4d N=1 from 6d (1,0)

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Motivation

• Compactification of 6d SCFT’s can be used to better understand dynamics of 4d SCFT’s.

• Has been successfully carried out for the case of the 6d SCFT living on $N$ M5-branes, for 4d $N=2$ and $N=1$ SCFT’s (class $S$).

• Recently extended also to the 6d SCFT living on $N$ M5-branes probing a $C^2/Z_k$ singularity, and 4d $N=1$ SCFT’s (class $S_k$).

• In this talk we will concentrate on better understanding the simpler case of $N = k = 2$. Particularly we shall present various expectations from 6d and compare against the 4d result.
Outline

1. Introduction
   - $N=2$ SCFT’s and Class $S$ theories
   - $N=1$ SCFT’s and Class $S$ theories
   - Class $S_k$ theories

2. 6d perspective

3. 4d perspective

4. Conclusions
Class $S$ theories

• Provide a systematic construction of 4d N=2 SCFT’s by compactification on a Riemann surface of the 6d SCFT living on $N$ M5-branes.

• Different realizations of the same Riemann surface imply duality relations for the 4d SCFT.

• For example: take the Riemann surface to be a torus. Leads to 4d maximally supersymmetric Yang-Mills. The complex structure moduli of the torus becomes the 4d complexified coupling constant.

• Modular invariance of the torus then leads to the $SL(2, \mathbb{Z})$ symmetry of 4d maximally supersymmetric Yang-Mills theory.
Class $S$ theories: three punctured spheres

- Three punctured spheres play an important role in class $S$ constructions.
- There are several types of punctures which are conveniently represented by Young diagrams.
- $T_N$ theory: corresponds to three maximal punctures. It has an $SU(N)^3$ global symmetry.
- Bifundamental: corresponds to two maximal punctures and a minimal one.
- Other theories can be generated from $T_N$ by Higgsing.

\[ T_N \quad SU(N) \times SU(N) \quad N-1 \]

[Gaiotto, 2009]
Class $S$ theories: general surfaces

- One can connect three punctured spheres with tubes to construct more general theories. Correspond to gauging the global symmetry associated to the punctures.
- The resulting 4d theory is the one associated with the surface.
- There may be more than one way to build a given surface from three punctured spheres. These different “pair of pants” decompositions correspond to dual descriptions of the same SCFT. [Gaiotto, 2009]
N=1 Class $S$ theories

• Can construct N=1 SCFT’s using Class $S$ building blocks.
• Done by giving an N=1 preserving deformation to an N=2 SCFT.
• Example: mass term for the adjoint chiral in the N=2 vector multiplet.
• Seiberg duality [Seiberg, 1995]:

\[ SU(N) + 2NF, \ W = \rho(QQ)^2 \quad \leftrightarrow \quad SU(N) + 2NF, \ W = q\bar{q}M + \rho M^2 \]

• In 6d: corresponds to N=1 preserving compactifications of the (2,0) theory.
6d construction

• The curvature of the Riemann surface breaks SUSY.
• In order to preserve SUSY we must perform a twist $SO(2)_S \rightarrow SO(2)_S - U(1)_R$ for $U(1)_R$ a subgroup of $U(1)_R \times SU(2)_J \subset SO(4) \subset SO(5)_R$.
• The twist makes some of the spinors covariantly constant and preserves N=1 SUSY.
• This breaks $SO(5)_R$ to $U(1)_R \times SU(2)_J$.
• We can also give flux for the $U(1)_J$ Cartan of the $SU(2)_J$.
• This is necessary to get N=2 SUSY, but for N=1 we have a choice for the total flux.
6d construction of N=1 class $S$

• The resulting theories are labeled by the Riemann surface and flux value.
• Gaiotto case: flux chosen to preserve N=2. Gives the N=2 class $S$ theories.
• Sicilian case: flux is zero. Have an extra $SU(2)_J$ global symmetry [Benini, Tachikawa, Wecht, 2010].
• BBBW case: generic case. Have an extra $U(1)_J$ global symmetry [Bah, Beem, Bobev, Wecht, 2012].
6d construction of N=1 class $S$

- Theories on a general Riemann surface can be constructed from three punctured spheres with fluxes.
- When connecting components with tubes the total flux is the sum of fluxes.
- Punctures have a discrete label: sign. Determines the charges of operators under $U(1)_J$. Can be either positive or negative.
- Gluing punctures with the same sign is done with N=2 vector multiplets. While gluing punctures with opposite signs is done with N=1 vector multiplets.
N=1 Class $S$ theories

- Using these rules we can construct N=1 SCFT’s.
- Example: N=1 $SU(N) + 2NF$ SQCD correspond to a limit of a 4 punctured sphere with vanishing flux.
- Other degeneration limits give dual descriptions.

