IPMU
9 April 2009

A Holographic Approach to QCD

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Quantum Chromodynamics

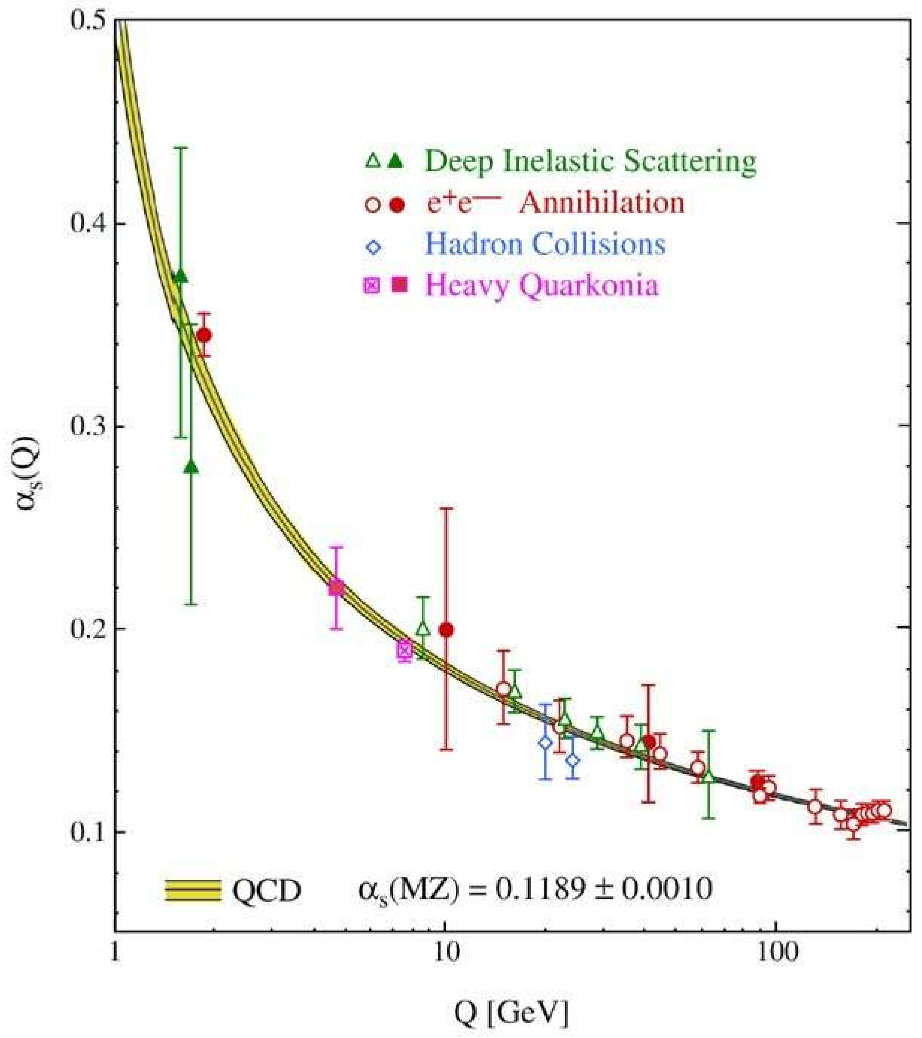
• Quantum Chromodynamics has been established as the correct theory of the strong interactions (in the past 35 years). Despite this, it is a theory beyond analytical and in many cases also numerical control

• It is described by a deceptively simple action:

$$S = \frac{1}{4g^2} Tr[F_{\mu\nu}F^{\mu\nu}] + \bar{q}_L(i\not{\partial} + \not{A})q_L + \bar{q}_R(i\not{\partial} + \not{A})q_R$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \quad \in \quad SU(3)_{\text{color}}$$

• Even in the absence of quarks the theory has defied analytical treatment so far.



• RG analysis indicates that the effective coupling constant becomes large in the IR while it becomes weak in the UV

$$\frac{1}{g^2_{eff}(E)} = \frac{1}{g^2_{eff}(\Lambda)} + b_0 \ \log \frac{E^2}{\Lambda^2} + \cdots$$

Confinement

• The theory exhibits confinement of color and a mass gap (this is one of the seven millenium problems of the Clay Mathematics Institute. To-date no proof of confinement exists).

• The force is "short range", and color flux is confined into thin flux tubes.

$$V_{q\bar{q}}(r) = \sigma\ r + \frac{1}{r} + \cdots \quad , \quad \sigma \quad \to \quad \text{string} \quad \text{tension}$$

• Quarks are permanently confined into colorless hadrons:

♠ Mesons of the $\bar{q}q$ type (pions, Kaons etc.)

♠ Baryons of qqq type (protons, neutrons etc) and their antiparticles.

De-confinement

• It has been speculated since a long time that at high-temperature confinement will be lost and the quarks and gluons will be liberated.

Collins+Perry 1975

• The resulting state of matter was thought to be a (weakly coupled) plasma similar to that of EM plasmas. It was named Quark-Gluon-Plasma (QGP).

Shuryak, 1978

• A phase transition was expected to separate the confined from the deconfined phase in the pure gauge theory.

• It took twenty years of lattice simulations and many false paths to eventually reach a conclusion in the pure gauge theory: the transition is first order.

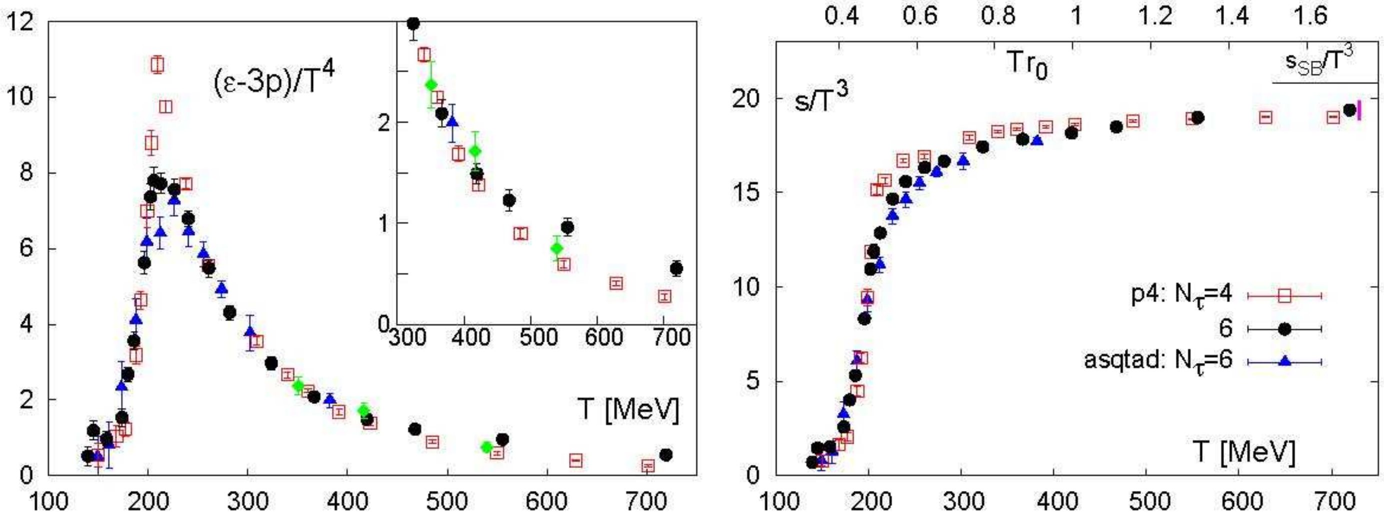


Figure 3: The trace anomaly, $\Theta^{\mu\mu}(T) \equiv \varepsilon - 3p$, in units of T^4 (left) and the entropy density, $s \equiv \varepsilon + p$, in units of T^3 calculated with the p4fat3 action [6] on lattices with temporal extent $N_\tau = 4$ and 6. For $\Theta^{\mu\mu}/T^4$ we also show results on $N_\tau = 8$ lattices (diamonds) obtained at high temperature. For $N_\tau = 6$ results from calculations with the asqtad action are also shown [5]. In the right hand figure we also show the temperature scale Tr_0 (upper x-axis) which has been obtained from an analysis of static quark potentials at zero temperature [6]. The MeV-scale shown on the lower x-axis has been extracted from this using $r_0 = 0.469$ fm.

F. Karsch, 2002

• It looks highly plausible that it is a crossover when quarks are added.

• QCD seems to have a complex phase diagram, most regions of which are unexplored and speculative.

The experimental hunt for QGP

- The energy density corresponding to the deconfinement transition is $E_c \sim 1\text{GeV}/\text{fm}^3$ (1 fm=10^{-15} m, radius of a proton $\simeq 0.8$ fm)

- The idea is to collide heavy-ion nuclei with the hope that they will create enough density and thermalize to probe the deconfined phase.

- The first attempt: 1 Gev/nucleon at LBL's Bevalac. No signals.

- Second attempt : AGS (Brookhaven) Si and Au nuclei on a fixed target. (5 GeV/nucleon in the collision rest frame) that was not enough!

- Third attempt : SPS (CERN). S and Pb nuclei were accelerated and collided on fixed target with 17 Gev/nucleon in the collision rest frame. That was still not enough!

- The CERN experiments after 15 years of running (in 2000) saw some hints of collective behavior beyond the known hadronic interactions.

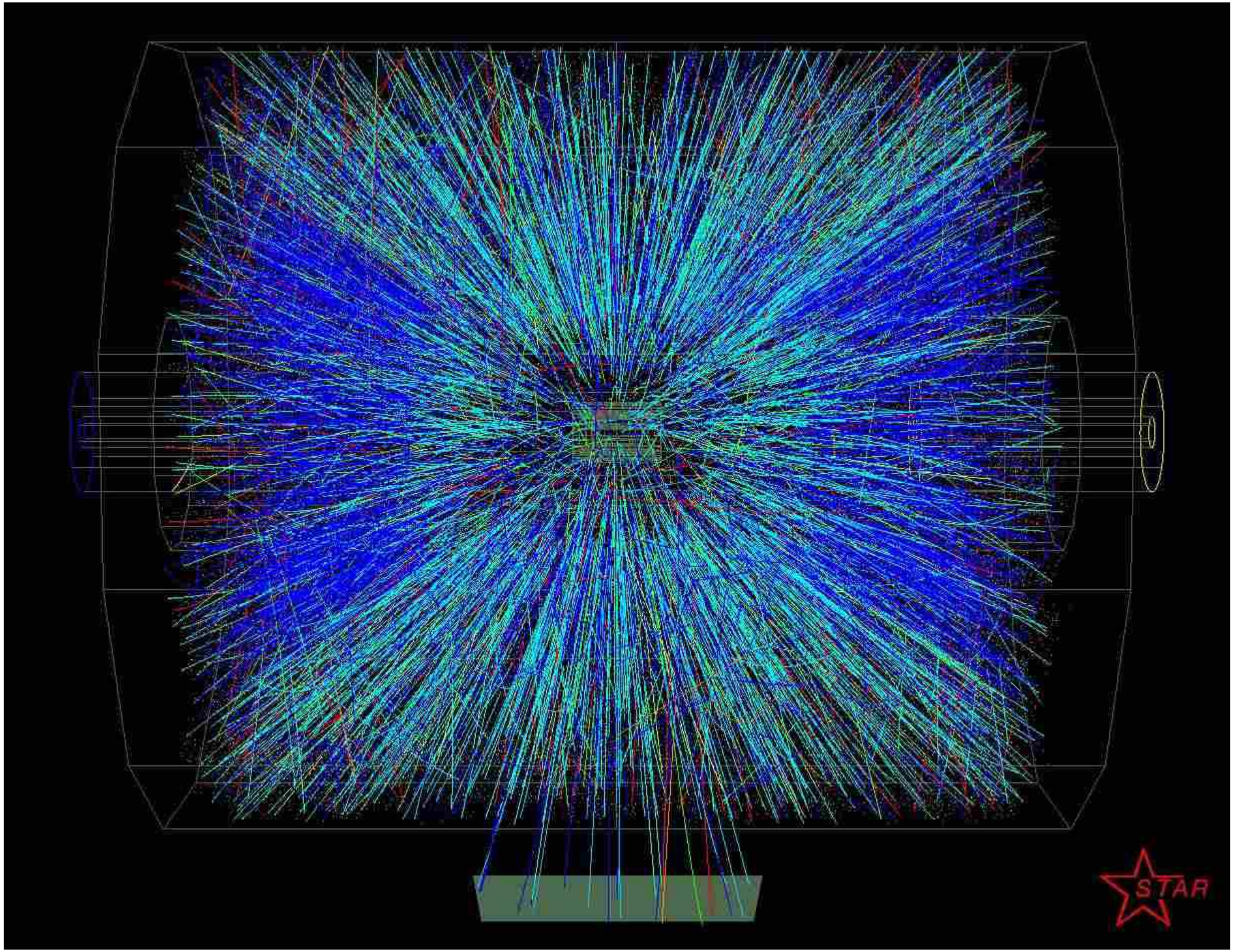
Relativistic Heavy Ion Collider (RHIC)

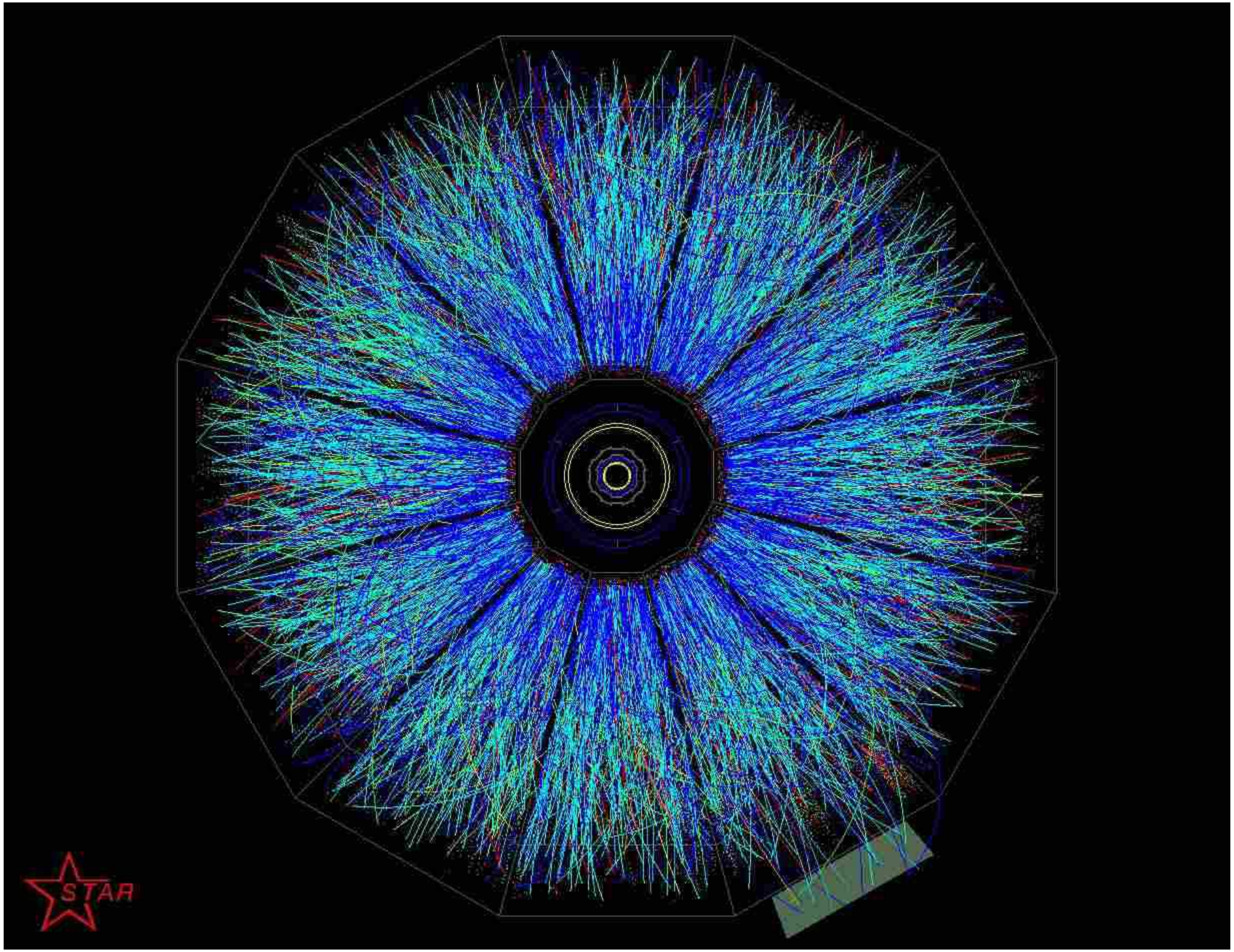
- The major breakthrough came at RHIC: two beams of Au or Cu nuclei colliding at 200 GeV/nucleon at the center-of-mass frame.

- Four experimental collaborations: BRAHMS, PHENIX, PHOBOS, STAR.

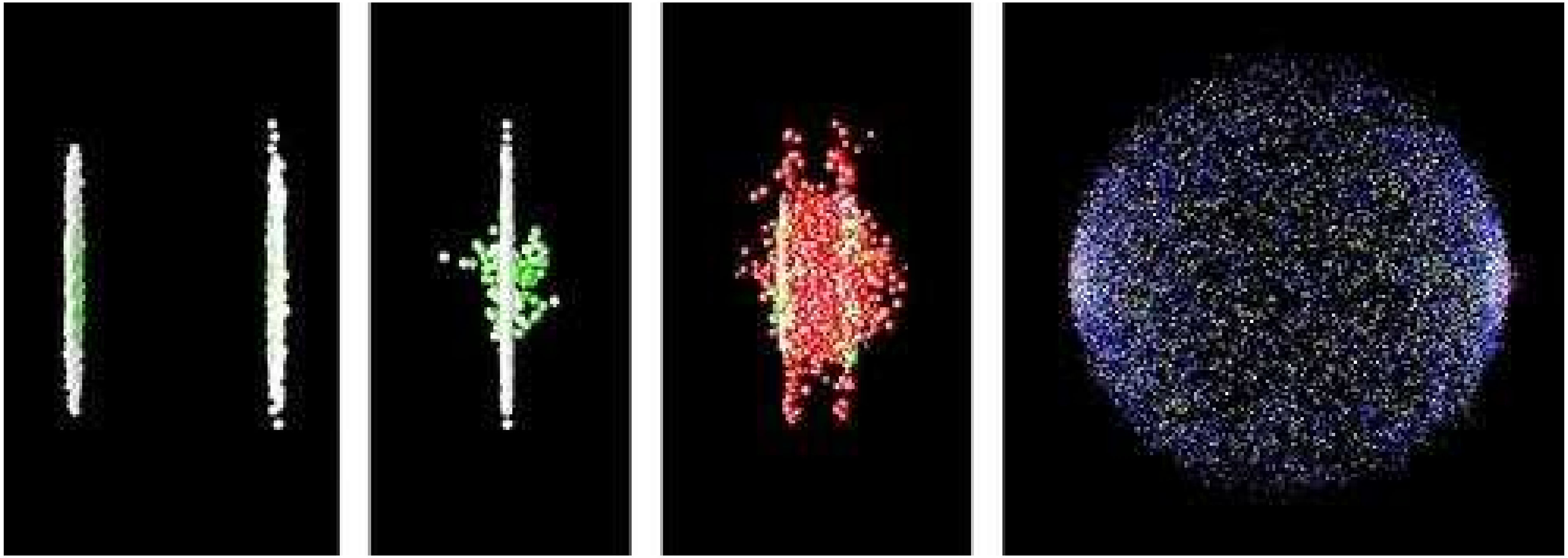
- For every almost central Au+Au collision we get about 7000 particles (fragments, most of them mesons).

RHIC head-on collision

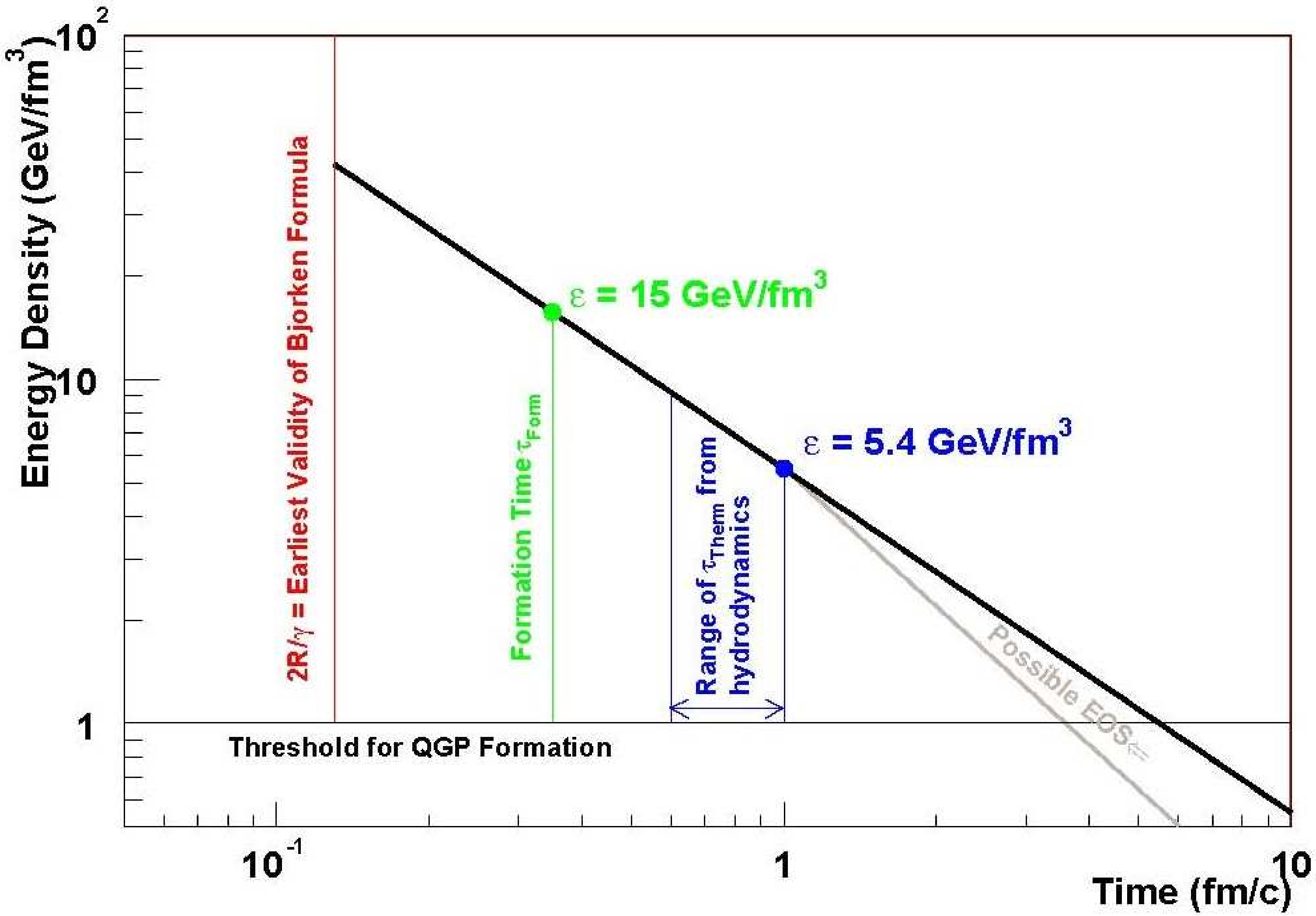



STAR

RHIC collision:another view



Phases of a collision



The "initial" energy density is given by the Bjorken formula

What we cannot calculate from first principles in QCD

- Observable rates for accelerator experiments. In particular, structure functions have to be measured. Hadronization is done by the Lund Monte Carlo model or the fragmentation model.

