Alternative production mechanism of sterile neutrino dark matter

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IPMU Seminar

Based on JCAP 1806, 036 (2018) [arXiv:1802.02973 [hep-ph]].

In collaboration with Johannes Herms, Alejandro Ibarra

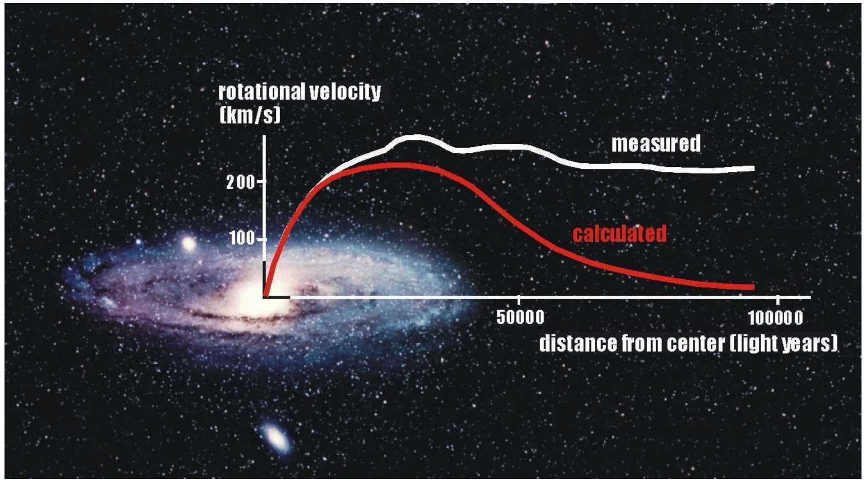
Outline

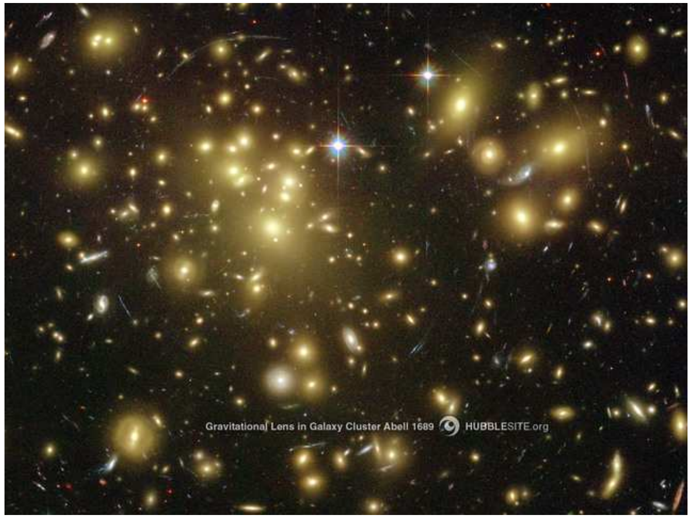
1. Introduction of dark matter
   - WIMPs (Weakly Interacting Massive Particles)
   - SIMPs (Strongly Interacting Massive Particles)
2. Sterile neutrino dark matter as a SIMP
   - Effective self-interaction
   - Model with a scalar mediator
3. Summary

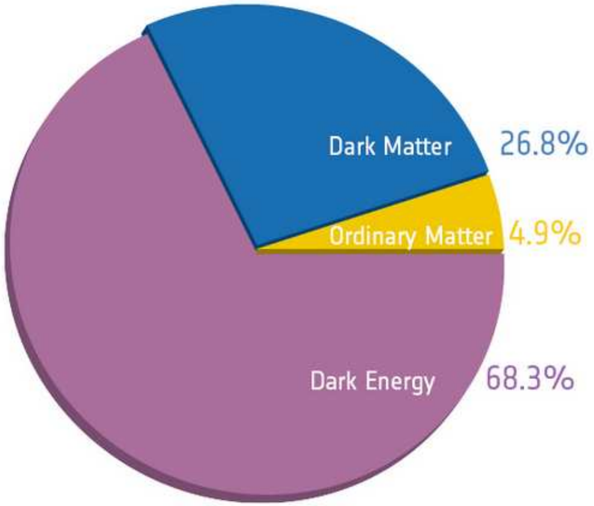
Dark Matter

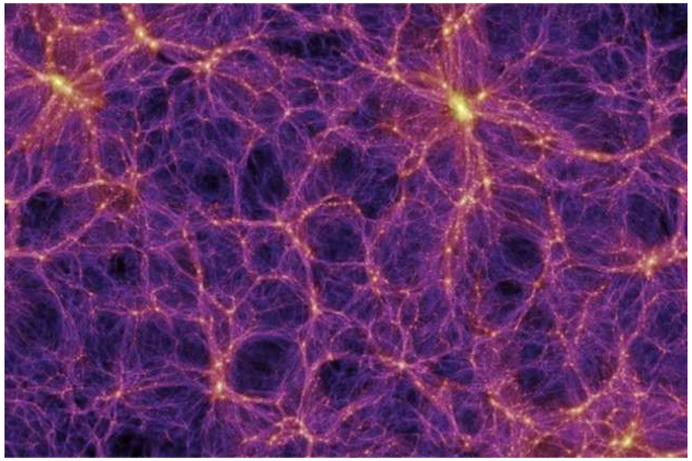
There are evidence of dark matter.

- Rotation curves of spiral galaxies
- CMB observation
- Gravitational lensing
- Large scale structure of universe
- Collision of bullet cluster









Dark Matter

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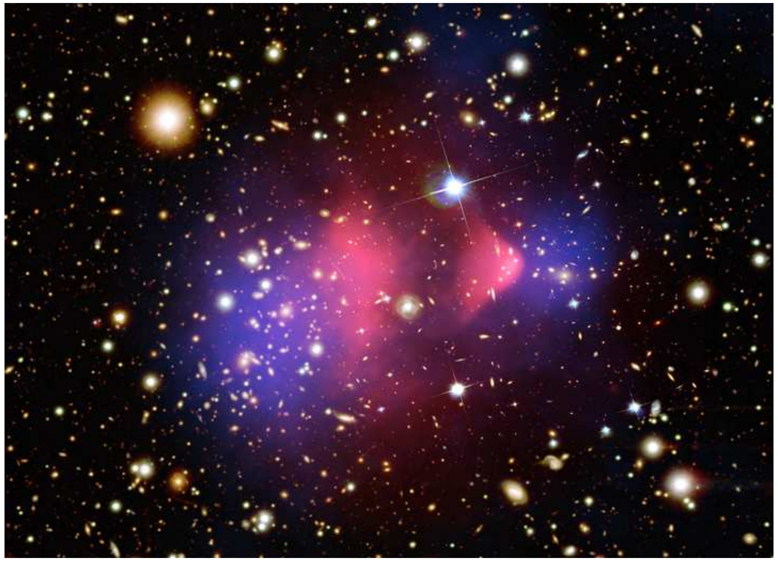
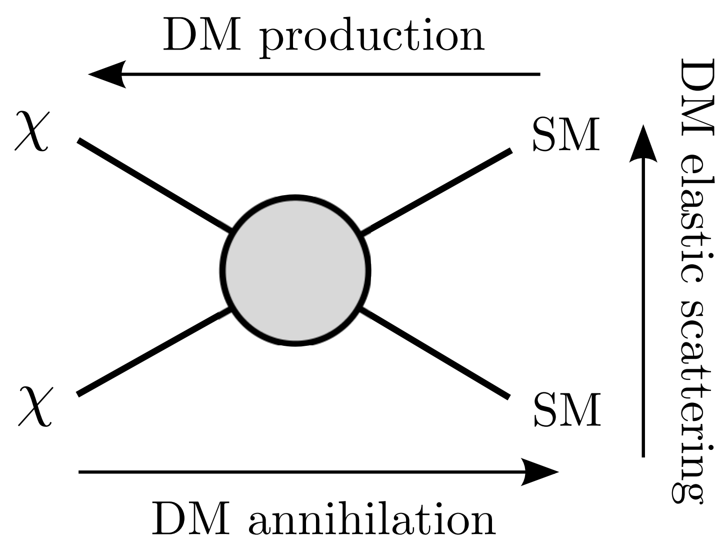


Existence of DM is crucial.

$\Rightarrow$ Plausible DM candidate: WIMPs

- Basic ways for WIMP detection
  - · Indirect detection
  - · Direct detection
  - · Collider search
  - $\rightarrow$ correlated with each other

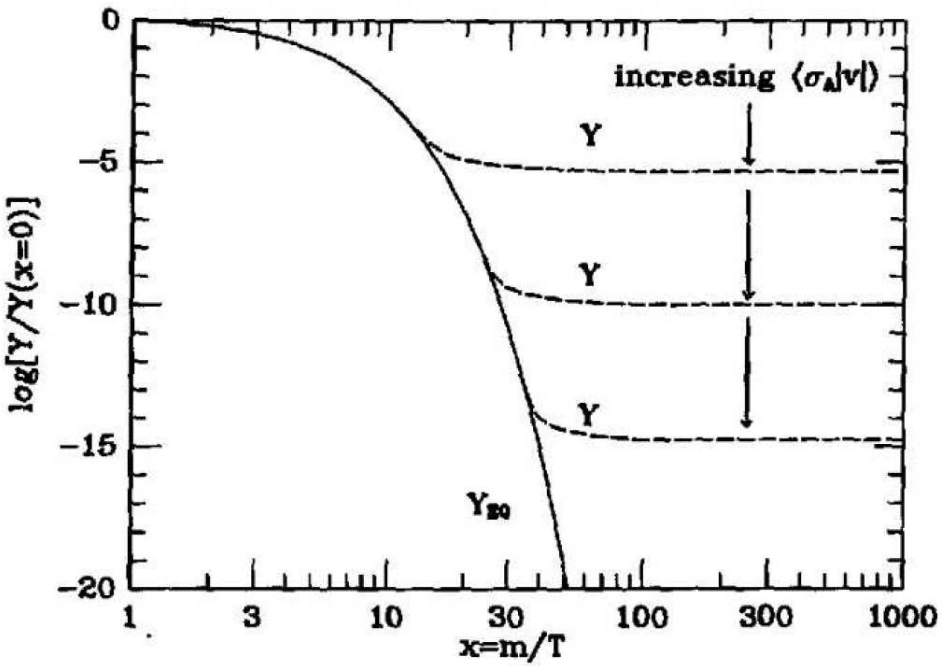


WIMP production (review)

