Is there eternal inflation in the cosmic landscape ?

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hep-th/0611148
ArXiv:0708.4374 [hep-th]
with Qing-Guo Huang, ArXiv:0803.0663 [hep-th]
with Dan Wohns and Yang Zhang, ArXiv:0811.3753 [hep-th]

IPMU, 04/01/09

Expansion of the universe

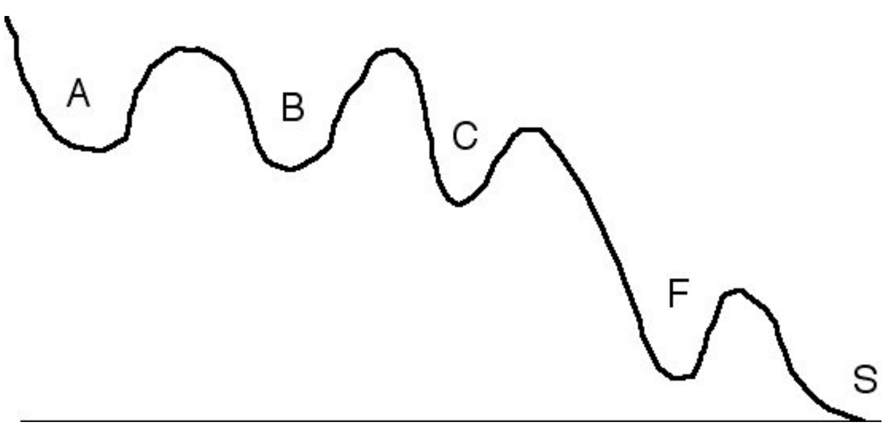
$$H^2 = \Lambda + \frac{k}{a^2} + \frac{\rho_m}{a^3} + \frac{\rho_r}{a^4}$$

- $a(t)$ = the cosmic scale factor ~ size of universe
- H = Hubble constant $=(da/dt)/a$
- Λ = dark energy ~ cosmological constant ~ effective potential
- k = curvature
- ρ_m = matter density at initial time
- ρ_r = radiation density at initial time
- H = constant $\Rightarrow$ $a = e^{Ht}$ $\rightarrow$ Inflation

Eternal Inflation

$$a(t) \simeq e^{Ht} \rightarrow V = a(t)^3 \simeq e^{3Ht} \qquad \tau > 1/H$$

Suppose the universe is sitting at a local minimum, with a lifetime longer than the Hubble time:
Then the number of Hubble patches will increase exponentially. Even after some Hubble patches have decayed, there would be many remaining Hubble patches that continue to inflate.



Eternal inflation implies that somewhere in the universe (outside our horizon), inflation is still happening today.

Flux compactification in Type II string theory

where all moduli of the 6-dim. "Calabi-Yau" manifold are stabilized

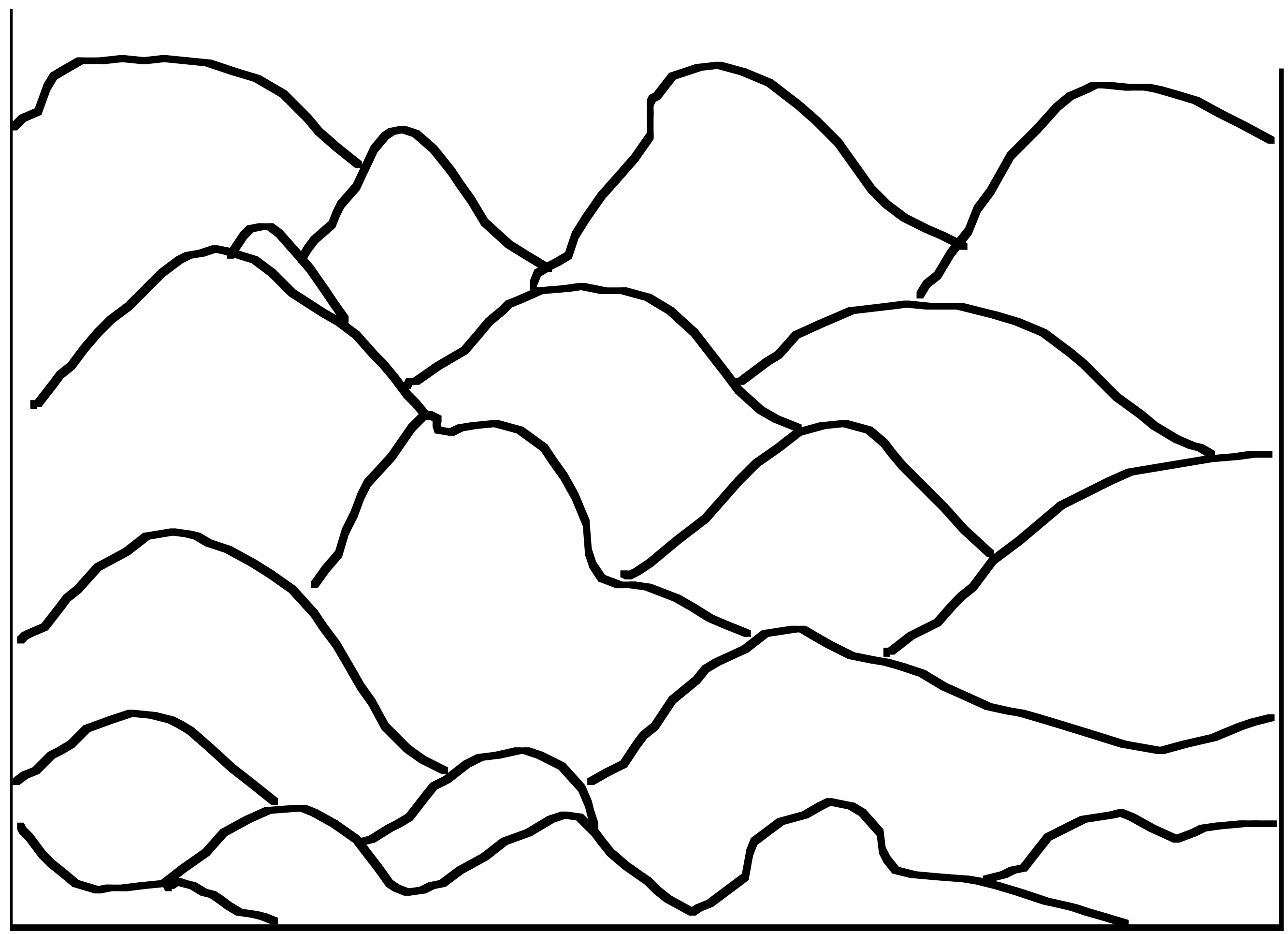
- There are many meta-stable manifolds/vacua, 10^{500} or more, probably infinite, with positive, zero, as well as negative cosmological constants.
- There are many (tens to hundreds or more) moduli (scalar modes) that are dynamically stabilized.

KKLT vacua

Giddings, Kachru, Polchinski, Kachru, Kallosh, Linde, Trivedi and many others, 2001....

A cartoon of the stringy landscape

many local minima



Pros : It seems that eternal inflation can be quite generic (unavoidable) in the cosmic landscape.
Its presence leads to a non-zero probability for every single site in the landscape, including our very own universe.

Cons : Since there are many (10^{500} or more) vacua, and some parts of the universe are still inflating, it is difficult to see why we end up where we are without invoking some strong version of the anthropic principle.

It is very difficult to make any testable prediction.

There is also a measure problem.

Scenarios with or without eternal inflation are very different.

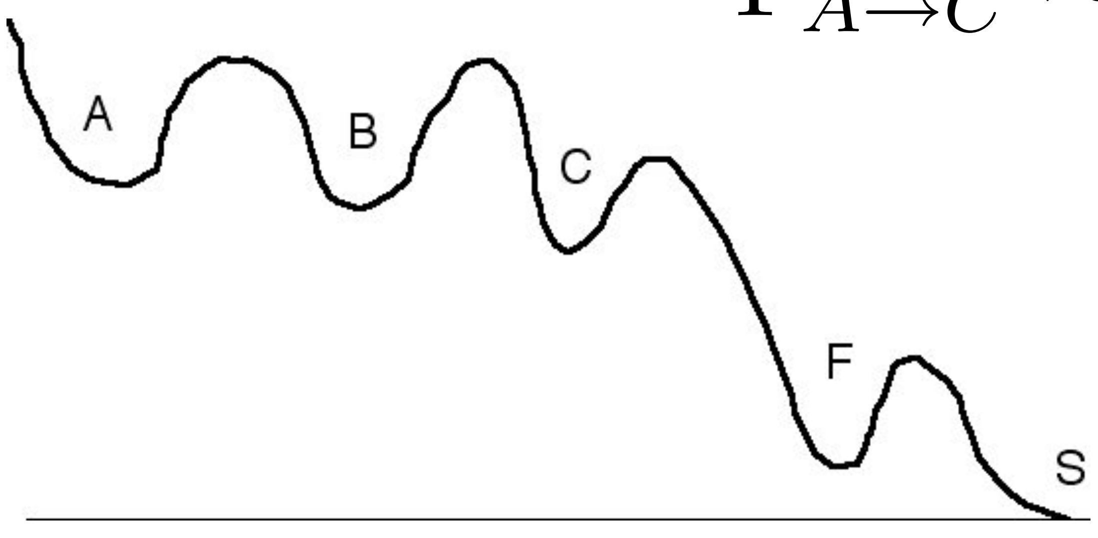
A simple question :

