Cheshire Cat Resurgence in QM and QFT

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with Philip Glass

thanks to Sungjay Lee

10th July - 2018

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Some physical motivations:

Generically ... Perturbation Theory in

- Asymptotic Nature of QM, QFT, String Theory
- IR Renormalon Puzzle in asymptotically free QFTs
- Non-perturbative phys. /wo Instantons

→ Role of non-BPS saddles?

The Bigger scheme:

- Non-pert. definition of asymptotically free QFTs
- Analytic continuation of path integrals

Lefschetz thimbles

Back to the Basics:

How do we compute physical quantities?

→ Unless **Magic** Happens (i.e. localization, integrability,...) : Perturbation Theory

Just by diagram counting
(Dyson, Lipatov)

$$f(g) = \sum_{n=0}^{\infty} c_n \, g^n \longrightarrow c_n \sim n!$$

Gevrey-1 Type

Idea: Insert factor $$1 = \frac{1}{n!}\int_0^{\infty} dt \, t^n \, e^{-t}$$

Commute Sum w/ Integral

Standard Borel Transform:

Take $$f(g) = \sum_{n=0}^{\infty} c_n \, g^n$$

Consider $$B[f](t) = \sum_{n=1}^{\infty} \frac{c_n}{(n-1)!} \, t^{n-1}$$

Germ of analytic functions at the origin

Obtain a possible Analytic Continuation for f(g)

$$\mathcal{S}[f](g) = c_0 + \int_0^{\infty} dt \, e^{-t/g} \, B[f](t)$$

Laplace transform back: Analytic for $\Re(g) > 0$

Formal Power Series

$$\tilde{\phi}(z) = \sum_{n=0}^{\infty} c_n z^{-n-1}$$

Borel

Transform

Germ of Analytic functions
in the origin

$$\mathcal{B}[\tilde{\phi}](\zeta) = \sum_{n=0}^{\infty} c_n \frac{\zeta^n}{n!}$$

Asymptotic

Expansion

$$z \to \infty$$

Laplace

Transform

Analytic Function
in the Region $\Re(z) > 0$

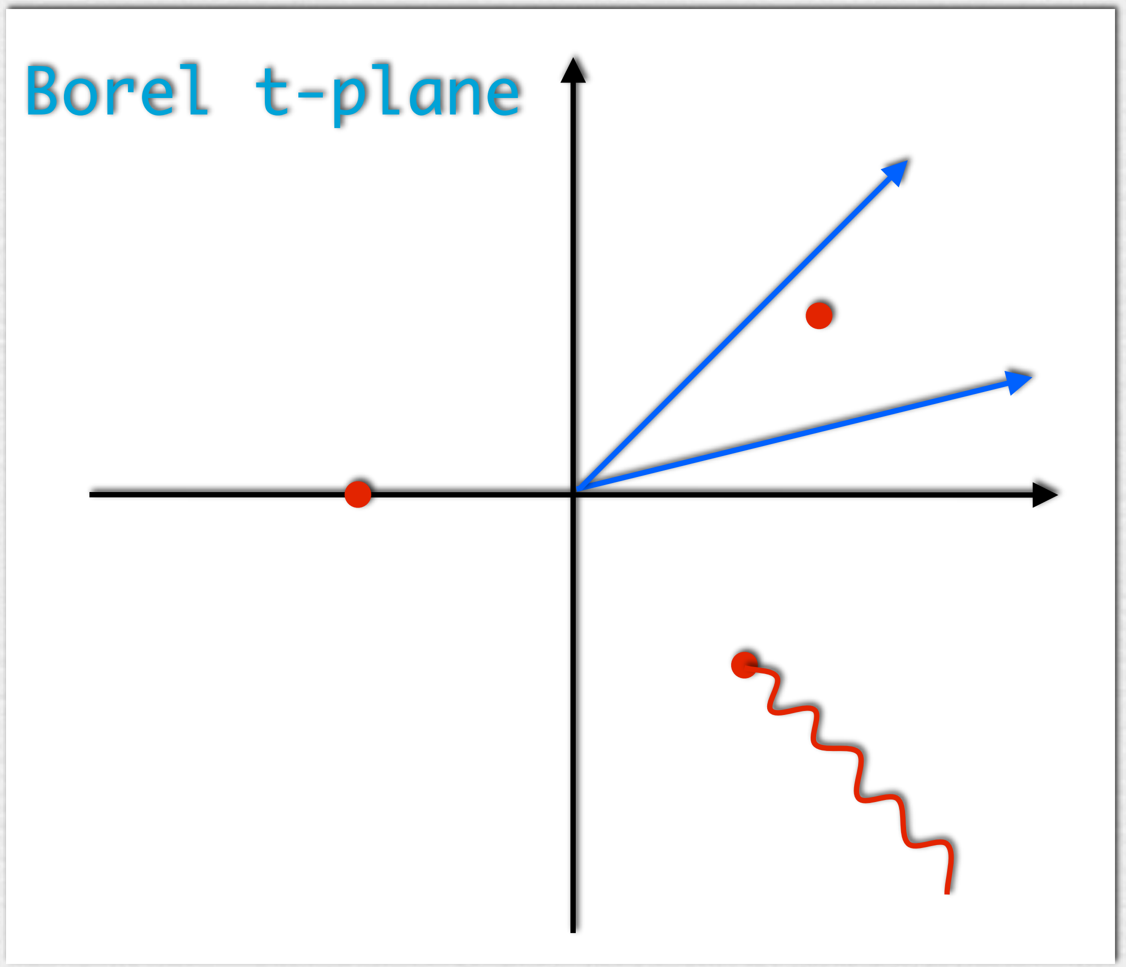
$$\mathcal{L}^0[\mathcal{B}[\tilde{\phi}]](z) = \int_0^{\infty} d\zeta \, e^{-z\,\zeta} \, \mathcal{B}[\tilde{\phi}](\zeta)$$