Evolution of DM number density is followed by Boltzmann equation

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma v \rangle \left(n_\chi^2 - n_\chi^{\mathrm{eq}2} \right)$$

- Rough estimate: $\Gamma > H \leftrightarrow$ thermal eq.
  $\Gamma < H \leftrightarrow$ decoupled (freeze-out)
- Freeze-out temperature: $x_f = m_\chi / T_f \sim 23$

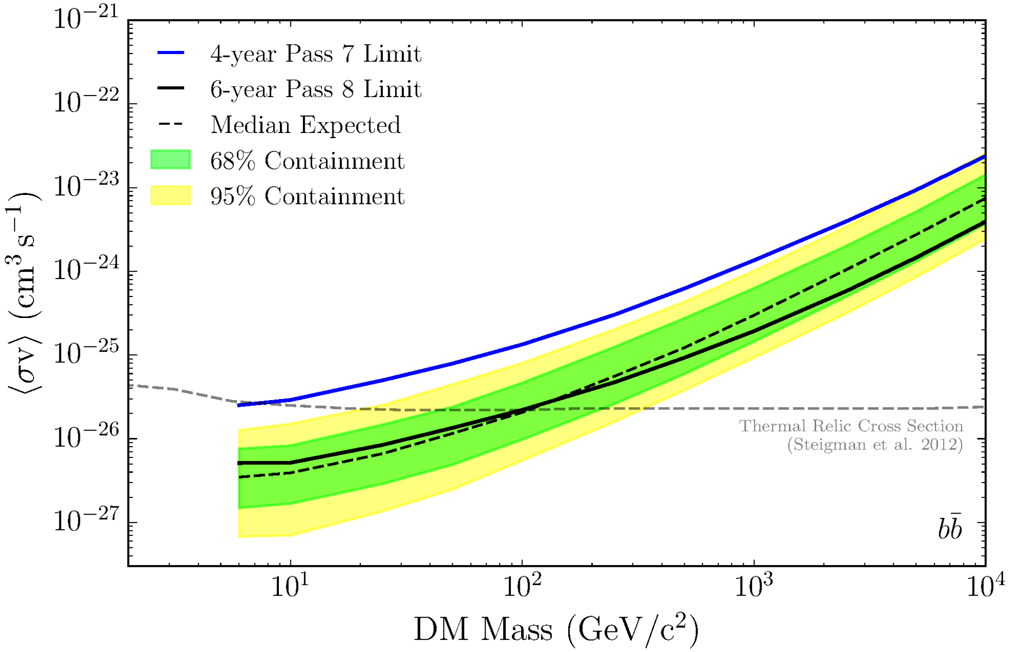


- Relic abundance is determined by $\langle \sigma v \rangle$.
- σv can be expanded by v.
  $\rightarrow \sigma v = a + bv^2 + \mathcal{O}\left(v^4\right)$
  a: s-wave, b: p-wave
- $\Omega h^2 \sim \dfrac{10^{-10}\ [\mathrm{GeV}^{-2}]}{\langle \sigma v \rangle} \sim 0.1$
  (Planck Coll.)

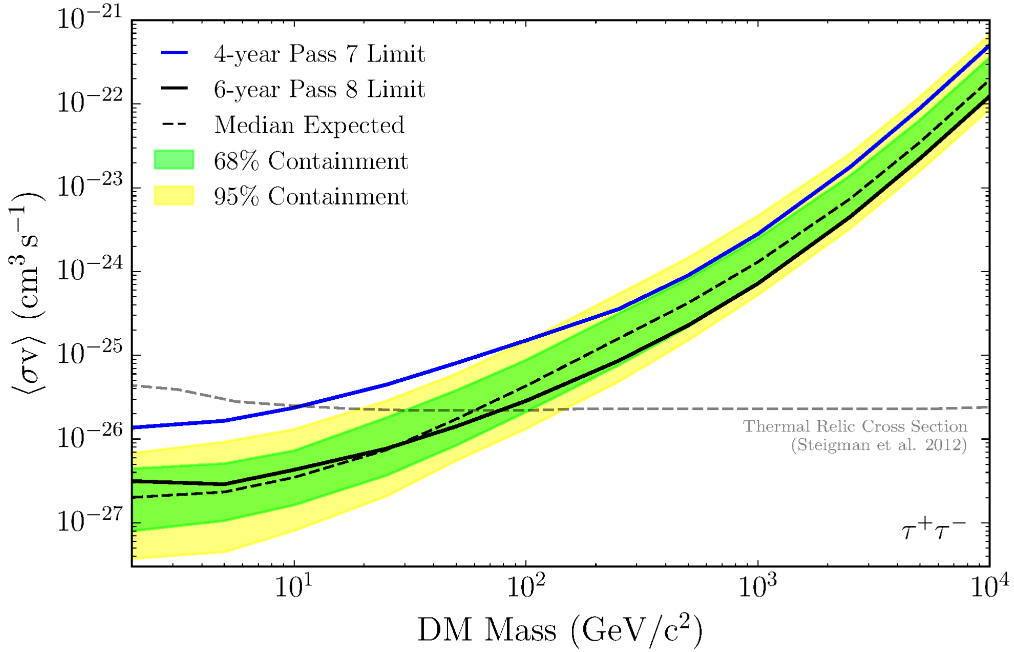
Experimental status for WIMP searches

Indirect detection (ex. gamma-rays from dSphs)

$\chi\chi \to b\bar{b}$



$\chi\chi \to \tau^+\tau^-$

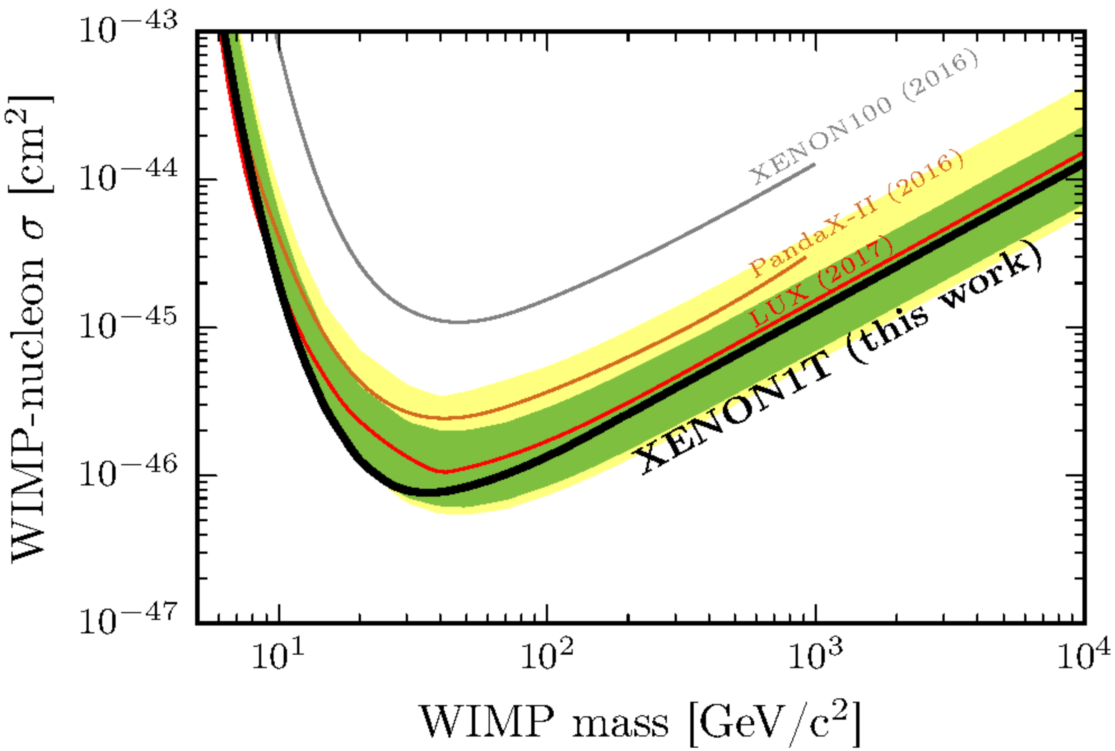


Fermi-LAT, PRL (2015) arxiv:1503.02641

- $m_{\rm DM} \lesssim 100$ GeV is excluded.