[Gadde, Maruyoshi, Tachikawa, Yan, 2013]
[Agarwal, Intriligator, Song, 2015]
N=1 class $S$: conformal manifold

- The conformal manifold is given by the complex structure moduli of the Riemann surface.
- In addition one can turn on flat connections, holonomies, for $SU(2)_J$ or $U(1)_J$ symmetry. These preserve only N=1 SUSY.
- For a genus $g > 1$ Riemann surface with no punctures, we can turn on such holonomies on each of the $2g$ cycles $A_i, B_j$. These are subject to one relation and the action of global flavor rotation.

$$\prod_{i=1}^{g} [A_i, B_i] = 1$$

- For Sicilian case: $\dim\mathcal{M}_g = (3g - 3) + (g - 1) \cdot 3 = 6g - 6$
- For BBBW case: $\dim\mathcal{M}_g = (3g - 3) + g = 4g - 3$

[Benini, Tachikawa, Wecht, 2010]
[Bah, Beem, Bobev, Wecht, 2012]
Class $S_k$ theories

- 4d SCFT’s can be constructed by the compactification of the 6d SCFT living on $N$ M5-branes probing a $C^2/Z_k$ singularity. This in general produces $N=1$ theories.

- A class of field theories arising from this construction was conjectured in [Gaiotto, Razamat, 2015].

- Lagrangian cases can be constructed using the free trinion: corresponding to a sphere with two maximal punctures and one minimal.

- Maximal punctures have an $SU(N)^k$ global symmetry.

- Minimal punctures have a $U(1)$ global symmetry.

- In addition there are also $2k - 1$ internal symmetries $U(1)_{\beta_i} \times U(1)_{\gamma_i} \times U(1)_t$. These are conjectured to come from the Cartan of the $SU(k) \times SU(k) \times U(1)$ global symmetry of the 6d theory.
Free trinion

- Contains $2kN^2$ free chiral fields $Q, \tilde{Q}$. 
Properties of class $S_k$ theories

• Can construct general theories corresponding to a Riemann surface and fluxes for the internal symmetries.

• Punctures have additional discrete labels: sign, color.
  
  ▪ Sign: determine the sign of the charges of the baryons and mesons under $U(1)_t \times U(1)_{\beta_i} \times U(1)_{\gamma_i}$. Can be positive or negative.
  
  ▪ Color: determine the charge ordering of the baryons and mesons under $U(1)_{\beta_i} \times U(1)_{\gamma_i}$. 
Properties of class $S_k$ theories

• Theories corresponding to a Riemann surface can be build by gluing three punctured spheres with fluxes.
• The fluxes of the different pieces are summed.
• Gluing punctures of different color breaks some of the internal symmetry.
• Gluing punctures of different signs does not break any symmetry, but the form of the gluing is different.
Gluing

\[ W = \Phi \cdot M - M' \cdot \Phi \]

\( M'_i \quad M_i \)

\( \Phi_i \)

- \( \Phi \) gluing: punctures of the same sign
- \( S \) gluing: punctures of different signs
Introduction summary

• We have seen that compactifying 6d theories to 4d helps understanding 4d dynamics.

• Examples: N=2 and N=1 theories in class S, N=1 theories in class $S_K$.

• Class $S_K$: construction is 4d. Interesting if:
  ▪ Relate the aspects observed in 4d to 6d.
  ▪ Test the suggested correspondence by direct computation of objects in 6d and 4d.
Plan

• Consider the reduction of the 6d SCFT on $N$ M5-branes probing a $\mathbb{C}^2/Z_k$ singularity, to 4d on a Riemann surface. This leads to various expectations for the resulting 4d theory.

