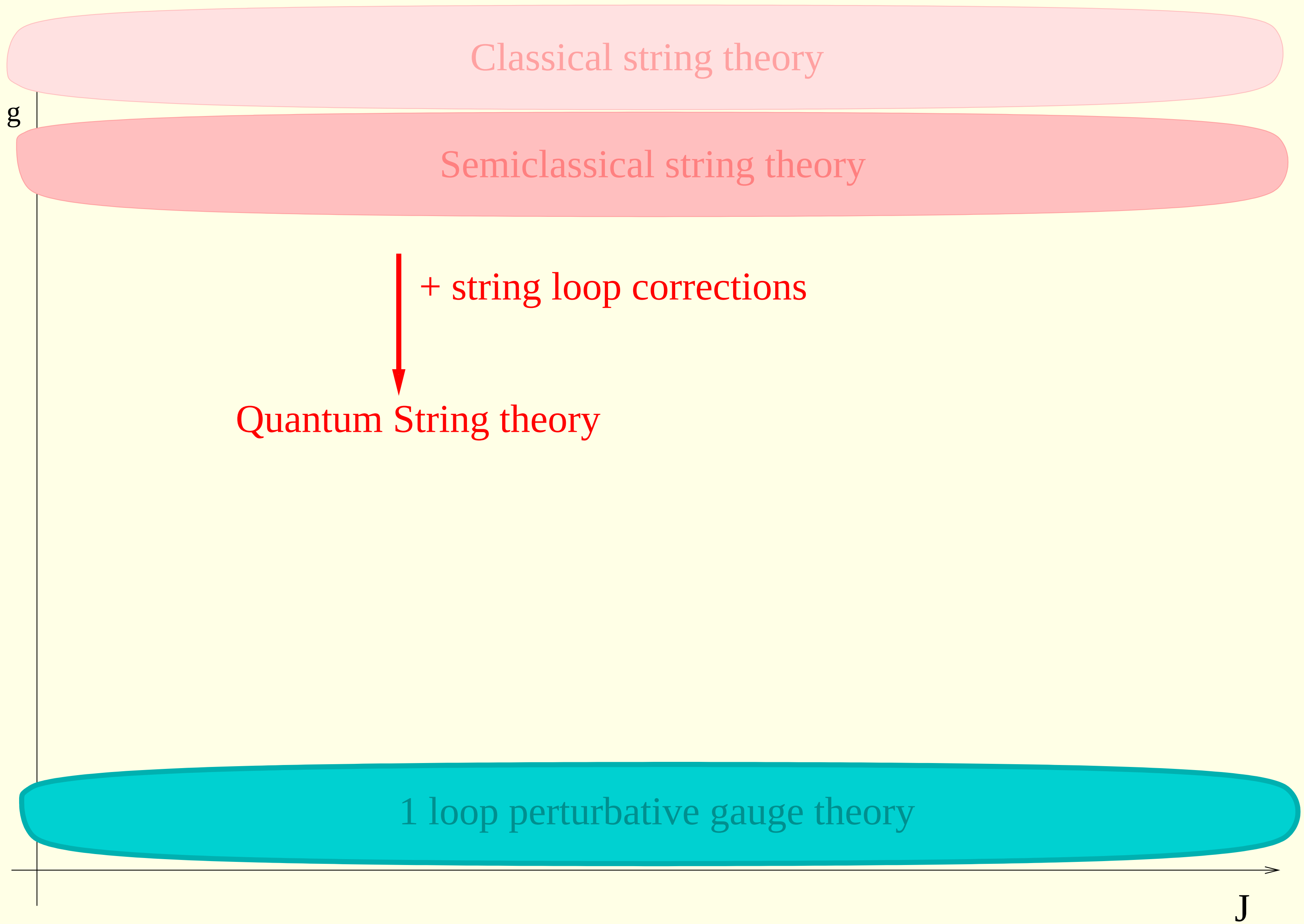

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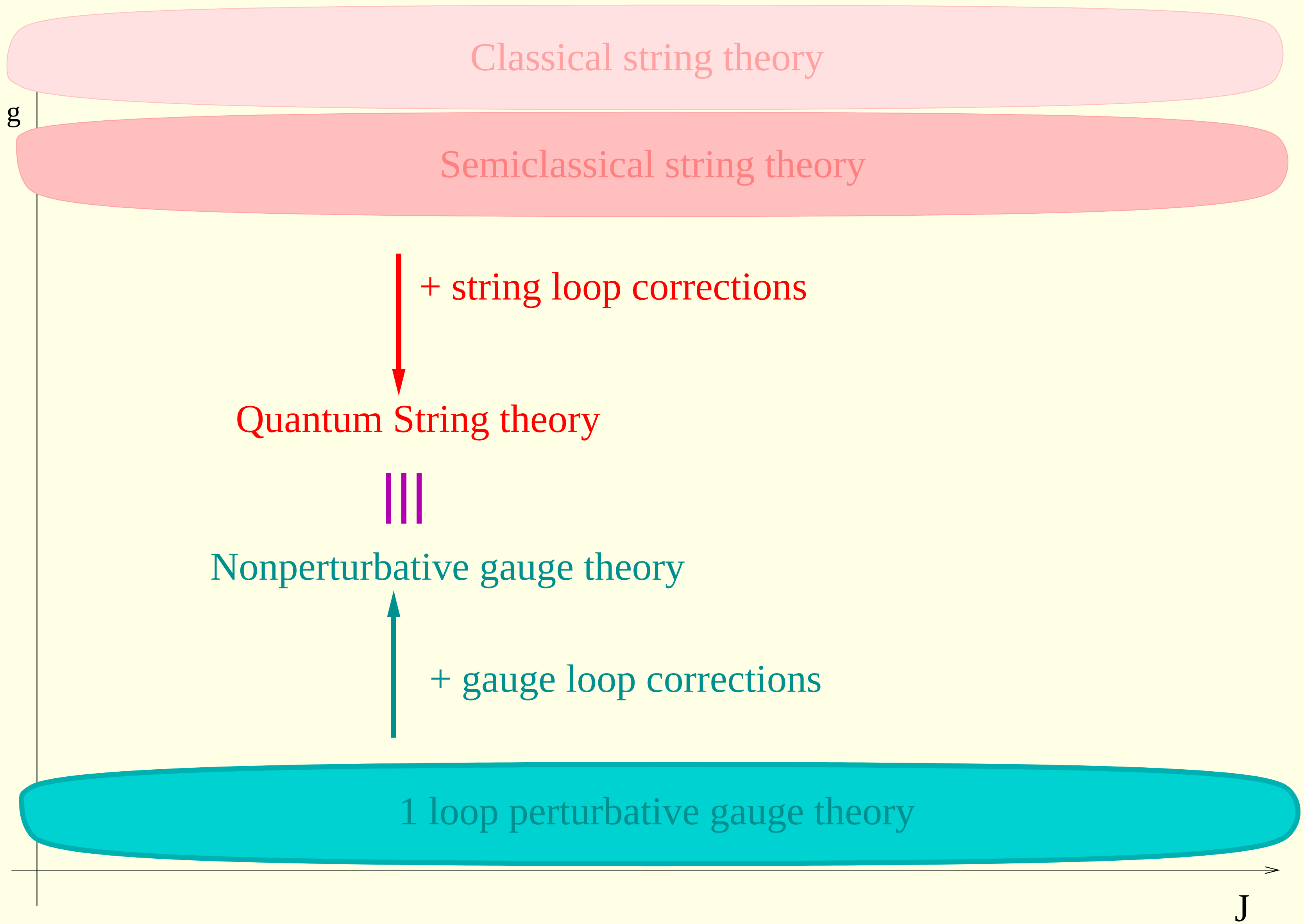

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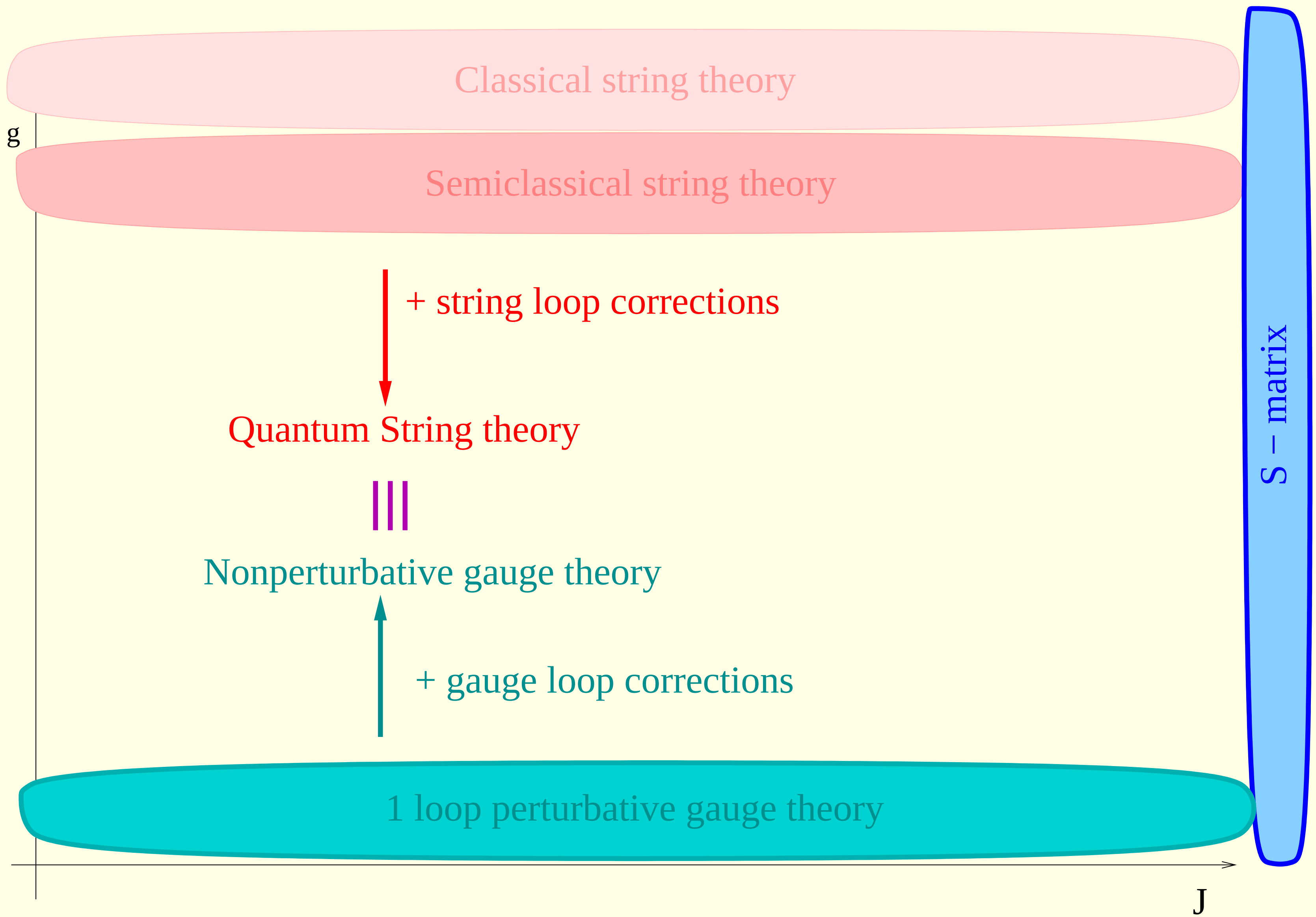

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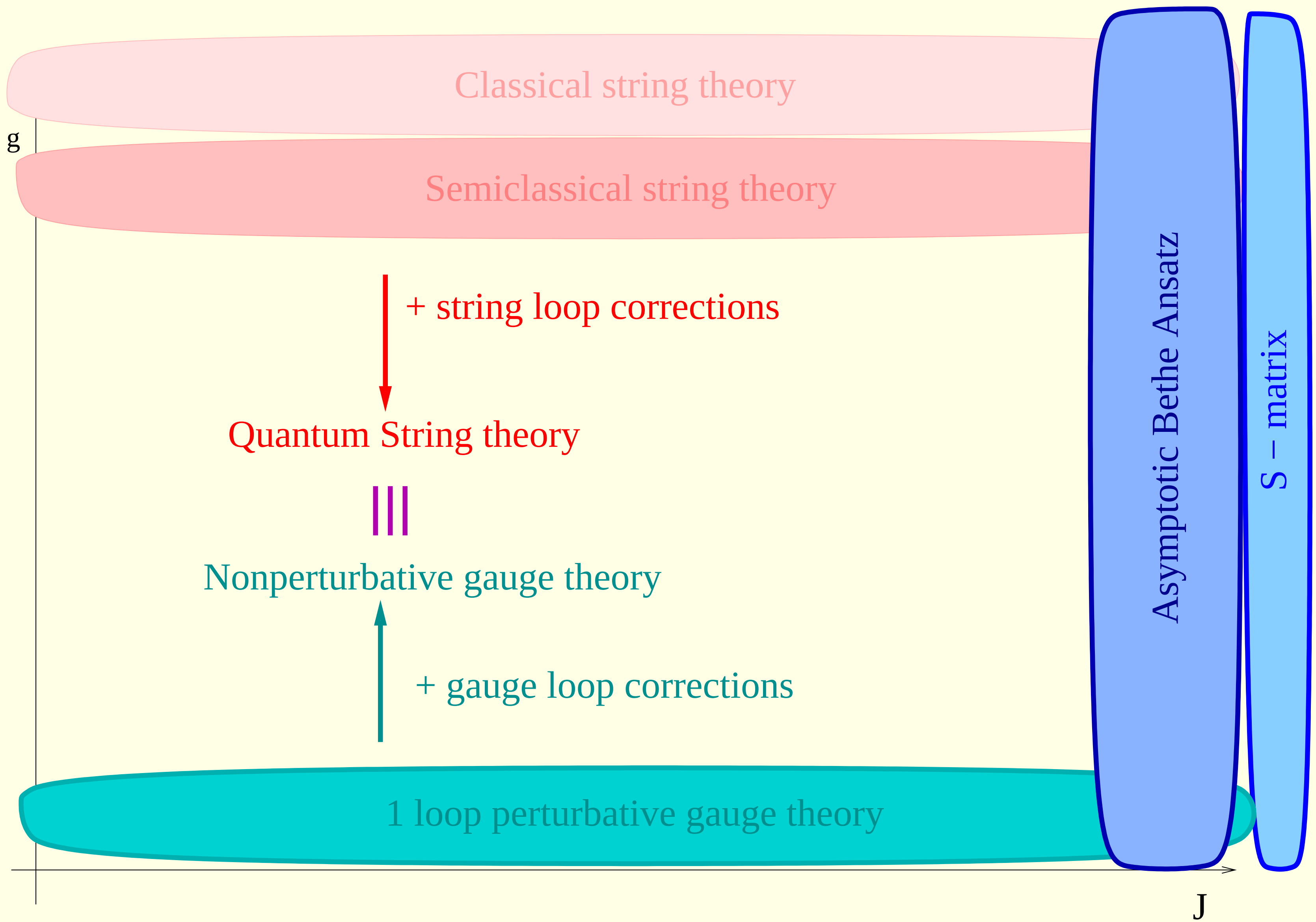

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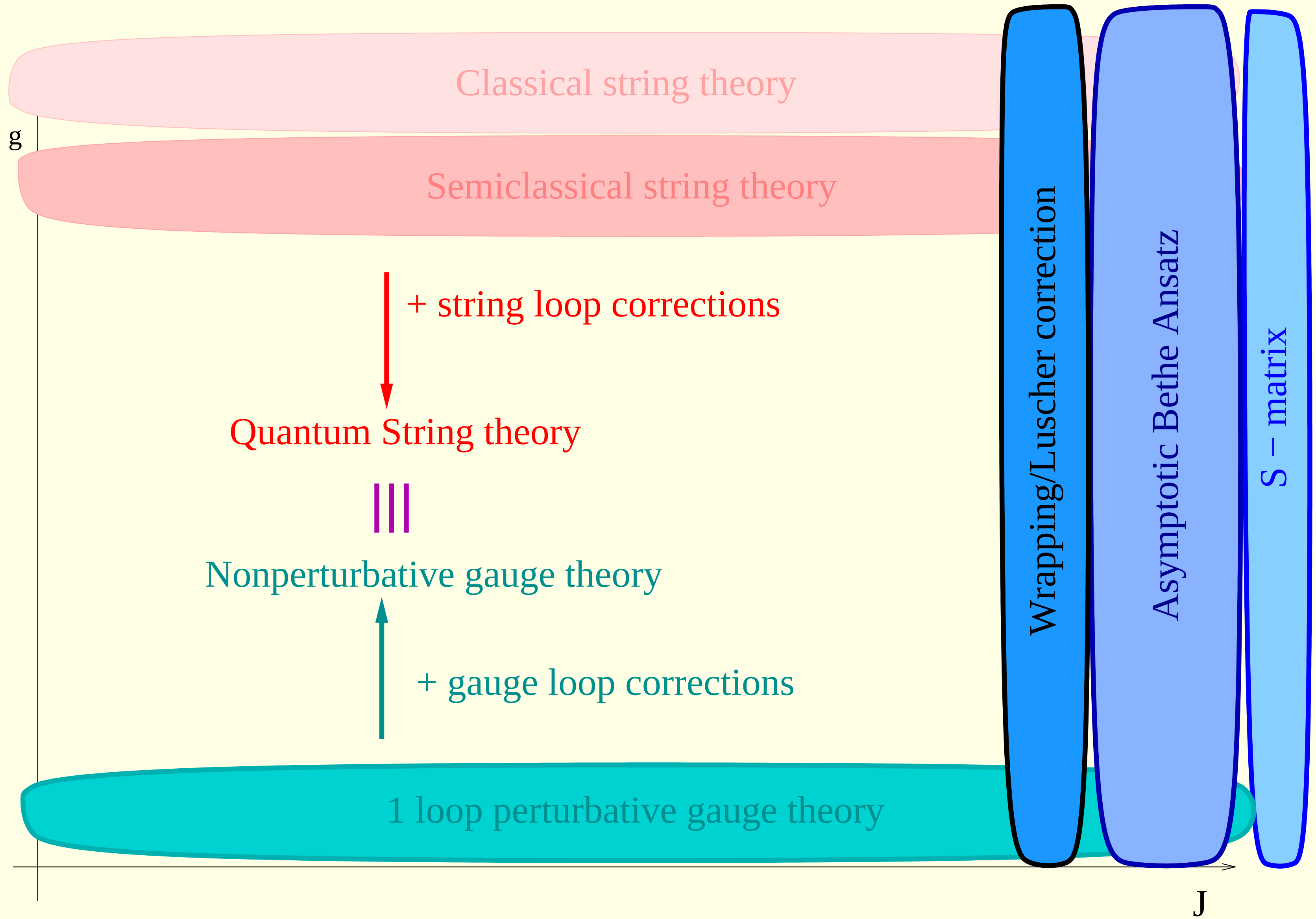

Need finite J (volume) solution of the spectral problem

Plan of talk I: Finite size effects in integrable models

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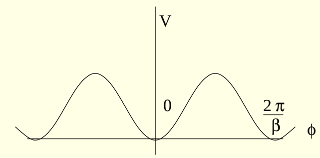
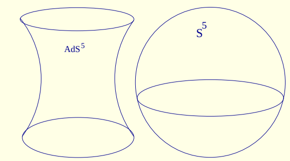
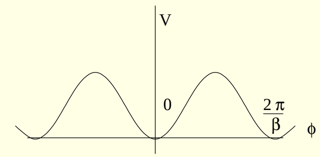
Classical integrable models: sine-Gordon theory

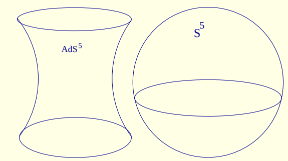




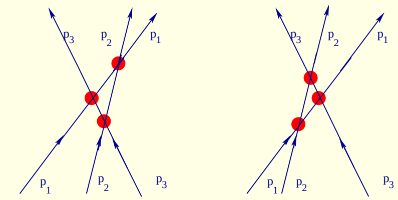
Plan of talk I: Finite size effects in integrable models

Classical integrable models: sine-Gordon theory





Bootstrap approach to quantum integrable models: S=scalar.matrix



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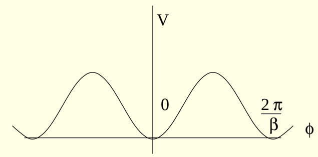
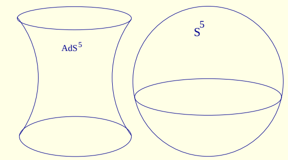
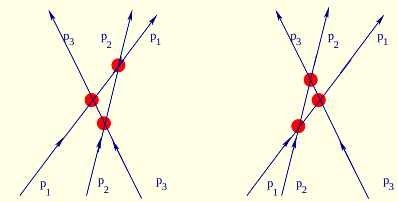
Classical integrable models: sine-Gordon theory







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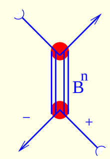
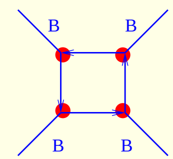
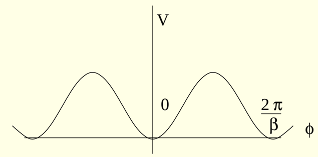


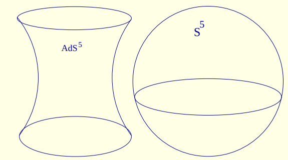


Lee-Yang, sinh-Gordon, sine-Gordon

Plan of talk I: Finite size effects in integrable models

Classical integrable models: sine-Gordon theory





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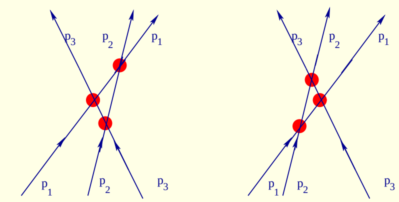

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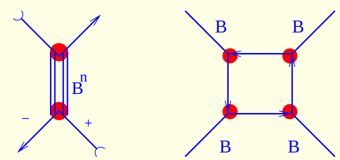
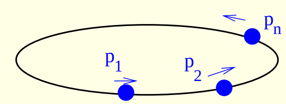
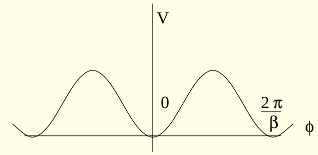


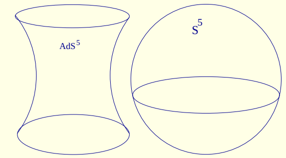
Finite volume: Asymptotic Bethe Ansatz:

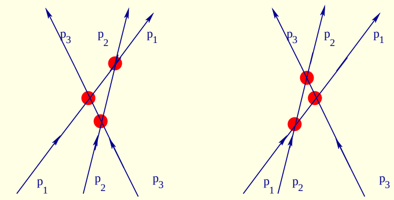


Plan of talk I: Finite size effects in integrable models

Classical integrable models: sine-Gordon theory







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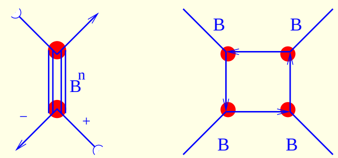
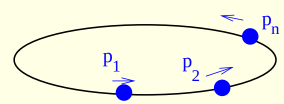


Lee-Yang, sinh-Gordon, sine-Gordon



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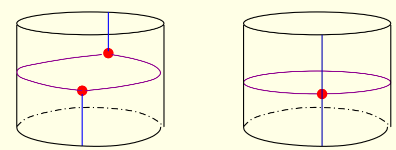
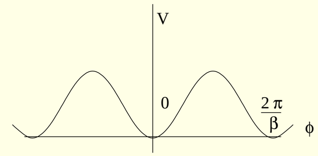
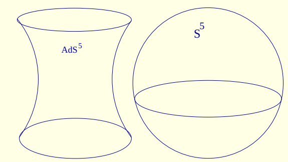


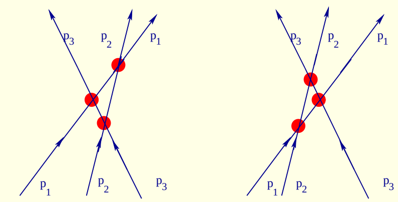
Luscher correction

Plan of talk I: Finite size effects in integrable models





Classical integrable models: sine-Gordon theory



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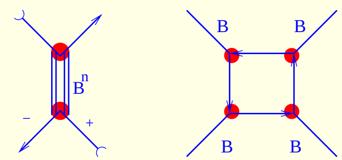

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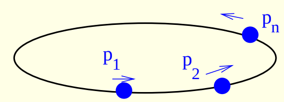

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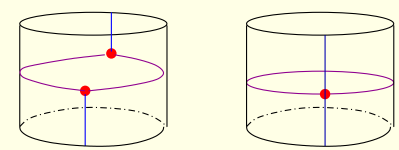

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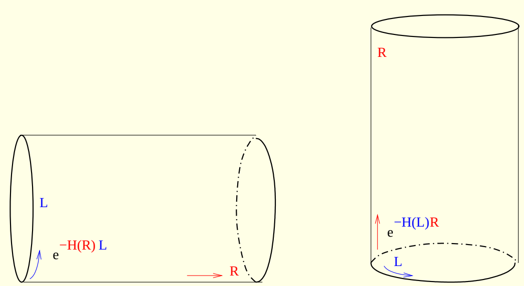
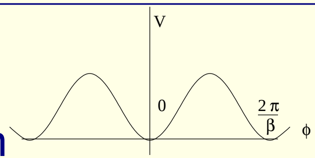
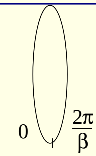
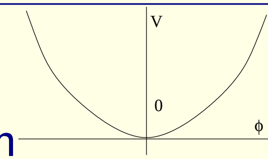
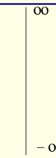
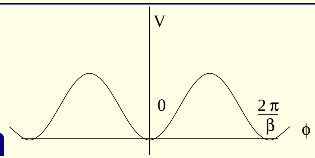
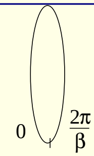
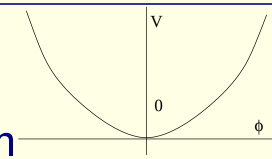
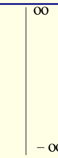


Exact groundstate: TBA,

Classical integrable models: sin(e/h)-Gordon theory

sine-Gordon  target 	$\beta \leftrightarrow ib$	sinh-Gordon  target 
$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{\beta^2}(1 - \cos\beta\phi)$		$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{b^2}(\cosh b\phi - 1)$

Classical integrable models: sin(e/h)-Gordon theory

sine-Gordon  target 	$\beta \leftrightarrow ib$	sinh-Gordon  target 
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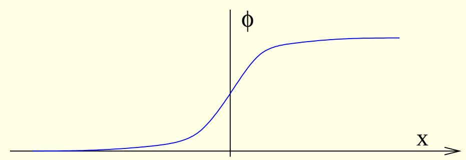
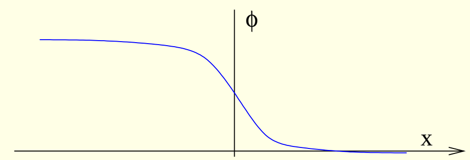
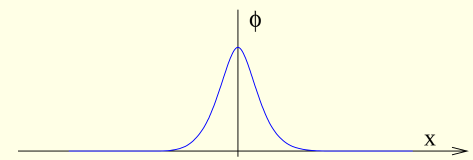
Classical
finite energy
solutions:
sine-Gordon theory



soliton

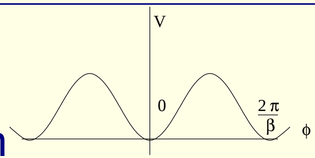
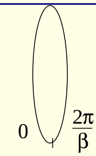
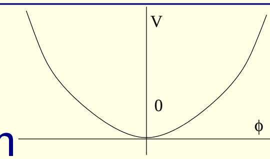
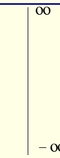


anti-soliton



breather

Classical integrable models: sin(e/h)-Gordon theory

sine-Gordon  target 	$\beta \leftrightarrow ib$	sinh-Gordon  target 
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Classical
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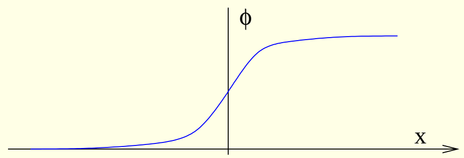
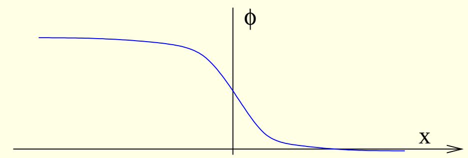
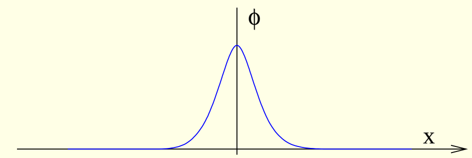


soliton

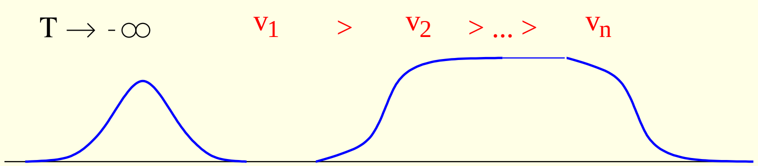


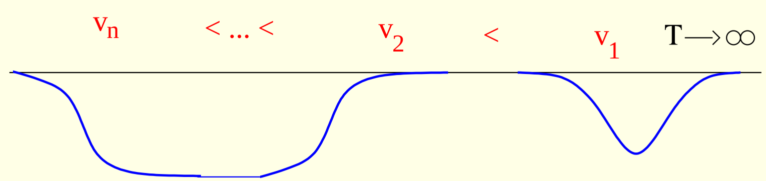
anti-soliton



breather

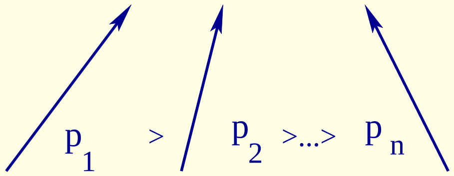
Classical factorized scattering: time delays sums up $\Delta T_1(v_1, v_2, \ldots, v_n) = \sum_i \Delta T_1(v_1, v_i)$





Bootstrap program

Asymptotic states $|p_1, p_2, \ldots, p_n\rangle_{in/out}$
form a representation of global symmetry:



Bootstrap program

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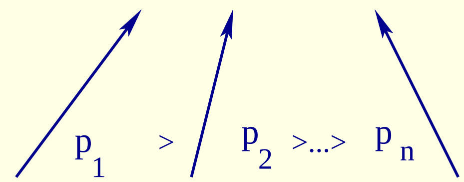

Lorentz: $P = \sum_i p_i \;\; E = \sum_i E(p_i)$
dispersion relation $E(p) = \sqrt{m^2 + p^2}$

Bootstrap program

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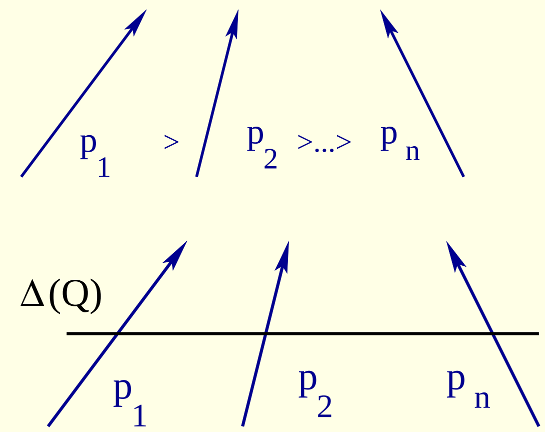
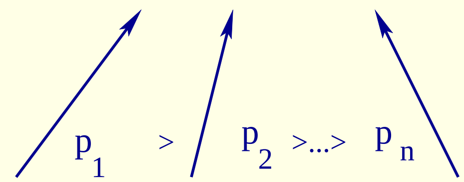


Bootstrap program

Asymptotic states $|p_1, p_2, \ldots, p_n\rangle_{in/out}$
form a representation of global symmetry:



Lorentz: $P = \sum_i p_i \quad E = \sum_i E(p_i)$
dispersion relation $E(p) = \sqrt{m^2 + p^2}$

Scattering matrix S: $|out\rangle \to |in\rangle$
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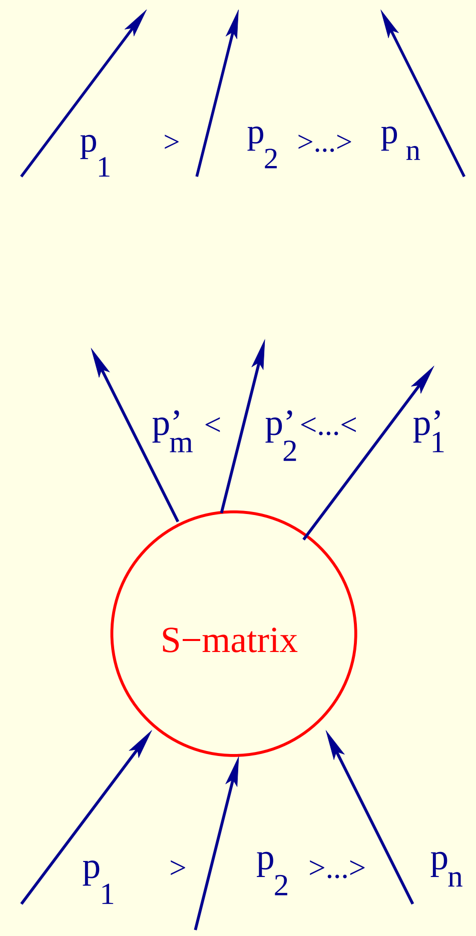

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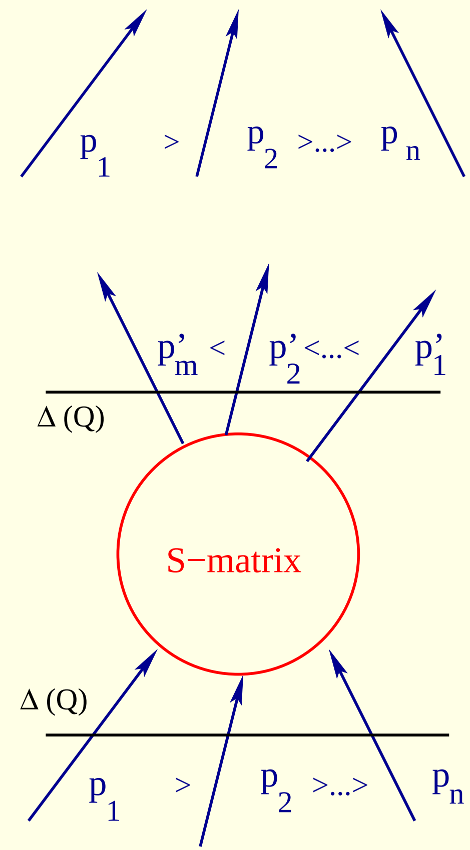

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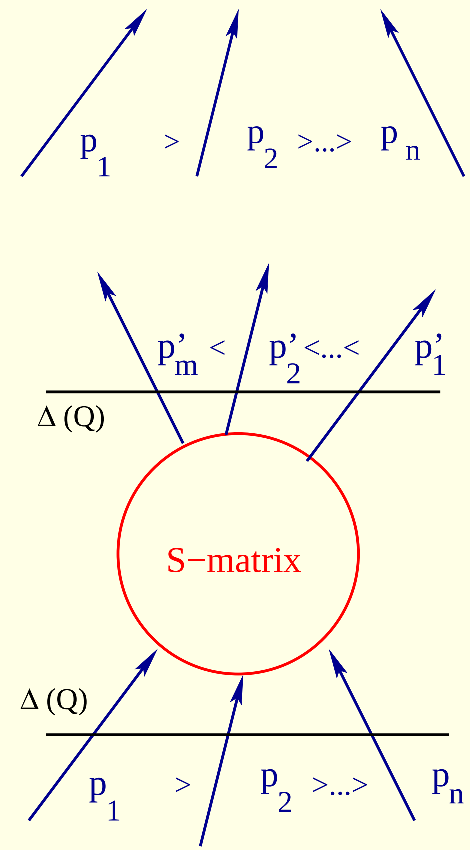

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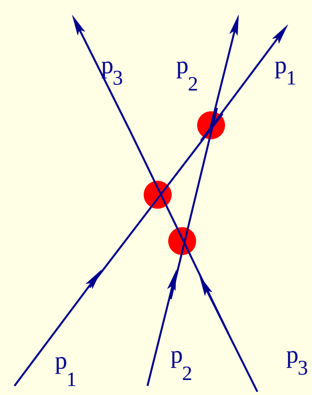
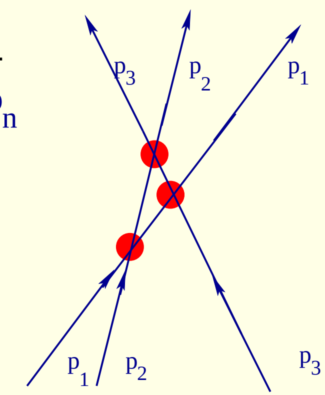
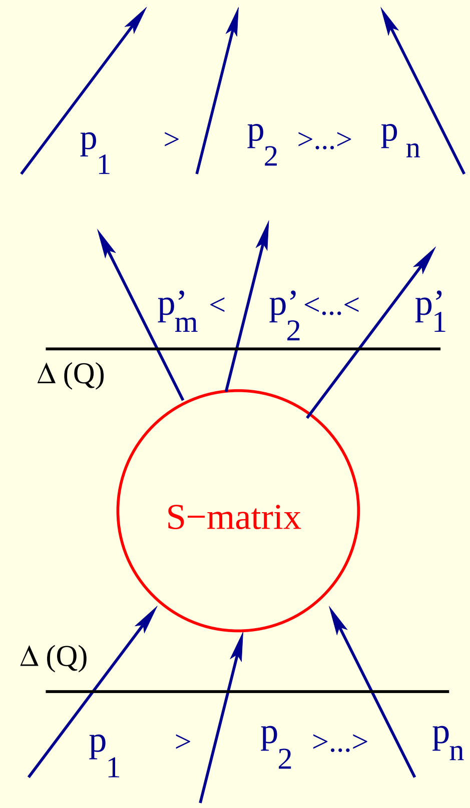
Higher spin concerved charge
factorization + Yang-Baxter equation
$S_{123} = S_{23}S_{13}S_{12} = S_{12}S_{13}S_{23}$





S-matrix = scalar . Matrix

Bootstrap program



Asymptotic states $|p_1, p_2, \ldots, p_n\rangle_{in/out}$
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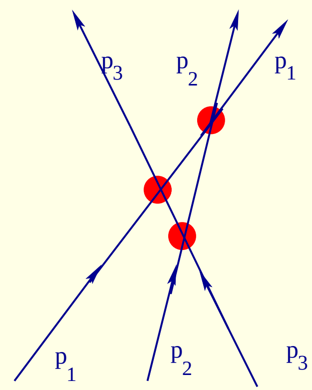