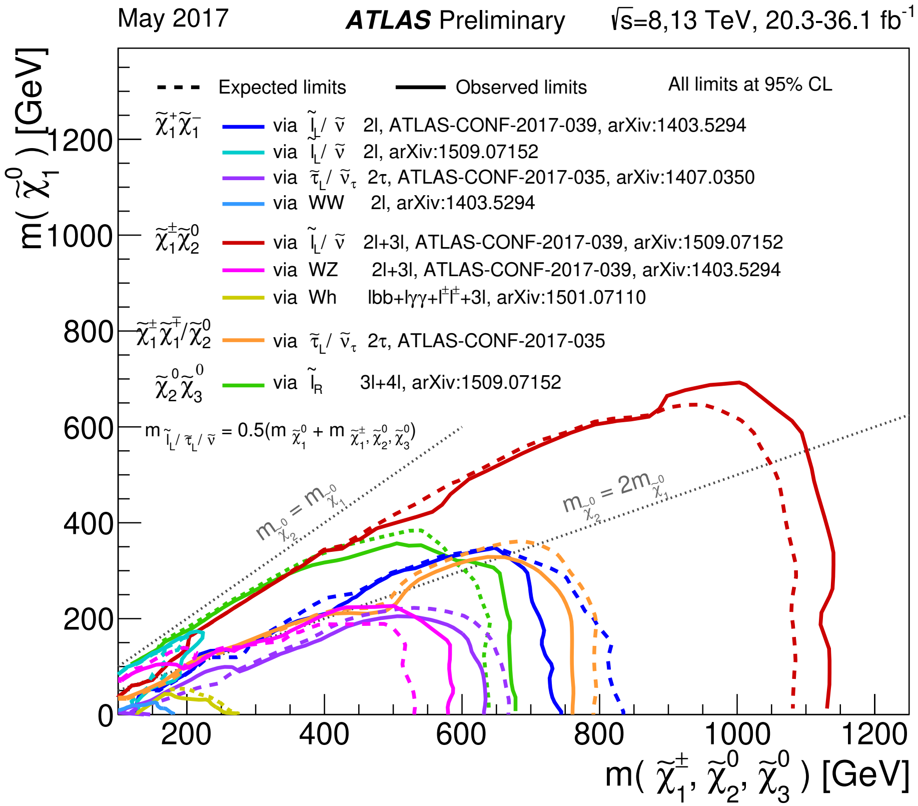
Experimental status for WIMP searches

Direct detection



XENON1T, PRL (2017) arxiv:1705.06655

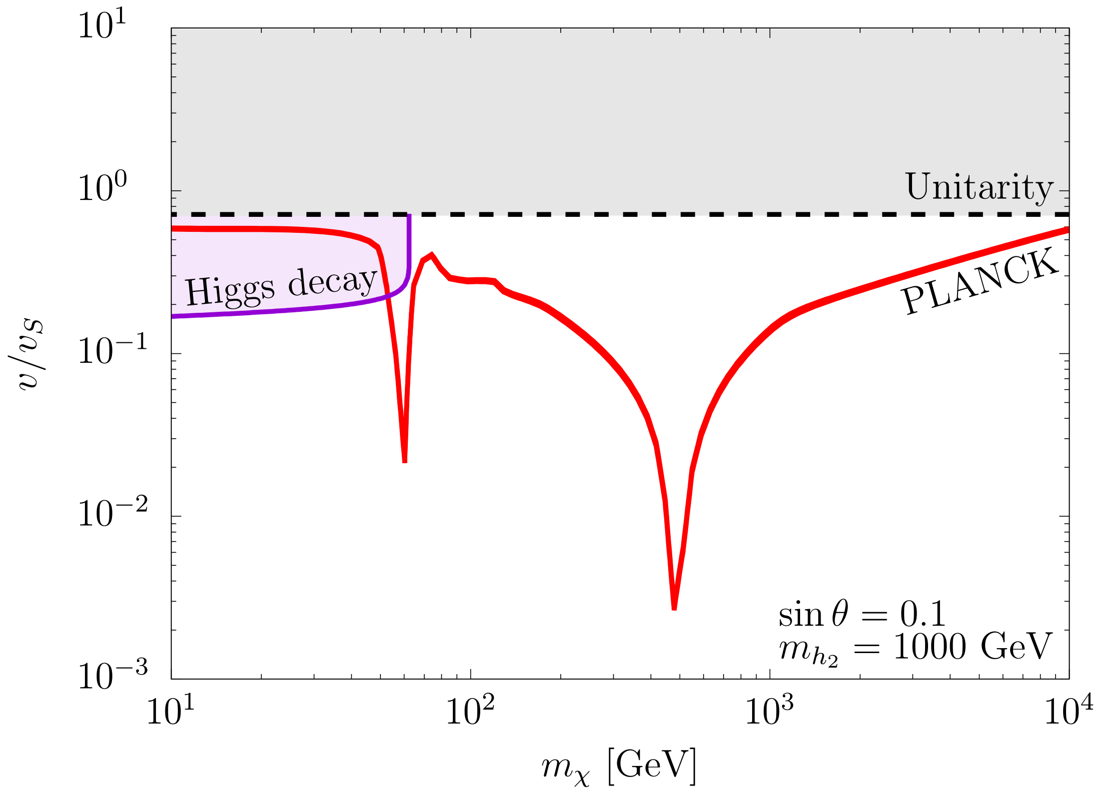
Collider search

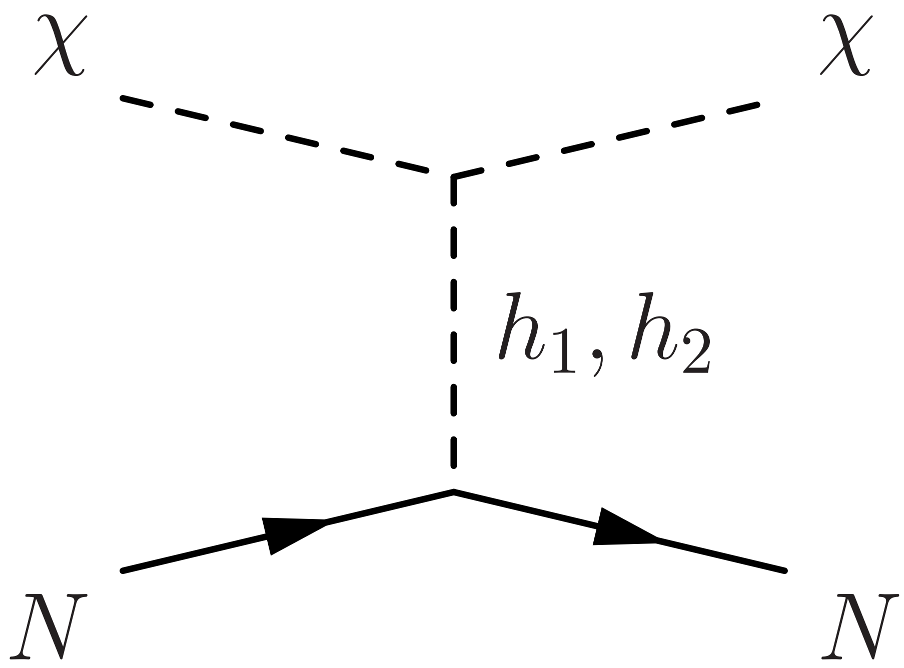


Summary plot from ATLAS

- Experimental constraints are stronger and stronger.
- Interactions between DM and SM are weak enough?

One possibility to evade strong DD constraint





C. Gross, O. Lebedev, TT, PRL (2017) [arXiv:1708.02253]

- SM + complex scalar + global U(1)
- Pseudo-scalar component is DM $[S = (v_s + s + i\chi)/\sqrt{2}]$
- Cancellation of total amplitude mediated by Higgs bosons

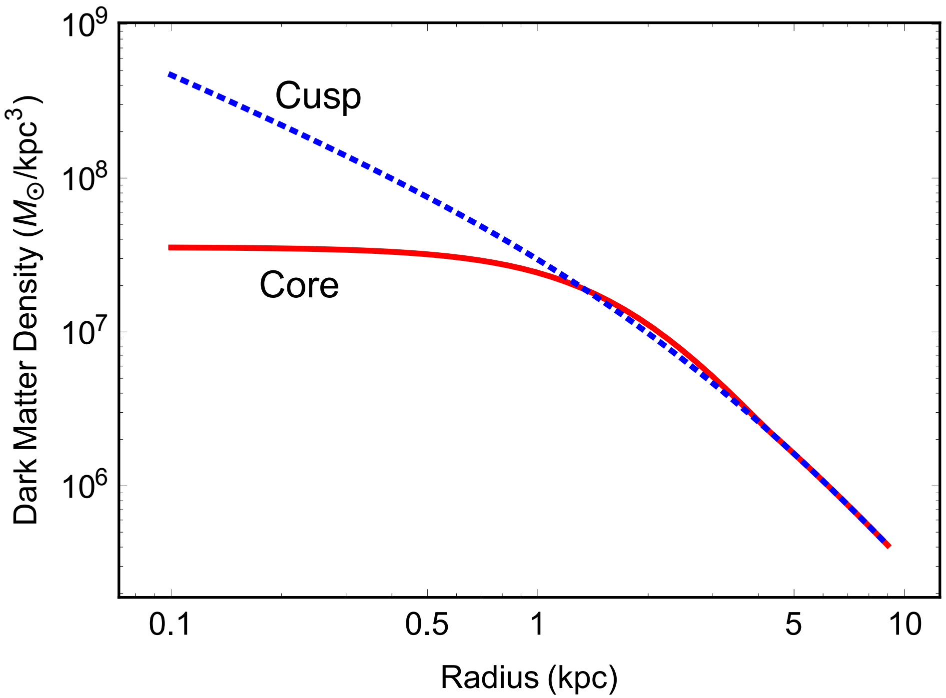
Small scale problems

- Cusp vs core problem
  N-body simulation infers cusp DM profile at centre of galaxies $\rho_{\rm DM} \propto r^{-1}$.
  But rotation of spiral galaxies prefers core profile $\rho_{\rm DM} \sim$ const.