• Consider the expected 4d theory in class $S_k$.

• Compare the two expectations.

• Limitations:
  ▪ Consider only the $N = k = 2$ case.
  ▪ Riemann surface with no continuous isometries.
  ▪ In this talk we concentrate on Riemann surfaces with no punctures.
6d perspective

• Take the 6d SCFT on 2 M5-branes probing a $C^2/Z_2$ singularity and compactify it to 4d on a genus $g > 1$ Riemann surface.

• To preserve SUSY must twist: $SO(2)_S \rightarrow SO(2)_S - U(1)_R$, for $U(1)_R \subset SU(2)_R$.

• The $SU(k) \times SU(k) \times U(1)$ global symmetry is thought to enhance to $SO(7)$ for the $N = k = 2$ case [Ohmori, Shimizu, Tachikawa, Yonekura, 2014].

• Can also have non-zero flux in an abelian subgroup of $SO(7)$. 
6d perspective: fluxes

- Decompose the flavor symmetry: \( SO(7) \rightarrow SO(3) \times SO(4) \rightarrow SO(3)_{t} \times SU(2)_{\beta} \times SU(2)_{\gamma} \)
- Define a flux vector \( \mathcal{F} = (\beta, \gamma, t) \).
- The flux breaks \( SO(7) \) to a subgroup \( G^{\text{max}} \) with abelian part \( L \).
- The possible values for \( G^{\text{max}} \) are:

<table>
<thead>
<tr>
<th>( G^{\text{max}} )</th>
<th>( u(1)^3 )</th>
<th>( su(2)u(1)^2 )</th>
<th>( su(2)_{\text{diag}}u(1)^2 )</th>
<th>( su(2)su(2)u(1) )</th>
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<tbody>
<tr>
<td>( L )</td>
<td>( u(1)^3 )</td>
<td>( u(1)^2 )</td>
<td>( u(1)^2 )</td>
<td>( u(1) )</td>
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<tr>
<td>( \mathcal{F} )</td>
<td>( (a, b, c) )</td>
<td>( (a, 0, b)/(0, a, b) )</td>
<td>( (a, \pm a, b) )</td>
<td>( (a, 0, 0)/(0, a, 0) )</td>
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<th>( G^{\text{max}} )</th>
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<th>( so(7) )</th>
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<td>( L )</td>
<td>( u(1) )</td>
<td>( u(1) )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( \mathcal{F} )</td>
<td>( (0, 0, a) )</td>
<td>( (a, 0, \pm a)/(0, a, \pm a) )</td>
<td>( (0, 0, 0) )</td>
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6d perspective: conformal manifold

• Complex structure moduli.

• Can also turn on flat connections for global symmetries. Conformal manifold:

\[
\dim M_{g,0} = 3g - 3 + (g - 1) \dim G^{\max} + \dim L
\]

• \(G^{\max}\) is the global symmetry preserved by the flux. In 4d should be the maximal global symmetry on the conformal manifold.

• \(L\) is the abelian part of \(G^{\max}\). In 4d should be the global symmetry on a generic point on the conformal manifold.
6d perspective: anomalies

• Can calculate the 4d anomaly polynomial by integrating the 6d one.
• 6d anomaly polynomial [Ohmori, Shimizu, Tachikawa, Yonekura, 2014]:

\[
I_{8}^{so(7)} = \frac{11C_2^2(R)}{12} - \frac{C_2(R)p_1(T)}{24} + \frac{C_2(so(7))_8p_1(T)}{24} - \frac{C_2(R)C_2(so(7))_8}{2} \\
+ \frac{7C_2^2(so(7))_8}{48} - \frac{C_4(so(7))_8}{6} + \frac{29p_1^2(T) - 68p_2(T)}{2880}
\]

• Decompose the characteristic classes:
  ▪ Space-time parts broken to parallel \( t \), or orthogonal to the surface.
  ▪ The \( SU(2)_R \) bundle is broken to \( U(1)_R \) by the twist. Decompose it to its Chern roots \( n_1, n_2 \).
6d perspective: anomalies

• Without the twist we take $n_1 = -n_2$. Twist changes this by shifting: $n_2 \rightarrow n_2 - t$.