- Spectra for higher glueballs, mesons and baryons. Decay widths for all of the above.

- There are at least two weak matrix elements that cannot be computed so far reliably enough by lattice computations: The $\Delta I = \frac{1}{2}$ matrix elements of type $\langle K|\mathcal{O}_{\Delta I=1/2,3/2}|\pi\pi\rangle$, and the $B_K \sim \langle K|\mathcal{O}_{\Delta S=2}|\bar{K}\rangle$.

- Data associated to the chiral symmetry breaking (like the quark condensate), or its restauration at higher temperatures.

- In general matrix elements with at least two particle final states.

- Real time finite temperature correlation functions (associated to QGP dynamics)
  Also strong interactions between energetic quarks and the dense plasma.

- Nuclear interactions at low energy, and at finite chemical potential.

- Finite temperature physics at finite baryon density.

♠ Several complementary semi-phenomenological techniques have been developed to deal with the above (chiral perturbation theory, perturbation theory resummation schemes, SD equations, bag models, etc.) with varied success.

Gauge theories with many colors

• Gauge theories with N-colors (SU(N) gauge group) have a single continuous parameter: the gauge coupling constant g_{YM}.

• When N is large ($N \to \infty$) there is another way of reorganizing the theory:

't Hooft, 1974

$$N \to \infty \quad , \qquad \text{keep} \quad \lambda \equiv g_{YM}^2 N \quad \text{fixed}$$

• The expansion in powers of $1/N$ is similar to the topological expansion of a string theory with $g_{\text{string}} \sim \frac{1}{N}$

$$Z(\lambda, N) = \sum_{g=0}^{\infty} Z_g(\lambda)\; N^{2-2g} = N^2 \sum_{g=0}^{\infty} Z_g(\lambda)\; \frac{1}{N^{2g}}$$

• When $N \to \infty$ and $\lambda \to 0$ we can use perturbation theory to calculate.

• When $N \to \infty$ and λ is large, we are at strong coupling.

The gauge-theory/gravity duality

• The gauge-theory/gravity duality is a duality that relates a string theory with a (conformal) gauge theory.

• The prime example is the AdS/CFT correspondence

Maldacena 1997

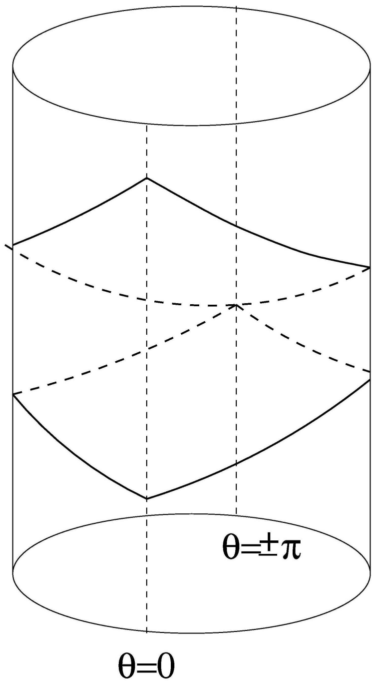
• It states that N=4 four-dimensional SU(N) gauge theory (gauge fields, 4 fermions, 6 scalars) is equivalent to ten-dimensional IIB string theory on $\mathrm{AdS}_5 \times S^5$

$$ds^2 = \frac{\ell^2_{AdS}}{r^2}\left[dr^2 + dx^\mu dx_\mu\right] + \ell^2_{AdS}\,(d\Omega_5)^2$$

This space (AdS_5) has a single boundary, at $r = 0$.

• The string theory has as parameters, g_{string}, ℓ_{string}, ℓ_{AdS}. They are related to the gauge theory parameters as

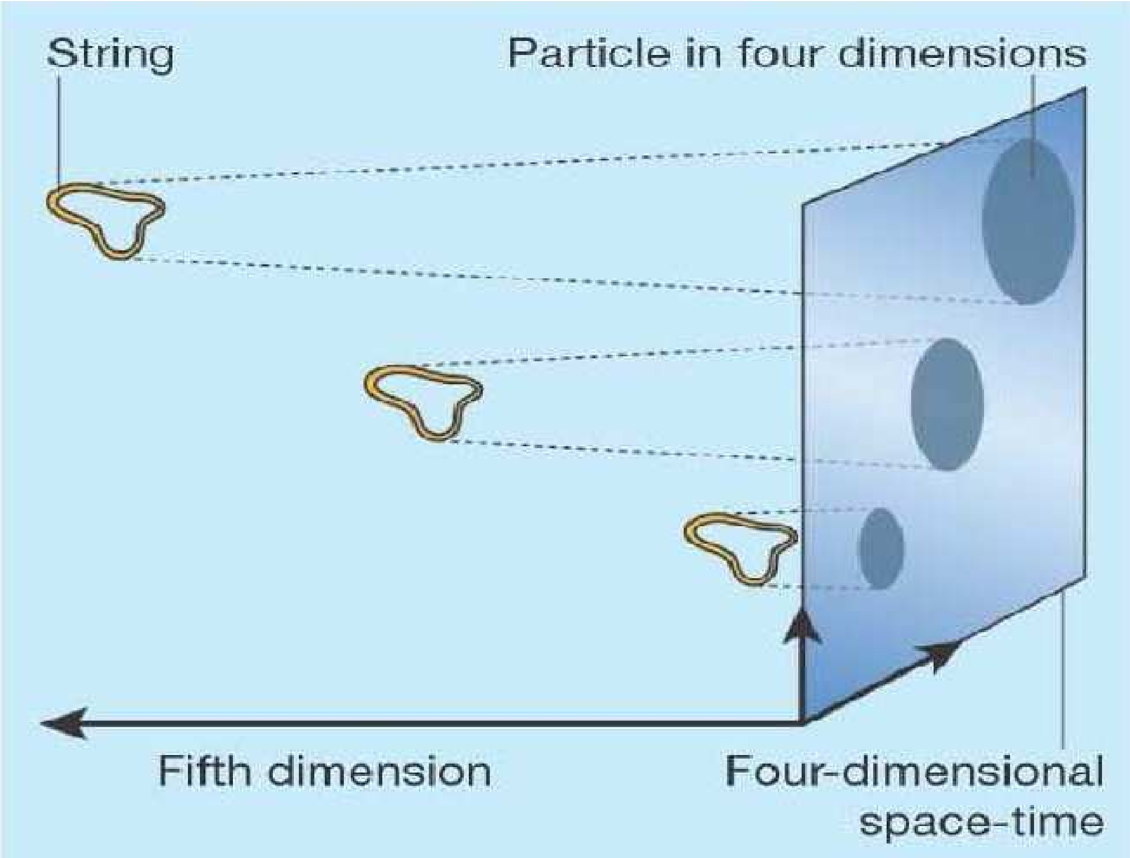
$$g_{YM}^2 = 4\pi\ g_{\text{string}} \quad , \quad \lambda = g_{YM}^2\ N = \frac{\ell_{AdS}^4}{\ell_{\text{string}}^4}$$



- As $N \to \infty$, $g_{\text{string}} \sim \frac{\lambda}{N} \to 0$.

- As $N \to \infty$, $\lambda \gg 1$ implies that $\ell_{\text{string}} \ll \ell_{AdS}$ and the geometry is very weakly curved. String theory can be approximated by gravity in that regime and is weakly coupled.

- As $N \to \infty$, $\lambda \ll 1$ the gauge theory is weakly coupled, but the string theory is strongly curved.



• There is one-to-one correspondence between on-shell string states $\Phi(r, x^\mu)$ and gauge-invariant (single-trace) operators $O(x^\mu)$ in the sYM theory

• In the string theory we can compute the "S-matrix" , $S(\phi(x^\mu))$ by studying the response of the system to boundary conditions $\Phi(r = 0, x^\mu) = \phi(x^\mu)$

• The correspondence states that this is equivalent to the generating function of c-correlators of O

$$\langle e^{\int d^4x \ \phi(x) \ O(x)} \rangle \ = \ e^{-S(\phi(x))}$$

The gauge-theory at finite temperature

• The finite temperature ground state of the gauge theory corresponds to a different solution in the dual string theory: the AdS-Black-hole solution

E. Witten, 1998

$$ds^2 = \frac{\ell_{AdS}^2}{r^2}\left[\frac{dr^2}{f(r)} + f(r)dt^2 + dx^i dx_i\right] + \ell_{AdS}^2\,(d\Omega_5)^2 \quad , \quad f(r) = 1 - (\pi T)^4 r^4$$

• The horizon is at $r = \frac{1}{\pi T}$

• The dynamics of low-energy gravitational fluctuations is governed by the relativistic Navier-Stokes equation.

A quick preview for the rest

- Although the AdS/CFT correspondence works well for N=4 sYM, for several issues it may not be close to to QCD.

- A direct fully-controlable holographic description of QCD is so far lacking (but several similar models exist, like D4-SS model)

- Bottom-up approaches have also been developed which use phenomenologically motivated holographic models

- Our approach is a hybrid between string theory description and gravity approximation, and results in a phenomenological model.

$$S = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V(\lambda) \right] + M^3 \int d^5x \sqrt{g} \; Z(\lambda) \; (\partial a)^2$$

- This model is capturing both asymptotic freedom and confinement in the IR.

- The adjustable parameters are included in the dilaton potential.

- With two adjustable parameters all known lattice data at zero and finite temperature can be accommodated (pure gauge theory)

- Many further predictions on spectra and transport coefficients.

End of Part I

Break

What are we after?

• Interactions of hadrons at medium or low energy (little or no help from lattice, partial help from chiral perturbation theory)

• Transport coefficients of the deconfined phase (not computable directly from lattice, crucial for understanding current (RHIC) and future (LHC) heavy-ion data)

• The phase structure and properties of dense matter (not computable from lattice, important for understanding properties of nuclei, and dense nuclear matter, like neutron stars)

• Exploring the strong dynamics of other QCD-like theories, eg.

♠ N=1 super- QCD. (a very interesting toy model and may be relevant for nature)

♠ Technicolor theories

A string theory for QCD:basic expectations

• Pure SU(N_c) d=4 YM at large N_c is expected to be dual to a string theory in 5 dimensions only. Essentially a single adjoint field $\rightarrow$ a single extra dimension.

• The theory becomes asymptotically free and conformal at high energy $\rightarrow$ we expect the classical saddle point solution to asymptote to AdS_5.

♠ Operators with lowest dimension (or better: lowest bulk masses) are expected to be the only important non-trivial bulk fields in the large-N_c saddle-point

• Scalar YM operators with $\Delta_{UV} > 4 \rightarrow m^2 > 0$ fields near the AdS_5 boundary $\rightarrow$ vanish fast in the UV regime and do not affect correlators of low-dimension operators.

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• Their dimension may grow large in the IR so they are also irrelevant there. The large 't Hooft coupling is expected to suppress the effects of such operators.

• This is suggested by the success of low-energy SVZ sum rules as compared to data.

♠ Therefore we will consider

$$T_{\mu\nu} \leftrightarrow g_{\mu\nu}, \; tr[F^2] \leftrightarrow \phi, \; tr[F \wedge F] \leftrightarrow a$$

bosonic string or superstring?

• The string theory must have no on-shell fermionic states at all because there are no gauge invariant fermionic operators in pure YM. (even with quarks modulo baryons).

• There is a direct argument that the axion, dual to the instanton density $F \wedge F$ must be a RR field (as in $\mathcal{N} = 4$).

• Therefore the string theory must be a 5d-superstring theory resembling the II-0 class.

♠ Another RR field we expect to have is the RR 4-form, as it is necessary to "seed" the D_3 branes responsible for the gauge group.

• It is non-propagating in 5D

• We will see later however that it is responsible for the non-trivial IR structure of the gauge theory vacuum.

The minimal effective string theory spectrum

• NS-NS $\rightarrow$ $g_{\mu\nu} \leftrightarrow T_{\mu\nu}$, $B_{\mu\nu} \leftrightarrow Tr[F]^3$, $\phi \leftrightarrow Tr[F^2]$

• RR $\rightarrow$ Spinor$_5\times$Spinor$_5$=$F_0 + F_1 + F_2 + (F_3 + F_4 + F_5)$

♠ $F_0 \leftrightarrow F_5 \rightarrow$ C_4, background flux $\rightarrow$ no propagating degrees of freedom.

♠ $F_1 \leftrightarrow F_4 \rightarrow$ $C_3 \leftrightarrow C_0$: C_0 is the axion, C_3 its 5d dual that couples to domain walls separating oblique confinement vacua.

♠ $F_2 \leftrightarrow F_3 \rightarrow$ $C_1 \leftrightarrow C_2$: They are associated with baryon number (as we will see later when we add flavor). C_2 mixes with B_2 because of the C_4 flux, and is massive.

• In an ISO(3,1) invariant vacuum solution, only $g_{\mu\nu}, \phi, C_0 = a$ can be non-trivial.

$$ds^2 = e^{2A(r)}(dr^2 + dx_4^2) \quad , \quad a(r), \phi(r)$$

The effective action, I

- as $N_c \to \infty$, only string tree-level is dominant.

- Relevant field for the vacuum solution: $g_{\mu\nu}, a, \phi, F_5$.

- The vev of $F_5 \sim N_c\ \epsilon_5$. It appears always in the combination $e^{2\phi}F_5^2 \sim \lambda^2$, with $\lambda \sim N_c\ e^{\phi}$ All higher derivative corrections $(e^{2\phi}F_5^2)^n$ are $\mathcal{O}(1)$. A non-trivial potential for the dilaton will be generated already at string tree-level.

- This is not the case for all other RR fields: in particular for the axion as $a \sim \mathcal{O}(1)$

$$(\partial a)^2 \sim \mathcal{O}(1) \quad , \quad e^{2\phi}(\partial a)^4 = \frac{\lambda^2}{N_c^2}(\partial a)^4 \sim \mathcal{O}\left(N_c^{-2}\right)$$

Therefore to leading order $\mathcal{O}(N_c^2)$ we can neglect the axion.

The UV regime

- In the far UV, the space should asymptote to AdS$_5$.

- The 't Hooft coupling should behave as ($r \to 0$)

$$\lambda \sim \frac{1}{\log(r\Lambda)} + \cdots \quad \to \quad 0 \quad , \quad r \sim \frac{1}{E}$$

The effective action to leading order in N_c is

$$S_{eff} \sim \int d^5x \sqrt{g}\; e^{-2\phi}\; Z(\; \ell_s^2 R \;,\; \ell_s^2 (\partial\phi)^2 \;,\; e^{2\phi}\ell_s^2 F_5^2 \;)$$

Solving the equation of motion of F_5 amounts to replacing

$$e^{2\phi}\; \ell_s^2\; F_5^2 \sim e^{2\phi} N_c^2 \equiv \lambda^2$$

$$S_{eff} \sim N_c^2 \int d^5x \sqrt{g}\; \frac{1}{\lambda^2}\; H(\; \ell_s^2 R \;,\; \ell_s^2 (\partial\lambda)^2 \;,\; \lambda^2 \;)$$

• As $r \to 0$

$$\text{Curvature} \to \text{finite} \quad , \quad \Box\phi \sim (\partial\phi)^2 \sim \frac{(\partial\lambda)^2}{\lambda^2} \sim \lambda^2 \sim \frac{1}{\log^2(r\Lambda)} \to 0$$

• For $\lambda \to 0$ the potential in the Einstein frame starts as $V(\lambda) \sim \lambda^{\frac{4}{3}}$ and cannot support the asymptotic AdS_5 solution.

• Therefore asymptotic AdS_5 must arise from curvature corrections:

$$S_{eff} \simeq \int d^5x \, \frac{1}{\lambda^2} \, H\left(\ell_s^2 \, R, 0, 0\right)$$

• Setting $\lambda = 0$ at leading order we can generically get an AdS_5 solution coming from balancing the higher curvature corrections.

INTERESTING QUESTION: Is there a good toy example of string vacuum (CFT) which is not Ricci flat, and is supported only by a metric?

• There is a "good" (but hard to derive the coefficients) perturbative expansion around this asymptotic AdS_5 solution by perturbing inwards :

$$e^A = \frac{\ell}{r}[1+\delta A(r)] \quad , \quad \lambda = \frac{1}{b_0 \log(r\Lambda)} + \cdots$$

• This turns out to be a regular expansion of the solution in powers of

$$\frac{P_n(\log\log(r\Lambda))}{(log(r\Lambda))^{-n}}$$

• Effectively this can be rearranged as a "perturbative" expansion in $\lambda(r)$. In the case of running coupling, the radial coordinate can be substituted by $\lambda(r)$.

• Using λ as a radial coordinate the solution for the metric can be written

$$E \equiv e^A = \frac{\ell}{r(\lambda)}\left[1 + c_1\lambda + c_2\lambda^2 + \cdots\right] = \ell\,(e^{-\frac{b_0}{\lambda}})\left[1 + c_1'\lambda + c_2'\lambda^2 + \cdots\right] \quad , \quad \lambda \to 0$$

.

Conclusion 1: The asymptotic AdS_5 is stringy, but the rest of the geometry is "perturbative around the asymptotics". We cannot however do computations even if we know the structure.

Conclusion 2: It has been a mystery how can one get free field theory at the boundary. This is automatic here since all non-trivial connected correlators are proportional to positive powers of λ that vanishes in the UV.

The IR regime

• Here the situation is more obscure. The constraints/input will be: confinement, discreteness of the spectrum and mass gap.

• We do expect that $\lambda \to \infty$ (or becomes large) at the IR bottom.

• Intuition from N=4 and other 10d strongly coupled theories suggests that in this regime there should be an (approximate) two-derivative description of the physics.

• The simplest solution with this property is the linear dilaton solution with

$$\lambda \sim e^{Qr} \quad , \quad V(\lambda) \sim \delta c = 10 - D \quad \to \quad \text{constant} \quad , \quad R = 0$$

• This property persists with potentials $V(\lambda) \sim (\log \lambda)^P$. Moreover all such cases have confinement, a mass gap and a discrete spectrum (except the P=0 case).

• At the IR bottom (in the string frame) the scale factor vanishes, and 5D space becomes (asymptotically) flat.

Improved Holographic QCD: a model

The simplification in this model relies on writing down a two-derivative action

$$S_{\text{Einstein}} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V(\lambda) \right]$$

with a monotonic potential (no extrema).

$$\lim_{\lambda\to 0} V(\lambda) = \frac{12}{\ell^2}\left(1 + \sum_{n=1}^{\infty} c_n \lambda^n\right) \quad , \quad \lim_{\lambda\to\infty} V(\lambda) = \lambda^{\frac{4}{3}} \sqrt{\log\lambda} + \text{subleading}$$

- The small λ asymptotics "simulate" the UV expansion around AdS_5:

$$\frac{1}{\lambda} = -b_0 \log(r\Lambda) - \frac{b_1}{b_0} \log\left[-b_0 \log(r\Lambda)\right] + \cdots \quad , \quad e^A = \frac{\ell}{r}\left[1 + \frac{2}{9\log(r\Lambda)} + \cdots\right]$$

- There is a 1-1 correspondence between the YM β-function, $\beta(\lambda)$ and W:

$$\left(\frac{3}{4}\right)^3 V(\lambda) = W^2 - \left(\frac{3}{4}\right)^2 \left(\frac{\partial W}{\partial \log\lambda}\right)^2 \quad , \quad \beta(\lambda) = -\frac{9}{4}\lambda^2 \frac{d\log W(\lambda)}{d\lambda}$$

once a choice of energy is made (here $\log E = A_E$).

.

Not everything is perfect: There are some shortcomings localized at the UV

- The conformal anomaly (proportional to the curvature) is incorrect.

- Shear viscosity ratio is constant and equal to that of N=4 sYM.

(This is not expected to be a serious error in the experimentally interesting $T_c \leq T \leq 4T_c$ range.)

Both of the above need Riemann curvature corrections.

- We shall see that other observables can come out very well both at T=0 and finite T

An assessment of IR asymptotics

- We define the superpotential W as

$$V(\lambda) = \frac{4}{3}\lambda^2 \left(\frac{dW}{d\lambda}\right)^2 + \frac{64}{27}W^2$$

- We parameterize the UV ($\lambda \to 0$) and IR asymptotics ($\lambda \to \infty$) as

$$V(\lambda) = \frac{12}{\ell^2}\left[1 + \mathcal{O}(\lambda)\right] \quad , \quad V(\lambda) \sim V_\infty \lambda^Q (\log\lambda)^P$$

- All confining solutions have an IR singularity.

There are three types of solution for W:

- The "Good type" (single solution)

$$W(\lambda) \sim (\log\lambda)^{\frac{P}{2}}\ \lambda^{\frac{Q}{2}}$$

It leads to a "good" IR singularity, confinement, a mass gap, discrete spectrum of glueballs and screening of magnetic charges if

$$\frac{8}{3} > Q > \frac{4}{3} \quad or \quad Q = \frac{4}{3} \quad and \quad P > 0$$

.

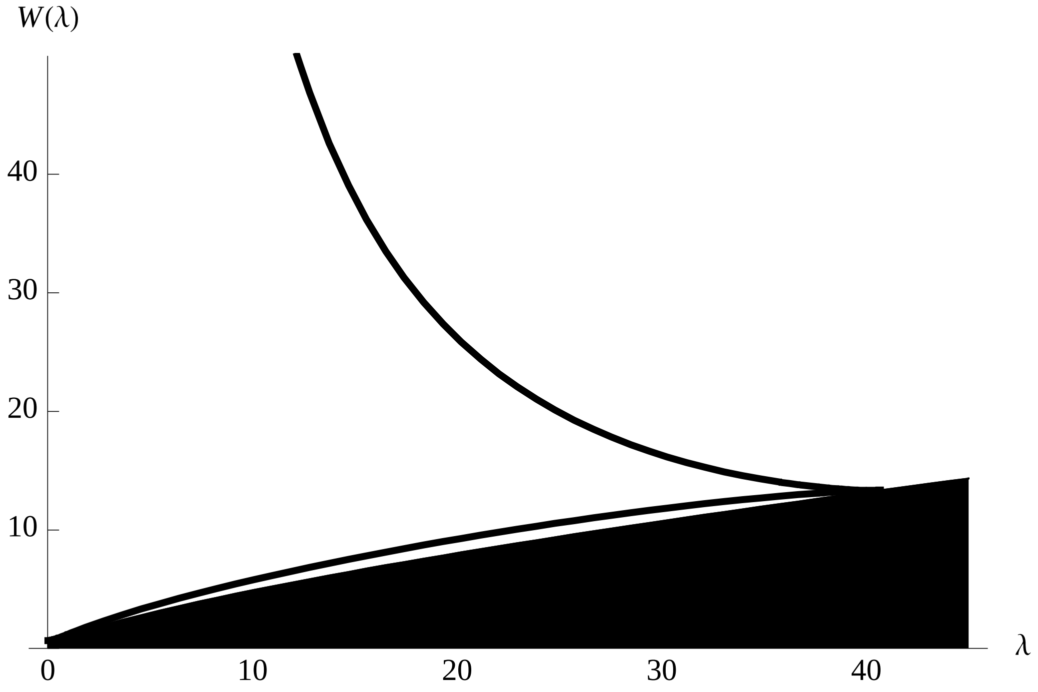
- The asymptotic spectrum of glueballs is linear if $Q = \frac{4}{3}$ and $P = \frac{1}{2}$.

