Search for Dark Neutrino via Vacuum Magnetic Birefringence Experiment

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X. Fan (Harvard Univ.), S. Kamioka, S. Asai (Tokyo Univ.) experiment
A. Sugamoto (Ochanomizu Univ., OUJ) theory

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(arXiv:1707.03308)

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Kavli IPMU

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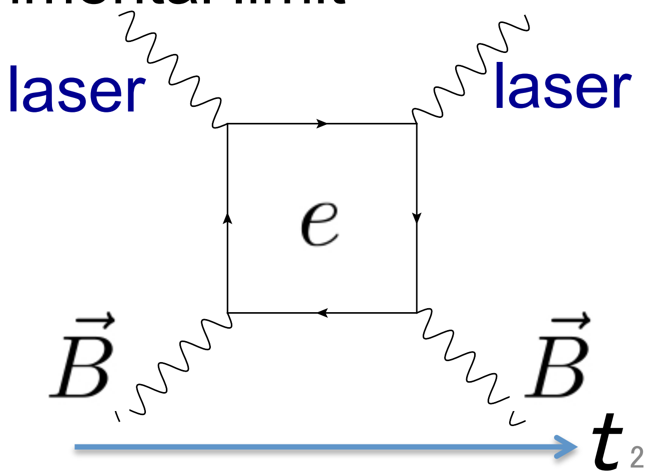
Phenomenology and Proposal for the Experiment

1. Introduction: Vacuum as a “medium”

1. Vacuum is a “medium” in which particles and anti-particles are pair-created and pair-annihilated quantum mechanically
2. We study the magneto-active vacuum in which the constant magnetic field exists as a background
3. We derived a low energy effective Lagrangian including a parity violated term for light-by-light scattering by integrating out the dark fermion and gave constraints on model parameters from a current table-top, low-energy experimental limit

Dark Sectors are collections of fields whose couplings with Standard model particles are extremely weak

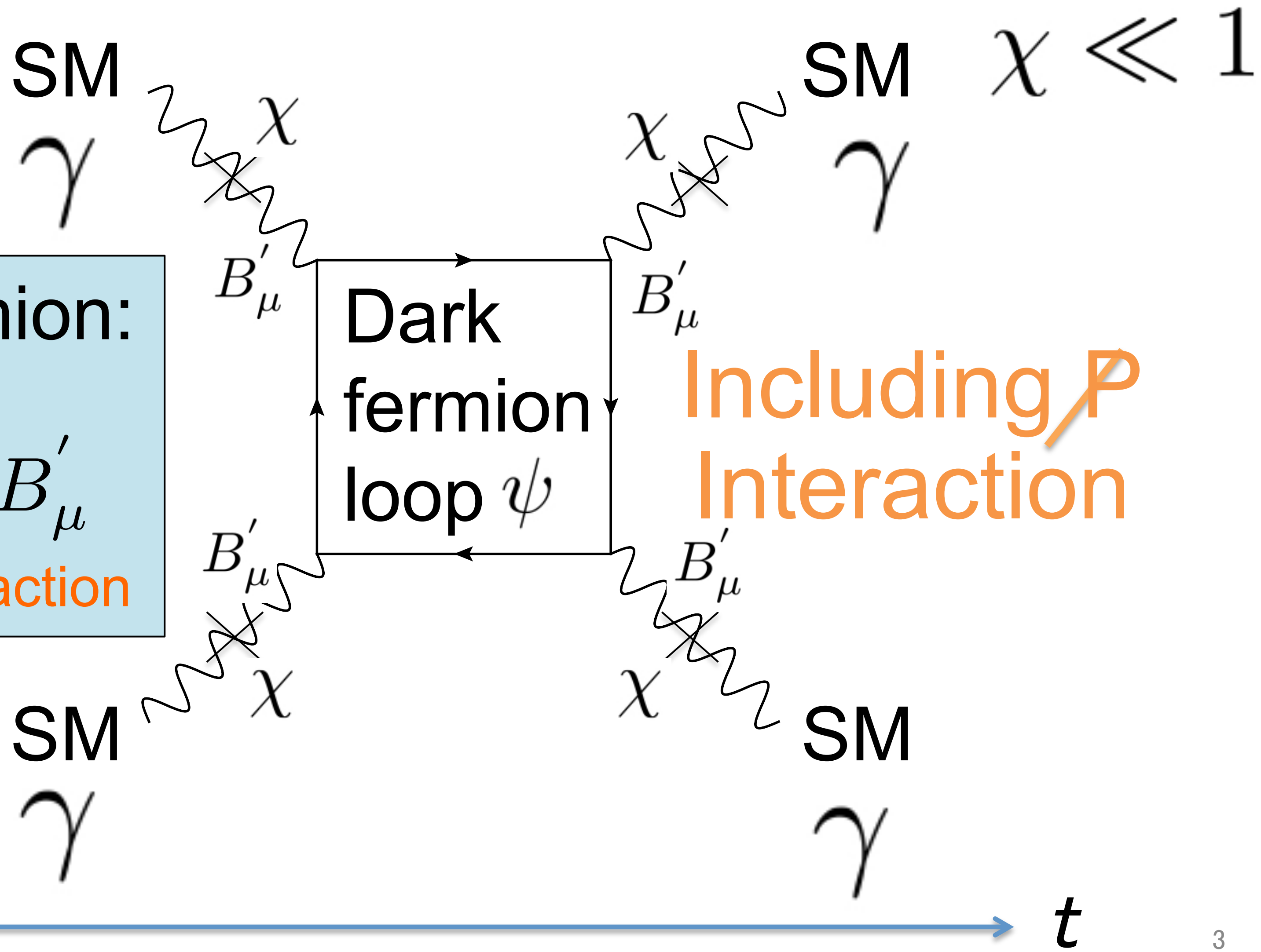


1. Introduction: Dark Sector Search

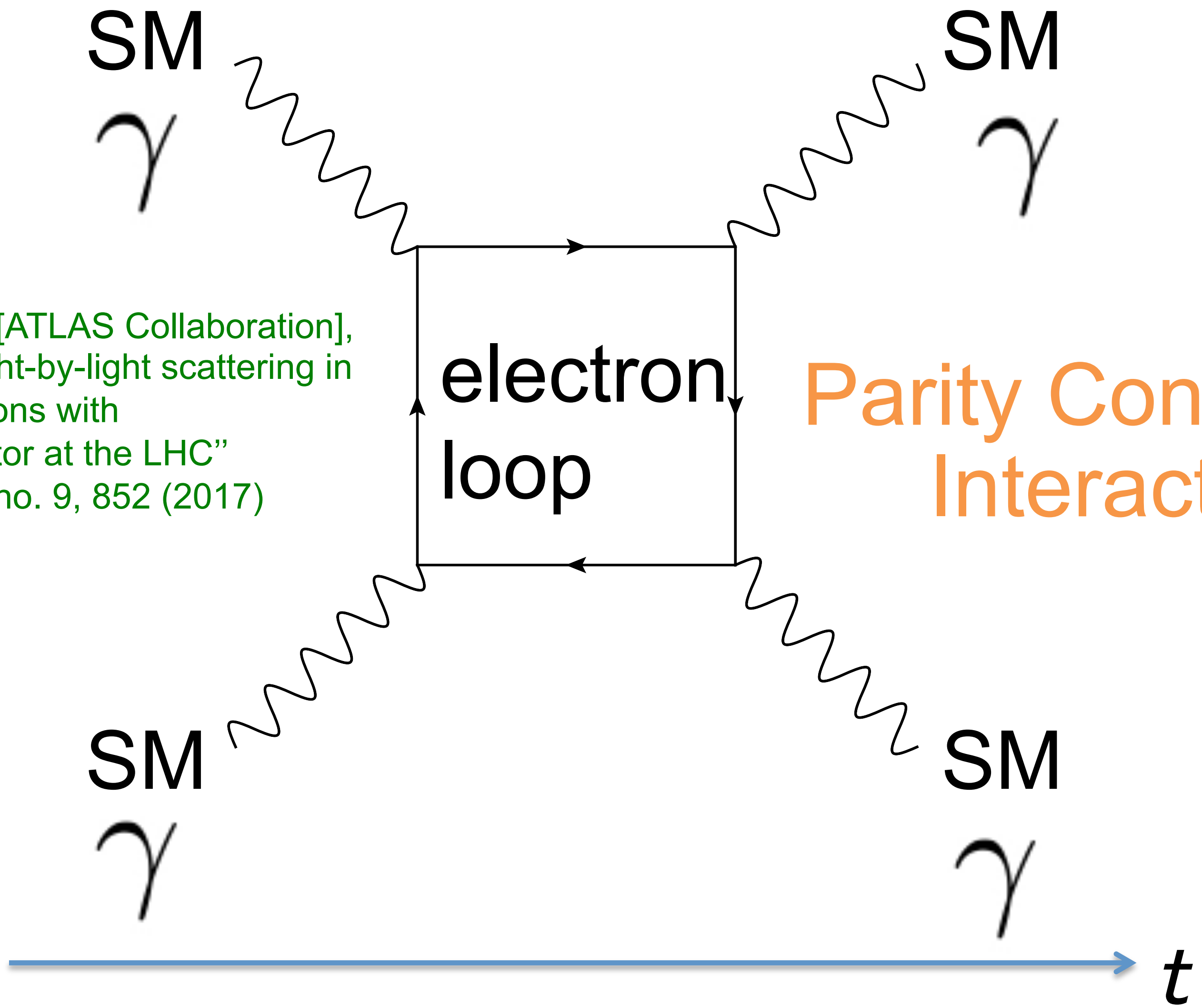
Dark fermion:

- ψ×1
- $\cancel{P}$ with B'_μ

  V-A interaction



1. Introduction: cf. QED interaction



M. Aaboud *et al*. [ATLAS Collaboration], ``Evidence for light-by-light scattering in heavy-ion collisions with the ATLAS detector at the LHC'' Nature Phys.**13**, no. 9, 852 (2017)

Parity Conserving Interaction

Need to Calculate Effective Lagrangian

→ Vacuum Birefringence Experiment

1. Introduction: QED Case

already known

W. Heisenberg, H. Euler, Z. Phys. **98**, 714 (1936)

Heisenberg-Euler Lagrangian:

$$\mathfrak{L} = -\mathfrak{F} - \frac{1}{8\pi^2}\int_0^\infty ds\, s^{-3}\exp(-m^2 s) \times\left[(es)^2\mathfrak{G}\frac{\mathrm{Re}\cosh esX}{\mathrm{Im}\cosh esX} - 1 - \tfrac{2}{3}(es)^2\mathfrak{F}\right]$$

$$= \tfrac{1}{2}(\mathbf{E}^2-\mathbf{H}^2) + \frac{2\alpha^2}{45}\frac{(\hbar/mc)^3}{mc^2}\times[(\mathbf{E}^2-\mathbf{H}^2)^2+7(\mathbf{E}\cdot\mathbf{H})^2]+\cdots$$

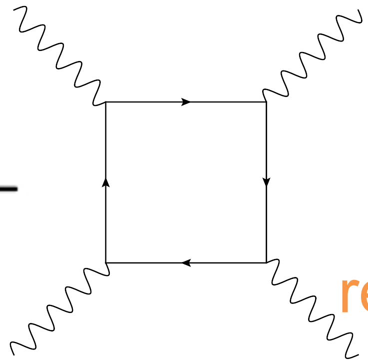
$$X = \sqrt{2(\mathcal{F}+i\mathcal{G})},$$

$$\mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\left(\vec{H}^2 - \vec{E}^2\right)$$

$$\mathcal{G} = \frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \vec{E}\cdot\vec{H}$$