Consider tunneling from A to C :

assume for simplicity : $\Gamma_{A\to B} = \Gamma_{B\to C} = \Gamma_0 \sim e^{-S}$

$$\Gamma_{A\to C} \sim \Gamma_{A\to B}\Gamma_{B\to C} = \Gamma_0^2$$

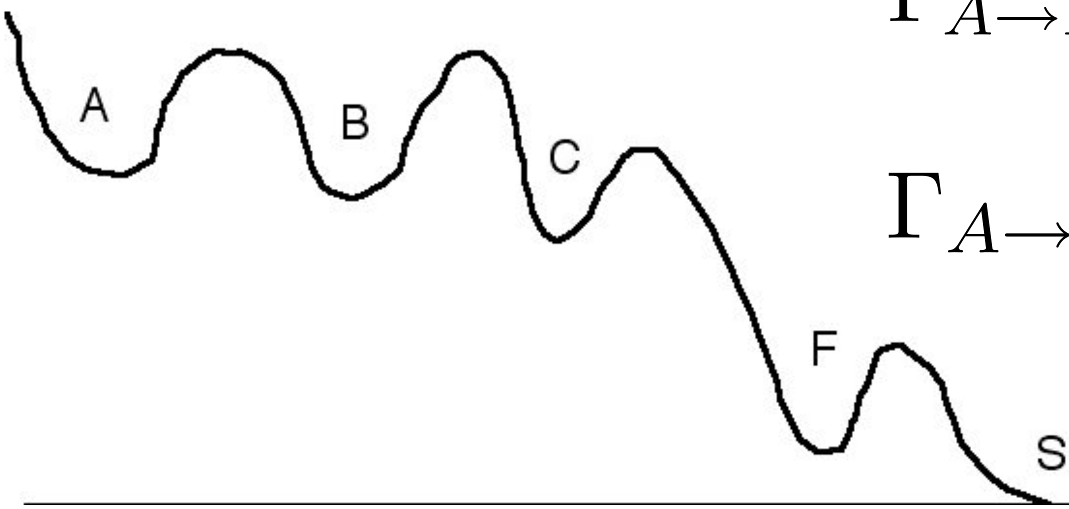
$$\sim e^{-2S}$$



?

A simple question :

Consider tunneling from A to C :



$$\Gamma_{A\to B} = \Gamma_{B\to C} = \Gamma_0 \sim e^{-S}$$

$$\Gamma_{A\to C} \sim \Gamma_{A\to B}\Gamma_{B\to C} = \Gamma_0^2$$

?

Time :

$$t_{A\to C} = t_{A\to B} + t_{B\to C} = \frac{1}{\Gamma_{A\to B}} + \frac{1}{\Gamma_{B\to C}} \sim \frac{2}{\Gamma_0}$$

$$\Gamma_{A\to C} = \frac{\Gamma_0}{2}$$

?

Which is correct ?

?

$$\Gamma(2) \sim e^{-2S} \Rightarrow \Gamma(n) \sim e^{-nS}$$ negligible

$$\Gamma(2) \sim e^{-S} \Rightarrow \Gamma(n) \sim e^{-S}$$

$$\Gamma(1) = \Gamma_0 \sim e^{-S} \sim T_0$$ Transmission coefficient

Consider the second case :

$$T_{A\to C} = T_0/2 \quad \rightarrow \quad T(n) \simeq T_0/n$$

$$\Gamma(1) = \Gamma_0 \sim e^{-S}$$

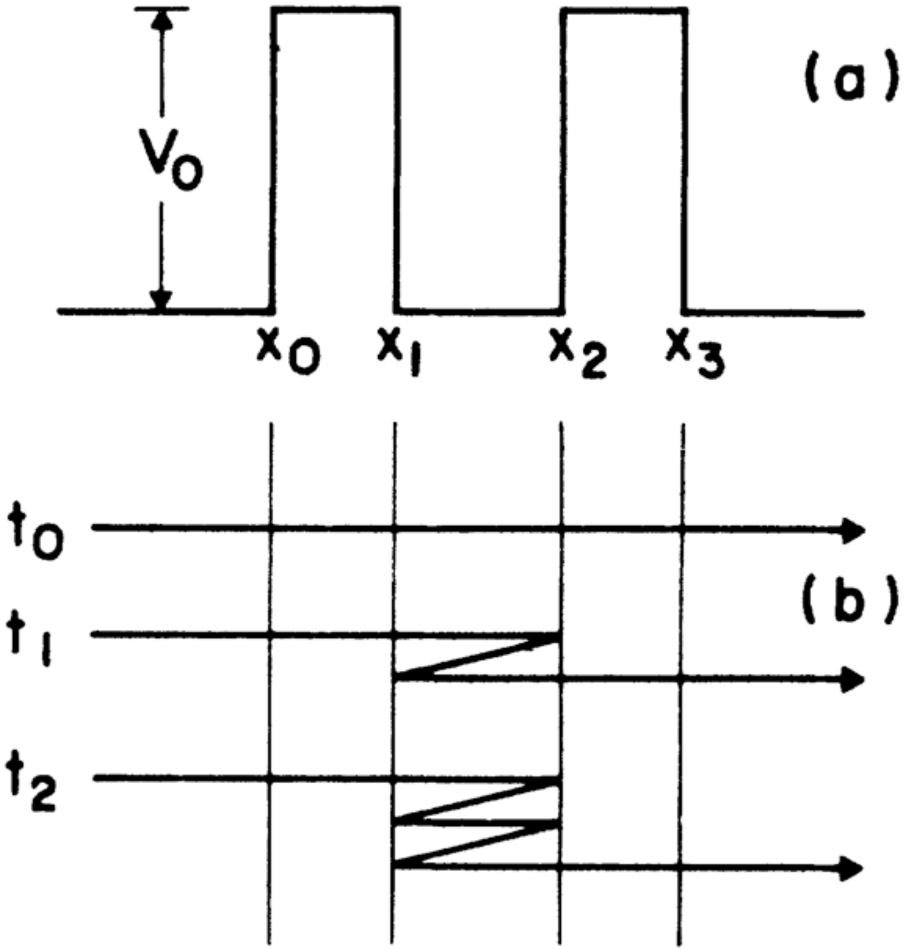
$$\Gamma(2) \sim \cancel{e^{-2S}} \Rightarrow \Gamma(n) \quad \text{negligible}$$

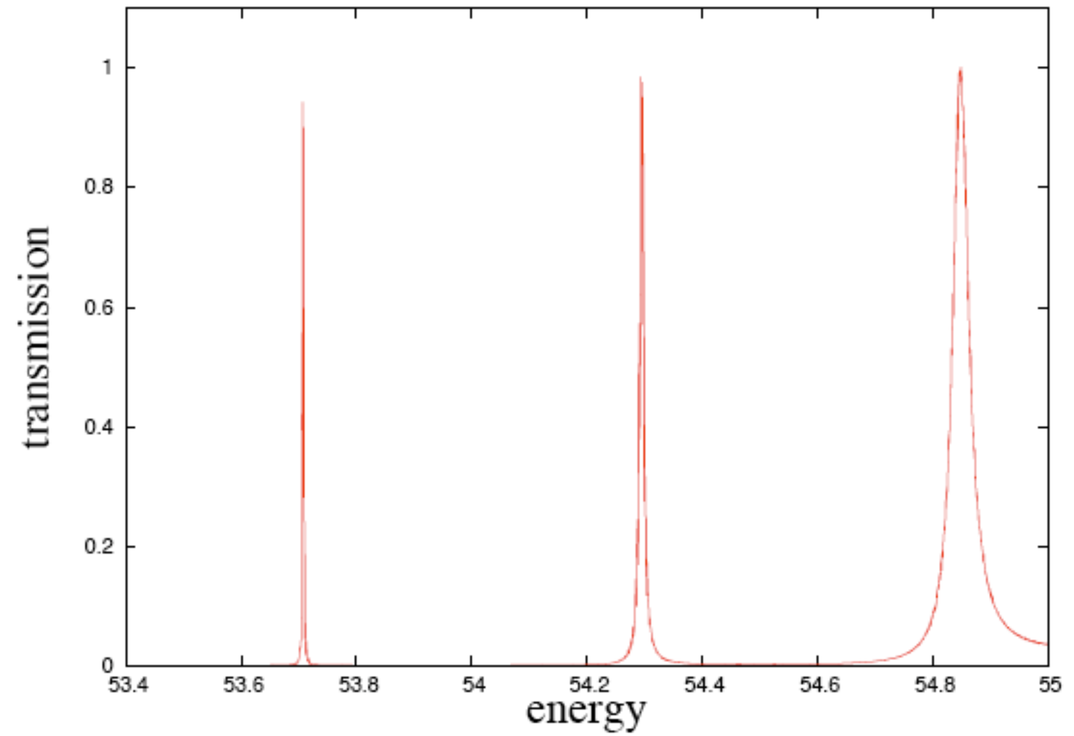
$$\Gamma(2) \sim e^{-S} \Rightarrow \Gamma(n) \sim e^{-S}$$

correct

$$T_{A \to C} = T_0/2 \qquad T(n) \simeq T_0/n$$

To understand why : resonance tunneling





Y. Zhota, PRB 41 (1990) 7879.

Resonance tunneling in semi-conductors

Wikipedia

A **resonant tunnel diode** (RTD) is a device which uses quantum effects to produce negative differential resistance (NDR). As an RTD is capable of generating a terahertz wave at room temperature, it can be used in ultra high-speed circuitry. Therefore the RTD is extensively studied.

