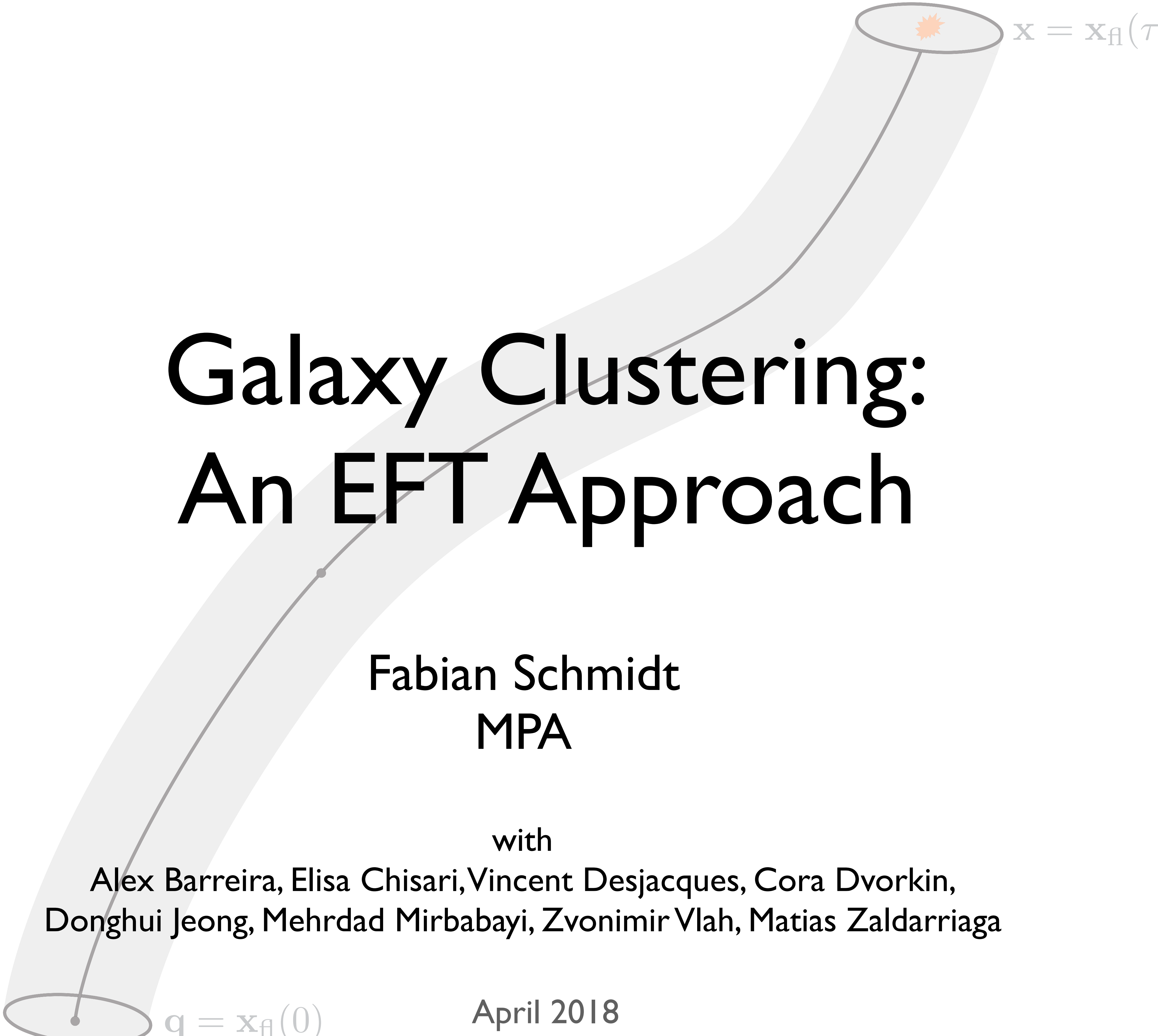

Galaxy Clustering: An EFT Approach

Fabian Schmidt
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with
Alex Barreira, Elisa Chisari, Vincent Desjacques, Cora Dvorkin,
Donghui Jeong, Mehrdad Mirbabayi, Zvonimir Vlah, Matias Zaldarriaga

April 2018

Motivation

- The clustering of galaxies (large-scale structure, LSS) is historically one of the key probes of cosmology

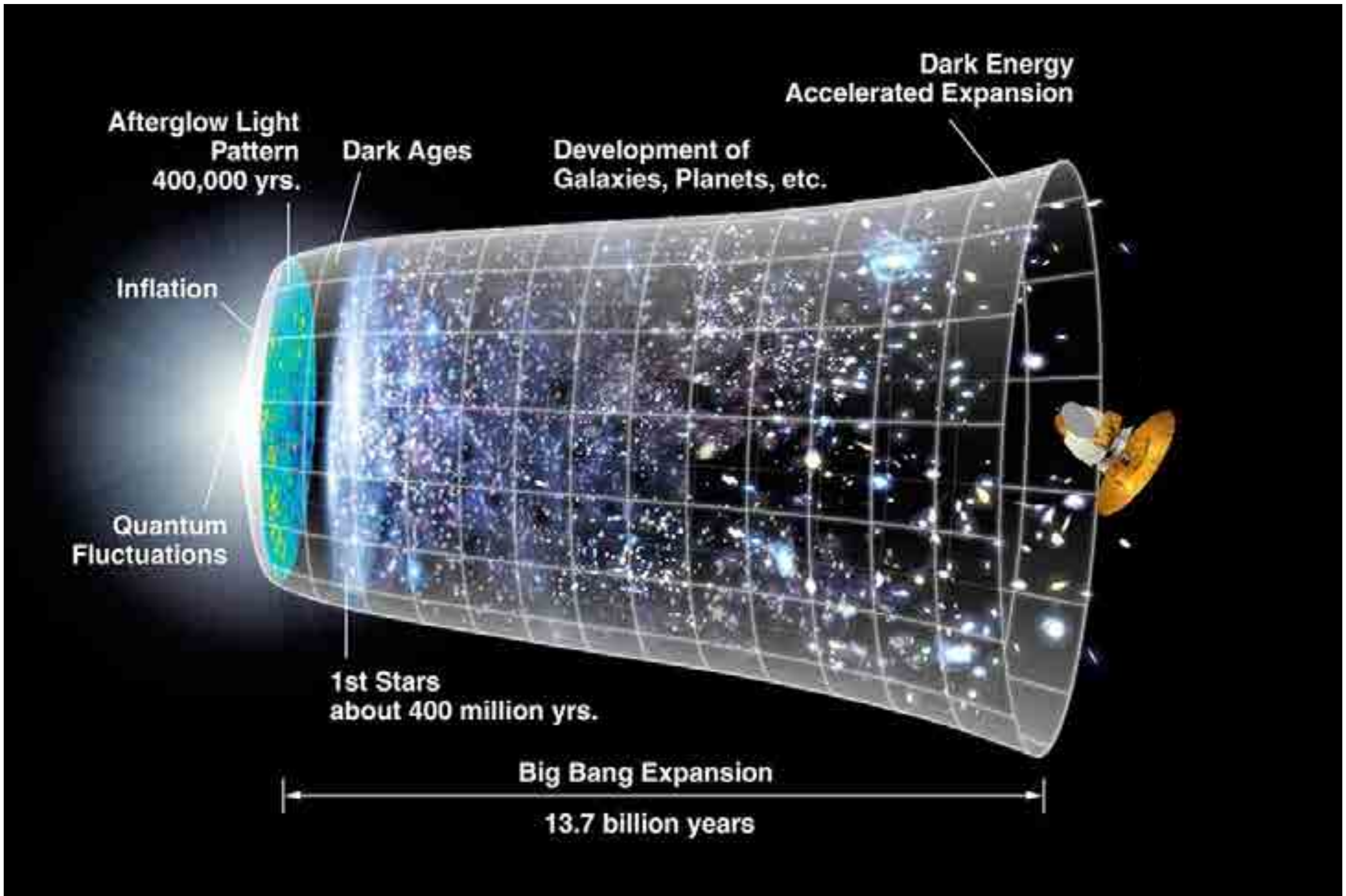
  Peebles; Efstathiou+ '90 predicted a positive cosmological constant Λ from LSS observations

- From ~1998 until recently, most spectacular results came from "cleaner" probes - Supernovae and the cosmic microwave background (CMB)

- Now, again, in a new golden age of LSS with plenty of experiments under way: BOSS, DES, DESI, PFS, SphereX, Euclid, WFIRST, ...

Motivation

- Using large-scale structure, we can learn about



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- Dark Energy and Gravity: the growth of structure depends sensitively on the expansion history of the Universe, and the nature of gravity

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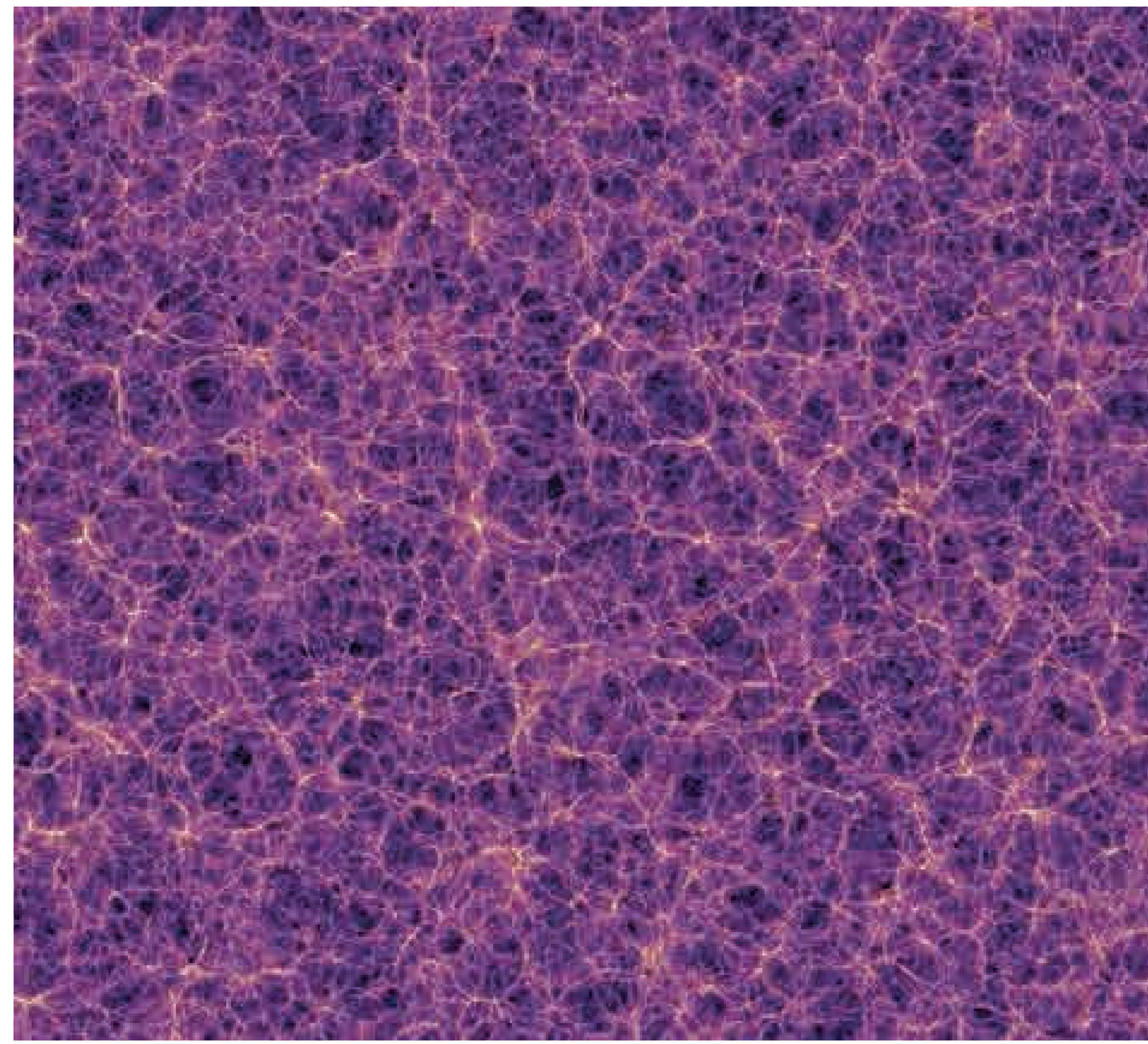
  Growth equation: $D'' + aH\,D' = 4\pi\,G\,\bar{\rho}\,D$