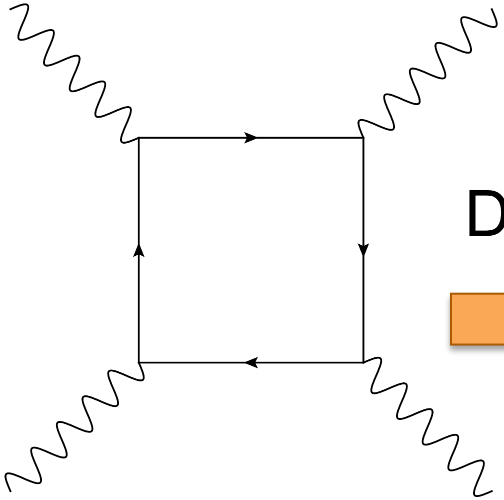
$$\tilde{F}_{\mu\nu} = \tfrac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$$

from J. Schwinger, Phys. Rev. **82**, 664 (1951)

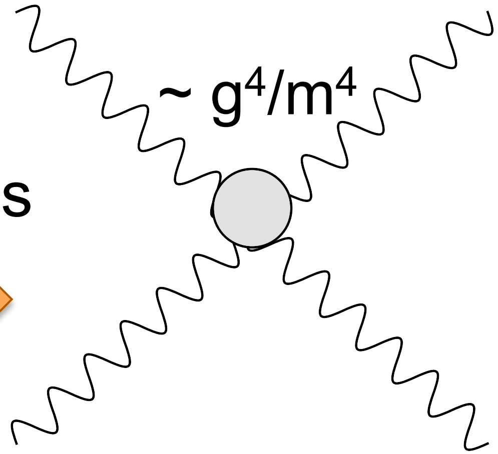
$$\mathcal{L}_{\rm eff} = -\tfrac{1}{4}F_{\mu\nu}^2 +$$  $$+ \quad \cdots$$

related to this

1. Introduction: Dark Sector Case



Photon Energy
^
^
Dark Fermion Mass



$+ \cdots$
ignore
higher
dimensional
terms

$$\mathcal{F} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2}\left(\vec{B}^2 - \vec{E}^2\right)$$

$$\mathcal{G} = \frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} = \vec{E} \cdot \vec{B}$$

We calculated here including coefficients a,b,c

$E_\gamma \ll m_{DM}$

$$\mathcal{L}_{\text{eff}} = -\mathcal{F} + \boxed{a\mathcal{F}^2 + b\mathcal{G}^2 + ic\mathcal{F}\mathcal{G}}$$

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2-1. Effective Action in Proper-time Representation

Dark fermion:
- $\psi\times 1$
- $\cancel{P}$ with γ

$\cancel{P}$ Action:

Include axial current coupling

$$S_\psi(m) = \int d^4x\, \bar{\psi}\left[\gamma^\mu\left(i\partial_\mu - (g_V + g_A\gamma_5)A_\mu\right) - m\right]\psi$$

Effective Action:

$$S_{\text{eff}}[A_\mu] = \int d^4x\, \mathcal{L}_{\text{eff}}[A_\mu] = -i\ln\left[\underbrace{\int \mathcal{D}\psi(x)\mathcal{D}\bar{\psi}(x)e^{iS_\psi(m)}}_{\text{Integrated out}}\right]$$

$$= (-i)\frac{1}{2}\mathrm{Tr}\ln(\hat{H} + m^2)$$

$$\boxed{A_\mu(x) = \frac{1}{2}x^\lambda F_{\lambda\mu}}$$

$F_{\mu\nu}$: Constant
$\rightarrow$ easier to get L_{eff}

$$\hat{H} = -\left(i\partial_\mu - g_V\frac{1}{2}x^\nu F_{\nu\mu}(x)\right)^2 - \frac{1}{4}x^\mu\left(g_A^2 F_{\mu\lambda}F^{\lambda\nu}\right)x_\nu$$
$$+\frac{1}{2}(g_V + g_A\gamma_5)\sigma^{\mu\nu}F_{\mu\nu} + i\frac{1}{2}\sigma^{\mu\nu}g_A\gamma_5(x^\lambda F_{\lambda\mu}\, i\partial_\nu - x^\lambda F_{\lambda\nu}\, i\partial_\mu)$$

2-1. Effective Action in Proper-time Representation

x(0) = x(s)

$$S_{\text{eff}}(A) = (-i)\frac{1}{2}\mathrm{Tr}\ln(\hat{H}+m^2)$$

traces of x^μ and spin

Proper time description:

$$\boxed{= \frac{i}{2}\int_0^\infty \frac{ds}{s}\, e^{-im^2 s}\,\mathrm{Tr}(e^{-i\hat{H}s})}$$

transition amplitude

$$\int d^4x\ \mathrm{tr}\ \langle x(s), a(s) | x(0), b(0)\rangle$$

V. Fock,
Physik. Z. Sowjetunion, **12**, 404 (1937),
Y. Nambu,
Prog. Theor. Phys. **5**, 82 (1950)

Quantum mechanics of a point particle with position $x^\mu(s)$ and spin a(s) at a proper time s

2-2. Path Integral Representation

$$S_{\text{eff}}(A) = \frac{i}{2}\int_0^\infty \frac{ds}{s}\, e^{-im^2 s}\, \mathrm{Tr}(e^{-i\hat{H}s})$$

$$= \int d^4x \, \boxed{\underset{\text{spin}}{\mathrm{tr}} \int_{x^\mu(0)=x^\mu}^{x^\mu(s)=x^\mu} \mathcal{D}x^\mu(s')\, e^{i\int_0^s ds'\, \tilde{L}(x(s'),\, \dot{x}(s'))}}$$

$$\tilde{L}(x^\mu(s), \dot{x}^\mu(s))$$

$$= -\frac{1}{4}(\dot{x}^\mu)^2 + \boxed{\frac{1}{2}x^\mu F_{\mu\lambda}}\left(g_V\, g^{\lambda\nu} + \boxed{ig_A\, \gamma_5\sigma^{\lambda\nu}}\right)\dot{x}_\nu - \frac{1}{2}g_A^2\, x^\mu F_{\mu\lambda}F^{\lambda\nu}x_\nu$$

$$- \frac{1}{2}\sigma^{\mu\nu}(g_V + g_A\gamma_5)F_{\mu\nu}$$

This is the difficult part

If $g_A \neq 0$, x^μ and spin $\sigma^{\mu\nu}$ decouple (this is Heisenberg-Euler case), but <u>they do couple</u> when Parity is violated (our case).

Take "tr" for the spin easily, then

$$S_{\text{eff}}(A) = \frac{i}{2}\int_0^\infty \frac{ds}{s} e^{-im^2 s} \times$$

$$\boxed{\int \mathcal{D}x^\mu(s')\, e^{i\bar{A}(s)} \times 2\left(\cos \underbrace{\sqrt{2(\bar{\mathcal{F}}'(s) + i\bar{\mathcal{G}}'(s))}}_{=\bar{X}'_+(s)} + \cos \underbrace{\sqrt{2(\bar{\mathcal{F}}'(s) - i\bar{\mathcal{G}}'(s))}}_{=\bar{X}'_-(s)}\right)}$$

Lagrangian is separated into
A(s) (free part) and B(s) (interaction part): $\int_0^s ds' \tilde{L}(s') = \int_0^s ds' \left(A(s') + \frac{1}{2}B_{\mu\nu}(s')\sigma^{\mu\nu}\right)$

$$\bar{A}(s) = \int_0^s ds'\left[-\frac{1}{4}(\dot{x}^\mu)^2 + \frac{1}{2}g_V x^\mu (F_{\mu\nu})\dot{x}^\nu - \frac{1}{2}g_A^2 x^\mu (F_{\mu\lambda}F^{\lambda\nu})x_\nu\right]$$

$$\bar{\mathcal{F}}'(s) = \frac{1}{4}\bar{B}_{\mu\nu}(s)\bar{B}^{\mu\nu}(s), \quad \bar{\mathcal{G}}'(s) = \frac{1}{4}\bar{B}_{\mu\nu}(s)\bar{\tilde{B}}^{\mu\nu}(s).$$

$$\bar{B}_{\mu\nu}(s) = \int_0^s ds'\left[\underline{g_A \frac{1}{2}\epsilon_{\mu\nu\beta\gamma}x_\alpha F^{\alpha\beta}\dot{x}^\gamma} - (g_V F_{\mu\nu} - i g_A \tilde{F}_{\mu\nu})\right] \quad \gamma_5\sigma_{\mu\nu} = -\tfrac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}$$

The path integration can't be performed exactly.

A general expression of the effective action can be obtained, even in case of parity violation. However, the contractions by the propagator $\langle \cdots \rangle'$ remain.

= 1 in QED

$$\mathcal{L}_{\rm eff} = -\frac{1}{8\pi^2}\int_0^\infty \frac{ds}{s^3} e^{-m^2 s} \boxed{\frac{(g_+ s)^2 \mathcal{G}}{Im\cosh(g_+ X s)}} \times \boxed{\frac{(g_- s)^2 \mathcal{G}}{Im\cosh(g_- X s)}}$$
$$\boxed{\times \frac{1}{2}\left\langle \left(\cos \bar{\boldsymbol{X}}'_+(s\to -is) + \cos \bar{\boldsymbol{X}}'_-(s\to -is)\right)\right\rangle'}$$

$$g_\pm = \frac{1}{2}(g_V \pm \sqrt{g_V^2 + 2g_A^2})$$

g_V=-e, g_A=0 (g_+=0, g_-=-e)

$$\bar{\boldsymbol{X}}'_+ = s g_V X, \ \text{and} \ \bar{\boldsymbol{X}}'_- = s g_V X^\dagger$$

reproduce QED result.

$$\mathfrak{L} = -\mathfrak{F} - \frac{1}{8\pi^2}\int_0^\infty ds s^{-3}\exp(-m^2 s)$$
$$\times\left[(es)^2\mathfrak{G}\frac{\text{Re}\cosh esX}{\text{Im}\cosh esX} - 1 - \tfrac{2}{3}(es)^2\mathfrak{F}\right]$$

see J. Schwinger, Phys. Rev. **82**, 664 (1951)

$$X = \sqrt{2(\mathcal{F}+i\mathcal{G})},$$

3. Effective Lagrangian of Fourth Order

$$\mathcal{L}_{\rm eff} = -\frac{1}{8\pi^2}\int_0^\infty \boxed{\frac{ds}{s^3}} e^{-m^2 s} \frac{(g_+ s)^2\mathcal{G}}{Im\cosh(g_+ X s)} \times \frac{(g_- s)^2\mathcal{G}}{Im\cosh(g_- X s)}$$

$$\times \frac{1}{2}\left\langle \left(\cos\bar{X}'_+(s\to -is) + \cos\bar{X}'_-(s\to -is)\right)\right\rangle'$$

$$\|$$

$$1 - \langle \bar{\mathcal{F}}'(s)\rangle' + \frac{1}{6}\langle (\bar{\mathcal{F}}'(s))^2 - (\bar{\mathcal{G}}'(s))^2\rangle' + \cdots$$

dimension 4

extract s^4 terms because

$O(s^4)$ corresponds to $O(F^4)$

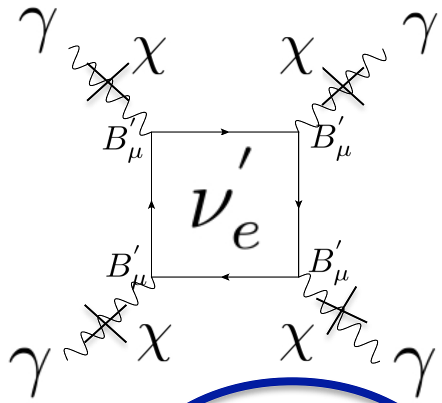
($sF_{\mu\nu}$ makes no dimension)

$$\mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\left(\vec{B}^2 - \vec{E}^2\right)$$

$$\mathcal{G} = \frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \vec{E}\cdot\vec{B}$$

$$\boxed{a\mathcal{F}^2 + b\mathcal{G}^2 + ic\mathcal{F}\mathcal{G}}$$

3. Effective Lagrangian of Fourth Order



$$\mathcal{L}_{\rm eff} = -\mathcal{F} + \boxed{\chi^4 (a\mathcal{F}^2 + b\mathcal{G}^2 + ic\mathcal{F}\mathcal{G})}$$