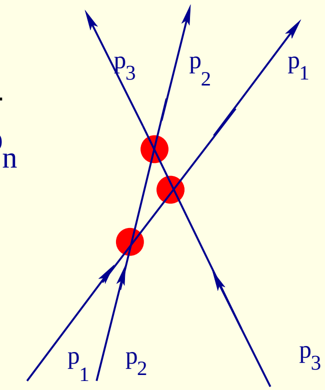

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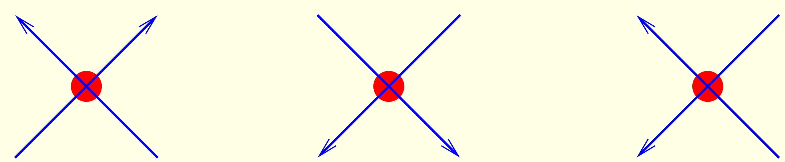
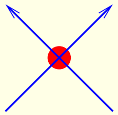


Unitarity $S_{12} S_{21} = Id$
Crossing symmetry $S_{12} = S_{2\bar{1}}$
Maximal analyticity: all poles have physical origin $\longrightarrow$ boundstates, anomalous thresholds

Bootstrap program: diagonal

Diagonal scattering: S-matrix = scalar $S(p_1,p_2) = S(\theta_1 - \theta_2)$  $p = m \sinh\theta$

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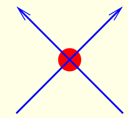
Diagonal scattering: S-matrix = scalar $S(p_1, p_2) = S(\theta_1 - \theta_2)$  $p = m \sinh \theta$

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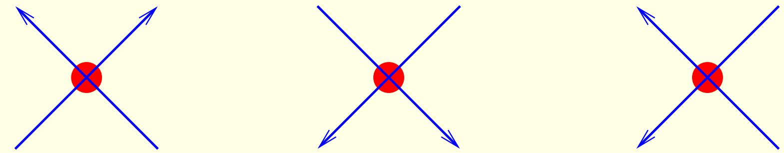

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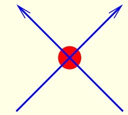
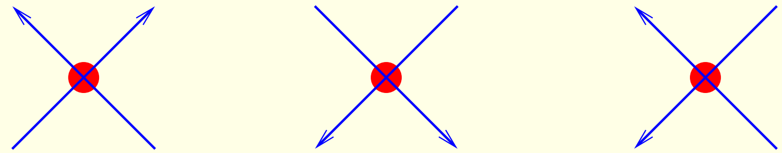
Diagonal scattering: S-matrix = scalar $S(p_1,p_2)=S(\theta_1-\theta_2)$  $p=m\sinh\theta$



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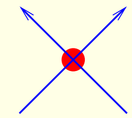
Diagonal scattering: S-matrix = scalar $S(p_1,p_2)=S(\theta_1-\theta_2)$  $p=m\sinh\theta$

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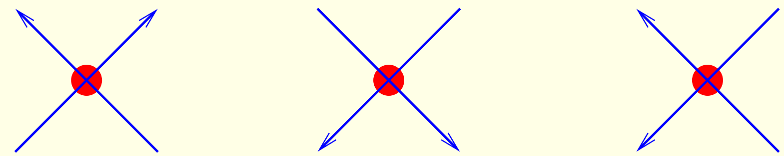

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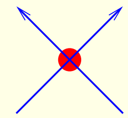
Minimal solution: $S(\theta)=1$ Free boson

$V\sim\cosh b\phi \leftrightarrow \frac{b^2}{8\pi+b^2}=p>0$

CDD factor: $S(\theta)=\frac{\sinh\theta-i\sin p\pi}{\sinh\theta+i\sin p\pi}$ Sinh-Gordon

Bootstrap program: diagonal

Diagonal scattering: S-matrix = scalar $S(p_1,p_2)=S(\theta_1-\theta_2)$  $p=m\sinh\theta$

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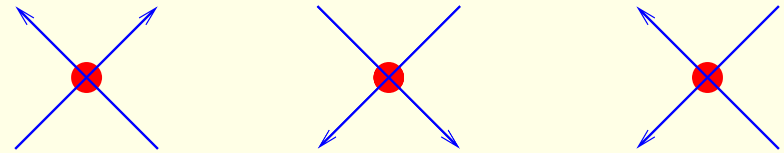

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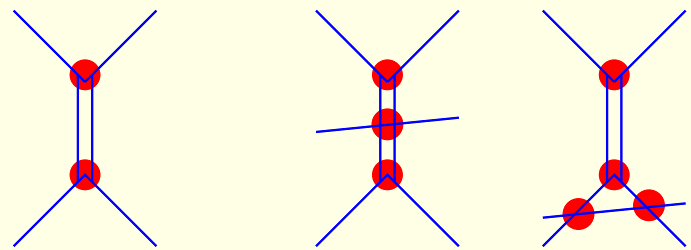
Maximal analyticity: $S(\theta)=\frac{\sinh\theta+i\sin p\pi}{\sinh\theta-i\sin p\pi}$

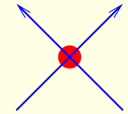
pole at $\theta=ip\pi$ $\rightarrow$boundstate B^2



bootstrap: $S_{12}(\theta)=S_{11}(\theta-\frac{ip\pi}{2})S_{11}(\theta+\frac{ip\pi}{2})$

new particle if $p\neq\frac{2}{3}$ Lee-Yang

Bootstrap program: diagonal

Diagonal scattering: $\boxed{\text{S-matrix} = \text{scalar}}$ $S(p_1,p_2)=S(\theta_1-\theta_2)$  $p=m\sinh\theta$

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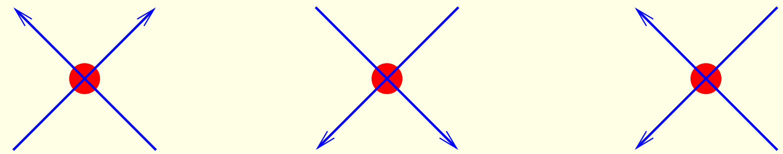

Maximal analyticity: all poles have physical origin

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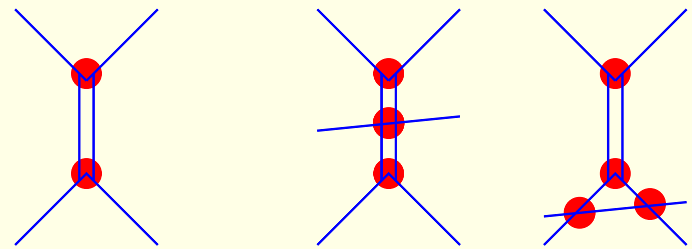

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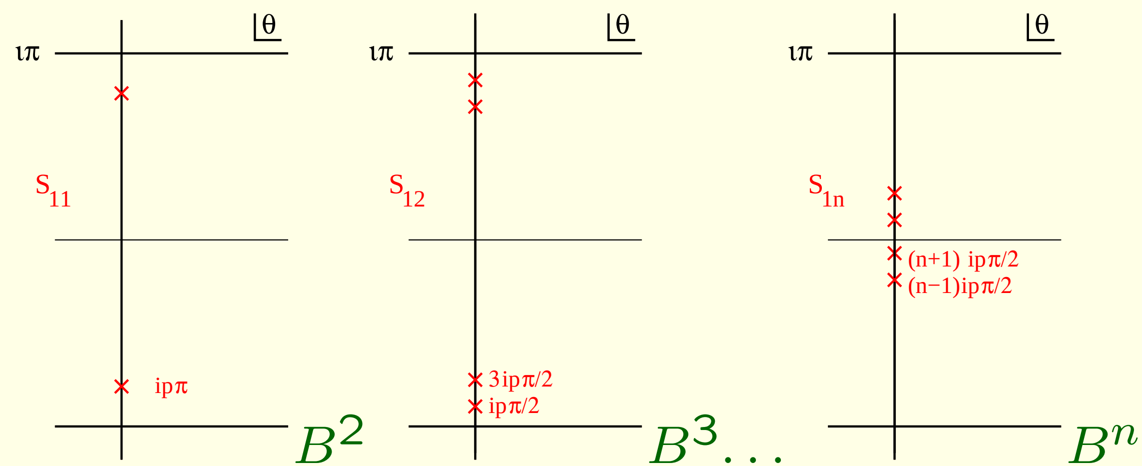
$\rightarrow$sine-Gordon solitons



Bootstrap program: sine-Gordon

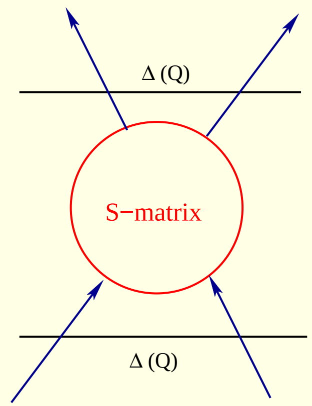
Nondiagonal scattering: $\boxed{\text{S-matrix} = \text{scalar . Matrix}}$ soliton doublet $\begin{pmatrix} s \\ \bar{s} \end{pmatrix}$

Matrix:

global symmetry $U_q(\hat{sl}_2)$

2d evaluation reps

$[S, \Delta(Q)] = 0$



$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{-\sin\lambda\pi}{\sin\lambda(\pi+i\theta)} & \frac{\sin i\lambda\theta}{\sin\lambda(\pi+i\theta)} & 0 \\ 0 & \frac{\sin i\lambda\theta}{\sin\lambda(\pi+i\theta)} & \frac{-\sin\lambda\pi}{\sin\lambda(\pi+i\theta)} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

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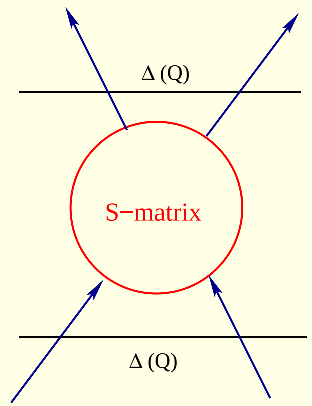

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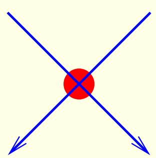
Unitarity

$\mathcal{S}(\theta)\mathcal{S}(-\theta) = 1$



$$\prod_{l=1}^{\infty}\left[\frac{\Gamma(2(l-1)\lambda+\frac{\lambda i\theta}{\pi})\Gamma(2l\lambda+1+\frac{\lambda i\theta}{\pi})}{\Gamma((2l-1)\lambda+\frac{\lambda i\theta}{\pi})\Gamma((2l-1)\lambda+1+\frac{\lambda i\theta}{\pi})}/(\theta\to-\theta)\right]$$

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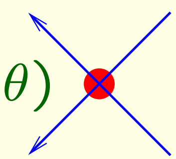

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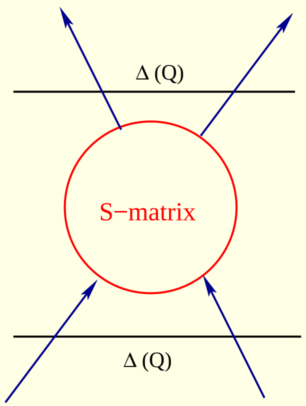

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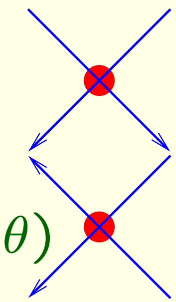


$$\prod_{l=1}^{\infty} \left[\frac{\Gamma(2(l-1)\lambda + \frac{\lambda i\theta}{\pi})\Gamma(2l\lambda + 1 + \frac{\lambda i\theta}{\pi})}{\Gamma((2l-1)\lambda + \frac{\lambda i\theta}{\pi})\Gamma((2l-1)\lambda + 1 + \frac{\lambda i\theta}{\pi})} / (\theta \to -\theta) \right]$$

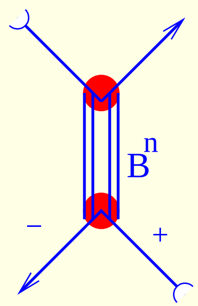
Maximal analyticity:

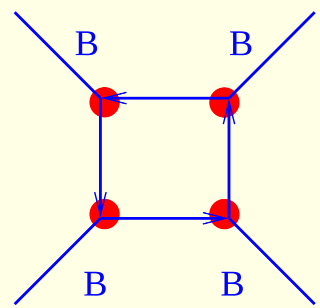
all poles have physical origin

either boundstates or

anomalous thresholds

$p = \lambda^{-1}$





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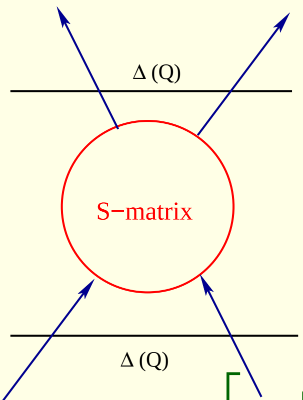

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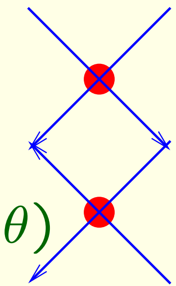

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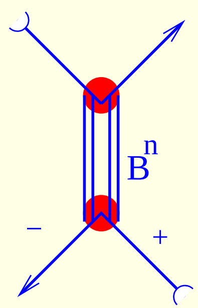
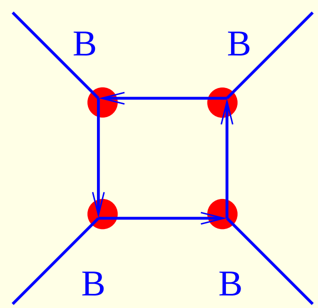
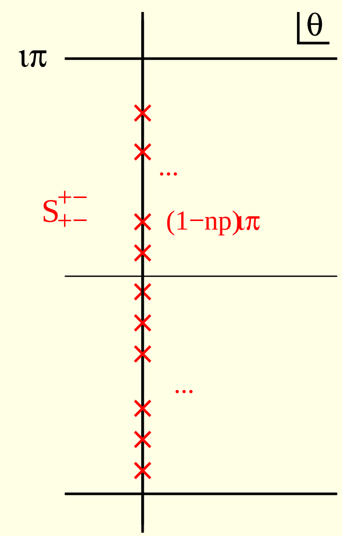
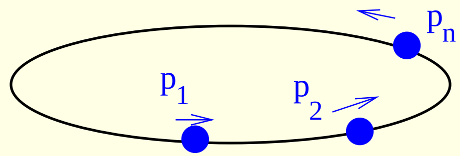






QFTs in finite volume

Finite volume spectrum



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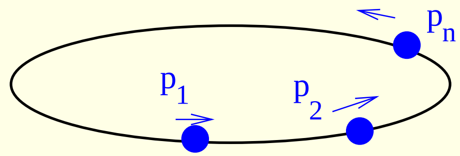

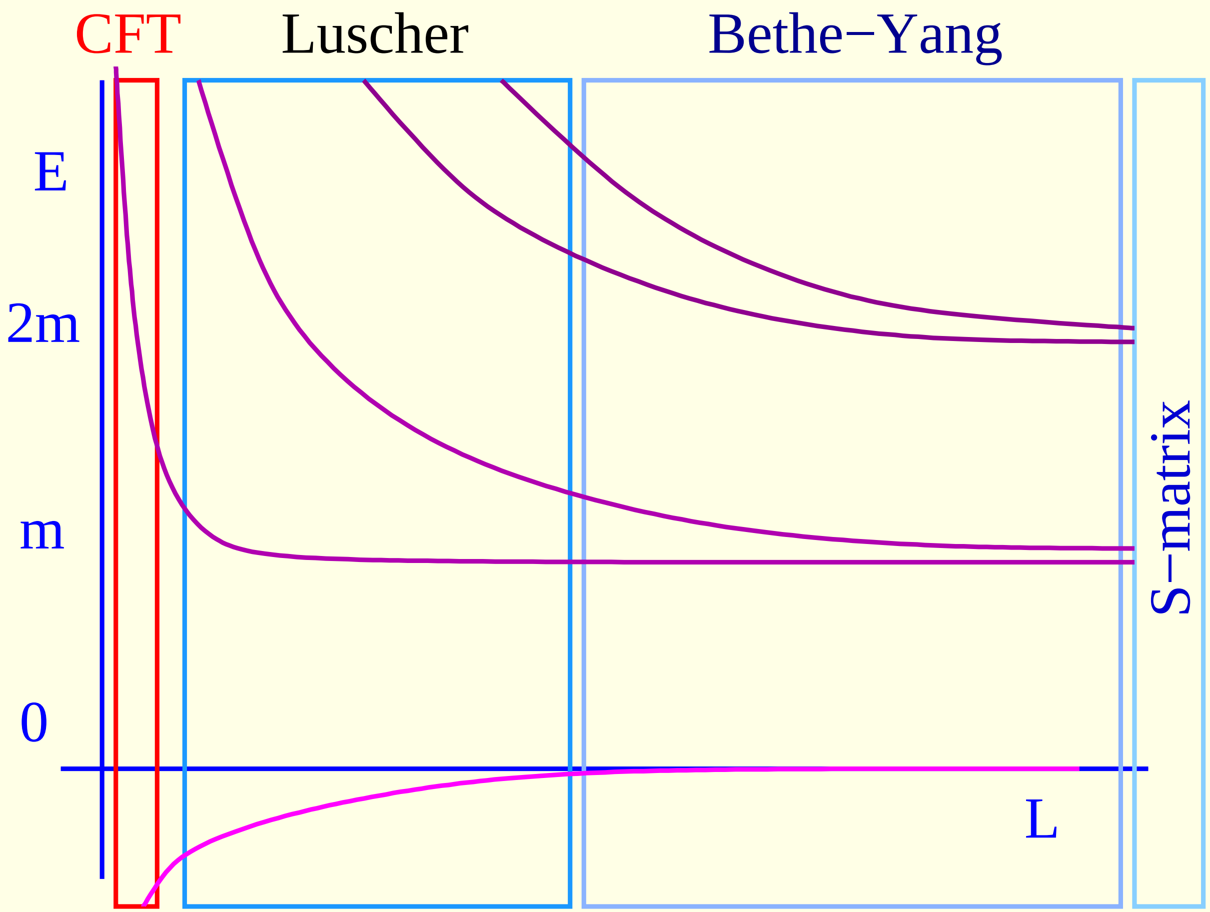

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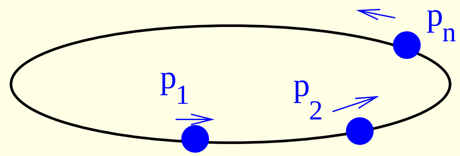

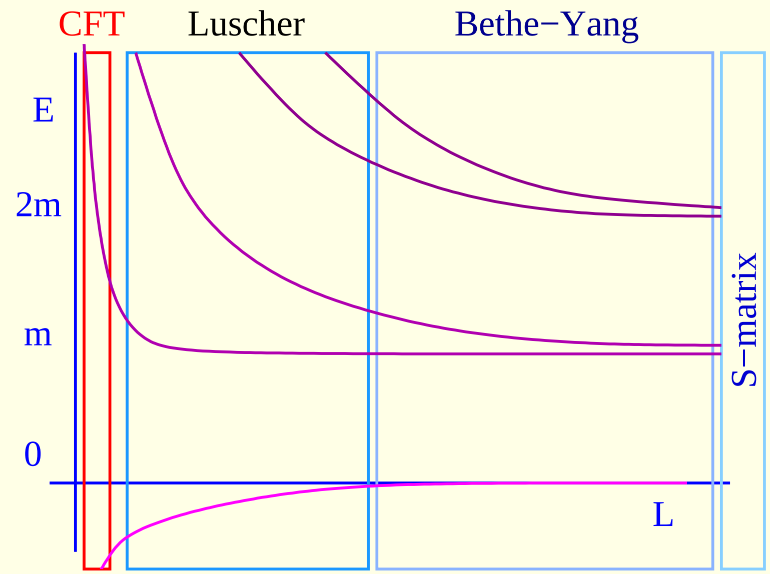

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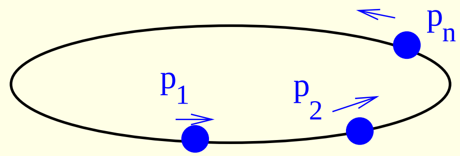
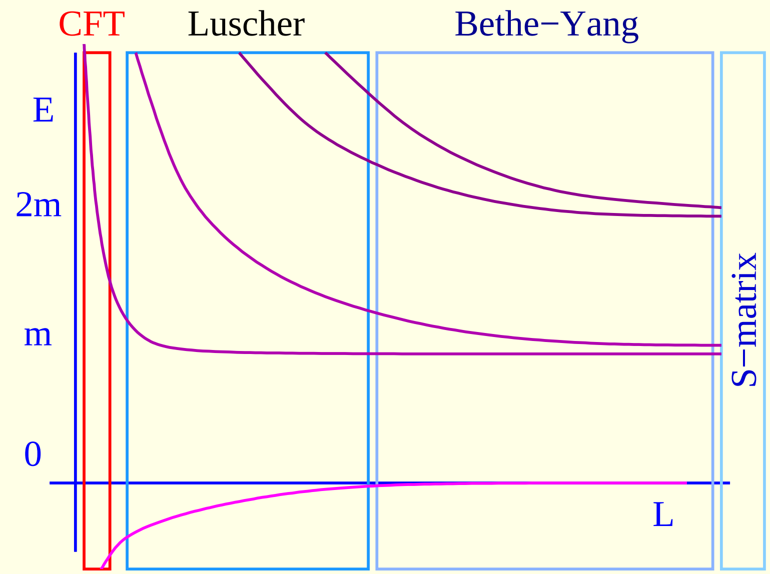


Infinite volume spectrum:

$E(p_1,\ldots,p_n)=\sum_i E(p_i) \qquad p_i\in R$



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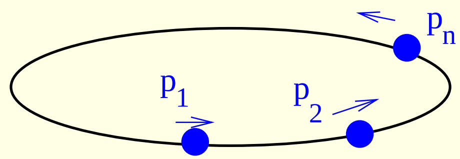
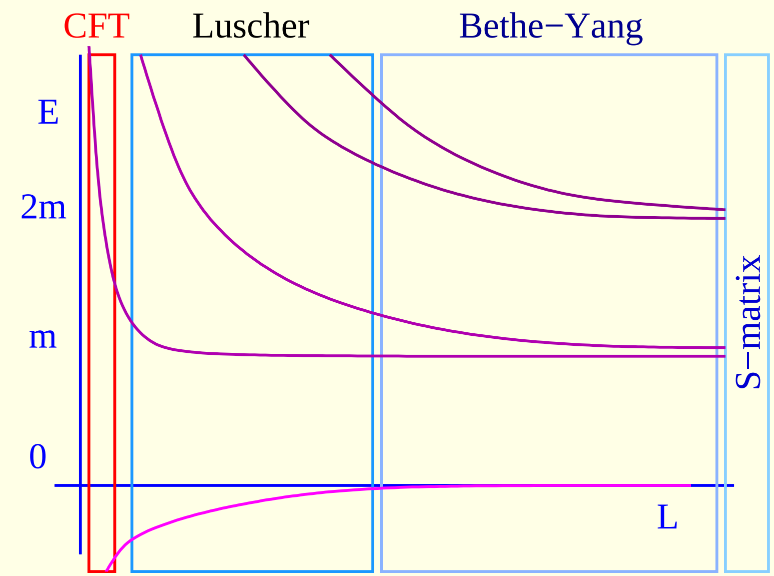
Finite volume spectrum





Infinite volume spectrum:

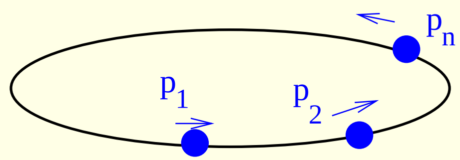
$E(p_1, \dots, p_n) = \sum_i E(p_i) \qquad p_i \in R$

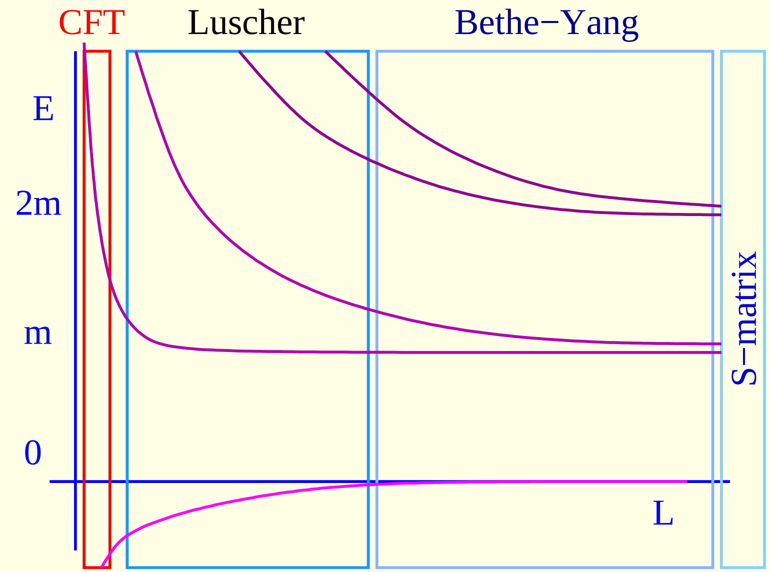
Polynomial volume corrections:

Bethe-Yang; p_i quantized. Diagonal

QFTs in finite volume

Finite volume spectrum





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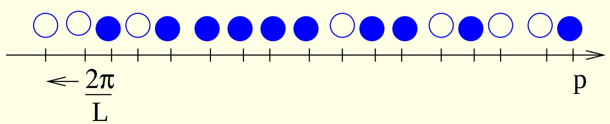

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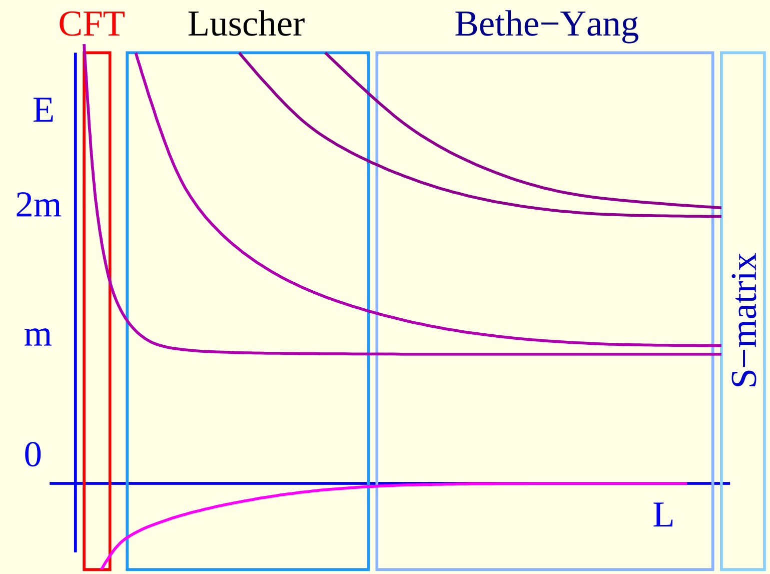
QFTs in finite volume



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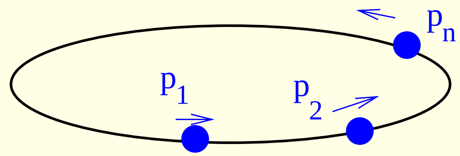

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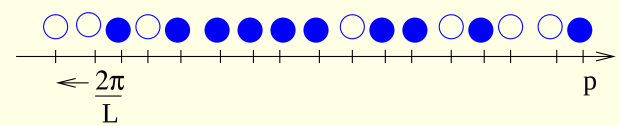

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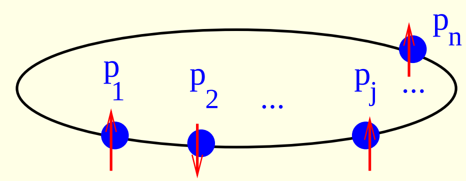
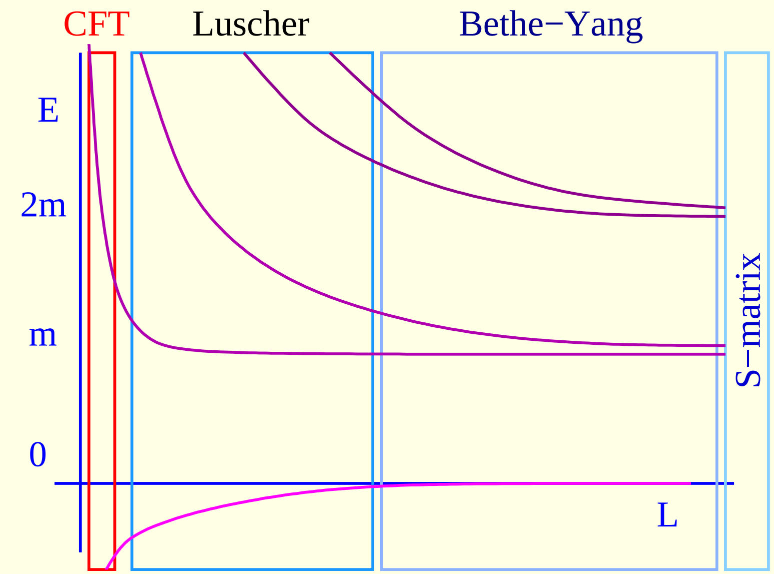


QFTs in finite volume



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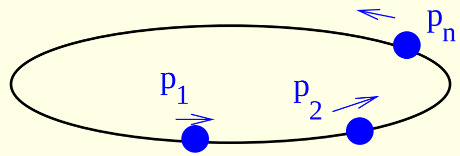

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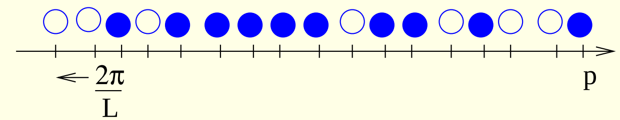

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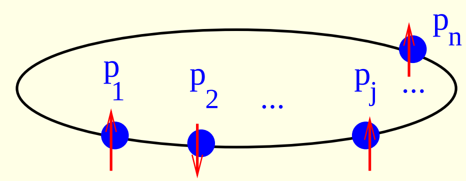
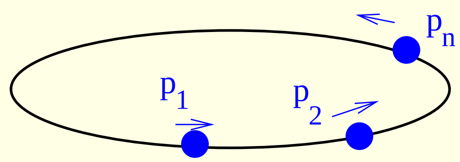
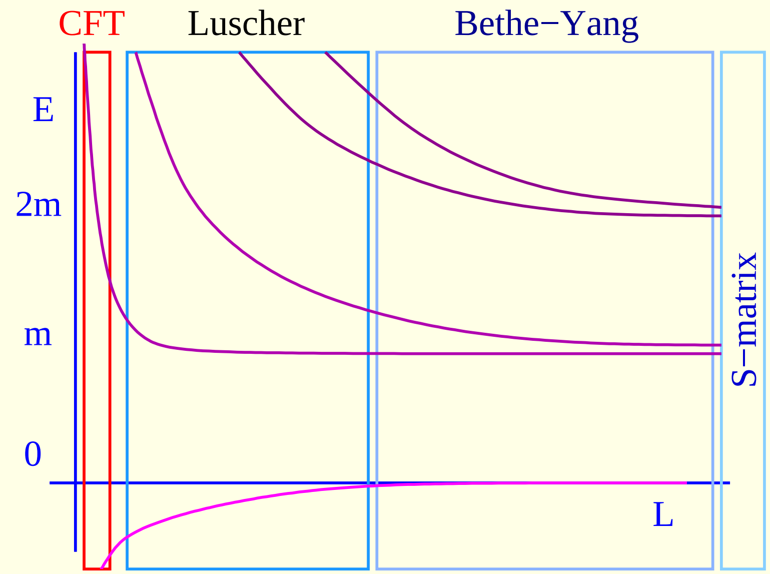


Inhomogenous XXZ spin-chain spectral problem $e^{iL\sinh\theta_j} T(\theta_j) S_0(\theta_j) = -1$

QFTs in finite volume

Finite volume spectrum





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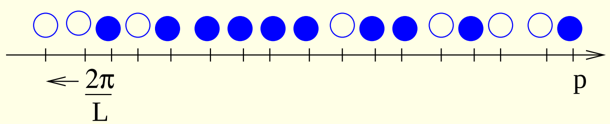

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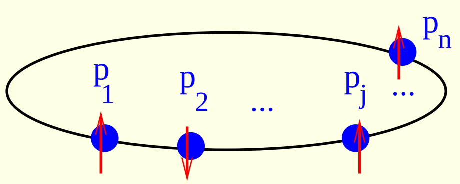

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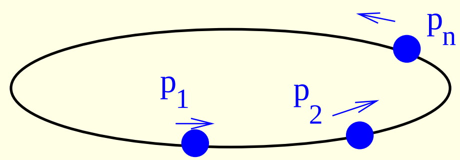
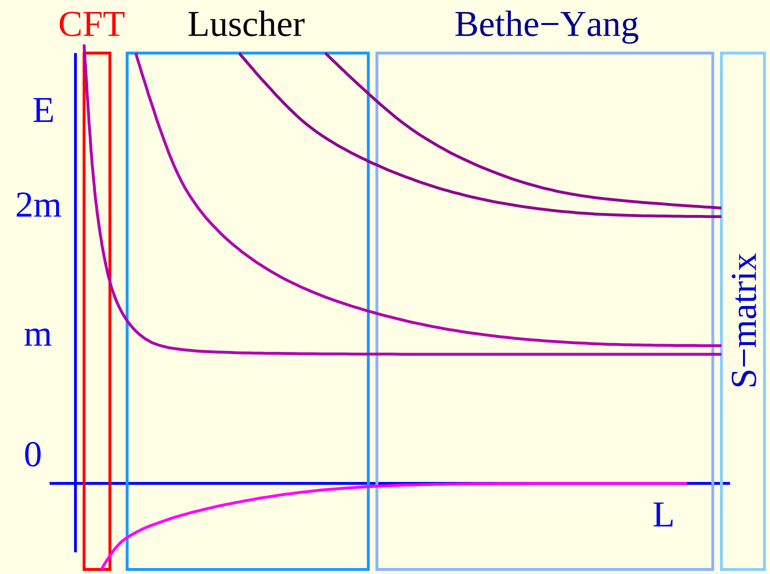
QFTs in finite volume

Finite volume spectrum





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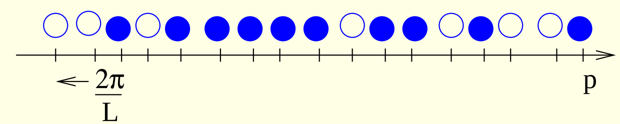

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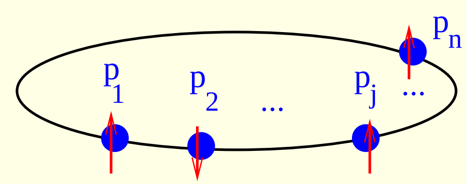

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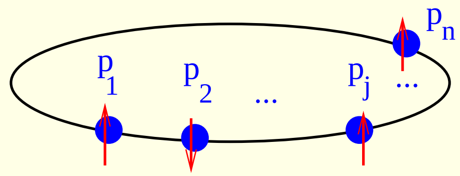
$Q(\theta)=\prod_\beta \sinh(\lambda(\theta-w_\beta))$
$T_0(\theta)=\prod_j \sinh(\lambda(\theta-\theta_j))$

Bethe Ansatz: $\dfrac{T_0(w_\alpha-\frac{i\pi}{2})Q(w_\alpha+i\pi)}{T_0(w_\alpha+\frac{i\pi}{2})Q(w_\alpha-i\pi)}=\dfrac{T_0^-Q^{++}}{T_0^+Q^{--}}|_\alpha=-1$

Lüscher correction of multiparticle states

Finite volume spectrum



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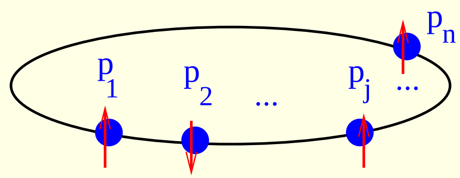

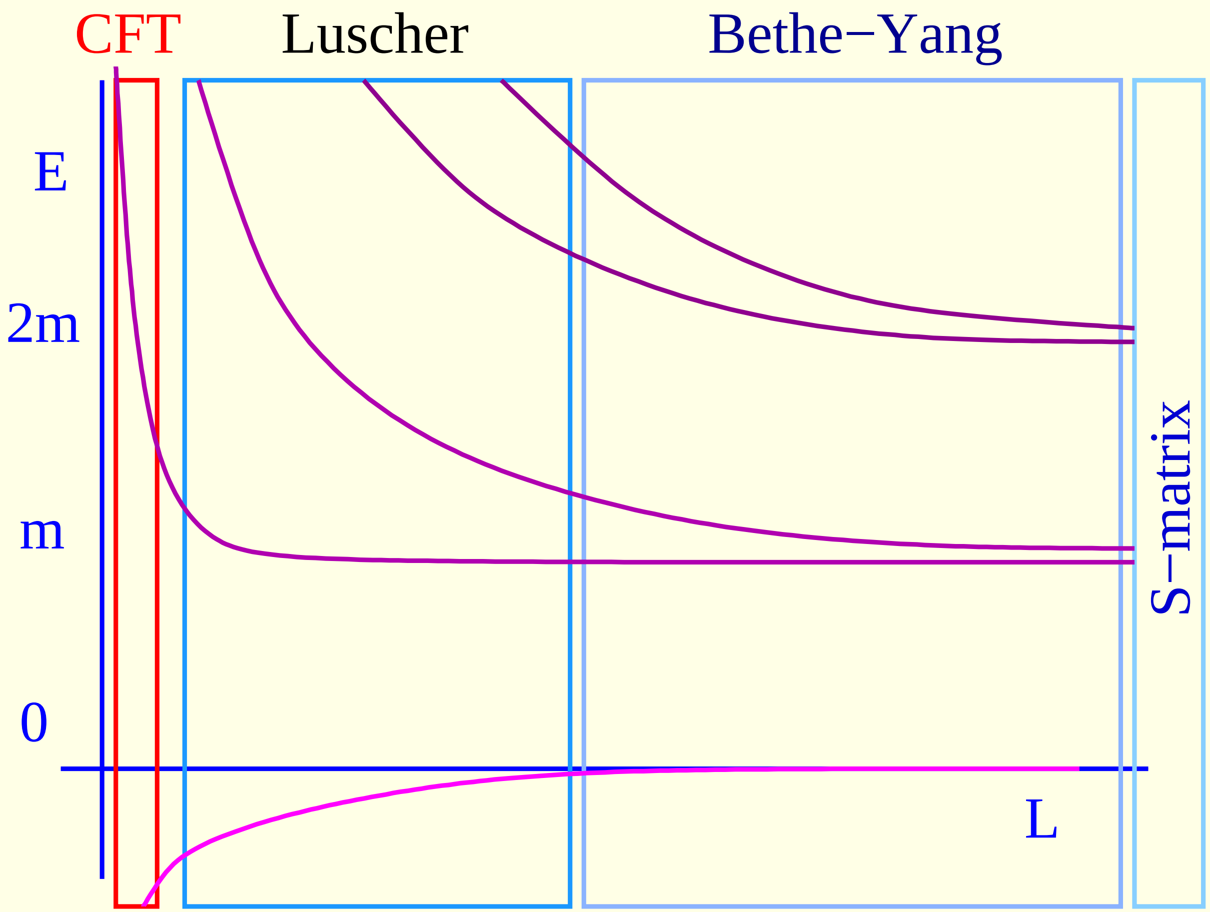

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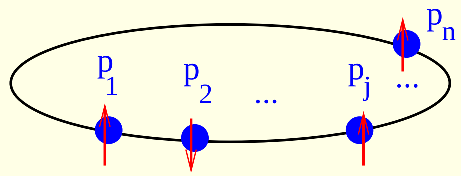