Different analytic continuations of the SAME physical observable (in pert.theory)

$$\mathcal{S}_\theta[f](g) = c_0 + \int_0^{e^{i\theta}\infty} dt\, e^{-t/g}\, B[f](t)$$

Same weak coupling expansion

Directional Borel Resummations



Whenever we cross a Stokes Line (i.e. singular direction)

$$\mathcal{S}_{\theta_1}[f](g) - \mathcal{S}_{\theta_2}[f](g) \neq 0$$

Different analytic continuations of the SAME physical observable (in pert.theory)

$$\mathcal{S}_\theta[f](g) = c_0 + \int_0^{e^{i\theta}\infty} dt\, e^{-t/g}\, B[f](t)$$

Different continuations of the same perturbative series ⟶ Ambiguities

On a Stokes line $\quad \mathcal{S}_+[f](g) - \mathcal{S}_-[f](g) \sim 2\pi i\, e^{-S/g}$

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Non-perturbative - non-analytic and Imaginary

Resurgence:

Location of singularities in Borel plane:

Non-Perturbative Objects with that particular action

-QM Instantons

-D-branes [Shenker]

Behaviour close to singularities

Fluctuations on top of Non-Perturbative Objects

(Large Order Perturbation Theory)

What about QFT:

- IR renormalons in 2d QFT:

  CP^{N-1} , O(N), Grassmanian models; [Argyres, Dunne, Ünsal see also Yamazaki -Yonekura]

  PCM; [Cherman, DD, Dunne, Ünsal]

  η-deformed PCM Role of Complex Saddles; [Demulder, DD, Thompson]

- 3d Chern-Simons; [Garoufalidis - Gukov, Marino, Putrov]
- 4d $\mathcal{N}$=2 From Localization; [Russo - Aniceto , Russo, Schiappa - Honda]
- 4d $\mathcal{N}$=4 SYM @ strong coupling

  Cusp Anomaly; [Aniceto - DD, Hatsuda]

  Dressing Phase; [Arutyunov, DD, Savin]

- Path integral interpretation (Lefshetz Thimbles).

  [Behtash, Dunne, Sulejmanpasic, Ünsal, Nitta,Sakai,...]

Is Perturbation theory ALWAYS Asymptotic?

Is Perturbation theory ALWAYS Asymptotic?

Generically Yes, unless **Magic** cancellations happen:

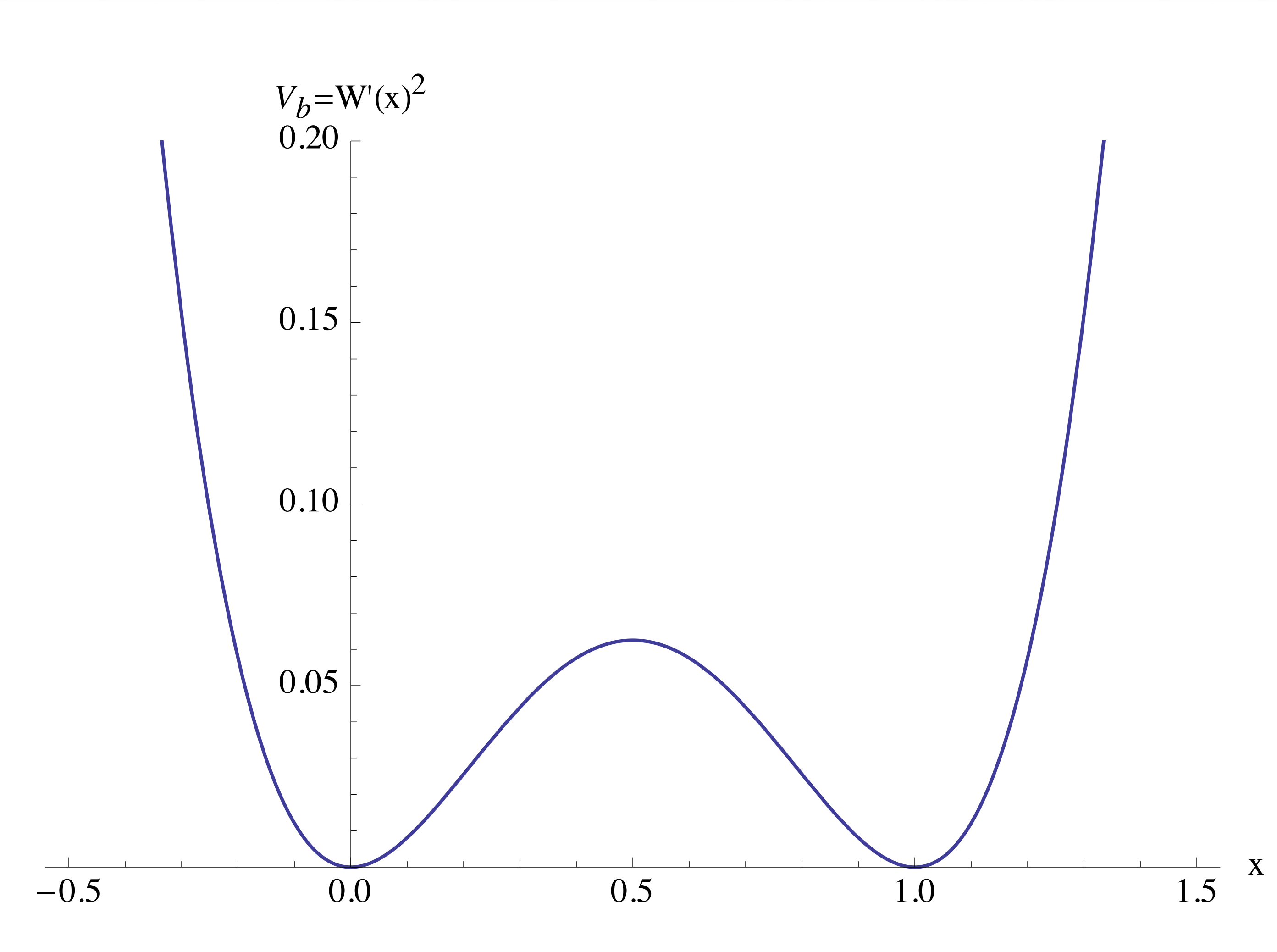
e.g. Supersymmetry

Susy QM:

Consider simplest susy QM with superpotential W(x):

$$\mathcal{L} = \frac{1}{2g}\dot{x}^2 - \frac{1}{2g}W'(x)^2 + i\,\bar{\psi}\dot{\psi} + \frac{1}{2}W''(x)\,\bar{\psi}\psi$$

For example susy double-well $W(x) = x^3/3 - x^2/2$



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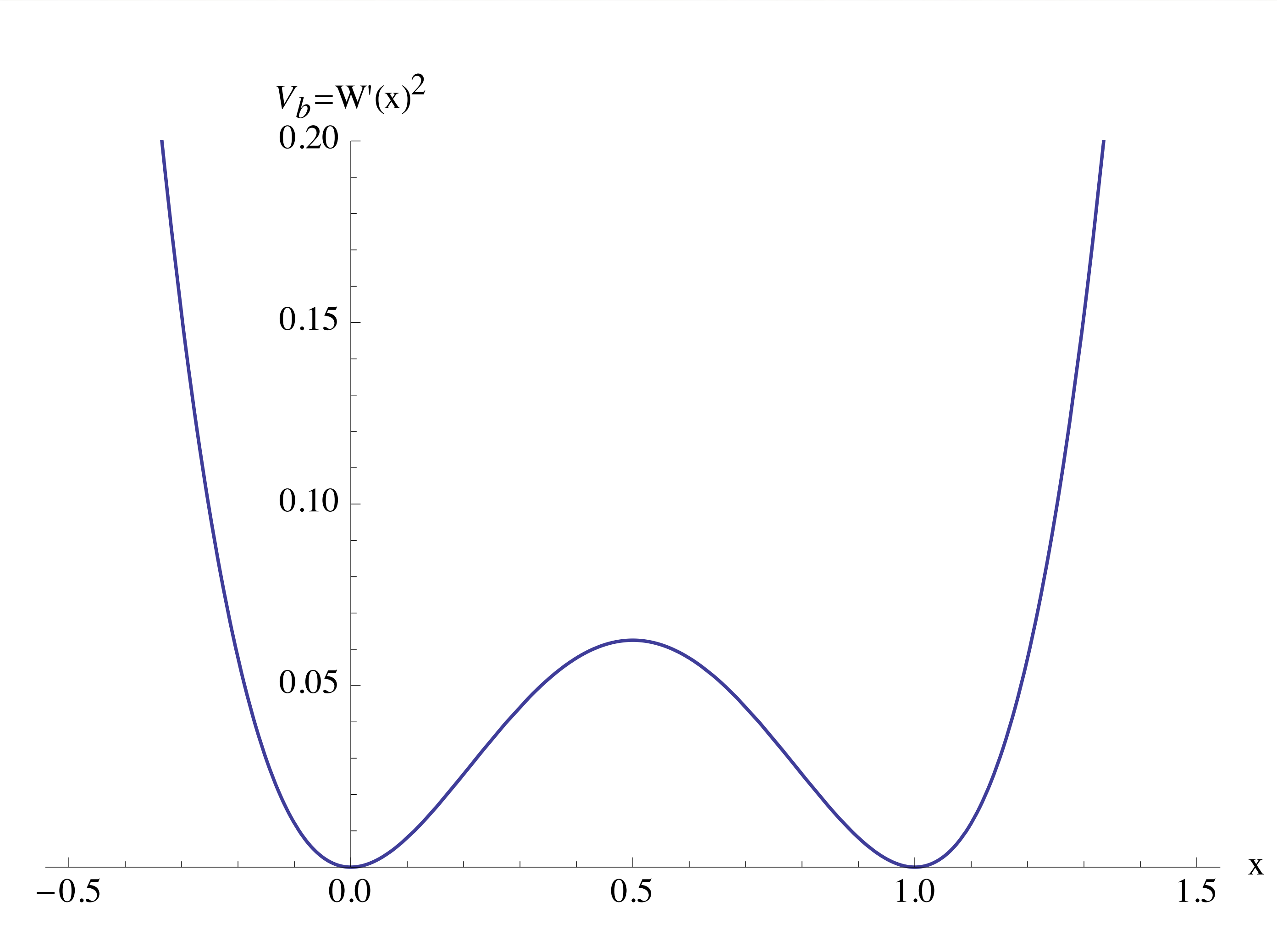