$\cancel{P}$

$$\mathcal{F} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2}\left(\vec{B}^2 - \vec{E}^2\right) \quad \mathcal{G} = \frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} = \vec{E} \cdot \vec{B}$$

$$a = \frac{1}{(4\pi)^2 m^4} \left(\frac{8}{45}\ g_V^4 - \frac{4}{5}\ g_V^2 g_A^2 - \frac{1}{45}\ g_A^4 \right)$$

$$b = \frac{1}{(4\pi)^2 m^4} \left(\frac{14}{45}\ g_V^4 + \frac{34}{15}\ g_V^2 g_A^2 + \frac{97}{90}\ g_A^4 \right)$$

$$c = \frac{1}{(4\pi)^2 m^4} \left(\frac{4}{3}\ g_V^3 g_A + \frac{28}{9}\ g_V g_A^3 \right)$$

c=0 when g_A or g_V is 0

We followed a method developed by Schwinger J. Schwinger, Phys. Rev.**82**, 664 (1951)

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Phenomenology and Proposal for the Experiment

4. Dark Sector Model

SM + U'(1)$_{Y'}$ + 1 Complex Scalar

$$\mathcal{L}_S = \left| \left(i\partial_\mu - g_1 Y_s B_\mu - g_1' Y_s' B_\mu' \right) S(x) \right|^2$$

spontaneously broken $\langle S \rangle = v_s/\sqrt{2}$,

$$\mathcal{L}_{\text{mixing}} = \frac{1}{2} m_{B'}^2 \left(\varepsilon^2 B_\mu B^\mu + 2\varepsilon B_\mu B'^\mu + B'_\mu B'^\mu \right)$$

$$m_{B'} = g_1' Y_s' v_s \qquad \varepsilon \equiv \frac{g_1 Y_s}{g_1' Y_s'}$$

Including $A_\mu^3(x)$ the 3rd component of SU(2)L gauge boson

$$\mathcal{L}_{mass} = \frac{v^2}{8}\left(A^{3\mu}(x),\ B^\mu(x),\ B'^\mu(x) \right) \begin{pmatrix} g_2^2 & -g_1 g_2 & 0 \\ -g_1 g_2 & g_1^2 + \alpha'\varepsilon^2 & \alpha'\varepsilon \\ 0 & \alpha'\varepsilon & \alpha' \end{pmatrix} \begin{pmatrix} A_\mu^3(x) \\ B_\mu(x) \\ B'_\mu(x) \end{pmatrix}$$

mass diagonalization $\alpha' = 4(m_{B'}/v)^2$

$$(m_{\tilde{A}})^2 = 0,\ (m_{\tilde{Z}})^2 = \frac{1}{4}v^2(g_1^2 + g_2^2) + \varepsilon^2 \frac{g_1^2}{g_1^2 + g_2^2 - \alpha'}(m_{B'})^2, \text{ and}$$

$$(m_{\tilde{B}'})^2 = (m_{B'})^2 \left(1 + \varepsilon^2 \frac{g_2^2 - \alpha'}{g_1^2 + g_2^2 - \alpha'}\right).$$

$$\tilde{A}_\mu = \frac{g_1 A_\mu^3 + g_2 B_\mu}{\sqrt{g_1^2 + g_2^2}} - \varepsilon \frac{g_2}{\sqrt{g_1^2 + g_2^2}} B'_\mu$$

$$\chi$$

We assume $\epsilon \ll 1$

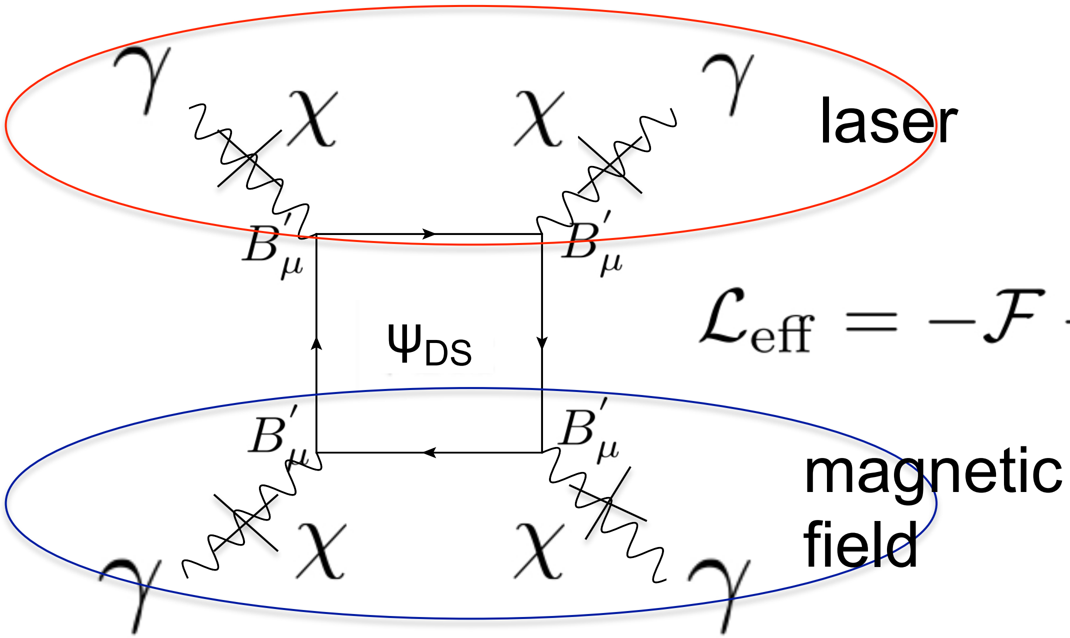
$$\tilde{A}^{\mu} = A^{\mu}_{\mathrm{SM}} + \chi B'^{\mu} \qquad \chi \ll 1$$

photon in our theory (massless)

ordinary photon in SM

extra U(1) gauge boson

$$\mathcal{L} = \bar{\psi}_{\mathrm{DS}}\left[\gamma^{\mu}(i\partial_{\mu} - (g_V + g_A\gamma_5)B'_{\mu}) - m_{\mathrm{DS}}\right]\psi_{\mathrm{DS}}$$



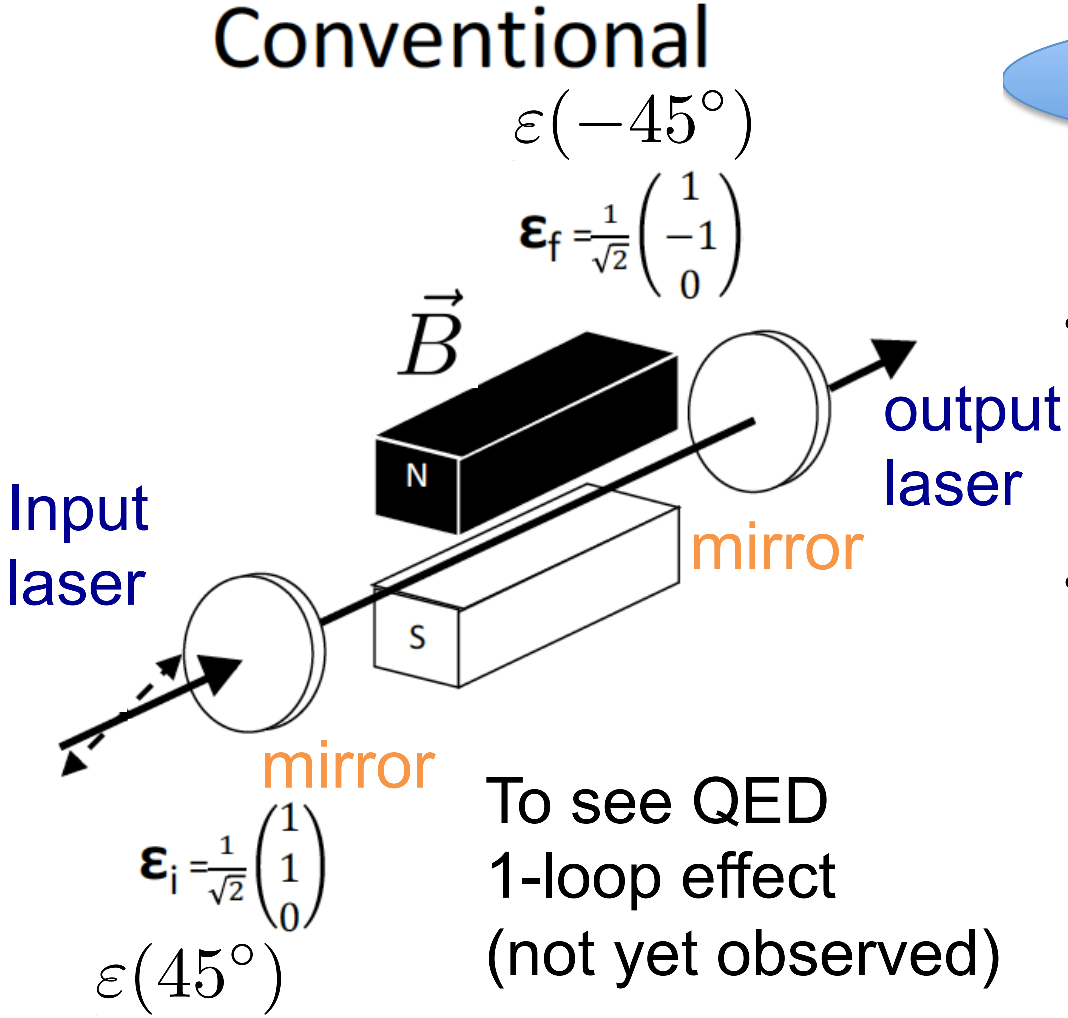
4γ   4γ   4γ, $\not{P}$

$$\mathcal{L}_{\mathrm{eff}} = -\mathcal{F} + \boxed{\chi^4(a\mathcal{F}^2 + b\mathcal{G}^2 + ic\mathcal{F}\mathcal{G})}$$