- Missing satellite problem
- Too big to fail problem

  etc

  arXiv:1705.02358, Tulin and Yu



Possible solutions

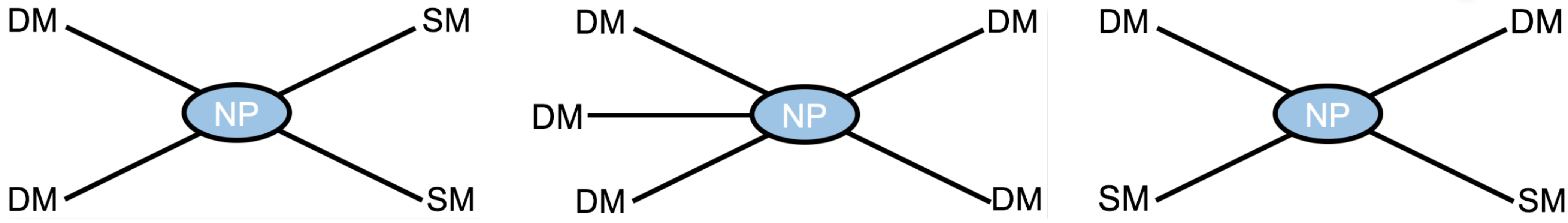
- Add baryon contribution
- DM self-interaction

SIMP (Strongly Interacting Massive Particle)

Y. Hochberg et al. PRL (2014) [arxiv:1402.5143]

- DM abundance is determined by $3 \to 2$ or $4 \to 2$ annihilations (final state is also DM).
- Assume that DM and SM are thermal eq. $\Rightarrow$ DM temperature is same with SM one

Thermal SIMP condition: $\Gamma_{\text{ann}} < \Gamma_{3\to 2} < \Gamma_{\text{kin}}$



Normal annihilation 3 to 2 process Elastic scattering

- Typical thermal SIMP mass: $m_\chi \sim \mathcal{O}(10)$ MeV for $3 \to 2$ process ($m_\chi \sim \mathcal{O}(100)$ keV for $4 \to 2$ process)

Small scale problems can be alleviated by large self-interaction of DM.

Models for SIMPs

- Dark QCD (Bound state DM)

  arXiv:1411.3727, Y. Hochberg, E. Kuflik, H. Murayama, T. Volansky and J. G. Wacker

  arXiv:1512.07917, Y. Hochberg, E. Kuflik and H. Murayama etc
- Large, but still perturbative couplings

  arXiv:1501.01973, N. Bernal, C. Garcia-Cely, R. Rosenfeld

  arXiv:1505.00960, S. M. Choi and H. M. Lee etc
- Model combining neutrino masses generation

  arXiv:1705.00592, S. Ho, TT, K. Tsumura
- Dirac SIMP (arXiv:1604.02401, 1704.05359, M. Heikinheimo et al.)

SIMP candidate is a scalar or vector boson in most of the models.

Note that 3-to-2 does not work for fermion because of Lorentz invariance (simplest case).

Sterile neutrino SIMP (singlet Majorana fermion)

Singlet fermion SIMP (Effective Model)

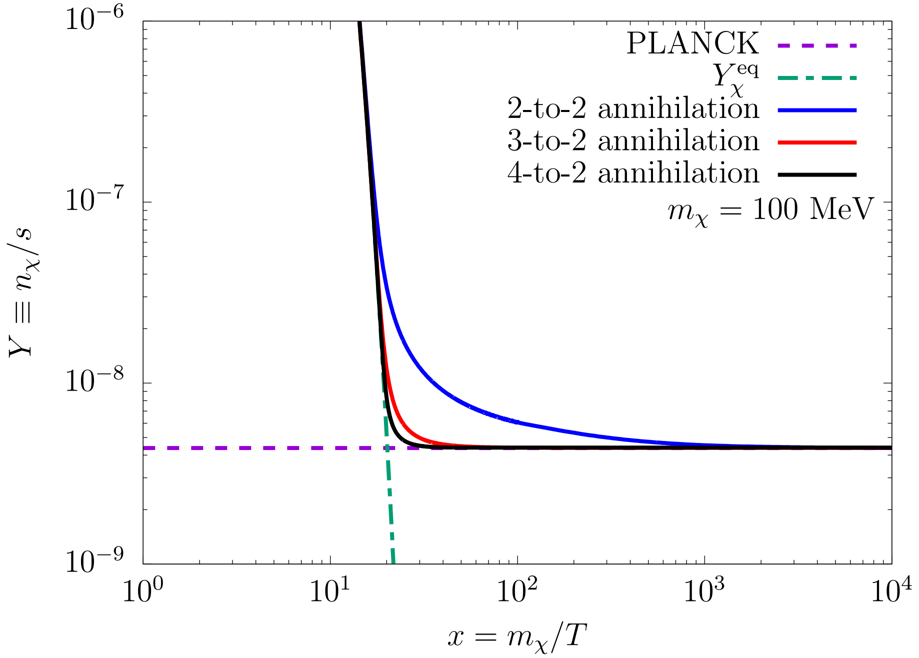
- Consider effective interaction of Majorana fermion χ (DM).

$$\mathcal{L} = \frac{1}{4!\Lambda^2} \left(\overline{\chi^c}\chi\right) \left(\overline{\chi^c}\chi\right) - y_\nu H \overline{\chi} L$$

- χ is weakly coupled with SM via y_ν.
- DM abundance is determined by 4-to-2 process ($\chi\chi\chi\chi \to \chi\chi$).
- If DM is in thermal eq. with SM
  $\to m_\chi \sim 100$ keV. $\to$ freeze-out temperature $T_f < 1$ MeV
  $\to$ conflict with BBN, CMB observation.
- Non-thermal eq. ($T \neq T'$) via small y_ν.
- Boltzmann equation

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma v^3 \rangle \left(n_\chi^4 - n_\chi^2 n_\chi^{\mathrm{eq}2}\right)$$

Instantaneous Freeze-out Approximation

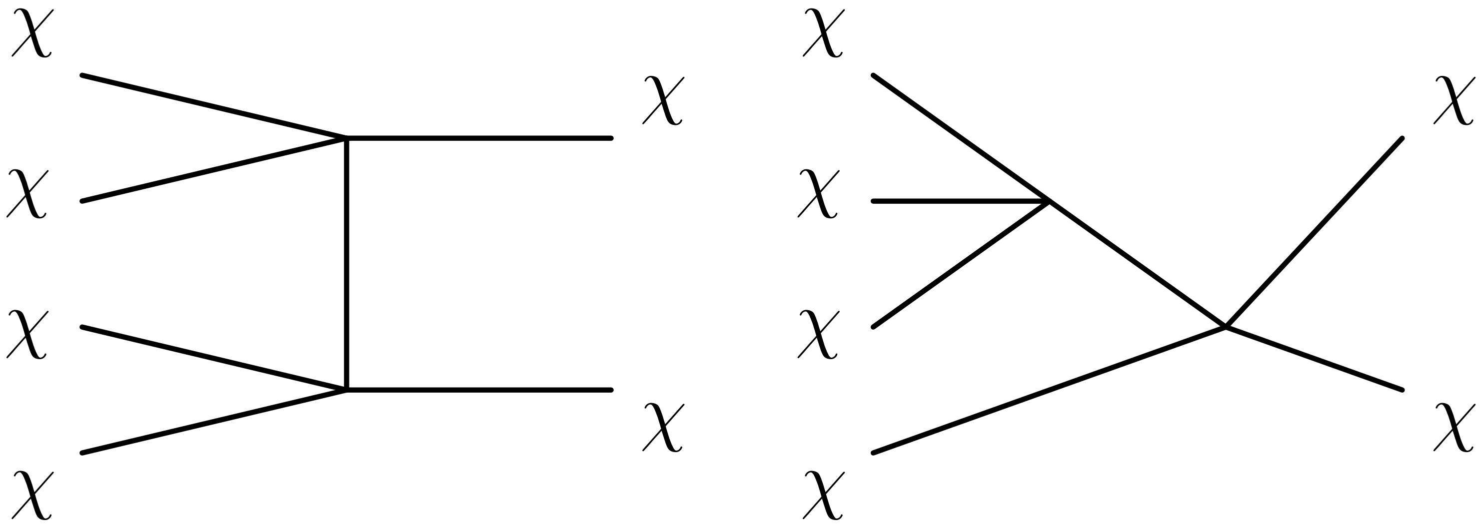


- Generally coupled Boltzmann equations have to be solved.
- Instantaneous freeze-out approximation

$$\Gamma_{4\to 2}(x'_f) = H(x_f),$$
$$Y_\chi(\infty) = Y_\chi(x'_f)$$

- Instantaneous freeze-out approximation is very accurate.
  $\Gamma_{4\to 2} \equiv \langle \sigma v^3 \rangle n_\chi^3 \propto T'^2 T^9$ (cf: $\Gamma_{2\to 2} \equiv \langle \sigma v \rangle n_\chi \propto T^3$)
- For a given $(\langle \sigma v^3 \rangle,\ x'_f) \Rightarrow \Omega_\chi h^2$
  DM abundance is determined by two parameters $\langle \sigma v^3 \rangle$ and freeze-out temperature x'_f