- The Bad type. This is a one parameter family of solutions with

$$W(\lambda) \sim \lambda^{\frac{4}{3}}$$

It has a bad IR singularity.

♠ The Ugly type. This is a one parameter family of solutions. In such solutions there are two branches but they never reach the IR $\lambda \to \infty$. Instead λ goes back to zero



Selecting the IR asymptotics

The $Q = 4/3$, $0 \leq P < 1$ solutions have a singularity at $r = \infty$. They are compatible with

- Confinement (it happens non-trivially: a minimum in the string frame scale factor)
- Mass gap+discrete spectrum (except P=0)
- good singularity
- $R \to 0$ justifying the original assumption. More precisely: the string frame metric becomes flat at the IR .

♠ It is interesting that the lower endpoint: P=0 corresponds to linear dilaton and flat space (string frame). It is confining with a mass gap but continuous spectrum.

- For linear asymptotic trajectories for fluctuations (glueballs) we must choose $P = 1/2$

$$V(\lambda) =\sim \lambda^{\frac{4}{3}}\sqrt{\log\lambda} + \text{subleading} \qquad \text{as} \qquad \lambda \to \infty$$

Adding flavor

• To add N_f quarks q_L^I and antiquarks $q_R^{\bar{I}}$ we must add (in 5d) space-filling N_f D_4 and N_f $\bar{D}_4$ branes.
(tadpole cancellation=gauge anomaly cancellation)

• The q_L^I are the "zero modes" of the $D_3 - D_4$ strings while $q_R^{\bar{I}}$ are the "zero modes" of the $D_3 - \bar{D}_4$

• The low-lying fields on the D_4 branes ($D_4 - D_4$ strings) are U$(N_f)_L$ gauge fields A_μ^L. The low-lying fields on the $\bar{D}_4$ branes ($\bar{D}_4 - \bar{D}_4$ strings) are U$(N_f)_R$ gauge fields A_μ^R. They are dual to the J_L^μ and J_μ^R

$$\delta S_A \sim \bar{q}_L^I \ \gamma^\mu \ (A_\mu^L)^{IJ} \ q_L^J + \bar{q}_R^{\bar{I}} \ \gamma^\mu \ (A_\mu^R)^{\bar{I}\bar{J}} \ q_R^{\bar{J}} = Tr[J_L^\mu \ A_\mu^L + J_R^\mu \ A_\mu^R]$$

• There are also the low lying fields of the ($D_4 - \bar{D}_4$ strings), essentially the string-theory "tachyon" $T_{I\bar{J}}$ transforming as $(N_f, \bar{N}_f)$ under the chiral symmetry $U(N_f)_L \times U(N_f)_R$. It is dual to the quark mass terms

$$\delta S_T \sim \bar{q}_L^I \ T_{I\bar{J}} \ q_R^{\bar{J}} + \text{complex congugate}$$

• The interactions on the flavor branes are weak, so that $A_\mu^{L,R}, T$ are as sources for the quarks.

• Integrating out the quarks, generates an effective action $S_{flavor}(A_\mu^{L,R}, T)$, so that $A_\mu^{L,R}, T$ can be thought as effective $q\bar{q}$ composites, that is : mesons

• On the string theory side: integrating out $D_3 - D_4$ and $D_3 - \bar{D}_4$ strings gives rise to the DBI action for the $D_4 - \bar{D}_4$ branes in the D_3 background:

$$S_{flavor}(A_\mu^{L,R}, T) \quad \longleftrightarrow \quad S_{DBI}(A_\mu^{L,R}, T) \quad \text{holographically}$$

• In the "vacuum" only T can have a non-trivial profile: $T^{I\bar{J}}(r)$. Near the AdS_5 boundary ($r \to 0$)

$$T^{I\bar{J}}(r) = M_{I\bar{J}}\, r + \cdots + \langle \bar{q}_L^I\, q_R^{\bar{J}} \rangle r^3 + \cdots$$

Casero+Kiritsis+Paredes

• A typical solution is T vanishing in the UV and $T \to \infty$ in the IR. At the point $r = r_*$ where $T = \infty$, the D_4 and $\bar{D}_4$ branes "fuse". The true vacuum is a brane that enters folds on itself and goes back to the boundary. A non-zero T breaks chiral symmetry.

• A GOR relation is satisfied (for an asymptotic AdS$_5$ space)

$$m_\pi^2 = -2\frac{m_q}{f_\pi^2}\ \langle\bar{q}q\rangle \quad , \quad m_q \to 0$$

• We can derive formulae for the anomalous divergences of flavor currents, when they are coupled to an external source.

• When $m_q = 0$, the meson spectrum contains N_f^2 massless pseudoscalars, the $U(N_f)_A$ Goldstone bosons.

• The WZ part of the flavor brane action gives the Adler-Bell-Jackiw $U(1)_A$ axial anomaly and an associated Stuckelberg mechanism gives an $O\left(\frac{N_f}{N_c}\right)$ mass to the would-be Goldstone boson η', in accordance with the Veneziano-Witten formula.

• Fluctuations around the T solution for $T, A_\mu^{L,R}$ give the spectra (and interactions) of various meson trajectories.

• Studying the spectrum of highly excited mesons, we find the expected property of linear confinement: $m_n^2 \sim n$.

• The detailed spectrum of mesons remains to be worked out

The axion background

- The axion action is down by $1/N_c^2$

$$S_{axion} = -\frac{M_p^3}{2}\int d^5x\sqrt{g}\ Z(\lambda)\ (\partial a)^2$$

$$\lim_{\lambda\to 0} Z(\lambda) = Z_0\left[1 + c_1\lambda + c_2\lambda^2 + \cdots\right] \quad , \quad \lim_{\lambda\to\infty} Z(\lambda) = c_a\lambda^4 + \cdots$$

- The equation of motion is

$$\ddot{a} + \left(3\dot{A} + \frac{\dot{Z}(\lambda)}{Z(\lambda)}\right)\dot{a} = 0 \quad \to \quad \dot{a} = \frac{C\ e^{-3A}}{Z(\lambda)}$$

- The full solution is

$$a(r) = \theta_{UV} + 2\pi k + C\ \int_0^r dr\frac{e^{-3A}}{Z(\lambda)} \quad , \quad C = \langle Tr[F\wedge F]\rangle$$

- $a(r)$ is a running effective θ-angle. Its running is non-perturbative,

$$a(r) \sim r^4 \sim e^{-\frac{4}{b_0\lambda}}$$

- The vacuum energy is

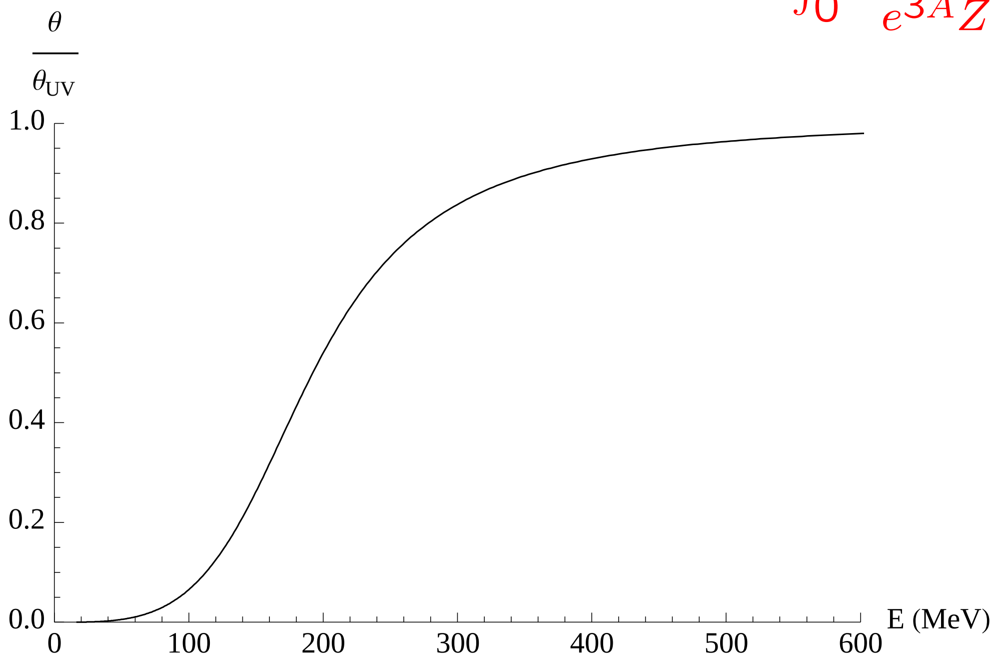
$$E(\theta_{UV}) = -\frac{M^3}{2N_c^2}\int d^5x\sqrt{g}\ Z(\lambda)\ (\partial a)^2 = -\frac{M^3}{2N_c^2}Ca(r)\ \Big|_{r=0}^{r=r_0}$$

- Consistency requires to impose that $a(r_0) = 0$. This determines C and

$$E(\theta_{UV}) = \frac{M^3}{2}\mathrm{Min}_k \frac{(\theta_{UV} + 2\pi k)^2}{\int_0^{r_0} \frac{dr}{e^{3A}Z(\lambda)}} \quad , \quad \frac{a(r)}{\theta_{UV} + 2\pi k} = \frac{\int_r^{r_0} \frac{dr}{e^{3A}Z(\lambda)}}{\int_0^{r_0} \frac{dr}{e^{3A}Z(\lambda)}}$$

- The topological susceptibility is given by

$$E(\theta) = \frac{1}{2}\chi\ \theta^2 + \mathcal{O}(\theta^4) \quad , \quad \chi = \frac{M_p^3}{\int_0^{r_0} \frac{dr}{e^{3A}Z(\lambda)}}$$



We take: $Z(\lambda) = Z_0(1 + c_a\lambda^4)$

Finite temperature

The theory at finite temperature can be described by:
(1) The "thermal vacuum solution". This is the zero-temperature solution we described so far with time periodically identified with period β.

(2) "black-hole" solutions

$$ds^2 = b(r)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^i dx^i \right], \qquad \lambda = \lambda(r)$$

♠ We need VERY UNUSUAL boundary conditions: The dilaton (scalar) is diverging at the boundary so that $\lambda \sim e^{\phi} \to \frac{1}{\log r} \to 0$

♠ The boundary AdS is NOT at a minimum of the potential.

• Such type of solutions have not been analyzed so far in the literature.

General phase structure

• For a general potential (with no minimum) the following can be shown :

i. *There exists a phase transition at finite $T = T_c$, if and only if the zero-T theory confines.*

ii. *This transition is of the first order for all of the confining geometries, with a single exception described in iii:*

iii. *In the limit confining geometry $b_0(r) \to e^{-Cr}$, $\lambda_0 \to e^{\frac{3}{2}Cr}$, (as $r \to \infty$), the phase transition is of the second order and happens at $T = 3C/4\pi$.*
This is the linear dilaton vacuum solution in the IR.

iv. *All of the non-confining geometries at zero T are always in the black hole phase at finite T. They exhibit a second order phase transition at $T = 0^+$.*

Finite-T Confining Theories

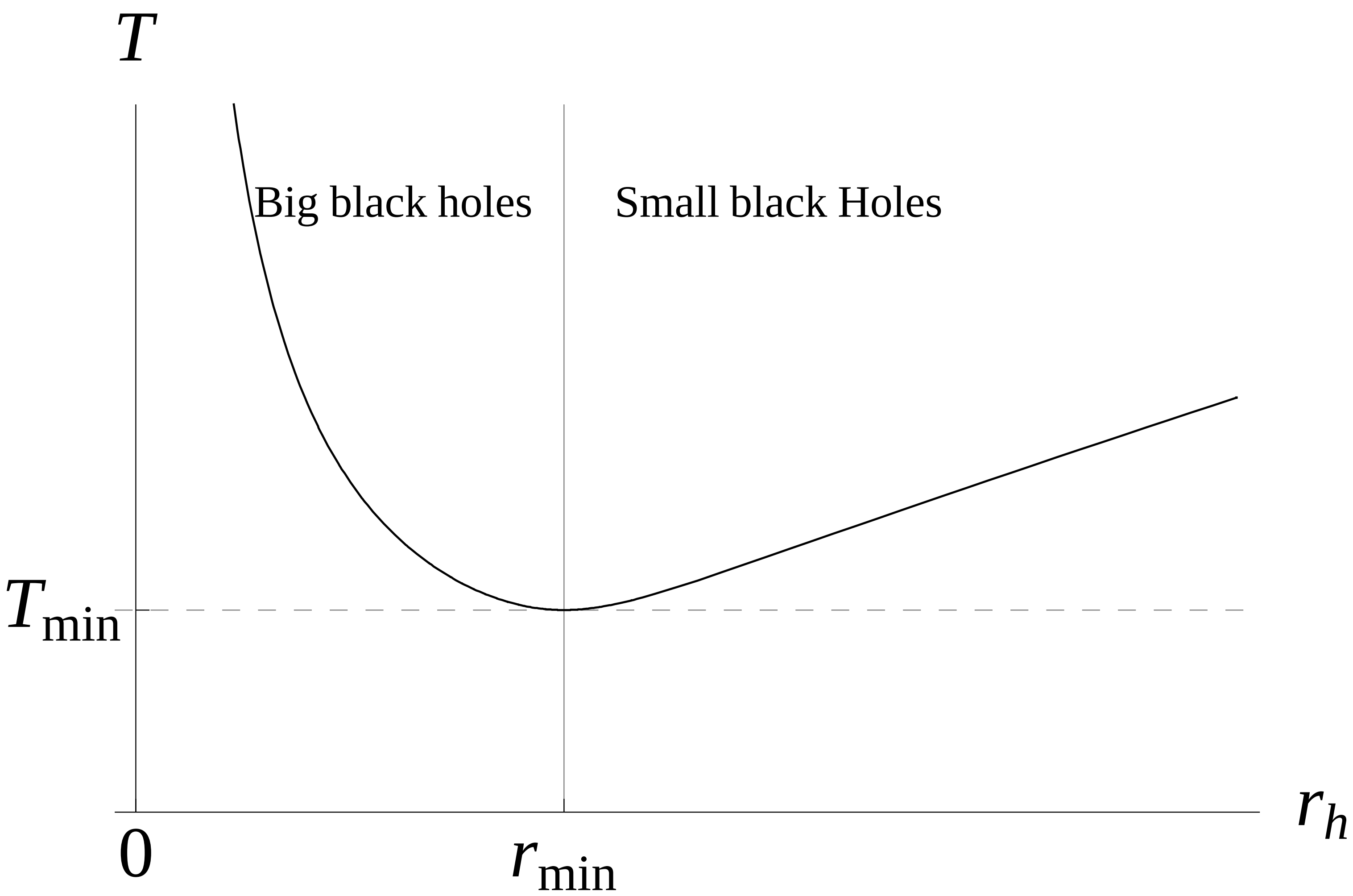
• There is a minimal temperature T_{min} for the existence of Black-hole solutions

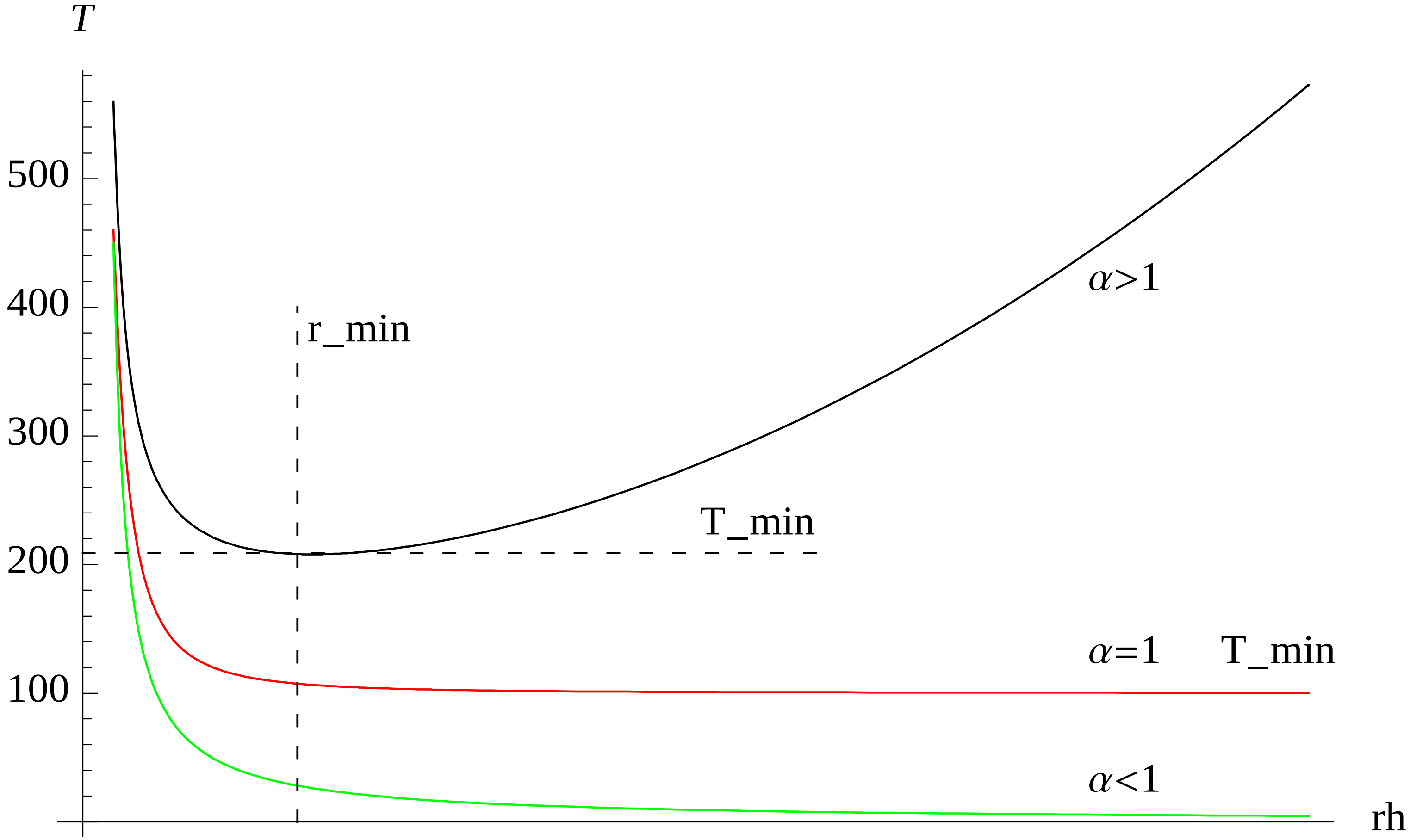
• When $T < T_{\text{min}}$ only the "thermal vacuum solution" exists: it describes the confined phase at small temperatures.

• For $T > T_{\text{min}}$ there are two black-hole solutions with the same temperature but different horizon positions. One is a "large" BH the other is "small".

• When $T > T_{\text{min}}$ three competing solutions exist. The large BH has the lowest free energy for $T > T_c > T_{min}$. It describes the deconfined "Gluon-Glass" phase.

Temperature versus horizon position





We plot the relation $T(r_h)$ for various potentials parameterized by a. $a = 1$ is the critical value below which there is only one branch of black-hole solutions.

The free energy

• The free energy is calculated from the action as a boundary term for both the black-holes and the thermal vacuum solution. They are all UV divergent but their differences are finite.

$$\frac{\mathcal{F}}{M_p^3 V_3} = 12\mathcal{G}(T) - T\ S(T)$$

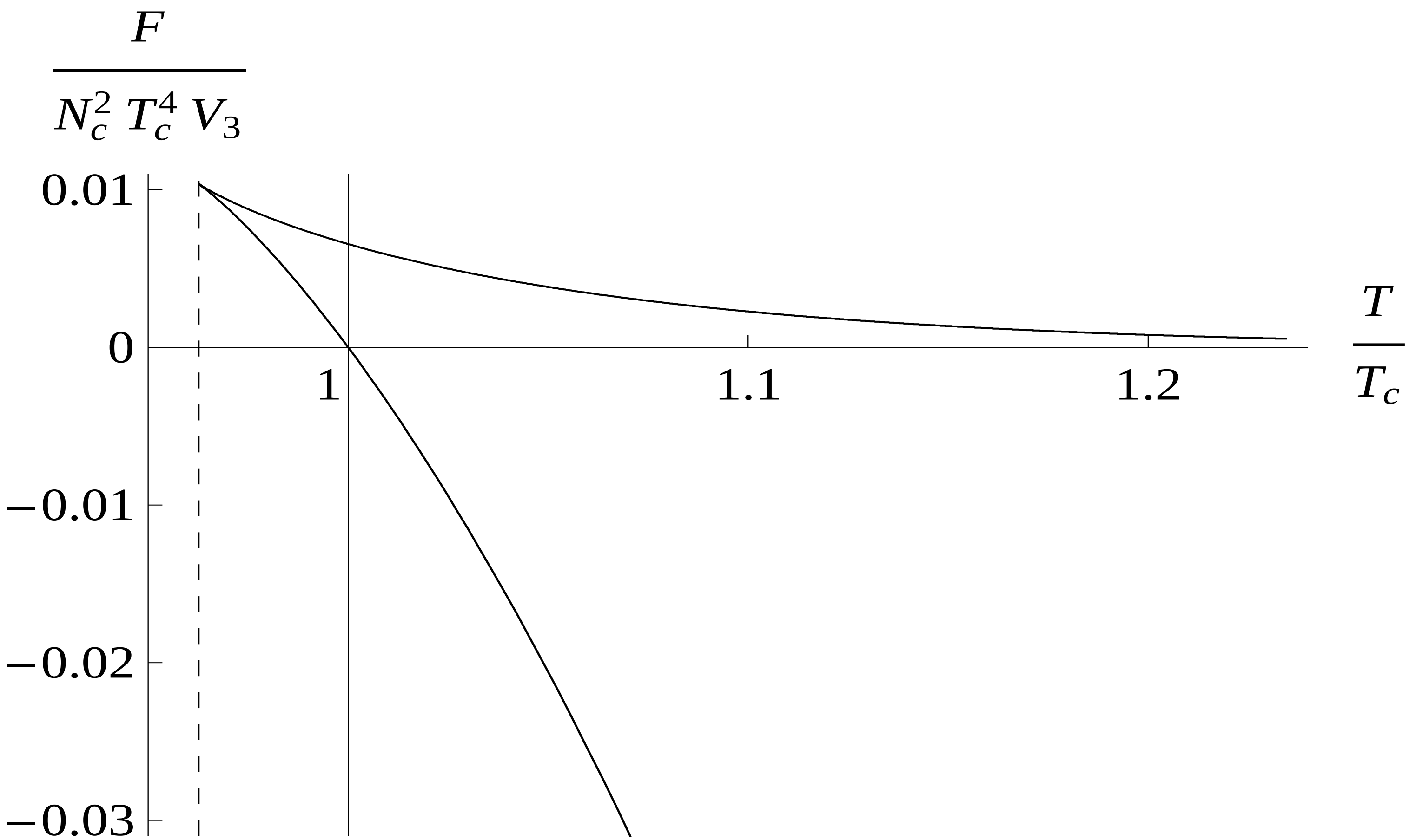
• $\mathcal{G}$ is the temperature-depended gluon condensate $\langle Tr[F^2]\rangle_T - \langle Tr[F^2]\rangle_{T=0}$ defined as

$$\lim_{r\to 0} \quad \lambda_T(r) - \lambda_{T=0}(r) = \mathcal{G}(T)\ r^4 + \cdots$$

• It is $\mathcal{G}$ the breaks conformal invariance essentially and leads to a non-trivial deconfining transition (as $S > 0$ always)

• The axion solution must be constant above the phase transition (black-hole). Therefore $\langle F \wedge F\rangle$ vanishes.