- Dark Matter: how "cold" is cold dark matter ? What is the sum of neutrino masses ?

Cold Dark Matter cosmology in a nutshell

- Assume scale-invariant, adiabatic, approx. Gaussian initial conditions
- Large-scale fluctuations are small (still linear today)
- Structure forms *hierarchically from small to large* scales
- *Perturbative expansion* in fluctuations on large scales



Millennium simulation / MPA

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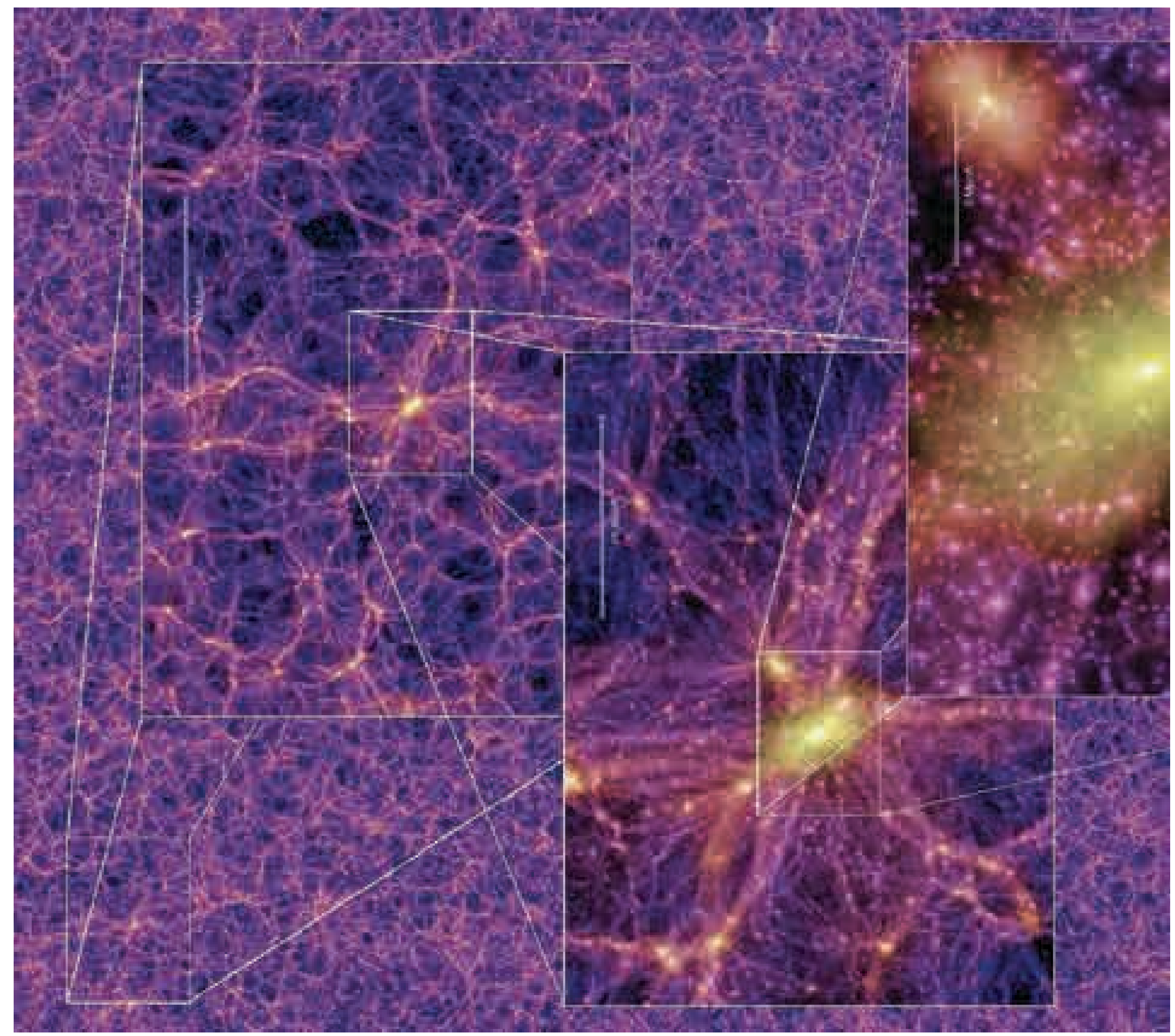
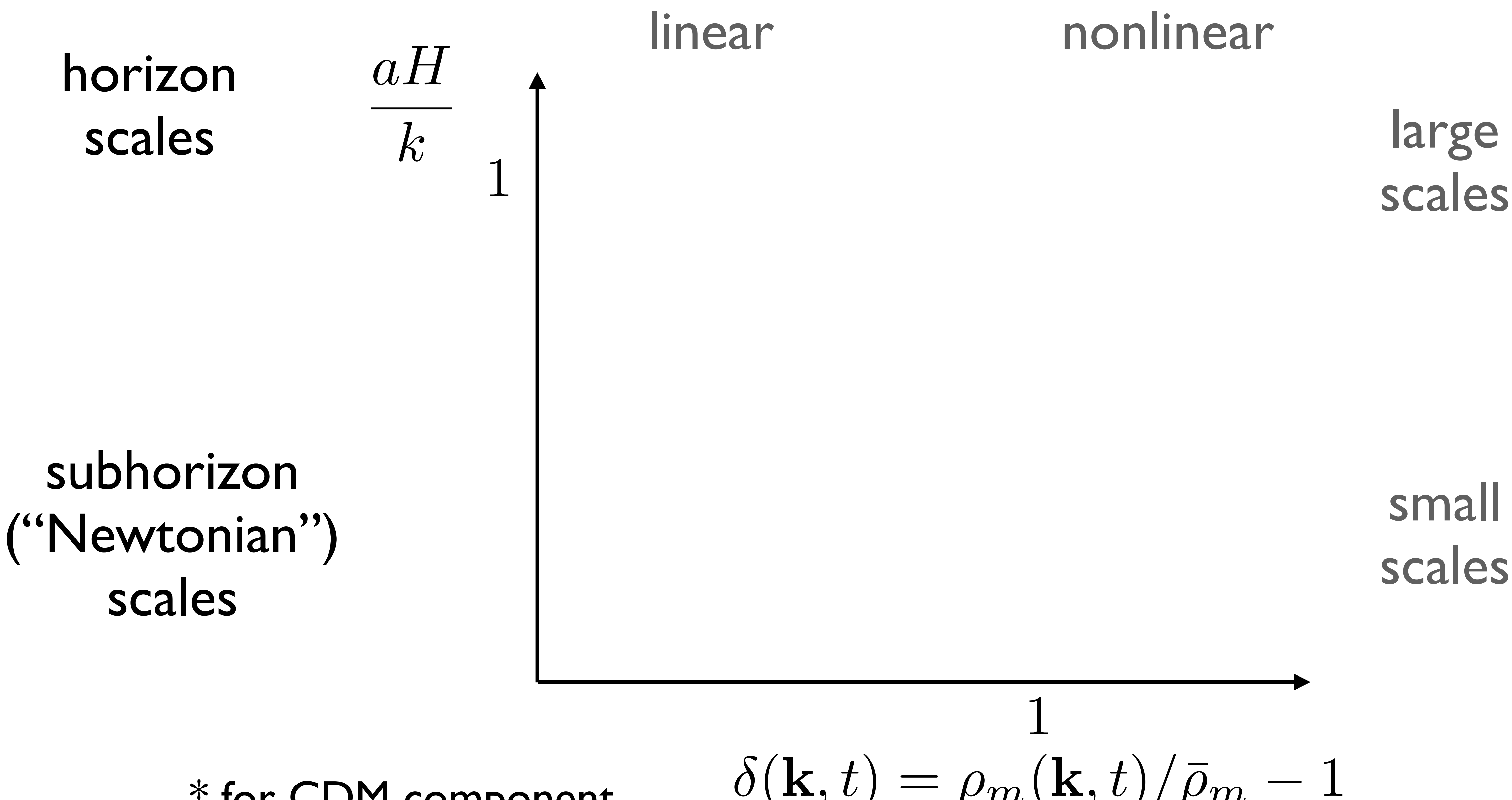