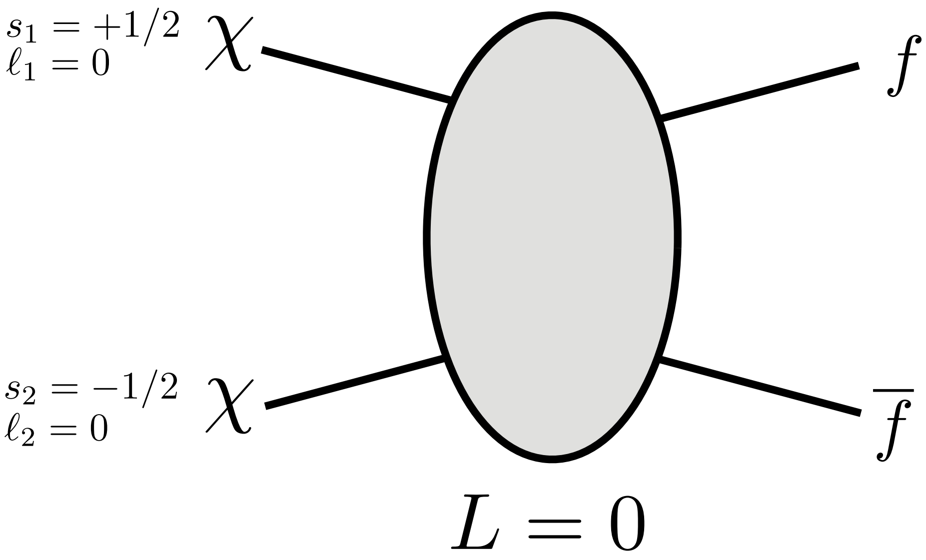
Computation of 4-to-2 cross section

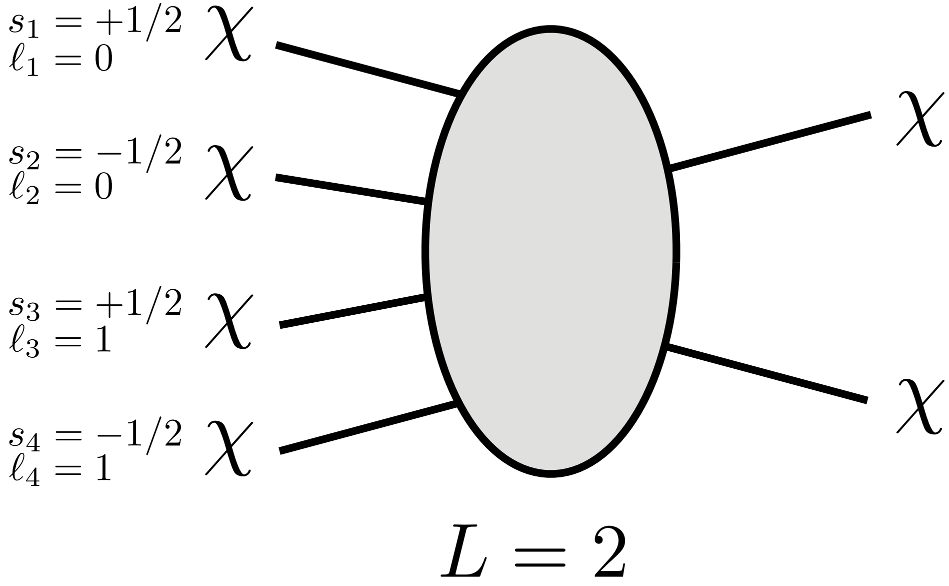


- ~ 100 Feynman diagrams exist.
  $\rightarrow$ classified to 2 topology.
- Hard to compute by hand.
- Possible to compute with FeynCalc if non-relativistic.
- First diagram $\sim \overline{v}(p_4)u(p_3)\overline{v}(p_2)u(p_1) \sim v^2 \rightarrow \overline{|\mathcal{M}|^2} \sim v^4$
  Naively second diagram is $\rightarrow \overline{|\mathcal{M}|^2} \propto v^2$,
- But total amplitude square is proportional to v^4 (d-wave).

Computation of 4-to-2 cross section

- 4-to-2 cross section of Majorana DM is suppressed by d-wave due to Pauli exclusion principle.
- Do not depend on interactions





- cf: Helicity suppression
  annihilation for Majorana DM: $\chi\chi \to f\overline{f}$ is suppressed by p-wave if $m_f \to 0$

Thermal average of cross section

Choi, Lee, Seo, JHEP [arXiv:1702.07860]

Definition of thermal average

$$\langle \sigma v^3 \rangle = \frac{\int d^3v_1 d^3v_2 d^3v_3 d^3v_4 \left(\sigma v^3\right) e^{-\frac{x'}{2}\left(v_1^2+v_2^2+v_3^2+v_4^2\right)}}{\int d^3v_1 d^3v_2 d^3v_3 d^3v_4 e^{-\frac{x'}{2}\left(v_1^2+v_2^2+v_3^2+v_4^2\right)}}$$
$$= a + bx'^{-1} + cx'^{-2} + \mathcal{O}(x'^{-3})$$

where $x' = m_\chi / T'$.

- In case of Majorana DM $\Rightarrow a = b = 0$ (d-wave).
- $\sigma v^3 = a_i v_i^4 + a_{ij} v_i^2 v_j^2 + b_{ij} \left(\boldsymbol{v}_i \cdot \boldsymbol{v}_j\right)^2 + c_{ijk} v_i^2 \left(\boldsymbol{v}_j \cdot \boldsymbol{v}_k\right) + \cdots$

Thermal averaged of cross section

Choi, Lee, Seo, JHEP [arXiv:1702.07860]

Velocity averages:

$$\langle v_i^4 \rangle = \frac{135}{16}\frac{1}{x'^2}, \qquad \langle v_i^2 v_j^2 \rangle = \frac{87}{16}\frac{1}{x'^2}$$

$$\langle v_i^2 (\boldsymbol{v}_i \cdot \boldsymbol{v}_j) \rangle = -\frac{45}{16}\frac{1}{x'^2}, \qquad \langle v_i^2 (\boldsymbol{v}_j \cdot \boldsymbol{v}_k) \rangle = -\frac{21}{16}\frac{1}{x'^2},$$

$$\langle (\boldsymbol{v}_i \cdot \boldsymbol{v}_j)^2 \rangle = \frac{39}{16}\frac{1}{x'^2}, \qquad \langle (\boldsymbol{v}_i \cdot \boldsymbol{v}_j)(\boldsymbol{v}_i \cdot \boldsymbol{v}_k) \rangle = \frac{3}{16}\frac{1}{x'^2},$$

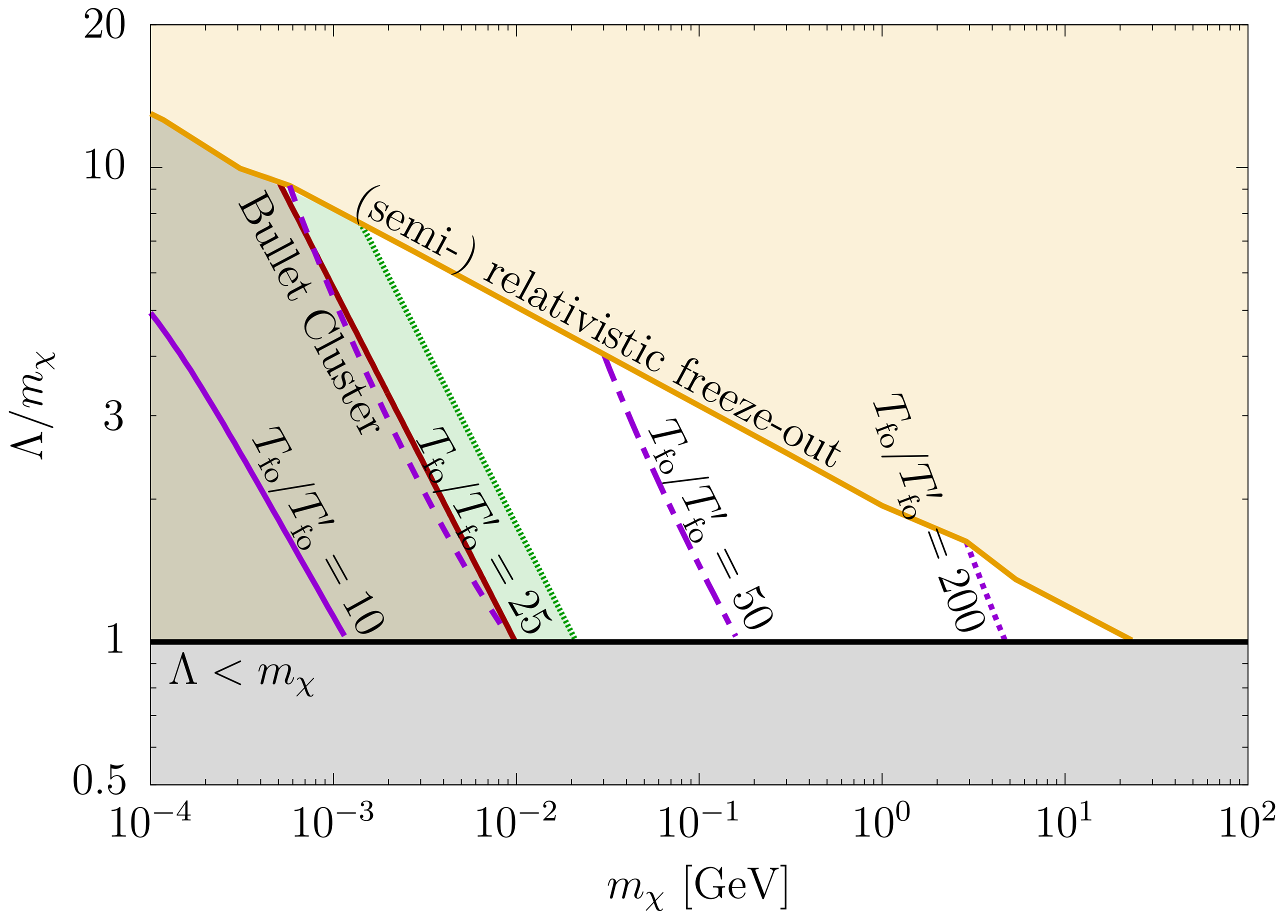
$$\langle (\boldsymbol{v}_i \cdot \boldsymbol{v}_j)(\boldsymbol{v}_k \cdot \boldsymbol{v}_\ell) \rangle = \frac{15}{16}\frac{1}{x'^2}, \qquad \langle \frac{(\boldsymbol{v}_i \cdot \boldsymbol{v}_j)(\boldsymbol{v}_i \cdot \boldsymbol{v}_k)(\boldsymbol{v}_i \cdot \boldsymbol{v}_\ell)}{v_i^2} \rangle = \frac{7}{16}\frac{1}{x'^2}$$

where $i, j, k, \ell = 1, 2, 3, 4$ and $i \neq j \neq k \neq \ell$.