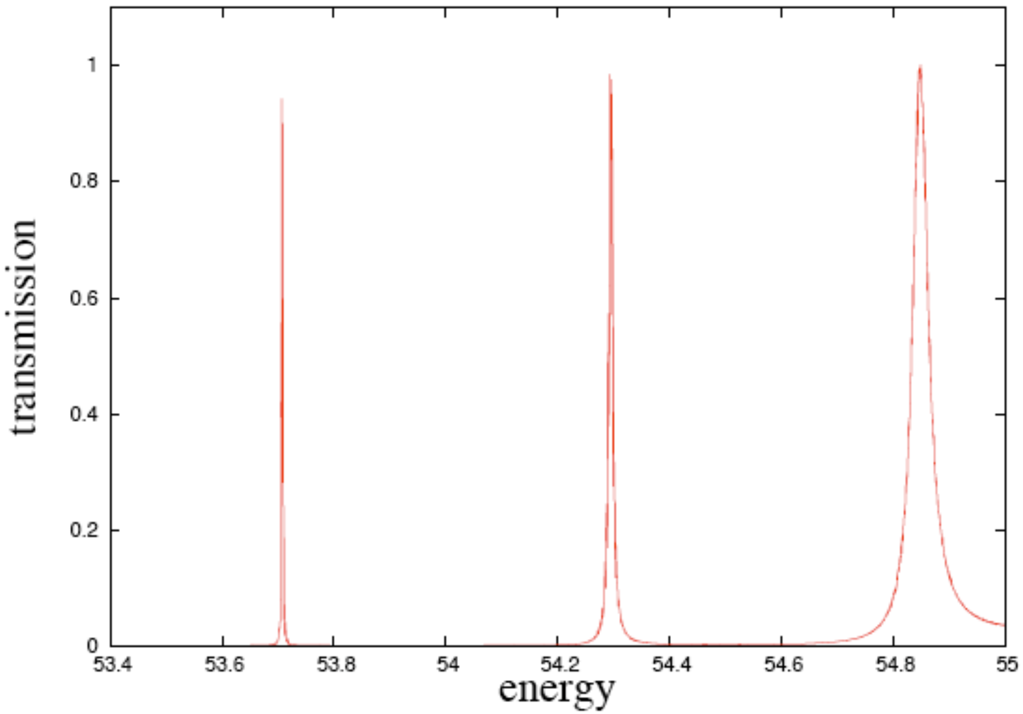
Chang, Esaki, Tsu, 1974

Generic wavefunction spread in energy :

$$T(A \to C) = \frac{T(A \to B)T(B \to C)}{T(A \to B) + T(B \to C)} \sim T_0/2$$

naive

$$T(n) \simeq T_0/n$$



In any meta-stable site in the cosmic landscape, we cannot focus only at its nearest neighbors.

Resonance tunneling in QFT :
Saffin, Padilla and Copeland, arXiv:0804.3801 [hep-th]
Sarangi, Shiu, Shlaer, arXiv:0708.4375 [hep-th]

In a d-dimensional potential :

naive : $$\Gamma_t^{nr} \sim 2d\, \Gamma_0$$

The time for 1 e-fold of inflation is Hubble time 1/H, so the lifetime of a typical site seems to be much longer than the Hubble scale, and eternal inflation is unavoidable.

Very roughly : $$\Gamma_t \sim n^{d-1} \Gamma_0$$

$n \sim 1/Hs$

For large enough d (and maybe n) the tunneling can be fast.

Outline

- Tunneling in the cosmic landscape is quite different from tunneling between 2 sites.
- It is harder to trap a wavefunction in higher dimensions. Also, the barriers in some directions in the landscape are known to be exponentially low.
- The vacuum energy in the landscape plays the role of a finite (Gibbons-Hawking) temperature effect, enabling faster tunneling.
- Treating the landscape as a random potential, we can borrow the knowledge developed in condensed matter physics to estimate the mobility of the wavefunction in the landscape.

The QM wavefunction is harder to be trapped in higher dimensions:

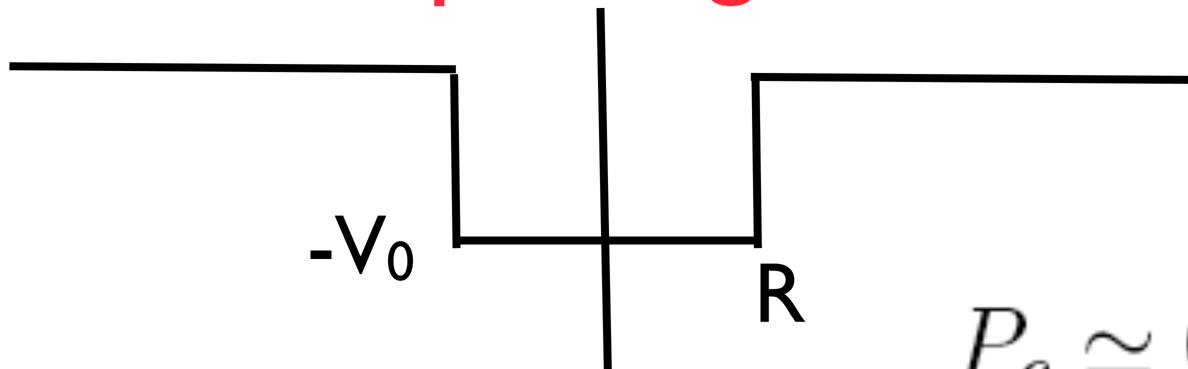
$$\eta(r)'' + \left(2m[E - V(r)] - \frac{(d-1)(d-3)}{4r^2}\right)\eta(r) = 0$$

Repulsive

A 1-dim. attractive delta-function potential always has a bound state but not a 3-dim one.

$$\psi(r) \;=\; r^{-(d-1)/2}\eta(r)$$

Harder to trap in higher dimensions



Spherical square well :

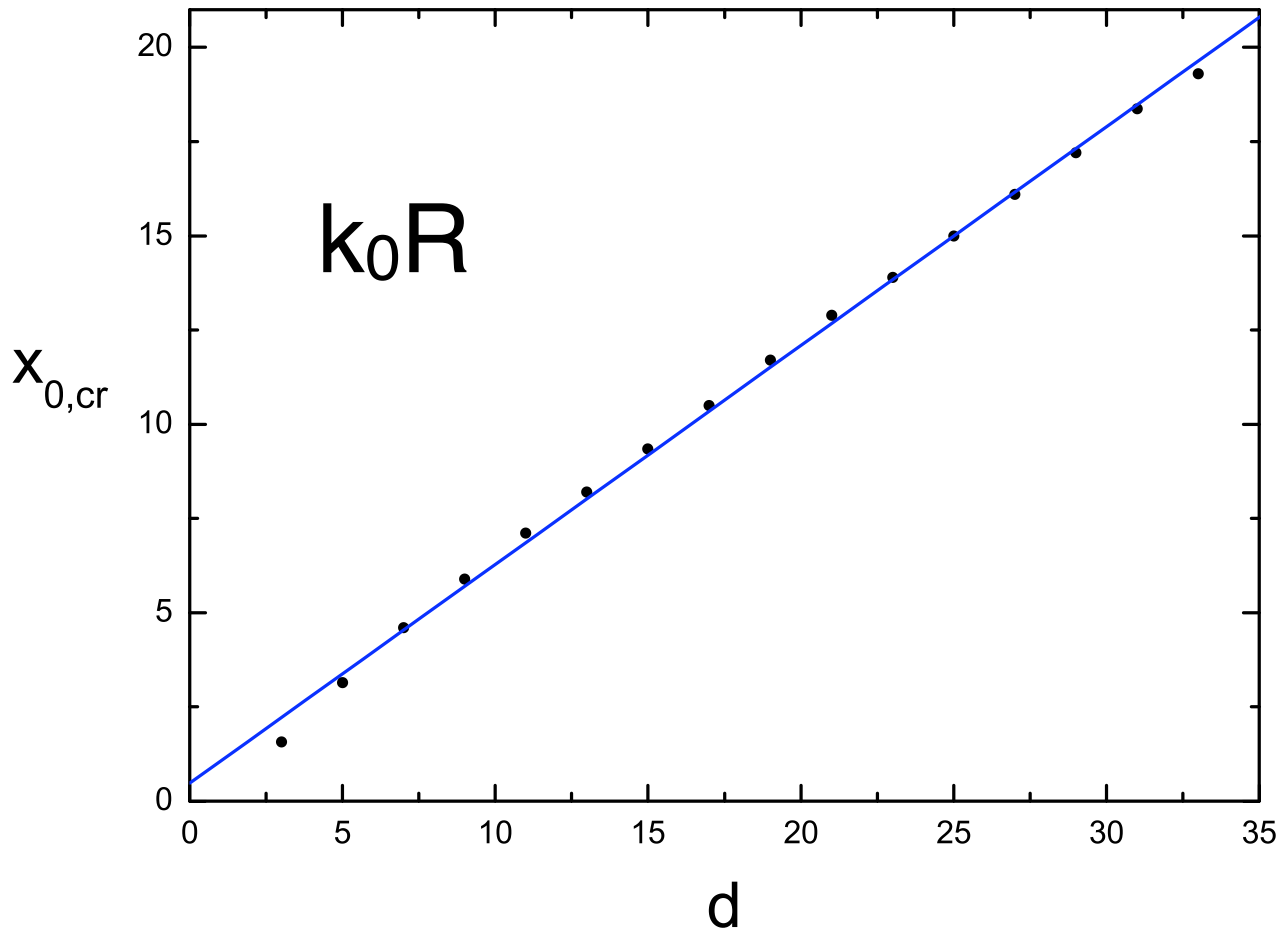
$$V(r) = -V_0 \quad r < R$$
$$= 0 \quad r > R$$

$$k_0^2 = 2mV_0$$

$$\psi(r) \sim e^{-ar}$$

$$a^2 = 2m|E|$$

$$P_c \simeq 0.58d + 0.5$$



Axionic potentials

$$V = V_0 + \sum_i \alpha_i \cos(2\pi\phi_i) + \sum_{i,j} \beta_{ij} \cos(2\pi\phi_i - 2\pi\phi_j)$$

$$\phi_i = \varphi_i / f_i \qquad \alpha_i \sim e^{-S_i}$$

$$|\beta_{ij}| < \alpha_i$$

For light axions, the potential heights are very low. In some directions, the barriers can easily be exponentially low.