Millennium simulation / MPA

Theory of Large-Scale Structure



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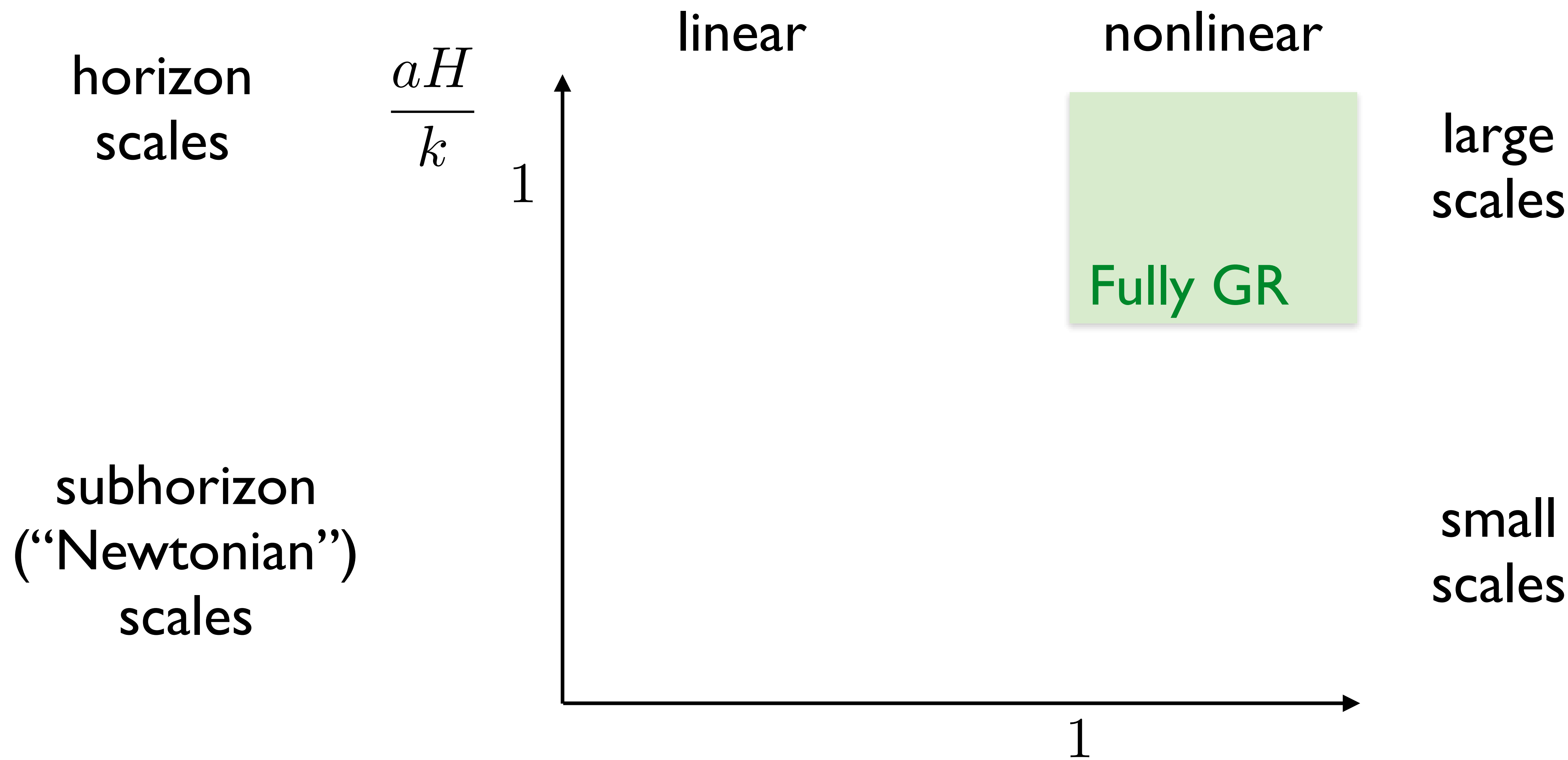

$$\delta(\mathbf{k}, t) = \rho_m(\mathbf{k}, t)/\bar{\rho}_m - 1$$

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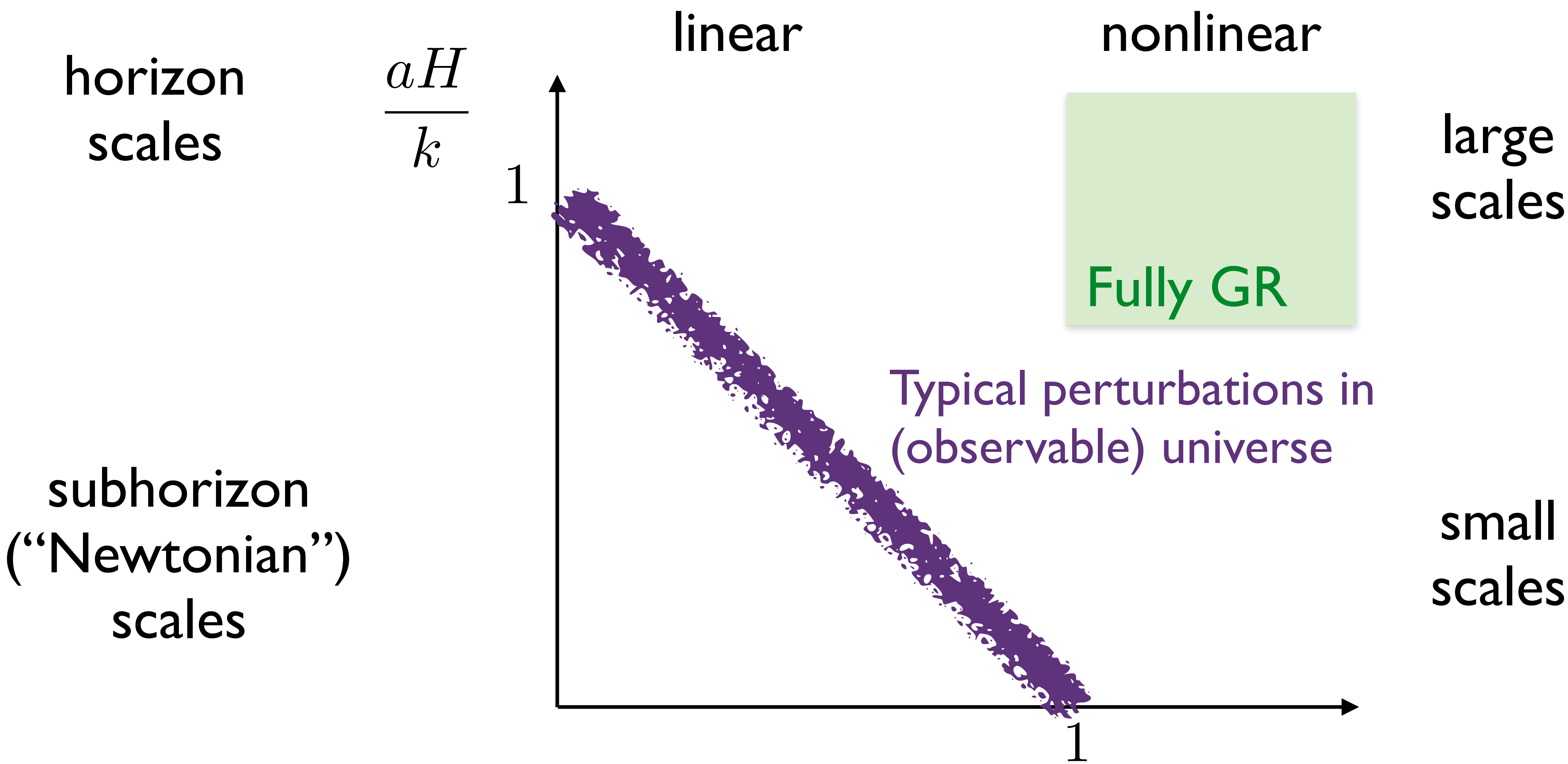

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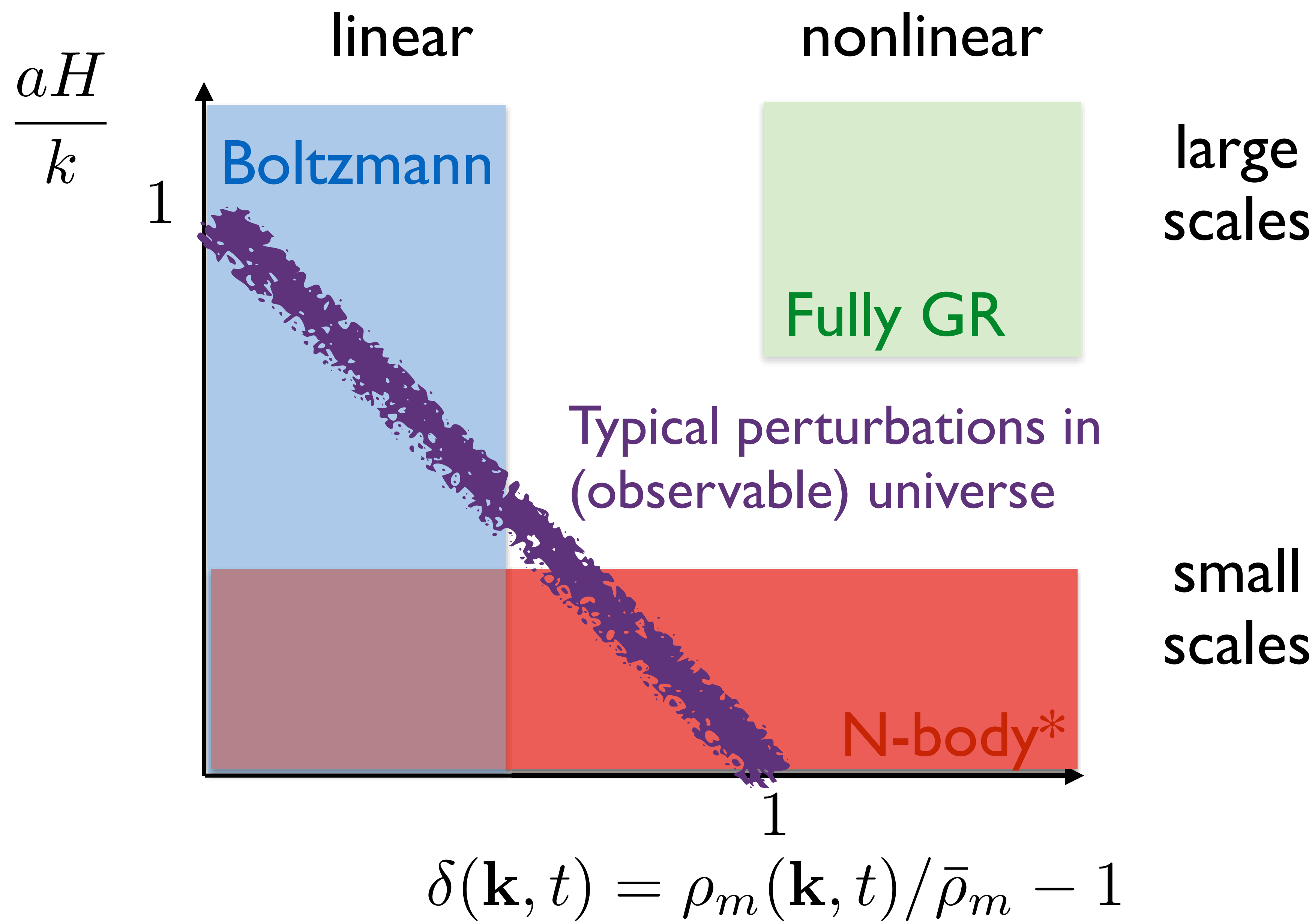
Theory of Large-Scale Structure