The transition in the free energy



Parameters

• We have 3 initial conditions in the system of graviton-dilaton equations:

♠ One is fixed by picking the branch that corresponds asymptotically to $\lambda \sim \frac{1}{\log(r\Lambda)}$

♠ The other fixes $\Lambda \to \Lambda_{QCD}$.

♠ The third is a gauge artifact as it corresponds to a choice of the origin of the radial coordinate.

• We parameterize the potential as

$$V(\lambda) = \frac{12}{\ell^2}\left\{1 + V_0\lambda + V_1\lambda^{4/3}\left[\log\left(1 + V_2\lambda^{4/3} + V_3\lambda^2\right)\right]^{1/2}\right\},$$

• We fix the one and two loop β-function coefficients:

$$V_0 = \frac{8}{9}b_0 \quad , \quad V_2 = b_0^4\left(\frac{23 + 36b_1/b_0^2}{81V_1^2}\right)^2, \quad \frac{b_1}{b_0^2} = \frac{51}{121}.$$

and remain with two leftover arbitrary (phenomenological) coefficients.

• We also have the Planck scale M_p

Asking for correct $T \to \infty$ thermodynamics (free gas) fixes

$$(M_p \ell)^3 = \frac{1}{45\pi^2} \quad , \quad M_{\text{physical}} = M_p N_c^{\frac{2}{3}} = \left(\frac{8}{45\pi^2 \ell^3}\right)^{\frac{1}{3}} \simeq 4.6 \text{ GeV}$$

• The fundamental string scale. It can be fixed by comparing with lattice string tension

$$\sigma = \frac{b^2(r_*)\lambda^{4/3}(r_*)}{2\pi \ell_s^2},$$

$\ell/\ell_s \sim \mathcal{O}(1)$.

• ℓ is not a parameter but a unit of length.

Fit and comparison

	HQCD	lattice $N_c = 3$	lattice $N_c \to \infty$	Parameter
$[p/(N_c^2 T^4)]_{T=2T_c}$	**1.2**	**1.2**	-	$V1 = 14$
$L_h/(N_c^2 T_c^4)$	**0.31**	0.28 (Karsch)	**0.31** (Teper+Lucini)	$V3 = 170$
$[p/(N_c^2 T^4)]_{T\to+\infty}$	$\pi^2/45$	$\pi^2/45$	$\pi^2/45$	$M_p \ell = [45\pi^2]^{-1/3}$
$m_{0^{++}}/\sqrt{\sigma}$	**3.37**	3.56 (Chen)	**3.37** (Teper+Lucini)	$\ell_s/\ell = 0.92$
$m_{0^{-+}}/m_{0^{++}}$	**1.49**	**1.49** (Chen)	-	$c_a = 0.26$
χ	$(191 MeV)^4$	$(191 MeV)^4$ (DelDebbio)	-	$Z_0 = 133$
$T_c/m_{0^{++}}$	0.167	-	0.177(7)	
$m_{0^{*++}}/m_{0^{++}}$	1.61	1.56(11)	1.90(17)	
$m_{2^{++}}/m_{0^{++}}$	1.36	1.40(4)	1.46(11)	
$m_{0^{*-+}}/m_{0^{++}}$	2.10	2.12(10)	-	

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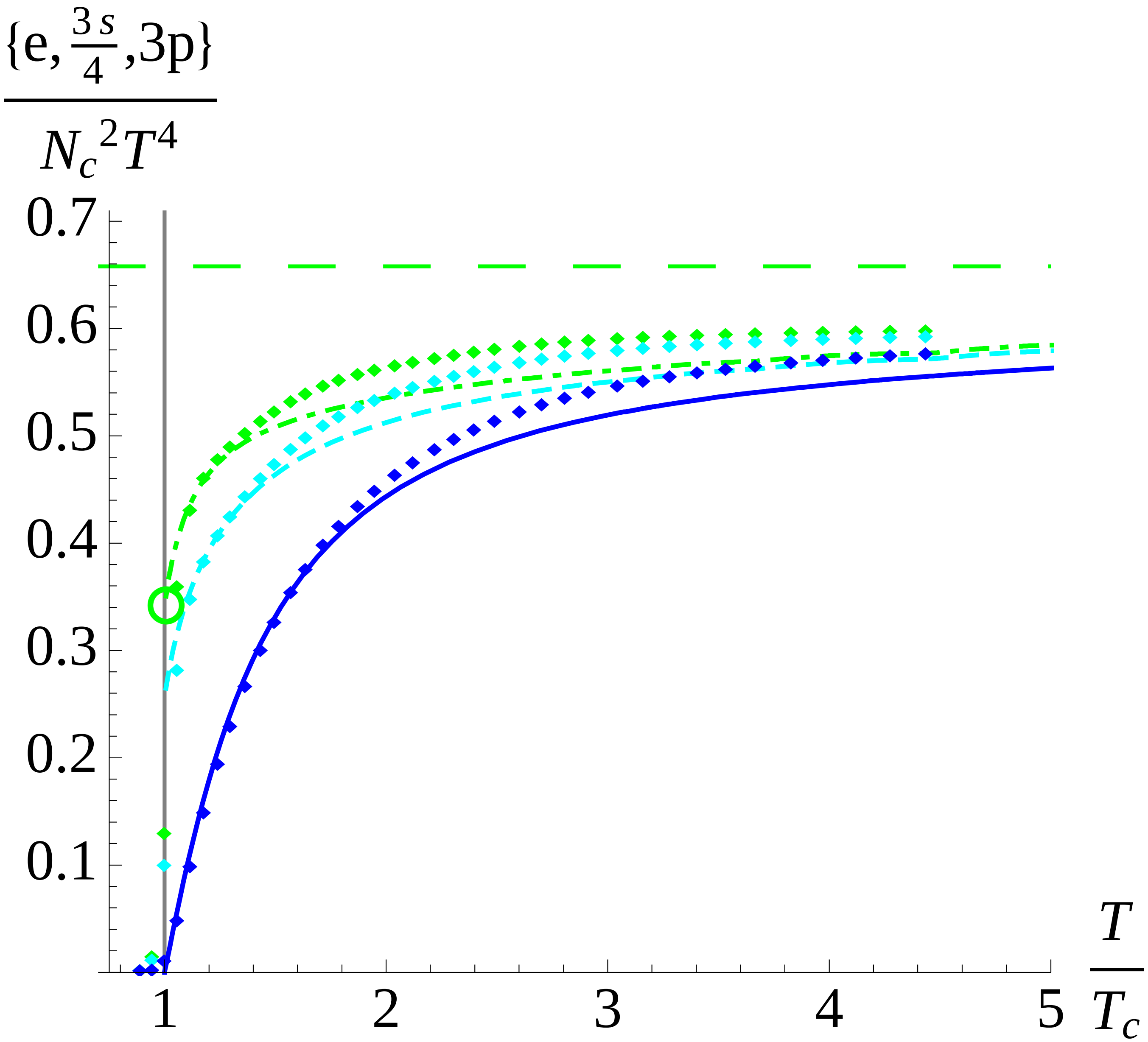
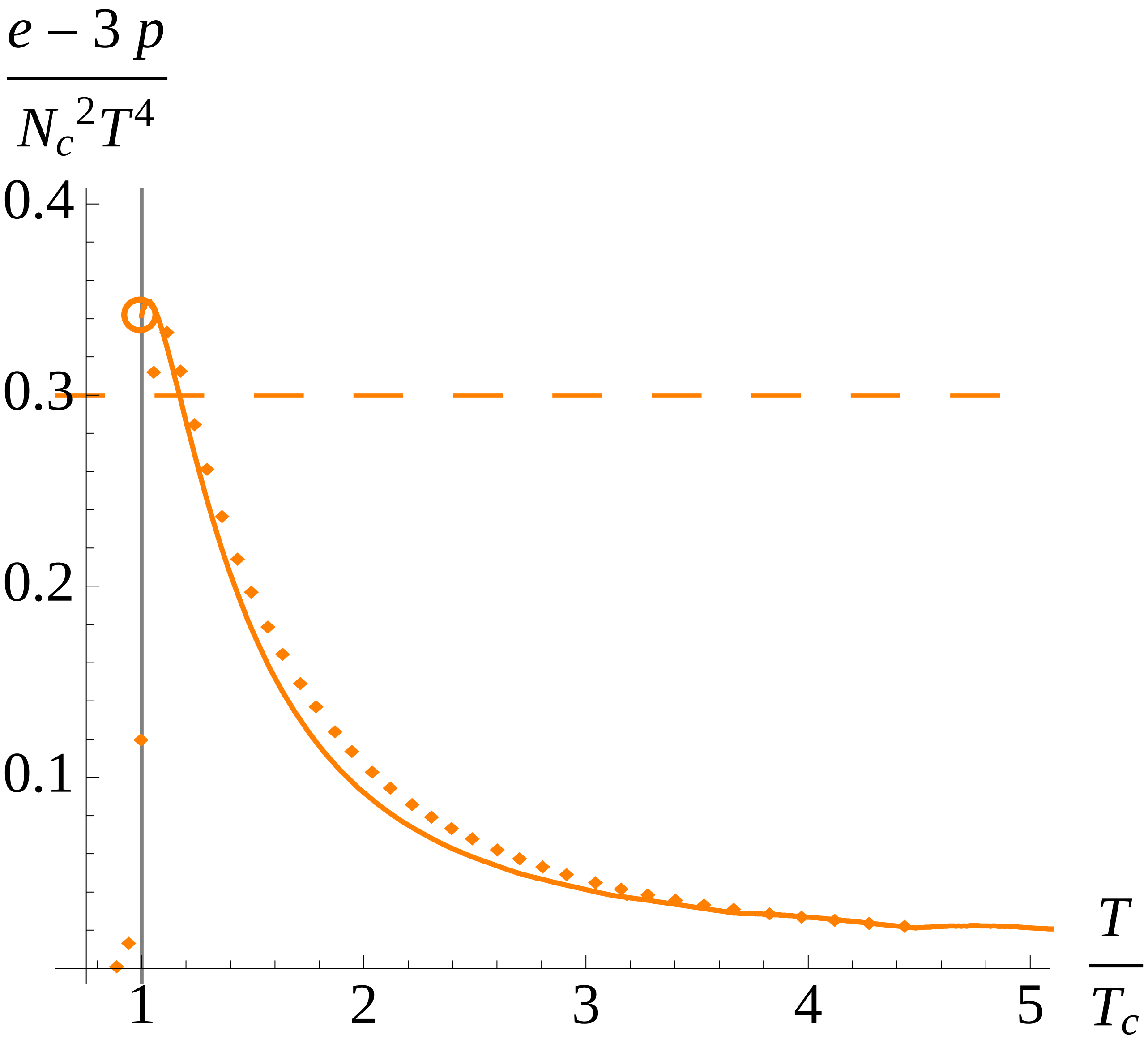
Thermodynamic variables



Equation of state



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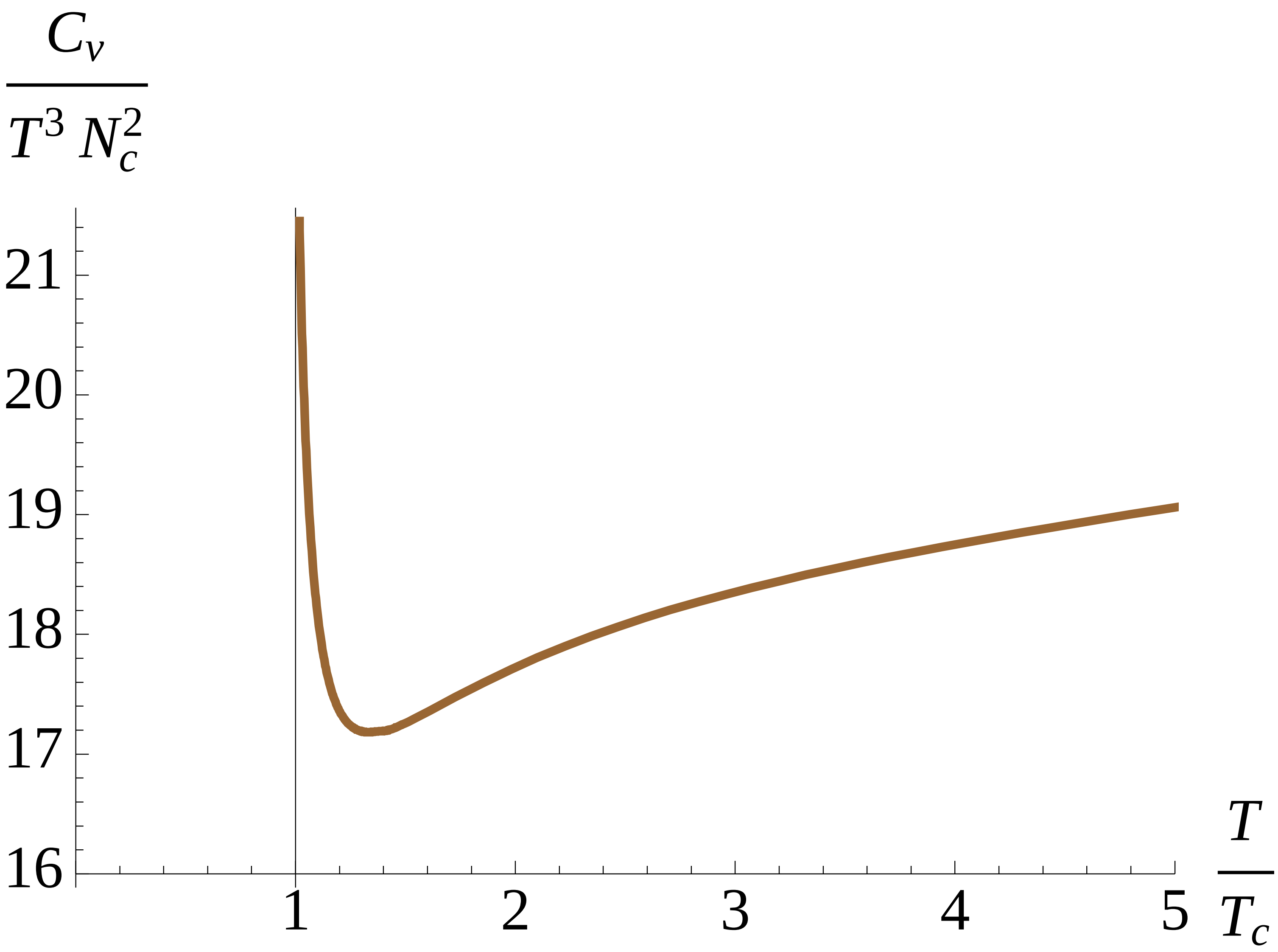
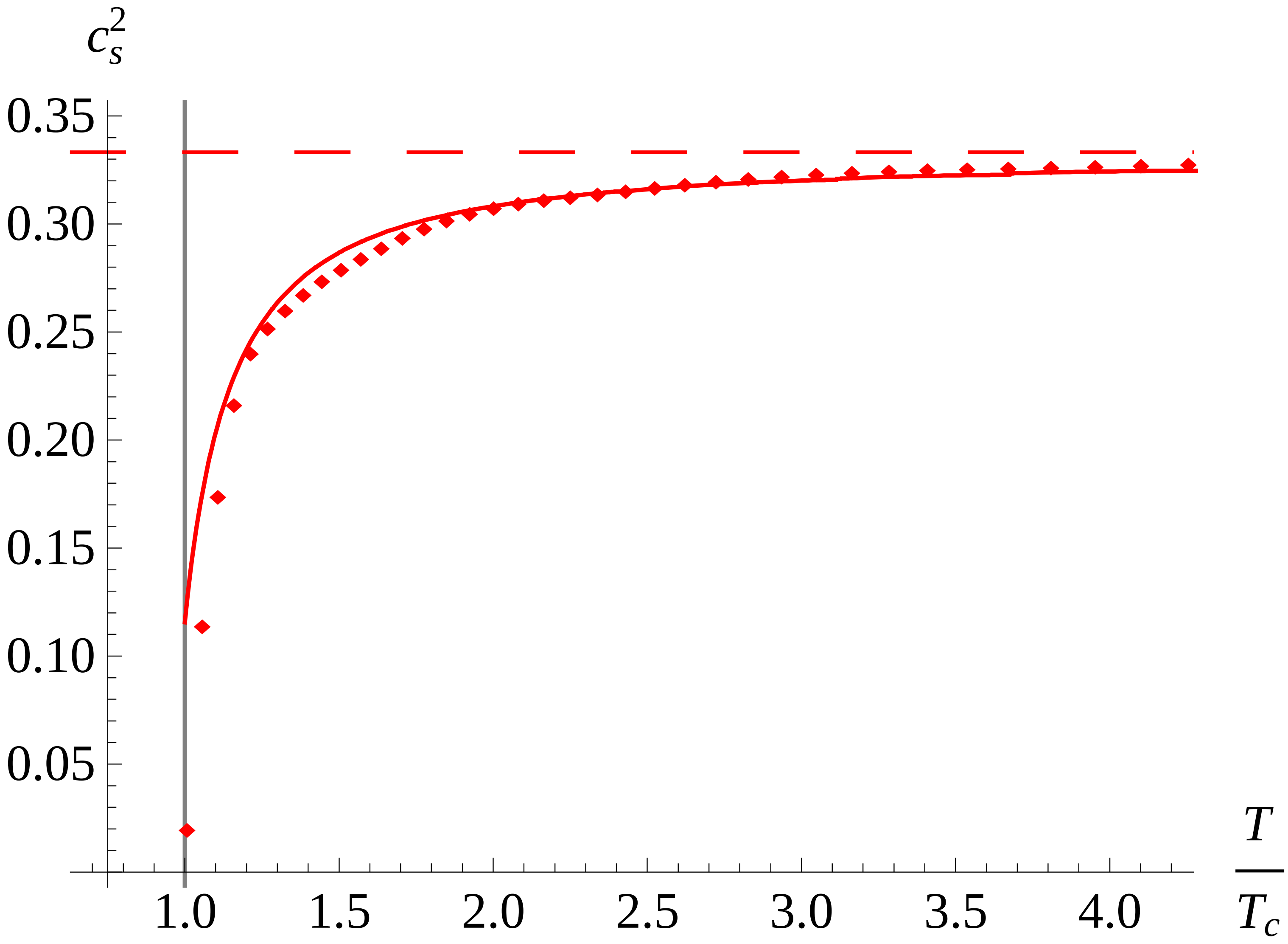
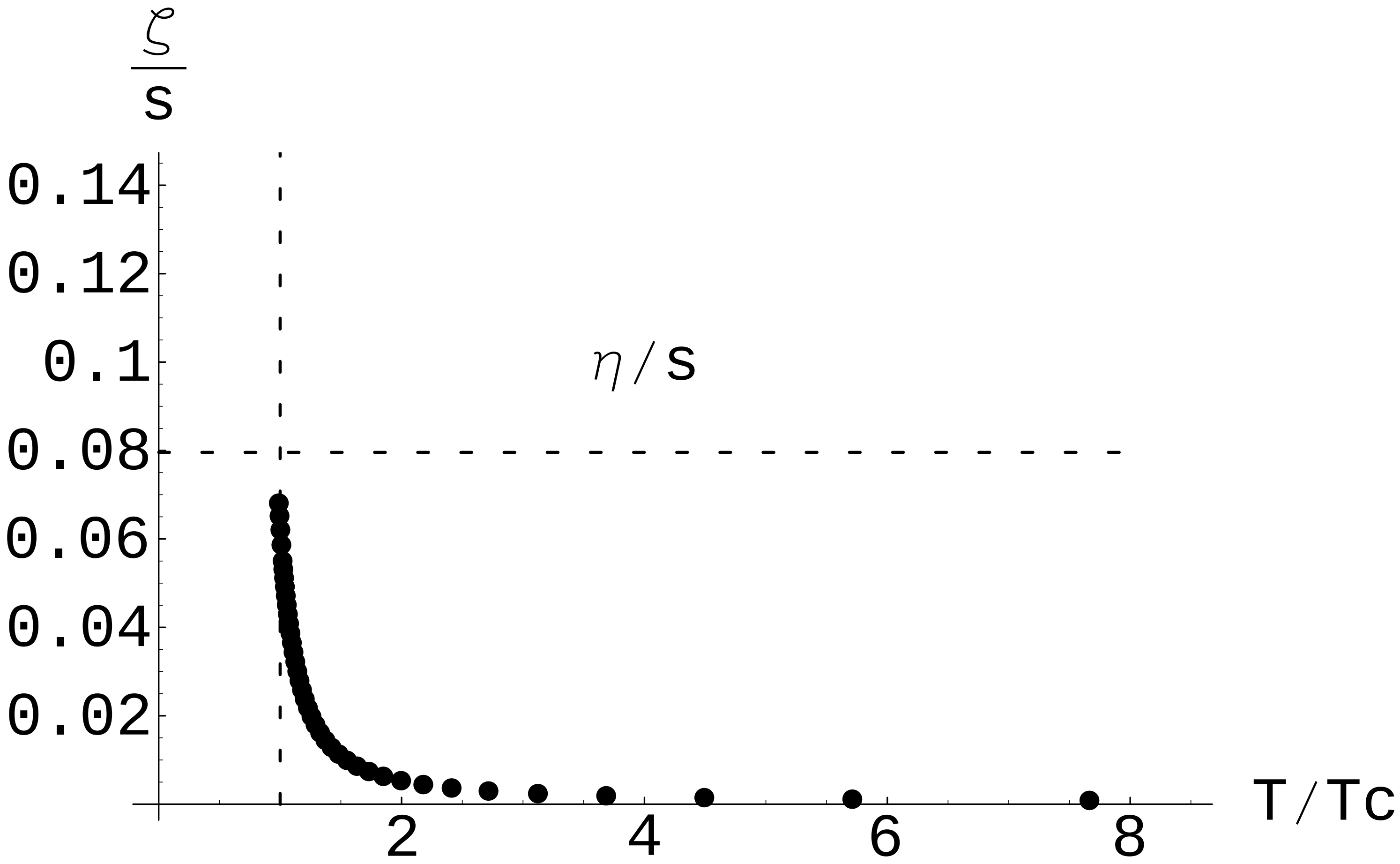
The specific heat