"Ground-State"

$$\psi_\pm = e^{\pm W(x)/g}$$

non-normalizable:

~~SUSY~~

[Witten]

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For example susy double-well $W(x) = x^3/3 - x^2/2$

$$E_0^{\text{pert}} = 0$$

"Ground-State"

$$\psi_\pm = e^{\pm W(x)/g}$$

However

non-normalizable:

~~SUSY~~

$$E_0 \sim \frac{1}{2\pi}e^{-2S_I/g}\left(1 - \frac{5}{6}g + O(g^2)\right)$$

$I\bar{I}$ events lift vacuum energy

How can resurgence possibly predict for us from perturbation theory $E_0^{\text{pert}} = 0$

the NP physics, i.e. $I\bar{I}$ contribution?

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Cheshire Cat Resurgence

[Dunne, Unsal - Kozcaz, Sulejmanpasic, Tanizaki, Unsal]

Deconstruct the "0" coming from perturbation theory

Cheshire Cat Resurgence in QM:

Idea:

In the Hilbert sector $\mathcal{H}_{(N_f,k)}$ with well-def fermion number k, the purely bosonic Hamiltonian is:

$$H_b = \frac{g}{2}p^2 + \frac{1}{2g}W'(x)^2 + \frac{1}{2}(2k - N_f)W''(x)$$

analytically continue in the number of fermions

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$$H_b = \frac{g}{2}p^2 + \frac{1}{2g}W'(x)^2 + \frac{1}{2}\underline{(2k - N_f)}W''(x)$$

$$H_b = \frac{g}{2}p^2 + \frac{1}{2g}W'(x)^2 + \frac{1}{2}\zeta\, W''(x) \quad \text{with } \zeta \in \mathbb{C}$$

Cheshire Cat Resurgence in QM:

Idea:

Use

$$H_b = \frac{g}{2}p^2 + \frac{1}{2g}W'(x)^2 + \frac{1}{2}\zeta W''(x) \quad \text{with } \zeta \in \mathbb{C}$$

and compute the ground state energy:

$$E(g) = \sum_{n=0}^{\infty} c_n(\zeta) g^n$$

for generic ζ we have $c_n(\zeta) \sim n!$

Use resurgence to extract NP physics and only at the end send the susy pt $\zeta \to 1$

The body of the Cheshire Cat Resurgence
is still present even if supersymmetry
has made it invisible

What about in susy QFT?

2d Susy $\mathcal{N} = (2,2)$ $\mathbb{CP}^{N-1}$

Matter

- ✤ U(1) gauge multiplet ⟶ Twisted Chiral Σ w/lowest Component σ
- ✤ N Chirals Φ_i ⟶ Charged +1 under U(1)

Parameters

- ✤ Gauge coupling w/ [e]=1
- ✤ FI term and theta angle $\tau = i\xi + \frac{\theta}{2\pi}$

$g^2_{\mathbb{CP}^{N-1}} = 1/\xi$ i.e. weak coupling $\xi \gg 1$

2d Susy $\mathcal{N} = (2,2)$ $\mathbb{CP}^{N-1}$

QCD-like Physics

- mass gap generation
- Expected IR renormalons
- Poles of Borel transform on positive half line [Dunne,Unsal,Argyres]

Unlike QCD we can apply Susy localisation

Susy Localization: (Duistermaat Heckman Formula)

Idea: (Apologies to the experts in the audience)

Suppose we have symmetry generator Q such that $Q^2=0$ then we add a Q-exact term to the path-integral

$$Z[t] = \int \mathcal{D}\phi \, e^{-S[\phi] - t\, QV}$$

$$\frac{dZ}{dt} = \int \mathcal{D}\phi \, QV \, e^{-S[\phi] - t\, QV} = 0$$

(Think of Q from BRST and gauge fixing)

So the original path integral wo/ QV term is also equal

$$Z[0] = \lim_{t\to\infty} Z[t]$$

in this limit the path-integral localizes on QV=0 and saddle point approximation becomes exact

Susy Localization: (Duistermaat Heckman Formula)

Idea:

$$Z[0] = \lim_{t \to \infty} Z[t]$$

in this limit the path-integral localizes on QV=0 and saddle point approximation becomes exact

$$Z = \sum_{\phi_0} e^{-S[\phi_0]} \left(\frac{\det \mathcal{O}_F}{\det \mathcal{O}_B} \right)$$