Svrcek and Witten, hep-th/0605206
McAllister and Easther, etc

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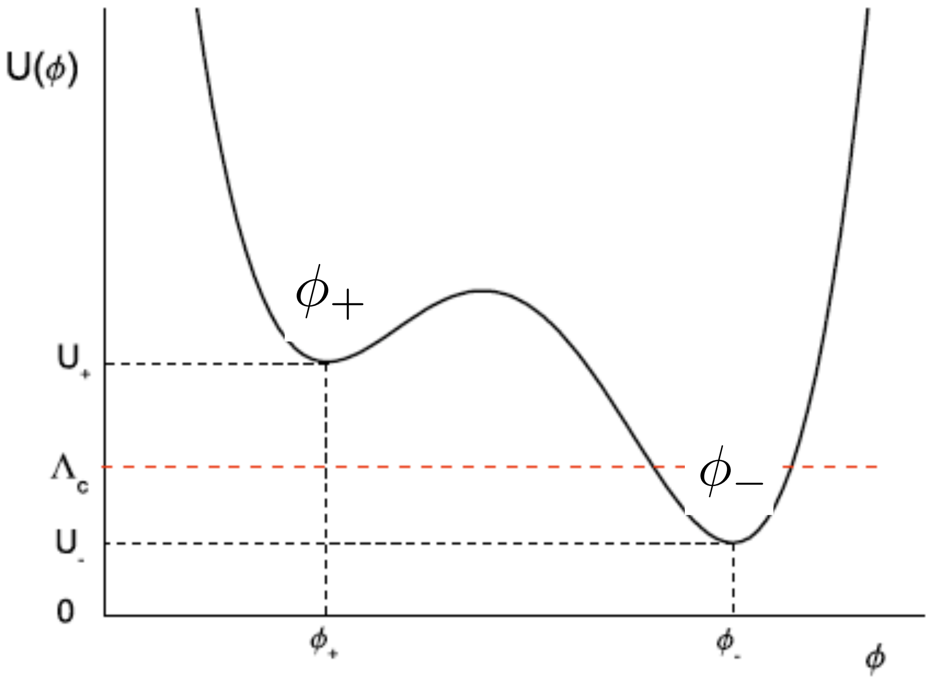
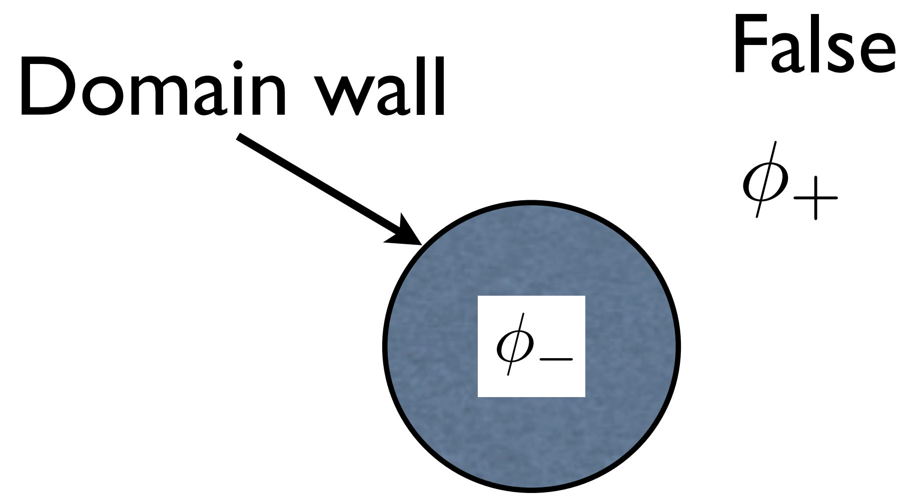
Coleman-de Luccia (CDL) Tunneling

$$\Gamma(CDL) \sim e^{-B}$$





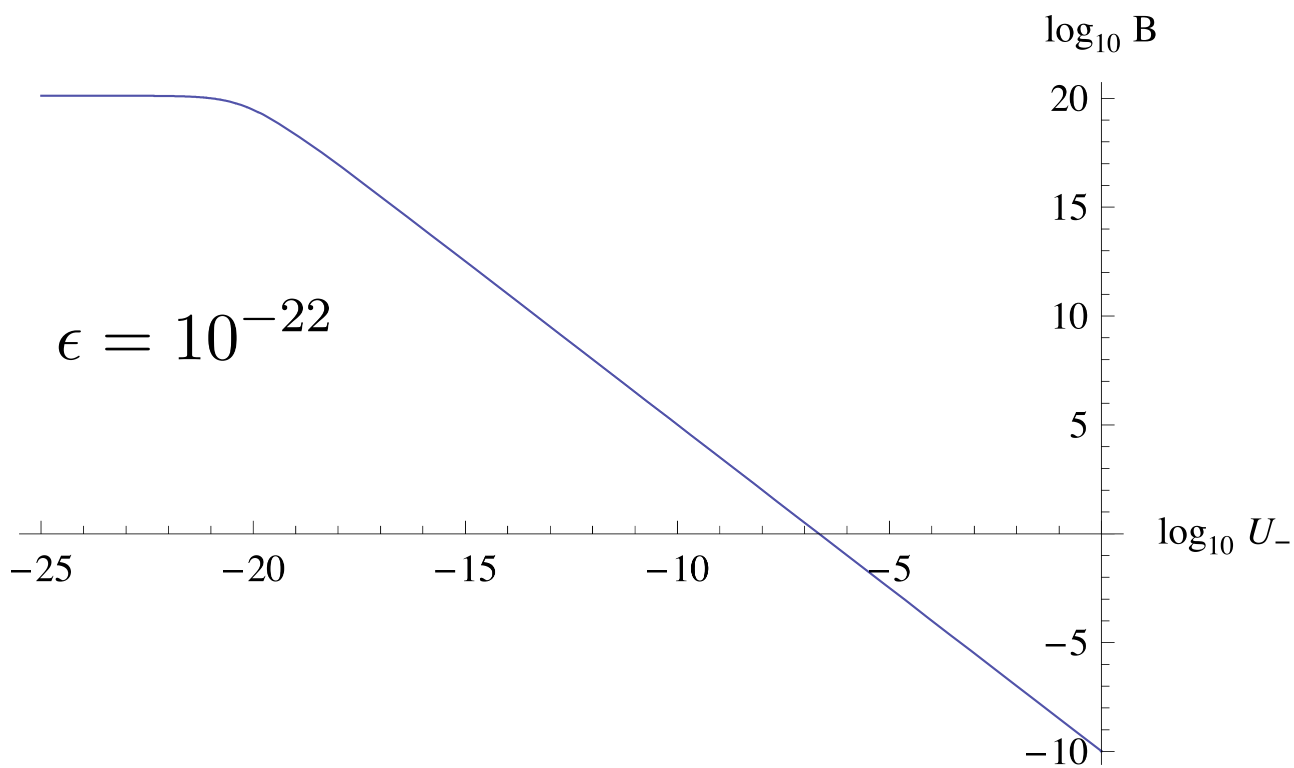
$$\epsilon = U(\phi_+) - U(\phi_-)$$

In Coleman-de Luccia Tunneling

$$\Gamma(CDL) \sim e^{-B}$$

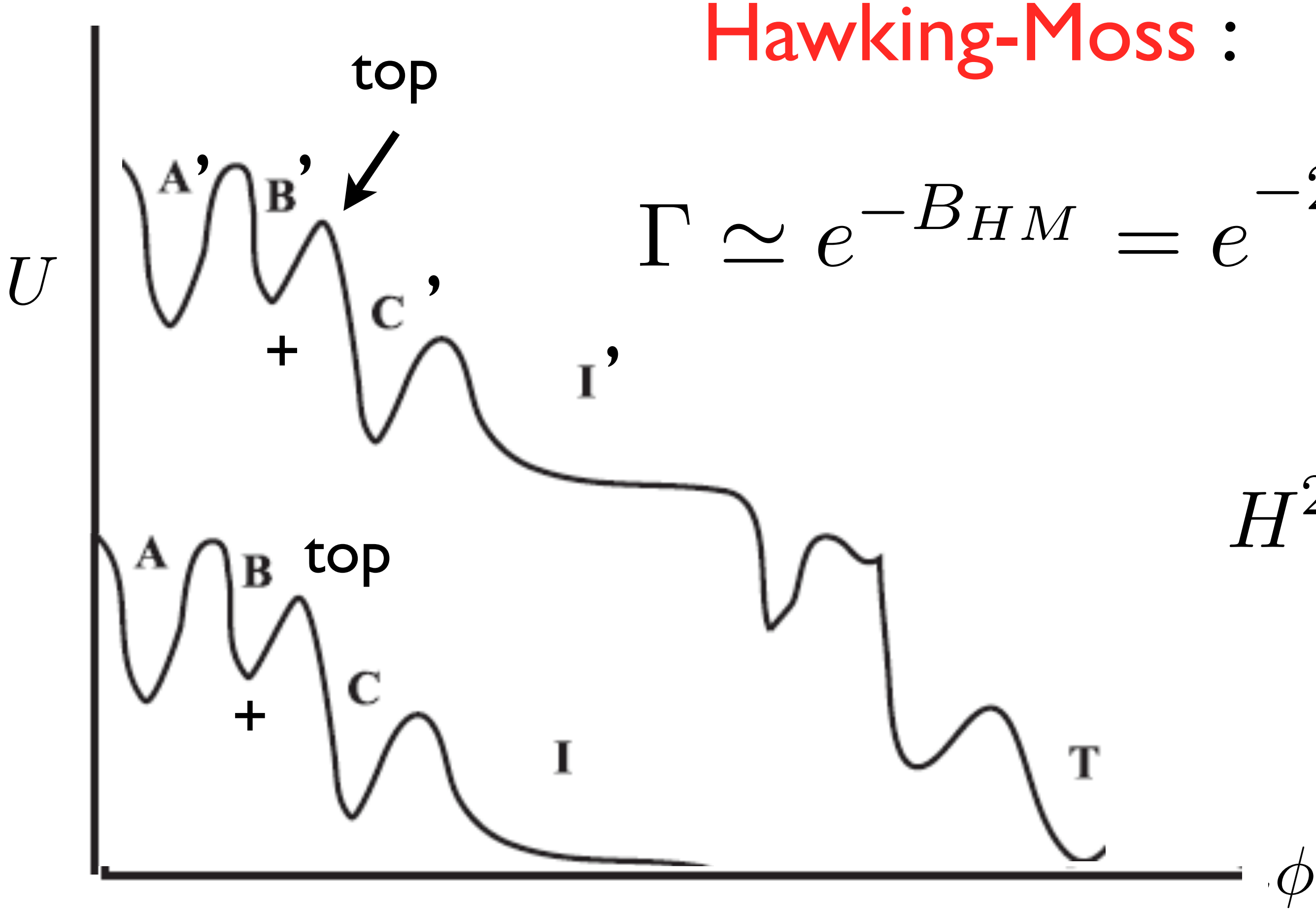
In the thin wall approximation :

$$\tau = 10^{-12}, \qquad \epsilon = 10^{-22}$$



for fixed domain wall tension τ and
(false-true) energy density difference $\epsilon = U(\phi_+) - U(\phi_-)$

Tunneling is much faster at higher C.C.