• If flux for $U(1) \subset SO(7)$ is turned on, must also decompose the $SO(7)$ characteristic classes. Need to perform a-maximization in 4d.

• Integrate the anomaly polynomial over the Riemann surface using:

$$\int t = 2(1 - g)$$

• Example for $SO(7)$ compactification:

$$Tr(R^3) = 22(g - 1), Tr(R) = -2(g - 1)$$

$$a = \frac{3}{32}(3TrR^3 - TrR) = \frac{51}{8}(g - 1), \quad c = \frac{1}{32}(9TrR^3 - 5TrR) = \frac{13}{2}(g - 1)$$
4d perspective: plan

• To build the analogous theories in class $S_2$ we need the theories corresponding to a three punctured sphere. These are strongly interacting theories.

• We can learn about these theories using duality relations.

• Consider a duality relating a gauging of these theories to a Lagrangian theory.

• The duality can then be “inverted” leading to information on the strongly coupled theory.
4d perspective: duality

- Connecting two free trinions leads to a Lagrangian SCFT.
- This SCFT has at least two dual descriptions involving different strongly coupled three punctured spheres.
4d perspective: duality

• To understand the second dual we can consider the analogue gauging but with the free trinion.
• Can close the maximal puncture down to a minimal one by giving a vev to a meson.
• The resulting theory is the one corresponding to the connecting the surface with two minimal punctures.
4d perspective: duality

• We find that the Lagrangian theory is dual to an $SU(2) + 2F$ gauging of the strongly interacting SCFT with 3 singlets connected via a cubic superpotential.

• The duality implies that the indices of the two theories are related as:

$$ I^{Lagr} = \int d \Delta_{Haar SU(2)} I^{\text{Vector + Flavor + Singlets}} \cdot I^{\text{Interacting SCFT}} $$
4d perspective: inversion

• There is a mathematical identity that allows to invert the relation: express the index of the strongly interacting theory as an $SU(2) + 1F$ gauging with singlets of the Lagrangian theory [Spiridonov, Warnaar, 2006].

\[ I_{\text{Interacting SCFT}} = \int d\Delta_{\text{Haar}SU(2)} I^{\text{Vector + Flavor + Singlets Lagr}} \]

• Take the further step and interpret the gauging as a physical process: duality. This gives a Lagrangian for the strongly interacting theory.

• Unfortunately Lagrangian is strongly coupled and only useful for the calculation of protected quantities: anomalies, index.
4d perspective: $SO(7)$

- Simple to generate a theory with $SO(7)$ global symmetry by connecting a trinion to the conjugate trinion.
4d perspective: $SO(7)$

- No mixing of $U(1)_\beta$, $U(1)_\gamma$ or $U(1)_t$ in $U(1)_R$.
- Can evaluate the index of this theory:
  \[
  \mathcal{I}^{so(7)}_{g,0} = 1 + \left(3g - 3 + (g - 1)21_{so(7)}\right|_{g=2})pq + \cdots.
  \]
- Can evaluate the conformal anomalies of this theory:
  \[
  a = \frac{51}{8}(g - 1), \quad c = \frac{13}{2}(g - 1)
  \]
- Same result also using $T_B$.  


4d perspective

- Can construct and match other theories. Build examples of all the $G^{\text{max}}$ cases.
- Example: $U(1) \times SU(2)^2$.
- Global symmetry, anomalies and number of marginal deformations agree with the 6d expectation.
- From matching we find: $\mathcal{F}_{T_A} = \left(\frac{1}{4}, \frac{1}{4}, 1\right)$, $\mathcal{F}_{T_B} = \left(-\frac{1}{4}, \frac{1}{4}, 1\right)$, $\mathcal{F}_{\text{Free}} = (0,0,\frac{1}{2})$. 
Conclusions

• Matched global symmetries, dimension of conformal manifold and anomalies for selected $N = 2$ class $S_2$ theories against those from the 6d construction.
• Provide a non-trivial test on class $S_k$ theories.
• Provide a bridge between field theory objects and properties of the compactification.

Open questions

• General $N$ and $k$:
  ▪ Concentrate on tori.
  ▪ Understanding the index of class $S_k$ theories.
  ▪ Generalizing the inversion procedure.
Thank you