Quadratic fluctuations, aka 1-loop det

ϕ_0 Critical points of QV such that QV=0

Susy Localization for $\mathcal{N} = (2,2)$ theories on S^2:

[Doroud, Gomis, Lee, Le Floch - Benini, Cremonesi]

Punch line: Susy loci
$$\begin{cases} \sigma(x) = \sigma \ \text{const.} \\ F_{12}(x) = \frac{B}{R^2}\,, \quad B \in \mathbb{Z} \end{cases}$$

R radius of S^2

$$\int [\mathcal{D}\Phi] \to \sum_{B\in\mathbb{Z}} \int_{-\infty}^{\infty} d\sigma$$

not a "path"-integral anymore

$$Z_{\mathbb{CP}^{N-1}} = \sum_{B\in\mathbb{Z}} \int_{-\infty}^{\infty} \frac{d\sigma}{2\pi}\, e^{-4\pi i\,\xi\,\sigma} \left[\frac{\Gamma(-i\sigma - B/2)}{\Gamma(1 + i\sigma - B/2)}\right]^N$$

"sum" over susy loci

on-shell action

one-loop determinant

E.g. $\mathbb{CP}^1$

$$Z_{\mathbb{CP}^1} = 2\left[I_0(2\sqrt{q})K_0(2\sqrt{\bar{q}}) + K_0(2\sqrt{q})I_0(2\sqrt{\bar{q}})\right]$$

$q = e^{2\pi i\tau} = e^{-2\pi\xi + i\theta}$ Instanton fugacity

Comment:

✤ Correct Chiral ring structure

$$\langle \Sigma^N \rangle = \frac{1}{Z_{\mathbb{CP}^{N-1}}} (q\partial_q)^N Z_{\mathbb{CP}^{N-1}} = q = \Lambda^N_{\mathbb{CP}^{N-1}}$$

✤ tt* Equations are not satisfied [Cecotti, Vafa - Gomis, Lee]

E.g. $\mathbb{CP}^1$

$$q\bar{q}\,\partial_q\partial_{\bar{q}} \log Z_{\mathbb{CP}^1} = q\bar{q}\, Z^2_{\mathbb{CP}^1} - \frac{1}{Z^2_{\mathbb{CP}^1}}$$

Two-Sphere localized partition function captures full physics, perturbative and non-perturbative

$q = e^{2\pi i\tau} = e^{-2\pi\xi + i\theta}$ Instanton fugacity

Vortex-Anti-Vortex configurations

Can we use resurgence applied to purely perturbative expansion, i.e. from Feynman diagrams on S^2 ?