The speed of sound



The bulk viscosity (preliminary)



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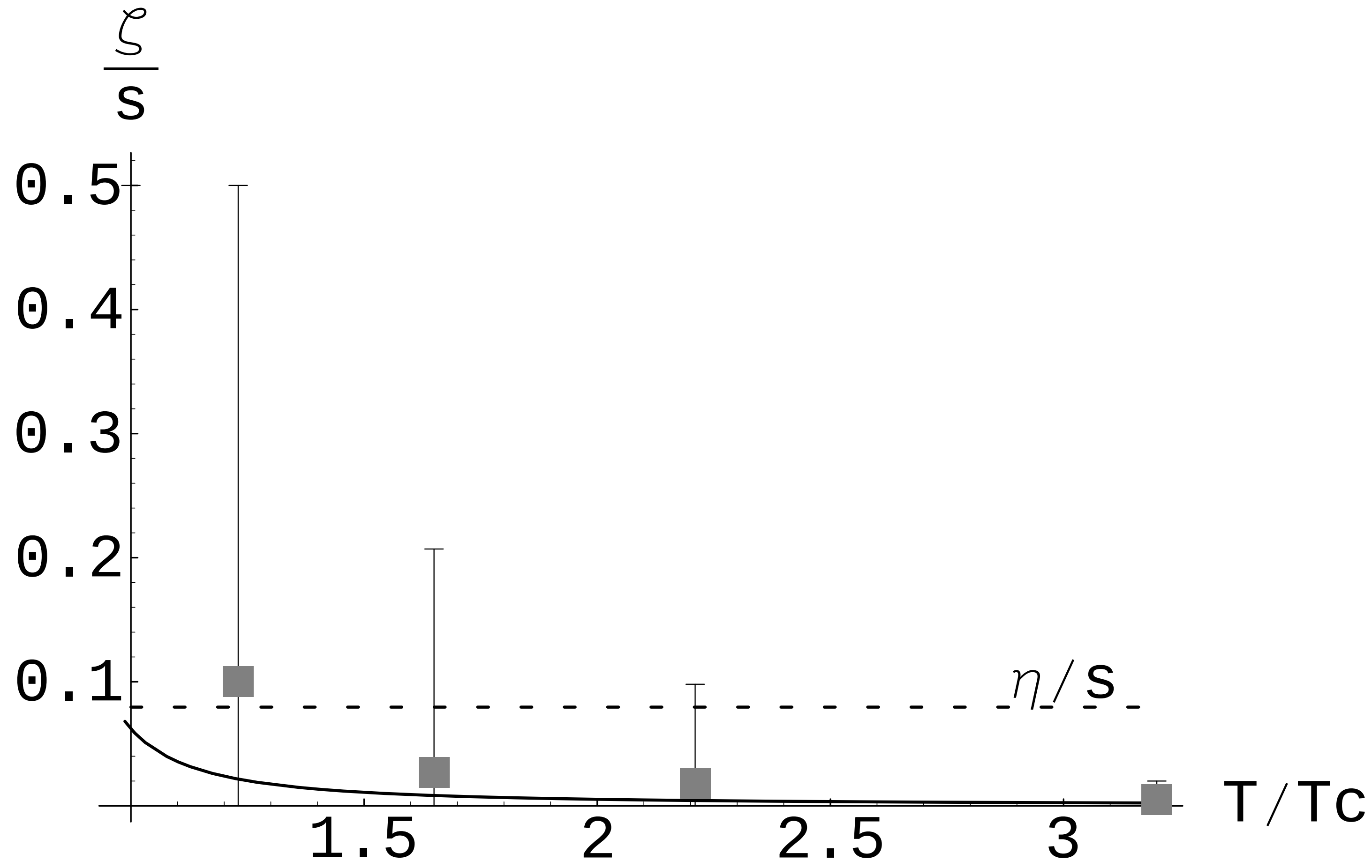
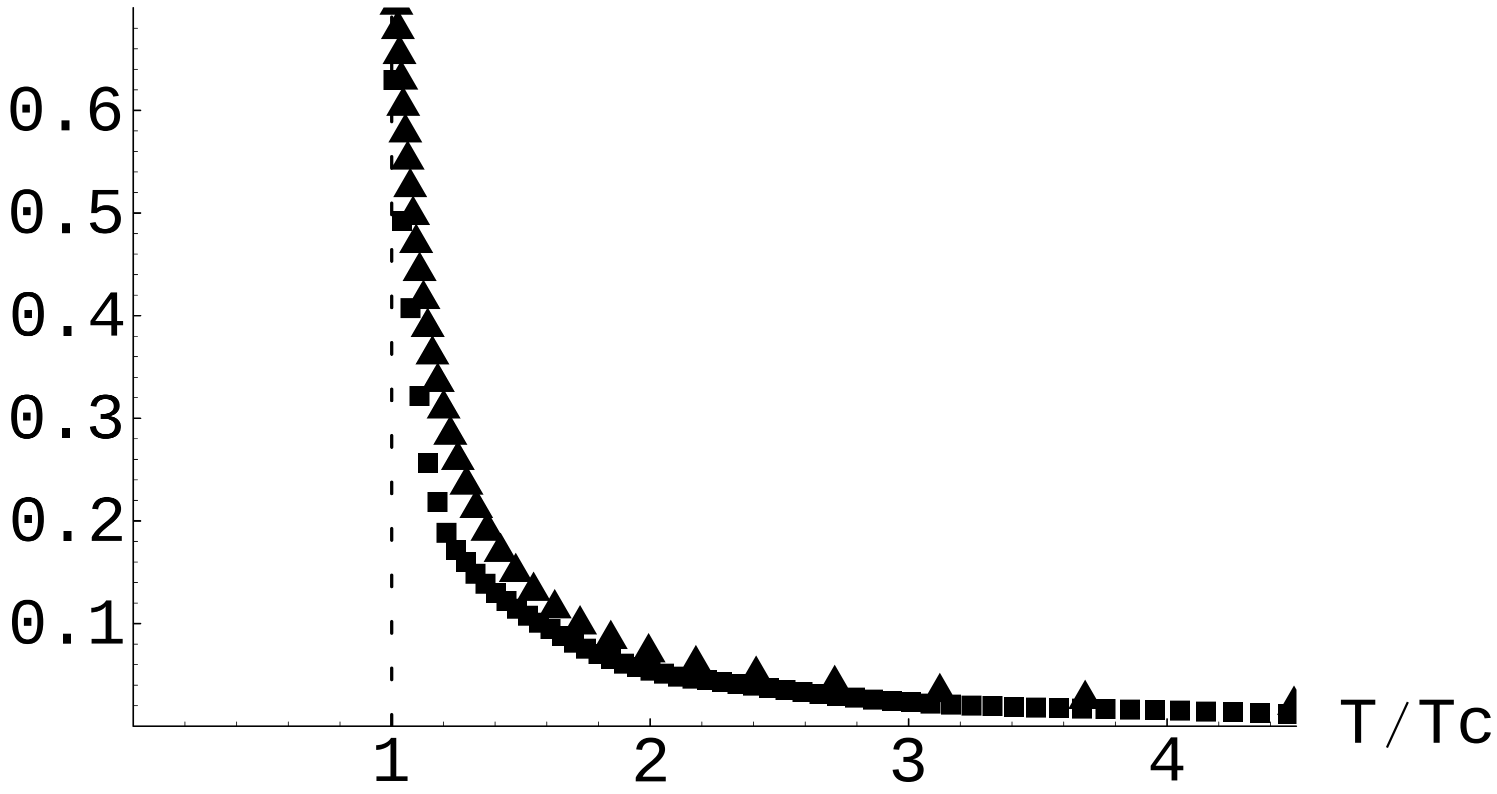


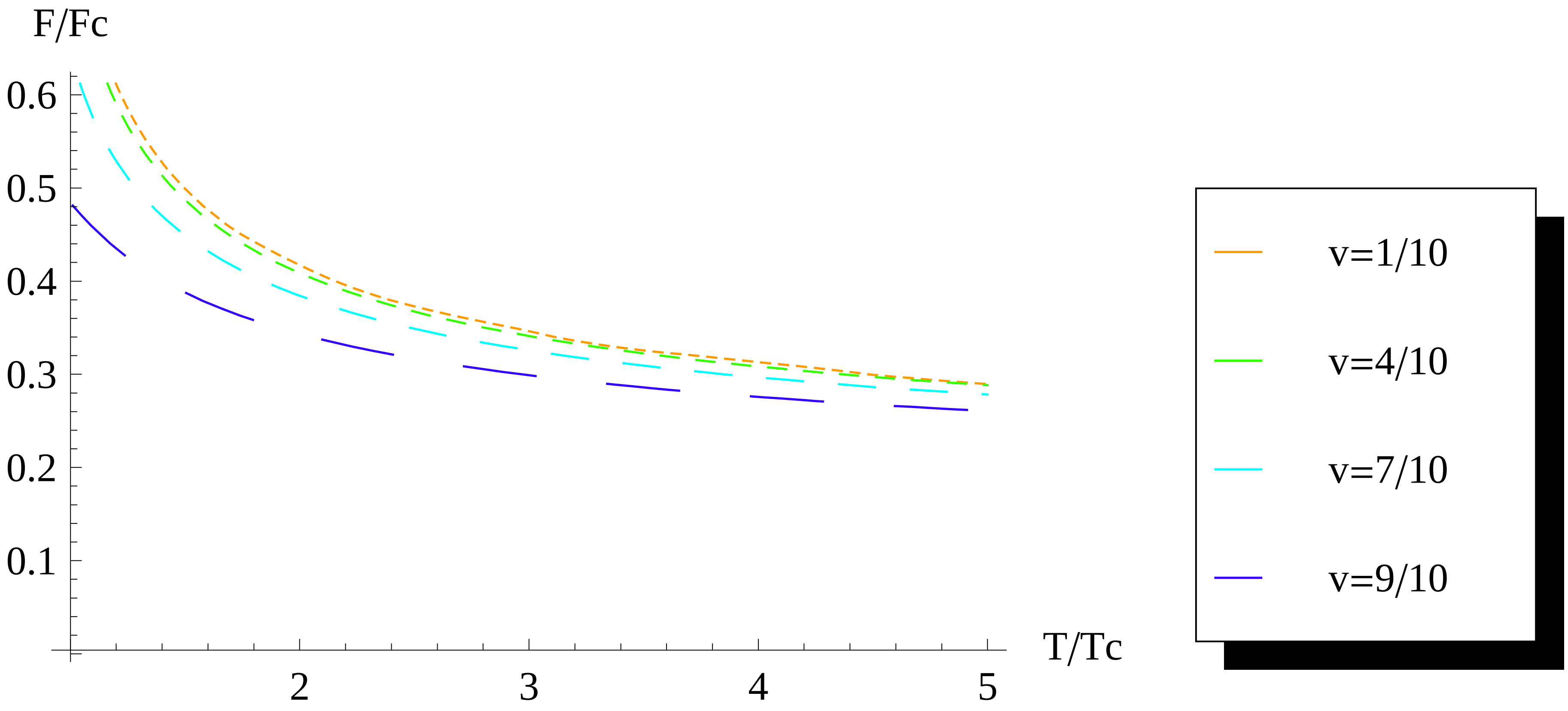
$\frac{\zeta}{s}$
0.5
0.4
0.3
0.2
0.1
η/s
T/Tc
1.5
2
2.5
3



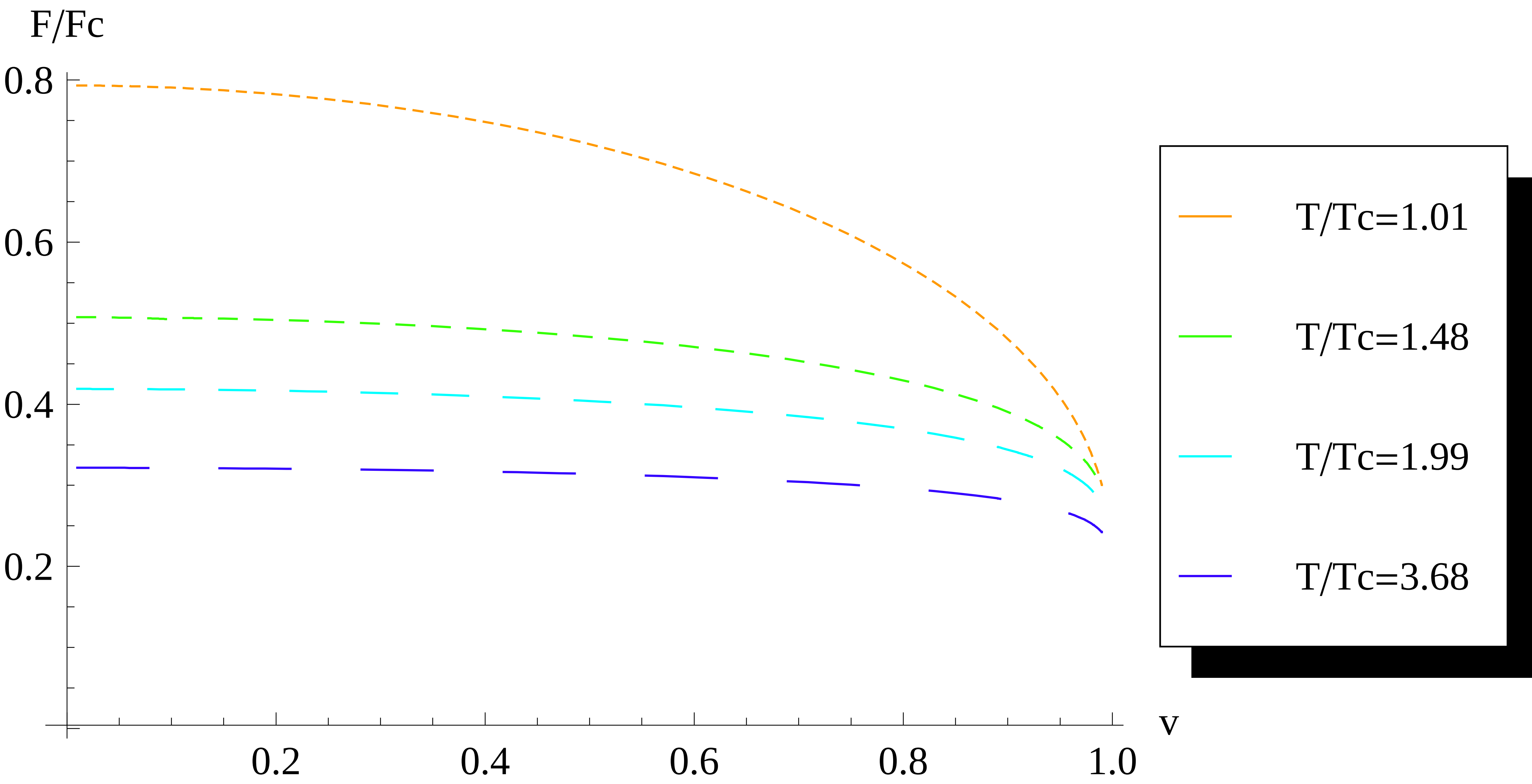
Comparing to the Buchel bound (triangles are ζ/η):

$$\frac{\zeta}{\eta} \geq 2\left(\frac{1}{3} - c_s^2\right).$$

The drag force (preliminary)



$$F_{conf} = -\frac{\pi\sqrt{\lambda}T^2}{2}\frac{v}{\sqrt{1-v^2}}$$


F/Fc
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0
v
T/Tc=1.01
T/Tc=1.48
T/Tc=1.99
T/Tc=3.68

Open problems

THEORETICAL:

• Investigate further the structure of the string dual of QCD. Try to control the UV physics (to which RR flux plays little role).

MORE PRACTICAL:

• Re-Calculate quantities relevant for heavy ion collisions: jet quenching parameter, drag force etc.

• Calculate the finite-temperature Polyakov loops and Debye screening lengths in various symmetry channels.

• Investigate quantitatively the meson sector

• Calculate the phase diagram in the presence of baryon number.

Collaborators

My Collaborators

- Roberto Casero (Milano)
- Umut Gursoy (Utrecht)
- Liuba Mazzanti (Ecole Polytechnique)
- George Michalogiorgakis (Ecole Polytechnique)
- Fransesco Nitti (APC, Paris)
- Angel Paredes (Utrecht)

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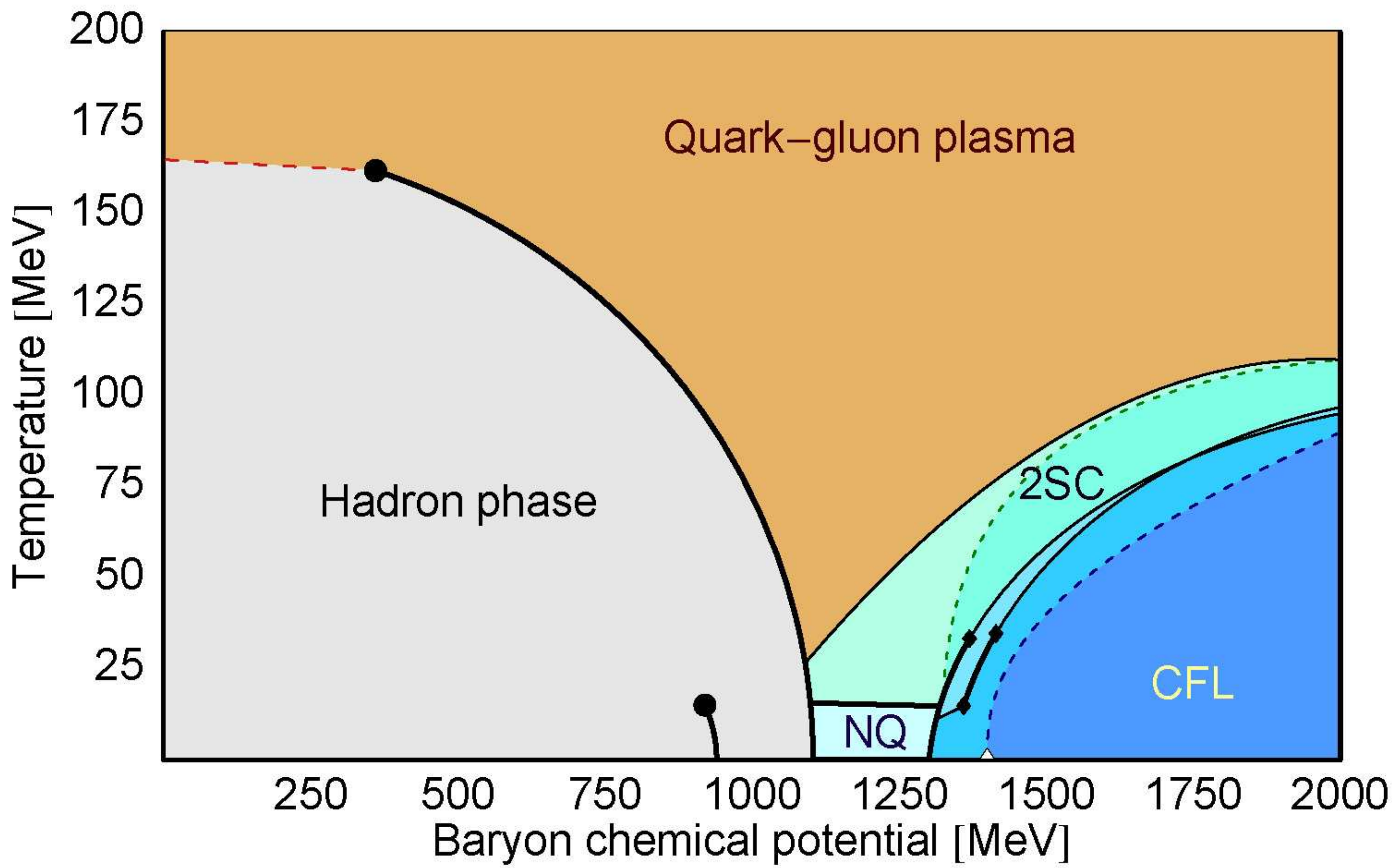
Thank you for your Patience

QCD phase diagram



(from Ruester et al., hep-ph/0503184)

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The Bjorken Relation

• Consider that after the collision of the nuclear pancakes a lot of particles are produced at $t = \tau$. These are confined in a slice of longitudinal width dz and transverse area A.

• The longitudinal velocities have a spread $dv_L = \frac{dz}{\tau}$.

• Near the middle region $v_L \to 0$

$$\frac{dy}{dv_L} = \frac{d}{dv_L}\left[\frac{1}{2}\log\frac{1+v_L}{1-v_L}\right] = \frac{1}{1-v_L^2} \simeq 1$$

• We may now write

$$dN = dv_L\,\frac{dN}{dv_L} \simeq \frac{dz}{\tau}\,\frac{dN}{dy} \quad \to \quad \frac{dN}{dz} \simeq \frac{1}{\tau}\,\frac{dN}{dy}$$

• If $\langle E_T\rangle \simeq \langle m_T\rangle$ is the average energy per particle then the energy density in this area at $t = \tau$ is given by the Bjorken formula:

$$\langle\epsilon(\tau)\rangle \simeq \frac{dN\langle m_T\rangle}{dz\ A} = \frac{1}{\tau}\frac{dN}{dy}\frac{\langle m_T\rangle}{A} = \frac{1}{\tau\ A}\frac{dE_T^{\rm total}}{dy}$$

• It is valid if (1) τ can be defined meaningfully (2) The crossing time $\ll \tau$.