Hawking-Moss :

$$\Gamma \simeq e^{-B_{HM}} = e^{-24\pi^2\left[\frac{M_P^4}{U(\phi_+)} - \frac{M_P^4}{U(\phi_t)}\right]}$$

$$H^2 = U/3M_P^2$$

$$B_{HM} = \frac{8\pi^2}{3}\frac{\delta U}{H_+^2 H_t^2}, \qquad \delta U = U(\phi_t) - U(\phi_+)$$

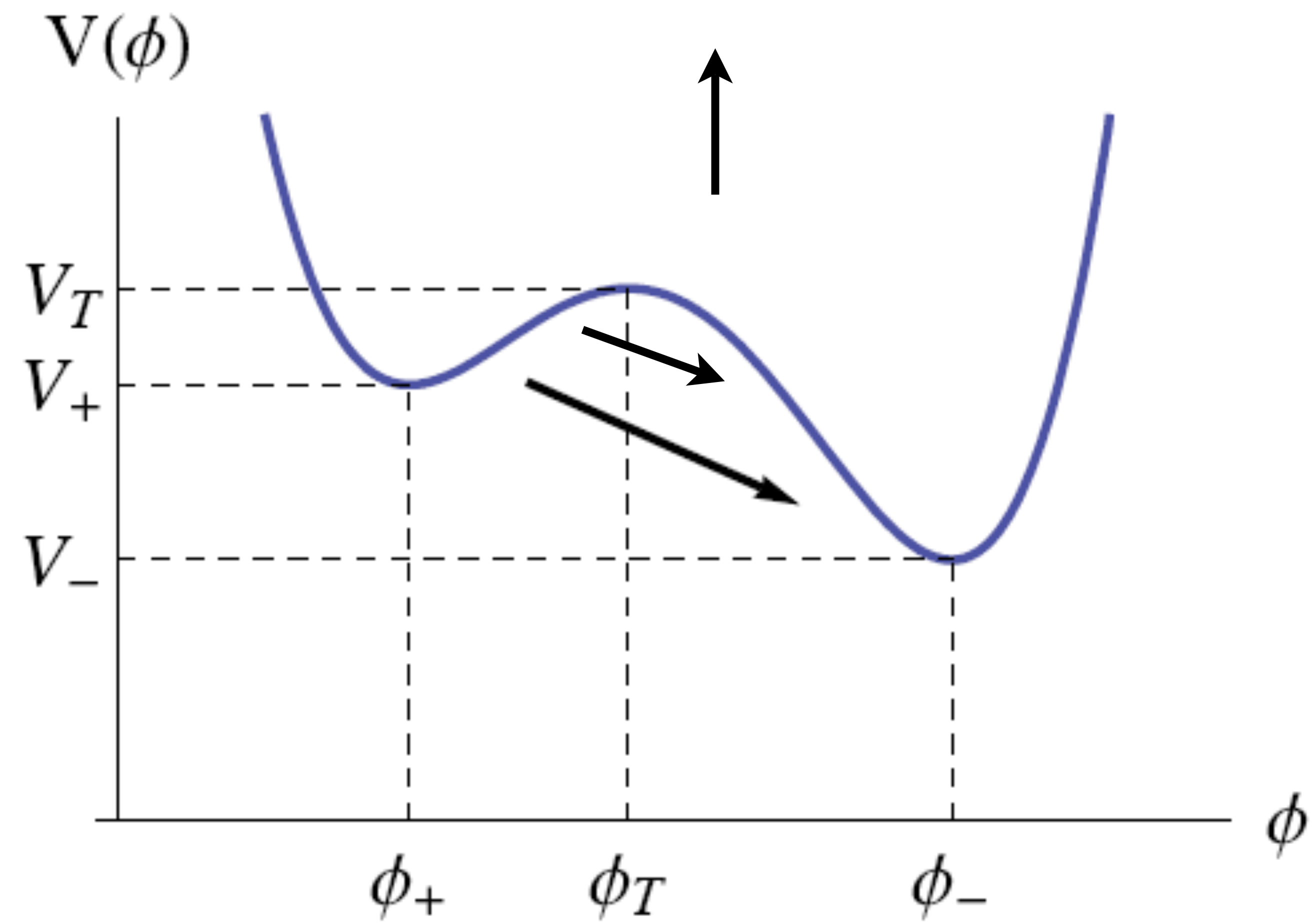
2 Formulae :

CDL thin wall : $$B \simeq \frac{27\pi^2\tau^4}{2\epsilon^3} \to \frac{2\pi^2\tau}{H^3}$$

Hawking-Moss : $$B_{HM} = \frac{8\pi^2}{3}\frac{\delta U}{H_+^2 H_t^2}$$

Gibbons-Hawking temperature $T_H = H/2\pi$

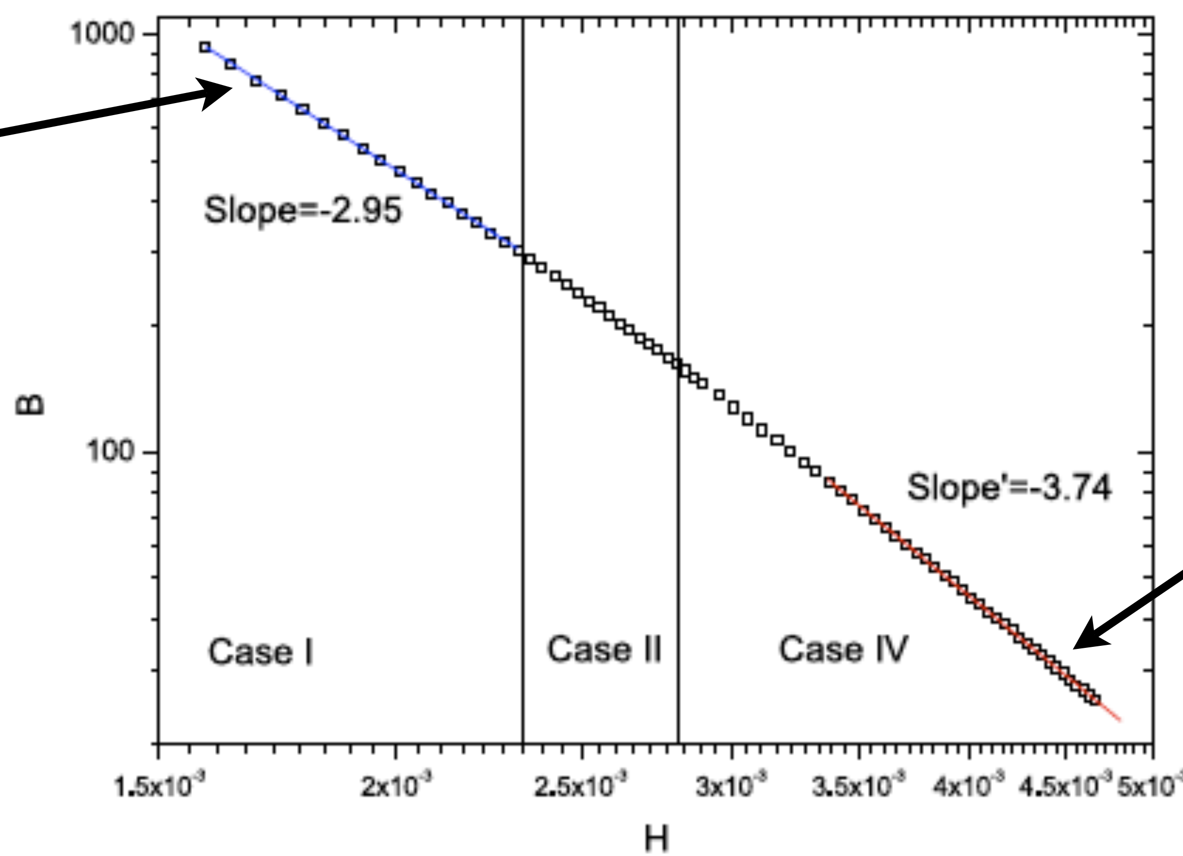
Coleman-de Luccia (CDL) tunneling



Bounce B as the vacuum energy is increased

while the shape of the barrier is fixed

CDL
thin wall



close to HM

$$\Gamma \simeq A e^{-B},$$

$$B = S_E(\phi) - S_E(\phi_+)$$

Thermal | Quantum

$$\Gamma \sim e^{-(S_E(\phi_{f+}) - S_E(\phi_+))} e^{-(S_E(\phi_{bounce}) - S_E(\phi_{f+}))}$$

HM | CDL

Affleck : $$\Gamma \sim \int dE e^{-(E - E(\phi_+))/T} e^{-J(E)}$$

$$J(E) = 2 \int_{\phi_{f+}}^{\phi_{t-}} d\phi \sqrt{2(V(\phi) - E)}$$

Brown and E. Weinberg
arXiv: 0706.1573

Finite Hawking temperature effect

$$U(\phi_i) \text{ should be replaced by } U(\phi_i, T)$$

Expanding about the top of a symmetric barrier :

$$U(\phi, T=0) = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4 + \ldots$$

$$\Rightarrow \quad U(\phi, T_H) = \frac{1}{2}\left(\frac{\lambda T_H^2}{24} - m^2\right)\phi^2 + \frac{\lambda}{4!}\phi^4 + \ldots$$

No trapping if for any modulus :

$$T_H^2 > \frac{24 m_i^2}{\lambda_i}, \qquad i = 1, 2, ..., d$$

d ~100

A comment :

- Tunneling of a D3-brane can be significantly enhanced by its Dirac-Born-Infeld action.