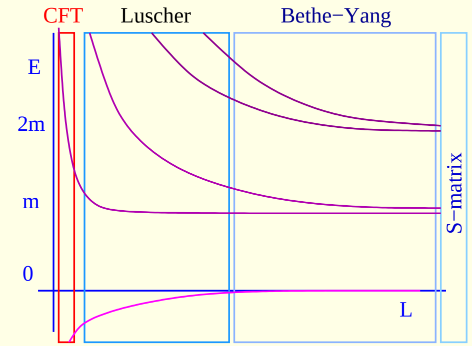

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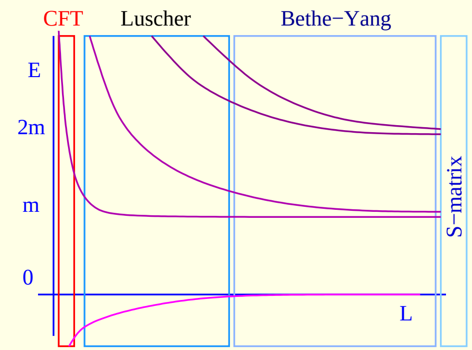

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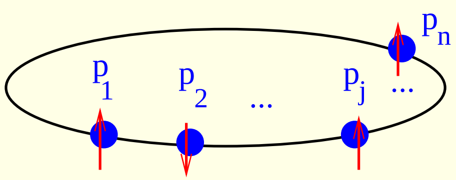

Lüscher originally: $O(e^{-mL})$ mass correction

$$m(L) = \frac{\sqrt{3}}{2} m \left(-i \mathrm{Res}_{\theta=\frac{2i\pi}{3}} S\right) e^{-\frac{\sqrt{3}}{2} mL}$$
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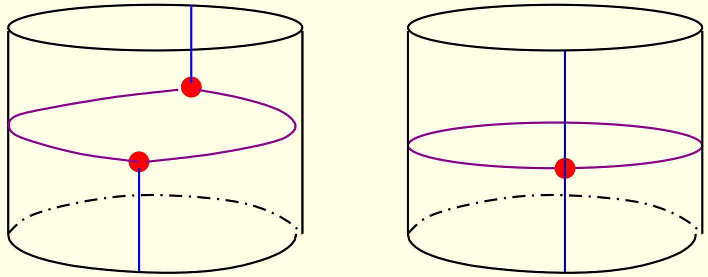

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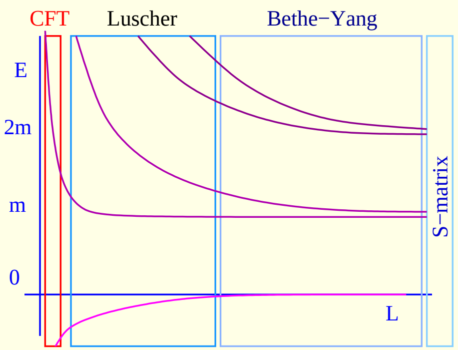

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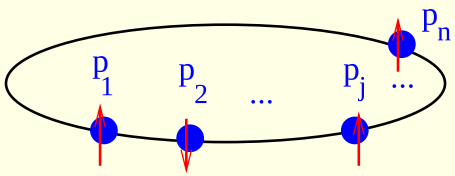

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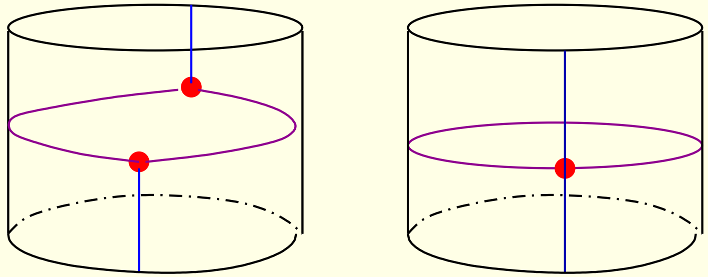
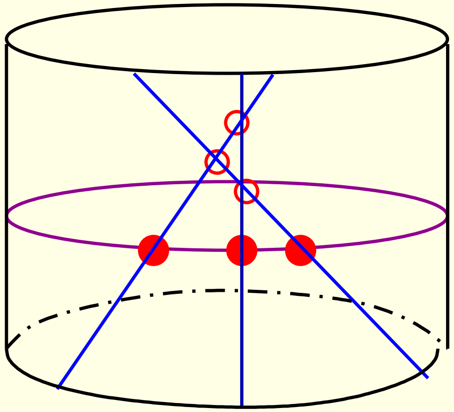


Multiparticle Lüscher correction



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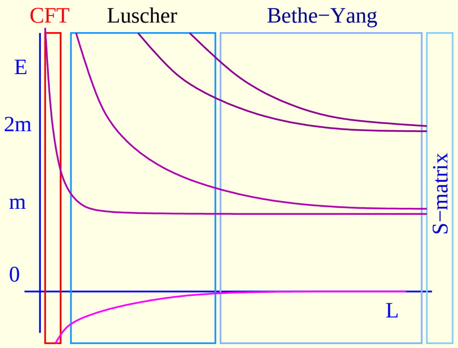

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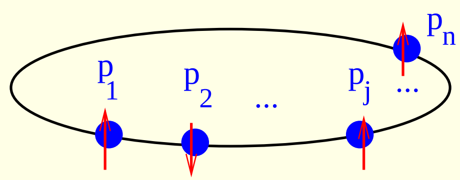

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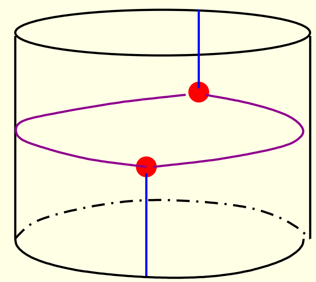

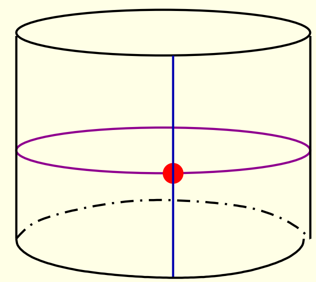

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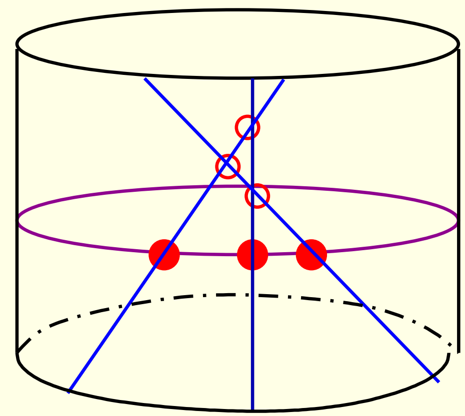

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Modified momenta:

$$p_jL - i\log t(p_j, p_1, \ldots, p_n, \Psi) = (2n+1)\pi + \Phi_j$$
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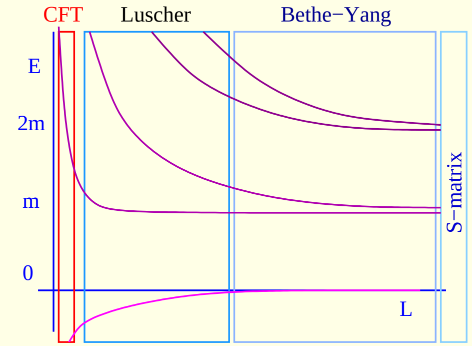
Lüscher correction of multiparticle states



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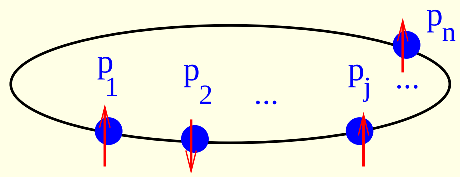

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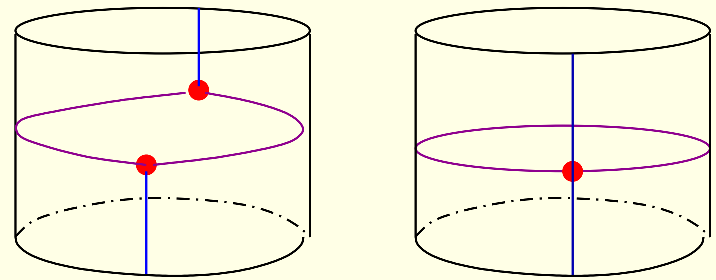

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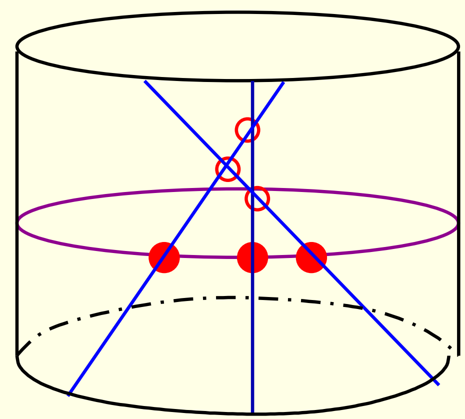

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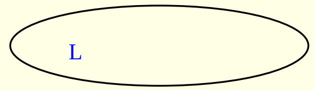
Thermodynamic Bethe Ansatz: diagonal

Ground-state energy exactly



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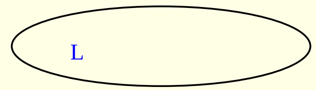
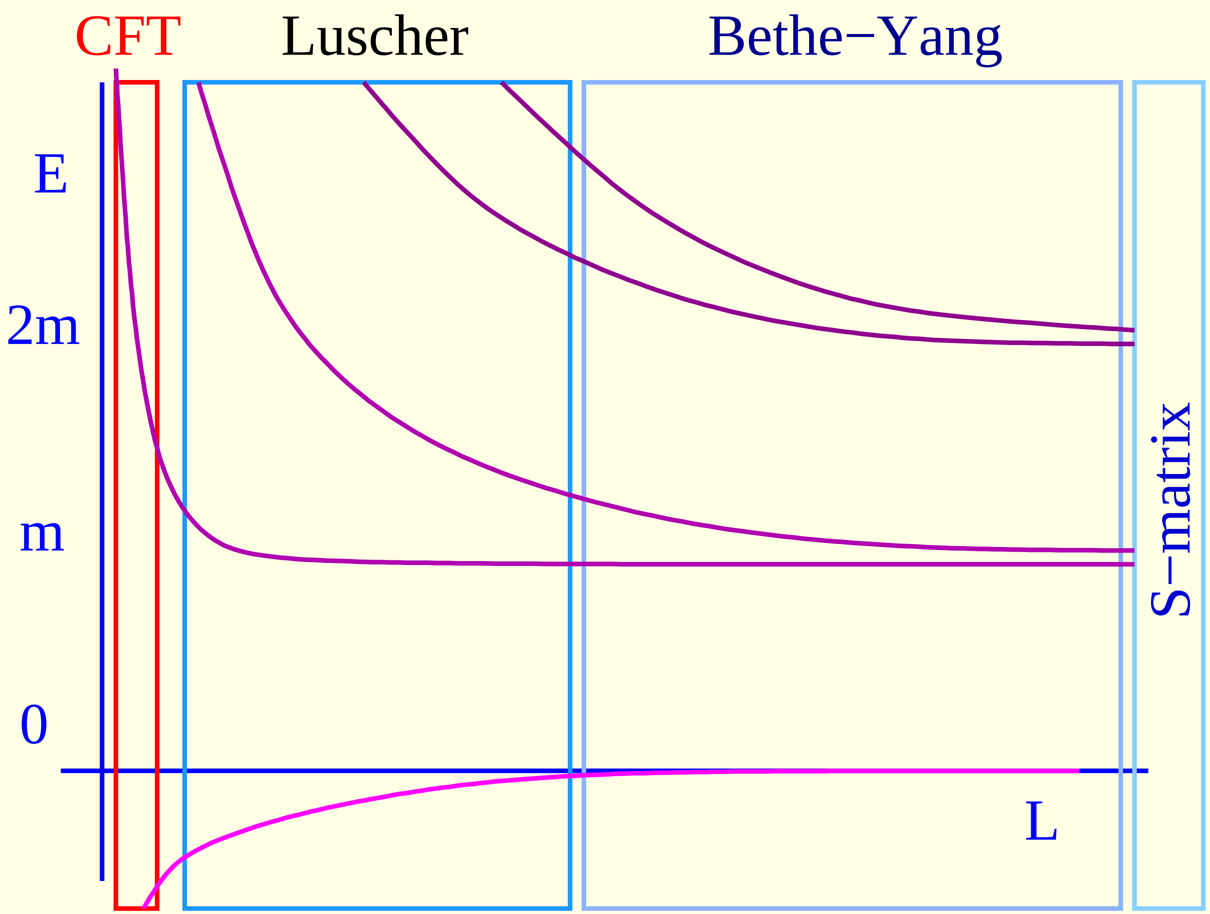
Ground-state energy exactly 



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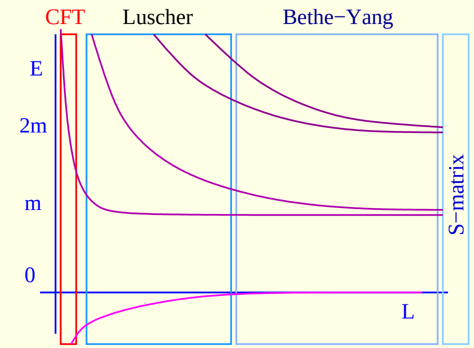

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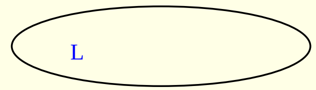

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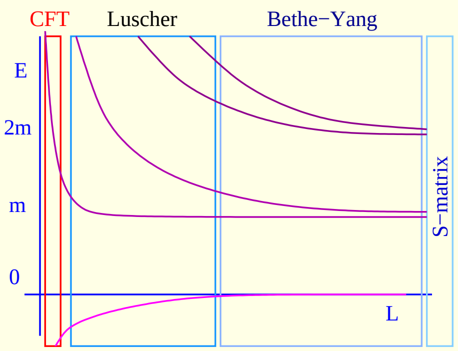

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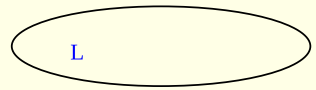
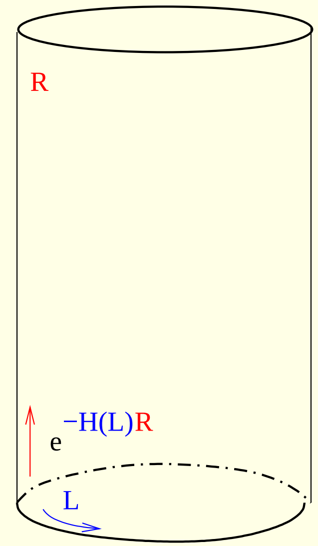


Euclidien partition function:

$$Z(L,R) =_{R\to\infty} \mathrm{Tr}(e^{-H(L)R})$$

$$Z(L,R) =_{R\to\infty} e^{-E_0(L)R}(1+e^{-\Delta E\, R})$$



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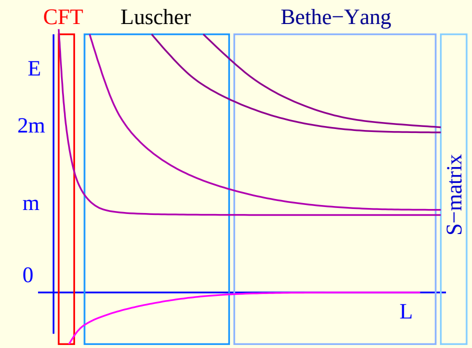
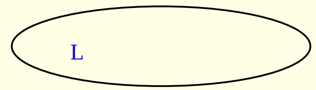
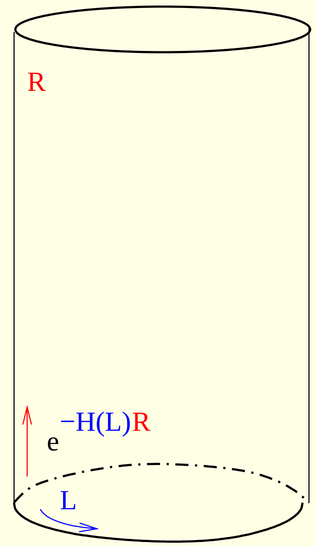


Ground-state energy exactly





Euclidien partition function:

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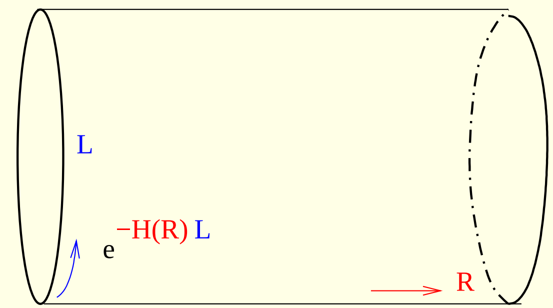

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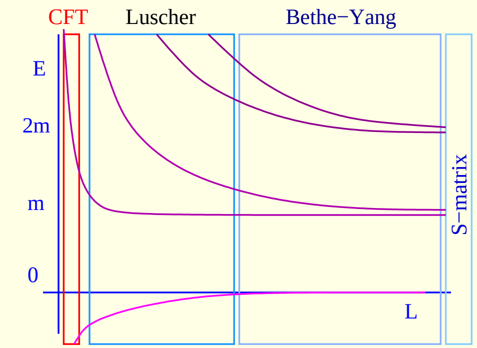

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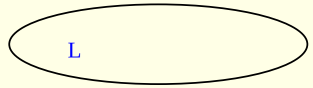

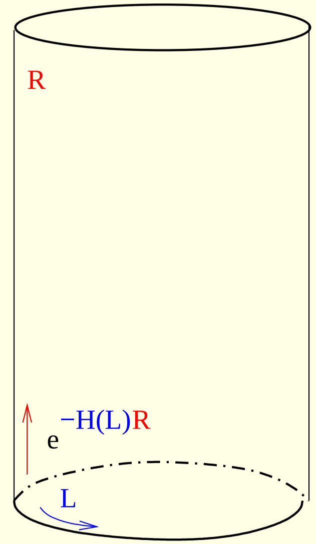

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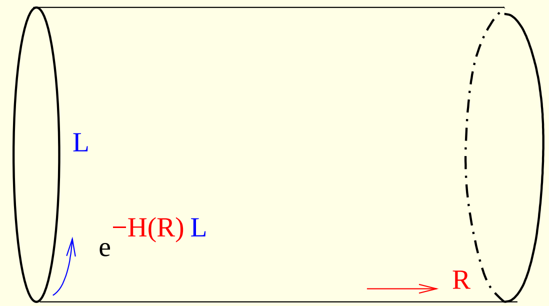

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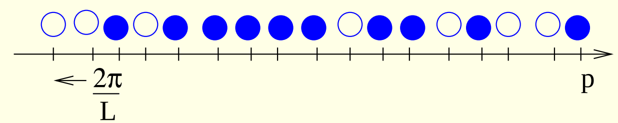

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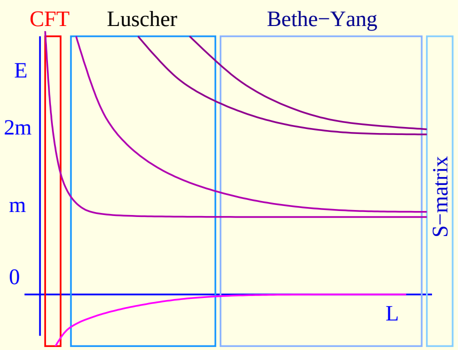

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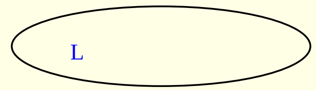

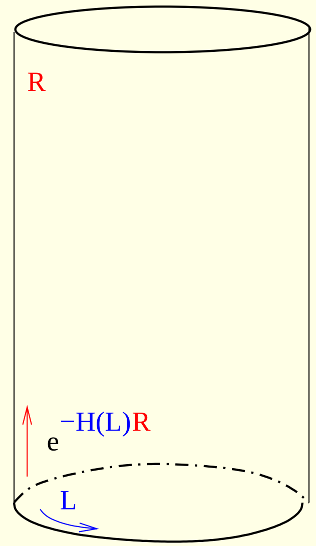

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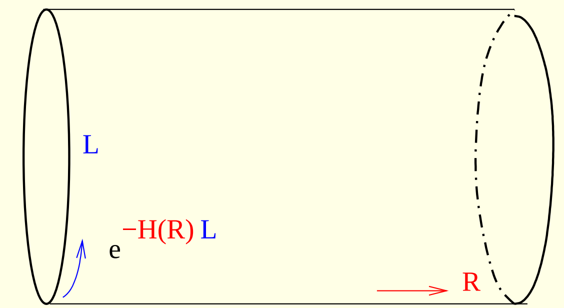

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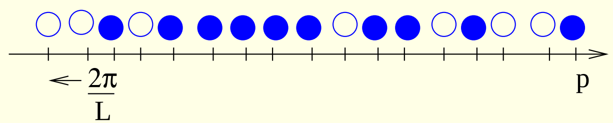

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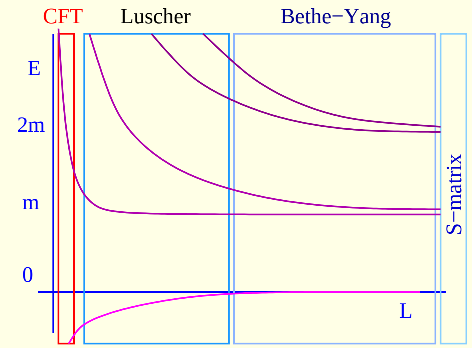

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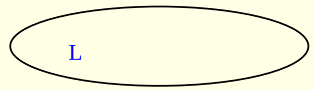

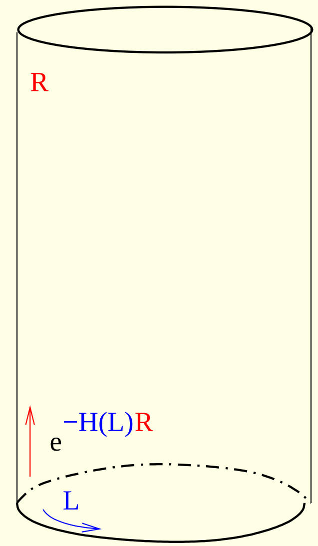

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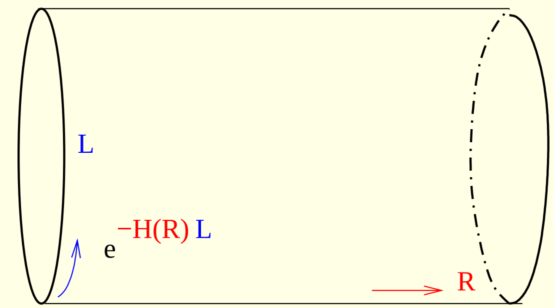

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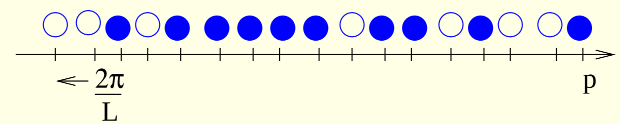

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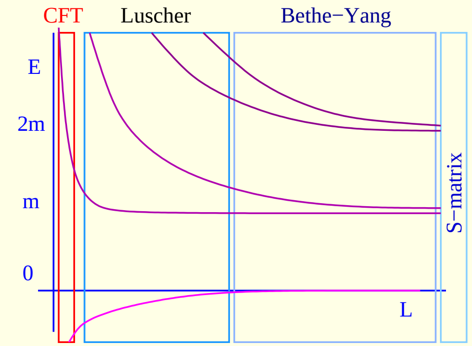

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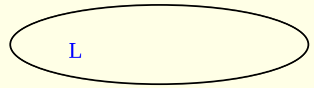

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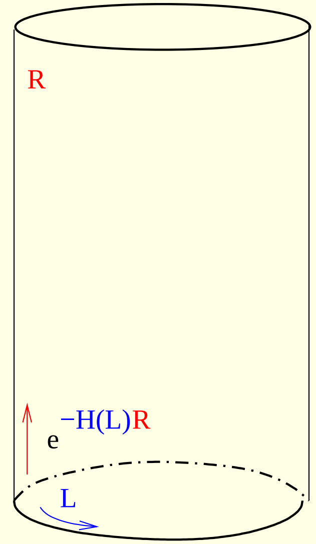

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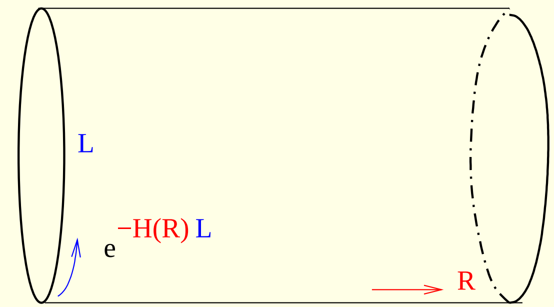

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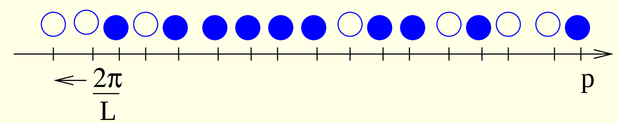

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Saddle point : $\epsilon(p) = \ln\frac{\rho_h(p)}{\rho(p)}$ $\quad \boxed{\epsilon(p) = E(p)L + \int \frac{dp}{2\pi} i d_p \log S(p',p)\log(1+e^{-\epsilon(p')})}$

Ground state energy exactly: $\boxed{E_0(L) = -\int \frac{dp}{2\pi}\log(1+e^{-\epsilon(p)})}$ Lee-Yang, sinh-Gordon

Plan of talk II: Planar AdS/CFT

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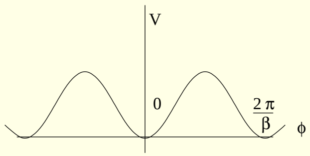
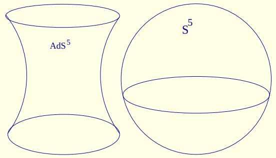
Classical integrable models: sine-Gordon theory 



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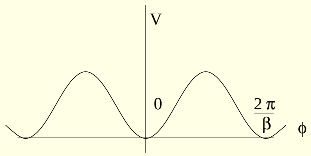

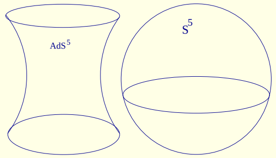

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Quantization of integrable models: sine-Gordon model: PCFT

Plan of talk II: Planar AdS/CFT

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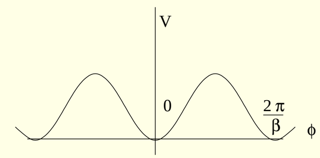
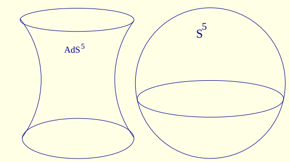
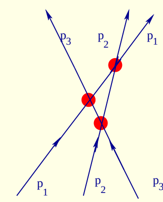
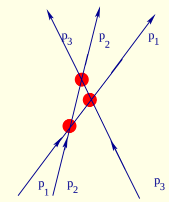




Quantization of integrable models: sine-Gordon model: PCFT

Bootstrap approach to quantum integrable models: S=scalar.matrix





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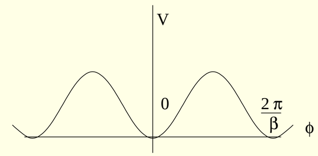

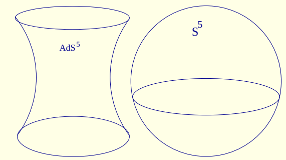

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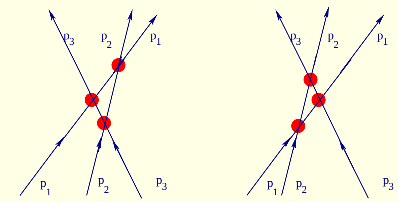
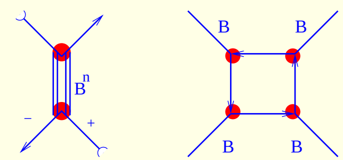


Lee-Yang, sinh-Gordon, sine-Gordon$\leftrightarrow$ AdS σmodel



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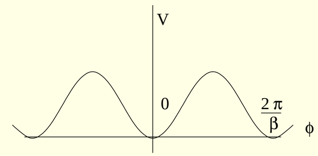

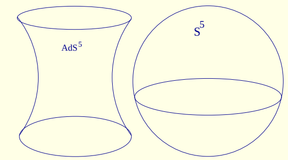

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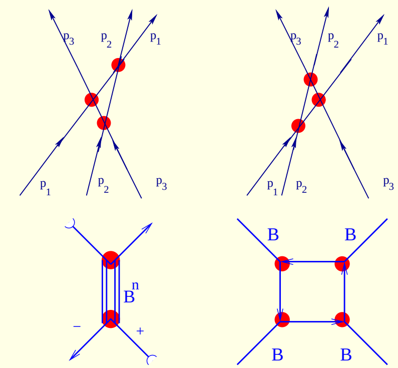

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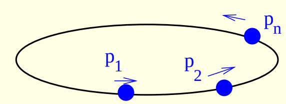
Finite volume: Asymptotic Bethe Ansatz:



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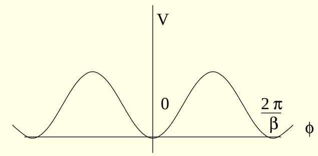

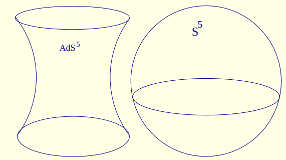

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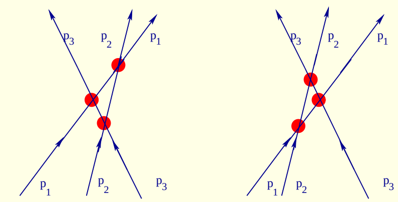

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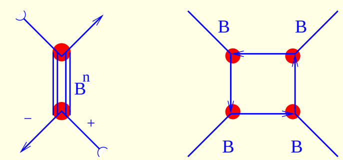

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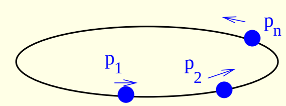

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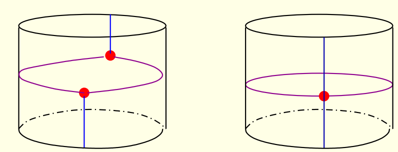

Luscher correction

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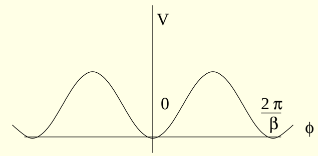
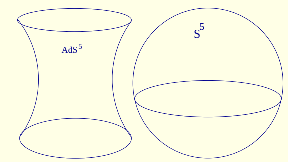
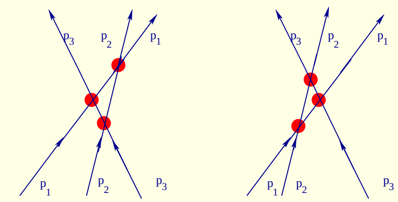




Quantization of integrable models: sine-Gordon model: PCFT



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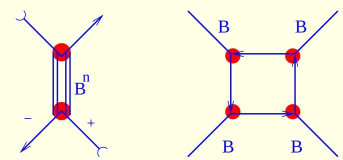

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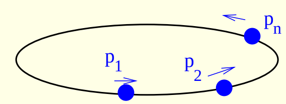

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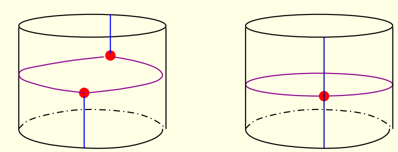

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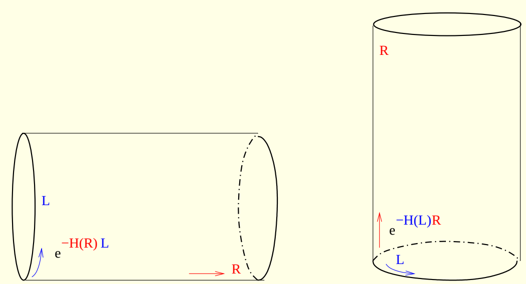

Exact: groundstate TBA,

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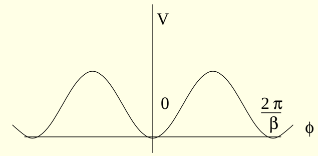

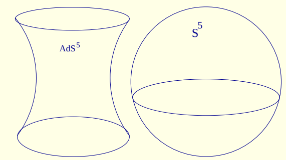

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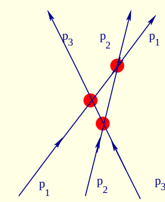

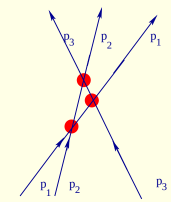

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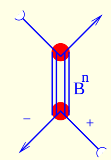

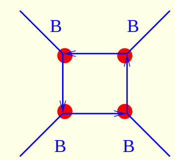

Lee-Yang, sinh-Gordon, sine-Gordon$\leftrightarrow$ AdS σmodel

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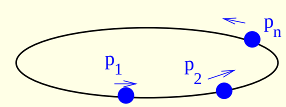
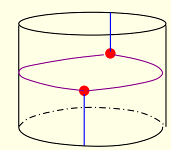
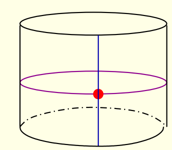
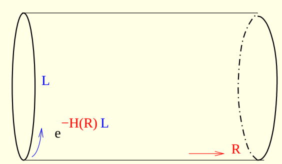
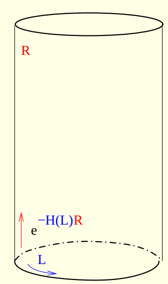






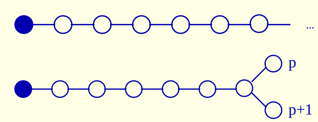
Luscher correction



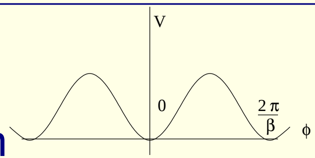
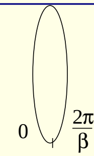
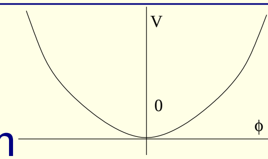
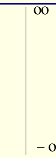


Exact: groundstate TBA,

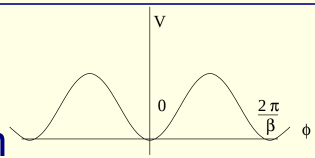
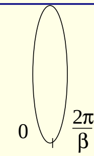
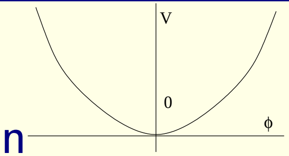
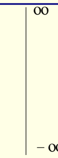
Excited TBA, Y-system, NLIE



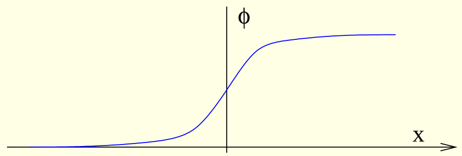
Classical integrable models: sin(e/h)-Gordon theory

sine-Gordon  target 	$\beta \leftrightarrow ib$	sinh-Gordon  target 
$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{\beta^2}(1 - \cos\beta\phi)$		$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{b^2}(\cosh b\phi - 1)$

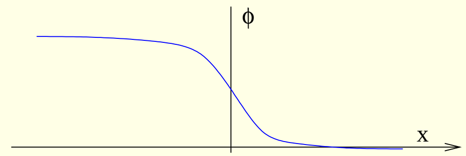
Classical integrable models: sin(e/h)-Gordon theory

sine-Gordon  target 	$\beta \leftrightarrow ib$	sinh-Gordon  target 
$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{\beta^2}(1 - \cos\beta\phi)$		$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{b^2}(\cosh b\phi - 1)$

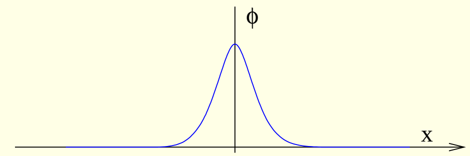
Classical
finite energy
solutions:
sine-Gordon theory



soliton

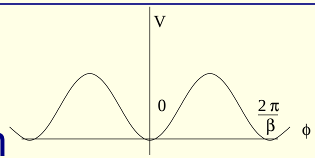
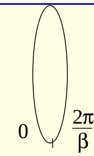
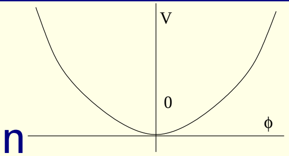
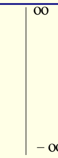


anti-soliton



breather

Classical integrable models: sin(e/h)-Gordon theory

sine-Gordon  target 	$\beta \leftrightarrow ib$	sinh-Gordon  target 
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Classical
finite energy
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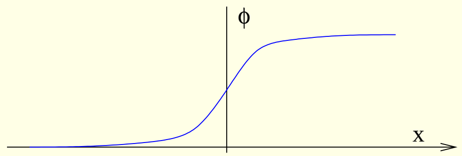
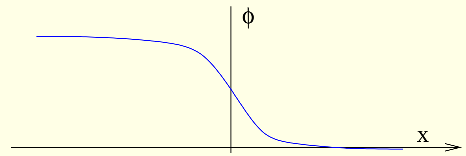
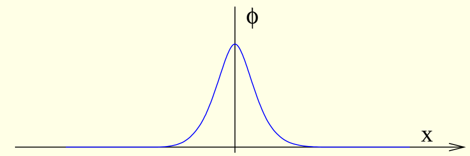


soliton



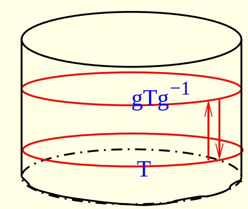
anti-soliton



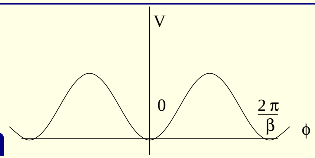
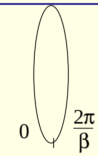
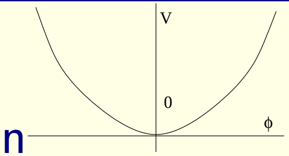
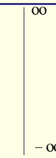
breather