- Well-established tools:
  - linear Boltzmann
  - N-body* methods



* and hydrodynamics

Theory of Large-Scale Structure

- Foundation: separation between nonlinear scale and horizon

  $k_{\rm NL} \simeq 0.1h\,{\rm Mpc}^{-1} \gg aH$

- Linear theory: Fourier modes evolve independently; *solved problem*
- However, bulk of information in LSS is on nonlinear scales ($N_{modes} \sim k_{max}^3$)

What we observe, is this, however…

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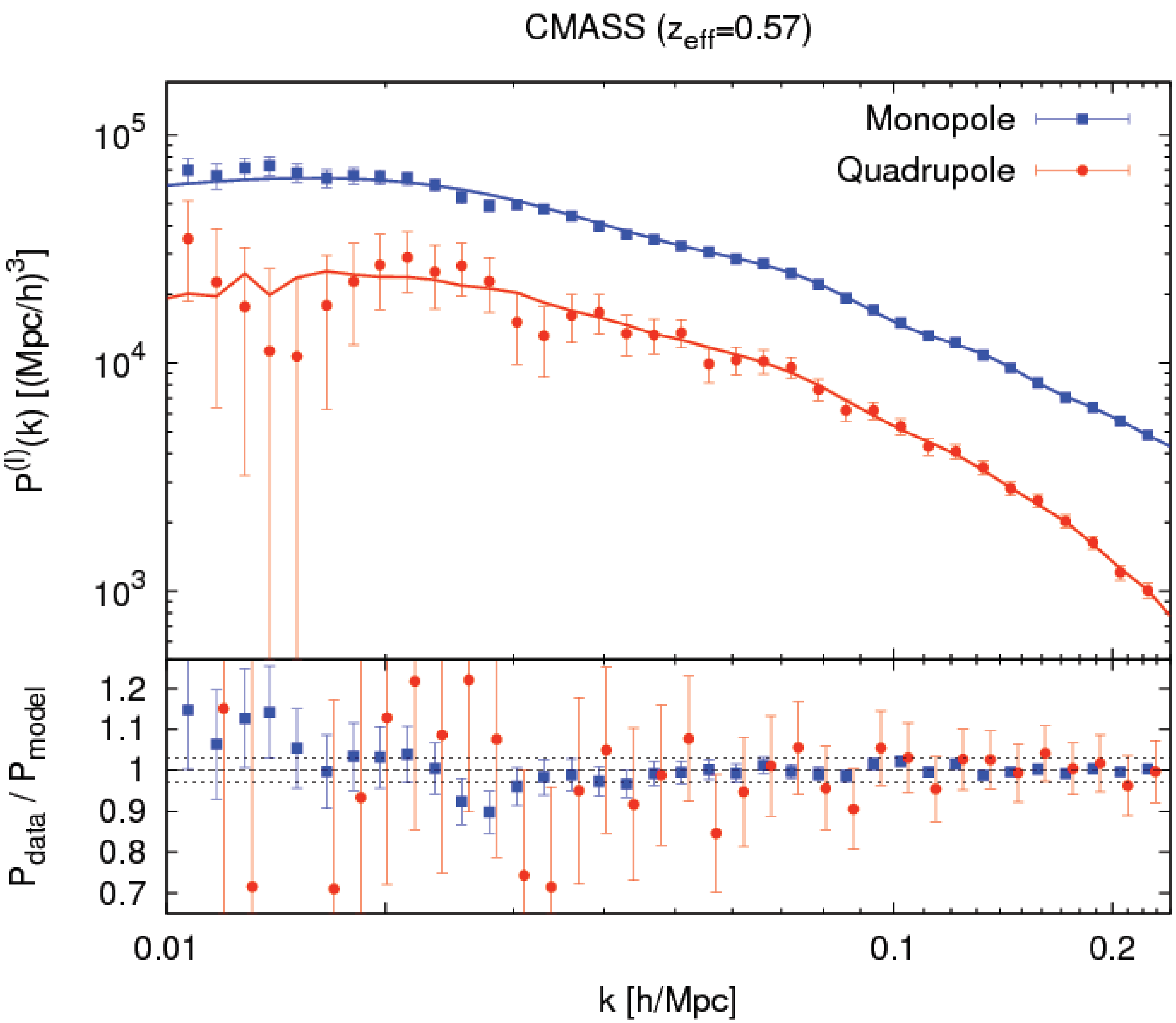

Gil-Marin et al, 2016

Theory of galaxy clustering

- We cannot yet simulate the formation of galaxies* fully realistically
- Need to abstract from the incomplete understanding on small scales
  - Only hope for rigorous results is on scales $k < k_{NL}$

* Of course, everything in following will apply to any tracer of LSS.

Theory of galaxy clustering

- We cannot yet simulate the formation of galaxies* fully realistically
- Need to abstract from the incomplete understanding on small scales
  - Only hope for rigorous results is on scales $k < k_{NL}$
- Goal: describe galaxy clustering up to a given scale and accuracy using a finite number of free bias parameters b_O:

$$\delta_g(\mathbf{x}) = \sum_O b_O \, O(\mathbf{x})$$ (at fixed time)

* Of course, everything in following will apply to any tracer of LSS.

All of what I will talk about, and much more, can be found in:

Large-Scale Galaxy Bias

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Physics Reports, in press

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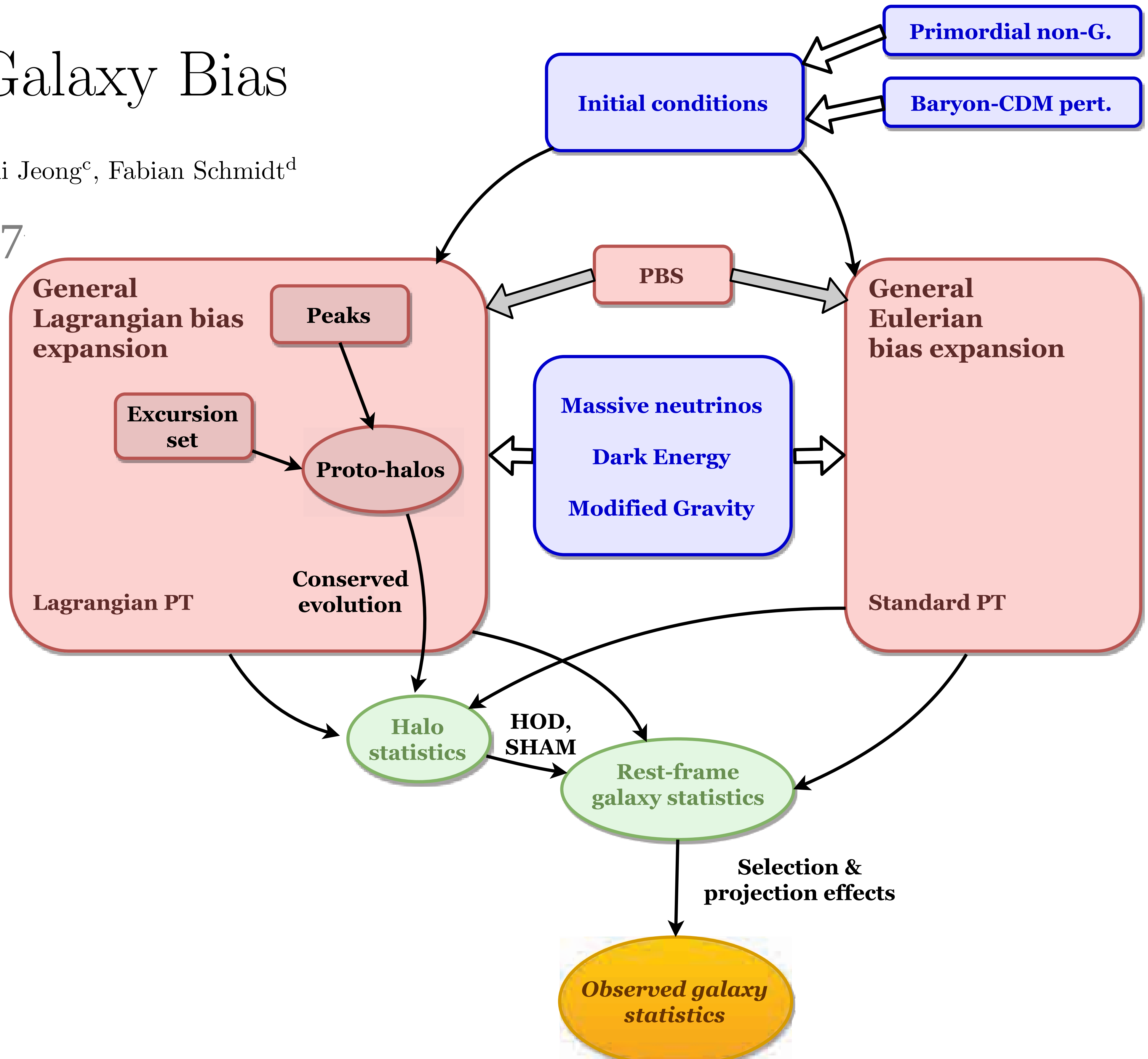

EFT approach in LSS

- Effective field theory: write down all terms (in Lagrangian or equations of motion) that are consistent with symmetries
  - Gravity: general covariance
  - Galaxy density: 0-component of 4-vector (momentum density)

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- Effective field theory: write down all terms (in Lagrangian or equations of motion) that are consistent with symmetries
  - Gravity: general covariance
  - Galaxy density: 0-component of 4-vector (momentum density)
- Order contributions by perturbative order, and number of spatial derivatives

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- LSS is non-relativistic: velocities v << c
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EFT approach in LSS

- LSS is non-relativistic: velocities v << c
- Only relevant metric component is time-time component: gravitational potential Φ
- Relevant remaining gauge symmetries:

$$\tau \to \tau + c(\tau) \Leftrightarrow \Phi \to \Phi + C(\tau)$$

Time rescaling

$$x^i \to x^i + \xi^i(\tau) \Leftrightarrow \Phi \to \Phi + A_i(\tau) x^i$$

$$v^i \to v^i + \xi^{i\prime}(\tau)$$

Time-dependent Lorentz boost ("generalized Galilei transformation")

$$x^i \to \boldsymbol{R}^i{}_j x^j$$

Rotations

EFT bias expansion

$$\tau \to \tau + c(\tau) \Leftrightarrow \Phi \to \Phi + C(\tau)$$

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$$\delta\,,\ \delta^2\,,\ \nabla^2\delta\,,\ \theta = \partial_i v^i\,,\ \frac{D\delta}{D\tau}\,,\ \cdots$$