CDL :

Brown, Sarangi, Shlaer and Weltman, ArXiv 0706.0485 [hep-th]

Hawking-Moss :

Tolley and Wyman, ArXiv 0801.1854 [hep-th]
Wohns, ArXiv 0802.0623 [hep-th]

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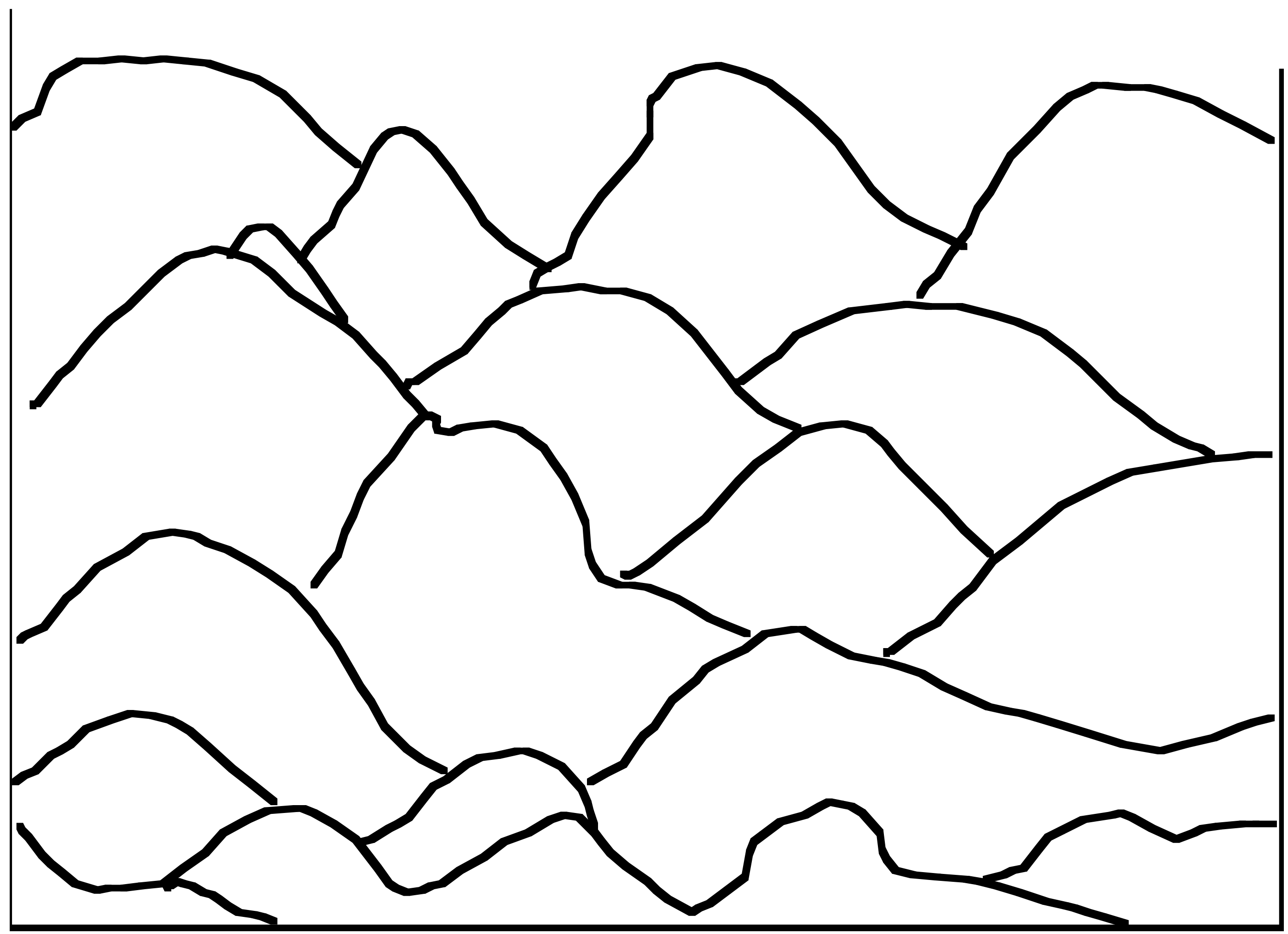
- Treating the landscape as a random potential, we can borrow the knowledge developed in condensed matter physics to estimate the mobility of the wavefunction in the landscape.

The vastness of cosmic landscape

- At a typical meta-stable site, count the number of parameters or the number of light scalar fields. This gives the number of moduli, or directions in the field space.
- This number at any neighborhood in the landscape may be taken as the dimension d of the landscape around that neighborhood.
- The landscape potential is not periodic (though it may be close to periodic along some axion directions). It is very complicated.
- Let us treat it as a random potential.

a cartoon :



Landscape : looks like
a random potential in multi-dimensions

The wavefunction of the universe

may be crudely approximated by that of a D3-brane

moduli open string modes

$$\Psi(a, \ldots, \varphi_j, \phi_i, \ldots)$$

cosmic scale factor

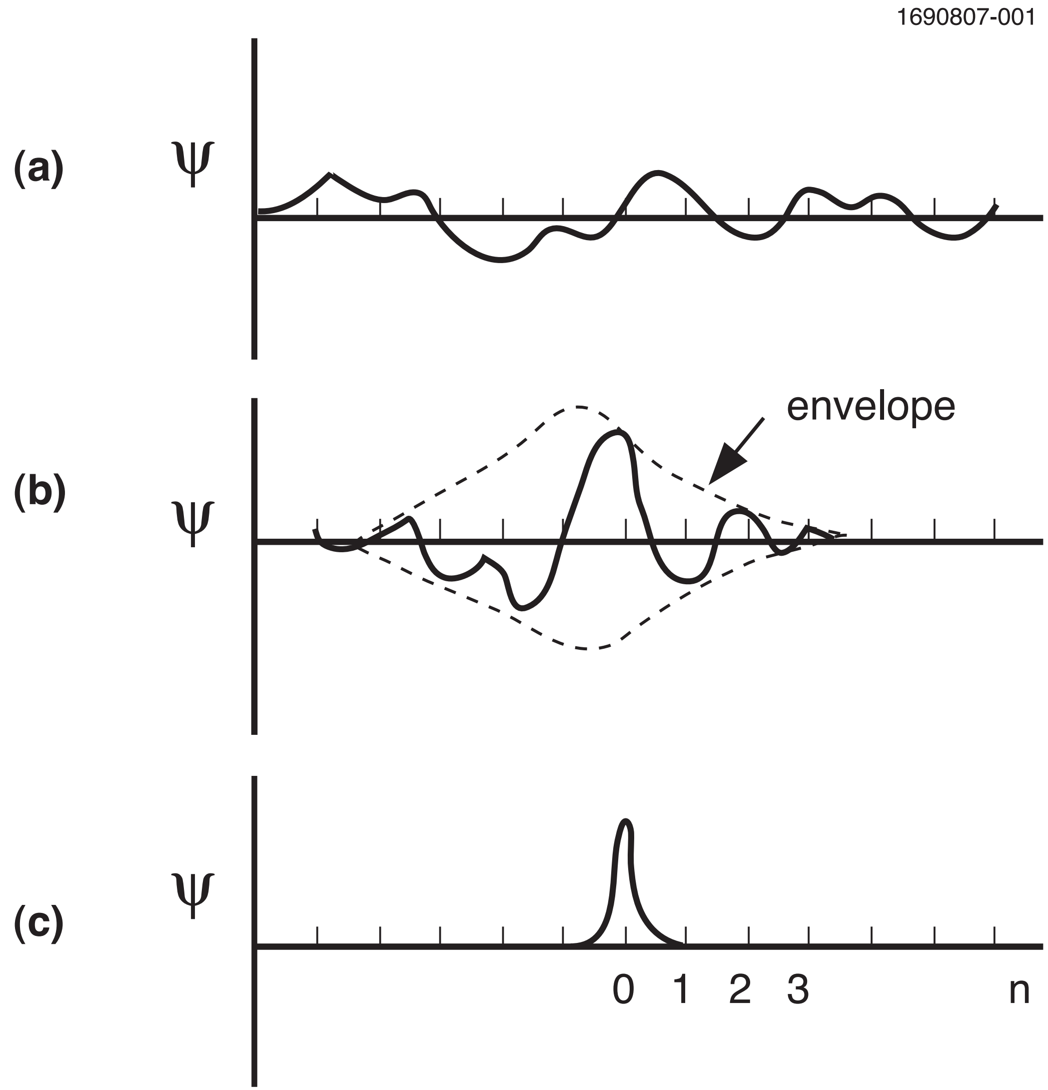
The behavior of the wavefunction in the landscape is a quantum diffusion and percolation problem.

Anderson localization transition

- random potential/disorder medium
- insulation-superconductivity transition
- quantum mesoscopic systems
- conductivity-insulation in disordered systems
- percolation
- strongly interacting electronic systems
- doped systems, alloys, ...