5. Vacuum Magnetic Birefringence Experiment

X. Fan *etal.* Eur. Phys. J. D **71**, no. 11, 308 (2017)

- OVAL (Observing Vacuum with Laser) experiment



Tabletop experiment

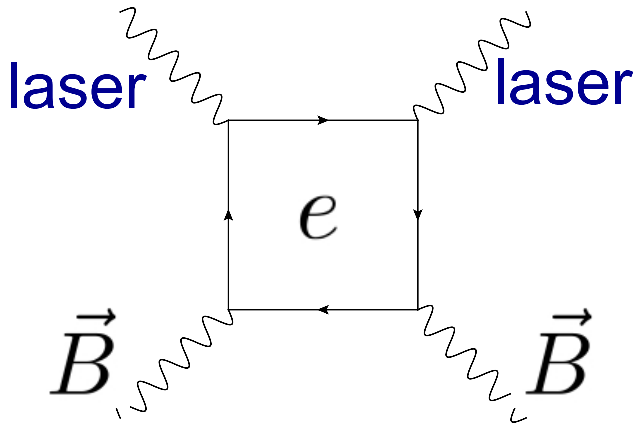
To see QED
1-loop effect
(not yet observed)

Eur. Phys. J. D (2014) **68**: 16

- BMV experiment

Eur. Phys. J. C (2016) **76**: 24

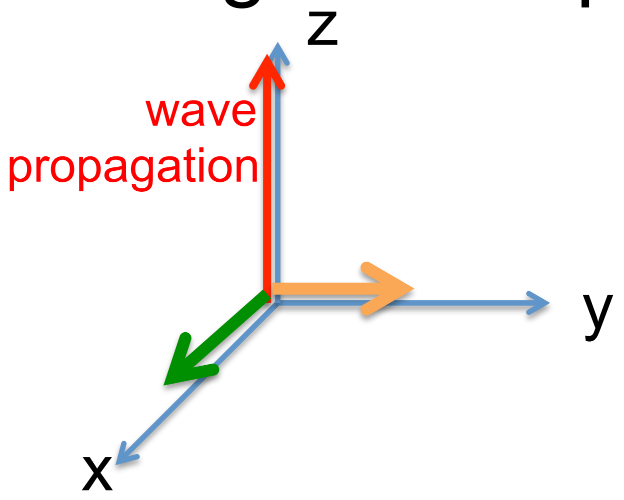
- PVLAS experiment



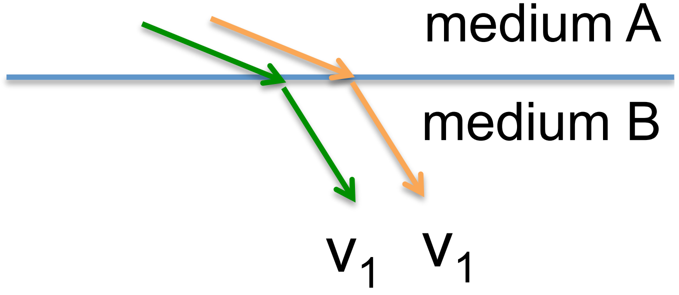
5. Vacuum Magnetic Birefringence Experiment

- refractive index: n
- phase velocity: v

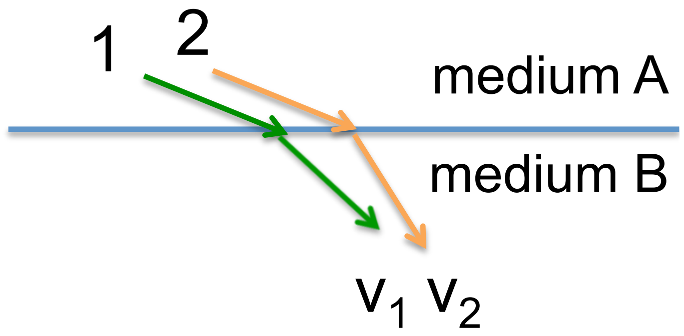
$\rightarrow n = 1/v$



refringence:
changing phase velocity of the light



birefringence:
changing phase velocity of 2 light polarizations in different ways



5. Vacuum Magnetic Birefringence Experiment

$$\mathcal{L}_{\text{eff}} = -\mathcal{F} + \boxed{a\mathcal{F}^2 + b\mathcal{G}^2 + ic\mathcal{F}\mathcal{G}}$$

Equation of Motion

refractive index: 1/(phase velocity of the laser):

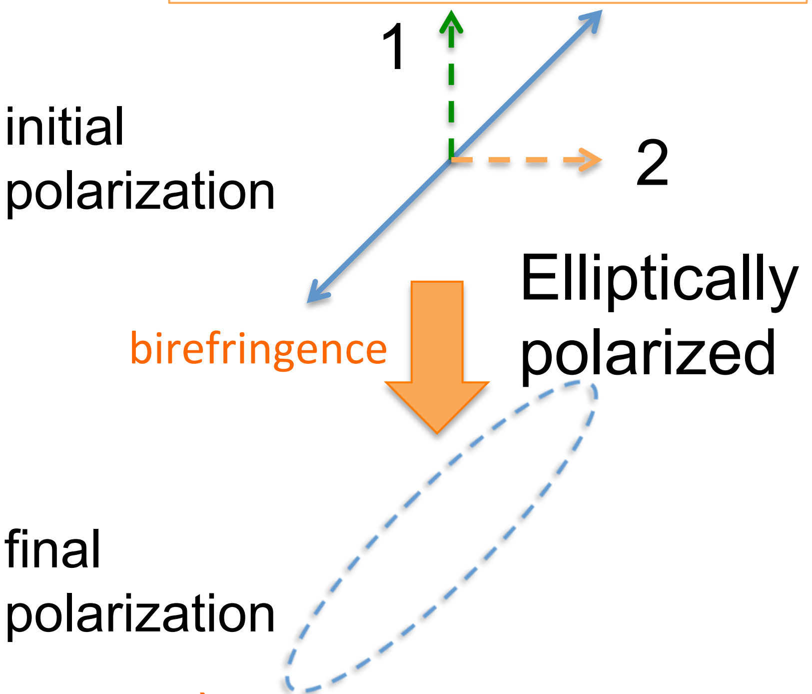
$$n_{\pm} \;=\; 1 + \frac{1}{2}\boldsymbol{B}^2 \left\{ (a+b) \pm \sqrt{\underbrace{\boxed{(a-b)^2 - c^2}}_{=D}} \right\}.$$

polarization vector $\varepsilon_{\pm}$

5. Vacuum Magnetic Birefringence Experiment

To detect birefringence,
we observe a difference of polarization state

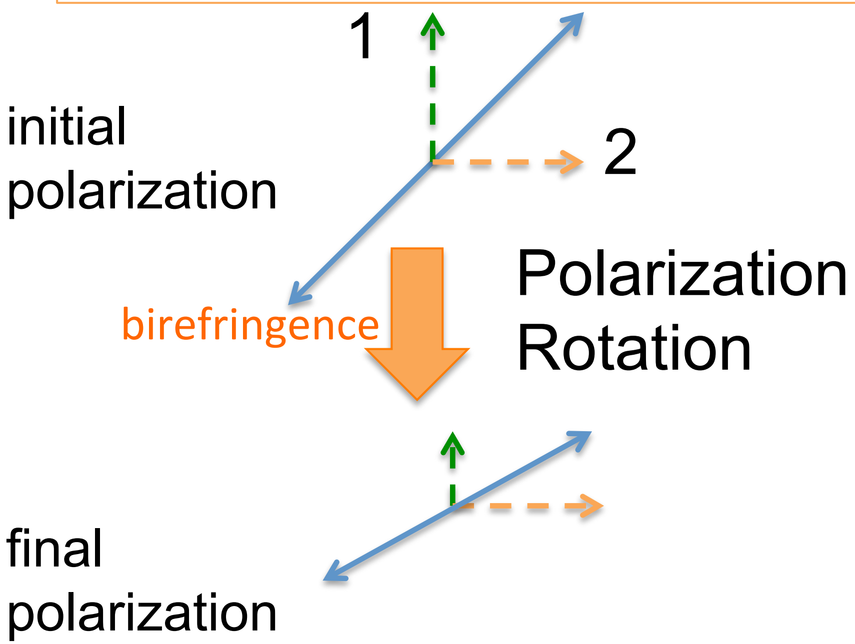
1) Ellipticity



ex)