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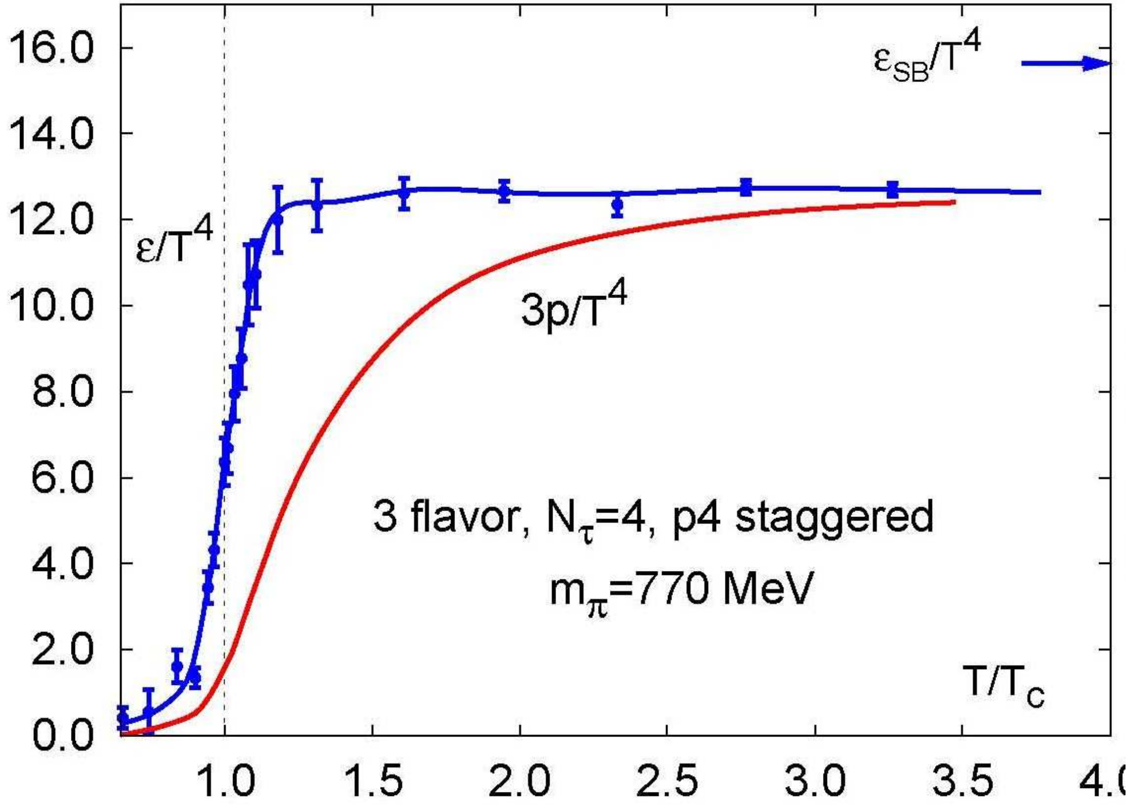
A weakly coupled plasma?

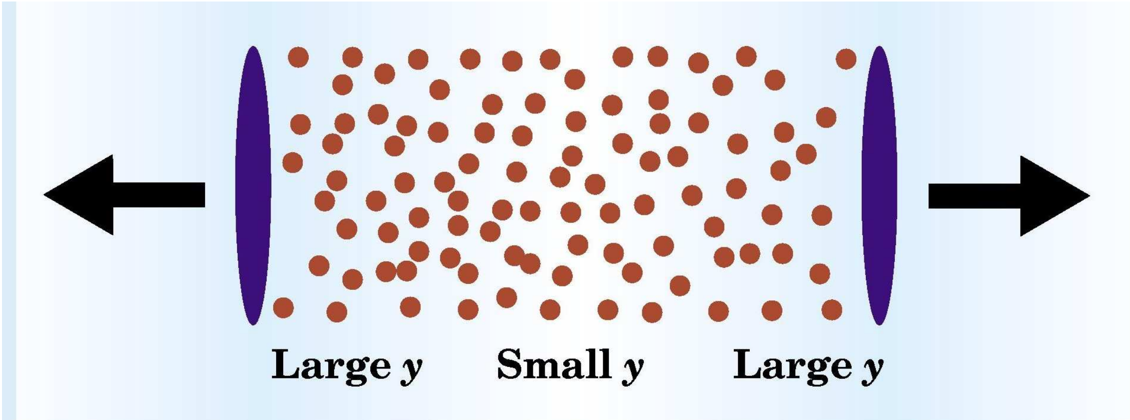


- The pure gauge theory (first-order) critical temperature is $T_c \simeq 240$ MeV.

- It is interesting that the lightest bound state (glueball) in the pure gauge theory has a mass 1700 MeV so that $\frac{T_c}{M_{0^{++}}} \simeq 0.14$

- The crossover with almost physical quarks is at $T_c \simeq 175$ MeV $\simeq 10^{12}$ ^{0}K. $\rightarrow$ 10^{-6} sec

The mid-rapidity range

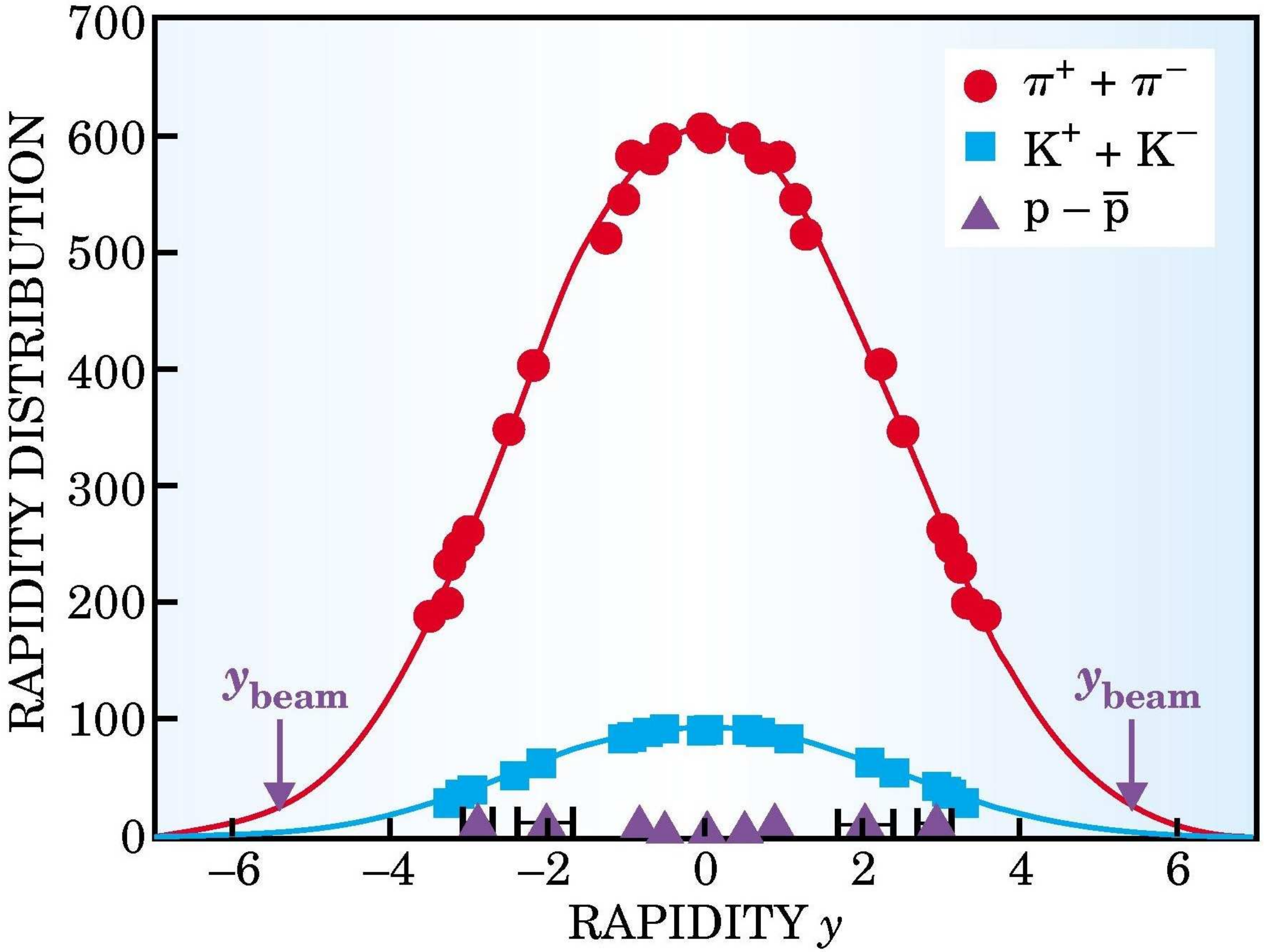


- The crossing time for Au nuclei (with radius 8 fm) is ~ 0.1fm/c $\simeq 3 \times 10^{-25}$ seconds.

- The particles with small v_L (at the center) are produced after 1 fm/c $\simeq 3 \times 10^{-24}$ seconds.

- The "new matter" (free of fragments) is produced near zero rapidity $y \simeq 0$. This is what we are looking for.

- This can be tested by looking at how much "baryon" number is at mid-rapidity



• Each beam nucleon looses 73 ± 6 GeV on the average that goes into creating new particles. Therefore there is 26 TeV worth of energy available for particle production.

The low dimension spectrum

- What are all gauge invariant YM operators of dimension 4 or less?

- They are given by $Tr[F_{\mu\nu}F_{\rho\sigma}]$.

Decomposing into U(4) reps:

$$(\yng(1,1) \otimes \yng(1,1))_{\text{symmetric}} = \yng(2,2) \oplus \yng(1,1,1,1) \tag{1}$$

We must remove traces to construct the irreducible representations of O(4):

$$\yng(2,2) = \underline{\yng(2,2)} \oplus \underline{\yng(2)} \oplus \bullet \quad , \quad \yng(1,1,1,1) = \bullet$$

The two singlets are the scalar (dilaton) and pseudoscalar (axion)

$$\phi \leftrightarrow Tr[F^2] \quad , \quad a \leftrightarrow Tr[F \wedge F]$$

The traceless symmetric tensor

$$\underline{\yng(2)} \quad \rightarrow \quad T_{\mu\nu} = Tr\left[F^2_{\mu\nu} - \frac{1}{4}g_{\mu\nu}F^2\right]$$

is the conserved stress tensor dual to a massless graviton in 5d reflecting the translational symmetry of YM.

$$\underline{\yng(2,2)} \quad \rightarrow \quad T^4_{\mu\nu;\rho\sigma} = Tr[F_{\mu\nu}F_{\rho\sigma} - \frac{1}{2}(g_{\mu\rho}F^2_{\nu\sigma} - g_{\nu\rho}F^2_{\mu\sigma} - g_{\mu\sigma}F^2_{\nu\rho} + g_{\nu\sigma}F^2_{\mu\rho}) + \frac{1}{6}(g_{\mu\rho}g_{\nu\sigma} - g_{\nu\rho}g_{\mu\sigma})F^2]$$

It has 10 independent d.o.f, it is not conserved and it should correspond to a similar massive tensor in 5d. We do not expect it to play an non-trivial role in the large-N_c, YM vacuum also for reasons of Lorentz invariance.

- Therefore the nontrivial fields are expected to be:

$$g_{\mu\nu}, \phi, a$$

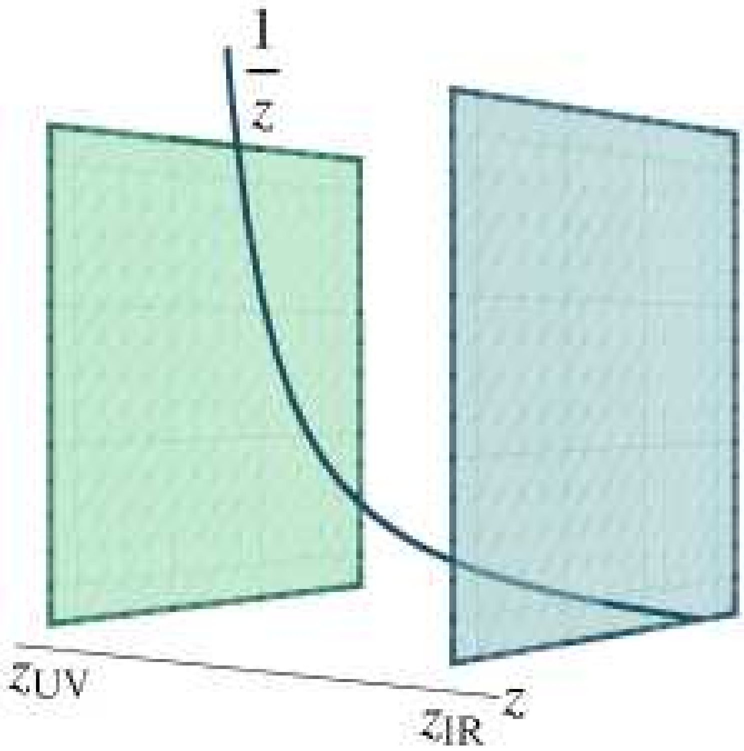
AdS/QCD

♠ A basic phenomenological approach: use a slice of AdS_5, with a UV cutoff, and an IR cutoff.
Polchinski+Strassler

♠ It successfully exhibits confinement (trivially via IR cutoff), and power-like behavior in hard scattering amplitudes

♠ It may be equipped with a bifundamental scalar, T, and $U(N_f)_L \times U(N_f)_R$, gauge fields to describe mesons.
Erlich+Katz+Son+Stepanov, DaRold+Pomarol

Chiral symmetry is broken by hand, via IR boundary conditions. The low-lying meson spectrum looks "reasonable".



♠ Shortcomings:

- The glueball spectrum does not fit very well the lattice calculations. It has the wrong asymptotic behavior $m_n^2 \sim n^2$ at large n.

- Magnetic quarks are confined instead of screened.

- Chiral symmetry breaking is input by hand.

- The meson spectrum has also the wrong UV asymptotics $m_n^2 \sim n^2$.

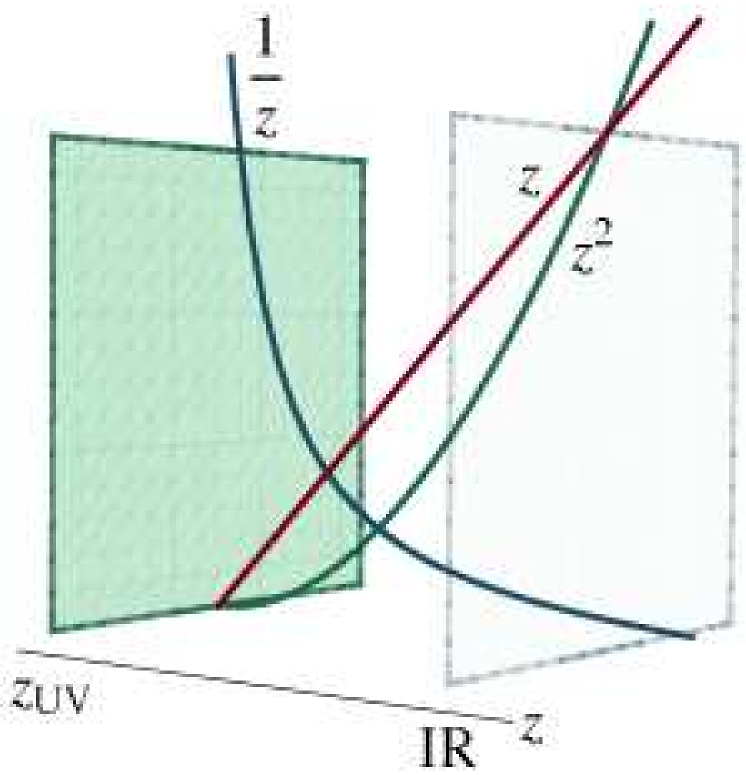
- at finite temperature there is a deconfining transition but the equation of state is trivial (conformal) (e-2p) and the speed of sound is $c_s^2 = \frac{1}{3}$.

The "soft wall"

♠ The asymptotic spectrum can be fixed by introducing a **non-dynamical** dilaton profile $\Phi \sim r^2$ (soft wall)

Karch+Katz+Son+Stephanov

• It is not a solution of equations of motion: the metric is still AdS: Neither $g_{\mu\nu}$ nor Φ solves the equations of motion.



Bosonic string or superstring?

• Consider the axion a dual to $Tr[F \wedge F]$. We can show that it must come from a RR sector.

In large-N$_c$ YM, the proper scaling of couplings is obtained from

$$\mathcal{L}_{YM} = N_c \; Tr\left[\frac{1}{\lambda}F^2 + \frac{\theta}{N_c}F \wedge F\right] \quad , \quad \zeta \equiv \frac{\theta}{N_c} \sim \mathcal{O}(1)$$

It can be shown

$$E_{YM}(\theta) \simeq C_0 \; N_c^2 + C_1\theta^2 + C_2\frac{\theta^4}{N_c^2} + \cdots$$

Witten

In the string theory action

$$S \sim \int e^{-2\phi}\,[R + \cdots] + (\partial a)^2 + e^{2\phi}(\partial a)^4 + \cdots \quad , \quad e^{\phi} \sim g_{YM}^2 \quad , \quad \lambda \sim N_c e^{\phi}$$

$$\sim \int \frac{N_c^2}{\lambda^2}\,[R + \cdots] + (\partial a)^2 + \frac{\lambda^2}{N_c^2}(\partial a)^4 + \cdots \quad , \quad a = \theta[1 + \cdots]$$

The relevant "defects"

• $B_{\mu\nu} \to$ Fundamental string (F_1). This is the QCD (glue) string: fundamental tension $\ell_s^2 \sim \mathcal{O}(1)$

• Its dual $\tilde{B}_\mu \to NS_0$: Tension is $\mathcal{O}(N_c^2)$. It is an effective magnetic baryon vertex binding N_c magnetic quarks.

• $C_5 \to D_4$: Space filling flavor branes. They must be introduced in pairs: $D_4 + \bar{D}_4$ for charge neutrality/tadpole cancelation $\to$ gauge anomaly cancelation in QCD.

• $C_4 \to D_3$ branes generating the gauge symmetry.

.

• $C_3 \to D_2$ branes : domain walls separating different oblique confinement vacua (where $\theta_{k+1} = \theta_k + 2\pi$). Its tension is $\mathcal{O}(N_c)$

• $C_2 \to D_1$ branes: These are the magnetic strings:
(strings attached to magnetic quarks) with tension $\mathcal{O}(N_c)$

• $C_1 \to D_0$ branes. These are the baryon vertices: they bind N_c quarks, and their tension is $\mathcal{O}(N_c)$.
Its instantonic source is the (solitonic) baryon in the string theory.

• $C_0 \to D_{-1}$ branes: These are the Yang-Mills instantons.

Further α' corrections

There are further dilaton terms generated by the 5-form in:

• The kinetic terms of the graviton and the dilaton $\sim \lambda^{2n}$.

• The kinetic terms on probe D_3 branes that affect the identification of the gauge-coupling constant, $\sim \lambda^{2n+1}$. There is also a multiplicative factor relating g_{YM^2} to e^{ϕ}, (not known). Can be traded for b_0.

• Corrections to the identification of the energy. At $r = 0$, $E = 1/r$. There can be log corrections to our identification $E = e^A$, and these are a power series in $\sim \lambda^{2n}$.

• It is a remarkable fact that all such corrections affect the higher that the first two terms in the β-function (or equivalently the potential), that are known to be non-universal!

the metric is also insensitive to the change of b_0 by changing Λ.

Organizing the vacuum solutions

A useful variable is the phase variable

$$X \equiv \frac{\Phi'}{3A'} = \frac{\beta(\lambda)}{3\lambda} \quad , \quad e^{\Phi} \equiv \lambda$$

and a superpotential

$$W^2 - \left(\frac{3}{4}\right)^2 \left(\frac{\partial W}{\partial \Phi}\right)^2 = \left(\frac{3}{4}\right)^3 V(\Phi).$$

with

$$A' = -\frac{4}{9}W \quad , \quad \Phi' = \frac{dW}{d\Phi}$$

$$X = -\frac{3}{4}\frac{d\log W}{d\log\lambda} \quad , \quad \beta(\lambda) = -\frac{9}{4}\lambda\frac{d\log W}{d\log\lambda}$$

♠ The equations have three integration constants: (two for Φ and one for A) One corresponds to the "gluon condensate" in the UV. It must be set to zero otherwise the IR behavior is unacceptable. The other is Λ. The third one is a gauge artifact (corresponds to overall translation in the radial coordinate).

The IR regime

For any asymptotically AdS$_5$ solution ($e^A \sim \frac{\ell}{r}$):

• The scale factor $e^{A(r)}$ is monotonically decreasing

Girardelo+Petrini+Porrati+Zaffaroni
Freedman+Gubser+Pilch+Warner

• Moreover, there are only three possible, mutually exclusive IR asymptotics:

♠ *there is another asymptotic AdS_5 region, at $r \to \infty$, where* $\exp A(r) \sim \ell'/r$, *and* $\ell' \le \ell$ (equality holds if and only if the space is exactly AdS_5 everywhere);

♠ there is a curvature singularity at some finite value of the radial coordinate, $r = r_0$;

♠ **there is a curvature singularity at $r \to \infty$, where the scale factor vanishes and the space-time shrinks to zero size.**

Wilson-Loops and confinement

• Calculation of the static quark potential using the vev of the Wilson loop calculated via an F-string worldsheet.

Rey+Yee, Maldacena

$$T\ E(L) = S_{minimal}(X)$$

We calculate

$$L = 2\int_0^{r_0} dr \frac{1}{\sqrt{e^{4A_S(r)-4A_S(r_0)} - 1}}.$$

It diverges when e^{A_s} has a minimum (at $r = r_*$). Then

$$E(L) \sim T_f\ e^{2A_S(r_*)}\ L$$

• Confinement $\to$ $A_s(r_*)$ is finite. This is a more general condition that considered before as A_S `is not monotonic in general`.

• Effective string tension

$$T_{\text{string}} = T_f\ e^{2A_S(r_*)}$$

General criterion for confinement

• **the geometric version**:
A geometry that shrinks to zero size in the IR is dual to a confining 4D theory if and only if the Einstein metric in conformal coordinates vanishes as (or faster than) e^{-Cr} as $r \to \infty$, for some $C > 0$.

• It is understood here that a metric vanishing at finite $r = r_0$ also satisfies the above condition.

♠ **the superpotential**
A 5D background is dual to a confining theory if the superpotential grows as (or faster than)

$$W \sim (\log \lambda)^{P/2} \lambda^{2/3} \quad \text{as} \quad \lambda \to \infty \quad , \quad P \geq 0$$

♠ **the β-function** A 5D background is dual to a confining theory if and only if

$$\lim_{\lambda \to \infty} \left(\frac{\beta(\lambda)}{3\lambda} + \frac{1}{2} \right) \log \lambda = K, \qquad -\infty \leq K \leq 0$$

(No explicit reference to any coordinate system) Linear trajectories correspond to $K = -\frac{3}{16}$

Classification of confining superpotentials

Classification of confining superpotentials $W(\lambda)$ as $\lambda \to \infty$ in IR:

$$W(\lambda) \sim (\log \lambda)^{\frac{P}{2}} \lambda^{Q} \quad , \quad \lambda \sim E^{-\frac{9}{4}Q} \left(\log \frac{1}{E}\right)^{\frac{P}{2Q}} , \qquad E \to 0.$$

- $Q > 2/3$ or $Q = 2/3$ and $P > 1$ leads to confinement and a singularity at finite $r = r_0$.

$$e^{A}(r) \sim \begin{cases} (r_0 - r)^{\frac{4}{9Q^2-4}} & Q > \frac{2}{3} \\ \exp\left[-\frac{C}{(r_0-r)^{1/(P-1)}}\right] & Q = \frac{2}{3} \end{cases}$$

- $Q = 2/3$, and $0 \leq P < 1$ leads to confinement and a singularity at $r = \infty$ The scale factor e^A vanishes there as

$$e^{A}(r) \sim \exp[-C r^{1/(1-P)}].$$

- $Q = 2/3, P = 1$ leads to confinement but the singularity may be at a finite or infinite value of r depending on subleading asymptotics of the superpotential.

♠ If $Q < 2\sqrt{2}/3$, no *ad hoc* boundary conditions are needed to determine the glueball spectrum → One-to-one correspondence with the β-function This is unlike standard AdS/QCD and other approaches.

