Theories of Class F and Their Anomalies

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Roadmap

1. Introduction: Theories of Class S
2. Part I: Theories of Class F [CL, Martelli, Schäfer-Nameki]
   → M5-brane wrapping $C_{g,n}$ fibration
   → 4d theories with varying couplings and duality defects
   → determine anomaly polynomials, I_6, from I_8
3. Part II: Compactifcations of Class F
   → class F on complex curve C with topological duality twist
   → 2d chiral SCFTs with anomaly polynomial I_4
   → compare with AdS_3 duals of D3-branes on C with varying τ
   [Couzens, CL, Martelli, Schäfer-Nameki, Wong]
4. Part III: Extensions
   → more examples of fibral dimensional reductions

Introduction: Class S

Theories of Class S

Class S are 4d $\mathcal{N} = 2$ theories [Gaiotto]
$\rightarrow$ twisted dimensional reduction of the 6d $\mathcal{N} = (2, 0)$ SCFT on $C_{g,n}$
$\rightarrow$ $C_{g,n}$ a complex curve, with genus g and n punctures

$$M_6 = M_4 \times C_{g,n}$$

- Couplings in 4d depend on complex structure moduli of $C_{g,n}$
- Class S are generically non-Langrangian and strongly coupled
- Information about degrees of freedom of these theories can be obtained from their anomalies

Anomalies

A theory with an anomaly under group G

$\Rightarrow Z$ not invariant under G-transformation of fields with parameter δ

$$\mathcal{A}_\delta = \int_{M_D} I_D$$

An anomaly can be captured by a formal $D+2$-form on spacetime M_D via Wess–Zumino descent

$$I_{D+2} = dI^{(0)}_{D+1} \,, \quad \delta I^{(0)}_{D+1} = dI^{(1)}_D$$

The contribution to the anomaly polynomial, I_{D+2} from a chiral fermion is

$$ch(F_{\mathbf{R}})\widehat{A}(M_D)|_{D+2\text{-form}} \,,$$

for fermion in represention $\mathbf{R}$ of symmetry group G

Anomalies for 6d $(2,0)$ SCFT

The 6d $(2,0)$ SCFT of type G has anomaly polynomial [Witten; (Freed), Harvey, Minasian, Moore; Intriligator; Yi; Ohmori, Shimizu, Tachikawa, Yonekura]

$$I_8 = \frac{r_G}{48}\left[p_2(N_5) - p_2(M_6) + \frac{1}{4}(p_1(N_5) - p_1(M_6))^2\right] + \frac{h_G^\vee d_G}{24} p_2(N_5)$$

N_5: $SO(5)$ R-symmetry bundle
M_6: 6d spacetime
r_G, d_G, $h_G^\vee$: rank, dimension, dual Coxeter number of G

I_8 is formal expansion in Chern roots of

$$T_{M_6} \leftrightarrow \pm\lambda_1, \pm\lambda_2, \pm\lambda_3$$
$$N_5 \leftrightarrow \pm n_1, \pm n_2, 0$$

Anomalies for Class S

Anomalies for class S from $C_{g,n}$ have been computed

[Gaiotto, (Maldacena); Chacaltana, Distler, (Tachikawa); Tachikawa]

$\rightarrow$ without punctures can be determined from I_8

[Bah, Beem, Bobev, Wecht; Alday, Benini, Tachikawa]

(See [Bah, Nardoni] for extensions including punctures)

Let $M_6 = M_4 \times C_g$ and $\pm\lambda_3$ Chern roots of T_{C_g}

Topological twist

$$n_1 = 2r - \lambda_3\,, \quad n_2 = 2\alpha\,,$$

where α is Chern root of $SU(2)_R$ and r of $U(1)_R$

$$\int_{C_g} I_8 = (g-1)\left[-\left(\frac{4}{3}d_G h_G^\vee + r_G\right)c_2(R)c_1(R) + \frac{1}{3}r_G c_1(R)^3 - \frac{1}{12}r_G c_1(R)p_1(M_4)\right]$$

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   [Couzens, CL, Martelli, Schäfer-Nameki, Wong]
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   → more examples of fibral dimensional reductions

Part I: Theories of Class F

Theories of Class F

Consider the 6d $(2,0)$ theory on a fibration M_6,

$$\begin{array}{ccc} C_{g,n} & \hookrightarrow & M_6 \\ & & \downarrow \\ & & M_4 \end{array}$$

$\rightarrow$ 4d theory when $\mathrm{vol}(C_{g,n}) \rightarrow 0$

  $\rightarrow$ fibration is trivial $\Rightarrow$ class S

$\rightarrow$ complex structure of $C_{g,n}$ varies over spacetime

  $\Rightarrow$ couplings vary over spacetime

  $\rightarrow$ global fibration $\Rightarrow$ monodromies in mapping class group $\mathfrak{MCG}_g$