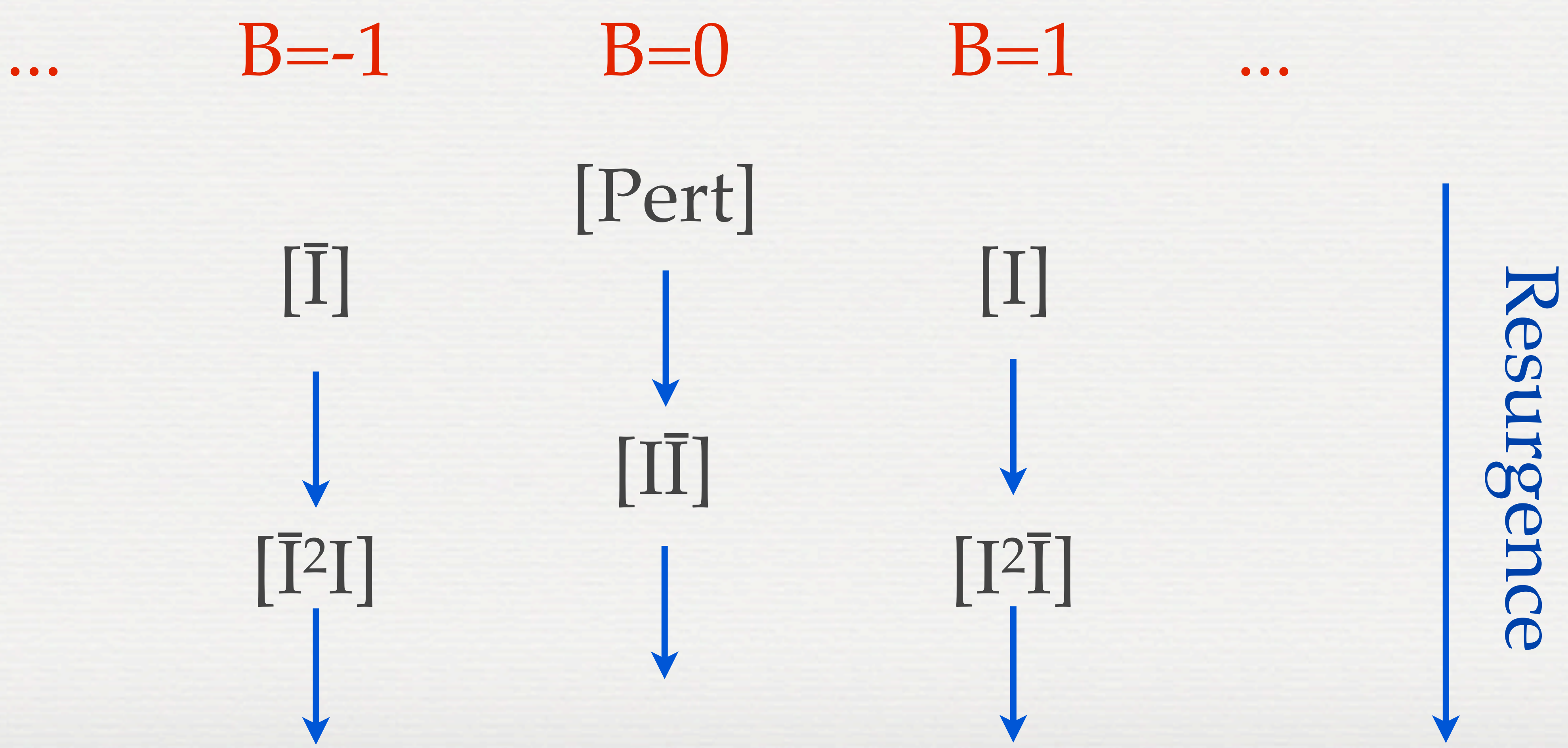
Weak coupling: $\xi \gg 1$

For concreteness $\mathbb{CP}^1$ sum over Fourier modes, i.e. topological sectors

$$Z_{\mathbb{CP}^1} = \sum_{B \in \mathbb{Z}} e^{-2\pi\xi|B| + i\theta B} \zeta_B(\xi)$$

Resurgence triangle:

[Dunne, Unsal]



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Resurgence triangle:

[Dunne, Unsal]

In each topological sector we have perturbative piece plus infinite tower of $I\bar{I}$ contributions, e.g. B=0 here

$$\zeta_0(\xi) = \sum_{k=0}^{\infty} e^{-4\pi k\xi}(4\pi\xi)^2\left[\frac{1}{(k!)^4}\frac{1}{(4\pi\xi)} + \frac{4H_k-\gamma}{(k!)^4}\frac{1}{(4\pi\xi)^2}\right]$$

$(I\bar{I})^k$ factor

Perturbation theory in $(I\bar{I})^k$ sector

Weak coupling: $\xi \gg 1$

In each topological sector, and for every $I\bar{I}$ contribution on top of that, perturbation theory truncates after N orders

Does the resurgence program fail?

Idea: Analytically continue in the number of fermions!

Weak coupling: $\xi \gg 1$

Idea: Analytically continue in the number of fermions!
AFTER having localized

Go back to one-loop determinant for matter fields

$$Z_{mat} = \left(\frac{\det\mathcal{O}_\psi}{\det\mathcal{O}_\phi}\right)^N$$

$$\det\mathcal{O}_\phi = \prod_{j=|B|/2}^{\infty} (j - i\sigma)^{2j+1}(j+1+i\sigma)^{2j+1}$$

No Index-theo,
just eigenvalues of quadratic fluctuations.
Similarly for fermions

Introduce unbalance Boson/Fermions

$$\begin{cases} N_f = N \\ N_b = N_f - \Delta \end{cases} \qquad \tilde{Z}_{mat} = \left(\frac{\det\mathcal{O}_\psi}{\det\mathcal{O}_\phi}\right)^N \times (\det\mathcal{O}_\phi)^{-\Delta}$$

We consider the modified partition function

$$\tilde{Z}(\Delta,\xi) = \sum_{B\in\mathbb{Z}} \int_{\mathcal{C}} \frac{d\sigma}{2\pi} e^{-4\pi i\,\xi\sigma}\ \tilde{Z}_{mat}$$