Some references :

- P.W. Anderson, *Absence of Diffusion in Certain Random Lattices*, Phys. Rev. Lett. 109, 1492 (1958).
- E. Abrahams, P.W. Anderson, D. C. Licciardello and T.V. Ramakrishnan, *Scaling Theory of Localization : Absence of Quantum Diffusion in Two Dimensions*, Phys. Rev. Lett. 42, 673 (1979).
- B. Shapiro, *Renormalization-Group Transformation for Anderson Transition*, Phys. Rev. Lett. 48, 823 (1982).
- P. A. Lee and T.V. Ramakrishnan, Disordered electronic systems, Rev. Mod. Phys. 57, 287 (1985).
- M.V. Sadovskii, Superconductivity and Localization," (World Scientific, 2000).
- Les Houches 1994, Mesoscopic Quantum Physics.



$$|\psi(\mathbf{r})| \sim \exp(-|\mathbf{r}-\mathbf{r}_0|/\xi)$$

$$\Gamma_0 \sim |\psi(a)|^2 \sim e^{-2a/\xi}$$

Define a dimensionless conductance g in a d-dim. hypercubic region of size L

$$g_d(L) = \sigma L^{d-2}$$

conductivity

$$d = 3, \ g = \sigma(\text{Area})/L \sim \sigma L$$

Thouless $$g(L) \sim (L/a)^{(d-2)}$$

Conducting/mobile (metalic) with finite conductivity

Conductance = Mobileness
Conductivity = Mobility

How does g scale ?
Given g at scale a, what is g at scale L as L becomes large ?

$$g(a) \sim |\psi(a)| \sim e^{-a/\xi}$$

$$g_d(L) \sim e^{-L/\xi}$$

Insulating, localized, trapped, eternal inflation

zero conductivity

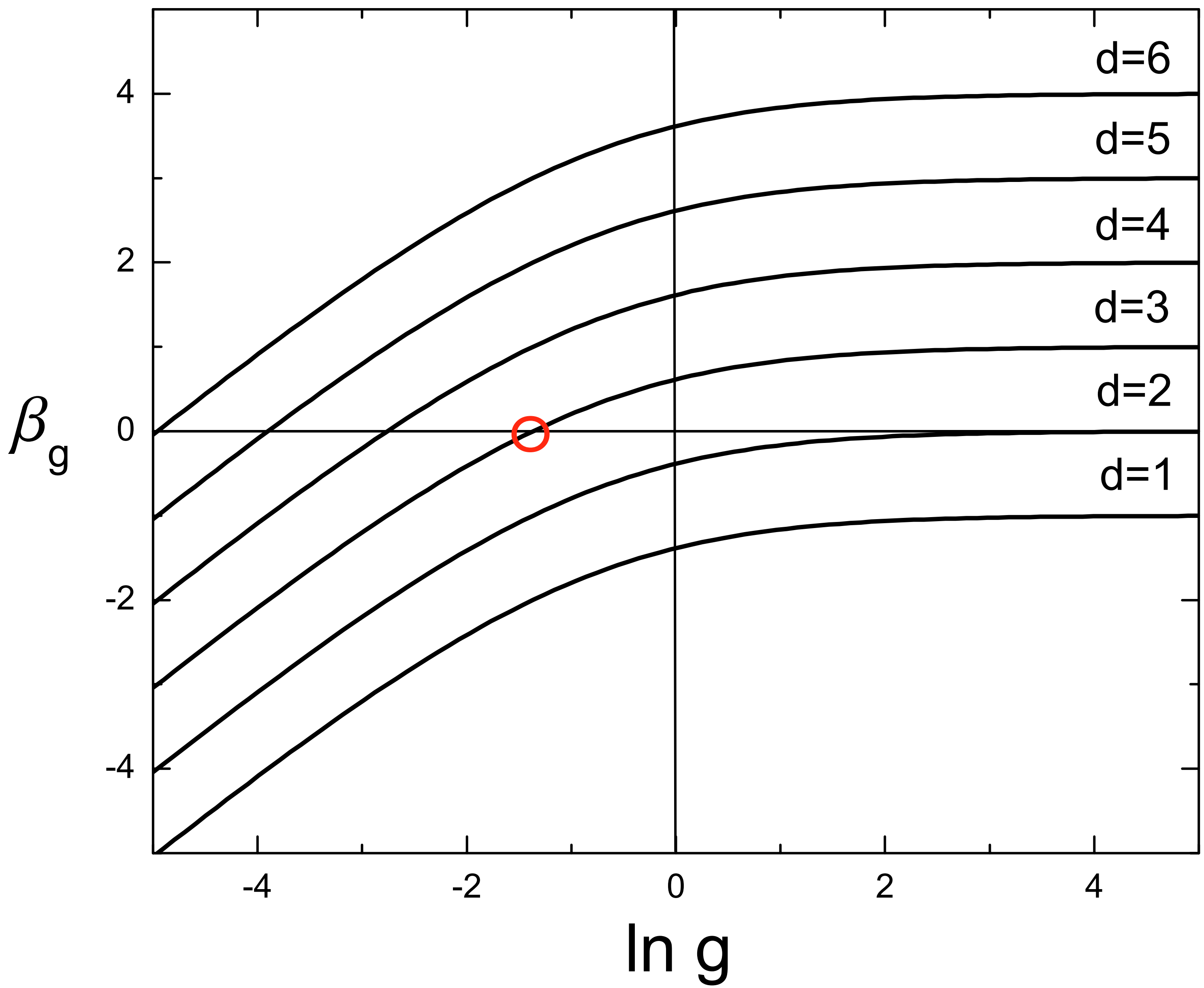
$$g_d(L) = \sigma L^{d-2}$$

Conducting, mobile

$$\beta_d(g_d(L)) = \frac{d\ln g_d(L)}{d\ln L}$$

$$\begin{array}{ccc} g_d(L) \sim e^{-L/\xi} & \qquad & g_d(L) = \sigma L^{d-2} \\ \ln g \sim -L = -e^{\ln L} \quad \Big\downarrow & & \Big\downarrow \\ \lim\limits_{g\to 0} \beta_d(g) \to \ln\dfrac{g}{g_c} & & \lim\limits_{g\to\infty} \beta_d(g) \to d-2 \end{array}$$

$$\beta_d(g_d(L)) = \frac{d \ln g_d(L)}{d \ln L}$$



$$\beta_d(g_c) = 0$$

$$\beta_d(g) \approx \frac{1}{\nu} \ln \frac{g}{g_c} \approx \frac{1}{\nu} \frac{g - g_c}{g_c}$$

this zero of $\beta_d(g)$ corresponds to an unstable fixed point

$g(a) < g_c$

$\downarrow$

$g_d(L) \sim e^{-L/\xi}$

$\downarrow$

Insulating, localized, trapped, eternal inflation

$g(a) > g_c$

$\downarrow$

$g_d(L) = \sigma L^{d-2}$

$\downarrow$

Conducting, mobile

What is the critical g_c ?

$$\Delta \ln g_c = \ln g_c(d) - \ln g_c(d+1) = k > 0$$

$$g_c(d) \simeq e^{-(d-3)k} g_c(3)$$

Shapiro :

$$\beta_d(g) = (d-1) - (g+1)\ln(1+1/g)$$

$$g_c = e^{-(d-1)}$$

$$\nu \to 1$$

d=1 :

Anderson, Thouless, Abraham, Fisher, Phys. Rev. B 22, 3519 (1980)

Condition for mobility

$$g_c = e^{-(d-1)} \qquad g(a) \sim |\psi(a)| \sim e^{-a/\xi}$$

$$d > \frac{a}{\xi} + 1$$

$$\Gamma_0 \sim |\psi(a)|^2 \sim e^{-2a/\xi}$$

d ~ 100

Generic decay rate to nearest neighbor in the landscape :