- when $Q > 2\sqrt{2}/3$, the spectrum is not well defined without extra boundary conditions in the IR because both solutions to the mass eigenvalue equation are IR normalizable.

Confining β-functions

A 5D background is dual to a confining theory if and only if

$$\lim_{\lambda\to\infty}\left(\frac{\beta(\lambda)}{3\lambda}+\frac{1}{2}\right)\log\lambda = K, \qquad -\infty \le K \le 0$$

(No explicit reference to any coordinate system). Linear trajectories correspond to $K = -\frac{3}{16}$

- We can determine the geometry if we specify K:

- $K = -\infty$: the scale factor goes to zero at some finite r_0, not faster than a power-law.

- $-\infty < K < -3/8$: the scale factor goes to zero at some finite r_0 faster than any power-law.

- $-3/8 < K < 0$: the scale factor goes to zero as $r \to \infty$ faster than $e^{-Cr^{1+\epsilon}}$ for some $\epsilon > 0$.

- $K = 0$: the scale factor goes to zero as $r \to \infty$ as e^{-Cr} (or faster), but slower than $e^{-Cr^{1+\epsilon}}$ for any $\epsilon > 0$.

The borderline case, $K = -3/8$, is certainly confining (by continuity), but whether or not the singularity is at finite r depends on the subleading terms.

Comments on confining backgrounds

• For all confining backgrounds with $r_0 = \infty$, although the space-time is singular in the Einstein frame, the string frame geometry is asymptotically flat for large r. Therefore only λ grows indefinitely.

• String world-sheets do not probe the strong coupling region, at least classically. The string stays away from the strong coupling region.

• Therefore: singular confining backgrounds have generically the property that the singularity is *repulsive*, i.e. only highly excited states can probe it. This will also be reflected in the analysis of the particle spectrum (to be presented later)

• The confining backgrounds must also screen magnetic color charges. This can be checked by calculating 't Hooft loops using D_1 probes:

♠ All confining backgrounds with $r_0 = \infty$ and most at finite r_0 screen properly

♠ In particular "hard-wall" AdS/QCD confines also the magnetic quarks.

Particle Spectra: generalities

- Linearized equation:

$$\ddot{\xi} + 2\dot{B}\dot{\xi} + \Box_4 \xi = 0 \quad , \quad \xi(r,x) = \xi(r)\xi^{(4)}(x), \qquad \Box\xi^{(4)}(x) = m^2\xi^{(4)}(x)$$

- Can be mapped to Schrodinger problem

$$-\frac{d^2}{dr^2}\psi + V(r)\psi = m^2\psi \quad , \quad V(r) = \frac{d^2B}{dr^2} + \left(\frac{dB}{dr}\right)^2 \quad , \quad \xi(r) = e^{-B(r)}\psi(r)$$

- Mass gap and discrete spectrum visible from the asymptotics of the potential.

- Large n asymptotics of masses obtained from WKB

$$n\pi = \int_{r_1}^{r_2} \sqrt{m^2 - V(r)}\; dr$$

- Spectrum depends only on initial condition for λ $(\sim \Lambda_{QCD})$ and an overall energy scale (e^A) that must be fixed.

• scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2}\log\frac{\beta(\lambda)^2}{9\lambda^2}$$

• tensor glueballs

$$B(r) = \frac{3}{2}A(r)$$

• pseudo-scalar glueballs

$$B(r) = \frac{3}{2}A(r) + \frac{1}{2}\log Z(\lambda)$$

• Universality of asymptotics

$$\frac{m^2_{n\to\infty}(0^{++})}{m^2_{n\to\infty}(2^{++})} \to 1 \quad , \quad \frac{m^2_{n\to\infty}(0^{+-})}{m^2_{n\to\infty}(0^{++})} = \frac{1}{4}(d-2)^2$$

predicts $d = 4$ via

$$\frac{m^2}{2\pi\sigma_a} = 2n + J + c,$$

Quarks ($N_f \ll N_c$) and mesons

• Flavor is introduced by N_f $D_4 + \bar{D}_4$ branes pairs inside the bulk background. Their back-reaction on the bulk geometry is suppressed by N_f/N_c.

• The important world-volume fields are

$$T_{ij} \leftrightarrow \bar{q}^i_a \frac{1+\gamma^5}{2} q^j_a \quad , \quad A^{ij\,L,R}_{\mu} \leftrightarrow \bar{q}^i_a \frac{1 \pm \gamma^5}{2} \gamma^{\mu} q^j_a$$

Generating the $U(N_f)_L \times U(N_f)_R$ chiral symmetry.

• The UV mass matrix m_{ij} corresponds to the source term of the Tachyon field. It breaks the chiral (gauge) symmetry. The normalizable mode corresponds to the vev $\langle \bar{q}^i_a \frac{1+\gamma^5}{2} q^j_a \rangle$.

• We show that the expectation value of the tachyon is non-zero and $T \sim \mathbf{1}$, breaking chiral symmetry $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$. The anomaly plays an important role in this (holographic Coleman-Witten)

• The fact that the tachyon diverges in the IR (fusing D with $\bar{D}$) constraints the UV asymptotics and determines the quark condensate $\langle \bar{q}q \rangle$ in terms of m_q. A GOR relation is satisfied (for an asymptotic AdS$_5$ space)

$$m_\pi^2 = -2\frac{m_q}{f_\pi^2}\ \langle \bar{q}q \rangle \quad , \quad m_q \to 0$$

• We can derive formulae for the anomalous divergences of flavor currents, when they are coupled to an external source.

• When $m_q = 0$, the meson spectrum contains N_f^2 massless pseudoscalars, the $U(N_f)_A$ Goldstone bosons.

• The WZ part of the flavor brane action gives the Adler-Bell-Jackiw $U(1)_A$ axial anomaly and an associated Stuckelberg mechanism gives an $O\left(\frac{N_f}{N_c}\right)$ mass to the would-be Goldstone boson η', in accordance with the Veneziano-Witten formula.

• Studying the spectrum of highly excited mesons, we find the expected property of linear confinement: $m_n^2 \sim n$.

• The detailed spectrum of mesons remains to be worked out

Tachyon dynamics

- In the vacuum the gauge fields vanish and $T \sim \mathbf{1}$. Only DBI survives

$$S[\tau] = T_{D_4} \int dr d^4x \, \frac{e^{4A_s(r)}}{\lambda} V(\tau) \sqrt{e^{2A_s(r)} + \dot{\tau}(r)^2} \quad , \quad V(\tau) = e^{-\frac{\mu^2}{2}\tau^2}$$

- We obtain the nonlinear field equation:

$$\ddot{\tau} + \left(3\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right)\dot{\tau} + e^{2A_S}\mu^2\tau + e^{-2A_S}\left[4\dot{A}_S - \frac{\dot{\lambda}}{\lambda}\right]\dot{\tau}^3 + \mu^2\tau \, \dot{\tau}^2 = 0.$$

- In the UV we expect

$$\tau = m_q \, r + \sigma \, r^3 + \cdots \quad , \quad \mu^2\ell^2 = 3$$

- We expect that the tachyon must diverge before or at $r = r_0$. We find that indeed it does `at the singularity`. For the $r_0 = \infty$ backgrounds

$$\tau \sim \exp\left[\frac{2}{a} \, \frac{R}{\ell^2} \, r\right] \quad \text{as} \quad r \to \infty$$

• Generically the solutions have spurious singularities: $\tau(r_*)$ stays finite but its derivatives diverges as:

$$\tau \sim \tau_* + \gamma\sqrt{r_* - r}.$$

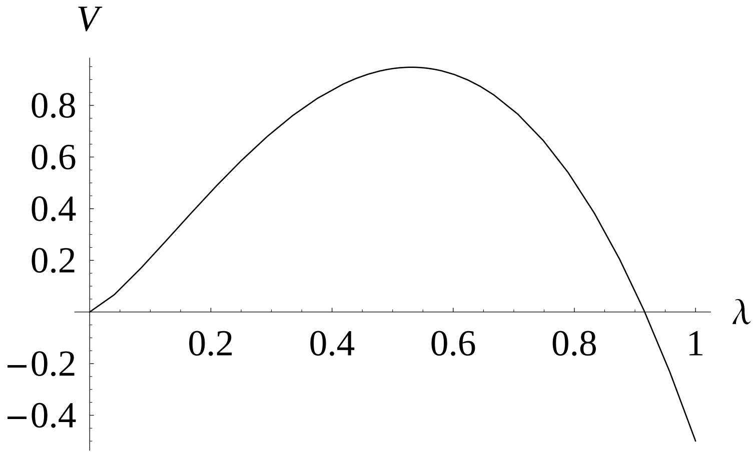
The condition that they are absent determines σ as a function of m_q.

• The easiest spectrum to analyze is that of vector mesons. We find $(r_0 = \infty)$

$$\Lambda_{glueballs} = \frac{1}{R}, \qquad \Lambda_{mesons} = \frac{3}{\ell}\left(\frac{\alpha\ell^2}{2R^2}\right)^{(\alpha-1)/2} \propto \frac{1}{R}\left(\frac{\ell}{R}\right)^{\alpha-2}.$$

This suggests that $\alpha = 2$. preferred also from the glue sector.

Fluctuations around the AdS$_5$ extremum



• In QCD we expect that

$$\frac{1}{\lambda} = \frac{1}{N_c e^{\phi}} \sim \frac{1}{\log r} \quad , \quad ds^2 \sim \frac{1}{r^2}(dr^2 + dx_\mu dx^\mu) \qquad \text{as} \qquad r \to 0$$

• Any potential with $V(\lambda) \sim \lambda^a$ when $\lambda \ll 1$ gives a power different that of AdS$_5$

• There is an AdS$_5$ minimum at a finite value λ_*. This cannot be the UV of QCD as dimensions do not match.

Near an AdS extremum

$$V = \frac{12}{\ell^2} - \frac{16\xi}{3\ell^2}\phi^2 + \mathcal{O}(\phi^3) \quad , \quad \frac{18}{\ell}\delta A' = \delta\phi'^2 - \frac{4}{\ell^2}\phi^2 = \mathcal{O}(\delta\phi^2) \quad , \quad \delta\phi'' - \frac{4}{\ell}\delta\phi' - \frac{4\xi}{\ell^2}\delta\phi = 0$$

where $\phi << 1$. The general solution of the second equation is

$$\delta\phi = C_+ e^{\frac{(2+2\sqrt{1+\xi})u}{\ell}} + C_- e^{\frac{(2-2\sqrt{1+\xi})u}{\ell}}$$

For the potential in question

$$V(\phi) = \frac{e^{\frac{4}{3}\phi}}{\ell_s^2}\left[5 - \frac{N_c^2}{2}e^{2\phi} - N_f\ e^{\phi}\right] \quad , \quad \lambda_0 \equiv N_c e^{\phi_0} = \frac{-7x + \sqrt{49x^2+400}}{10} \quad , \quad x \equiv \frac{N_f}{N_c}$$

$$\xi = \frac{5}{4}\left[\frac{400 + 49x^2 - 7x\sqrt{49x^2+400}}{100 + 7x^2 - x\sqrt{49x^2+400}}\right] \quad , \quad \frac{\ell_s^2}{\ell^2} = e^{\frac{4}{3}\phi_0}\left[\frac{100 + 7x^2 - x\sqrt{49x^2+400}}{400}\right]$$

The associated dimension is $\Delta = 2 + 2\sqrt{1+\xi}$ and satisfies

$$2 + 3\sqrt{2} < \Delta < 2 + 2\sqrt{6} \qquad \text{or} \quad \text{equivalently} \qquad 6.24 < \Delta < 6.90$$

It corresponds to an irrelevant operator. It is most probably relevant for the Banks-Zaks fixed points.

Bigazzi+Casero+Cotrone+Kiritsis+Paredes

RETURN

Concrete potential

- The superpotential chosen is

$$W = (3 + 2b_0\lambda)^{2/3}\left[18 + \left(2b_0^2 + 3b_1\right)\log(1+\lambda^2)\right]^{4/3},$$

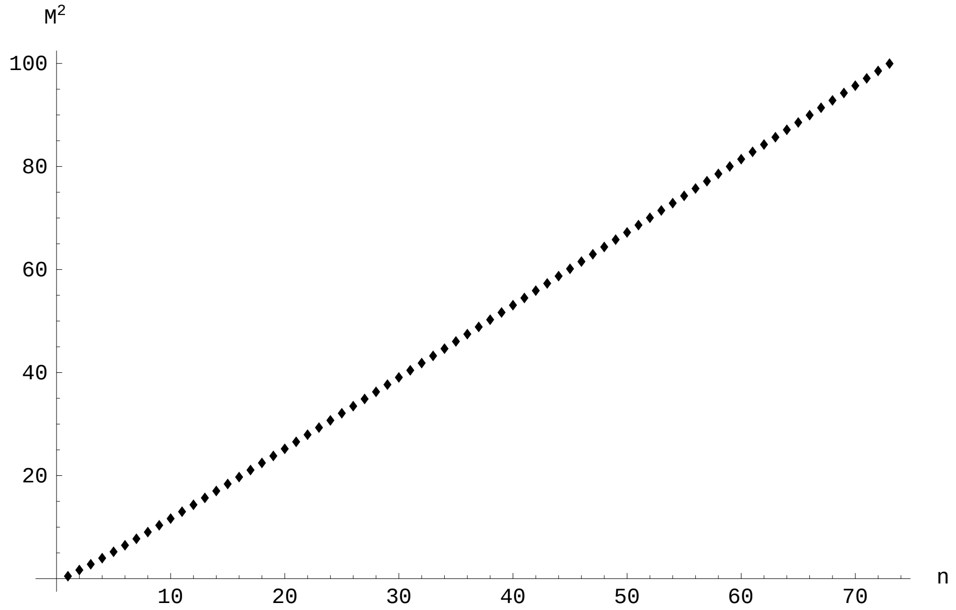
with corresponding potential

$$\beta(\lambda) = -\frac{3b_0\lambda^2}{3+2b_0\lambda} - \frac{6(2b_0^2+3b_1^2)\lambda^3}{(1+\lambda^2)\left(18+\left(2b_0^2+3b_1^2\right)\log(1+\lambda^2)\right)}$$

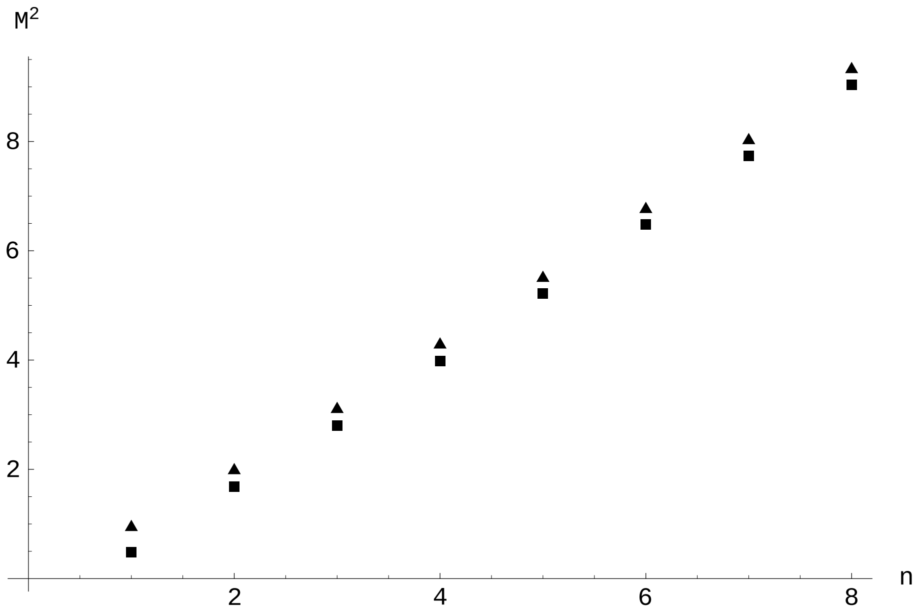
which is everywhere regular and has the correct UV and IR asymptotics.

- b_0 is a free parameter and b_1/b_0^2 is taken from the QCD β-function

Linearity of the glueball spectrum



(a)

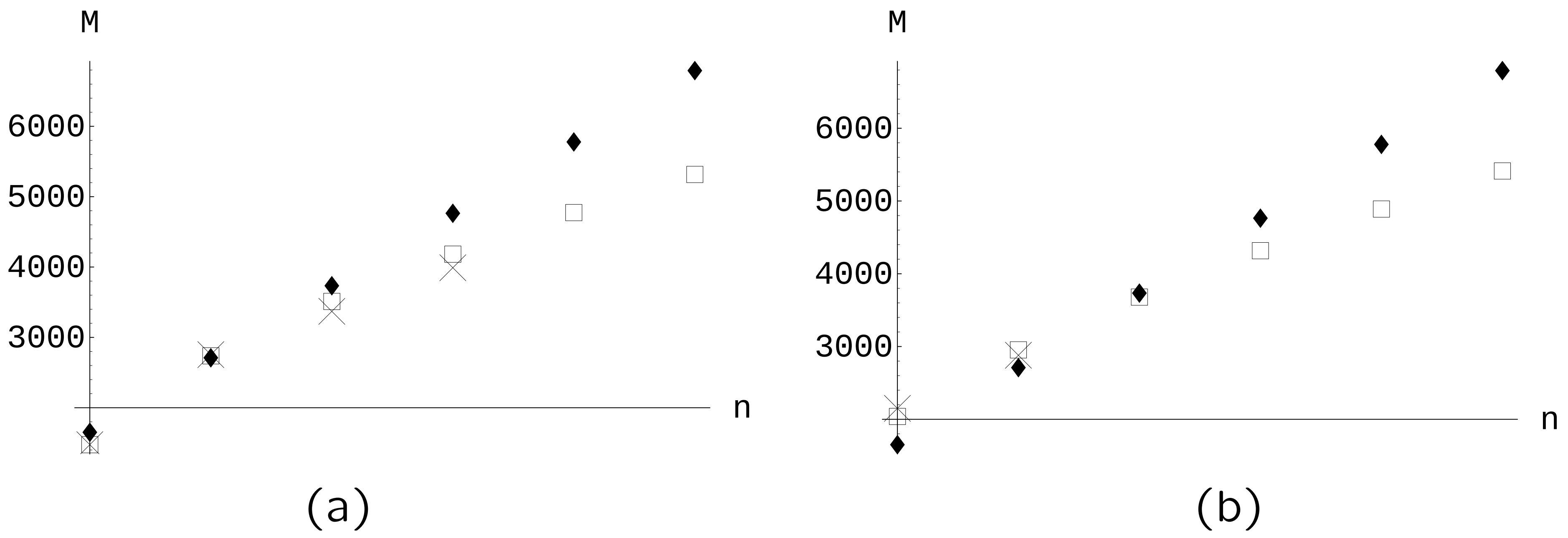


(b)

(a) Linear pattern in the spectrum for the first 40 0^{++} glueball states. M^2 is shown units of $0.015\ell^{-2}$.

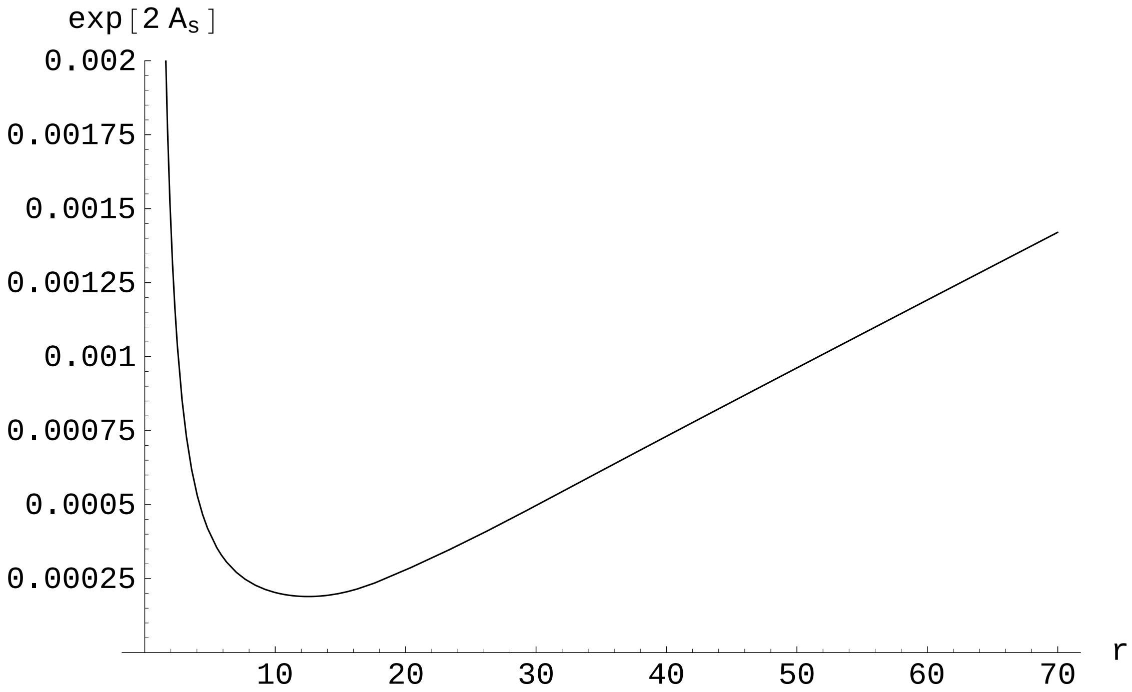
(b) The first 8 0^{++} (squares) and the 2^{++} (triangles) glueballs. These spectra are obtained in the background I with $b_0 = 4.2, \lambda_0 = 0.05$.

Comparison with lattice data (Meyer)



Comparison of glueball spectra from our model with $b_0 = 4.2, \lambda_0 = 0.05$ (boxes), with the lattice QCD data from Ref. I (crosses) and the AdS/QCD computation (diamonds), for (a) 0^{++} glueballs; (b) 2^{++} glueballs. The masses are in MeV, and the scale is normalized to match the lowest 0^{++} state from Ref. I.

$$\ell_{eff}^2 = 6.88\ \ell^2$$



The string frame scale factor in background I with $b_0 = 4.2$, $\lambda_0 = 0.05$.