    - But not velocity (forbidden by gauge symmetry)

$$\delta \equiv \frac{\rho_m - \bar{\rho}_m}{\bar{\rho}_m}$$

    - Time derivatives have to be convective: $\dfrac{D}{D\tau} = \partial_\tau + v^i \partial_i$

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    $$\nabla^2\Phi\,,\ (\partial_i\partial_j\Phi)^2\,,\ \frac{D}{D\tau}\nabla^2\Phi\,,\ \cdots$$

    - But not Φ or $\nabla\Phi$

EFT bias expansion

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- Many terms are redundant, as they are related through the equations of motion for matter and gravity
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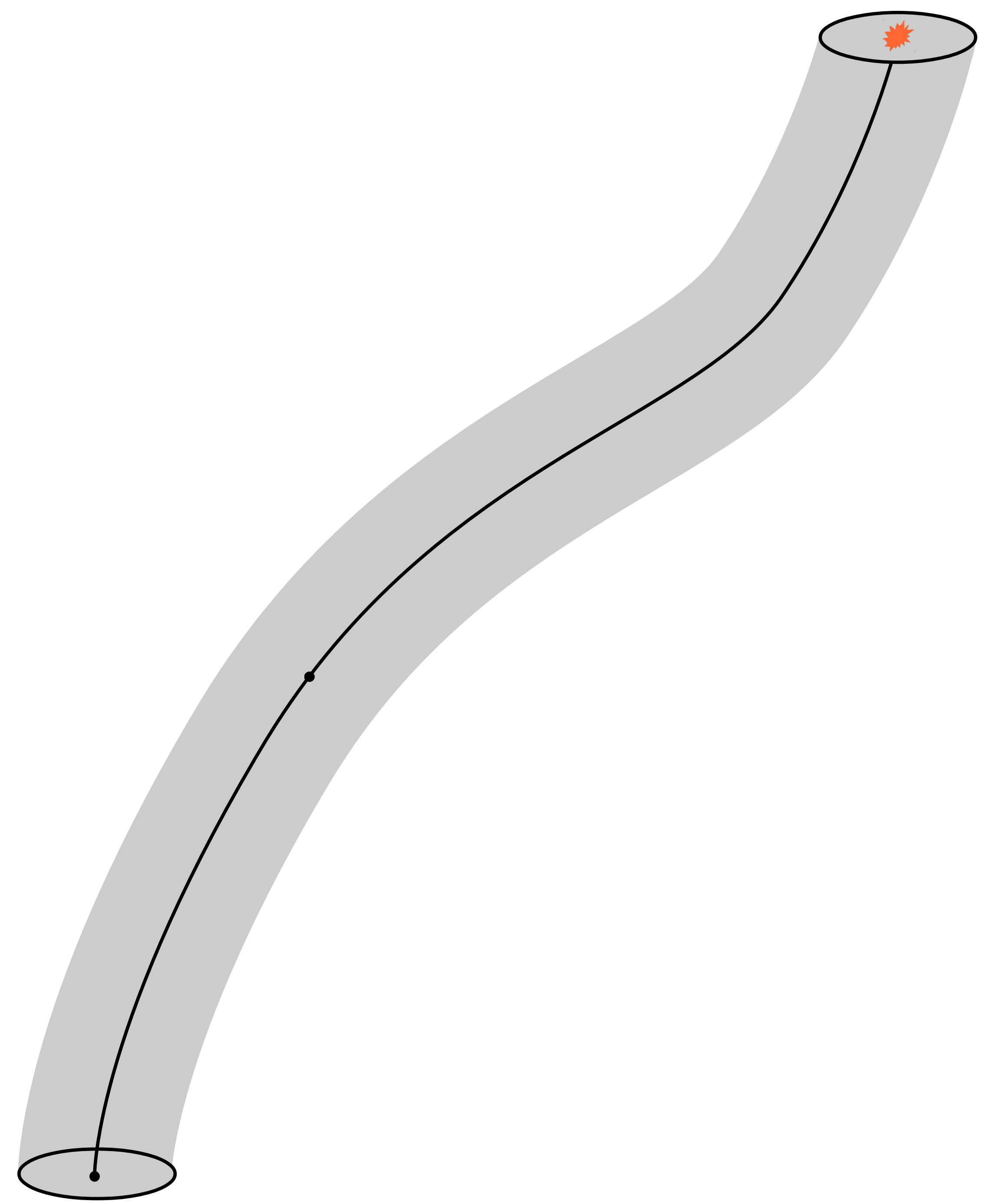
- We are not done yet however… Two issues:
- Many terms are redundant, as they are related through the equations of motion for matter and gravity
  - Cumbersome, but no problem - can eliminate redundant terms order by order in perturbations
- So far, we have written the EFT as local in time and space
  - Only makes sense if spatial and time derivatives are suppressed
  - True for spatial derivatives, but not for time derivatives! Galaxies form over many Hubble times (as does matter field)
  - Theory is "nonlocal in time"

Galaxy formation

- Consider coarse-grained (large scale) view of region that forms a galaxy at conformal time τ

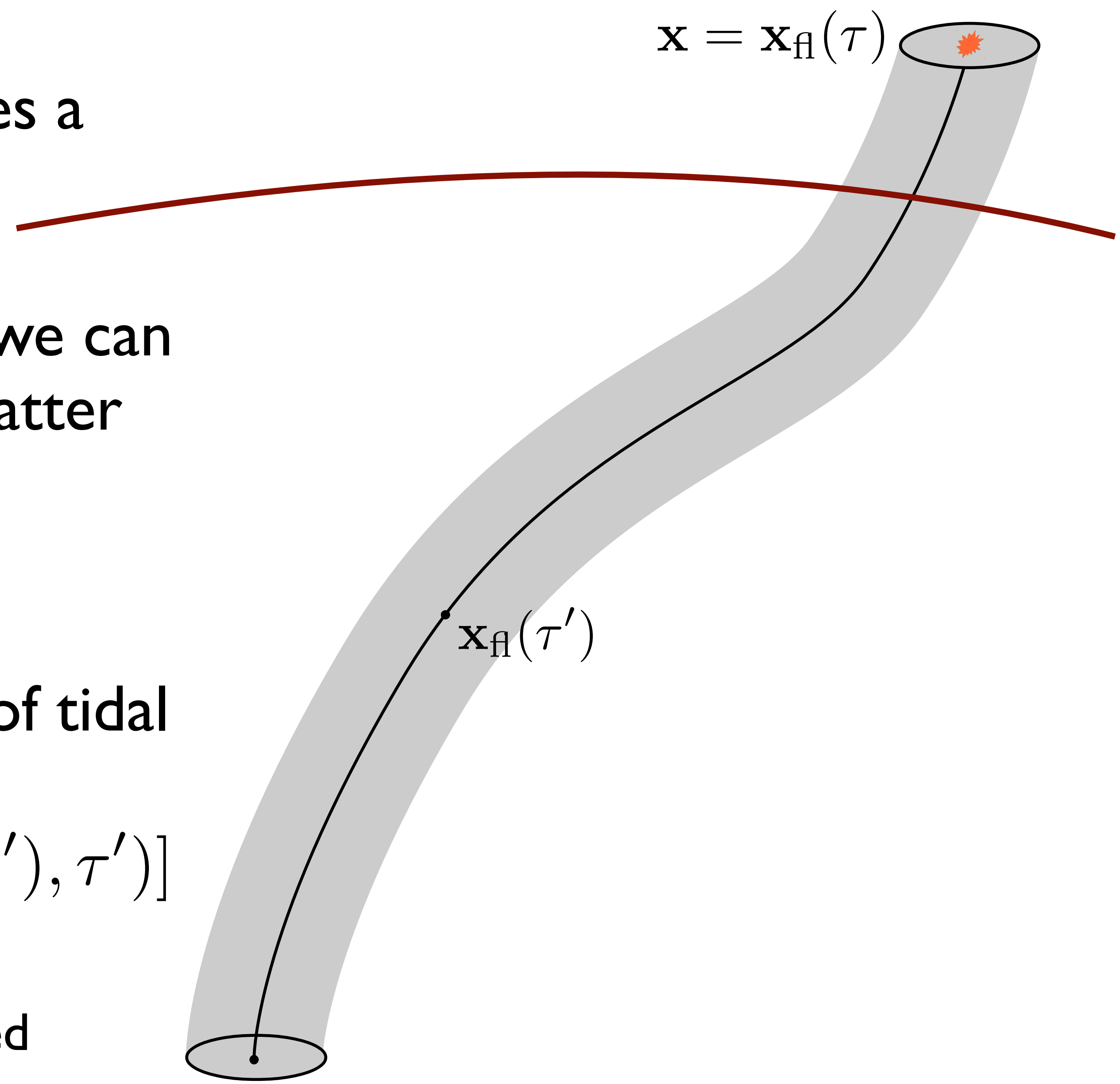
- Formation happens over long time scale, but small spatial scale R_*

  - For halos, expect $R_* \lesssim R_L$



Galaxy formation

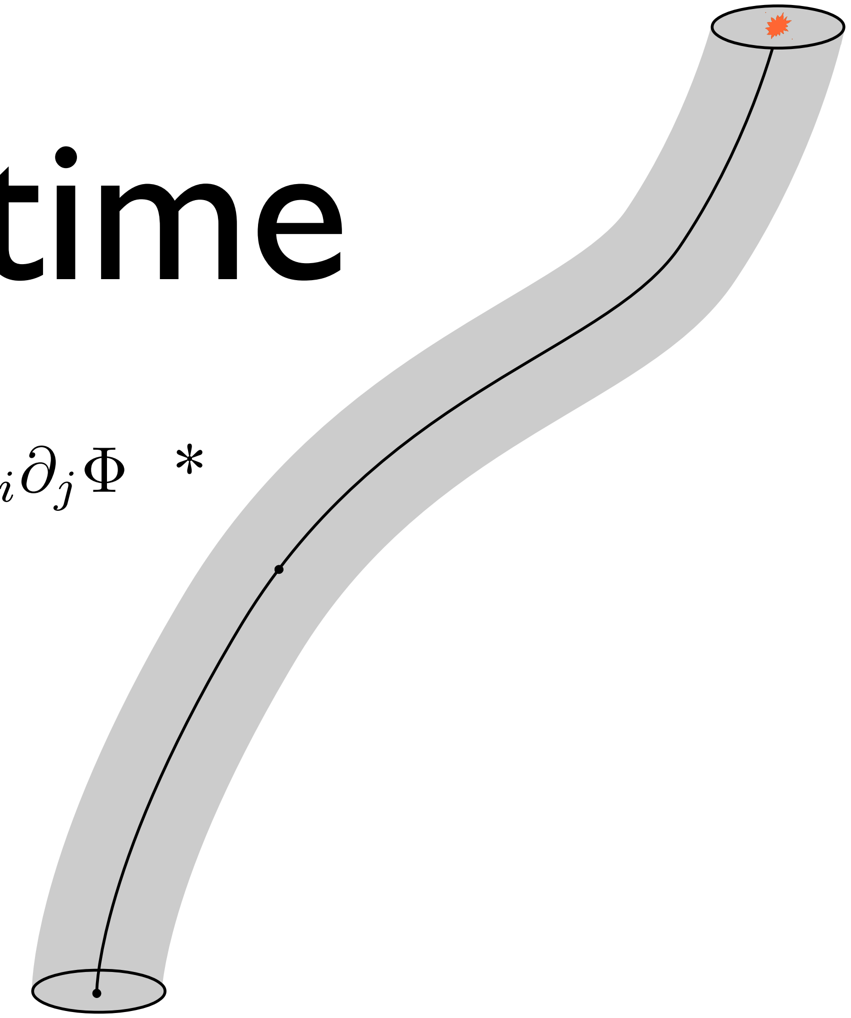
- Consider large-scale perturbations
- Galaxy density then becomes a local function *in space**
- Using equations of motion, we can eliminate dependence on matter density and velocity
- We are left with nonlinear, nonlocal-in-time functional of tidal tensor:
  $n_g(\mathbf{x}, \tau) = F_g \left[\partial_i \partial_j \Phi(\mathbf{x}_{\text{fl}}(\tau'), \tau')\right]$