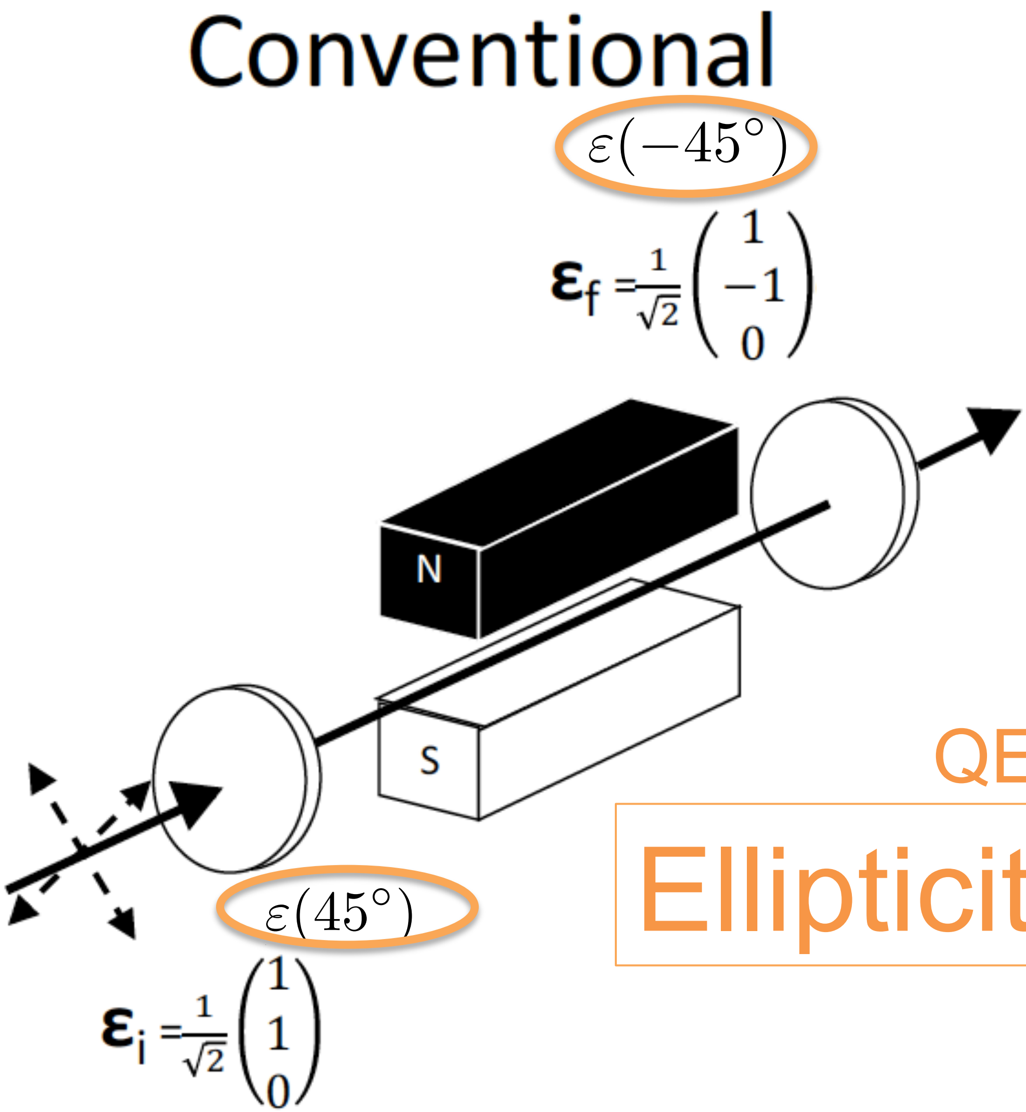
- QED
- dark sector in our model

2) Direction of the long axis of an ellipse



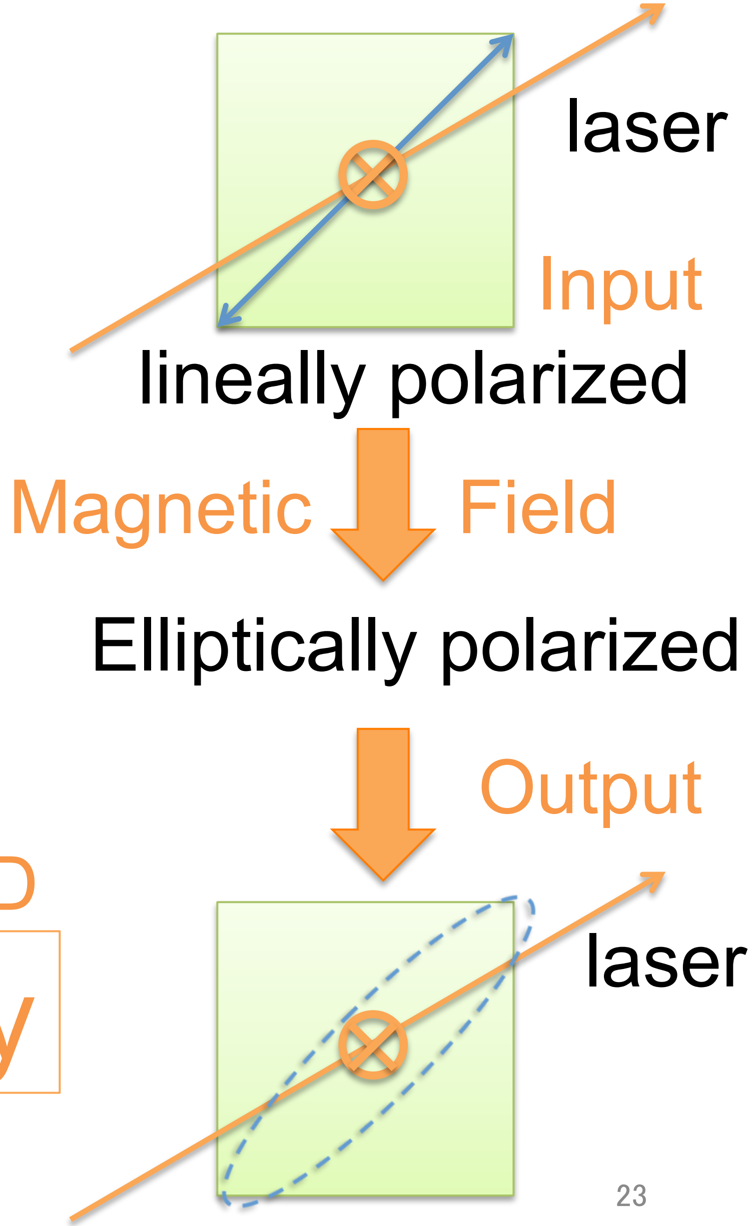
ex) dark sector in our model with $\cancel{P}$

5. Vacuum Magnetic Birefringence Experiment



QED

Ellipticity



5. Vacuum Magnetic Birefringence Experiment

After a distance L though the magnetic field

Conventional: ε(45°) for QED

$$\epsilon(45^\circ) \;\to\; \begin{cases} (\cos(\Psi - 2\phi)\epsilon(45^\circ) - i\sin\Psi\epsilon(-45^\circ))\,/\cos 2\phi & (D>0) \\ ((\cosh\theta\sinh\Psi - \cosh\Psi)\epsilon(45^\circ) - i\sinh\theta\sinh\Psi\epsilon(-45^\circ))\,/\cosh\theta & (D<0) \end{cases}$$

(coefficient of ε(-45°)) / (coefficient of ε(45°))

ellipticity *

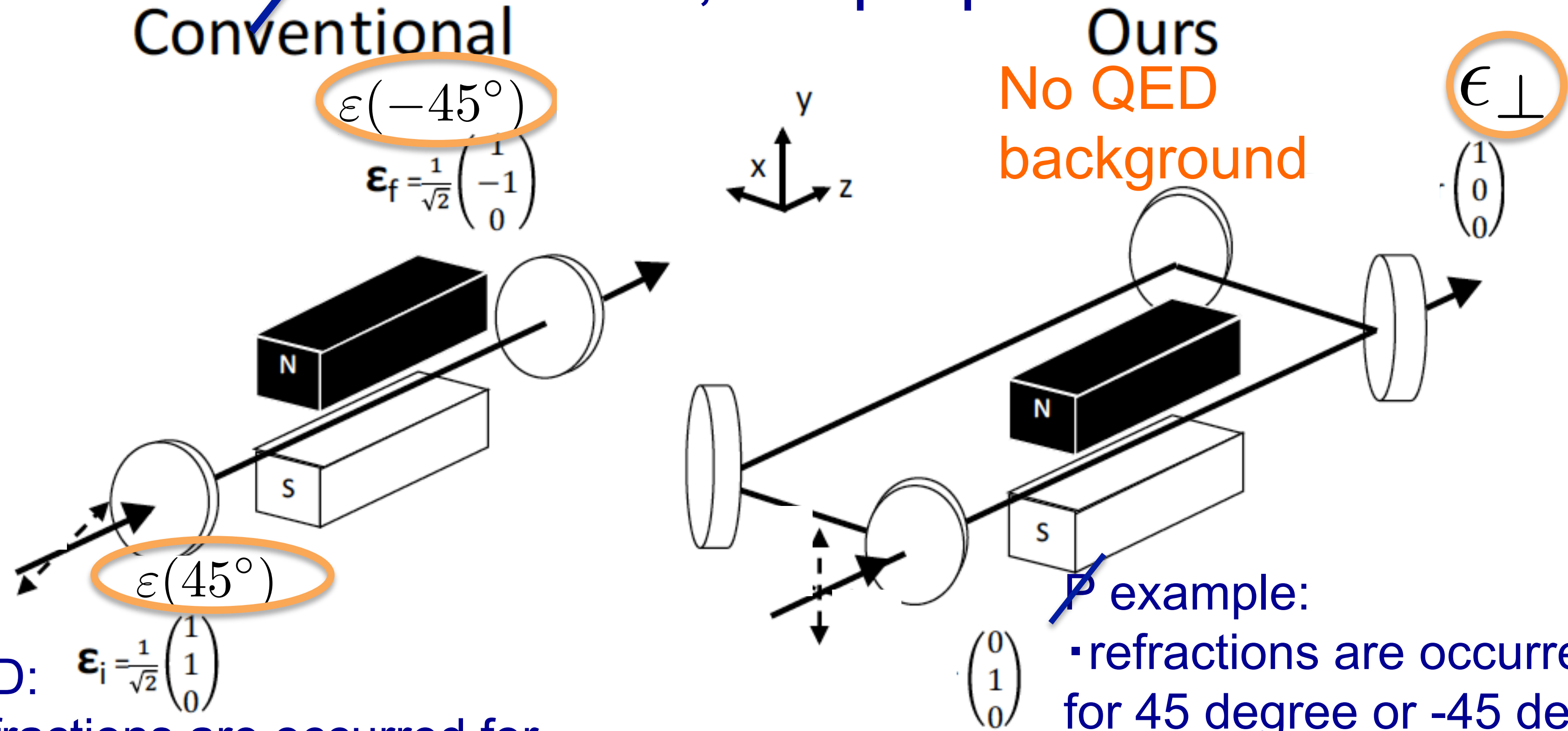
$$\boxed{\sin\Psi/\cos(\Psi - 2\phi) \quad \text{for} \quad \epsilon_i = \epsilon(45^\circ)\ (D>0)}$$

$$\sinh\theta\sinh\Psi/(\cosh\Psi - \cosh\theta\sinh\Psi) \quad \text{for} \quad \epsilon_i = \epsilon(45^\circ)\ (D<0)$$

(*) D > 0 in QED

5. Vacuum Magnetic Birefringence Experiment

To detect $\cancel{P}$ interaction, we propose a new method



QED:
- refractions are occurred for parallel (from magnetic field) or perpendicular polarization modes in different ways
- Polarization with 45 degrees includes both modes.

-> We detect -45 degrees to see refringence

$\cancel{P}$ example:
- refractions are occurred for 45 degree or -45 degree polarization modes in different ways
- Polarization with parallel includes both modes.

-> We detect perpendicular to see refringence

5. Vacuum Magnetic Birefringence Experiment

No QED background for the initial state $\varepsilon_{||}$

After a distance L though the magnetic field with initial state $\varepsilon_{||}$:

0!

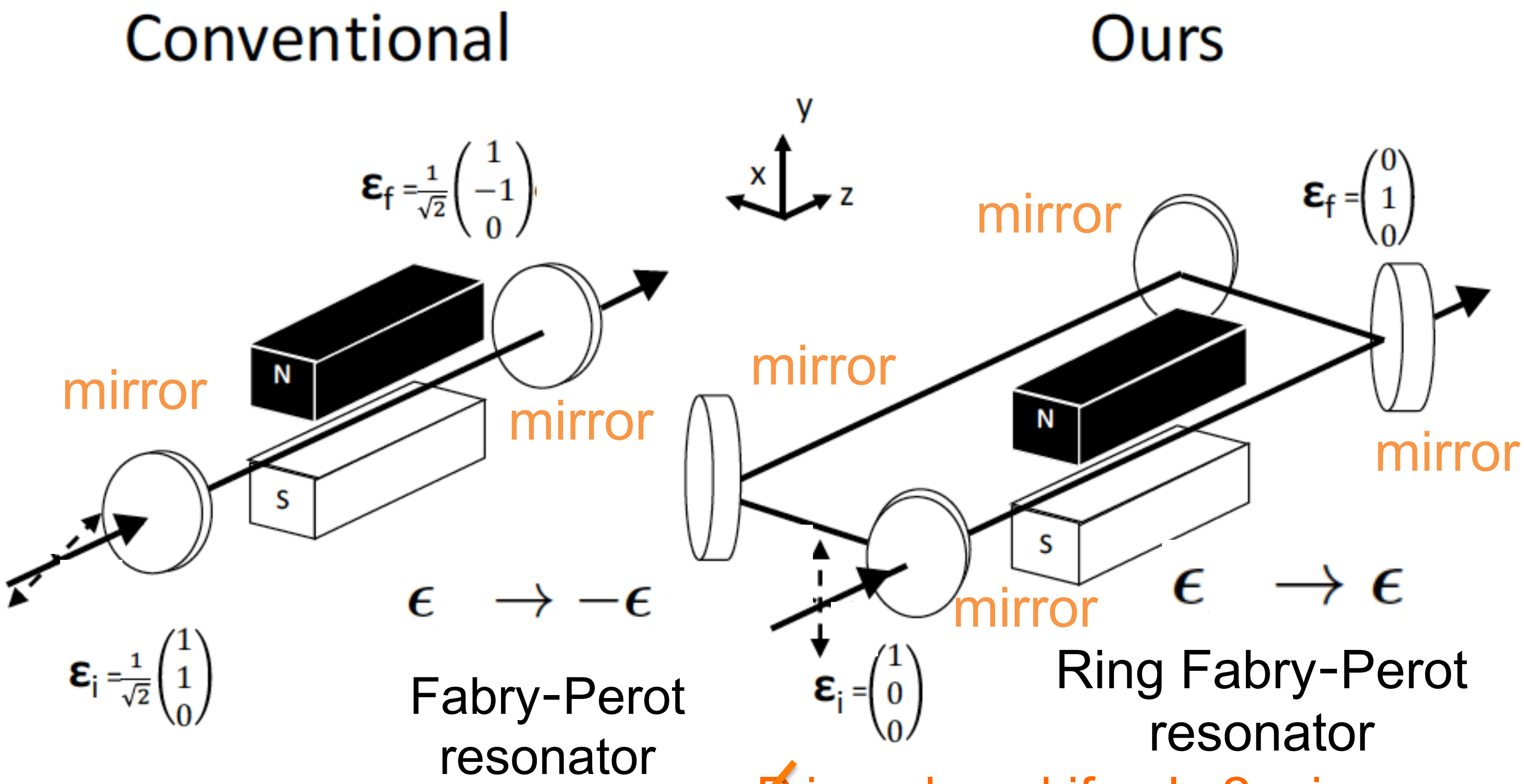
$$\epsilon_{||} \rightarrow \begin{cases} \boxed{\left((-i\sin\Psi + \cos 2\phi\cos\Psi)\epsilon_{||} + \sin\Psi\sin 2\phi\epsilon_{\perp}\right)/\cos 2\phi \quad (D>0)} \\ (\cosh\Psi + i\sinh\theta\sinh\Psi)\epsilon_{||} - \cosh\theta\sinh\Psi\epsilon_{\perp} \quad (D<0) \end{cases}$$