$\rightarrow$ Final result

$$\langle \sigma v^3 \rangle = \frac{1201}{245760\sqrt{3}\pi \Lambda^8 x'^2}$$

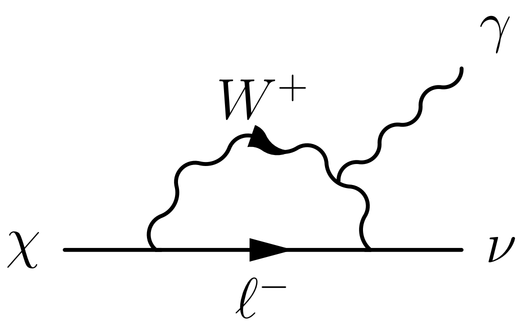
Numerical analysis

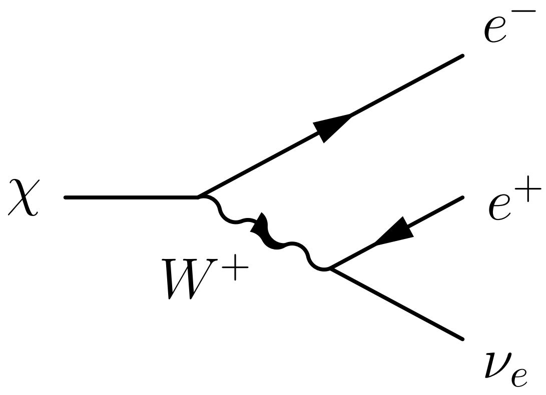


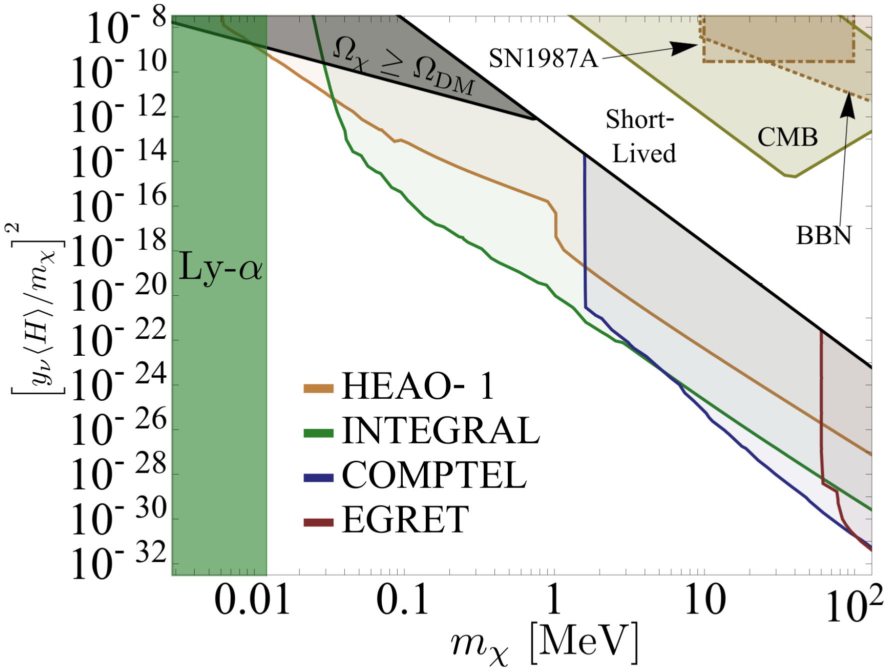
- Relativistic decoupling ($x'_f < 3$) is not considered here.
- Constraint of collision of bullet cluster: $\sigma_{\rm self}/m_\chi = m_\chi/(72\pi\Lambda^4) \leq 1\ {\rm cm}^2/{\rm g}$.
- DM mass range: $500\ {\rm keV} \lesssim m_\chi \lesssim 20\ {\rm GeV}$

Gamma-ray emission

$\chi \to \nu\gamma$, $\chi \to e^+e^-\nu$ occurs through y_ν, gamma-rays are produced.







Essig et al., arXiv:1309.4091

$$\Gamma_{\chi\to\nu\gamma} = \frac{9\alpha_{\rm em} G_F^2 m_\chi^5}{(4\pi)^4} \left(\frac{y_\nu \langle H \rangle}{m_\chi}\right)^2,$$

$$\Gamma_{\chi\to e^+e^-\nu} = \frac{0.59 G_F^2 m_\chi^5}{24\pi^3} \left(\frac{y_\nu \langle H \rangle}{m_\chi}\right)^2$$

$\to$ Constraint: $y_\nu \lesssim 10^{-16} \left(\dfrac{\rm MeV}{m_\chi}\right)^{3/2}$

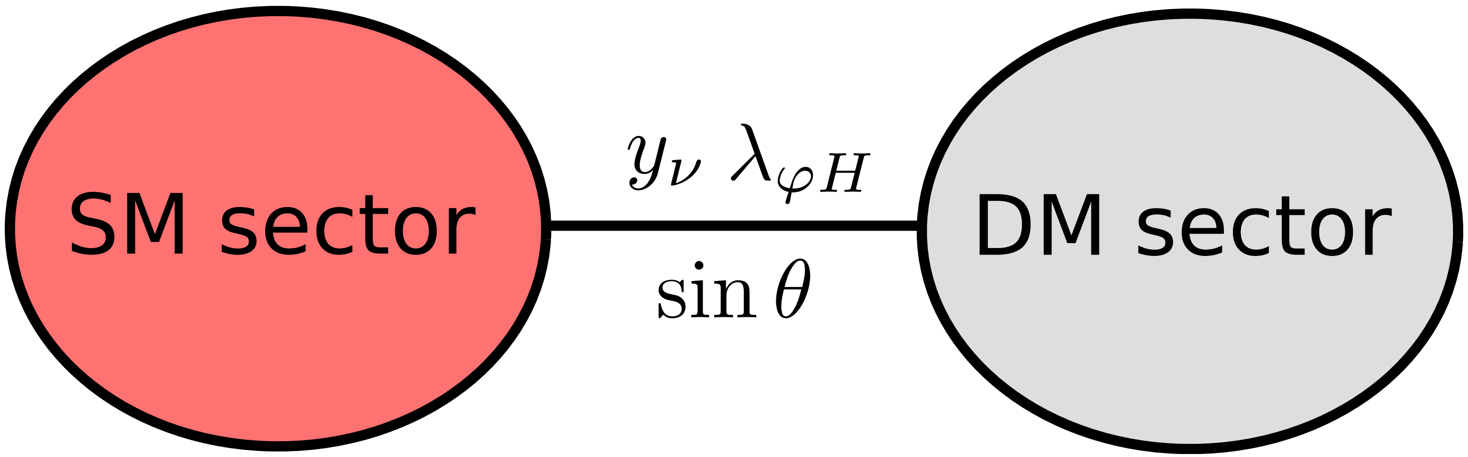
$\to$ $\mathbb{Z}_2$ symmetry?

Singlet fermion SIMP (toy model)

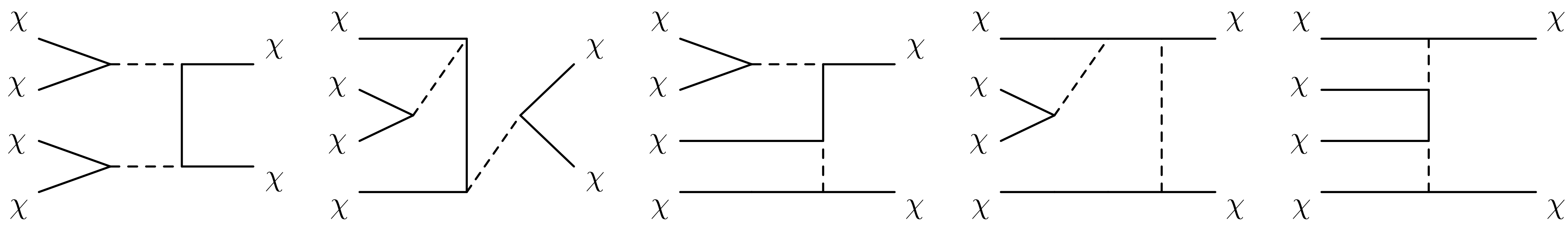
- Add Majorana fermion χ (DM) and scalar mediator φ.

$$\mathcal{L} = -\frac{y_\varphi}{2}\varphi\overline{\chi^c}\chi - y_\nu H\overline{\chi}L - \frac{\lambda_{\varphi H}}{2}\varphi^2|H|^2$$

- Assume dark sector (χ, φ) is weakly coupled with SM sector via y_ν, $\lambda_{\varphi H} \ll 1$.
- Mixing angle between h and φ ($\sin\theta \ll 1$) is also small.
- y_φ is large $\rightarrow$ SIMP



Computation of 4-to-2 cross section



- Classified into 5 topologies.
- First diagram $\sim \overline{v}(p_4)u(p_3)\overline{v}(p_2)u(p_1) \sim v^2 \rightarrow \overline{|\mathcal{M}|^2} \sim v^4$
  Naively second, third, fourth diagrams are $\rightarrow \overline{|\mathcal{M}|^2} \propto v^2$,
  Fifth diagram: $\rightarrow \overline{|\mathcal{M}|^2} \propto 1$
- But, total amplitude square is proportional to v^4 (d-wave) due to Pauli exclusion principle.