We can "measure"

$$\frac{\ell}{\ell_s} \simeq 6.26 \quad , \quad \ell_s^2 R \simeq -0.5 \tag{2}$$

and predict

$$\alpha_s(1.2GeV) = 0.34,$$

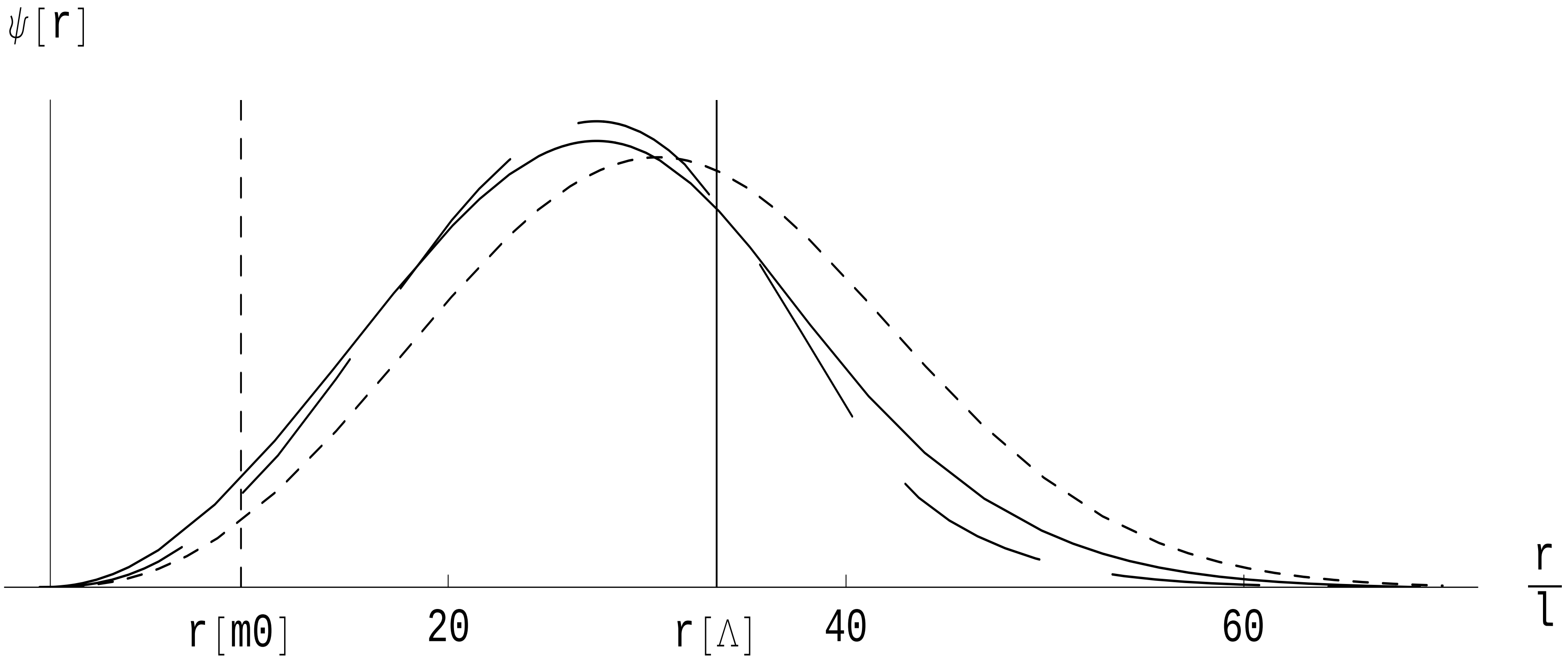
which is within the error of the quoted experimental value $\alpha_s^{(exp)}(1.2GeV) = 0.35 \pm 0.01$

The fit to glueball lattice data

J^{PC}	Ref I (MeV)	Our model (MeV)	Mismatch	$N_c \to \infty$	Mismatch
0^{++}	**1475 (4%)**	**1475**	0	**1475**	0
2^{++}	2150 (5%)	2055	4%	2153 (10%)	5%
0^{-+}	**2250 (4%)**	**2243**	0		
0^{++*}	**2755 (4%)**	**2753**	0	2814 (12%)	2%
2^{++*}	2880 (5%)	2991	4%		
0^{-+*}	3370 (4%)	3288	2%		
0^{++**}	3370 (4%)	3561	5%		
0^{++***}	3990 (5%)	4253	6%		

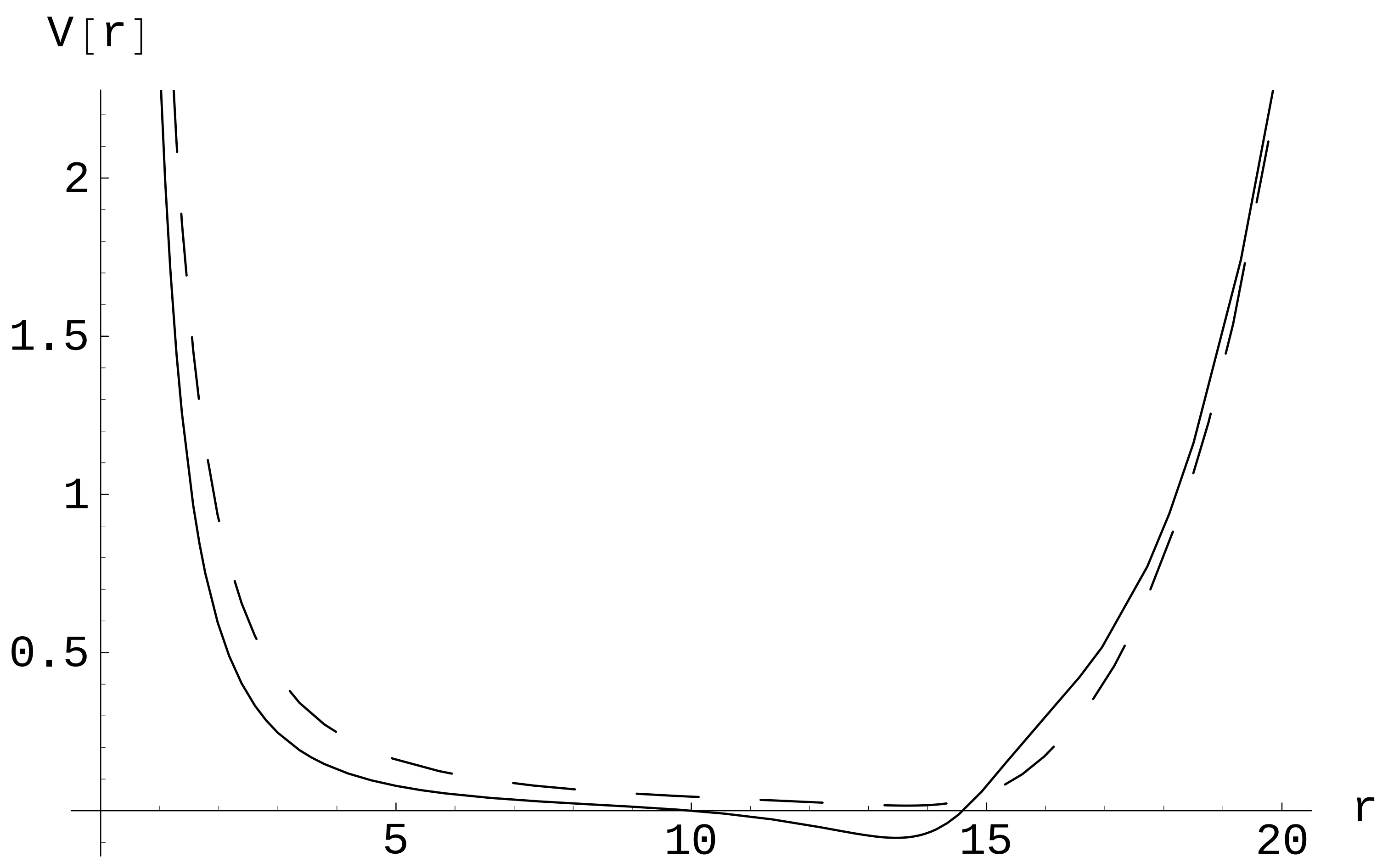
Comparison between the glueball spectra in Ref. I and in our model. The states we use as input in our fit are marked in red. The parenthesis in the lattice data indicate the percent accuracy.

The glueball wavefunctions



Normalized wave-function profiles for the ground states of the 0^{++} (solid line) ,0^{-+} (dashed line), and 2^{++} (dotted line) towers, as a function of the radial conformal coordinate. The vertical lines represent the position corresponding to $E = m_{0^{++}}$ and $E = \Lambda_p$.

Comparison of scalar and tensor potential



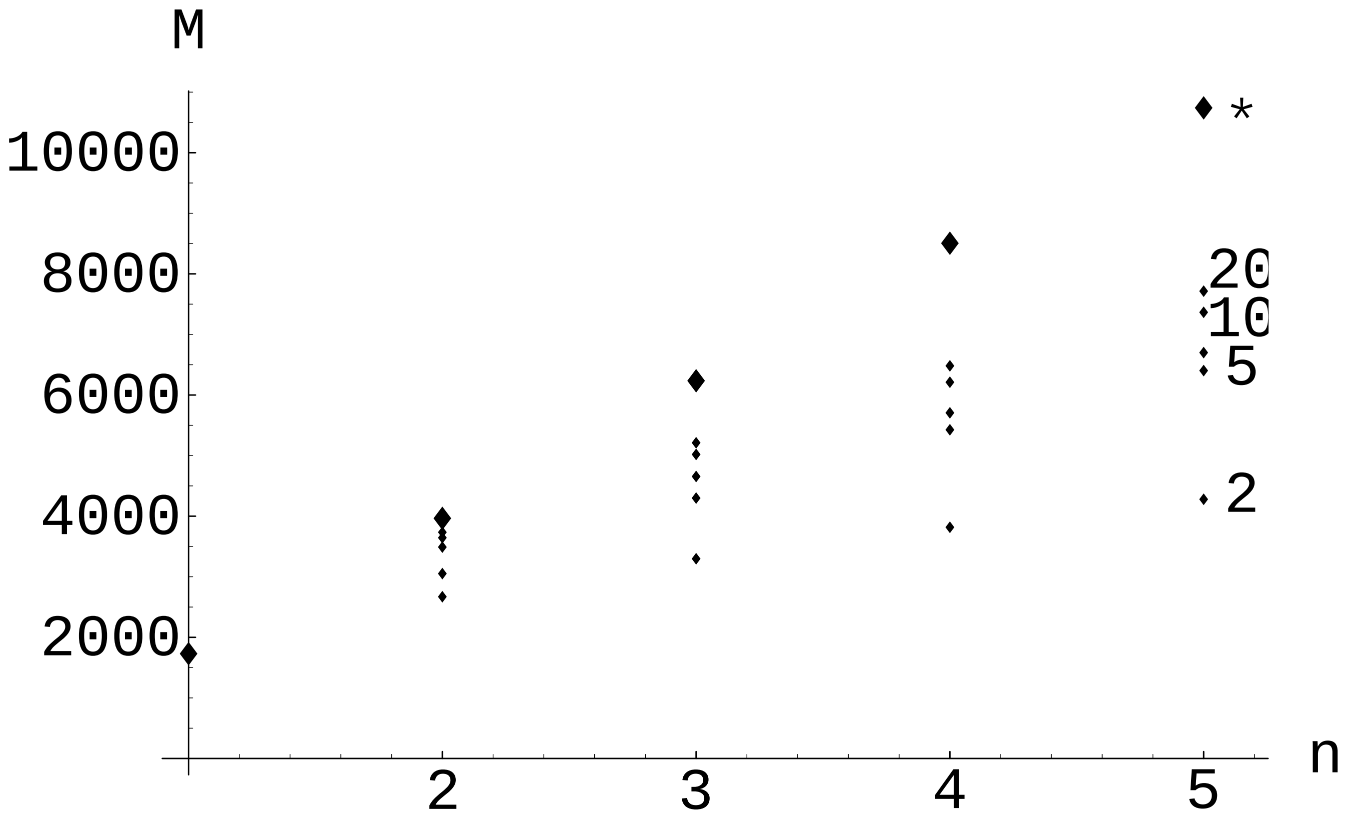
Effective Schrödinger potentials for scalar (solid line) and tensor (dashed line) glueballs. The units are chosen such that $\ell = 0.5$.

The lattice glueball data

J^{++}	Ref. I $(m/\sqrt{\sigma})$	Ref. I (MeV)	Ref. II (mr_0)	Ref. II (MeV)	$N_c \to \infty (m/\sqrt{\sigma})$
0	3.347(68)	1475(30)(65)	4.16(11)(4)	1710(50)(80)	3.37(15)
0^*	6.26(16)	2755(70)(120)	6.50(44)(7)	2670(180)(130)	6.43(50)
0^{**}	7.65(23)	3370(100)(150)	NA	NA	NA
0^{***}	9.06(49)	3990(210)(180)	NA	NA	NA
2	4.916(91)	2150(30)(100)	5.83(5)(6)	2390(30)(120)	4.93(30)
2^*	6.48(22)	2880(100)(130)	NA	NA	NA
R_{20}	1.46(5)	1.46(5)	1.40(5)	1.40(5)	1.46(11)
R_{00}	1.87(8)	1.87(8)	1.56(15)	1.56(15)	1.90(17)

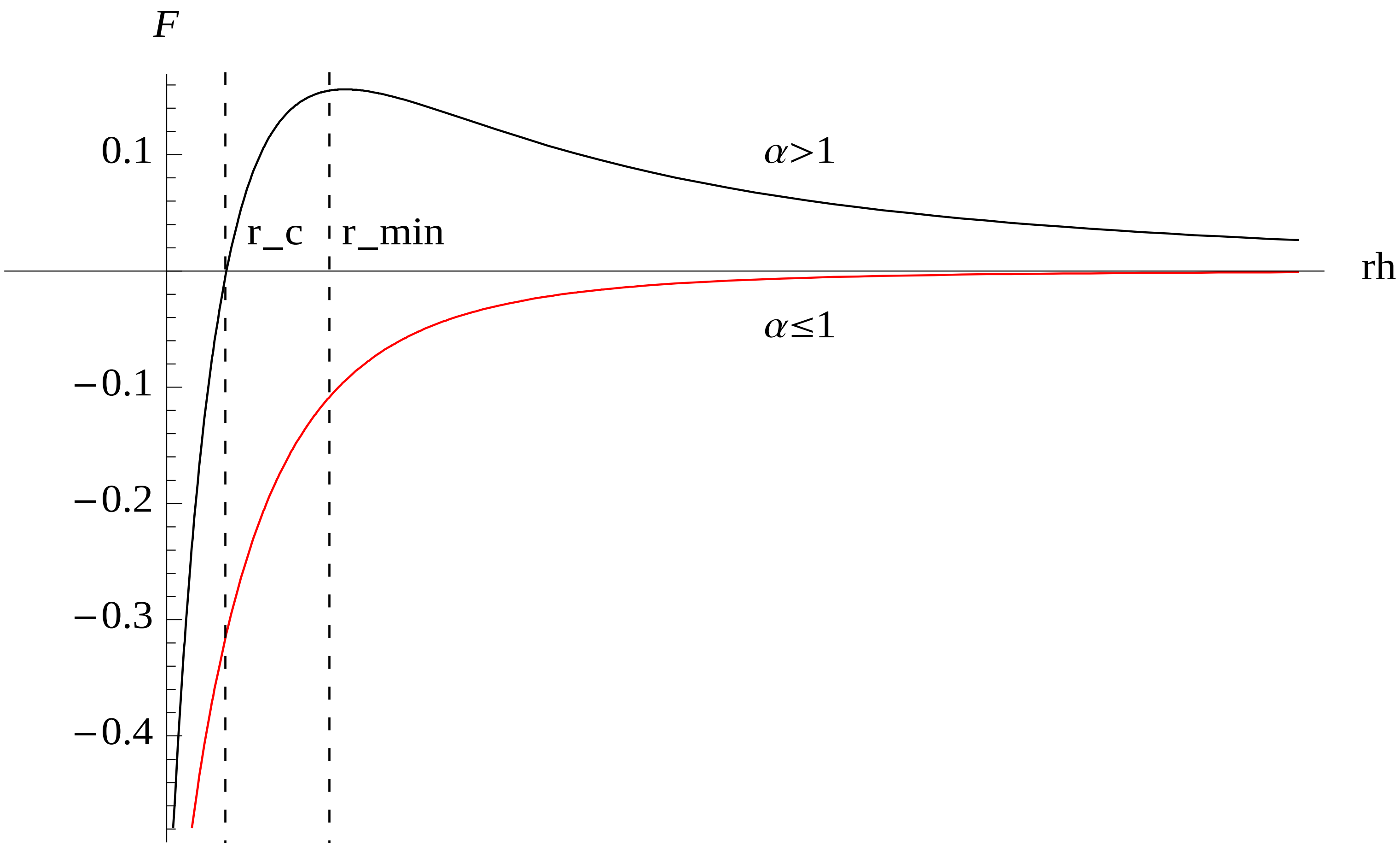
Available lattice data for the scalar and the tensor glueballs. Ref. I =H. B. Meyer, [arXiv:hep-lat/0508002]. and Ref. II = C. J. Morningstar and M. J. Peardon, [arXiv:hep-lat/9901004] + Y. Chen *et al.*, [arXiv:hep-lat/0510074]. The first error corresponds to the statistical error from the the continuum extrapolation. The second error in Ref.I is due to the uncertainty in the string tension $\sqrt{\sigma}$. (Note that this does not affect the mass ratios). The second error in the Ref. II is the estimated uncertainty from the anisotropy. In the last column we present the available large N_c estimates according to B. Lucini and M. Teper, [arXiv:hep-lat/0103027]. The parenthesis in this column shows the total possible error followed by the estimations in the same reference.

α-dependence of scalar spectrum



The 0^{++} spectra for varying values of α that are shown at the right end of the plot. The symbol $*$ denotes the AdS/QCD result.

Free energy versus horizon position



We plot the relation $\mathcal{F}(r_h)$ for various potentials parameterized by a. $a = 1$ is the critical value below which there is no first order phase transition .

The bulk viscosity: theory

- It is one of the important parameters for QGP hydrodynamics (along with the shear viscosity).
- It is related to entropy production (measurable at RHIC and LHC)
- It is defined from the Kubo formula

$$\zeta = \frac{1}{9}\lim_{\omega\to 0}\frac{1}{\omega} Im\ G_R(\omega) \quad , \quad G_R(\omega) \equiv \int d^3x \int dt\ e^{i\omega t}\theta(t)\ \langle 0|[T_{ii}(\vec{x},t),T_{ii}(\vec{0},0)]|0\rangle$$

Using a parametrization $ds^2 = e^{2A}(f dt^2 + d\vec{x}^2 + \frac{dr^2}{f})$ in a special gauge $\phi = r$ the relevant metric perturbation decouples

Gubser+Nellore+Pufu+Rocha

$$h''_{11} = -\left(-\frac{1}{3A'} - A' - \frac{f'}{f}\right) h'_{11} + \left(-\frac{\omega^2}{f^2} + \frac{f'}{6fA'} - \frac{f'}{f}A'\right) h_{11}$$

with

$$h_{11}(0) = 1 \quad , \quad h_{11}(r_h) \simeq C\ e^{i\omega t}\left|\log\frac{\lambda}{\lambda_h}\right|^{-\frac{i\omega}{4\pi T}}$$

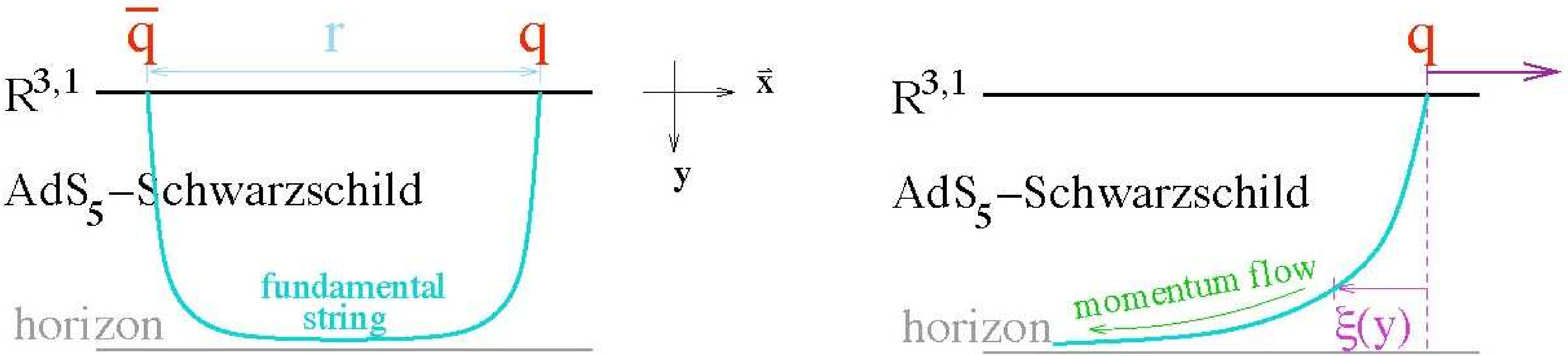
The correlator is given by the conserved number of h-quanta

$$Im\ G_R(\omega) = -4M^3\mathcal{G}(\omega) \quad , \quad \mathcal{G}(\omega) = \frac{e^{3A}f}{4A'^2}|Im[h^*_{11}h'_{11}]|$$

finally giving

$$\frac{\zeta}{s} = \frac{C^2}{4\pi}\left(\frac{V'(\lambda_h)}{V(\lambda_h)}\right)^2$$

The drag force (theory)



- We must find a solution to the string equations with

$$x^1 = vt + \xi(r) \quad , \quad x^{2,3} = 0 \quad , \quad \sigma^1 = t \quad , \quad \sigma^2 = r$$

Herzog+Karch+kovtun+Kozcac+Yaffe, Gubser

Casaldelrrey-Solana+Teaney, Liu+Rajagopal+Wiedeman

For a black-hole metric (in string frame)

$$ds^2 = b(r)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + d\vec{x} \cdot d\vec{x} \right]$$

the solution profile is

$$\xi'(r) = \frac{C}{f(r)}\sqrt{\frac{f(r)-v^2}{b^4(r)f(r)-C^2}} \quad , \quad C = vb(r_s)^2 \quad , \quad f(r_s) = v^2$$

• The induced metric on the world-sheet is a 2d black-hole with horizon at the turning point $r = r_s$.

• We can calculate the drag force:

$$F_{\text{drag}} = \pi_\xi = -\frac{b^2(r_s)\sqrt{f(r_s)}}{2\pi\ell_s^2}$$

• In $\mathcal{N} = 4$ sYM it is given by

$$F_{\text{drag}} = -\frac{\pi}{2}\sqrt{\lambda}\ T^2\frac{v}{\sqrt{1-v^2}} = -\frac{1}{\tau}\frac{p}{M} \quad , \quad \tau = \frac{2M}{\pi\sqrt{\lambda}\ T^2}$$

Detailed plan of the presentation

- Title page 1 minutes
- Quantum Chromodynamics 2 minutes
- Confinement 4 minutes
- Deconfinement 6 minutes
- The experimental hunt for QGP 8 minutes
- RHIC 9 minutes
- RHIC head-on collision 10 minutes
- RHIC collision: another view 11 minutes
- Phases of a collision 13 minutes
- What we cannot calculate from first principles 16 minutes
- Gauge theories with many colors 18 minutes
- The gauge-theory/gravity duality 22 minutes
- The gauge-theory at finite temperature 23 minutes
- A quick preview for the rest 24 minutes

- What are we after? 2 minutes
- A string theory for QCD:basic expectation 5 minutes
- Bosonic string or superstring? 7 minutes
- The minimal string theory spectrum 9 minutes
- Effective action I 11 minutes
- The UV regime 16 minutes
- The IR regime 19 minutes
- Improved Holographic QCD: a model 24 minutes
- An assessment of IR asymptotics 27 minutes
- Selecting the IR asymptotics 29 minutes
- Adding flavor 35 minutes
- The axion background 38 minutes

FINITE TEMPERATURE