D > 0 and Φ=0 in QED

To detect ~~P~~ interaction, we propose initial state as $\varepsilon_{||}$
-> We detect a perpendicular mode $\epsilon_{\perp}$
to see refringence

5. Vacuum Magnetic Birefringence Experiment

To detect $\not{P}$ interaction, we propose a new method



$\not{P}$ is reduced if only 2 mirrors

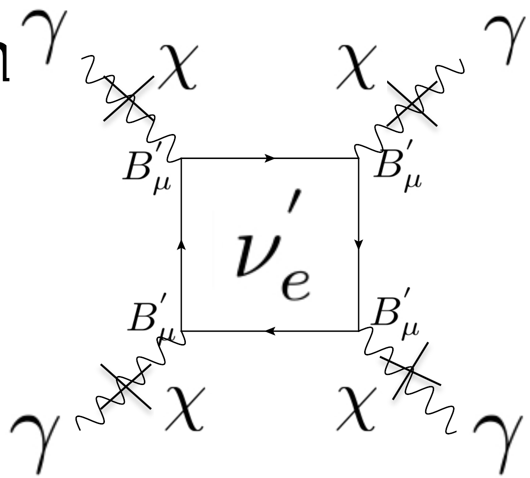
5. Vacuum Magnetic Birefringence Experiment

$$\mathcal{L} = \bar{\psi}_{\mathrm{DN}} [\gamma^{\mu}(i\partial_{\mu} + |e|(1 - \gamma_5)\, B'_{\mu}) - m_{\mathrm{DN}}]\psi_{\mathrm{DN}}$$

We assume $g_A = - g_V$ (= |e|) to obtain the experimental constraint

↓

V – A current: Dark neutrino



We examine the case, having both the electron and the lightest DS neutrino. For the DS search, QED forms the background to the DS signal.

$$\boxed{a = a_{\mathrm{QED}} + \chi^4 a_{\mathrm{DS}\nu'},\ b = b_{\mathrm{QED}} + \chi^4 b_{\mathrm{DS}\nu'},\ \text{and } c = \chi^4 c_{\mathrm{DS}\nu'}}$$

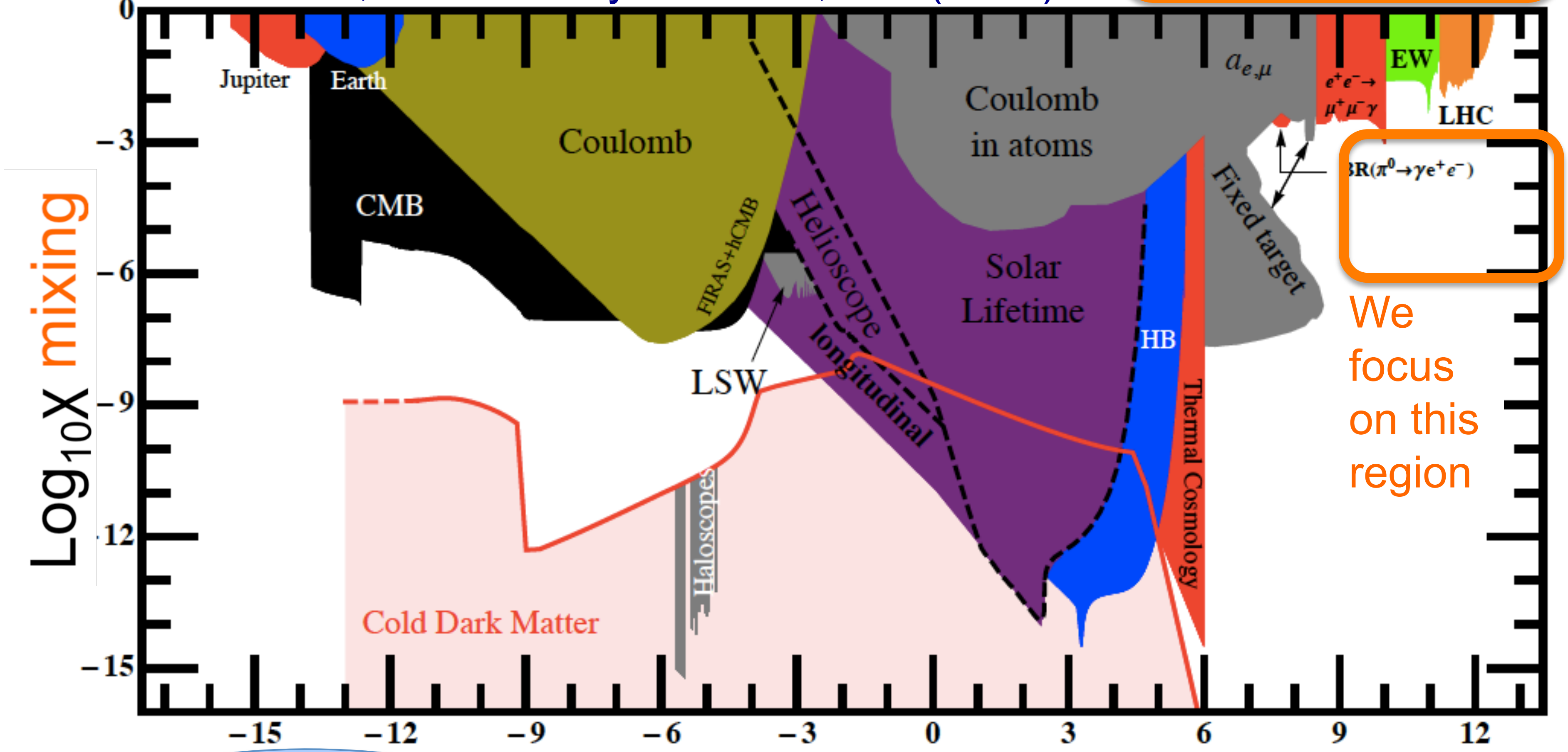
$$\mathcal{L}_{\mathrm{eff}} = -\mathcal{F} + \quad (a\mathcal{F}^2 + b\mathcal{G}^2 + ic\mathcal{F}\mathcal{G})$$

5. Vacuum Magnetic Birefringence Experiment: Allowed region

J. Jaeckel, Frascati Phys. Ser. 56, 172 (2012)

$$10^{-6} \leq \chi \leq 10^{-3}$$

$$m_{\tilde{B}'} \geq 1\ \text{GeV}$$



We focus on this region

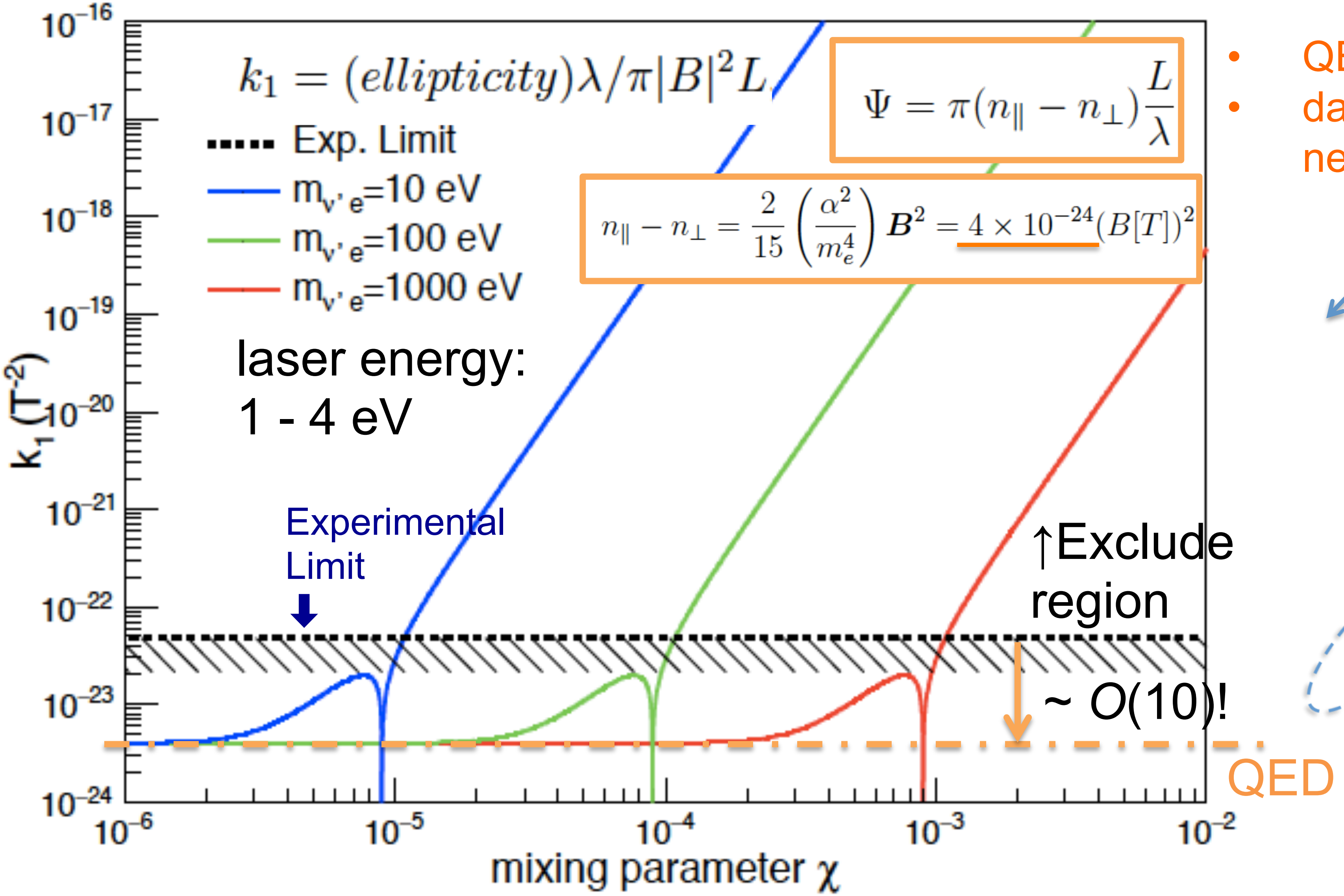
GeV

Log$_{10}$ $m_{\tilde{B}'}$ [eV]