Thermal average of cross section

Final result

$$\langle \sigma v^3 \rangle = \frac{27\sqrt{3} y_\varphi^8 \sum_{n=0}^{8} a_n \xi^n}{245760 \pi m_\chi^8 (16-\xi)^2 (4-\xi)^4 (2+\xi)^6 x'^2}$$

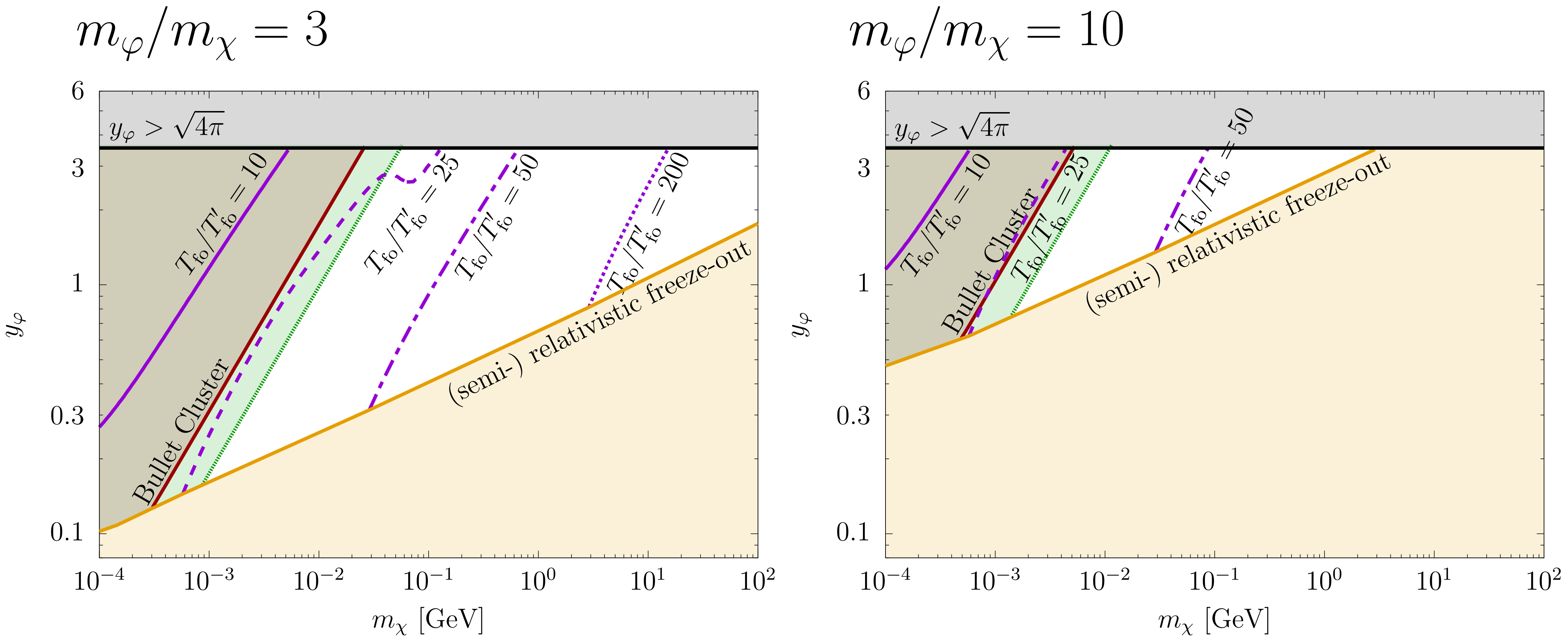
where $\xi = m_\varphi^2 / m_\chi^2 \geq 4$

$$\begin{aligned} a_0 &= 2467430400, & a_1 &= -1648072704, \\ a_2 &= 491804416, & a_3 &= -25463616, \\ a_4 &= 4824144, & a_5 &= -1528916, \\ a_6 &= 473664, & a_7 &= -35259, \\ a_8 &= 1201. \end{aligned}$$

When $\xi \gg 1$, this concides with effective self-interaction case

- Resonances at $m_\varphi = 2m_\chi, 4m_\chi$.
- Difficult to compute around resonances due to multi-dimensional integrals.

Numerical analysis



- Perturbativity $y_\varphi \leq \sqrt{4\pi} \approx 3.55$.
- Relativistic decoupling $(x'_f < 3)$ is not considered.
- Constraint of bullet cluster: $\sigma_{\rm self}/m_\chi = y_\varphi^4 m_\chi/(8\pi m_\varphi^4) \leq 1\ {\rm cm}^2/{\rm g}$.

Boltzmann equation

Assumptions:

- Quantum statistics is neglected
  $\rightarrow$ Assume always Boltzmann statistics
- Initial condition: $n_\chi = 0$, $\rho_\chi = 0$.
- DM and SM sectors are seperately thermal eq. $(T \neq T')$.

One can find DM temperature solving the Boltzmann eq.

$$\frac{dn_\chi}{dt} + 3Hn_\chi = 2\Gamma_h \frac{g_h m_h^2 m_\chi}{2\pi^2 x} K_2\left(\frac{m_h}{m_\chi}x\right) - \langle\sigma v^3\rangle\left(n_\chi^4 - n_\chi^2 n_\chi^{\mathrm{eq}2}\right)$$

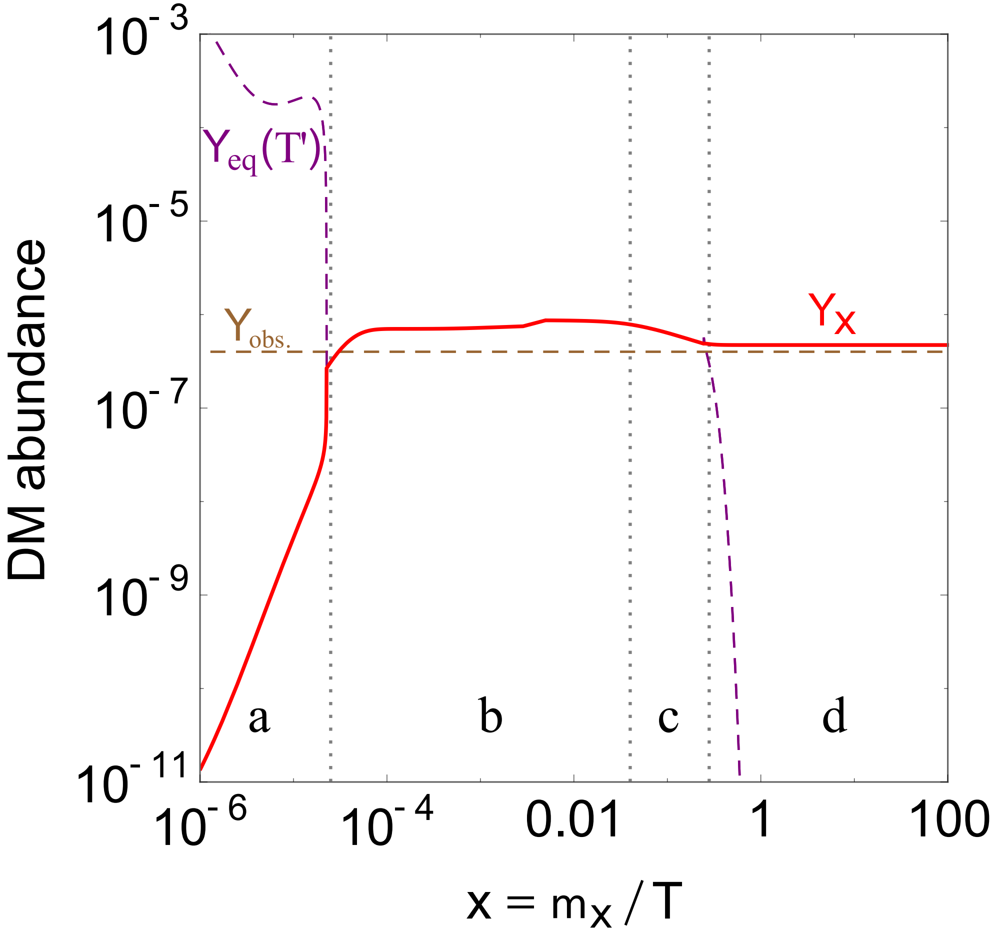
$$\frac{d\rho_\chi}{dt} + CH\rho_\chi = m_h \Gamma_h n_h^{\mathrm{eq}}$$

where $3 \leq C \leq 4$ and $\Gamma_h = \Gamma_{h\to\varphi\varphi} + \Gamma_{h\to\chi\chi}$.
$(\lambda_{\varphi H})$ $(\sin\theta)$