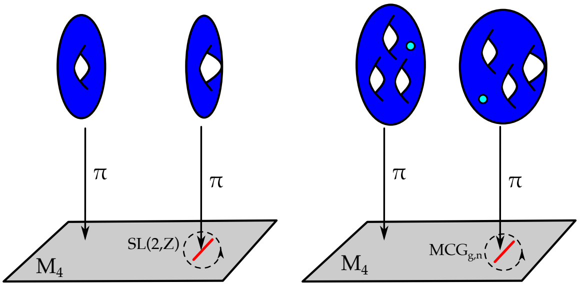
Such theories will be denoted

$$T[C_g, \mathcal{F}, G]$$

where $\mathcal{F}$ is the data describing the fibration

Duality Defects

The $C_{g,n}$ fibration has singular fibers
$\Rightarrow$ couplings undergo monodromy in $\mathfrak{MCG}_{g,n}$
$\rightarrow$ S-duality transformation



Class F: 4d supersymmetric field theories with consistent global network of duality defects

Torus Fibrations and $\mathcal{N} = 4$ SYM

Simplest case $\rightarrow g = 1$, $n = 0$

$\rightarrow \mathcal{N} = 4$ SYM with a non-trivial τ-profile over M_4

$\mathcal{N} = 4$ with spacetime dependent coupling has recently been studied in [Martucci; Haghighat, Murthy, Vafa, Vandoren; Assel, Schäfer-Nameki; CL, Schäfer-Nameki, Weigand; Choi, Fernandez-Melgarejo, Sugimoto; . . .]

Degrees of freedom living on duality defects was studied in

[Martucci; Assel, Schäfer-Nameki]

$\mathcal{N} = 4$ SYM arises as worldvolume theory of D3-brane stack in IIB

$\rightarrow$ coupling τ in 4d comes from 10d axio-dilaton τ^{IIB}

$\rightarrow$ varying axio-dilaton sourced by 7-branes

$\rightarrow$ duality defects are 3–7 strings

$\mathcal{N} = 4$ SYM with Varying Coupling

Consider $\mathcal{N} = 4$ SYM in a background with holomorphically varying τ

→ there is a local $U(1)$ symmetry from background sector

[Bergshoeff, de Roo, de Wit; Maxfield]

→ $SL(2, \mathbb{Z})$ action must be compensated by $U(1)$ transformation

[Bachas, Bain, Green; Kapustin, Witten; Martucci; Assel, Schäfer-Nameki]

$$\gamma : \tau \to \frac{a\tau + b}{c\tau + d}\,, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z}) \quad \to \quad e^{i\alpha(\gamma)} \equiv \frac{c\tau + d}{|c\tau + d|} \in U(1)_D$$

We have a $U(1)_D$ connection

$$\mathcal{Q} = -\frac{\mathrm{d}\tau_1}{2\tau_2}$$

and associated line bundle $\mathcal{L}_D \Rightarrow$ fermions transform as sections of $\mathcal{L}_D$

→ $\mathcal{L}_D$ is trivial ⇒ constant coupling

Anomaly from Bulk Spectrum of $\mathcal{N} = 4$ SYM with Varying Coupling

The gauginos transform under $Spin(1,3) \times SU(4)_R \times U(1)_D$ as

$$(\mathbf{2}, \mathbf{1}, \mathbf{4})_{1/2} \oplus (\mathbf{1}, \mathbf{2}, \overline{\mathbf{4}})_{-1/2}$$

A single vector multiplet of abelian $\mathcal{N} = 4$ contributes

$$\frac{1}{2}\left(ch(S_6^+ \otimes \mathcal{L}_D^{1/2}) - ch(S_6^+ \otimes \mathcal{L}_D^{-1/2})\right) \widehat{A}(M_4)$$

$S_6^\pm$: spin bundles on M_4 under which $\pm$ chirality fermions transform

The anomaly polynomial from each gaugino is

$$I_6^F = \frac{1}{2}c_3(S_6^+) - \frac{1}{2}c_2(S_6^+)c_1(\mathcal{L}_D) + \frac{1}{12}c_1(\mathcal{L}_D)^3 - \frac{1}{12}c_1(\mathcal{L}_D)p_1(M_4)$$