Integrability: $\partial_x A_t - \partial_t A_x + [A_x, A_t] = 0 \leftrightarrow T(\lambda) = \mathrm{Tr}\mathcal{P}\exp\oint A(x)_\mu dx^\mu$

$$A_x(\mu) = \frac{i}{2}\begin{pmatrix} 2\mu & \beta\partial_+\varphi \\ -\beta\partial_+\varphi & -2\mu \end{pmatrix} \qquad A_t(\mu) = \frac{1}{4i\mu}\begin{pmatrix} \cos\beta\varphi & -i\sin\beta\varphi \\ i\sin\beta\varphi & -\cos\beta\varphi \end{pmatrix}$$



Classical integrable models: sin(e/h)-Gordon theory

sine-Gordon  target 	$\beta \leftrightarrow ib$	sinh-Gordon  target 
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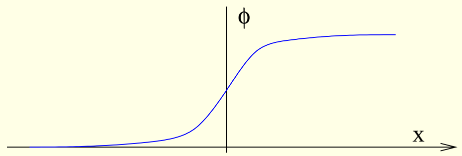
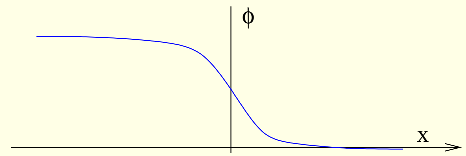
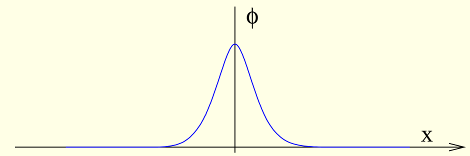
Classical finite energy solutions: sine-Gordon theory



soliton

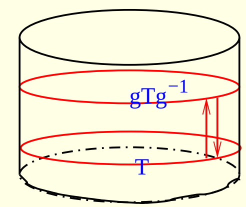


anti-soliton



breather

Integrability: $\partial_x A_t - \partial_t A_x + [A_x, A_t] = 0 \leftrightarrow T(\lambda) = \mathrm{Tr}\mathcal{P}\exp\oint A(x)_\mu dx^\mu$

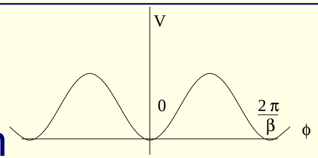
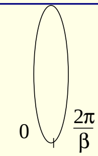
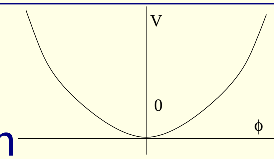
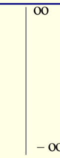


$$A_x(\mu) = \frac{i}{2}\begin{pmatrix} 2\mu & \beta\partial_+\varphi \\ -\beta\partial_+\varphi & -2\mu \end{pmatrix} \qquad A_t(\mu) = \frac{1}{4i\mu}\begin{pmatrix} \cos\beta\varphi & -i\sin\beta\varphi \\ i\sin\beta\varphi & -\cos\beta\varphi \end{pmatrix}$$

conserved $Q_{\pm 1}[\varphi] = E[\varphi] \pm P[\varphi] = \int \left\{ \frac{1}{2}(\partial_\pm\varphi)^2 + \frac{m^2}{\beta^2}(1 - \cos\beta\varphi) \right\} dx$

charges: $Q_{\pm 3}[\varphi] = \int \left\{ \frac{1}{2}(\partial_\pm^2\varphi)^2 - \frac{1}{8}(\partial_\pm\varphi)^4 + \frac{m^2}{\beta^2}(\partial_\pm\varphi)^2(1 - \cos\beta\varphi) \right\}$

Classical integrable models: sin(e/h)-Gordon theory

sine-Gordon  target 	$\beta \leftrightarrow ib$	sinh-Gordon  target 
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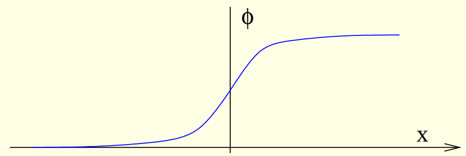
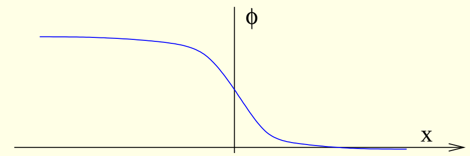
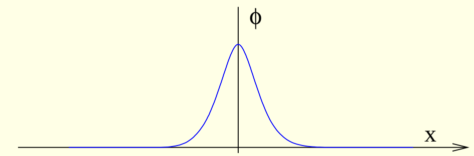
Classical finite energy solutions: sine-Gordon theory



soliton



anti-soliton



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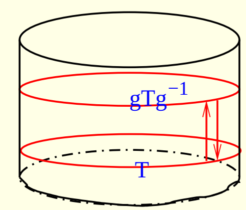

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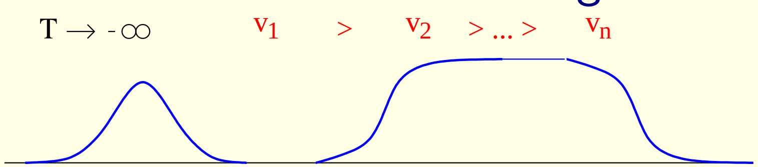
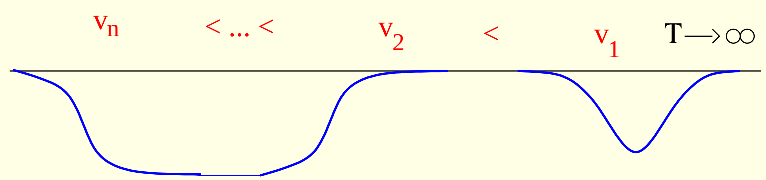
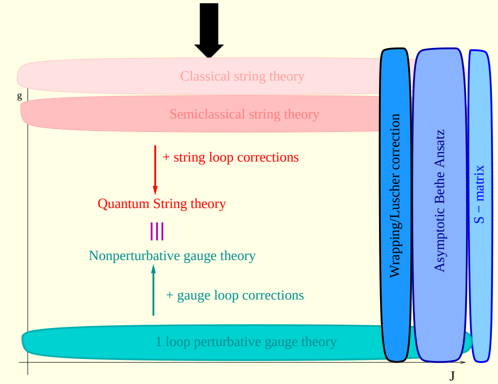
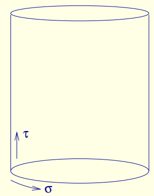
Classical factorized scattering: time delays sums up $\Delta T_1(v_1, v_2, \ldots, v_n) = \sum_i \Delta T_1(v_1, v_i)$

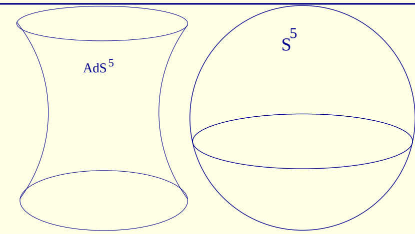




Classical integrability: AdS

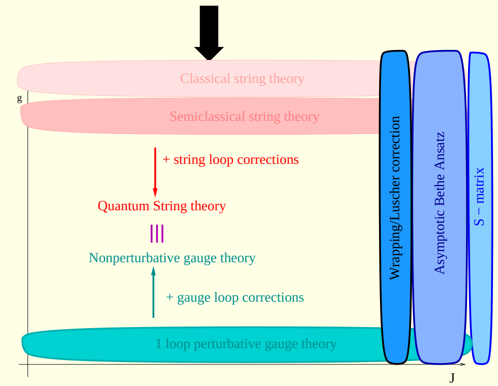


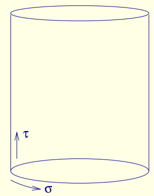
AdS σ model 

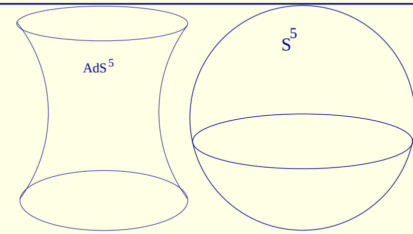
target 

$$\mathcal{L} = \frac{R^2}{\alpha'}\left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M\right) + fermions$$

Classical integrability: AdS



AdS σ model 

target 

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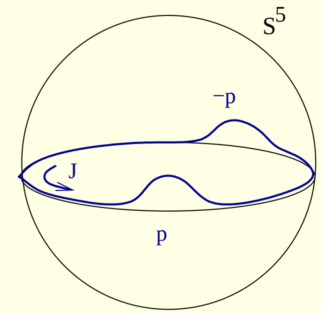
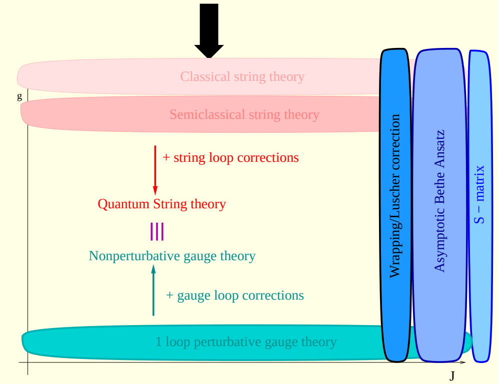
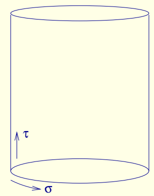
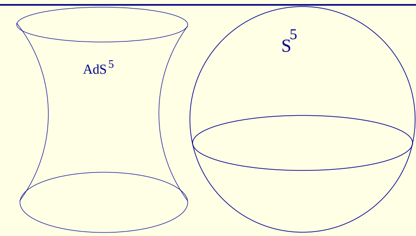


Classical solutions are found, for example magnon:

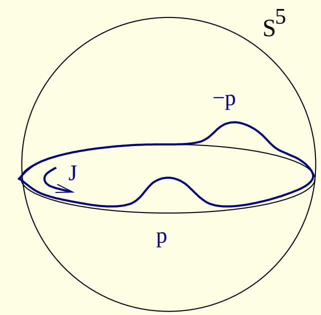
Classical integrability: AdS



AdS σ model  target 

$$\mathcal{L}=\frac{R^2}{\alpha'}\left(\partial_a X^M \partial^a X_M+\partial_a Y^M \partial^a Y_M\right)+fermions$$

Classical solutions are found, for example magnon:

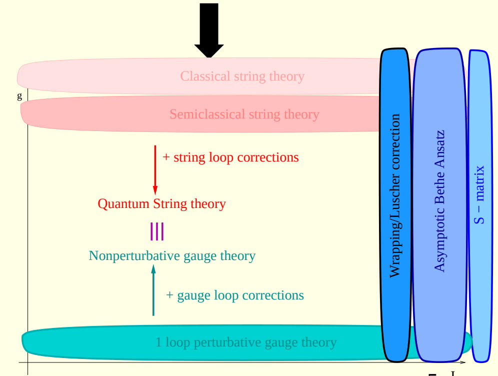


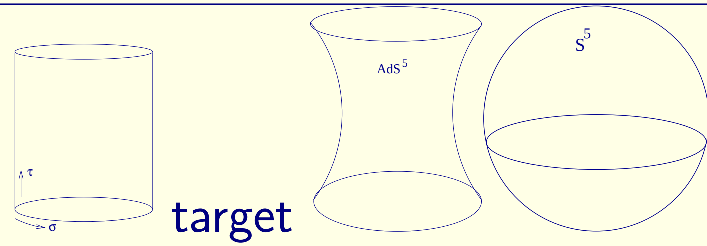
Coset NLσ model: $g\in\frac{PSU(2,2|4)}{SO(4,1)\times SO(5)}$ $\qquad J=g^{-1}dg=J_{||}+J_{\perp}$

Z_4 graded structure:

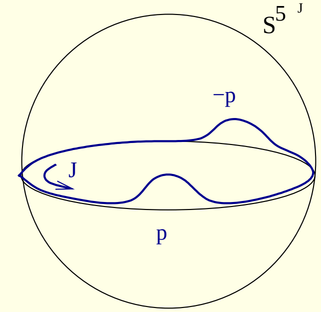
$J_{\perp}\to J_0,J_1,J_2,J_3$ $\qquad \mathcal{L}\propto \mathrm{STr}(J_2\wedge *J_2)-\mathrm{STr}(J_1\wedge J_3)$

Classical integrability: AdS



AdS σ model  target

$$\mathcal{L} = \frac{R^2}{\alpha'}\left(\partial_a X^M \partial^a X_M + \partial_a Y^M \partial^a Y_M\right) + fermions$$



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$$J = g^{-1}dg = J_{||} + J_\perp$$

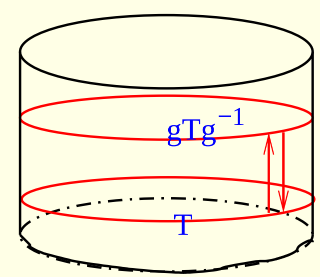
$$\mathcal{L} \propto \mathrm{STr}(J_2 \wedge *J_2) - \mathrm{STr}(J_1 \wedge J_3)$$

Integrability from flat connection: $dA - A \wedge A = 0$

$$A(\mu) = J_0 + \mu^{-1}J_1 + (\mu^2 + \mu^{-2})J_2/2 + (\mu^2 + \mu^{-2})J_2/2 + \mu J_3$$

Conserved charges from the trace of the monodromy matrix

$$T(\mu) = \mathcal{P}\exp\oint A(x)_\mu dx^\mu$$



Quantum integrability: sine-Gordon $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{\beta^2}(1-\cos\beta\phi)$

Perturbed Conformal Field Theory	Lagrangian perturbation theory
$\mathcal{L}_{CFT} + \lambda\mathcal{L}_{pert} = \frac{1}{2}(\partial\phi)^2 + \lambda(V_\beta + V_{-\beta})$	$\mathcal{L}_0 + V_{pert} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2 - \beta^2 U$
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Quantum conservation laws
$\partial_-\Lambda_4 = 0 \to \partial_-\Lambda_4 = \lambda\partial_+\Theta_2$
$[\lambda] = 2 - h_\beta$, $[\Lambda_4] = 4$,
Nonlocal symmetries $U_q(\widehat{sl}_2)$

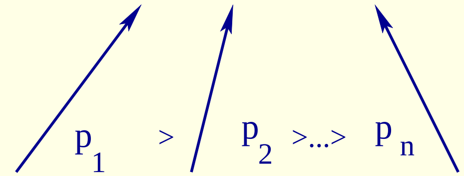
Correlators=$\sum_{loops} Feynman\, diagrams$
Asymptotic states $E(p) = \sqrt{p^2 + m^2}$
S-matrix↔correlators LSZ
unitarity, crossing symmetry, analyticity

Bootstrap scheme

Quantum integrability: AdS no proof !

Bootstrap program

Asymptotic states $|p_1, p_2, \ldots, p_n\rangle_{in/out}$
form a representation of global symmetry:



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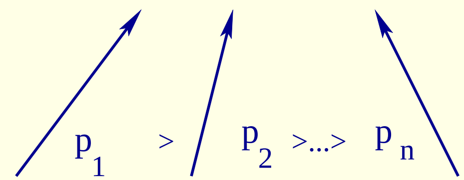

Lorentz: $P = \sum_i p_i \;\; E = \sum_i E(p_i)$
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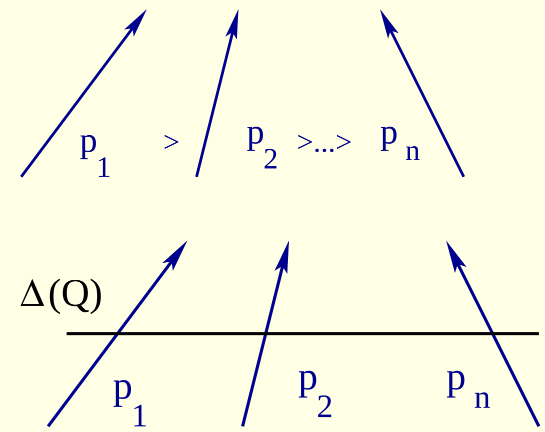

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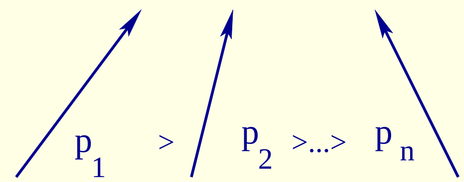

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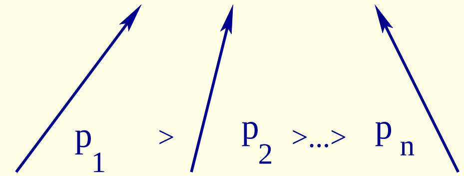

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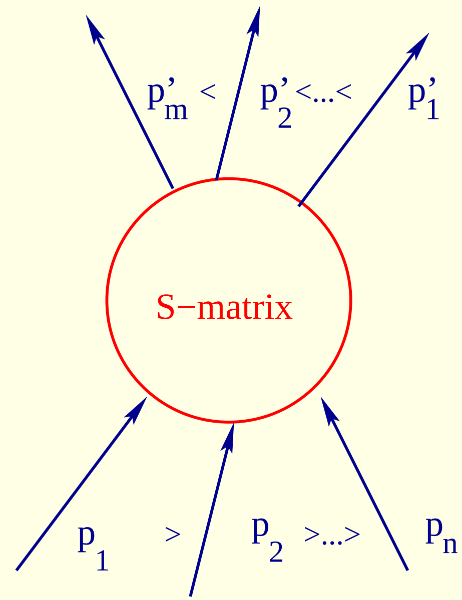

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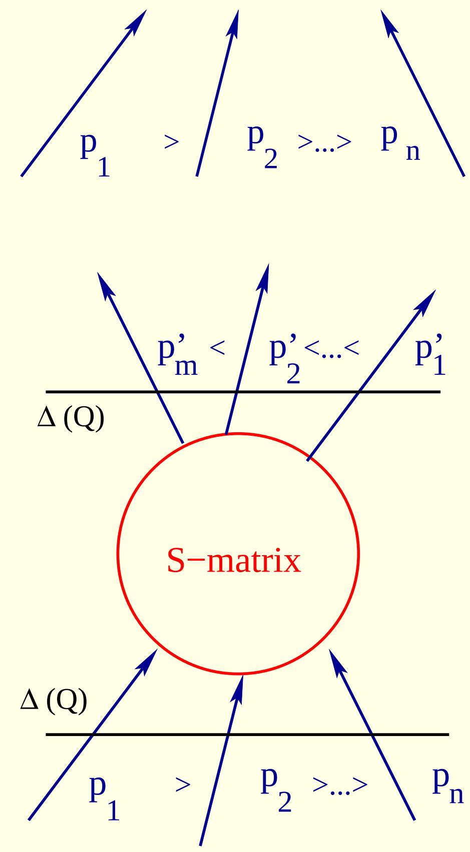

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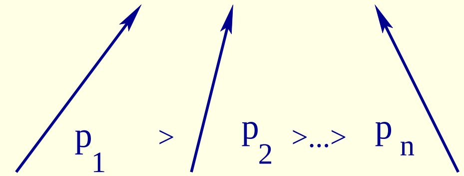

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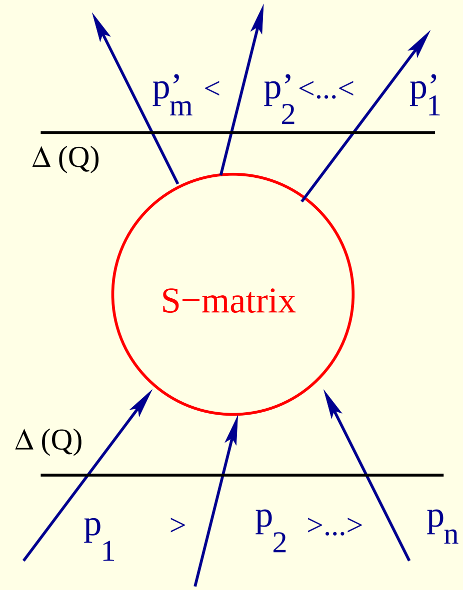

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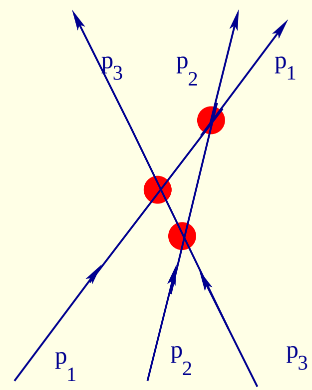
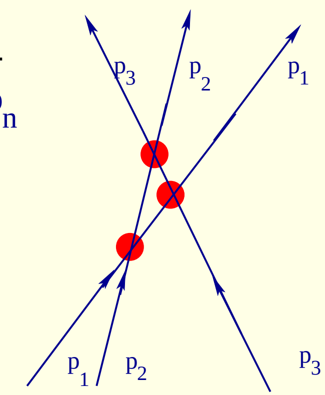
Higher spin concerved charge
factorization + Yang-Baxter equation
$S_{123} = S_{23}S_{13}S_{12} = S_{12}S_{13}S_{23}$





S-matrix = scalar . Matrix

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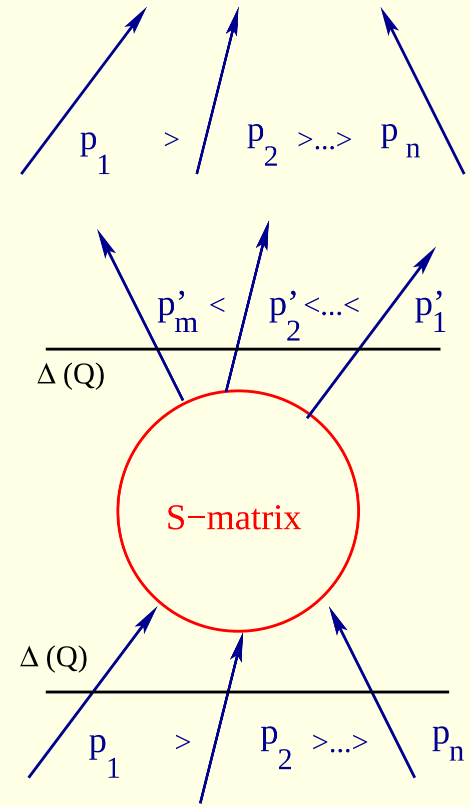

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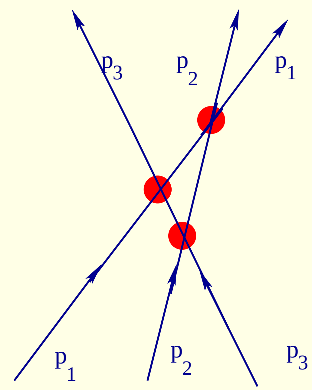

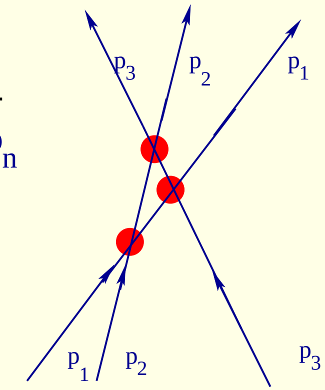

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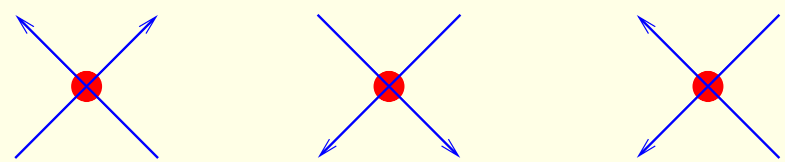
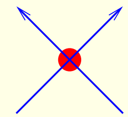


Unitarity $S_{12}S_{21} = Id$
Crossing symmetry $S_{12} = S_{2\bar{1}}$
Maximal analyticity: all poles have physical origin $\longrightarrow$ boundstates, anomalous thresholds

Bootstrap program: diagonal

Diagonal scattering: S-matrix = scalar $S(p_1, p_2) = S(\theta_1 - \theta_2)$  $p = m \sinh\theta$

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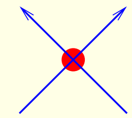
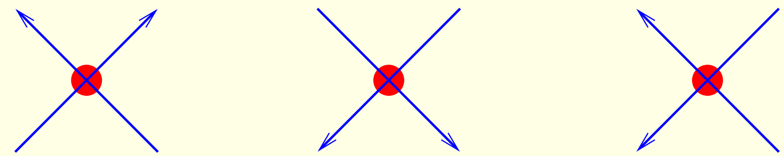
Diagonal scattering: S-matrix = scalar $S(p_1, p_2) = S(\theta_1 - \theta_2)$  $p = m \sinh \theta$



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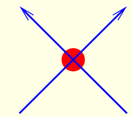
Diagonal scattering: S-matrix = scalar $S(p_1, p_2) = S(\theta_1 - \theta_2)$  $p = m \sinh\theta$

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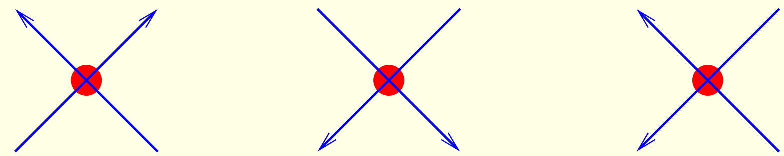

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Minimal solution: $S(\theta) = 1$ Free boson

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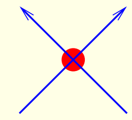
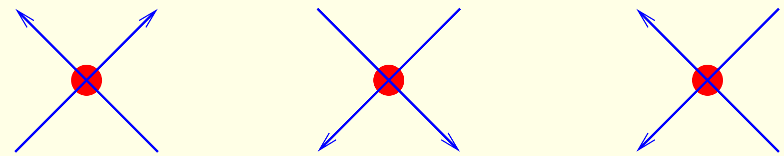
Diagonal scattering: $\boxed{\text{S-matrix} = \text{scalar}}$ $S(p_1, p_2) = S(\theta_1 - \theta_2)$  $p = m \sinh \theta$



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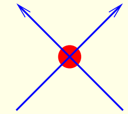
Diagonal scattering: S-matrix = scalar $S(p_1,p_2) = S(\theta_1-\theta_2)$  $p = m\sinh\theta$

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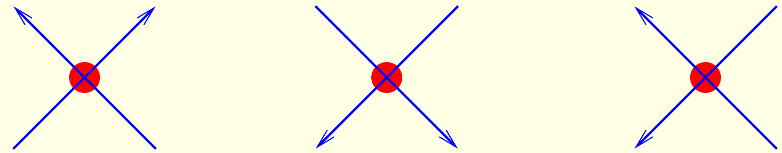

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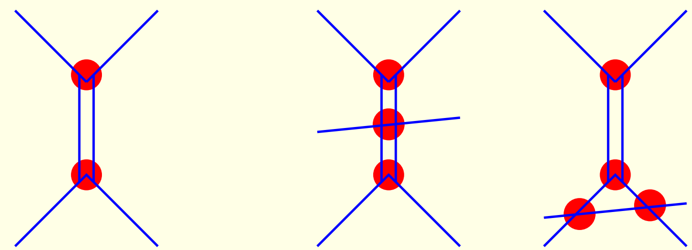
Maximal analyticity: $S(\theta) = \frac{\sinh\theta + i\sin p\pi}{\sinh\theta - i\sin p\pi}$

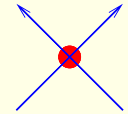
pole at $\theta = ip\pi$ →boundstate B^2

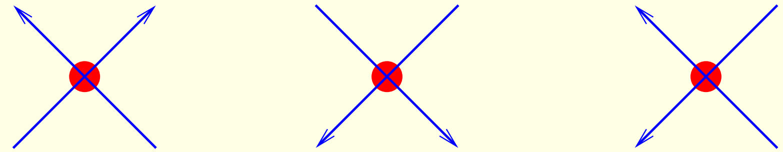


bootstrap: $S_{12}(\theta) = S_{11}(\theta - \frac{ip\pi}{2}) S_{11}(\theta + \frac{ip\pi}{2})$

new particle if $p \neq \frac{2}{3}$ Lee-Yang

Bootstrap program: diagonal

Diagonal scattering: S-matrix = scalar $S(p_1,p_2)=S(\theta_1-\theta_2)$  $p=m\sinh\theta$



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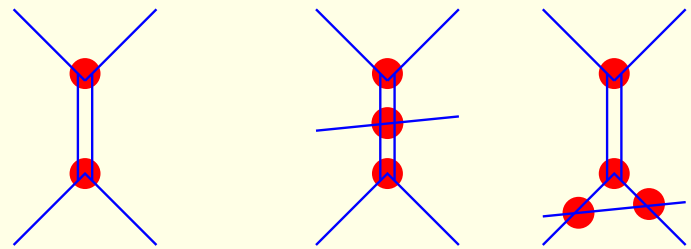

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pole at $\theta=ip\pi$ $\rightarrow$boundstate B^2

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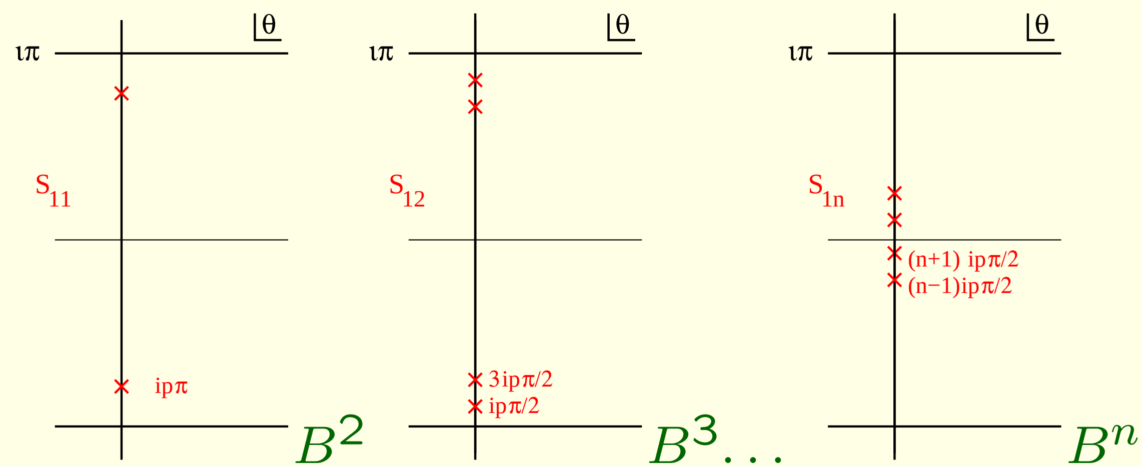
$\rightarrow$sine-Gordon solitons



Bootstrap program: sine-Gordon

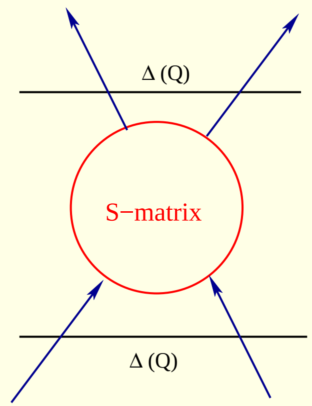
Nondiagonal scattering: **S-matrix = scalar . Matrix** soliton doublet $\begin{pmatrix} s \\ \bar{s} \end{pmatrix}$

Matrix:

global symmetry $U_q(s\hat{l}_2)$

2d evaluation reps

$[S, \Delta(Q)] = 0$



$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{-\sin\lambda\pi}{\sin\lambda(\pi+i\theta)} & \frac{\sin i\lambda\theta}{\sin\lambda(\pi+i\theta)} & 0 \\ 0 & \frac{\sin i\lambda\theta}{\sin\lambda(\pi+i\theta)} & \frac{-\sin\lambda\pi}{\sin\lambda(\pi+i\theta)} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

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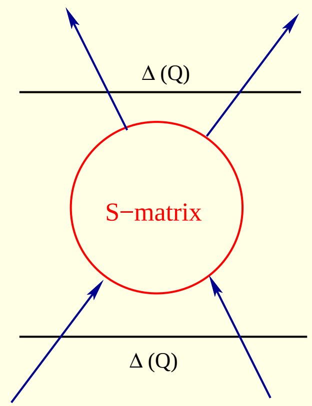

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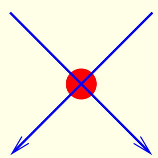
Unitarity

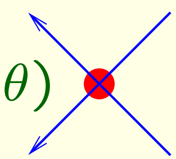
$\mathcal{S}(\theta)\mathcal{S}(-\theta) = 1$



$$\prod_{l=1}^{\infty}\left[\frac{\Gamma(2(l-1)\lambda+\frac{\lambda i\theta}{\pi})\Gamma(2l\lambda+1+\frac{\lambda i\theta}{\pi})}{\Gamma((2l-1)\lambda+\frac{\lambda i\theta}{\pi})\Gamma((2l-1)\lambda+1+\frac{\lambda i\theta}{\pi})}/(\theta\to-\theta)\right]$$

Crossing symmetry

$\mathcal{S}(\theta) = \mathcal{S}^{c_1}(i\pi-\theta)$



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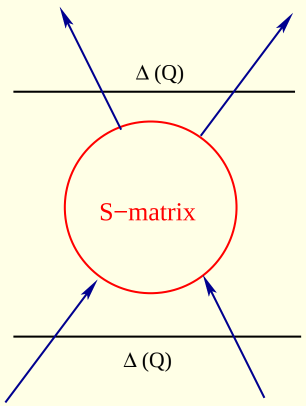

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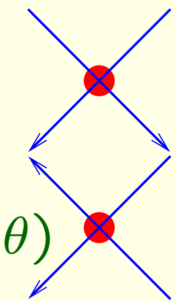

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either boundstates or

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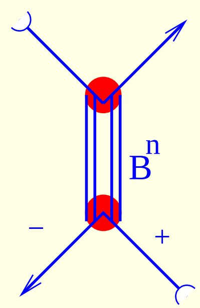

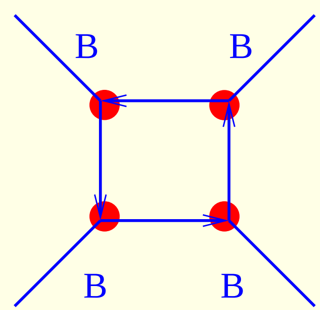

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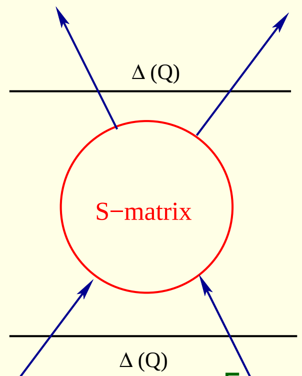

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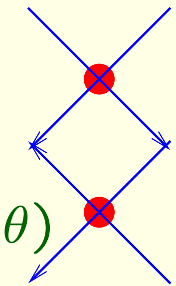

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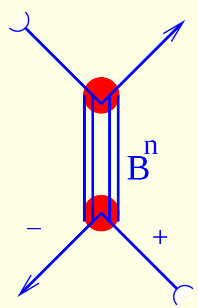
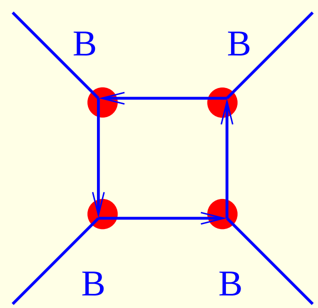
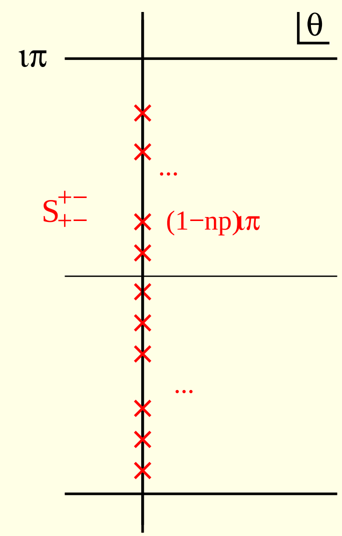
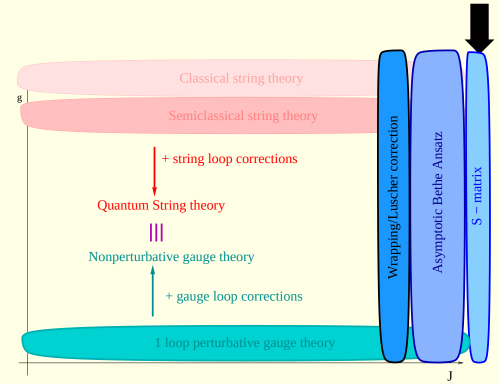




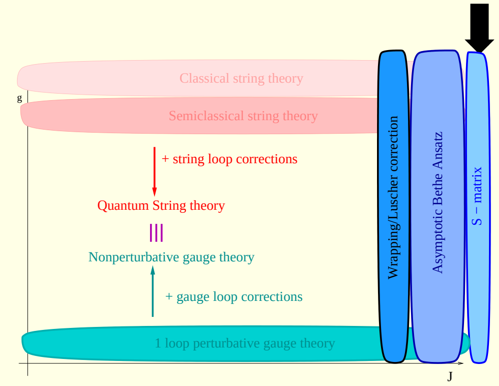




Bootstrap program: AdS

Nondiagonal scattering: S-matrix = scalar . Matrix

Bootstrap program: AdS



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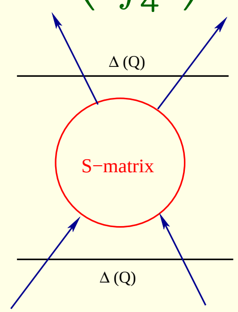
global symmetry $PSU(2|2)^2$

$$Q = 1 \text{ reps } \begin{pmatrix} b_1 \\ b_2 \\ f_3 \\ f_4 \end{pmatrix} \otimes \begin{pmatrix} b_{\dot{1}} \\ b_{\dot{2}} \\ f_{\dot{3}} \\ f_{\dot{4}} \end{pmatrix}$$

$$[S, \Delta(Q)] = 0$$



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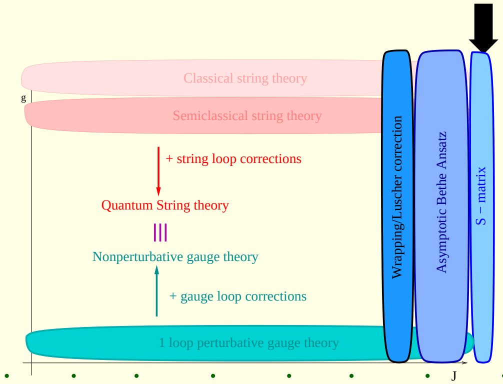