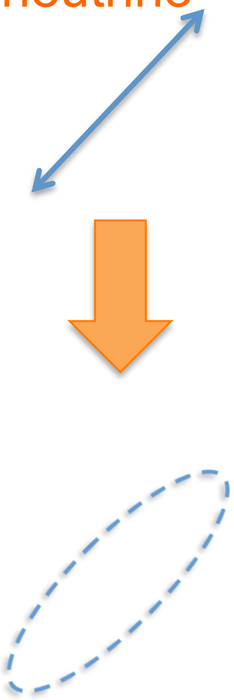
dark photon mass

At VMB experiment, the sensitivity does not depend on dark photon mass

5. Vacuum Magnetic Birefringence Experiment: Conventional, QED/Dark neutrino

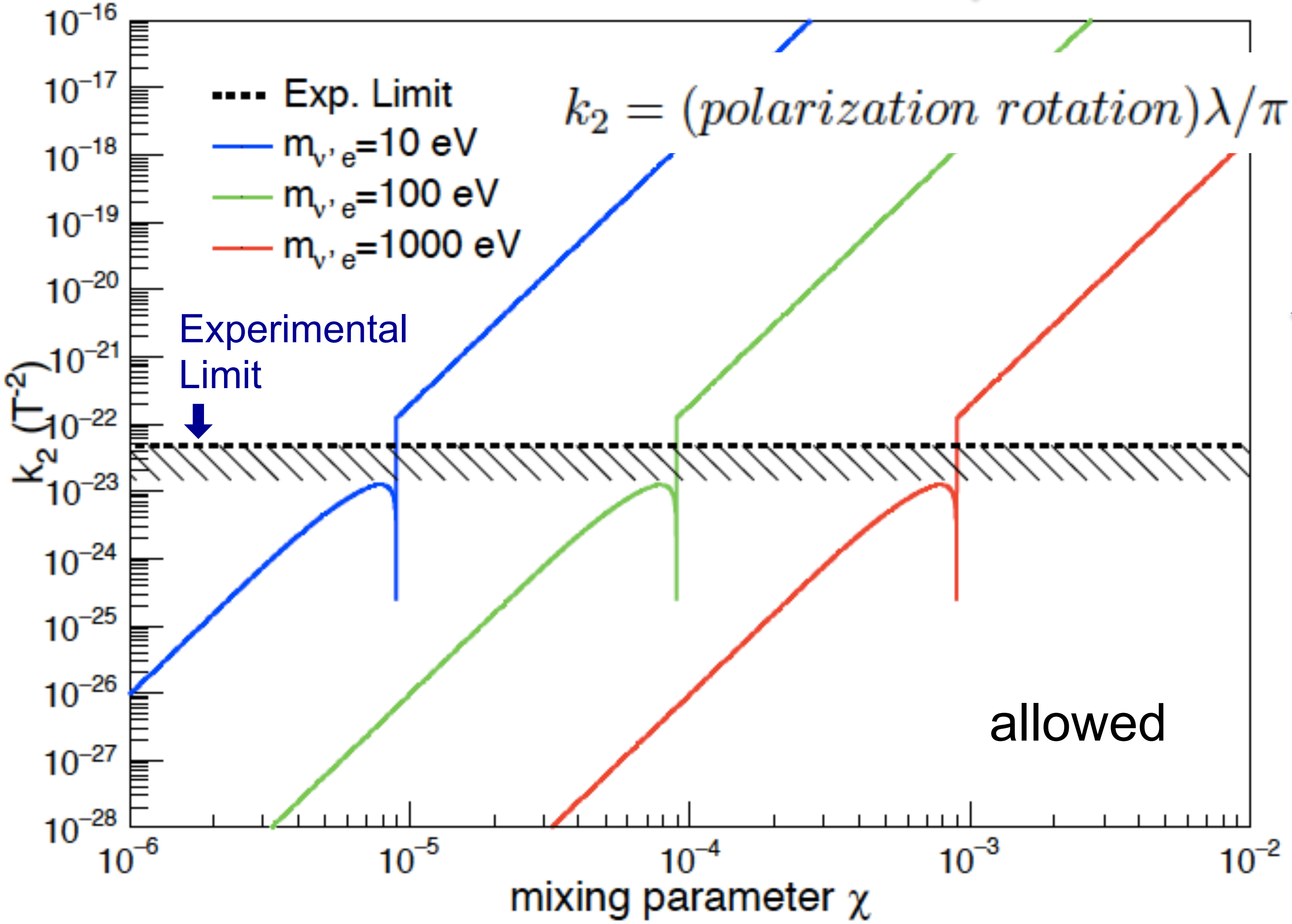


- QED
- dark neutrino



5. Vacuum Magnetic Birefringence Experiment – New Set Up, ~~P~~ DS only

$$k_2 = (polarization\ rotation)\lambda/\pi|B|^2L$$



$\cancel{P}$

6. Summary

1. We considered Parity violated dark sector model, and derived generalized Heisenberg-Euler formula
2. Our focus lay on light-by-light scattering effective Lagrangian of fourth order and gave a result:

$$\mathcal{L}_{\rm eff} = -\mathcal{F} + \boxed{a\mathcal{F}^2 + b\mathcal{G}^2 + ic\mathcal{F}\mathcal{G}}$$

$$\mathcal{F} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\left(\vec{B}^2 - \vec{E}^2\right) \quad \mathcal{G} = \frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \vec{E}\cdot\vec{B}$$

$$a = \frac{1}{(4\pi)^2 m^4}\left(\tfrac{8}{45}\ g_V^4 - \tfrac{4}{5}\ g_V^2 g_A^2 - \tfrac{1}{45}\ g_A^4\right)$$

$$b = \frac{1}{(4\pi)^2 m^4}\left(\tfrac{14}{45}\ g_V^4 + \tfrac{34}{15}\ g_V^2 g_A^2 + \tfrac{97}{90}\ g_A^4\right)$$

$$c = \frac{1}{(4\pi)^2 m^4}\left(\tfrac{4}{3}\ g_V^3 g_A + \tfrac{28}{9}\ g_V g_A^3\right)$$

3. We focused on Vacuum Magnetic Birefringence Experiment to probe the dark sector and proposed new polarization state and the ring resonator in stead of the usual Fabry-Perot resonator to measure the Parity violated term

Backup

title:
Search for Dark Neutrino via Vacuum Magnetic Birefringence Experiment

abstract:
We consider a dark sector model where a dark fermion couples to photons via an extra U(1) mediator and
can couple to the mediator with parity violation.
We derived a low energy effective Lagrangian including a parity violated term for light-by-light scattering by integrating out the dark fermion.
Our focus lies on Vacuum Magnetic Birefringence Experiment to probe the dark sector.
We propose the ring resonator (3-4 mirrors) with an appropriate polarization state of light in stead of
a usual Fabry-Perot resonator (2 mirrors) with a conventional polarization state of light to measure the Parity violated term. We assume that a dark neutrino is a dark fermion, i.e. V-A current, and
give constraints on model parameters from a current experimental limit.
PTEP 2017 no. 12, 123B03 (2017) (arXiv:1707.03308 [hep-ph]), arXiv:1707.03609 [hep-ph]

2-2. Path Integral Representation

$$S_{\rm eff}(A) = \frac{i}{2}\int_0^\infty \frac{ds}{s}\, e^{-im^2 s} \int d^4x \underline{\,\mathrm{tr}\,\langle x(s), a(s) | x(0), b(0)\rangle\,}$$

$$\|$$

$$\boxed{\int \mathcal{D}x^\mu(s')\, e^{i\bar{A}(s)} \times 2\left(\cos \underbrace{\sqrt{2(\bar{\boldsymbol{\mathcal{F}}}'(s) + i\bar{\boldsymbol{\mathcal{G}}}'(s))}}_{\boxed{=\bar{X}'_+(s)}} + \cos \underbrace{\sqrt{2(\bar{\boldsymbol{\mathcal{F}}}'(s) - i\bar{\boldsymbol{\mathcal{G}}}'(s))}}_{\boxed{=\bar{X}'_-(s)}}\right)}$$

Lagrangian is separated into
A(s) (free part) and B(s) (interaction part):

$$\bar{A}(s) = \int_0^s ds' \left[-\frac{1}{4}(\dot{x}^\mu)^2 + \frac{1}{2} g_V x^\mu (F_{\mu\nu}) \dot{x}^\nu - \frac{1}{2} g_A^2 x^\mu (F_{\mu\lambda} F^{\lambda\nu}) x_\nu \right]$$

$$\bar{\boldsymbol{\mathcal{F}}}'(s) = \frac{1}{4}\bar{B}_{\mu\nu}(s)\bar{B}^{\mu\nu}(s), \quad \bar{\boldsymbol{\mathcal{G}}}'(s) = \frac{1}{4}\bar{B}_{\mu\nu}(s)\tilde{\bar{B}}^{\mu\nu}(s).$$

$$\bar{B}_{\mu\nu}(s) = \int_0^s ds' \left[\underline{g_A \frac{1}{2}\epsilon_{\mu\nu\beta\gamma} x_\alpha F^{\alpha\beta} \dot{x}^\gamma} - (g_V F_{\mu\nu} - i g_A \tilde{F}_{\mu\nu}) \right]$$

depends on position, its derivative and
appears as non-Gaussian

2-2. Path Integral Representation

$$\tilde{L}(x^\mu(s), \dot{x}^\mu(s))$$

$$= -\frac{1}{4}(\dot{x}^\mu)^2 + \frac{1}{2}x^\mu F_{\mu\lambda}\left(g_V\ g^{\lambda\nu} + \underline{ig_A\ \gamma_5\sigma^{\lambda\nu}}\right)\dot{x}_\nu - \frac{1}{2}g_A^2\ x^\mu F_{\mu\lambda}F^{\lambda\nu}x_\nu$$

$$\underline{-\ \frac{1}{2}\sigma^{\mu\nu}(g_V + g_A\gamma_5)F_{\mu\nu}}$$

$$A(s') = -\frac{1}{4}(\dot{x}^\mu)^2 + \frac{1}{2}g_V x^\mu(F_{\mu\nu})\dot{x}^\nu - \frac{1}{2}g_A^2 x^\mu(F_{\mu\lambda}F^{\lambda\nu})x_\nu,$$

$$\underline{B_{\mu\nu}(s') = g_A\frac{1}{2}\epsilon_{\mu\nu\beta\gamma}x_\alpha F^{\alpha\beta}\dot{x}^\gamma - (g_V F_{\mu\nu} - ig_A\tilde{F}_{\mu\nu}),}$$

$$\gamma_5\sigma_{\mu\nu} = -\frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}$$

$$\begin{aligned}\mathrm{Tr}e^{-i\tilde{H}s} &= \int d^4x\ \mathrm{tr}\ \langle x(s), a(s)|x(0), b(0)\rangle \\ &= \int d^4x\ \mathrm{tr}\int_{x^\mu(0)=x^\mu}^{x^\mu(s)=x^\mu}\mathcal{D}x^\mu(s')\ e^{i\int_0^s\ ds'\ \tilde{L}(x(s'),\ \dot{x}(s'))},\end{aligned}$$

$$\tilde{S}(s) = \int_0^s ds'\tilde{L}(s') = \int_0^s ds'\left(A(s') + \frac{1}{2}B_{\mu\nu}(s')\sigma^{\mu\nu}\right),$$