* higher spatial derivatives are suppressed by $(\lambda/R_*)^2$ -> later

Mirbabayi, FS, Zaldarriaga '14

Non-locality in time



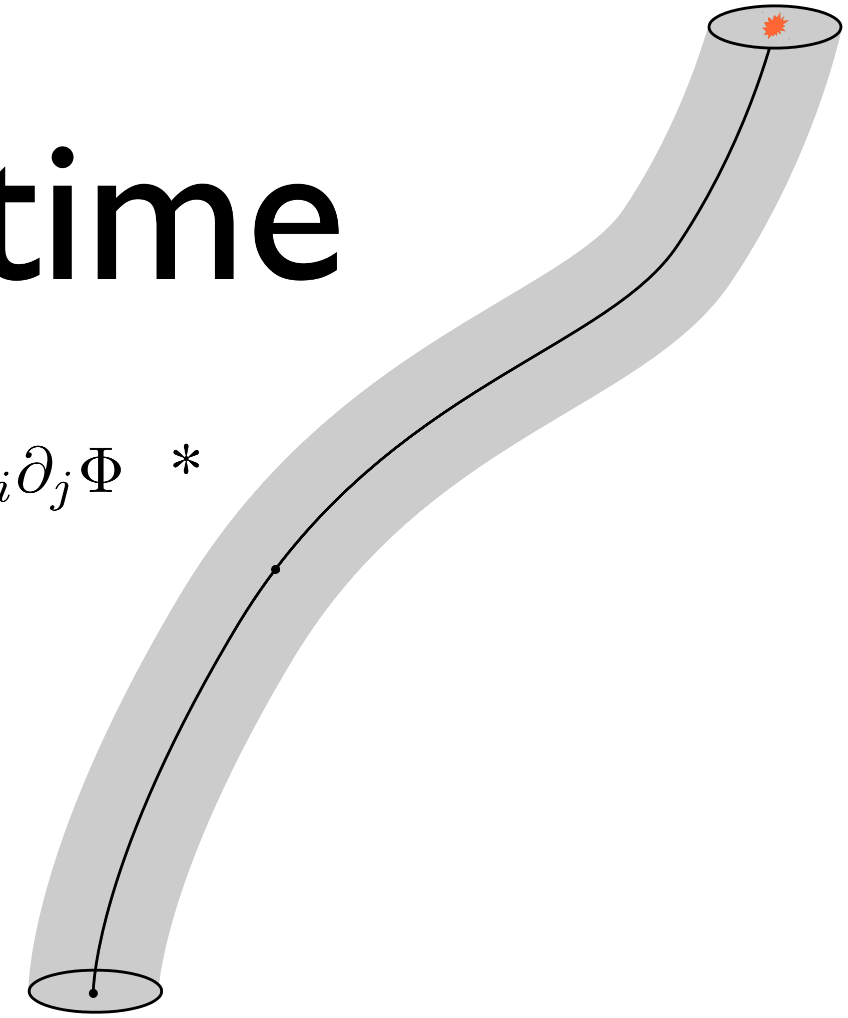
- Consider operator (field) O(x,t) that is constructed from $\partial_i \partial_j \Phi$ *
- For simplicity, consider linear dependence of galaxy on O
- Linear functional in time:

$$n_g(\boldsymbol{x}, \tau) = \int_{\tau_{\rm in}}^{\tau} \mathrm{d}\tau' \, f_O(\tau, \tau') \, O(\boldsymbol{x}_{\rm fl}(\tau'), \tau')$$

* From Poisson equation, $\delta \propto \nabla^2 \Phi$, so this includes "local bias" terms δ^n

Senatore '14; Mirbabayi, FS, Zaldarriaga '14

Non-locality in time



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$$n_g(\boldsymbol{x}, \tau) = \int_{\tau_{\rm in}}^{\tau} \mathrm{d}\tau' \; f_O(\tau, \tau') O(\boldsymbol{x}_{\rm fl}(\tau'), \tau')$$

- In perturbation theory, we know the time evolution of all these operators, e.g.

$$O(\boldsymbol{x}_{\rm fl}, \tau) = D^m(\tau) O^{(m)}(\boldsymbol{x}_{\rm fl}, \tau_0) + D^{m+1}(\tau) O^{(m+1)}(\boldsymbol{x}_{\rm fl}, \tau_0) + \cdots$$

- At n-th order, there can be at most n different time dependences, and hence *<= n independent terms!*
- Equivalently: arbitrarily high time derivatives can be written in terms of <= n terms

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Complete bias expansion

By reordering, we obtain a compact list of terms *up to any order,*

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Mirbabayi, FS, Zaldarriaga '14

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1^{st}	$\mathrm{tr}[\Pi^{[1]}]$
2^{nd}	$\mathrm{tr}[(\Pi^{[1]})^2]\,,\ (\mathrm{tr}[\Pi^{[1]}])^2$
3^{rd}	$\mathrm{tr}[(\Pi^{[1]})^3]\,,\ \mathrm{tr}[(\Pi^{[1]})^2]\,\mathrm{tr}[\Pi^{[1]}]\,,\ (\mathrm{tr}[\Pi^{[1]}])^3\,,\ \mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$
4^{th}	$\mathrm{tr}[(\Pi^{[1]})^4]\,,\ \mathrm{tr}[(\Pi^{[1]})^3]\,\mathrm{tr}[\Pi^{[1]}]\,,\ \left(\mathrm{tr}[(\Pi^{[1]})^2]\right)^2\,,\ (\mathrm{tr}[\Pi^{[1]}])^4\,,$
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where $\Pi^{[1]}_{ij} = \partial_i\partial_j\Phi(\mathbf{x},\tau)$

$$\Pi^{[n]}_{ij} \propto \frac{D}{D\tau}\Pi^{[n-1]}_{ij}$$

starts at n-th order in pert. theory

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starts at n-th order in pert. theory

Small-scale modes lead to *stochastic* contributions:

1^{st}	ϵ_1
2^{nd}	$\epsilon_2\,\mathrm{Tr}[\Pi_{ij}]$
3^{rd}	$\epsilon_3\,\mathrm{Tr}[(\Pi_{ij})^2]\,,\ \epsilon_4\,(\mathrm{Tr}[\Pi_{ij}])^2$
	$\ldots$

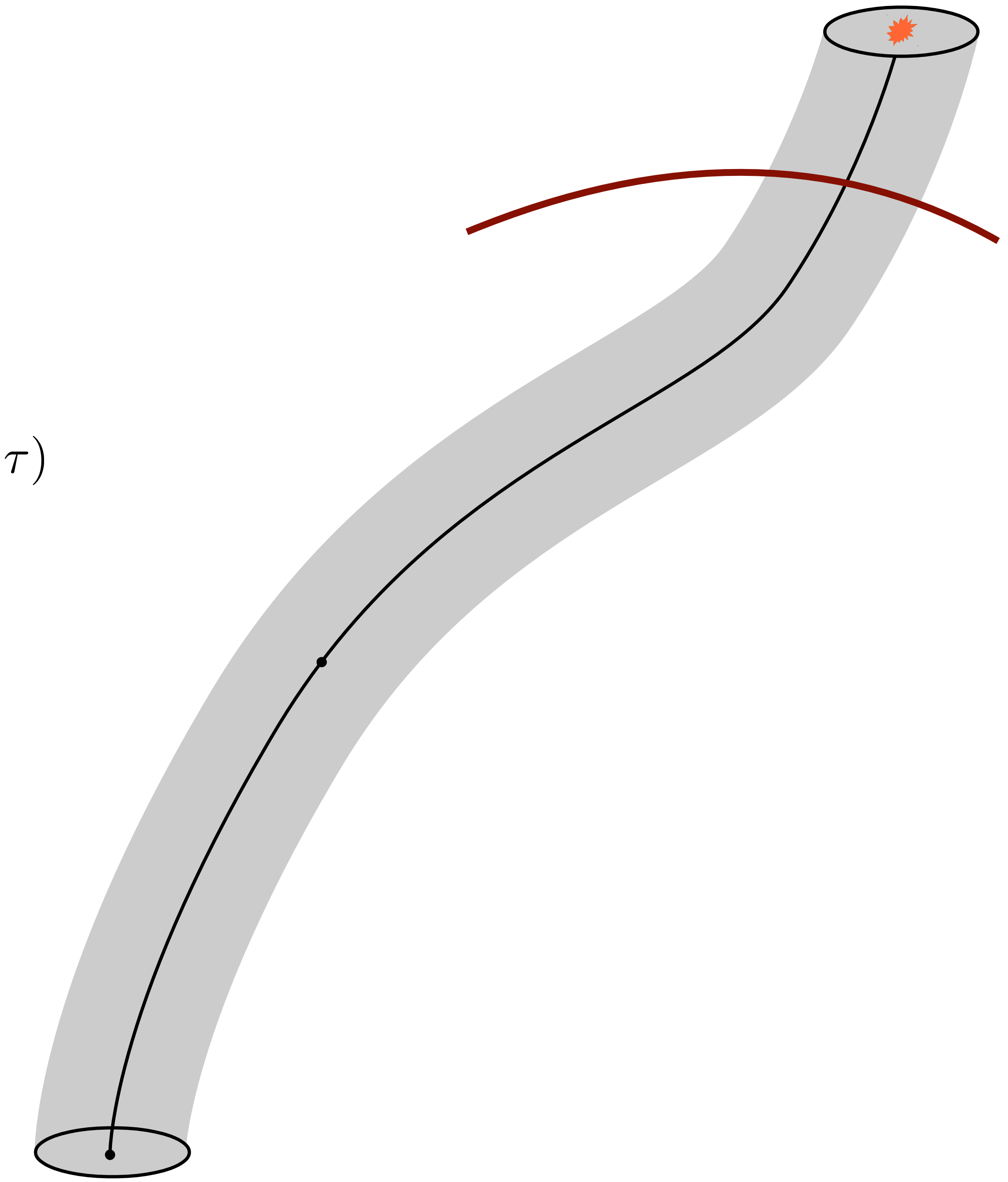
Mirbabayi, FS, Zaldarriaga '14

Complete bias expansion

- Some virtues of this expansion:
  - Linearly independent at each order
  - Complete in the EFT sense: closed under renormalization *(proof see MSZ)*
  - Equivalent expansions in Eulerian and Lagrangian space
    - Bias parameters can be mapped from one frame to another unambiguously