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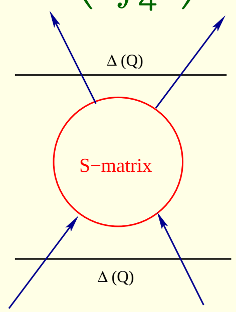

$$[S, \Delta(Q)] = 0$$

$$\begin{pmatrix}
1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & A & \cdot & \cdot & -b & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -c & \cdot & \cdot & c & \cdot \\
\cdot & \cdot & d & \cdot & \cdot & \cdot & \cdot & \cdot & e & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & -b & \cdot & d & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & e & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & A & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & c & \cdot & \cdot & -c & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d & \cdot & e & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d & \cdot & \cdot & \cdot & \cdot & \cdot & e & \cdot & \cdot \\
\cdot & \cdot & f & \cdot & \cdot & \cdot & \cdot & \cdot & g & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & f & \cdot & \cdot & g & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & -h & \cdot & \cdot & h & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & B & \cdot & \cdot & -i & \cdot \\
\cdot & \cdot & \cdot & f & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & g & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & f & \cdot & \cdot & \cdot & \cdot & \cdot & g & \cdot & \cdot \\
\cdot & h & \cdot & \cdot & -h & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -i & \cdot & \cdot & B & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a
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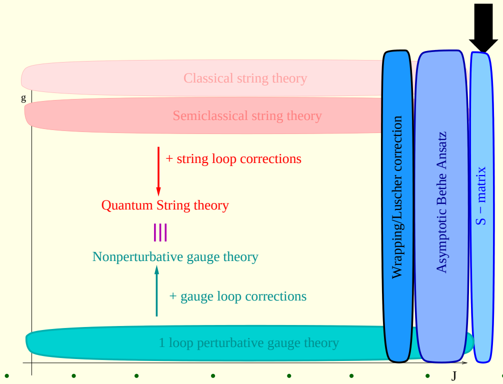

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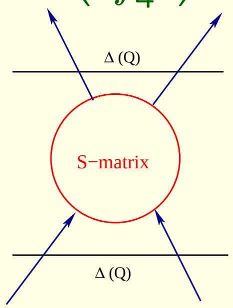


$$\begin{pmatrix}
1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & A & . & . & -b & . & . & . & . & . & . & -c & . & . & c & . \\
. & . & d & . & . & . & . & . & e & . & . & . & . & . & . & . \\
. & -b & . & d & . & . & . & . & . & . & . & . & e & . & . & . \\
. & . & . & . & A & . & . & . & . & . & . & c & . & . & -c & . \\
. & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & d & . & e & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & d & . & . & . & . & . & e & . & . \\
. & . & f & . & . & . & . & . & g & . & . & . & . & . & . & . \\
. & . & . & . & . & . & f & . & . & g & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & a & . & . & . & . & . \\
. & -h & . & . & h & . & . & . & . & . & . & B & . & . & -i & . \\
. & . & . & f & . & . & . & . & . & . & . & . & g & . & . & . \\
. & . & . & . & . & . & . & f & . & . & . & . & . & g & . & . \\
. & h & . & . & -h & . & . & . & . & . & . & -i & . & . & B & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & a
\end{pmatrix}$$

Unitarity

$$\mathcal{S}(z_1, z_2)\mathcal{S}(z_2, z_1) = 1$$

Crossing symmetry

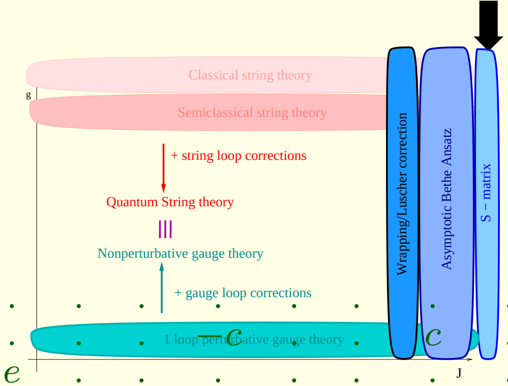
$$\mathcal{S}(z_1, z_2) = \mathcal{S}^{c_1}(z_2, z_1 + \omega_2)$$

$$S_{11}^{11} = \frac{u_1 - u_2 - i}{u_1 - u_2 + i} e^{i2\theta(z_1, z_2)}$$

$$u = \frac{1}{2} \cot \frac{p}{2} E(p)$$

$$p = 2\,\text{am}(z)$$

Bootstrap program: AdS



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\cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
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\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & d & \cdot & \cdot & \cdot & \cdot & \cdot & e & \cdot & \cdot \\
\cdot & \cdot & f & \cdot & \cdot & \cdot & \cdot & \cdot & g & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & f & \cdot & \cdot & g & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a & \cdot & \cdot & \cdot & \cdot & \cdot \\
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\cdot & h & \cdot & \cdot & -h & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -i & \cdot & \cdot & B & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a
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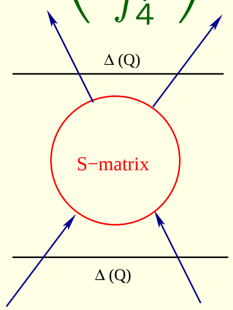

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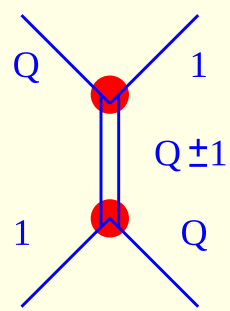
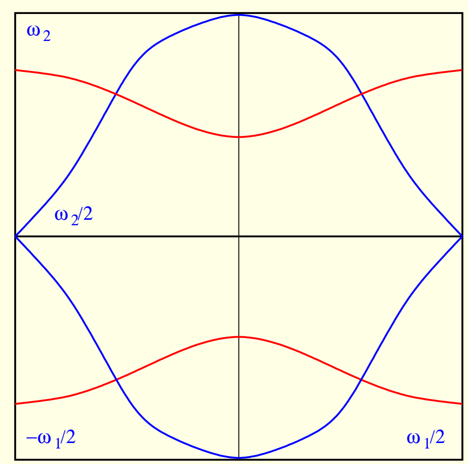
Maximal analyticity:

boundstates atyp symrep: $Q \in \mathbf{N}$

anomalous thresholds

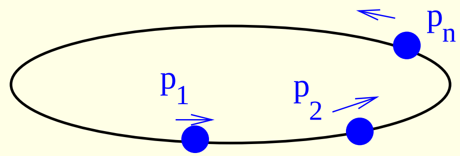


Physical domain?



QFTs in finite volume

Finite volume spectrum



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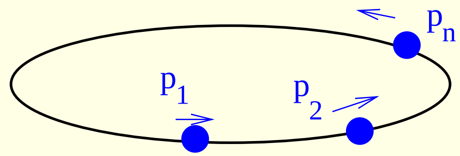

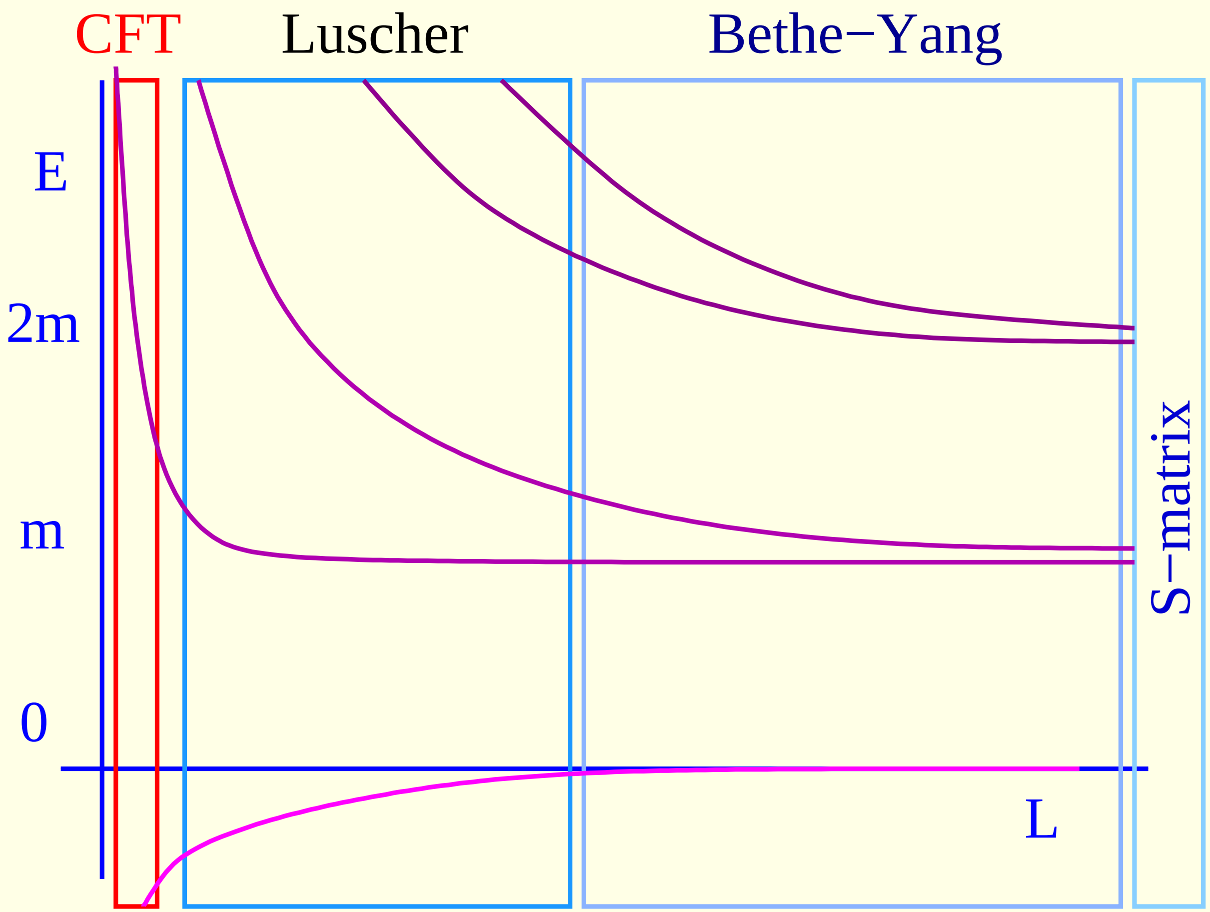

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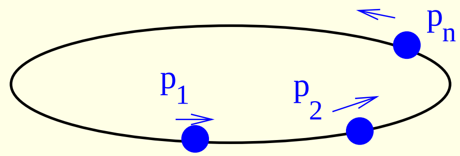

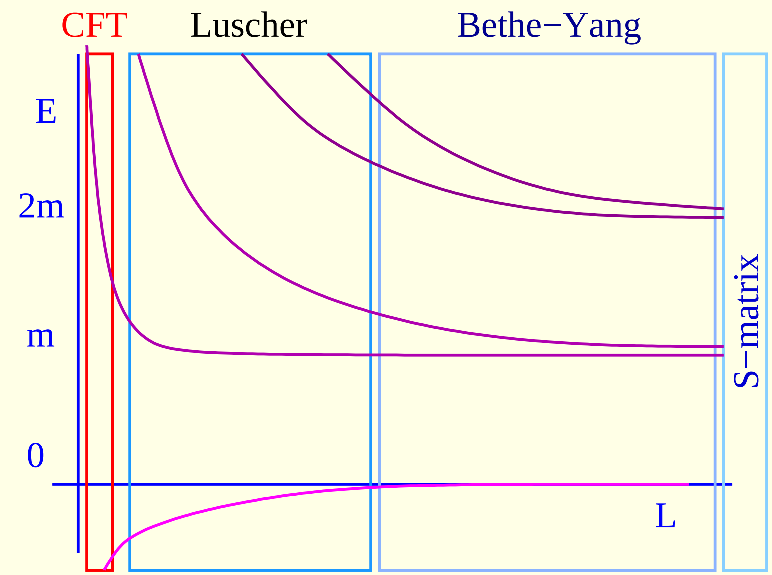

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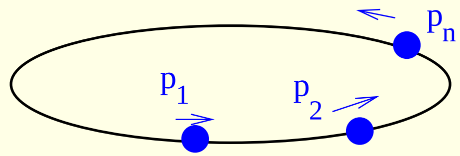
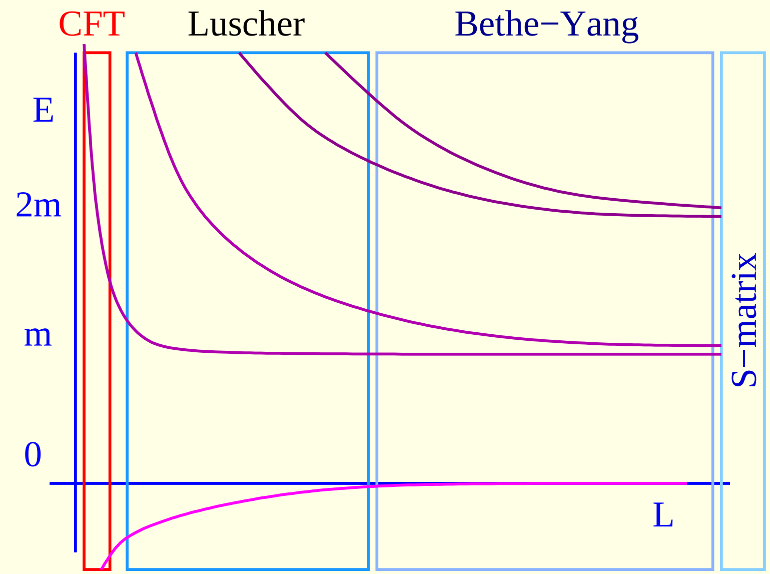


Infinite volume spectrum:

$E(p_1,\ldots,p_n)=\sum_i E(p_i) \qquad p_i\in R$



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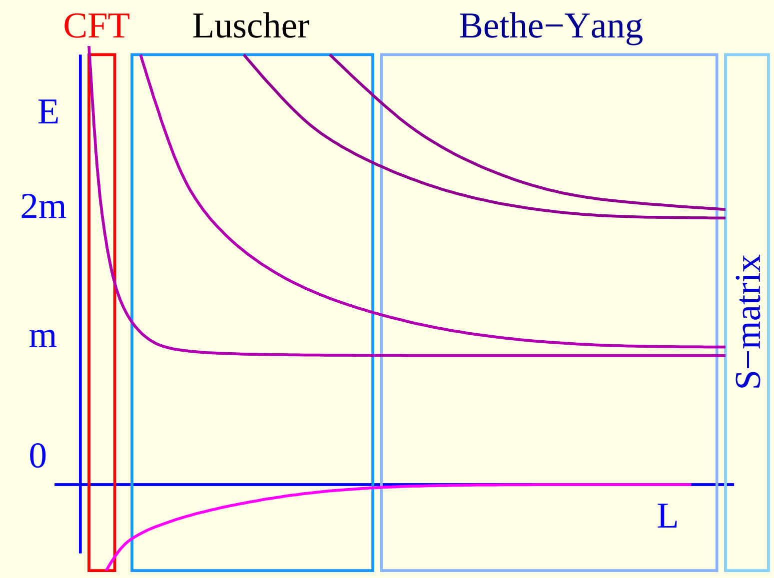

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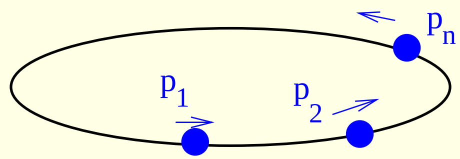

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Polynomial volume corrections:

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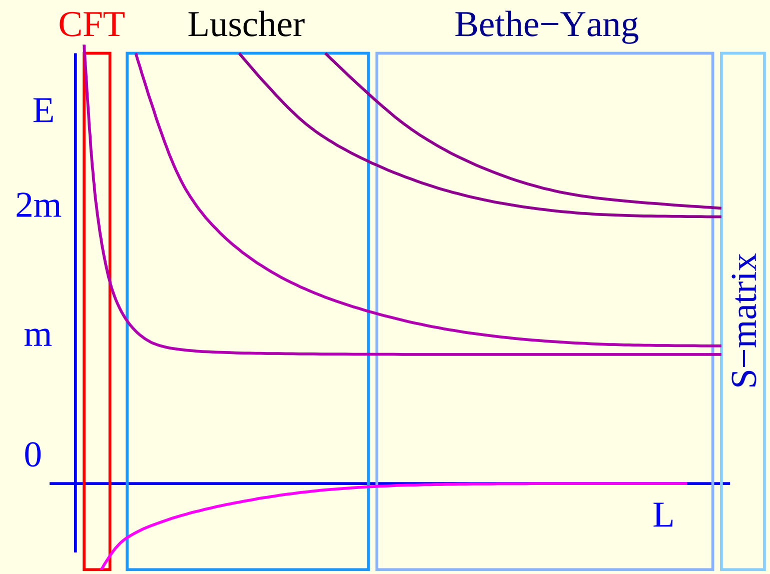
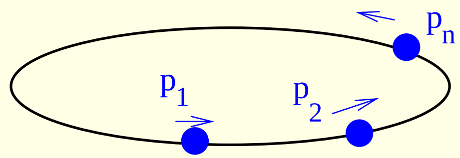
QFTs in finite volume



Finite volume spectrum



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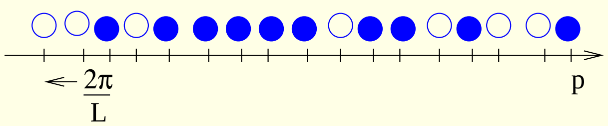
$$p_j L + \sum_k \frac{1}{i} \log S(p_j, p_k) = (2n+1) i\pi$$



QFTs in finite volume

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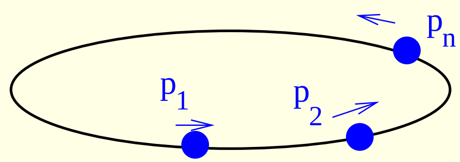

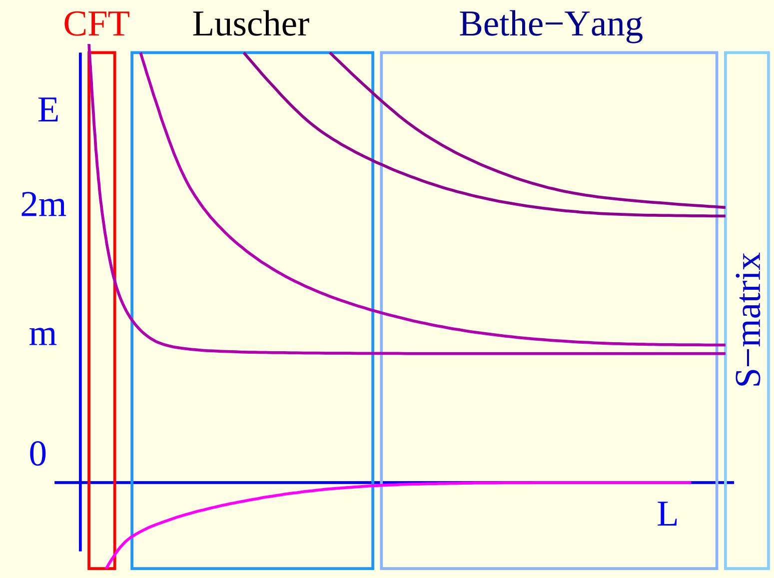

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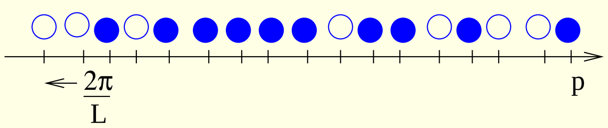

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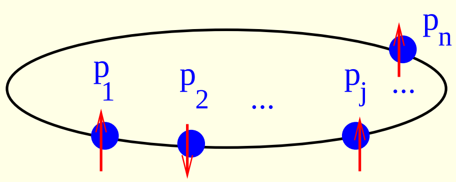
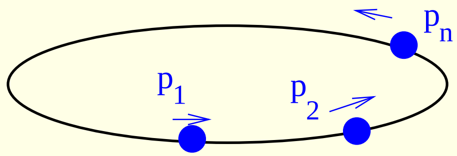
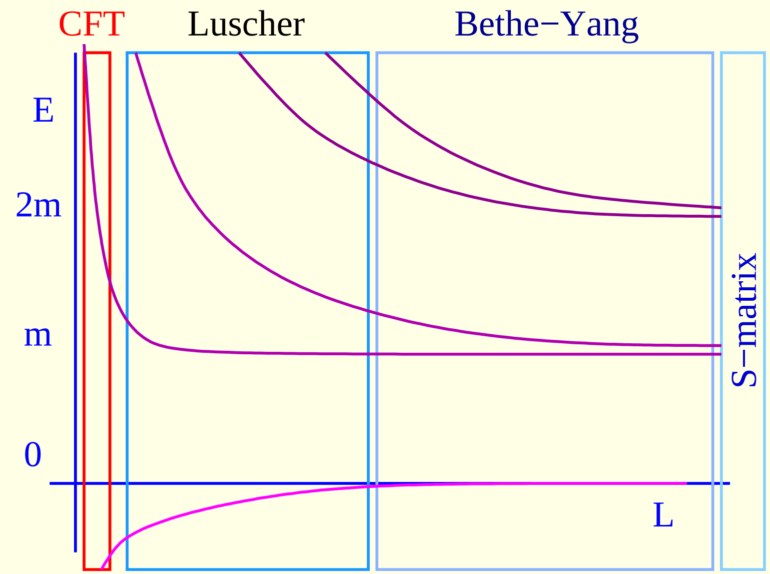


$e^{ip_jL}\mathcal{S}(p_j,p_1)\ldots\mathcal{S}(p_j,p_n)\Psi=-\Psi \quad S(0)=-P$

QFTs in finite volume

Finite volume spectrum





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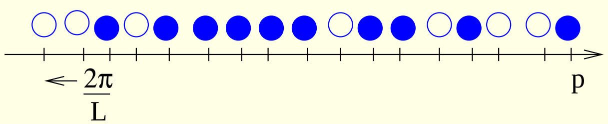

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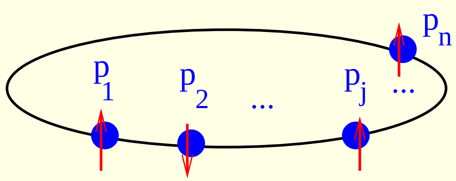
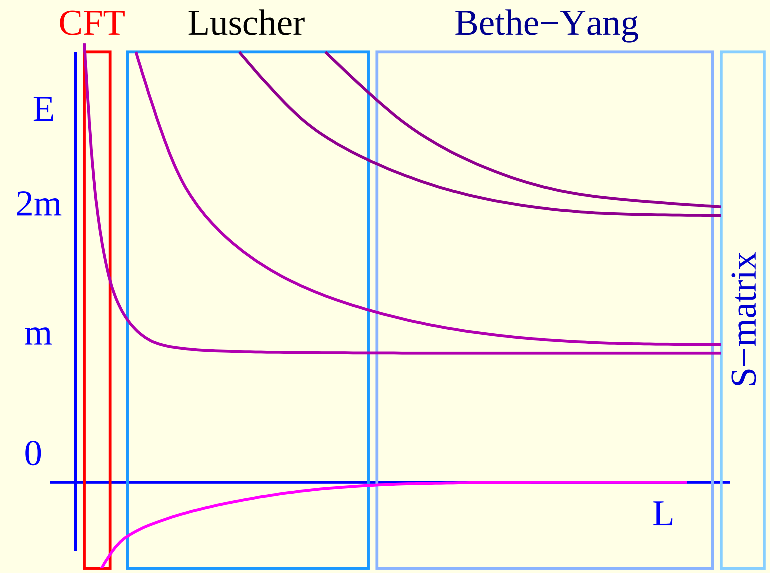
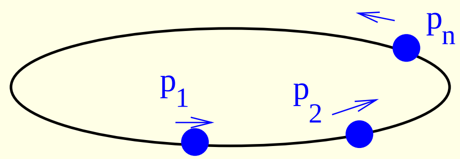


Inhomogenous XXZ spin-chain spectral problem $e^{iL\sinh\theta_j}T(\theta_j)S_0(\theta_j)=-1$

QFTs in finite volume



Finite volume spectrum



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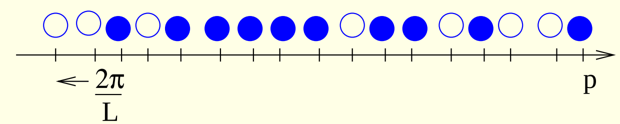

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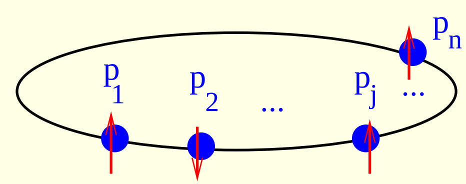

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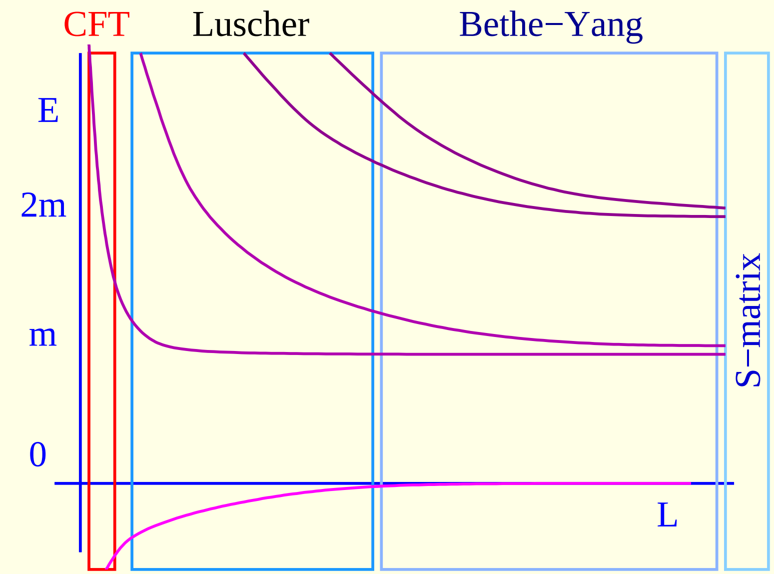
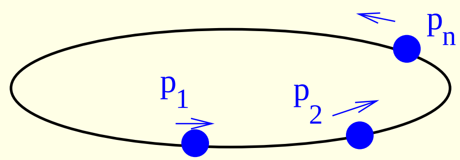
$T(\theta)Q(\theta)=Q(\theta+i\pi)T_0(\theta-\frac{i\pi}{2})+Q(\theta-i\pi)T_0(\theta+\frac{i\pi}{2})=Q^{++}T^-+Q^{--}T^+$

QFTs in finite volume



Finite volume spectrum



Infinite volume spectrum:

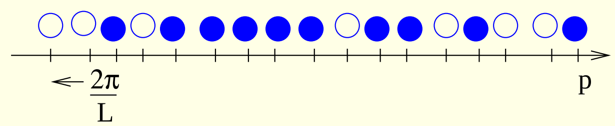
$E(p_1,\dots,p_n)=\sum_i E(p_i) \qquad p_i\in R$

Polynomial volume corrections:

Bethe-Yang; p_i quantized. Diagonal

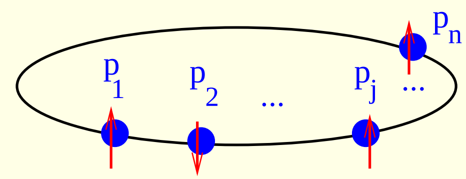
$e^{ip_jL}S(p_j,p_1)\dots S(p_j,p_n)=-1 \quad ; \quad S(0)=-1$

$p_jL+\sum_k \frac{1}{i}\log S(p_j,p_k)=(2n+1)i\pi$



Non-diagonal, sine-Gordon

$e^{ip_jL}\mathcal{S}(p_j,p_1)\dots \mathcal{S}(p_j,p_n)\Psi=-\Psi \quad S(0)=-P$



Inhomogenous XXZ spin-chain spectral problem $e^{iL\sinh\theta_j}T(\theta_j)S_0(\theta_j)=-1$

$T(\theta)Q(\theta)=Q(\theta+i\pi)T_0(\theta-\frac{i\pi}{2})+Q(\theta-i\pi)T_0(\theta+\frac{i\pi}{2})=Q^{++}T^{-}+Q^{--}T^{+}$

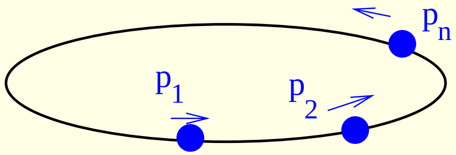
$Q(\theta)=\prod_\beta \sinh(\lambda(\theta-w_\beta))$

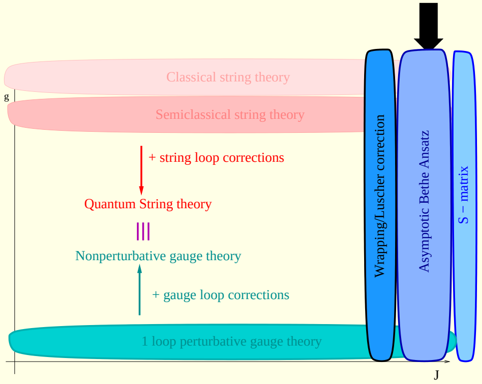
$T_0(\theta)=\prod_j \sinh(\lambda(\theta-\theta_j))$

Bethe Ansatz: $\frac{T_0(w_\alpha-\frac{i\pi}{2})Q(w_\alpha+i\pi)}{T_0(w_\alpha+\frac{i\pi}{2})Q(w_\alpha-i\pi)}=\frac{T_0^-Q^{++}}{T_0^+Q^{--}}|_\alpha=-1$

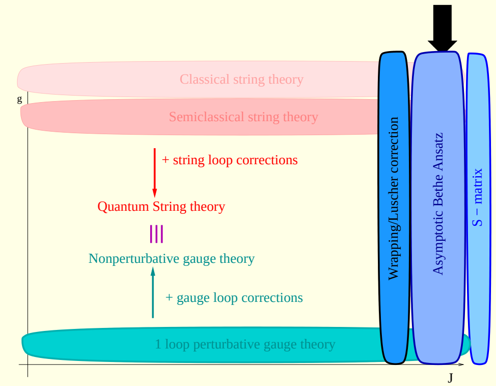
Bethe-Yang=Asymptotic Bethe Ansatz

Finite volume spectrum

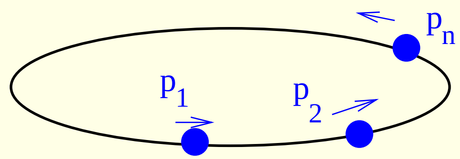




Bethe-Yang=Asymptotic Bethe Ansatz



Finite volume spectrum

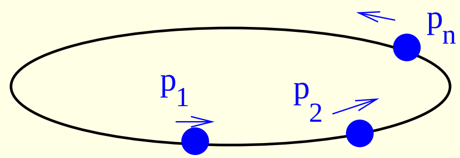


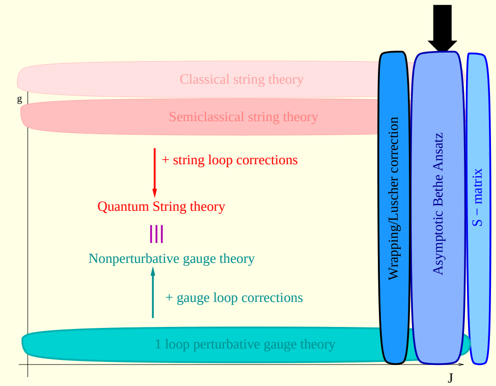
Infinite volume spectrum:

$E(p_1,\dots,p_n)=\sum_i E(p_i)$ $\qquad E(p)=\sqrt{1+(4g\sin\frac{p}{2})^2}$ $\qquad p_i\in[-\pi,\pi]$

Bethe-Yang=Asymptotic Bethe Ansatz

Finite volume spectrum





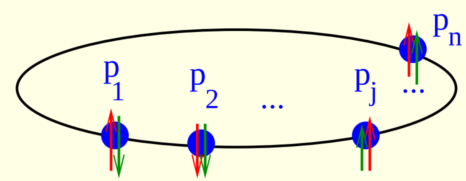
Infinite volume spectrum:

$E(p_1,\dots,p_n)=\sum_i E(p_i) \qquad E(p)=\sqrt{1+(4g\sin\frac{p}{2})^2} \qquad p_i\in[-\pi,\pi]$

Polynomial volume corrections:

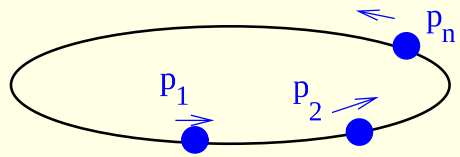
Asymptotic Bethe Ansatz; p_i quantized, .

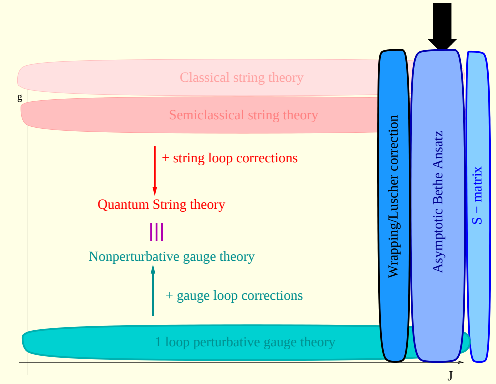
$e^{ip_jL}\mathcal{S}(p_j,p_1)\dots\mathcal{S}(p_j,p_n)\Psi=-\Psi \quad S(0)=-\hat{P}$



Bethe-Yang=Asymptotic Bethe Ansatz

Finite volume spectrum





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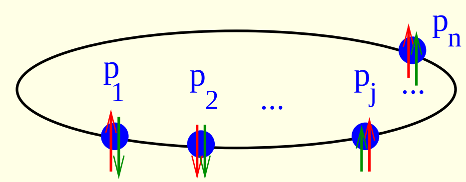
$$E(p_1,\dots,p_n)=\sum_i E(p_i) \qquad E(p)=\sqrt{1+(4g\sin\tfrac{p}{2})^2} \qquad p_i\in[-\pi,\pi]$$

Polynomial volume corrections:

Asymptotic Bethe Ansatz; p_i quantized, .