2. Path integral representation

$$S_{\text{eff}}(A) = \frac{i}{2}\int_0^\infty \frac{ds}{s}\, e^{-im^2 s} \int d^4x\, \underline{\mathrm{tr}\, \langle x(s), a(s) | x(0), b(0) \rangle}$$

$$\|$$

$$\boxed{\int \mathcal{D}x^\mu(s')\, e^{i\bar{A}(s)} \times 2\left(\cos \underbrace{\sqrt{2(\bar{\mathcal{F}}'(s) + i\bar{\mathcal{G}}'(s))}}_{\boxed{=\bar{X}'_+(s)}} + \cos \underbrace{\sqrt{2(\bar{\mathcal{F}}'(s) - i\bar{\mathcal{G}}'(s))}}_{\boxed{=\bar{X}'_-(s)}}\right)}$$

Free part

$$= \left[\int \mathcal{D}x^\mu(s')\, e^{i\bar{A}(s)}\right] \frac{\left[\int \mathcal{D}x^\mu(s')\, e^{i\bar{A}(s)} \times 2\left(\cos \bar{X}'_+(s) + \cos \bar{X}'_-(s)\right)\right]}{\cancel{\left[\int \mathcal{D}x^\mu(s')\, e^{i\bar{A}(s)}\right]}}$$

$$\|$$

$$\cancel{\left[\int \mathcal{D}x^\mu(s')\, e^{i\bar{A}(s)}\right]}\Big\{2\left(\cos \bar{X}'_+(s) + \cos \bar{X}'_-(s)\right)\left[x^\lambda(s') \to (-i)\delta/\delta j_\lambda(s')\right]$$

$$\times\; e^{-i\int_0^s ds' \int_0^s ds'' \sum_{\alpha\beta} j^\alpha(s')\Delta(s'-s'')_{\alpha\beta} j^\beta(s'')}\Big\}\Big|_{j_{\lambda=0}}$$

$\Delta(s' - s'')$:propagator of A(s)

2. Path integral representation

$$
\begin{aligned}
&\mathrm{Tr}\langle x(s), a(s)|x(0), b(0)\rangle \\
&= \int d^4x \; \langle x(s)|x(0)\rangle' \times \left\langle 2\left(\cos \bar{X}'_+(s) + \cos \bar{X}'_-(s)\right)\right\rangle' \\
&= \int d^4x \; \langle x(s)|x(0)\rangle' \times \left\{ 2\left(\cos \bar{X}'_+(s) + \cos \bar{X}'_-(s)\right) [x^\lambda(s') \to (-i)\delta/\delta j_\lambda(s')] \right. \\
&\quad \times \left. e^{-i\int_0^s ds' \int_0^s ds'' \sum_{\alpha\beta} j^\alpha(s')\Delta(s'-s'')_{\alpha\beta} j^\beta(s'')} \right\}\Bigg|_{j_\lambda=0},
\end{aligned}
$$

3. Effective Lagrangian of Fourth Order

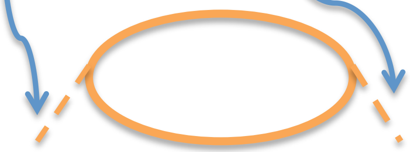
$$\mathcal{L}_{\text{eff}} = -\frac{1}{8\pi^2}\int_0^\infty \frac{ds}{s^3} e^{-m^2 s} \frac{(g_+ s)^2 \mathcal{G}}{Im \cosh(g_+ X s)} \times \frac{(g_- s)^2 \mathcal{G}}{Im \cosh(g_- X s)}$$

$$\times \underline{\frac{1}{2}\left\langle \left(\cos \bar{X}'_+(s \to -is) + \cos \bar{X}'_-(s \to -is) \right) \right\rangle'}$$

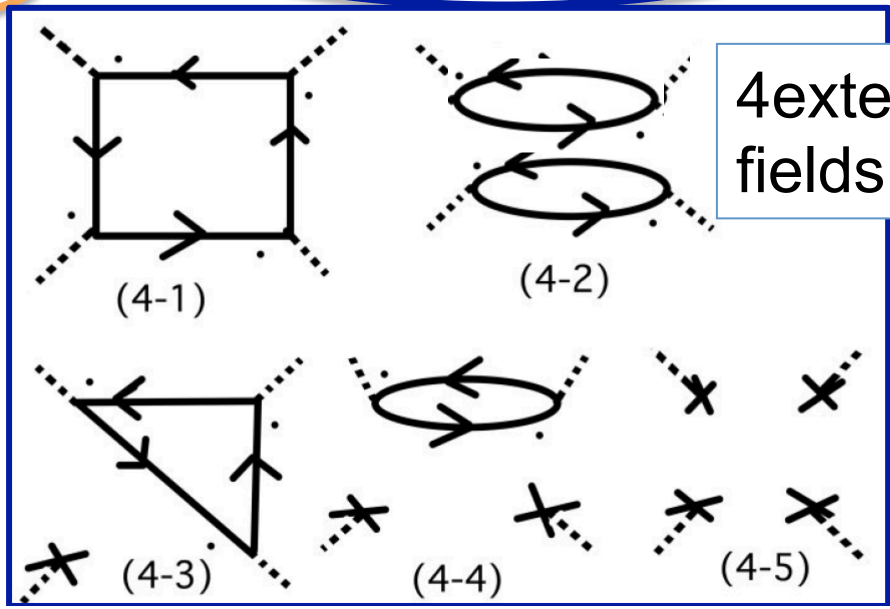
$$\|$$

$$1 - \langle \bar{\mathcal{F}}'(s) \rangle' + \frac{1}{6} \langle (\bar{\mathcal{F}}'(s))^2 - (\bar{\mathcal{G}}'(s))^2 \rangle' + \cdots$$

external field

1: 

2: 



4external fields

5. Vacuum Magnetic Birefringence Experiment

$$\sin\phi = \frac{c}{\left\{\left((a-b)+\sqrt{(a-b)^2-c^2}\right)^2+c^2\right\}^{\frac{1}{2}}}$$

$$\cos\phi = \frac{(a-b)+\sqrt{(a-b)^2-c^2}}{\left\{\left((a-b)+\sqrt{(a-b)^2-c^2}\right)^2+c^2\right\}^{\frac{1}{2}}}$$

$$\sinh\theta = \frac{a-b}{(c^2-(a-b)^2)^{\frac{1}{2}}} \times sign\,(c)$$

$$\cosh\theta = \frac{|c|}{(c^2-(a-b)^2)^{\frac{1}{2}}}$$

$$\Psi = \pi|B|^2\frac{L}{\lambda}\sqrt{|(a-b)^2-c^2|},$$

described by a,b,c,
B(magnetic field),
λ(laser beam wave length) ,
L(beam propagating distance with B)

5. Vacuum Magnetic Birefringence Experiment

$$\sin\phi = \frac{c}{\left\{\left((a-b)+\sqrt{(a-b)^2-c^2}\right)^2+c^2\right\}^{\frac{1}{2}}}$$

$$\cos\phi = \frac{(a-b)+\sqrt{(a-b)^2-c^2}}{\left\{\left((a-b)+\sqrt{(a-b)^2-c^2}\right)^2+c^2\right\}^{\frac{1}{2}}}$$

$$\sinh\theta = \frac{a-b}{(c^2-(a-b)^2)^{\frac{1}{2}}} \times sign\,(c)$$

$$\cosh\theta = \frac{|c|}{(c^2-(a-b)^2)^{\frac{1}{2}}}$$

$$\Psi = \pi|B|^2\frac{L}{\lambda}\sqrt{|(a-b)^2-c^2|},$$

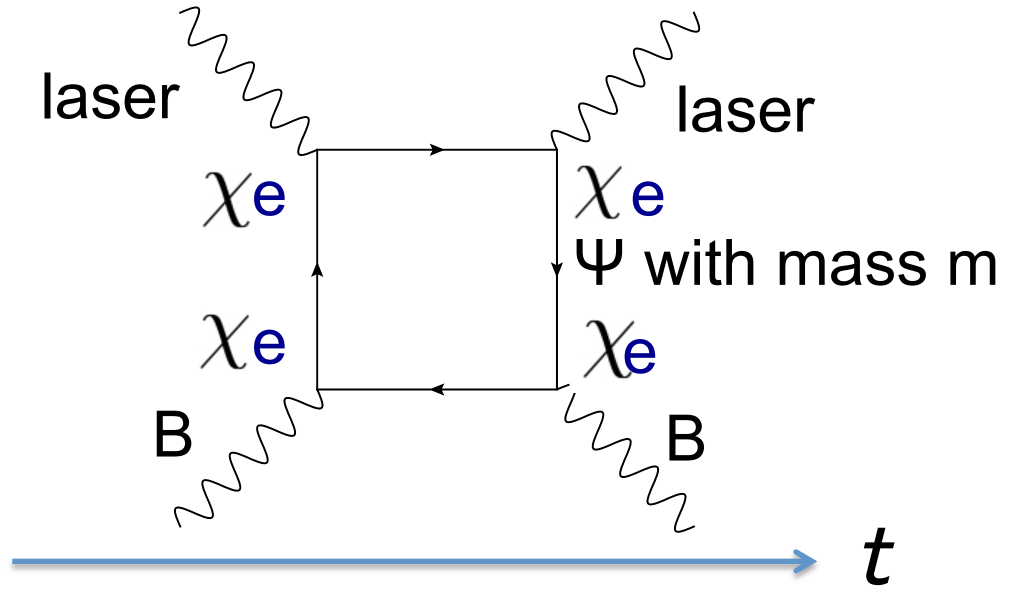
$$\cancel{P}$$

$$\mathcal{L}_{\text{eff}} = -\mathcal{F} + \boxed{a\mathcal{F}^2 + b\mathcal{G}^2 + ic\mathcal{F}\mathcal{G}}$$

2 conditions

$$\underbrace{h\nu_{\text{laser}}}_{\text{1 eV}} \ll mc^2$$

$$\hbar \chi e \underbrace{|\boldsymbol{B}|}_{\text{10 Tesla, (1 Tesla ~ 200 eV}^2\text{)}} \ll m^2 c^2$$



Vacuum Magnetic Birefringence Experiment: laser beam energy

beam energy 1.16 eV @OVAL experiment

For 2 mirrors system: 1 ~ 4 eV

laser energy itself:
m eV ~ <u>10 k eV</u> are available
thanks to X-ray Free Electron Laser