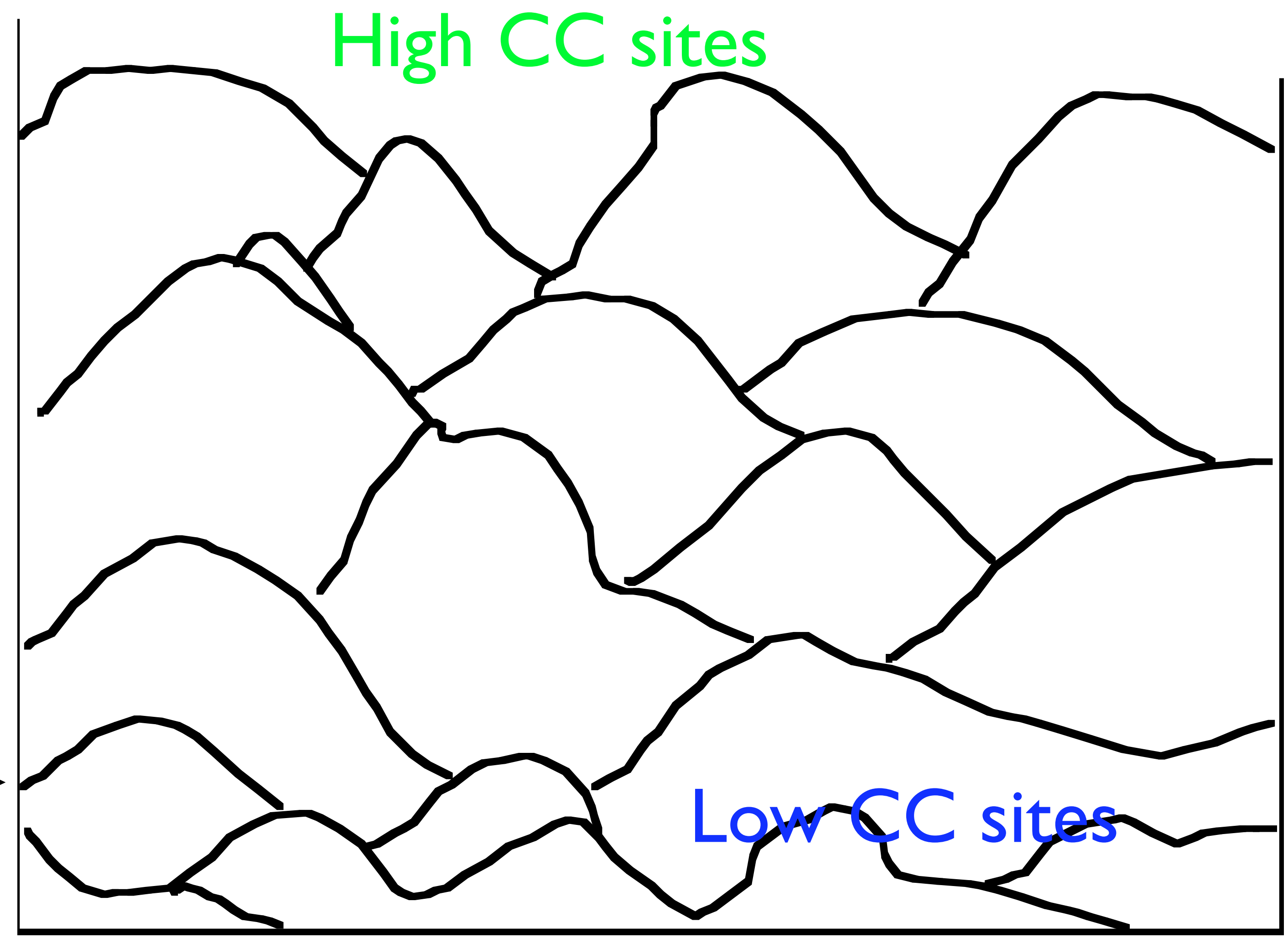
$$\Gamma_0 > e^{-2(d-1)}$$

Mobile

The picture from the scaling theory

mobile

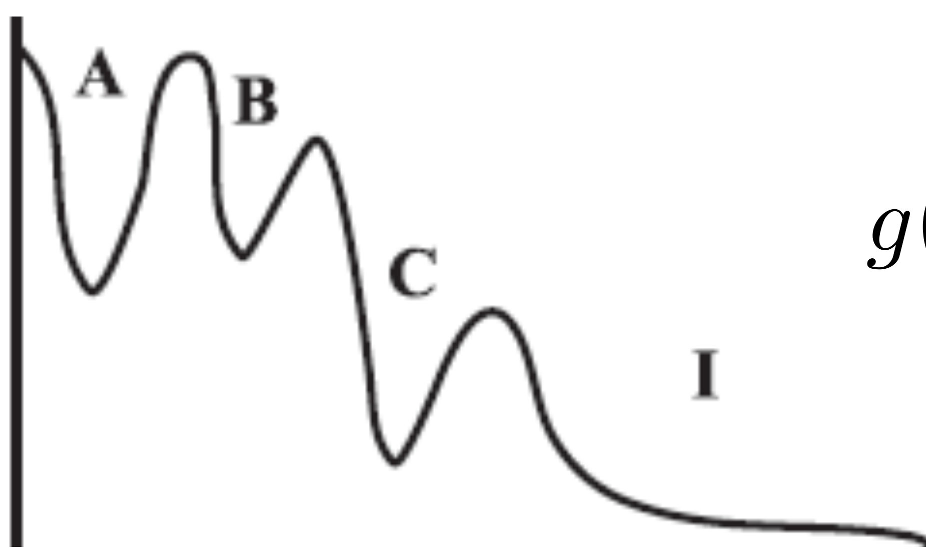
Sharp transition



trapped,
exponentially
long lifetimes

To estimate the transition CC in the cosmic landscape:

- Tunneling from a positive CC site to a negative CC site is ignored (CDL crunch).
- Tunneling from a dS site to another dS site with a larger CC is ignored.



$$g(a) \sim |\psi(a)| \sim e^{-a/\xi}$$

Estimate of critical C.C.

assume a random distribution : $\sim \Lambda^{q-1}$

fraction of sites : $f(\Lambda) = (\Lambda/\Lambda_s)^q$

$$N(\Lambda) = f(\Lambda) N_T = f(\Lambda) \left(\frac{L}{s(\Lambda_s)}\right)^d = \left(\frac{L}{s(\Lambda)}\right)^d$$

$$d = s(\Lambda_c)/\xi + 1$$

$$s(\Lambda) = s(\Lambda_s) \left(\frac{\Lambda}{\Lambda_s}\right)^{-q/d}$$

$$\xi \sim s(\Lambda_s) \sim \frac{1}{m_s} \sim \Lambda_s^{-1/4}$$

For flat distribution : $\Lambda_c \sim d^{-d} M_s^4$

$$d > 60$$

Is there eternal inflation ?

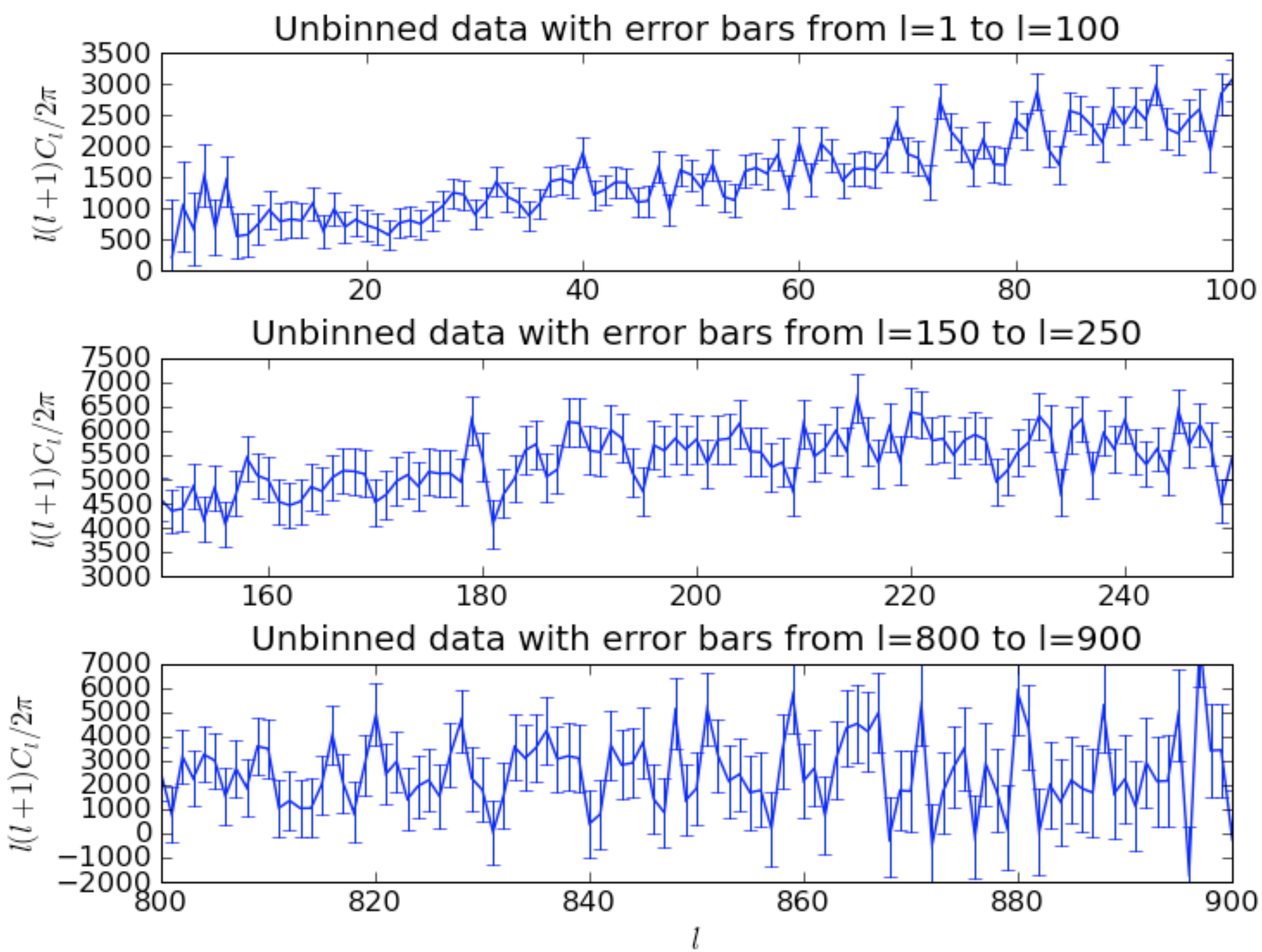
- Gibbons-Hawking temperature makes it harder to trap the wave function of the universe at a classical local vacuum site in the landscape.
- The presence of many moduli (tens to hundreds) makes it harder to trap the wavefunction. Furthermore, the moduli-axion potential has many low barrier directions.
- The vastness of the landscape (both in the number of moduli-axions and in the number of metastable sites) enhances the decay of any metastable site.
- Treating the cosmic landscape as a random potential, one can borrow the RG techniques developed in condensed matter physics to argue that there is no eternal inflation in the landscape, until the CC is exponentially small.

Inflation in the landscape

- Inflation takes place while the brane is moving in the landscape (and in the bulk).
- It rolls, scatters, percolates (bouncing around), hops and tunnels, happens at a rate of 20 to 10,000 times per e-fold. So it is like "rapid old inflation", but may look like slow roll. $t_F \simeq 10^{-5} H^{-1}$
- It is like inflation with a random multi-dimensional potential. This will lead to random fluctuations in the CMBR Power spectrum.

Freese, Spolyar, Liu   Huang . . . . . Davoudiasal, Sarangi, Shiu
Sarangi, Shlaer, Shiu,   Podolsky, Majumder, Jokela   Chialva, Danielsson
Watson, Perry, Kane, Adams   H.T., Zhang, Xu

Treating the landscape as a multi-dimensional random potential, inflation in such a potential will lead to random oscillations in the CMBR power spectrum.



WMAP

HT, J. Xu, Y Zhang, ArXiv 0812.1944

Summary

- The universe is freely moving in the stringy landscape when the vacuum energy density is above a critical value. Because of mobility, there is probably no eternal inflation.
- When the universe drops below the critical C.C. value, it stays there. Its lifetime there is exponentially long.
- The critical C.C. value is exponentially small compared to the string/Planck scale.
- This scenario suggests an inflationary scenario in the landscape, which hopefully leads to observable fluctuations in the CMBR power spectrum.