Systems of Uniformization Equations and Hyperelliptic Integrals

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Abstract: The main subject of this talk is systems of uniformization equations along logarithmic free divisors which have solutions expressed in terms of hyperelliptic integrals. Such a logarithmic free divisor in $\mathbf{C}^3$ is related with the discriminant of a dihedral group of order $2n$. We construct fundamental solutions by Gaussian hypergeometric functions in addition to a solution expressed by a hyperelliptic integral, say v. Let u_1, u_2 be fundamental solutions of the hypergeometric equation implied by the system in question. Then the image of the upper half plane by the map defined by the ratio u_1/u_2 is a hyperbolic triangle in the upper half plane with angles $0, \pi/2, \pi/n$. This is a fundamental domain of Hecke's triangle group. We now consider the case of reflection groups of types A_2, B_2, G_2. These are examples of dihedral groups. In these cases, the the inverse of the map $\varphi = (u_1, u_2, v)$ of $\mathbf{C}^3$ to $\mathbf{C}^3$ is concretely constructed in terms of elliptic functions and Eisenstein series. The case A_2 was already obtained by K. Saito who initiated and developed the theory of logarithmic free divisors and formulated the notion of the systems of uniformization equations along logarithmic free divisors extending the study of H. A. Schwarz on hypergeometric differential equations. The result in this talk is an extension of his work to the case of the dihedral groups.