Temperature T' $\Leftarrow$ $\dfrac{\rho_\chi}{m_\chi n_\chi} = F(x')$

Schematic picture

Evolution of DM number density



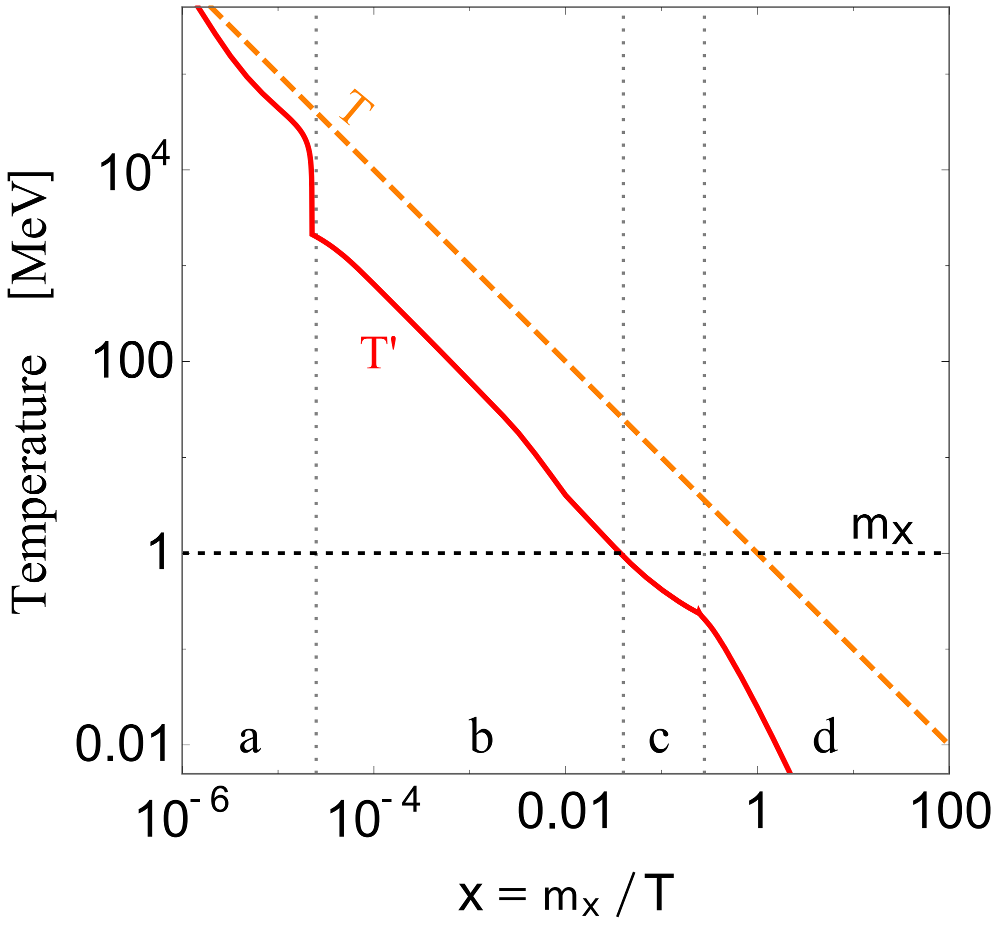
- Energy injection from SM sector to DM sector due to $h \to \varphi\varphi, \chi\chi$.
- $\chi\chi \to \chi\chi\chi\chi$ enters in dark thermal bath.

  $\to n_\chi$ rapidly increases, T' decreases.
- DM is in dark thermal bath $\to n_\chi = n_\chi^{\rm eq}$.
- When DM is non-relativtsic, freeze-out occurs in DM sector as same as WIMP case.

N. Bernal, X. Chu, arXiv:1510.08527

Schematic picture

Evolution of dark temperature T'



N. Bernal, X. Chu, arXiv:1510.08527

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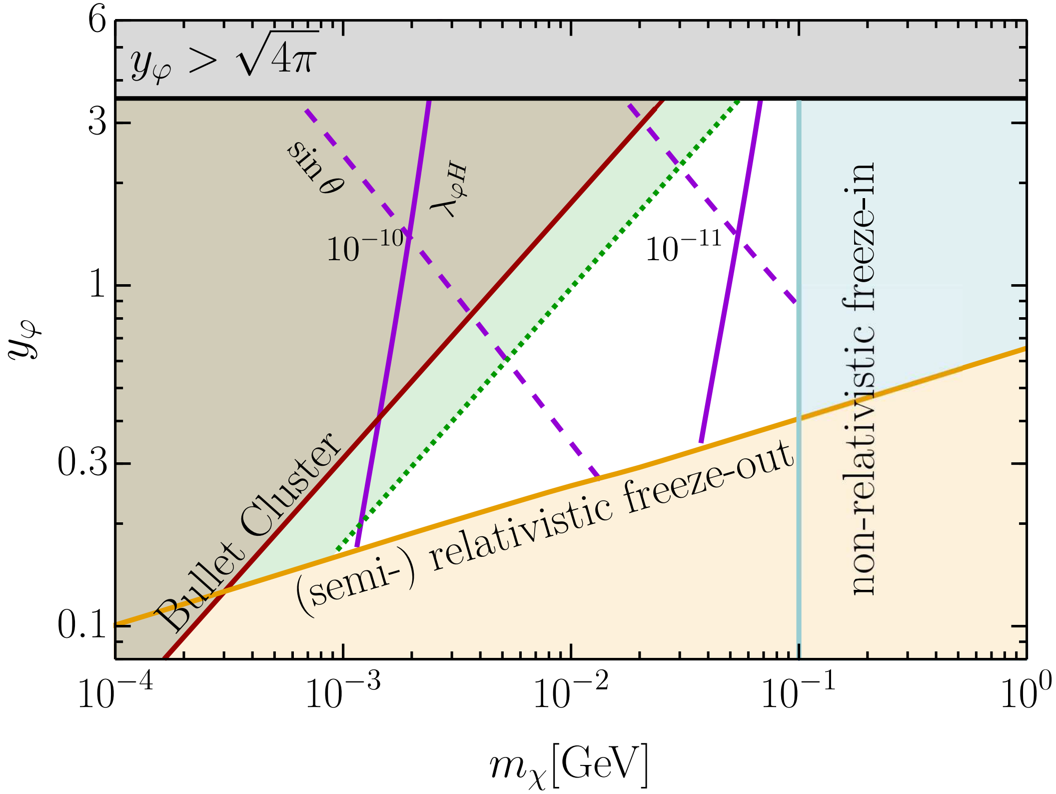
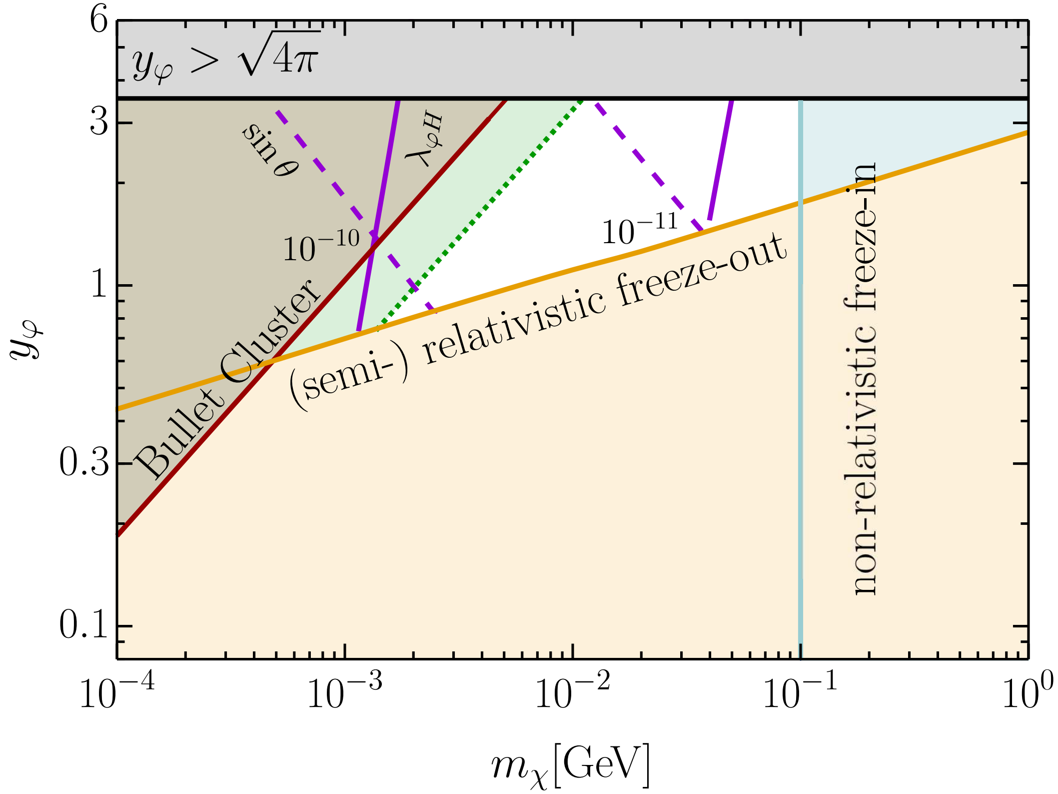
  $\to n_\chi$ rapidly increases, T' decreases.
- DM is in dark thermal bath $\to n_\chi = n_\chi^{\rm eq}$.
- When DM is non-relativtsic, freeze-out occurs in DM sector as same as WIMP case.

Magnitude of coupling

Contours of $\lambda_{\varphi H}$ and $\sin\theta$ in $(m_\chi,\, y_\varphi)$ plane

$m_\varphi/m_\chi = 3$ $m_\varphi/m_\chi = 10$





- Temperature ratio T'/T is controlled by $\lambda_{\varphi H}$, $\sin\theta$.
- $\lambda_{\varphi H} \lesssim 10^{-10}$, $\sin\theta \lesssim 10^{-9}$.

Summary

1. We have considered a singlet fermion DM with large self-interaction.
2. 4-to-2 cross section is always suppressed by d-wave due to Pauli exclusion principle.
   This does not depend on specific interactions.
3. We have considered very weak couplings between DM and SM ($T' \neq T$). Typical magnitude of the couplings are $\lesssim 10^{-9}$.