where using zeta function regularisation

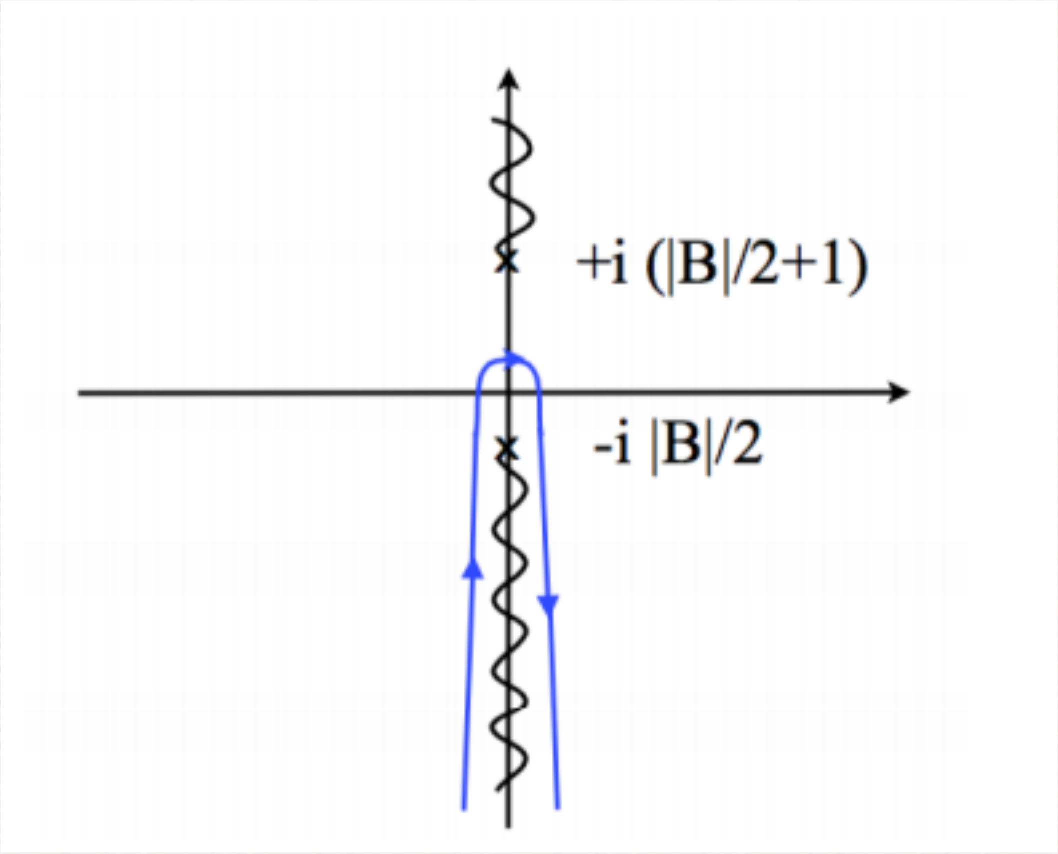
$$\tilde{Z}_{matter}(\sigma) = \left[(-1)^{B\theta(B)} \frac{\Gamma(-i\sigma+|B|/2)}{\Gamma(1+i\sigma+|B|/2)}\right]^N e^{-2\Delta\left(2\zeta'(-1)+\zeta'(0)(|B|+1)+|B|^2/4+i\sigma-\sigma^2\right)}$$
$$\times \exp\left[\Delta(2i\sigma+1)\left(\log\Gamma(1+i\sigma+|B|/2) - \log\Gamma(-i\sigma+|B|/2)\right)\right]$$
$$\times \exp\left[-2\Delta\left(\psi^{(-2)}(1+i\sigma+|B|/2) + \psi^{(-2)}(-i\sigma+|B|/2)\right)\right].$$

just some logGamma functions and digammas, don't panic, they are gone now

We consider the modified partition function

$$\tilde{Z}(\Delta, \xi) = \sum_{B \in \mathbb{Z}} \int_{\mathcal{C}} \frac{d\sigma}{2\pi} e^{-4\pi i\, \xi \sigma}\, \tilde{Z}_{mat}$$

and the contour of integration is



We consider the modified partition function

$$\tilde{Z}(\Delta,\xi) = \sum_{B\in\mathbb{Z}} \int_{\mathcal{C}} \frac{d\sigma}{2\pi} e^{-4\pi i\,\xi\sigma}\ \tilde{Z}_{mat}$$

using the known discontinuities properties for the logGamma and digamma functions we obtain e.g. B=0 sector of the full partition function

$$\tilde{\zeta}_0(\Delta,\xi) = \sum_{k=0}^{\infty} e^{-4\pi\xi k}\, e^{\pm i\pi k^2\Delta}\, \mathcal{S}_{\pm}[\Phi^{(k)}](\Delta,\xi)$$

with the modified directional Borel resummation

$$\mathcal{S}_{\pm}[\Phi^{(k)}](\Delta,\xi) = \int_0^{\infty\pm i\epsilon} dx\, e^{-4\pi\xi\, x} x^{-N+\Delta(2k+1)}\Phi^{(k)}(x,\Delta)$$

$$\tilde{\zeta}_0(\Delta, \xi) = \sum_{k=0}^{\infty} e^{-4\pi\xi k}\, e^{\pm i\pi k^2 \Delta}\, \mathcal{S}_{\pm}[\Phi^{(k)}](\Delta, \xi)$$

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$$\Phi^{(0)}(x,\Delta) = -\frac{(-1)^N \sin(\pi\Delta)}{\pi} \left[\frac{\pi x/\sin(\pi x)}{\Gamma(1+x)^2}\right]^N \exp\left[2\Delta(x + \psi^{(-2)}(1))\right] \times \tag{4.20}$$

$$\exp\left[\Delta(2x+1)\left(\log\Gamma(1+x) - \log\Gamma(1-x)\right) - 2\Delta\left(\psi^{(-2)}(1+x) + \psi^{(-2)}(1-x)\right)\right].$$

the exact form is <u>not</u> important!

$$\tilde{\zeta}_0(\Delta,\xi) = \sum_{k=0}^{\infty} e^{-4\pi\xi k}\, e^{\pm i\pi k^2 \Delta}\, \mathcal{S}_{\pm}[\Phi^{(k)}](\Delta,\xi)$$

with the modified directional Borel resummation

$$\mathcal{S}_{\pm}[\Phi^{(k)}](\Delta,\xi) = \int_0^{\infty \pm i\epsilon} dx\, e^{-4\pi\xi\, x} x^{-N+\Delta(2k+1)} \Phi^{(k)}(x,\Delta)$$