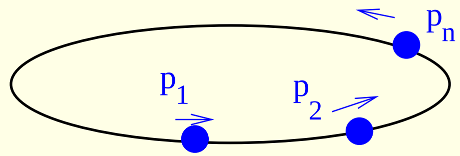
$$e^{ip_jL}\mathcal{S}(p_j,p_1)\dots\mathcal{S}(p_j,p_n)\Psi=-\Psi \quad S(0)=-\hat{P}$$

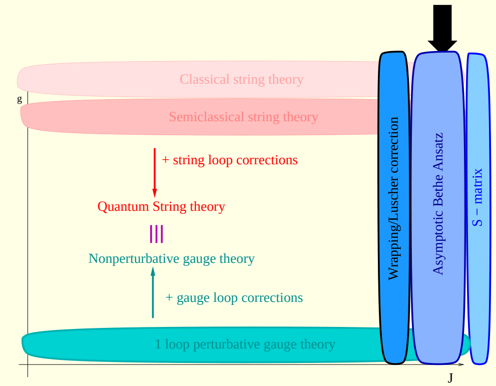


Inhomogenous Hubbard^2 spin-chain: $e^{iLp_j}S_0^2(u_j)\frac{Q_4^{++}(u_j)}{Q_4^{--}(u_j)}T(u_j)\dot{T}(u_j)=-1$

Bethe-Yang=Asymptotic Bethe Ansatz

Finite volume spectrum





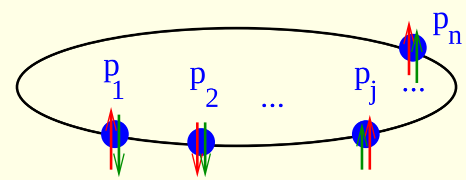
Infinite volume spectrum:

$$E(p_1,\dots,p_n)=\sum_i E(p_i) \qquad E(p)=\sqrt{1+(4g\sin\tfrac{p}{2})^2} \qquad p_i\in[-\pi,\pi]$$

Polynomial volume corrections:

Asymptotic Bethe Ansatz; p_i quantized, .

$$e^{ip_jL}\mathcal{S}(p_j,p_1)\dots\mathcal{S}(p_j,p_n)\Psi=-\Psi \quad S(0)=-\hat{P}$$



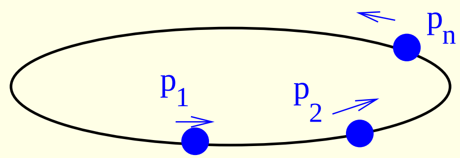
Inhomogenous Hubbard^2 spin-chain: $e^{iLp_j}S_0^2(u_j)\dfrac{Q_4^{++}(u_j)}{Q_4^{--}(u_j)}T(u_j)\dot{T}(u_j)=-1$

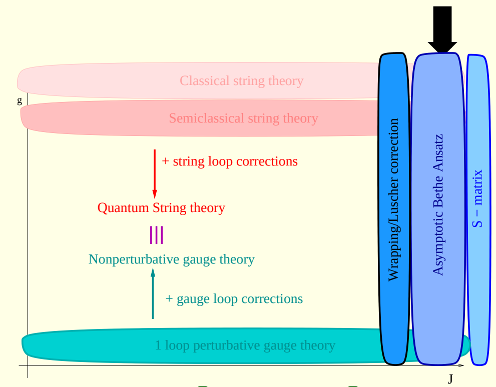
$$T(u)=\frac{B_1^-B_3^+R_4^{-(+)}}{B_1^+B_3^-R_4^{-(-)}}\left[\frac{Q_2^{--}Q_3^+}{Q_2Q_3^-}-\frac{R_4^{-(-)}Q_3^+}{R_4^{-(+)}Q_3^-}+\frac{Q_2^{++}Q_1^-}{Q_2Q_1^+}-\frac{B_4^{+(+)}Q_1^-}{B_4^{+(-)}Q_1^+}\right]$$

$Q_j(u)=-R_j(u)B_j(u)$ and $R_j^{(\pm)}=\prod_k\frac{x(u)-x_{j,k}^{\mp}}{\sqrt{x_{j,k}^{\mp}}} \qquad B_j^{(\pm)}=\prod_k\frac{\frac{1}{x(u)}-x_{j,k}^{\mp}}{\sqrt{x_{j,k}^{\mp}}}$

Bethe-Yang=Asymptotic Bethe Ansatz

Finite volume spectrum





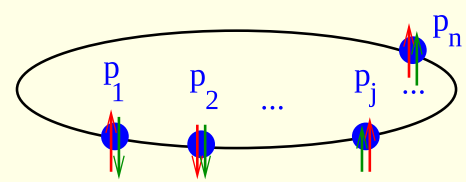
Infinite volume spectrum:

$$E(p_1,\ldots,p_n)=\sum_i E(p_i) \qquad E(p)=\sqrt{1+(4g\sin\tfrac{p}{2})^2} \qquad p_i\in[-\pi,\pi]$$

Polynomial volume corrections:

Asymptotic Bethe Ansatz; p_i quantized, .

$$e^{ip_jL}\mathcal{S}(p_j,p_1)\ldots\mathcal{S}(p_j,p_n)\Psi=-\Psi \quad S(0)=-\hat{P}$$



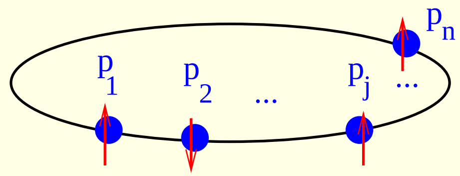
Inhomogenous Hubbard^2 spin-chain: $e^{iLp_j}S_0^2(u_j)\dfrac{Q_4^{++}(u_j)}{Q_4^{--}(u_j)}T(u_j)\dot{T}(u_j)=-1$

$$T(u)=\frac{B_1^-B_3^+R_4^{-(+)}}{B_1^+B_3^-R_4^{-(-)}}\left[\frac{Q_2^{--}Q_3^+}{Q_2Q_3^-}-\frac{R_4^{-(-)}Q_3^+}{R_4^{-(+)}Q_3^-}+\frac{Q_2^{++}Q_1^-}{Q_2Q_1^+}-\frac{B_4^{+(+)}Q_1^-}{B_4^{+(-)}Q_1^+}\right]$$

$Q_j(u)=-R_j(u)B_j(u)$ and $R_j^{(\pm)}=\prod_k\frac{x(u)-x_{j,k}^{\mp}}{\sqrt{x_{j,k}^{\mp}}}$ $\qquad B_j^{(\pm)}=\prod_k\frac{\frac{1}{x(u)}-x_{j,k}^{\mp}}{\sqrt{x_{j,k}^{\mp}}}$

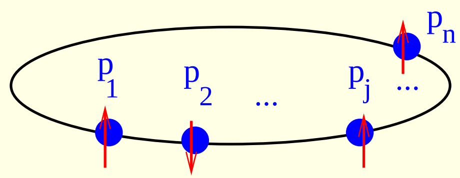
Bethe Ansatz: $\dfrac{Q_2^+B_4^{(-)}}{Q_2^-B_4^{(+)}}|_1=1 \quad \dfrac{Q_2^{--}Q_1^+Q_3^+}{Q_2^{++}Q_1^-Q_3^-}|_2=-1 \quad \dfrac{Q_2^+R_4^{(-)}}{Q_2^-R_4^{(+)}}|_3=1$

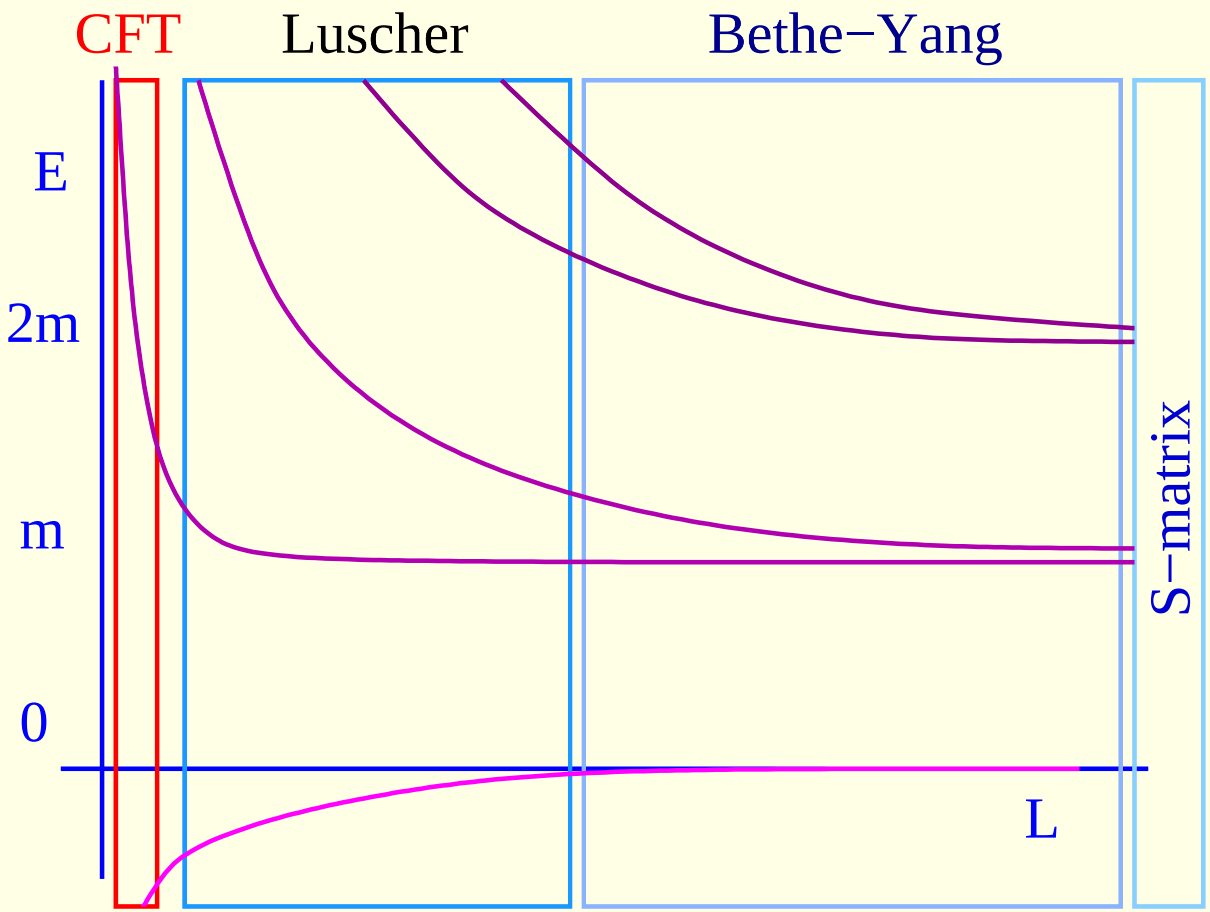
Lüscher correction of multiparticle states

Finite volume spectrum 

Lüscher correction of multiparticle states

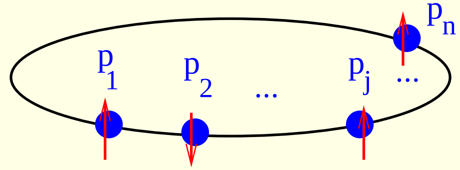
Finite volume spectrum

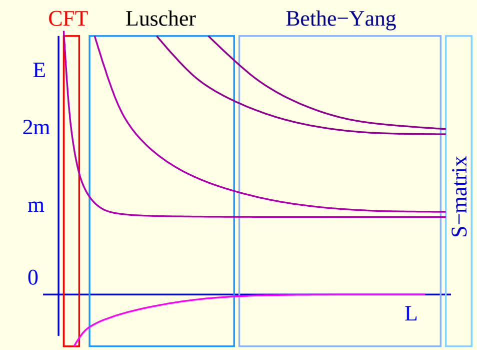




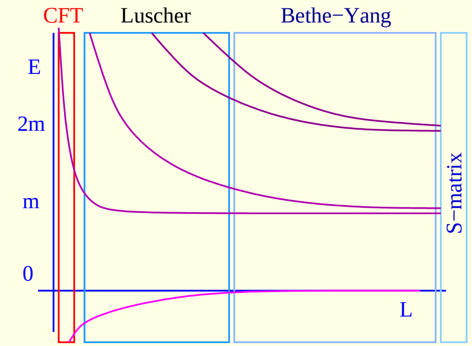
Lüscher correction of multiparticle states

Finite volume spectrum

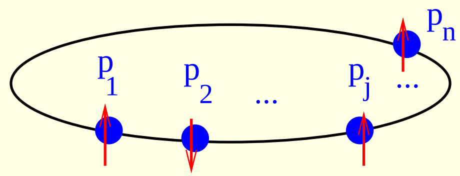




Lüscher correction of multiparticle states



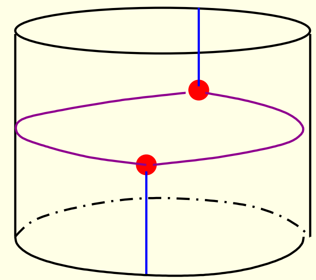
Finite volume spectrum

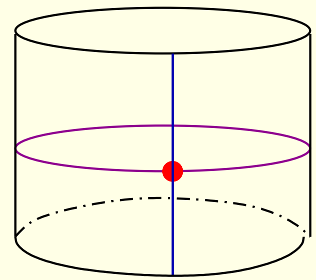


Lüscher originally: $O(e^{-mL})$ mass correction

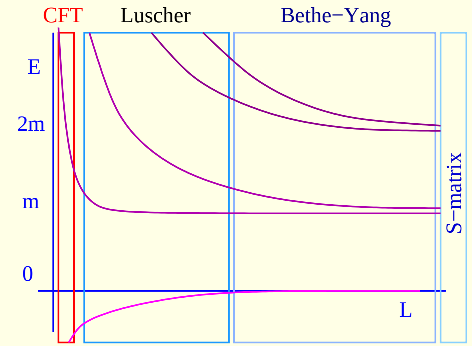
$$m(L) = \frac{\sqrt{3}}{2} m \left(-i \,\mathrm{Res}_{\theta=\frac{2i\pi}{3}} S\right) e^{-\frac{\sqrt{3}}{2} mL}$$

$$- \int \frac{d\theta}{2\pi} \cosh\theta \left(S\left(\theta + \frac{i\pi}{2}\right) - 1\right) e^{-mL\cosh\theta}$$

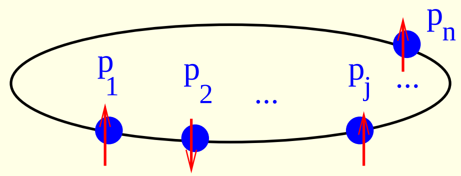




Lüscher correction of multiparticle states



Finite volume spectrum



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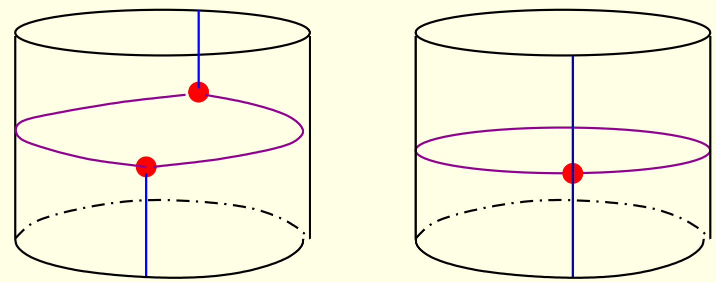
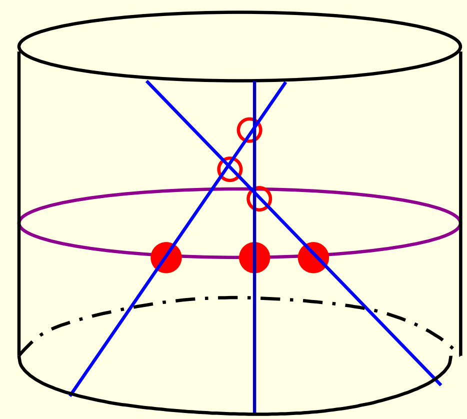
$$-\int \frac{d\theta}{2\pi} \cosh\theta (S(\theta + \frac{i\pi}{2}) - 1) e^{-mL\cosh\theta}$$



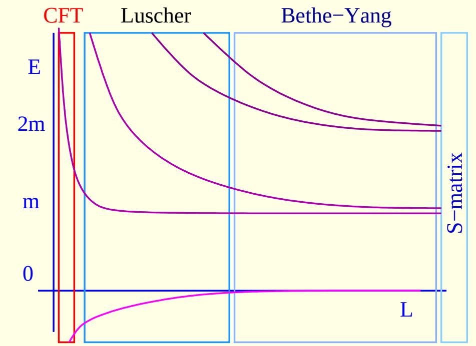
Multiparticle Lüscher correction



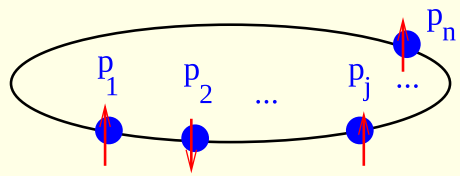
BY: $\mathcal{S}(p_j, p_1) \dots \mathcal{S}(p_j, p_n)\Psi = -e^{-ip_j L}\Psi$

$$T(p, p_1, \dots, p_n)\Psi = t(p, p_1, \dots, p_n, \Psi)\Psi$$

Lüscher correction of multiparticle states

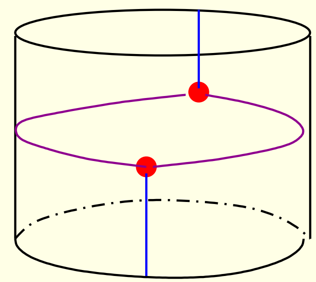


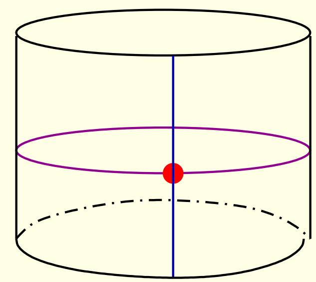
Finite volume spectrum



Lüscher originally: $O(e^{-mL})$ mass correction

$$m(L) = \frac{\sqrt{3}}{2} m(-i\text{Res}_{\theta=\frac{2i\pi}{3}} S) e^{-\frac{\sqrt{3}}{2}mL}$$
$$-\int \frac{d\theta}{2\pi} \cosh\theta (S(\theta + \tfrac{i\pi}{2}) - 1) e^{-mL\cosh\theta}$$

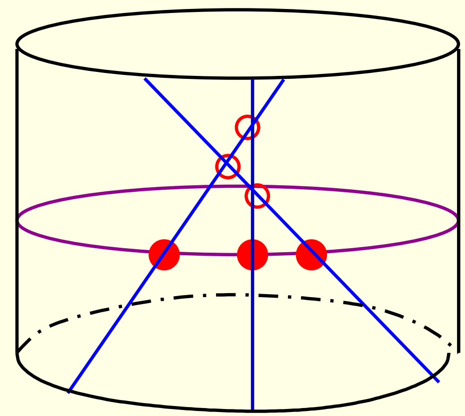




Multiparticle Lüscher correction

BY: $\mathcal{S}(p_j,p_1)\dots\mathcal{S}(p_j,p_n)\Psi = -e^{-ip_jL}\Psi$

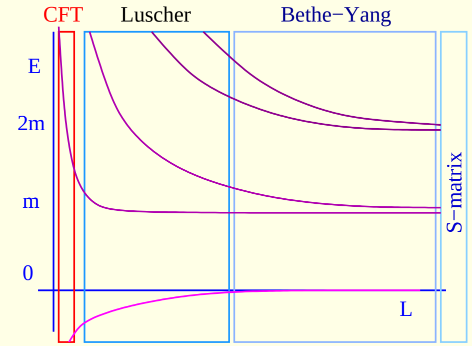
$T(p,p_1,\dots,p_n)\Psi = t(p,p_1,\dots,p_n,\Psi)\Psi$



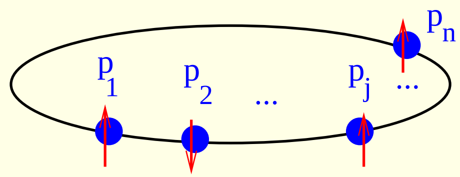
Modified momenta:

$$p_jL - i\log t(p_j,p_1,\dots,p_n,\Psi) = (2n+1)\pi + \Phi_j$$
$$\Phi_j = \int \frac{dq}{2\pi}\frac{d}{dp_j} t(q,p_1,\dots,p_n,\Psi)e^{-LE(q)}$$

Lüscher correction of multiparticle states



Finite volume spectrum



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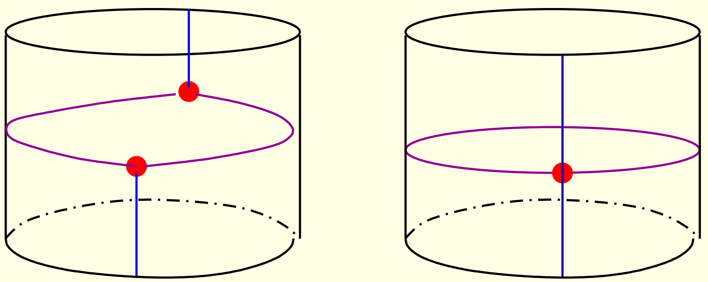
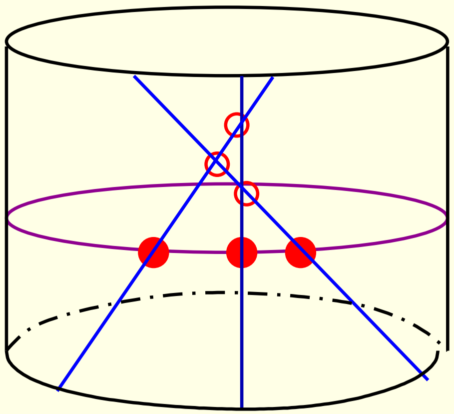
$$m(L) = \frac{\sqrt{3}}{2} m(-i\mathrm{Res}_{\theta=\frac{2i\pi}{3}} S) e^{-\frac{\sqrt{3}}{2} mL}$$

$$-\int \frac{d\theta}{2\pi} \cosh\theta (S(\theta + \frac{i\pi}{2}) - 1) e^{-mL\cosh\theta}$$



Multiparticle Lüscher correction



BY: $\mathcal{S}(p_j, p_1) \dots \mathcal{S}(p_j, p_n)\Psi = -e^{-ip_jL}\Psi$

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Modified momenta:

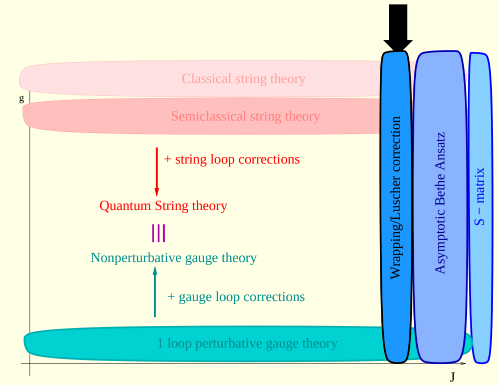
$$p_j L - i\log t(p_j, p_1, \dots, p_n, \Psi) = (2n+1)\pi + \Phi_j$$

$$\Phi_j = \int \frac{dq}{2\pi} \frac{d}{dp_j} t(q, p_1, \dots, p_n, \Psi) e^{-LE(q)}$$

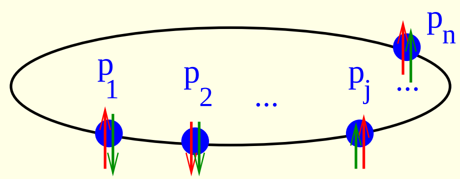
Modified energy:

$$E(p_1, \dots, p_n) = \sum_k E(p_k + \delta p_k) - \int \frac{dq}{2\pi} t(q, p_1, \dots, p_n, \Psi) e^{-LE(q)}$$

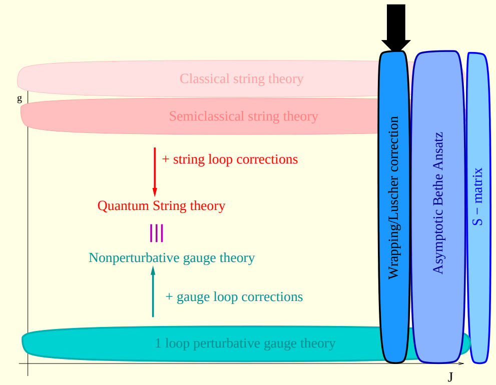
Lüscher/wrapping correction in AdS



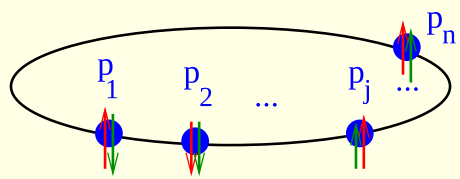
Finite volume spectrum



Lüscher/wrapping correction in AdS



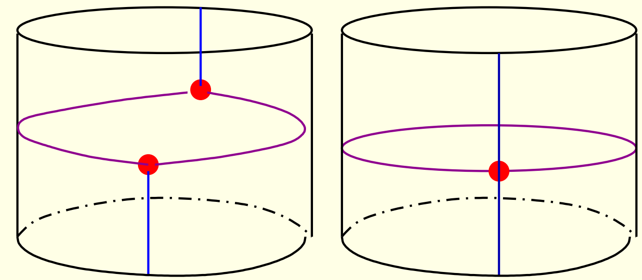
Finite volume spectrum



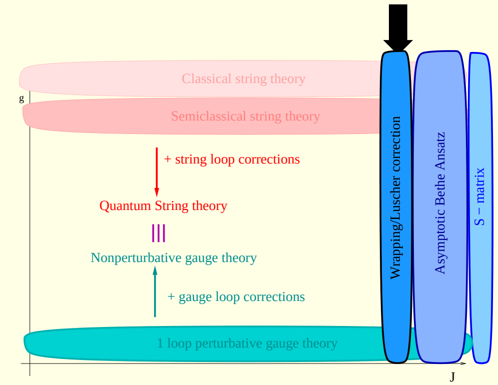
One particle correction:

$$\Delta(L) = \left(1 - E'(p)\tilde{E}'(\tilde{p}_0)\right)\left(-i\mathrm{Res}_{\tilde{p}=\tilde{p}_0}\sum_b S\right)e^{-\tilde{E}(\tilde{p})L}$$

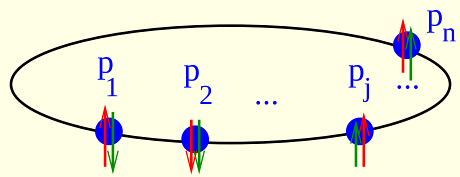
$$-\sum_b \int_{-\infty}^{\infty}\frac{d\tilde{p}}{2\pi}\left(1 - E'(p)\tilde{E}'(\tilde{p})\right)\left(S_{ba}^{ba}(\tilde{p},p) - 1\right)e^{-\tilde{E}(\tilde{p})L}$$



Lüscher/wrapping correction in AdS



Finite volume spectrum



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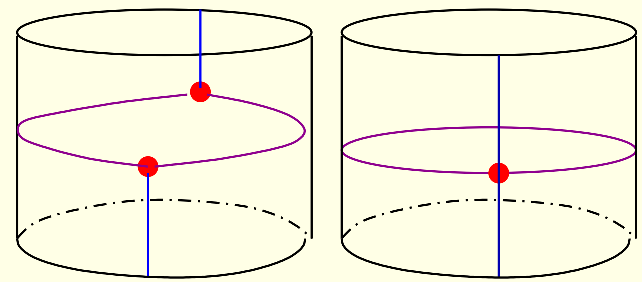
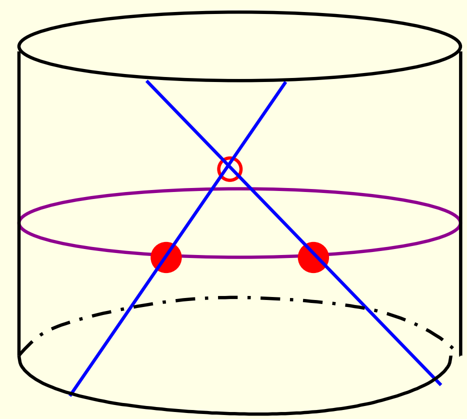


Two particle Lüscher correction (Konishi)



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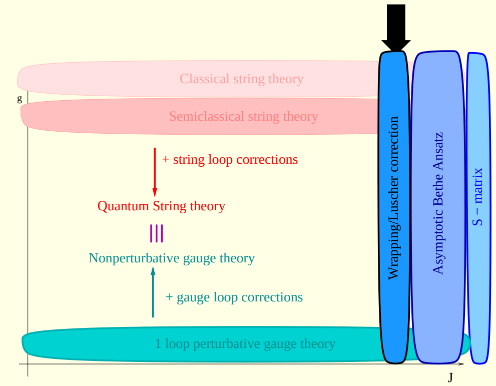

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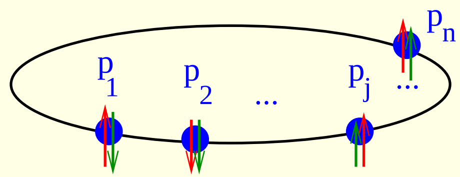

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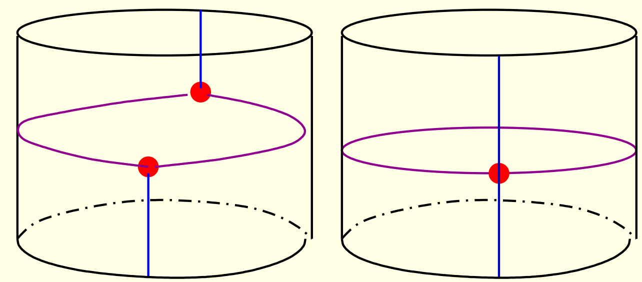

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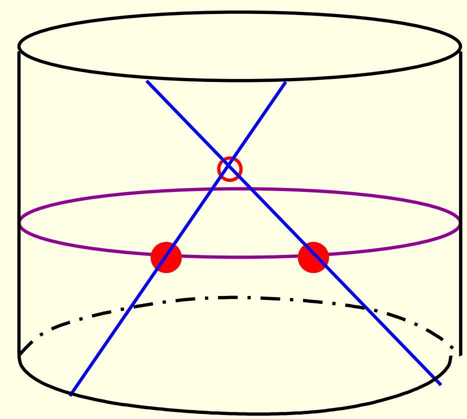

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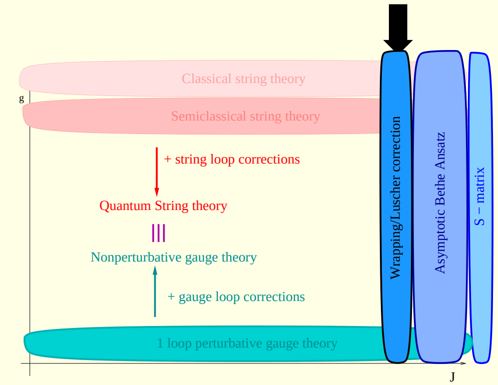
Modified momenta:

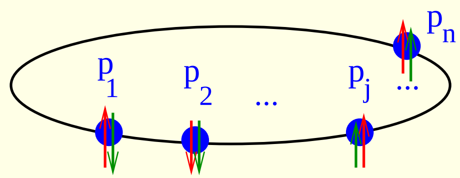
$$p_jL - i\log t(p_j,p_1,p_2,\Psi) = (2n+1)\pi + \Phi_j$$

$$\int\frac{d\tilde{p}}{2\pi}\frac{d}{dp_1}t(\tilde{p},p_1,p_2,\Psi)e^{-L\tilde{E}(\tilde{p})} \neq \Phi_1 = -\int\frac{d\tilde{p}}{2\pi}\left(\frac{d}{d\tilde{p}}S(\tilde{p},p_1)\right)S(\tilde{p},p_2)e^{-L\tilde{E}(\tilde{p})}$$

Lüscher/wrapping correction in AdS



Finite volume spectrum



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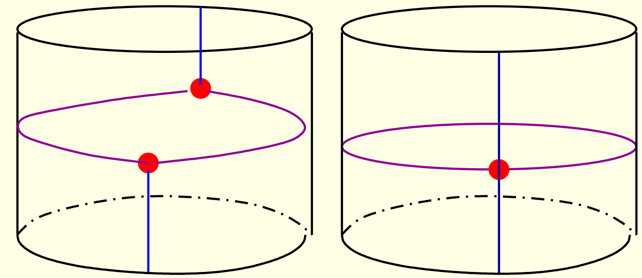

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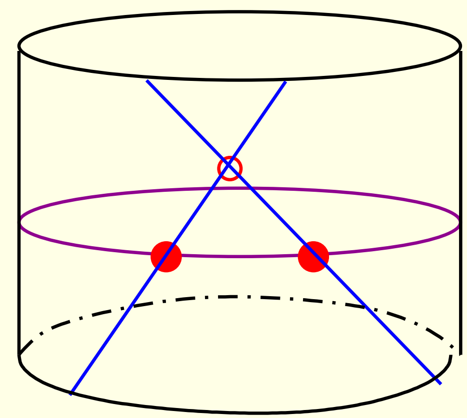

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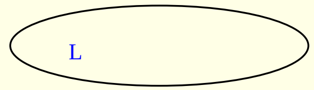
Modified energy:

$$E(p_1,p_2) = \sum_k E(p_k + \delta p_k) - \int \frac{dq}{2\pi}t(q,p_1,p_2,\Psi)e^{-LE(q)}$$

Thermodynamic Bethe Ansatz: diagonal

Ground-state energy exactly



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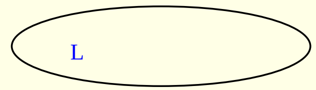
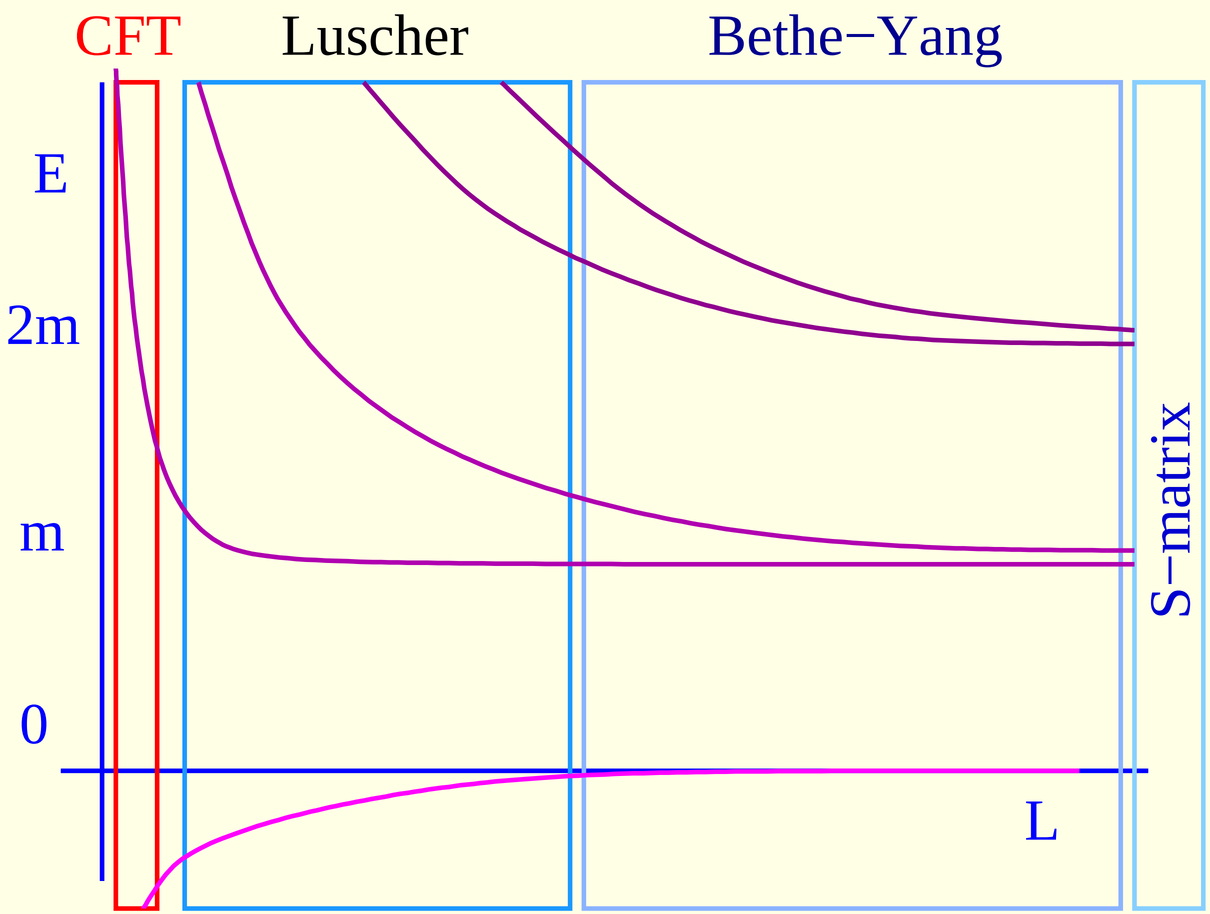
Ground-state energy exactly 



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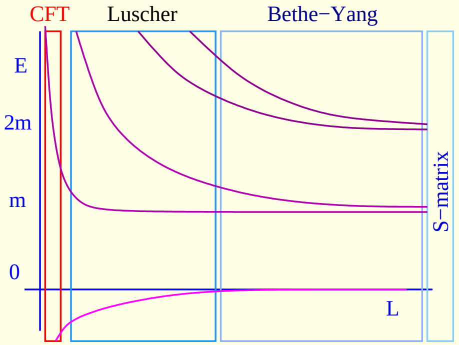

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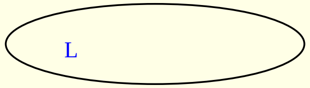

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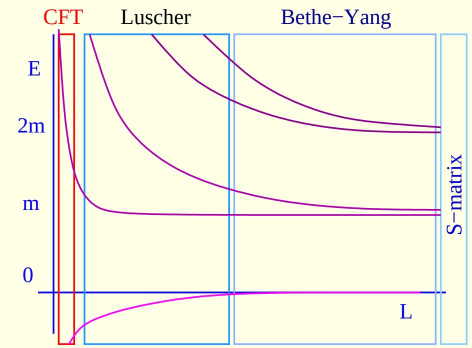

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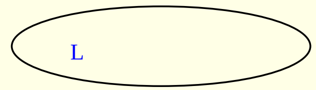
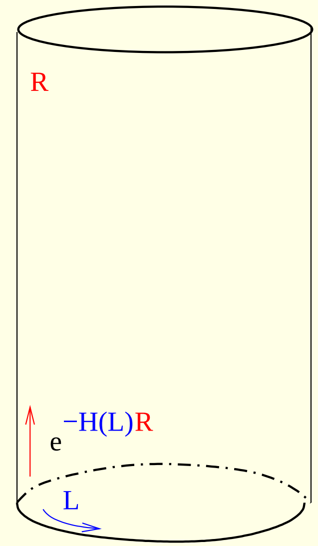
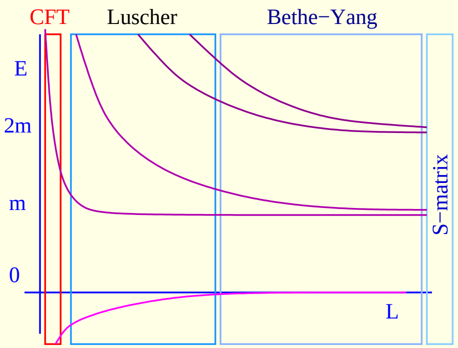


Euclidien partition function:

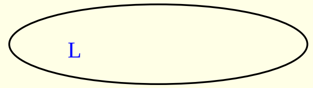


$$Z(L,R) =_{R\to\infty} \mathrm{Tr}(e^{-H(L)R})$$
$$Z(L,R) =_{R\to\infty} e^{-E_0(L)R}(1+e^{-\Delta E\,R})$$

Thermodynamic Bethe Ansatz: diagonal



Ground-state energy exactly



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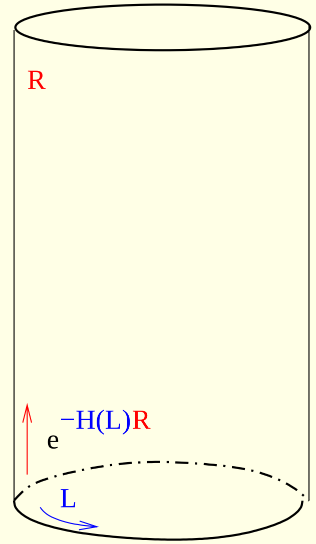

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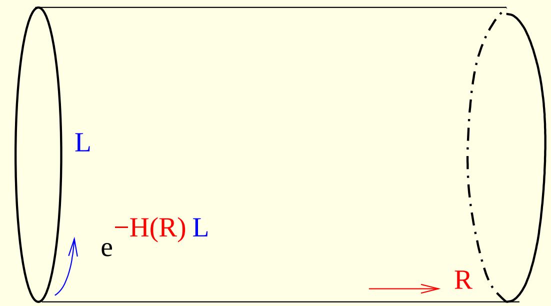
Exchange space and Euclidien time



$$Z(L,R) =_{R\to\infty} \mathrm{Tr}(e^{-H(R)L}) =_{R\to\infty} \sum_n e^{-E_n(L)R}$$

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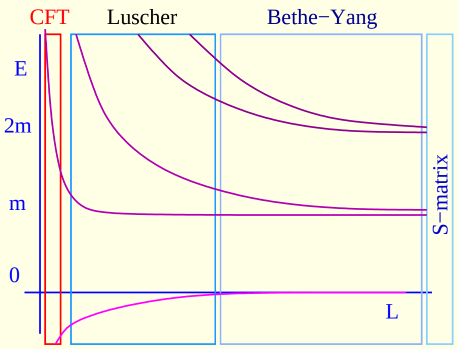

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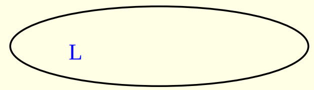

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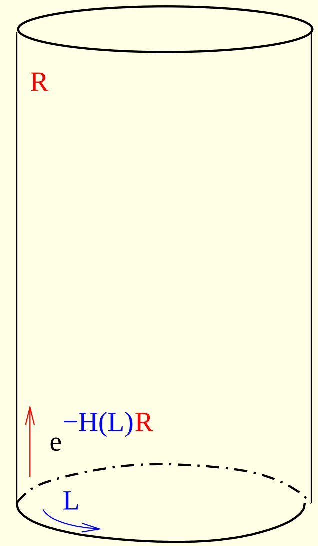

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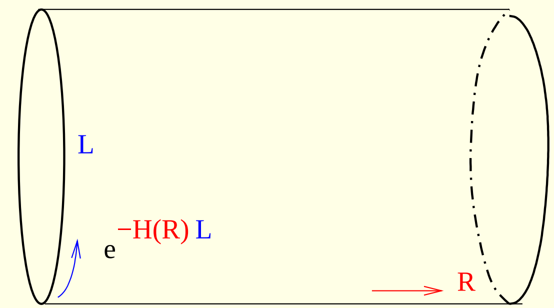