Spatial nonlocality and scale-dependent bias

- Beyond large-scale limit: need to expand *spatial nonlocality* of galaxy formation
- Higher derivative biases are suppressed with scale R_*
- E.g., $R_*^2 \nabla^2 \delta \longrightarrow \delta_g(\mathbf{k},\tau) = \left(b_1 + b_{\nabla^2\delta} k^2 R_*^2\right) \delta(\mathbf{k},\tau)$



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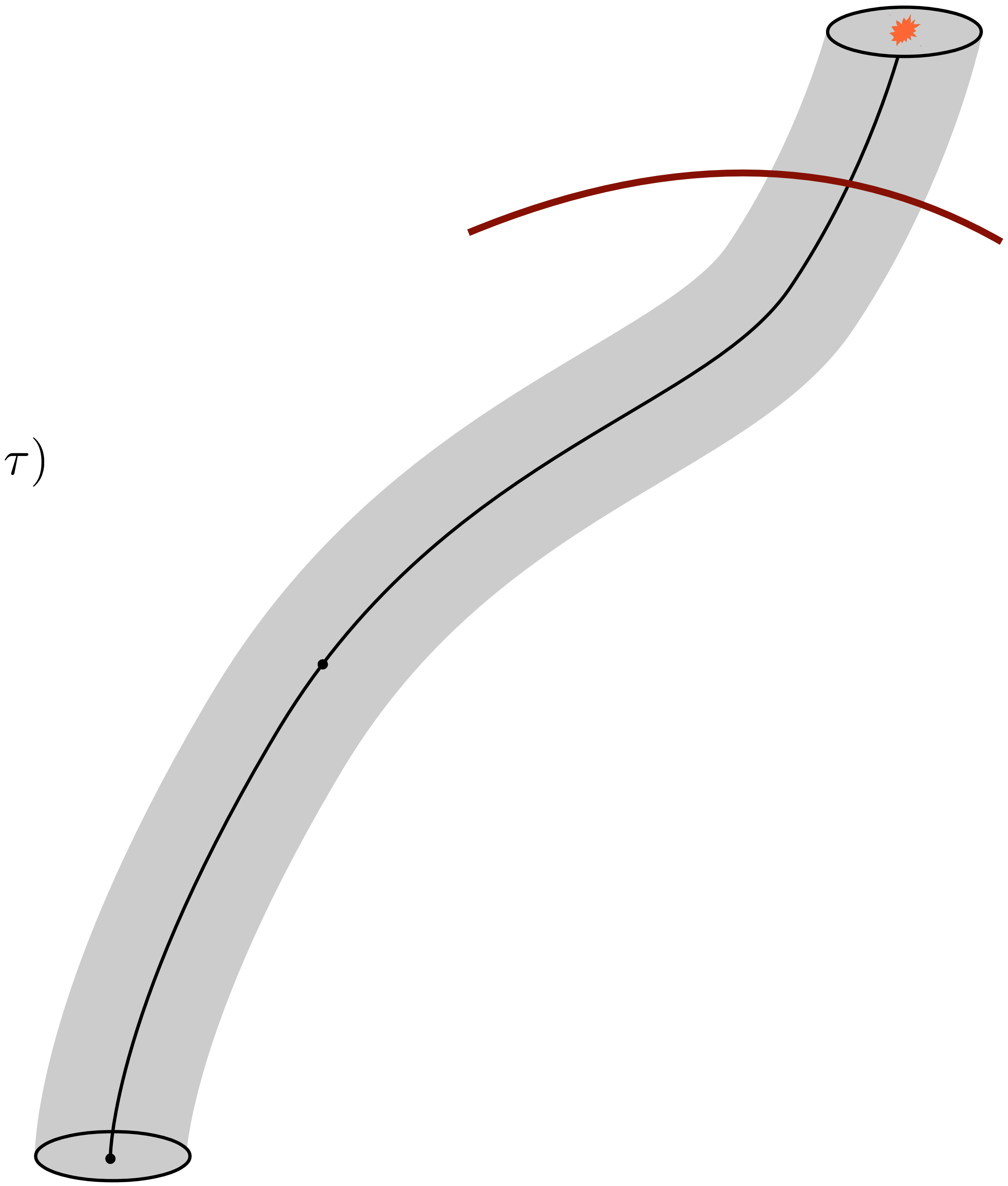
- This also allows for baryonic physics, which *has to come with additional derivatives*
  - Example: pressure perturbations $\delta p = c_s^2 \delta \rho$
  - Pressure force: $\boldsymbol{F} = \boldsymbol{\nabla} \delta p \propto \boldsymbol{\nabla} \delta$
- At higher order in derivatives, time evolution no longer determined by gravity alone



Velocity bias

- Galaxy velocities are important probe of cosmology - but how related to matter velocity?
- Recall that bias expansion for galaxy density cannot include $\nabla\Phi$
- The same is true for any observable - in particular also the relative velocity between matter and galaxies

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- Galaxy velocities are important probe of cosmology - but how related to matter velocity?
- Recall that bias expansion for galaxy density cannot include $\nabla\Phi$
- The same is true for any observable - in particular also the relative velocity between matter and galaxies
- Hence, relative velocity can be written as

$$v_g^i - v^i = \partial^i \left\{ \delta\,,\ (\partial_i\partial_j\Phi)^2\,,\ \cdots \right\}$$

- Necessarily higher derivative ~ R_*^2 ! Cf. pressure forces $\boldsymbol{F} = \boldsymbol{\nabla}\delta p \propto \boldsymbol{\nabla}\delta$
  - Also small-scale stochastic velocities, with power spectrum ~ k^4, which captures virial motions

- Summary: Galaxy velocities are unbiased on large scales.

Transformation to redshift space

- Observed galaxy statistics are obtained from rest-frame statistics via coordinate transformation to redshift space

$$\boldsymbol{x}_{\rm obs} = \boldsymbol{x} + \frac{\boldsymbol{v}\cdot\hat{\boldsymbol{n}}}{aH}\hat{\boldsymbol{n}}$$

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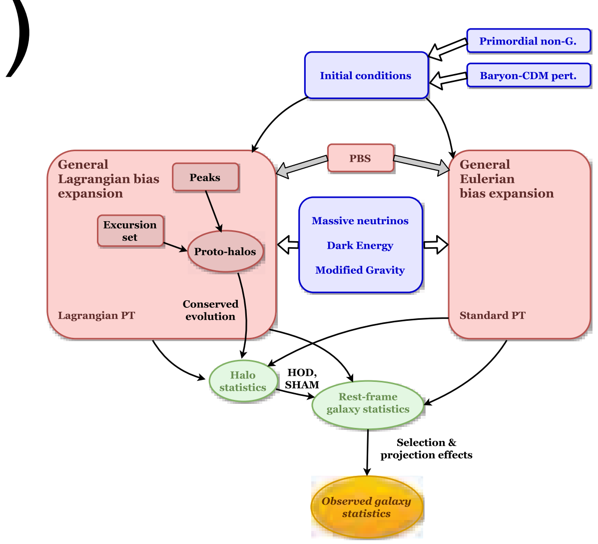
- By combining three ingredients, we can obtain consistent theoretical description for observed galaxy statistics (n-point functions in redshift space):
    - Perturbation theory for matter
    - Bias expansion
    - Velocity bias expansion

Impact of initial conditions

- So far, assumed Gaussian, adiabatic initial conditions
- If these assumptions are violated at most weakly (as indicated by CMB), can perturbatively include these:
  - Primordial non-Gaussianity
  - Relative density/velocity perturbations between CDM and baryons (from pre-recombination)
- In each case, obtain well-defined finite set of additional terms in bias expansion



Application: galaxy power spectrum

- Assume we can measure rest-frame galaxy density
  - That is, neglect redshift-space distortions and other projection effects
- Leading-order galaxy power spectrum at fixed time:

$$P_{gg}(k) = b_1^2 P_{\mathrm{L}}(k) + P_{\varepsilon}^{\{0\}}$$

  - Valid on very large scales
  - 2 free parameters

Application: galaxy power spectrum

- Next-to-leading order (NLO): involve 2 additional quadratic, 1 cubic, and 2 higher-derivative parameters:

$$
\begin{aligned}
P_{hm}^{\rm NLO}(k) &= b_1\left[P_{mm}^{\rm NLO}(k) - 2C_{s,\rm eff}^2 k^2 P_{\rm L}(k)\right] + \hat{P}_{hm}^{\rm NLO}(k) \\
\hat{P}_{hm}^{\rm NLO}(k) &\equiv b_{\delta^2}\mathcal{I}^{[\delta^{(2)},\delta^2]}(k) + b_{K^2}\mathcal{I}^{[\delta^{(2)},K^2]}(k) + \left(b_{K^2} + \frac{2}{5}b_{\rm td}\right) f_{\rm NLO}(k) P_{\rm L}(k) \\
&\quad - b_{\nabla^2\delta} k^2 P_{\rm L}(k) + k^2 P_{\varepsilon\varepsilon_m}^{\{2\}} \\
P_{hh}^{\rm NLO}(k) &= (b_1)^2\left[P_{mm}^{\rm NLO}(k) - 2C_{s,\rm eff}^2 k^2 P_{\rm L}(k)\right] + 2b_1\hat{P}_{hm}^{\rm NLO}(k) + \sum_{O,O'\in\{\delta^2,K^2\}} b_O b_{O'}\mathcal{I}^{[O,O']}(k) + k^2 P_{\varepsilon}^{\{2\}},
\end{aligned}
$$

with

$$
\begin{aligned}
f_{\rm NLO}(k) &= 4\int_{\boldsymbol{p}}\left[\frac{[\boldsymbol{p}\cdot(\boldsymbol{k}-\boldsymbol{p})]^2}{p^2|\boldsymbol{k}-\boldsymbol{p}|^2} - 1\right] F_2(\boldsymbol{k},-\boldsymbol{p}) P_{\rm L}(p) \\
\mathcal{I}^{[O,O']}(k) &= 2\Bigg[\int_{\boldsymbol{p}} S_O(\boldsymbol{k}-\boldsymbol{p},\boldsymbol{p}) S_{O'}(\boldsymbol{k}-\boldsymbol{p},\boldsymbol{p}) P_{\rm L}(p) P_{\rm L}(|\boldsymbol{k}-\boldsymbol{p}|) \\
&\qquad - \int_{\boldsymbol{p}} S_O(-\boldsymbol{p},\boldsymbol{p}) S_{O'}(-\boldsymbol{p},\boldsymbol{p}) P_{\rm L}(p) P_{\rm L}(p)\Bigg],
\end{aligned}
$$

$$
\text{where}\quad S_O(\boldsymbol{k}_1,\boldsymbol{k}_2) = \begin{cases} F_2(\boldsymbol{k}_1,\boldsymbol{k}_2), & O = \delta^{(2)} \\ 1, & O = \delta^2 \\ (\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_2)^2 - 1/3, & O = K^2 \end{cases}.
$$