The important thing is:

$$\Phi^{(k)}(x,\Delta) \sim \sin[\pi\Delta(2k+1)] \sum_{n=0}^{\infty} c_n^{(k)}(\Delta) x^n$$

Polynomials in Δ

The grin of the cat:

focus on the purely perturbative part of $\tilde{\zeta}_0(\Delta, \xi)$

(i.e. purely perturbative part of the B=0 contribution to the partition function, i.e. perturbation theory)

$$\tilde{\zeta}_0^{pert}(\Delta, \xi) = (4\pi\xi)^{N-\Delta} \frac{\sin(\pi\Delta)}{\pi} \sum_{n=0}^{\infty} \frac{\Gamma(n+1+\Delta-N)}{(4\pi\xi)^{n+1}} c_n^{(0)}(\Delta)$$

as soon as $\Delta \notin \mathbb{Z}$ perturbation theory is asymptotic

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however we have to be careful with the limit $\Delta \to 0$

$\sin(\pi\Delta)\Gamma(n+1+\Delta-N)$ is non-zero only for n≤N

i.e. at the susy point $\Delta = 0$
Perturbation theory truncates dramatically

The grin of the cat:

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$$\tilde{\zeta}_0^{pert}(\Delta, \xi) = (4\pi\xi)^{N-\Delta} \sum_{n=0}^{\infty} \frac{C_n^{pert}(\Delta)}{(4\pi\xi)^{n+1}}$$

For generic Δ: $C_n^{pert}(\Delta) \sim n!$

This is an example of Cheshire cat resurgence!

- Work at Δ non-integer,
- use resurgence to extract NP info from P data,
- Send Δ to 0,

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Large orders in perturbation theory:

$$C_n^{pert}(\Delta) \sim \sin(\pi\Delta) \frac{\Gamma(n-2\Delta)}{(+1)^{n-2\Delta}} \left(C_0^{(1)}(\Delta) + \frac{C_1^{(1)}(\Delta)}{n-2\Delta-1} + O(n^{-2}) \right)$$

$$+ \sin(\pi\Delta)\cos(3\pi\Delta) \frac{\Gamma(n-4\Delta)}{(+2)^{n-4\Delta}} \left(C_0^{(2)}(\Delta) + O(n^{-1}) \right) + \ldots$$

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Stokes Constants

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$$+ \sin(\pi\Delta)\cos(3\pi\Delta) \frac{\Gamma(n-4\Delta)}{(+2)^{n-4\Delta}} \left(C_0^{(2)}(\Delta) + O(n^{-1}) \right) + \ldots$$

Instantons-anti-Instantons actions

The grin of the cat:

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$$+ \sin(\pi\Delta)\cos(3\pi\Delta) \frac{\Gamma(n-4\Delta)}{(+2)^{n-4\Delta}} \left(C_0^{(2)}(\Delta) + O(n^{-1}) \right) + \ldots$$

Perturbative coefficients in the $I\bar{I}$ sector

In particular as Δ goes to 0 only a finite number of coefficients $C_p^{(1)}(0), C_p^{(2)}(0), \ldots$ remains non-zero in each $(I\bar{I})^k$ sector

$$\tilde{\zeta}_0^{pert}(\Delta,\xi) = (4\pi\xi)^{N-\Delta} \sum_{n=0}^{\infty} \frac{C_n^{pert}(\Delta)}{(4\pi\xi)^{n+1}}$$

And from the limit of the purely perturbative expansion we can derive the perturbation coefficients in $(I\bar{I})^k$ sector for the SUSY theory

$$\zeta_0(\xi) = \sum_{k=0}^{\infty} e^{-4\pi k\xi} (4\pi\xi)^2 \left[\overline{\frac{1}{(k!)^4} \frac{1}{(4\pi\xi)} + \frac{4H_k - \gamma}{(k!)^4} \frac{1}{(4\pi\xi)^2}} \right]$$

$(I\bar{I})^k$ factor

SUSY non-asymptotic expansion

$\Delta \to 0$

$$\zeta_0(\xi) = \sum_{k=0}^{\infty} e^{-4\pi k \xi} (4\pi\xi)^2 \left[\frac{1}{(k!)^4} \frac{1}{(4\pi\xi)} + \frac{4H_k - \gamma}{(k!)^4} \frac{1}{(4\pi\xi)^2} \right]$$

non-SUSY resurgent transseries

$$\tilde{\zeta}_0(\Delta, \xi) = \sum_{k=0}^{\infty} e^{-4\pi\xi k}\, e^{\pm i\pi k^2 \Delta}\, \mathcal{S}_{\pm}[\Phi^{(k)}](\Delta, \xi)$$

Resurgence:

- Large orders in perturbation theory;
- Cancellations of ambiguities;
- Reconstruct NP physics out of P data

Conclusions & Outlook:

- Even in the case of a truncating perturbative expansion resurgence is still there;

- As soon as we break slightly SUSY, i.e. $\Delta \neq 0$ the body of the Cheshire cat reappears;

- Interpretation of the deformation? $\Delta \to n \in \mathbb{Z}$

- Obtain similar results considering $\mathbb{CP}^{r-1}$

- How general is Cheshire resurgence in SUSY theories?

Thanks for Listening!