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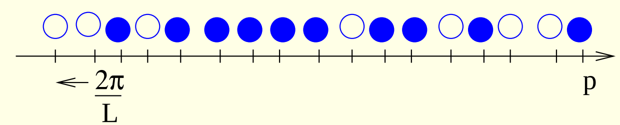
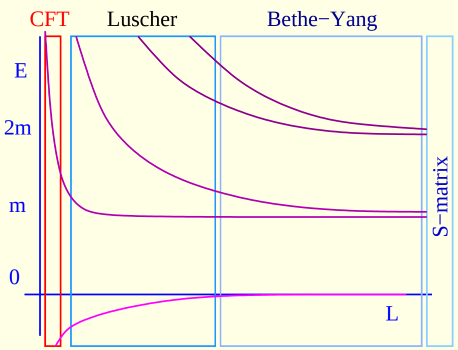
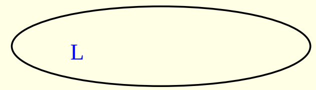


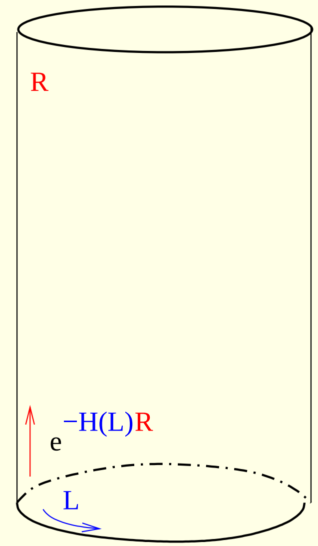
Dominant contribution: finite particle/hole density ρ, ρ_h:

Thermodynamic Bethe Ansatz: diagonal



Ground-state energy exactly





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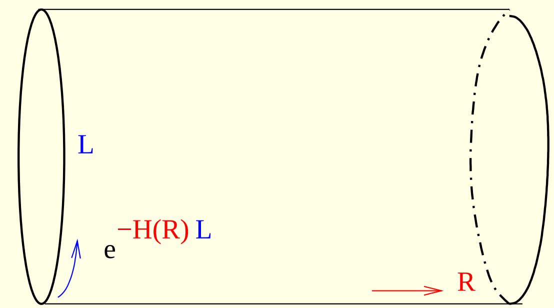
Exchange space and Euclidien time



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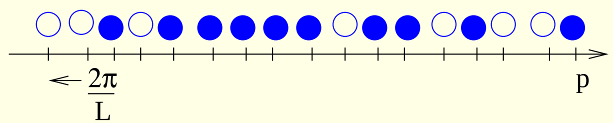

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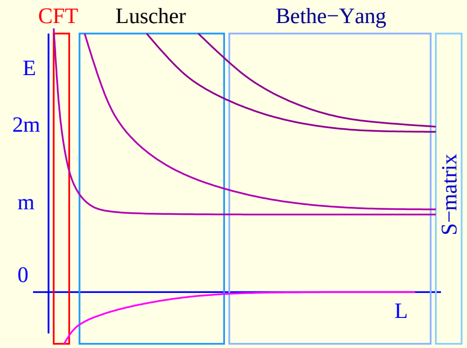

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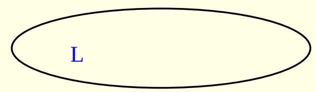

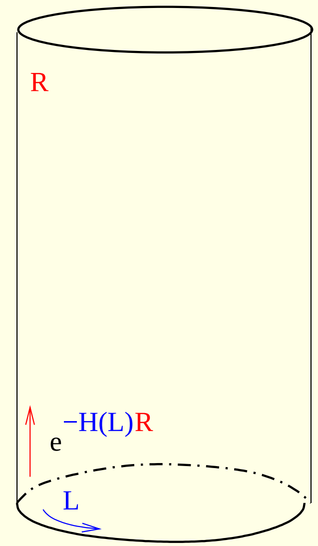

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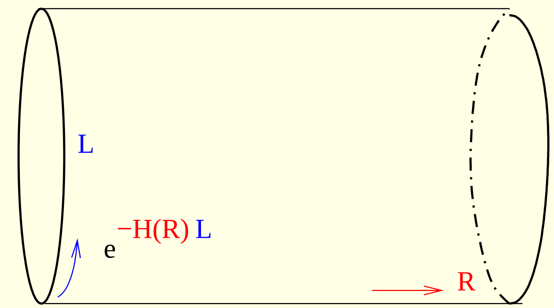

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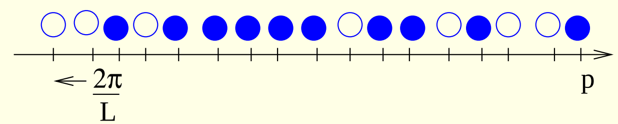

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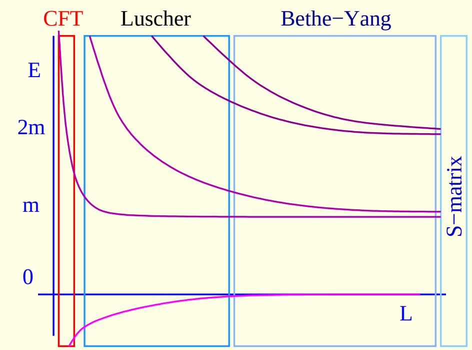
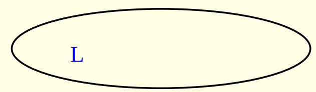
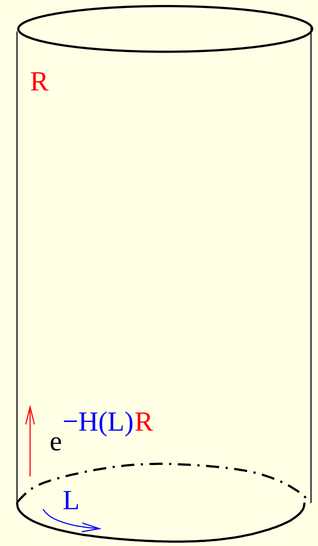


Ground-state energy exactly





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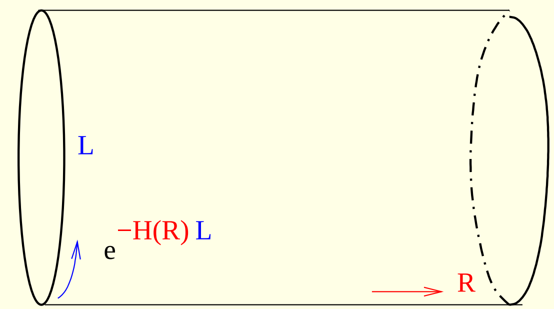
Exchange space and Euclidien time



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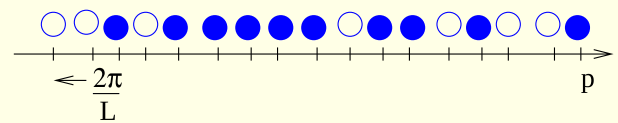

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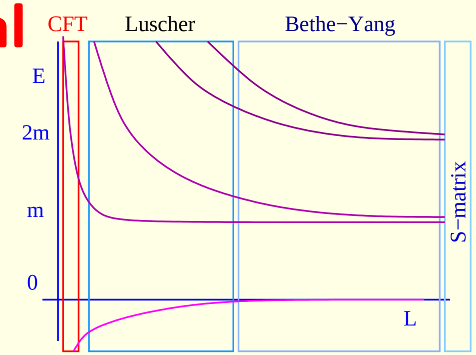
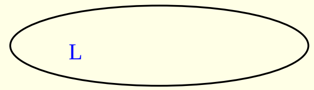
Saddle point : $\epsilon(p) = \ln\frac{\rho_h(p)}{\rho(p)}$ $\boxed{\epsilon(p) = E(p)L + \int\frac{dp}{2\pi} i d_p \log S(p',p)\log(1+e^{-\epsilon(p')})}$

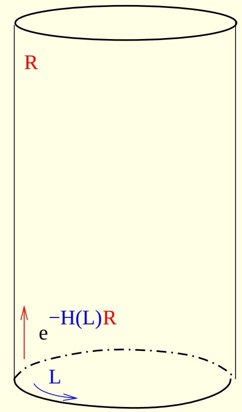
Ground state energy exactly: $\boxed{E_0(L) = -\int\frac{dp}{2\pi}\log(1+e^{-\epsilon(p)})}$ Lee-Yang, sinh-Gordon

Thermodynamic Bethe Ansatz: non-diagonal



Ground-state energy exactly





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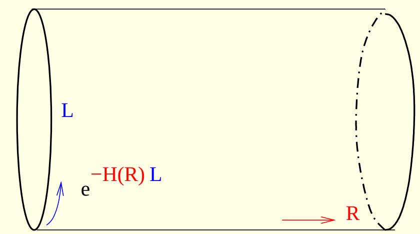
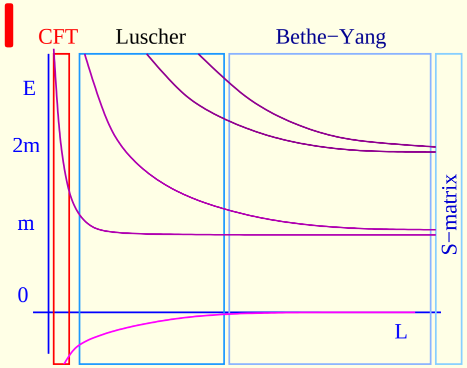
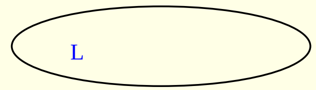
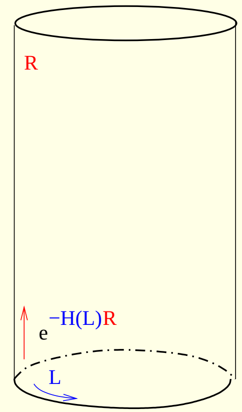


Thermodynamic Bethe Ansatz: non-diagonal



Ground-state energy exactly





Euclidien partition function:

$$Z(L,R) =_{R\to\infty} e^{-E_0(L)R}(1+e^{-\Delta E\, R})$$

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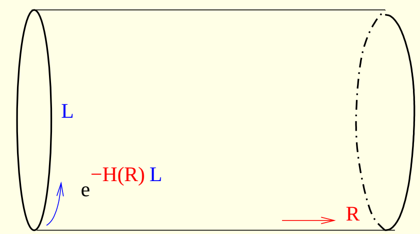

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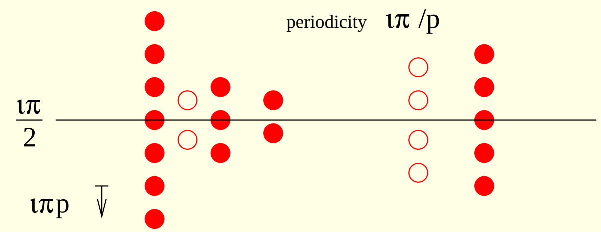

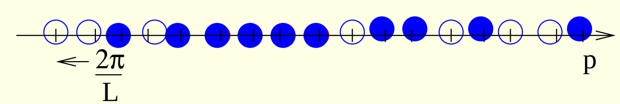

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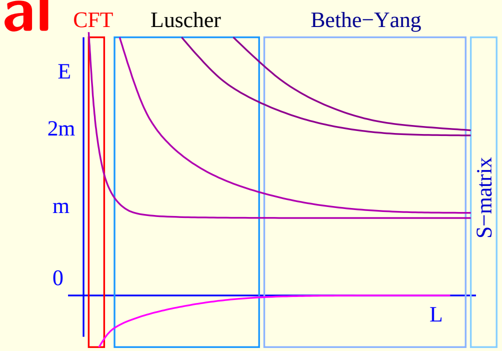

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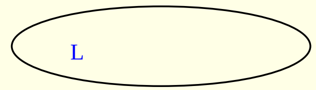

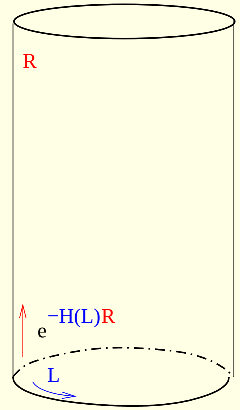

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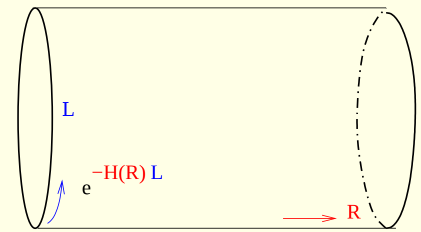

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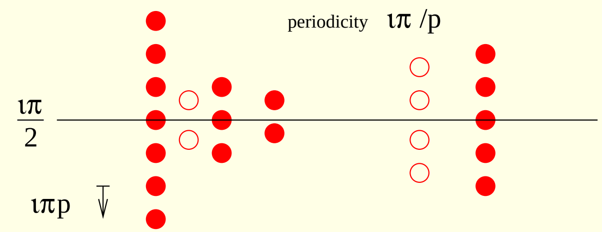

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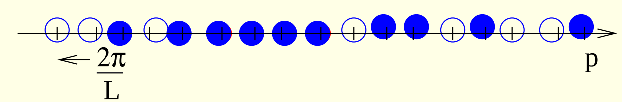
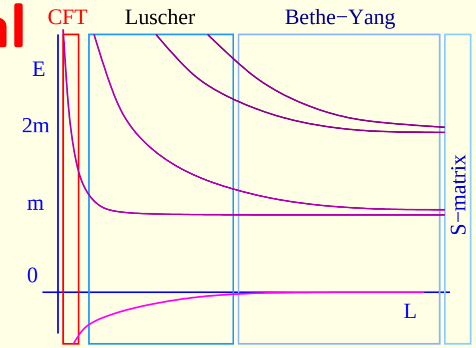


$$R\delta_0^m + \int K_n^m(p,p')\rho^n(p')dp' = 2\pi(\rho^m + \rho_h^m)$$

Thermodynamic Bethe Ansatz: non-diagonal



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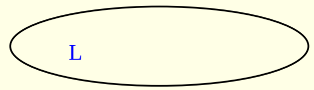

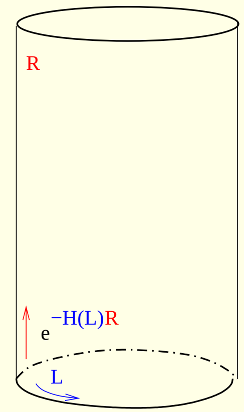

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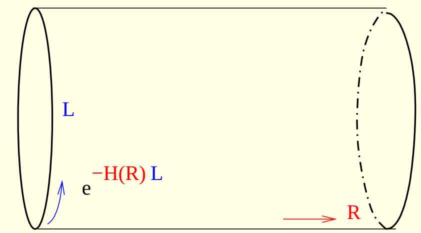

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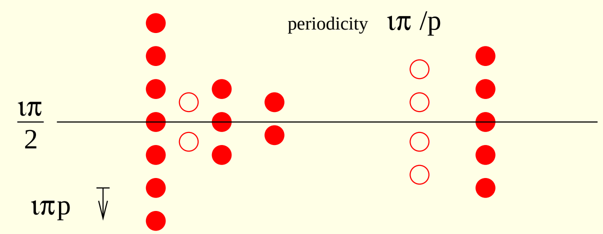

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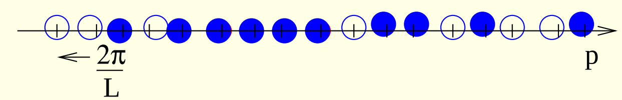

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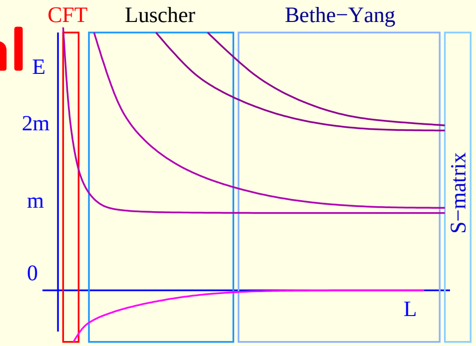
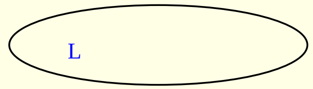
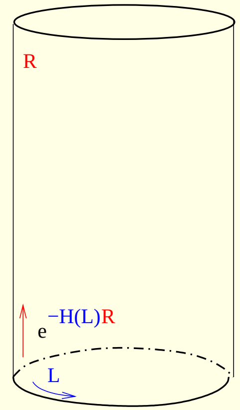
Thermodynamic Bethe Ansatz: non-diagonal



Ground-state energy exactly





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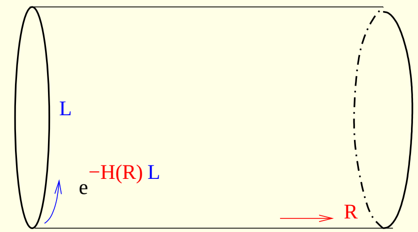
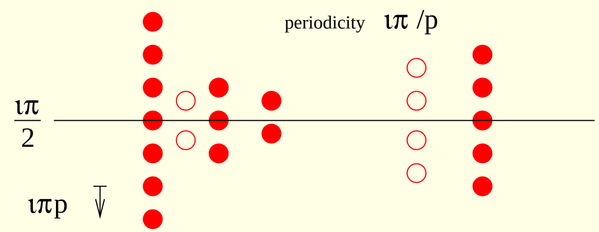


Finite particle/hole + Bethe root density $\rho^0, \rho^0_h, \rho^i, \rho^i_h$:



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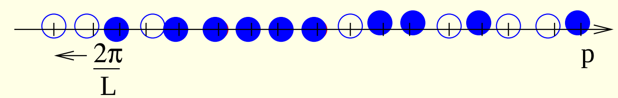


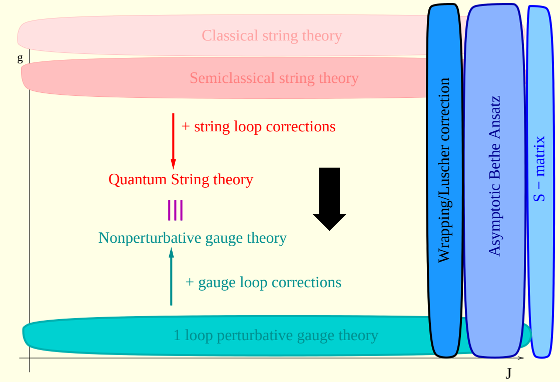
$$R\delta^m_0 + \int K^m_n(p,p')\rho^n(p')dp' = 2\pi(\rho^m + \rho^m_h)$$

$$Z(L,R) = \int d[\rho^i, \rho^i_h] e^{-LE(R) - \sum_i \int((\rho^i + \rho^i_h)\ln(\rho^i + \rho^i_h) - \rho^i \ln \rho^i - \rho^i_h \ln \rho^i_h)dp}$$

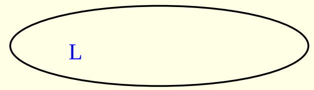
Saddle point : $\epsilon^i(\theta) = -\ln \frac{\rho^i(p)}{\rho^i_h(p)}$ $\quad \boxed{\epsilon^j(\theta) = \delta^j_0 E(p)L - \int K^j_i(p',p)\log(1 + e^{-\epsilon^i(p')})dp'}$

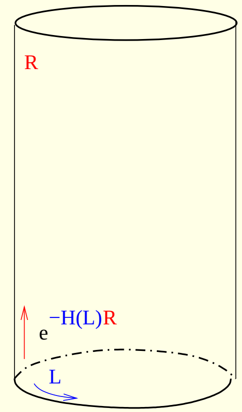
Ground state energy exactly: $\boxed{E_0(L) = -\int \frac{dp}{2\pi}\log(1 + e^{-\epsilon_0(\theta)})d\theta}$

Thermodynamic Bethe Ansatz: AdS



Ground-state energy exactly





Euclidien $E^2 + (4g \sin \frac{p}{2})^2 = 1$ partition function:

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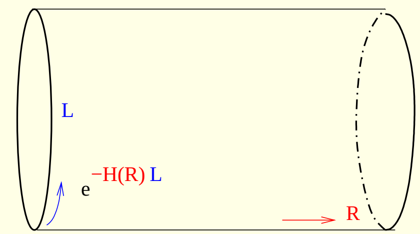

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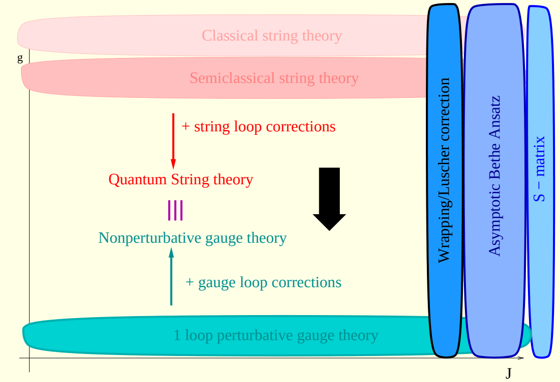

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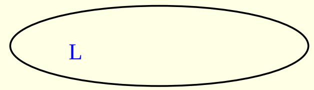

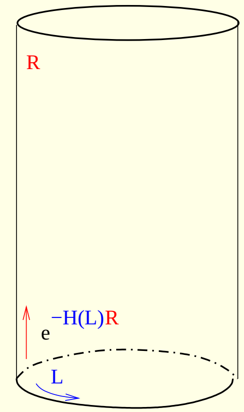

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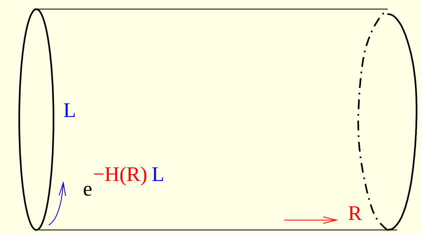

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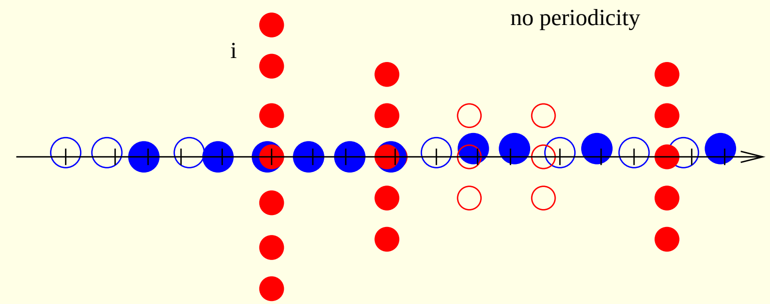

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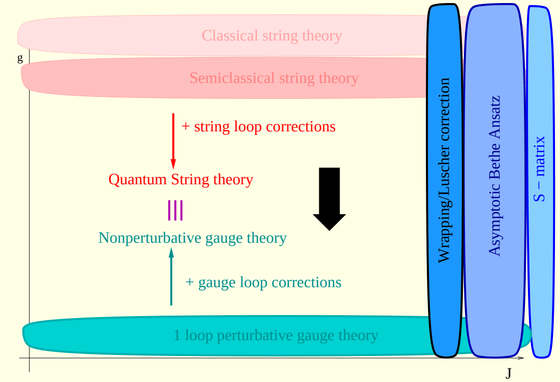

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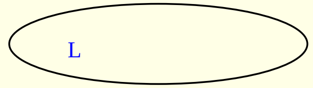

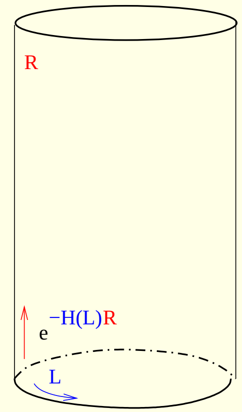

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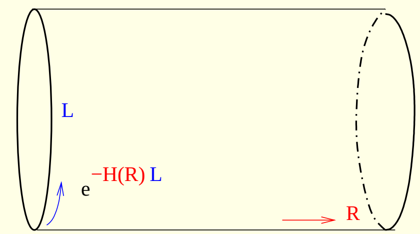

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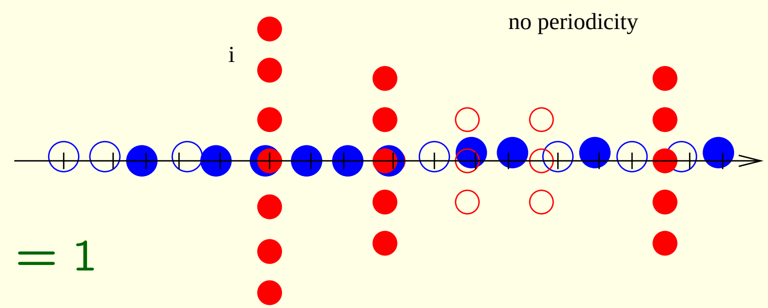

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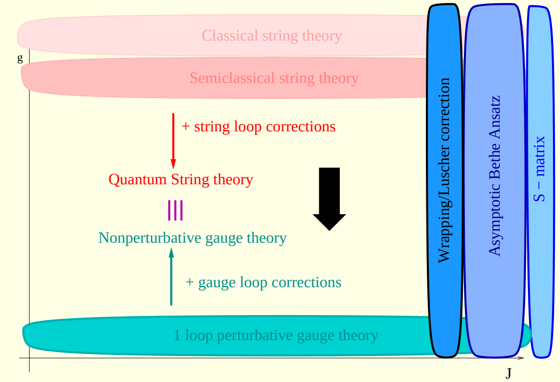
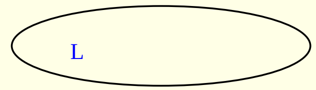
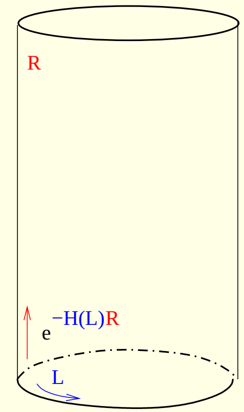
Thermodynamic Bethe Ansatz: AdS



Ground-state energy exactly





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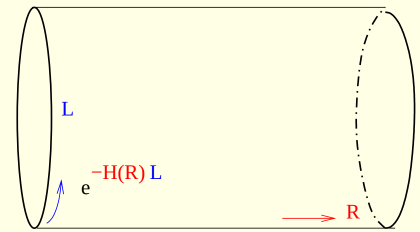

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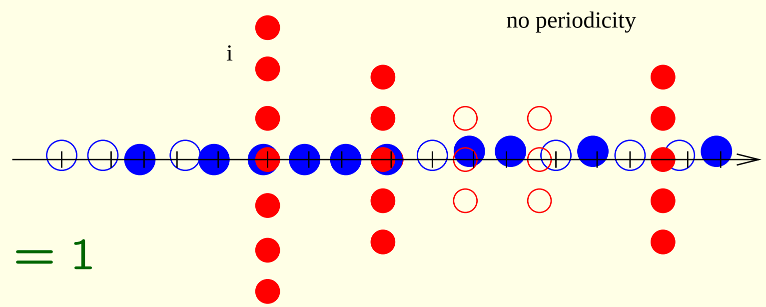

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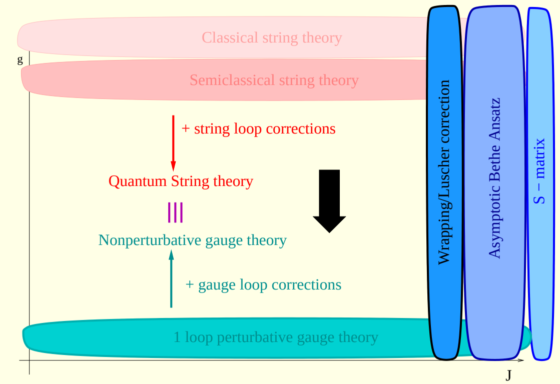
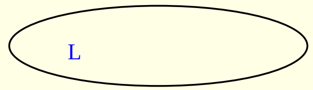
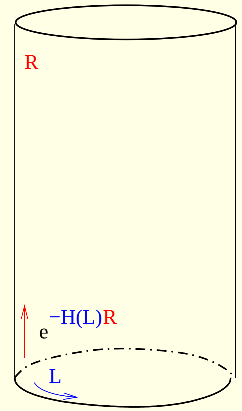
Thermodynamic Bethe Ansatz: AdS



Ground-state energy exactly





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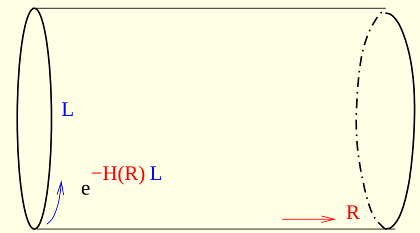

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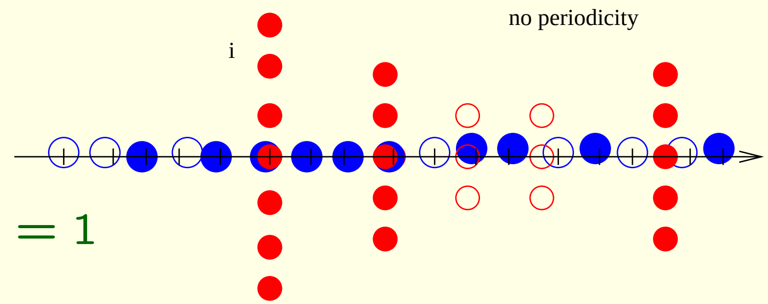

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$$e^{iLp} S_0^2 \frac{Q_4^{++}}{Q_4^{--}} T\dot{T}|_4 = -1 \quad \frac{Q_2^+ B_4^{(-)}}{Q_2^- B_4^{(+)}}|_1 = 1 \quad \frac{Q_2^{--} Q_1^+ Q_3^+}{Q_2^{++} Q_1^- Q_3^-}|_2 = -1 \quad \frac{Q_2^+ R_4^{(-)}}{Q_2^- R_4^{(+)}}|_3 = 1$$

$$\int K_n^m(\tilde{p},\tilde{p}')\rho^n(\tilde{p}')d\tilde{p}' = 2\pi(\rho^m + \rho_h^m)$$

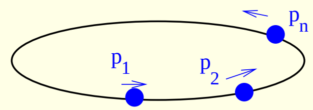
$$Z(L,R) = \int d[\rho^i,\rho_h^i] e^{-LE(R) - \sum_i \int ((\rho^i+\rho_h^i)\ln(\rho^i+\rho_h^i) - \rho^i \ln\rho^i - \rho_h^i \ln\rho_h^i)dp}$$

Saddle point : $\epsilon^i(\tilde{p}) = -\ln\frac{\rho^i(\tilde{p})}{\rho_h^i(\tilde{p})}$
$$\boxed{\epsilon^j(\theta) = \delta_Q^j \tilde{E}_Q(\tilde{p})L - \int K_i^j(\tilde{p},\tilde{p}')\log(1+e^{-\epsilon^i(\tilde{p}')})d\tilde{p}'}$$

Ground state energy exactly:
$$\boxed{E_0(L) = -\sum_Q \int \frac{d\tilde{p}}{2\pi}\log(1+e^{-\epsilon_Q(\tilde{p})})d\tilde{p}}$$

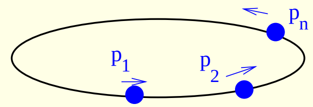
Excited states TBA, Y-system: diagonal

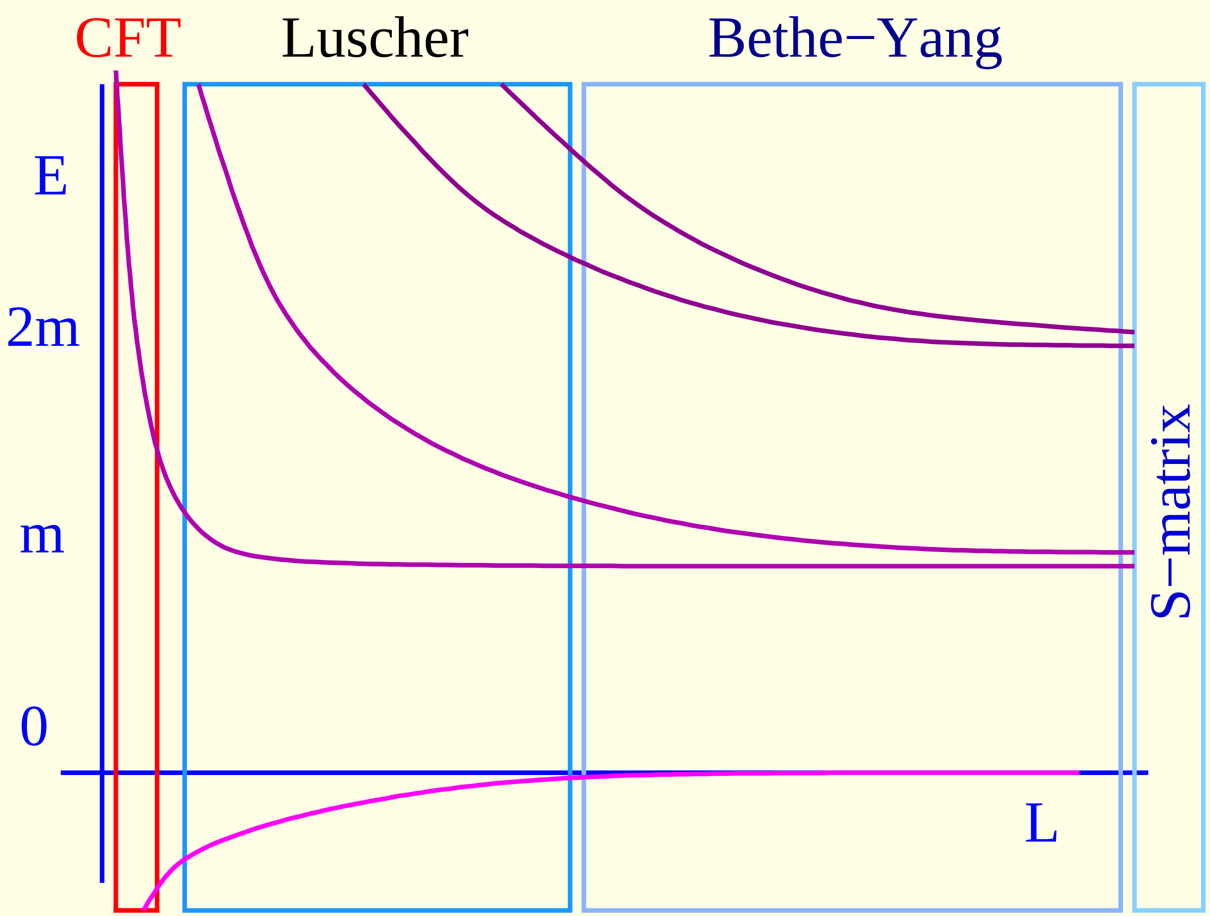
Excited states exactly



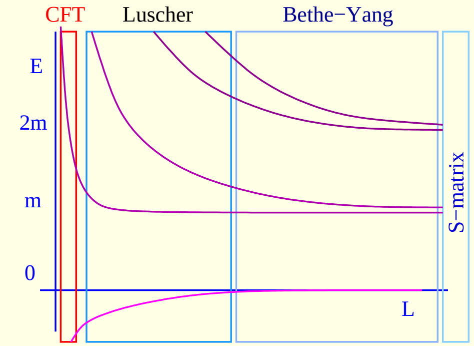
Excited states TBA, Y-system: diagonal

Excited states exactly

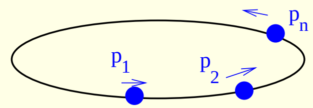


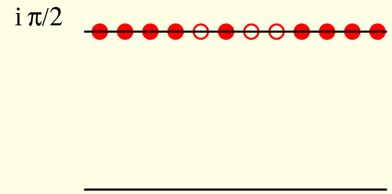


Excited states TBA, Y-system: diagonal



Excited states exactly



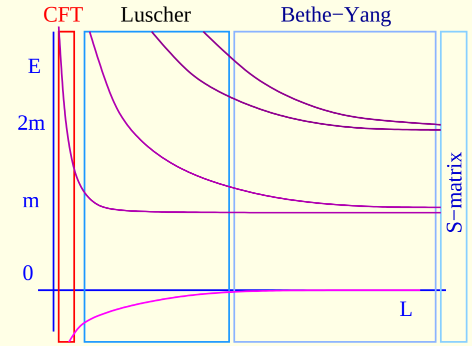


By lattice regularization: sinh-Gordon

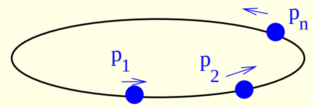
$$\epsilon(\theta) = mL\cosh\theta \qquad + \int \frac{d\theta'}{2\pi} i d_\theta \log S(\theta-\theta') \log(1+e^{-\epsilon(\theta')})$$

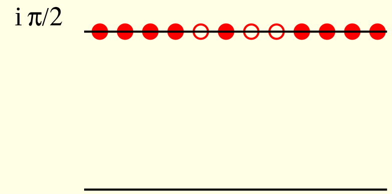
$$E_{\{n_j\}}(L) = \qquad -m\int \frac{d\theta}{2\pi}\cosh\theta \, \log(1+e^{-\epsilon(\theta)})$$

Excited states TBA, Y-system: diagonal



Excited states exactly



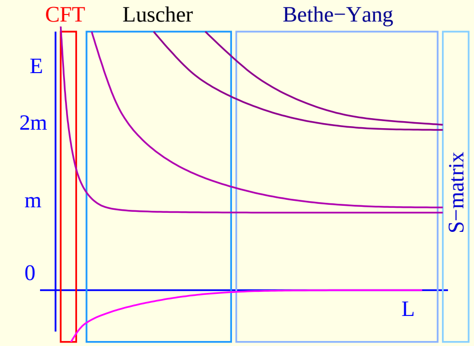


By lattice regularization: sinh-Gordon

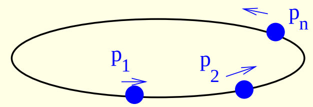
$$\epsilon(\theta) = mL\cosh\theta + \sum \log S(\theta - \theta_j) + \int \frac{d\theta'}{2\pi} i d_\theta \log S(\theta - \theta') \log(1 + e^{-\epsilon(\theta')})$$

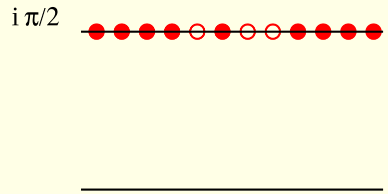
$$E_{\{n_j\}}(L) = \qquad -m\int \frac{d\theta}{2\pi} \cosh\theta \log(1 + e^{-\epsilon(\theta)})$$

Excited states TBA, Y-system: diagonal



Excited states exactly

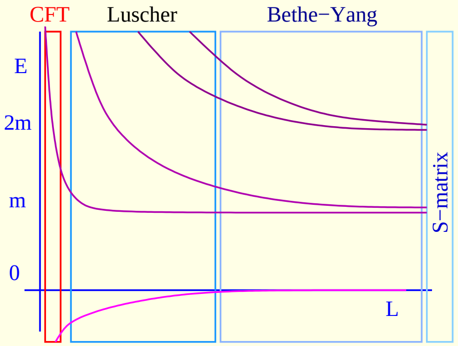




By lattice regularization: sinh-Gordon

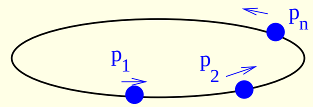
$$\epsilon(\theta) = mL\cosh\theta + \sum \log S(\theta-\theta_j) + \int \frac{d\theta'}{2\pi} i d_\theta \log S(\theta-\theta')\log(1+e^{-\epsilon(\theta')})$$