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$$P_{hm}^{\rm NLO}(k) = b_1\left[P_{mm}^{\rm NLO}(k) - 2C_{s,\rm eff}^2 k^2 P_{\rm L}(k)\right] + \hat{P}_{hm}^{\rm NLO}(k)$$

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with

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Application: galaxy power spectrum

- Quadratic and cubic terms scale like

$$\epsilon_{\text{loop}} \equiv \left(\frac{k}{k_{\text{NL}}}\right)^{3+n} \approx \left(\frac{k}{0.25\,h\,\text{Mpc}^{-1}}\right)^{1.3}$$

  - Controlled by shape of P(k) and nonlinear scale

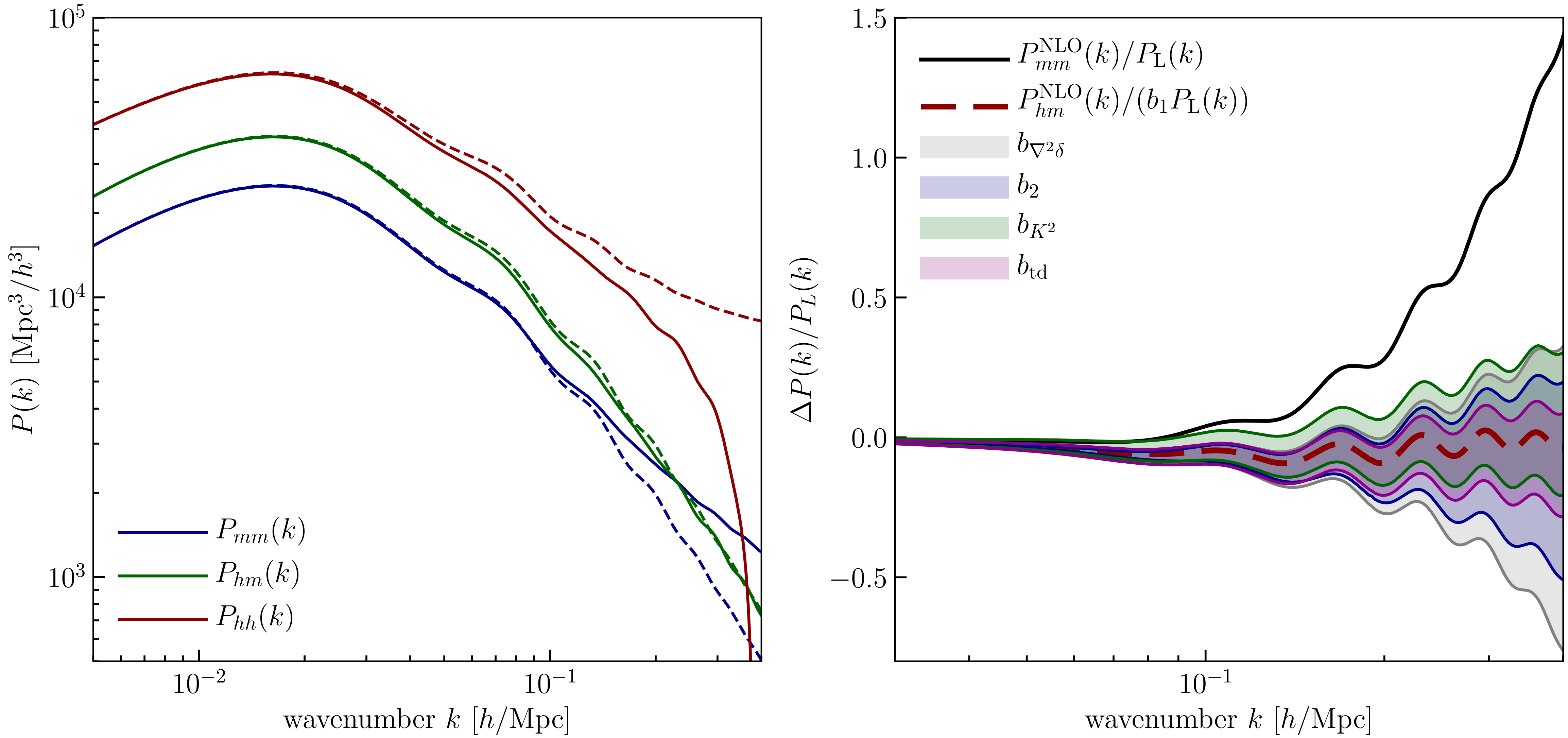
- Higher-derivative contributions scale as

$$\epsilon_{\text{deriv.}} \equiv k^2 R_*^2$$

- Obviously, NLO corrections become important toward smaller scales (higher k)

- Importantly: Two independent expansion parameters!

Illustration of NLO contributions to galaxy power spectrum



- Many contributions have very similar shape
- If only interested in power spectrum, can significantly reduce number of free parameters

Further applications

- Galaxy density and velocity are not the only application of EFT approach/general bias expansion:

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1. Galaxy shapes = intrinsic alignments: symmetric, trace-free 2-tensor

   FS, Chisari, Dvorkin 2015
   Vlah, Chisari, FS, in prep.

2. Complete description of counterterms in EFT of matter

   Zaldarriaga & Mirbabayi, 2015

3. Small scale power spectrum = "responses": symmetric 2-/4-/6-/… tensor

   - Describes nonlinear matter n-point functions in squeezed limit (bispectrum, covariance, …)

   Wagner, FS et al 2015
   Barreira & FS 2017a,b

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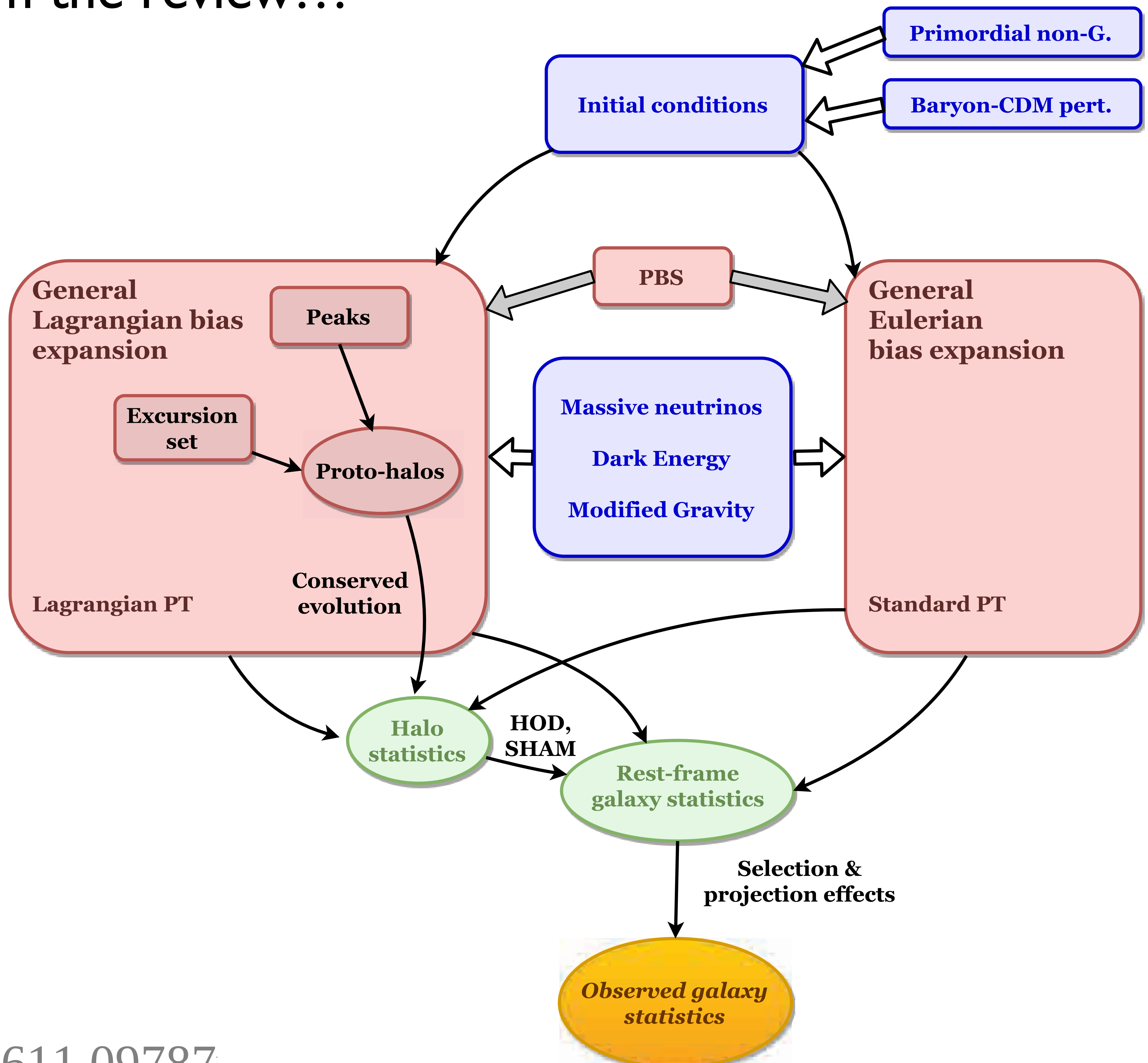
- Leads to well-defined prediction for all n-point functions of galaxies
  - "Just compute"
  - Lots of free parameters, but many of them quasi-degenerate
- Next challenges:
  - How much information in nonlinear galaxy clustering, given many free parameters?
  - How best to extract it?

- From the review…



arXiv:1611.09787