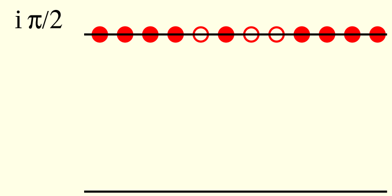
$$E_{\{n_j\}}(L) = \frac{m}{i}\sum \sinh\theta_j - m\int \frac{d\theta}{2\pi}\cosh\theta\,\log(1+e^{-\epsilon(\theta)})$$



Excited states TBA, Y-system: diagonal

Excited states exactly



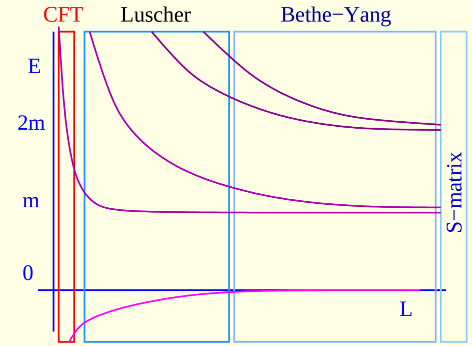


By lattice regularization: sinh-Gordon

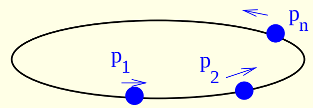
$$\epsilon(\theta) = mL\cosh\theta + \sum \log S(\theta-\theta_j) + \int \frac{d\theta'}{2\pi} i d_\theta \log S(\theta-\theta')\log(1+e^{-\epsilon(\theta')})$$

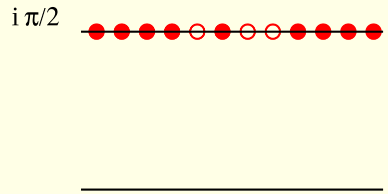
$$E_{\{n_j\}}(L) = \frac{m}{i}\sum \sinh\theta_j - m\int\frac{d\theta}{2\pi}\cosh\theta\,\log(1+e^{-\epsilon(\theta)})\;;\; Y(\theta_j) = e^{\epsilon(\theta_j)} = -1$$

Excited states TBA, Y-system: diagonal



Excited states exactly





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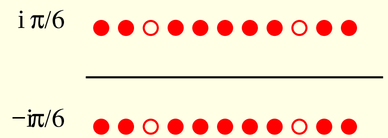
$$E_{\{n_j\}}(L) = \frac{m}{i}\sum \sinh\theta_j - m\int\frac{d\theta}{2\pi}\cosh\theta\,\log(1+e^{-\epsilon(\theta)})\,;\; Y(\theta_j) = e^{\epsilon(\theta_j)} = -1$$



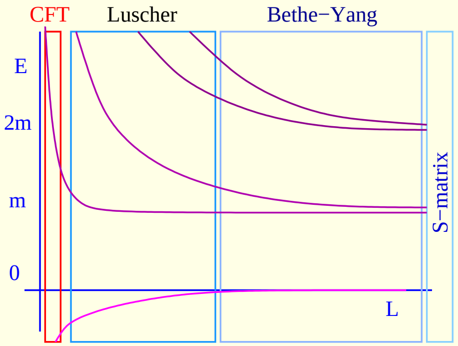
By analytical continuation: Lee-Yang

$$\epsilon(\theta) = mL\cosh\theta \qquad - \int_{-\infty}^{\infty}\frac{d\theta'}{2\pi}\phi(\theta-\theta')\log(1+e^{-\epsilon(\theta')})$$

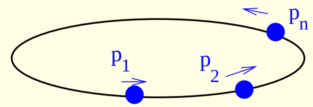
$$E_{\{n_j\}}(L) = \qquad - m\int_{-\infty}^{\infty}\frac{d\theta}{2\pi}\cosh\theta\,\log(1+e^{-\epsilon(\theta)})$$

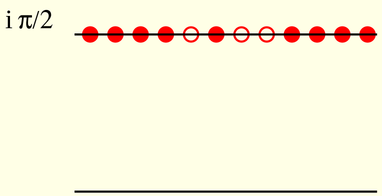
Lüscher corrections: differ by μ term

Excited states TBA, Y-system: diagonal



Excited states exactly

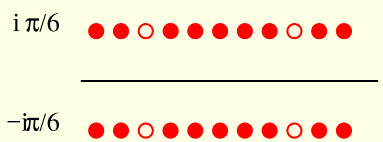




By lattice regularization: sinh-Gordon

$$\epsilon(\theta) = mL\cosh\theta + \sum \log S(\theta-\theta_j) + \int \frac{d\theta'}{2\pi} i d_\theta \log S(\theta-\theta')\log(1+e^{-\epsilon(\theta')})$$

$$E_{\{n_j\}}(L) = \frac{m}{i}\sum \sinh\theta_j - m\int\frac{d\theta}{2\pi}\cosh\theta\,\log(1+e^{-\epsilon(\theta)})\ ;\ Y(\theta_j) = e^{\epsilon(\theta_j)} = -1$$



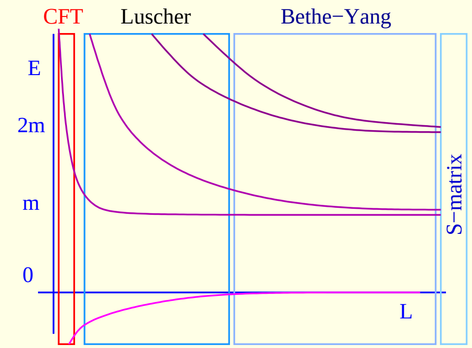
By analytical continuation: Lee-Yang

$$\epsilon(\theta) = mL\cosh\theta + \sum_{j=1}^{N}\log\frac{S(\theta-\theta_j)}{S(\theta-\theta_j^*)} - \int_{-\infty}^{\infty}\frac{d\theta'}{2\pi}\phi(\theta-\theta')\log(1+e^{-\epsilon(\theta')})$$

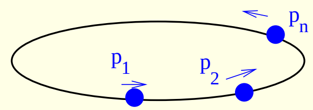
$$E_{\{n_j\}}(L) = \qquad - m\int_{-\infty}^{\infty}\frac{d\theta}{2\pi}\cosh\theta\,\log(1+e^{-\epsilon(\theta)})$$

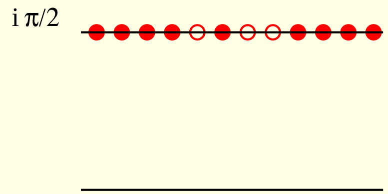
Lüscher corrections: differ by μ term

Excited states TBA, Y-system: diagonal



Excited states exactly

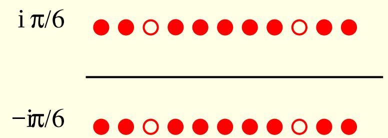




By lattice regularization: sinh-Gordon

$$\epsilon(\theta)=mL\cosh\theta+\sum\log S(\theta-\theta_j)+\int\frac{d\theta'}{2\pi}id_\theta\log S(\theta-\theta')\log(1+e^{-\epsilon(\theta')})$$

$$E_{\{n_j\}}(L)=\frac{m}{i}\sum\sinh\theta_j-m\int\frac{d\theta}{2\pi}\cosh\theta\,\log(1+e^{-\epsilon(\theta)})\;;\;Y(\theta_j)=e^{\epsilon(\theta_j)}=-1$$



By analytical continuation: Lee-Yang

$$\epsilon(\theta)=mL\cosh\theta+\sum_{j=1}^{N}\log\frac{S(\theta-\theta_j)}{S(\theta-\theta_j^*)}-\int_{-\infty}^{\infty}\frac{d\theta'}{2\pi}\phi(\theta-\theta')\log(1+e^{-\epsilon(\theta')})$$

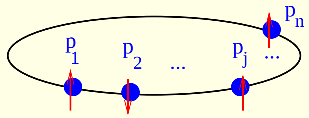
$$E_{\{n_j\}}(L)=-im\sum(\sinh\theta_j-\sinh\theta_j^*)-m\int_{-\infty}^{\infty}\frac{d\theta}{2\pi}\cosh\theta\,\log(1+e^{-\epsilon(\theta)})$$

Lüscher corrections: differ by μ term

$$S(\theta-\tfrac{i\pi}{3})S(\theta+\tfrac{i\pi}{3})=S(\theta)\to Y(\theta+\tfrac{i\pi}{3})Y(\theta-\tfrac{i\pi}{3})=1+Y(\theta)$$

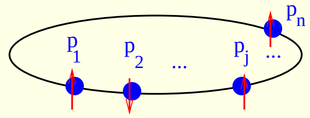
Y-system+analyticity=TBA <-> scalar . Matrix

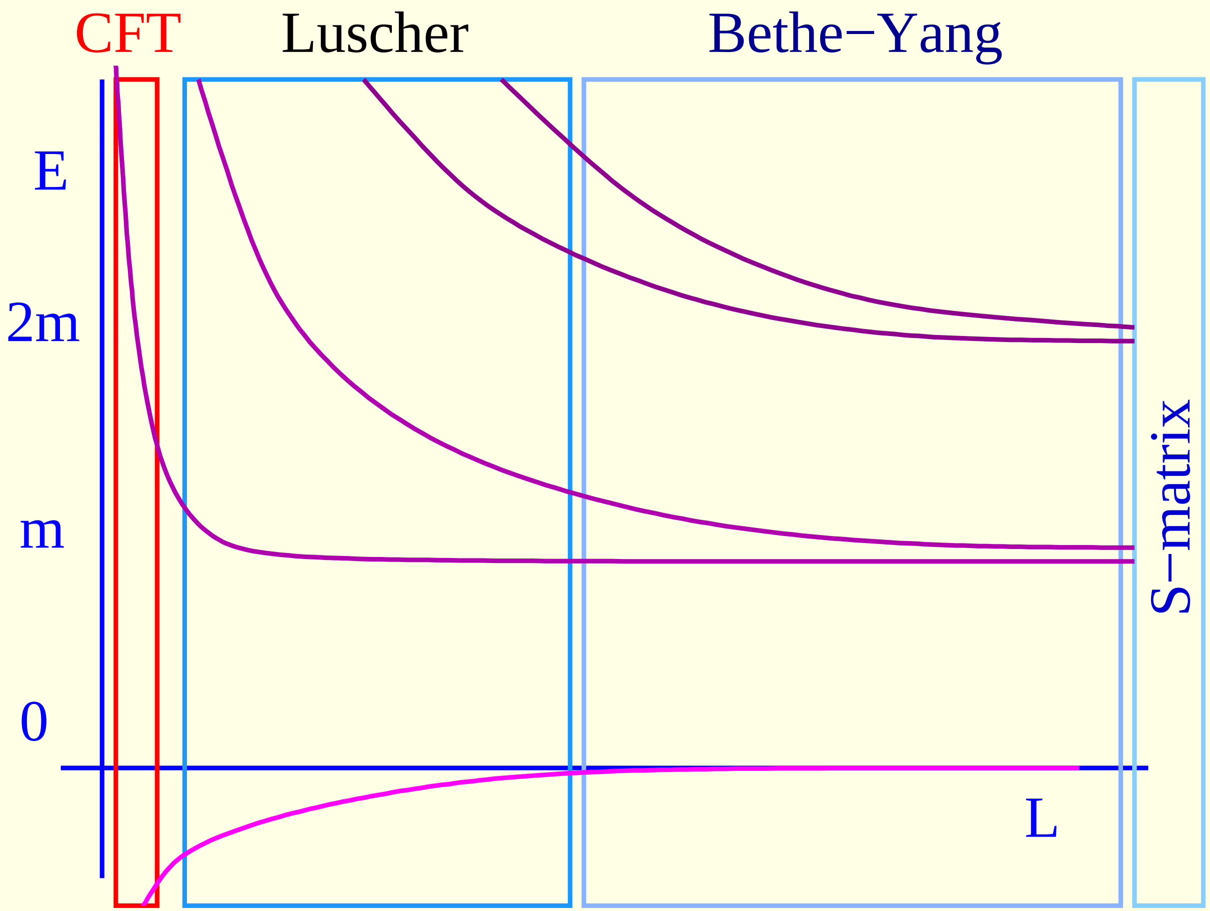
Excited states TBA, Y-system: Non-diagonal

Excited states exactly 

Excited states TBA, Y-system: Non-diagonal

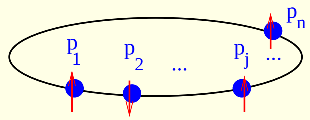
Excited states exactly

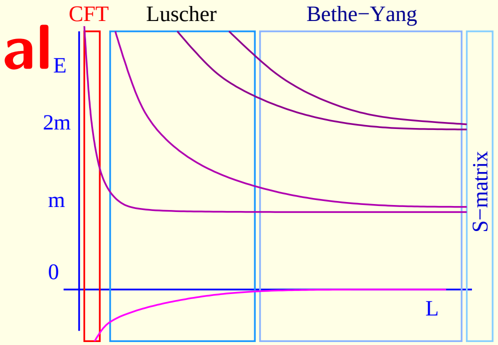




Excited states TBA, Y-system: Non-diagonal

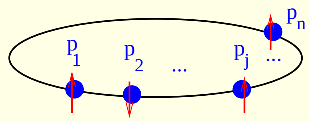
Excited states exactly

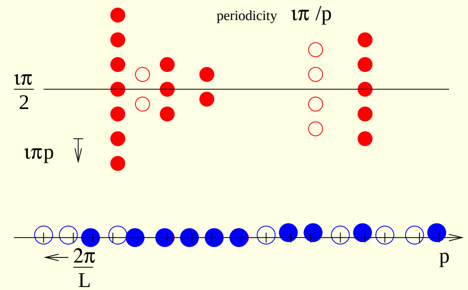


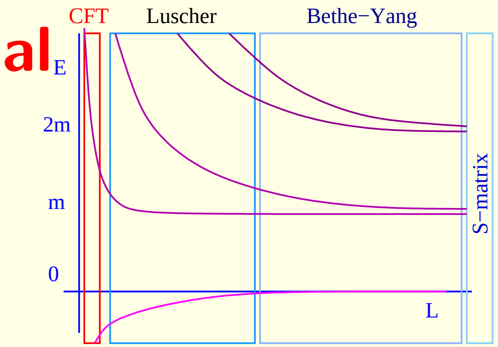


Excited states TBA, Y-system: Non-diagonal

Excited states exactly

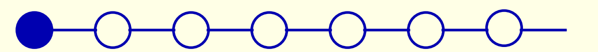




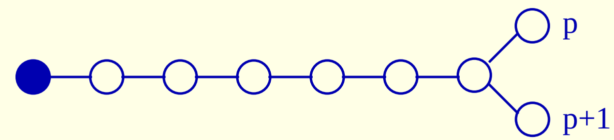


Y-system: sine-Gordon

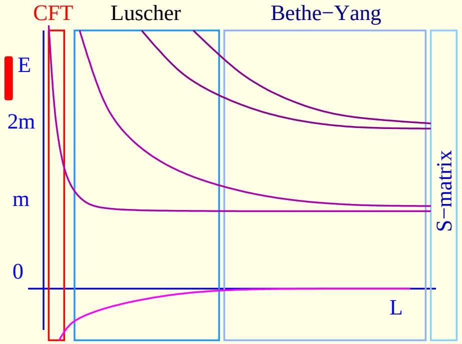
$$Y_s(\theta+\frac{i\pi p}{2})Y_s(\theta-\frac{i\pi p}{2})=(1+Y_{s-1})(1+Y_{s+1})$$



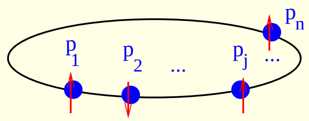
$$(e^{\frac{i\pi p}{2}\partial}+e^{-\frac{i\pi p}{2}\partial})\log Y_s=\sum_r I_{sr}\log(1+Y_r)$$

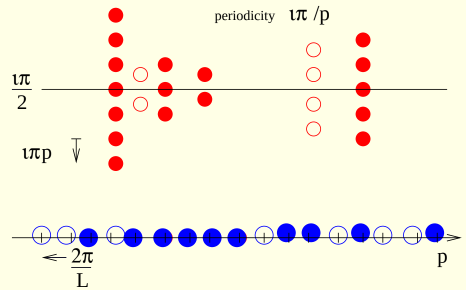


Excited states TBA, Y-system: Non-diagonal



Excited states exactly

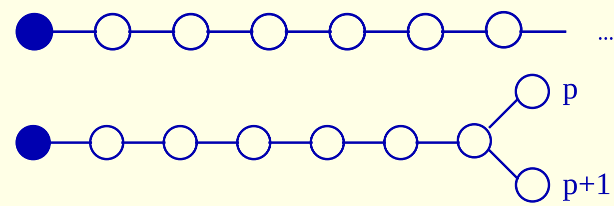




Y-system: sine-Gordon

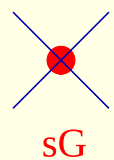
$$Y_s(\theta + \frac{i\pi p}{2})Y_s(\theta - \frac{i\pi p}{2}) = (1 + Y_{s-1})(1 + Y_{s+1})$$

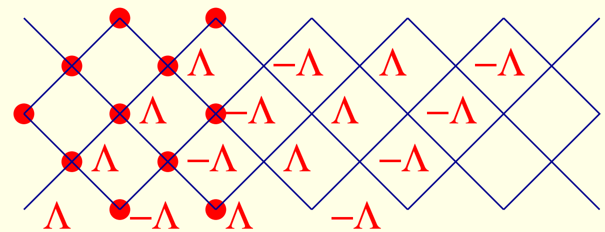
$$(e^{\frac{i\pi p}{2}\partial} + e^{-\frac{i\pi p}{2}\partial}) \log Y_s = \sum_r I_{sr} \log(1 + Y_r)$$



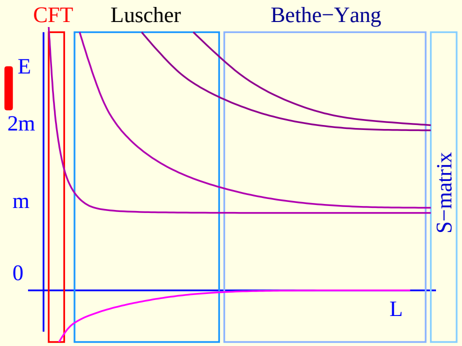
Excited states: analyticity from Lüscher

Lattice regularization:

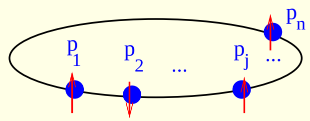


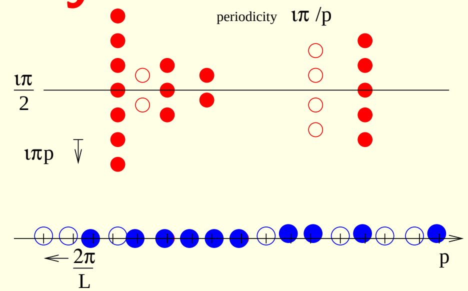


Excited states TBA, Y-system: Non-diagonal



Excited states exactly





Y-system: sine-Gordon

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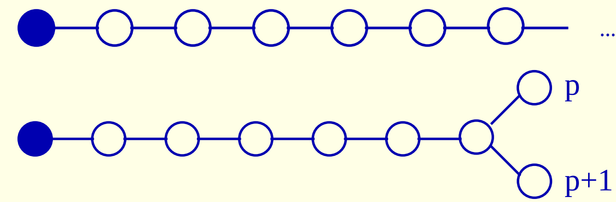
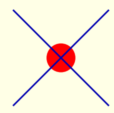
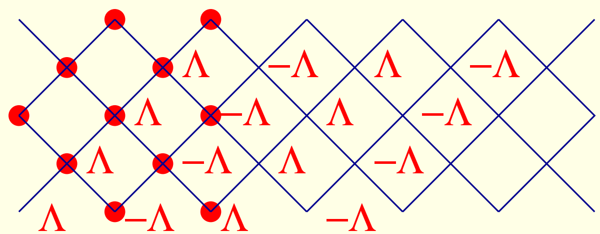
$$(e^{\frac{i\pi p}{2}\partial}+e^{-\frac{i\pi p}{2}\partial})\log Y_s=\sum_r I_{sr}\log(1+Y_r)$$



Excited states: analyticity from Lüscher

Lattice regularization:





$$Z(\theta)=ML\sinh\theta+\text{source}(\theta|\{\theta_k\})+2\Im m\int dx G(\theta-x-i\epsilon)\log\left[1-(-1)^{\delta}e^{iZ(x+i\epsilon)}\right]$$

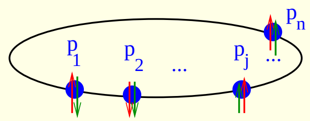
$$\text{source}(\theta|\{\theta_k\})=\sum_k \text{sgn}_k(-i)\log S_{++}^{++}(\theta-\theta_k)$$ kernel: $G(\theta)=-i\partial_\theta\log S_{++}^{++}(\theta)$

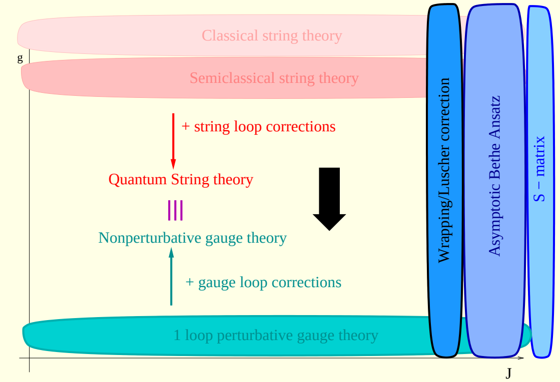
Energy: $E=M\sum_k \text{sgn}_k\cosh\theta_k-2M\Im m\int dx G(\theta+i\epsilon)\log\left[1-(-1)^{\delta}e^{iZ(x+i\epsilon)}\right]$

Bethe-Yang $e^{iZ(\theta_k)}=-1$

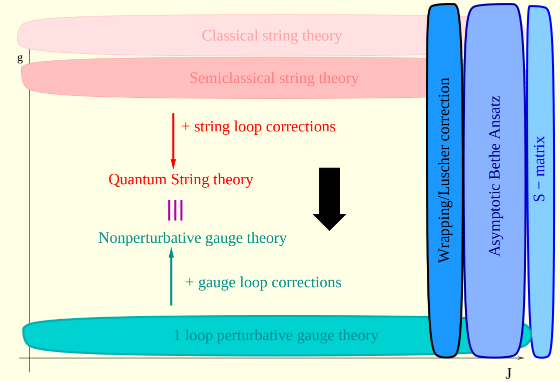
Excited states TBA, Y-system: AdS

Excited states exactly

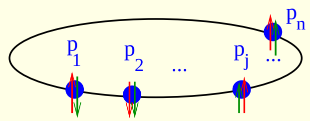


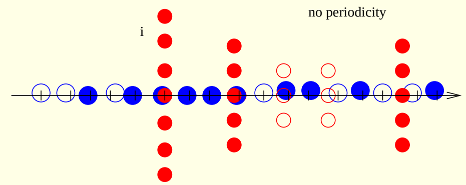


Excited states TBA, Y-system: AdS



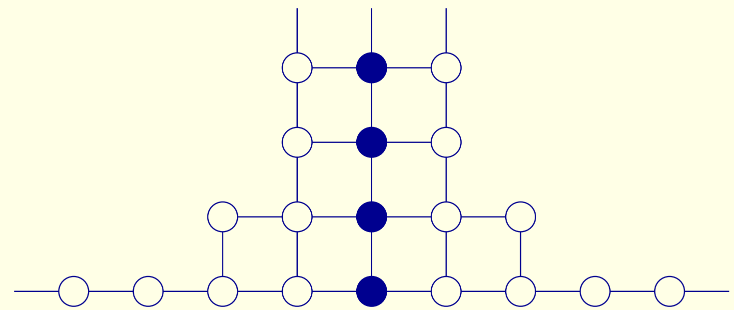
Excited states exactly



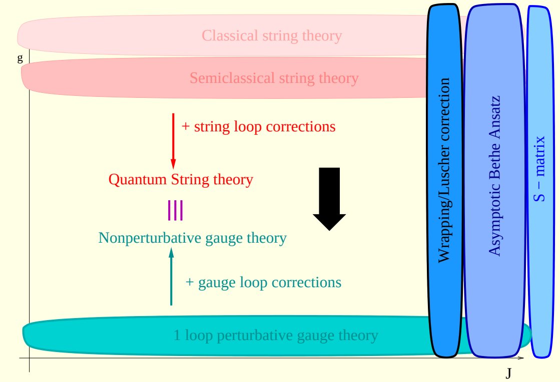


Y-system: AdS

$$\frac{Y_{a,s}(\theta+\frac{i}{2})Y_{a,s}(\theta-\frac{i}{2})}{Y_{a+1,s}Y_{a-1,s}} = \frac{(1+Y_{a,s-1})(1+Y_{a,s+1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}$$



Excited states TBA, Y-system: AdS



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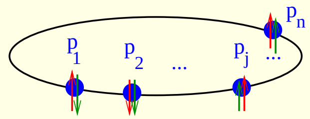

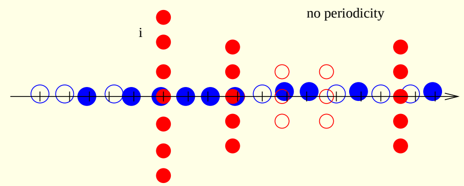

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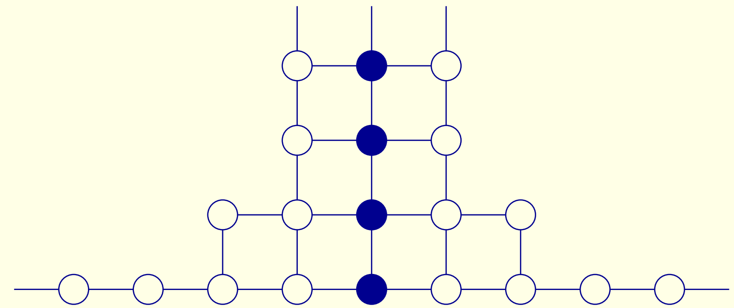
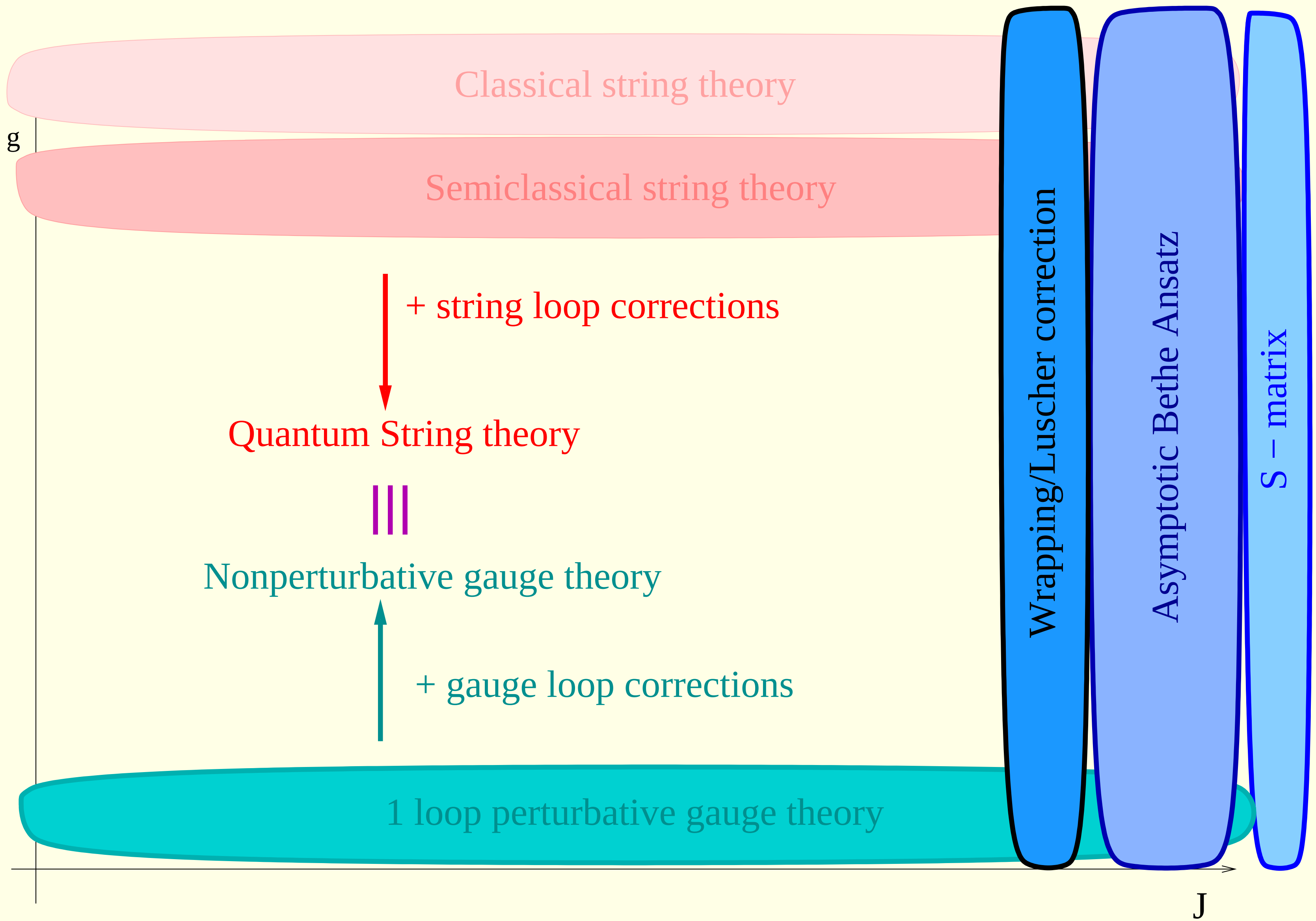


Excited states: analyticity from Lüscher

Lattice regularization: ?

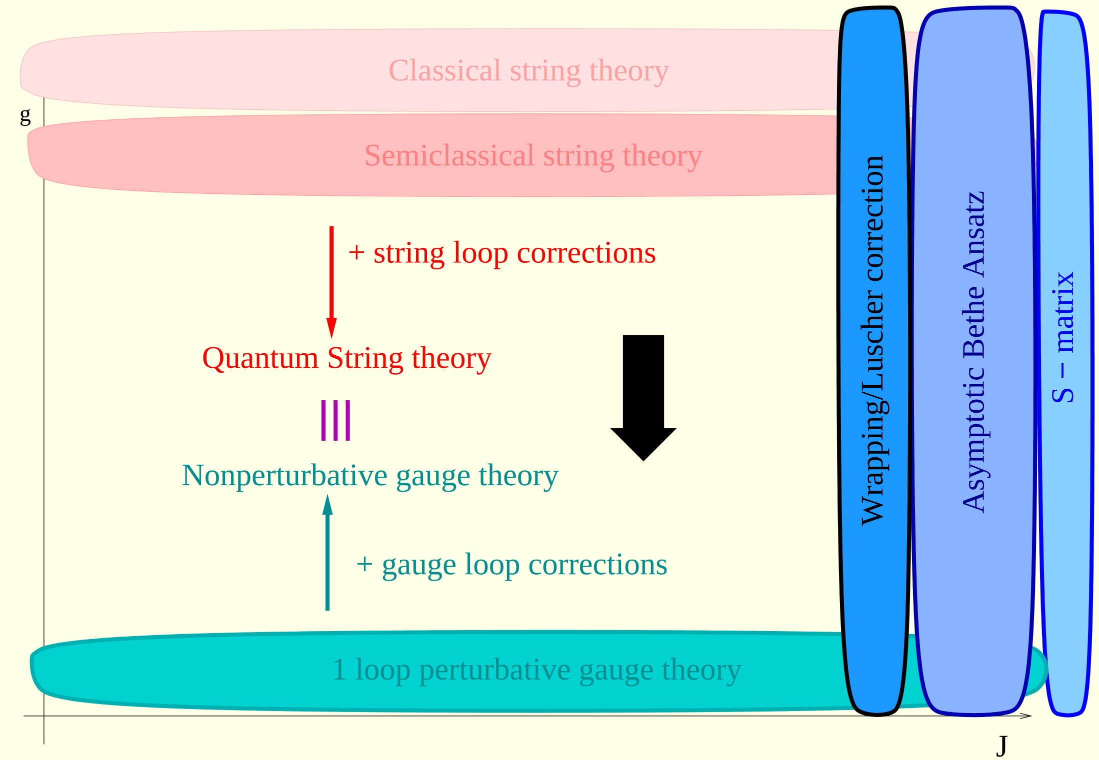
Conclusion

Conclusion



Classical string theory
g
Semiclassical string theory
+ string loop corrections
Quantum String theory
Nonperturbative gauge theory
+ gauge loop corrections
1 loop perturbative gauge theory
J
Wrapping/Luscher correction
Asymptotic Bethe Ansatz
S – matrix

Conclusion



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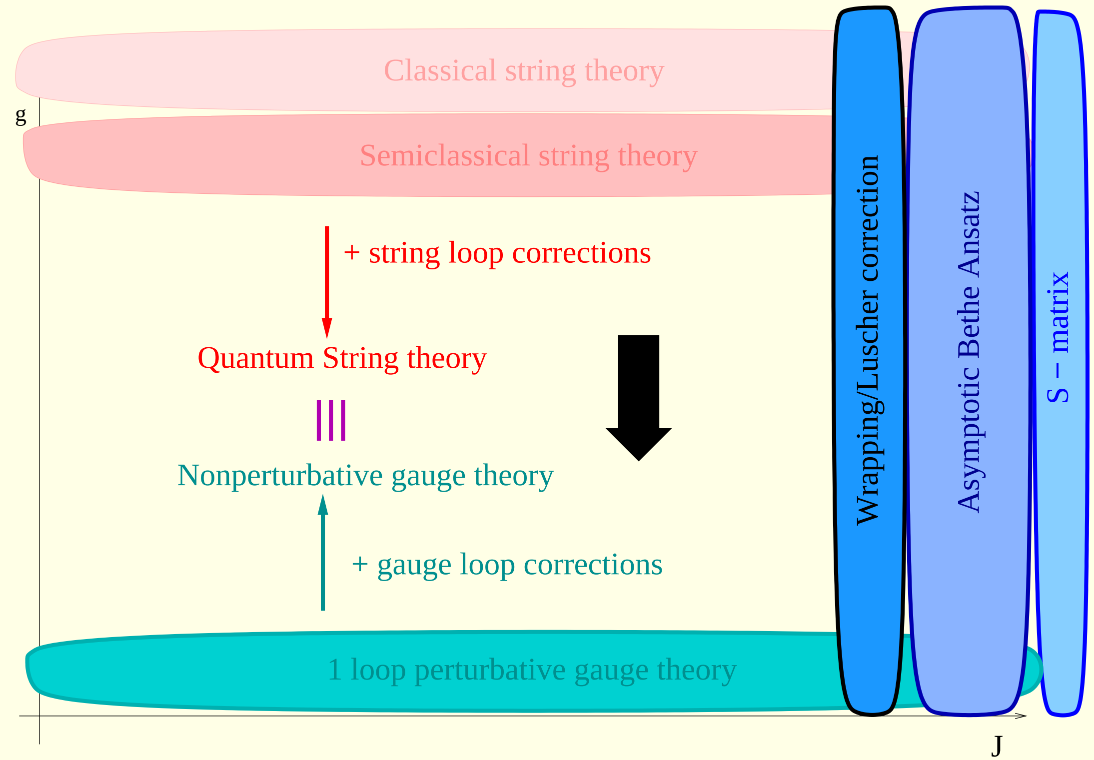
Conclusion

S-matrix = scalar . Matrix

physical sheet, explanation of all the poles



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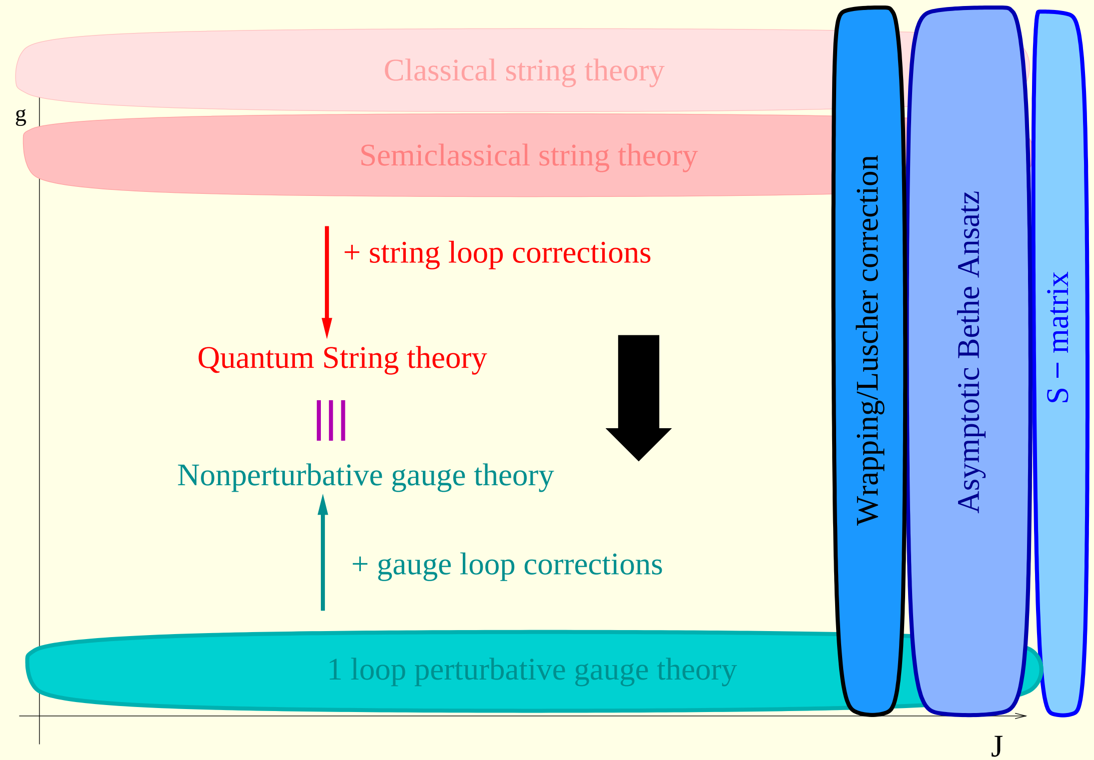
Excited states = analiticity . Y-system

Analytical structure of all excited states



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