Wess–Zumino Interactions

For non-abelian $\mathcal{N} = 4$ SYM with varying coupling
$\rightarrow$ consider theory on Coulomb branch: $G \rightarrow U(1)^{r_G}$
Integrating out massive fermions
$\Rightarrow$ Wess–Zumino interaction terms $\rightarrow$ contribute to the anomaly
$\rightarrow$ for constant coupling contribution is [Intriligator]

$$I_6^{WZ} = \frac{1}{2}(d_G - r_G)c_3(S_6^+)$$

Conjecture: interaction terms contribute in the same way for varying τ
Anomaly polynomial from vector multiplets is

$$I_6 = \frac{1}{2}d_G c_3(S_6^+) - \frac{1}{2}r_G c_2(S_6^+)c_1(\mathcal{L}_D) + \frac{1}{12}r_G c_1(\mathcal{L}_D)^3 - \frac{1}{12}r_G c_1(\mathcal{L}_D)p_1(M_4)$$

There are, in addition, defect modes
$\rightarrow$ this is not the complete anomaly polynomial

Anomaly from M5 on T^2 Fibration

A theory of class F is specificed by $T[C_g, \mathcal{F}, G]$

→ $\mathcal{F}$ denotes data describing the fibration

→ when $C_g = T^2$

$$\mathcal{F} = (f,\, g,\, \mathbb{L})$$

→ $\mathbb{L}$ is the *Weierstrass line bundle*

→ f, g are sections of appropriate powers of $\mathbb{L}$

→ describes τ-profile: how complex structure of T^2 varies over M_4

Note: $\mathbb{L}$ is related to the relative tangent bundle of the fibration

⇒ the connection on $\mathbb{L}$ enters in the 6d spin connection

[Assel, Schäfer-Nameki]

Integration over the Fiber

For a fibration $\pi : M_6 \to M_4$ with $\pi^{-1}(x) = C_g$ then

$$I_6 \supset \int_{\text{fiber}} I_8 = \pi_* I_8$$

For an elliptic fibration $\pi : M_6 \to M_4$ we

$$\pi_* I_8 = \frac{r_G}{48} \left[\pi_* \left(-p_2(M_6) + \frac{1}{4} p_1(M_6)^2 \right) - \frac{1}{2} p_1(N_5) \pi_* p_1(M_6) \right]$$

The spin bundle associated to N_5, $S(N_5)$, will become an $SO(5)$ subbundle of the $SO(6)$ R-symmetry bundle of the $\mathcal{N} = 4$ SYM

$\to$ we only see anomalies sensitive to this subbundle

$\to$ the $SO(6)$ R-symmetry is emergent $\Rightarrow$ not seen directly

Determining $I_6 \Rightarrow$ we must calculate these pushforwards

Pushforwards of Chern Classes

To determine the integral over the fiber of I_8 we need to know the pushforward of polynomials in the Chern classes of M_6 onto M_4

$$\pi_* c(M_6) = 12 c_1(\mathbb{L}) c(M_4)(1 + \mathcal{O}(c_1(\mathbb{L}), \cdots))$$

The prefactor is *universal* – it is identical for any elliptic fibration
$\rightarrow$ can compute universal part of I_6 – independent of τ-profile

Anomaly Polynomial for $T[T^2, \mathcal{F}]$

One finds

$$I_6 = \pi_* I_8 = \left[-\frac{1}{2} r_G c_2(S(N_5)) c_1(\mathbb{L}) - \frac{1}{24}(-6 r_G) c_1(\mathbb{L}) p_1(M_4) + \cdots \right]$$

Comparison with the direct, bulk(!), 4d anomaly polynomial

$$I_6 = \left[-\frac{1}{2} r_G c_2(S_6^+) c_1(\mathcal{L}_D) - \frac{1}{24}(2 r_G) c_1(\mathcal{L}_D) p_1(M_4) + \cdots \right]$$

Given $\mathbb{L} = \mathcal{L}_D$ [Assel, Schäfer-Nameki]

- $\rightarrow$ first term is identical
- $\rightarrow$ second term differs by $8 r_G$
  - $\rightarrow$ defect degrees of freedom contribute to $c_1(\mathcal{L}_D) p_1(M_4)$ anomaly
  - $\rightarrow$ later: on compactification to 2d, known defect contribution

[CL, Schäfer-Nameki, Weigand]

$\mathcal{N} = 2$ Theories of Class F

There is not a universal construction of all $g > 1$ C_g fibrations
$\rightarrow$ consider special case of plane curve fibrations

Take a line bundle $\mathcal{J} \rightarrow M_4$ and hypersurface M_6 in the projective bundle

$$\mathbb{P}(\mathcal{O} \oplus \mathcal{J}^a \oplus \mathcal{J}^b)$$

of degree $dH + ec_1(\mathcal{J})$
$\rightarrow$ the genus of the fibered curve is

$$g = \frac{(d-1)(d-2)}{2}$$

$\rightarrow$ for $g = 1$ line bundle $\mathcal{J} = \mathbb{L}$
$\rightarrow$ bundle measures the non-triviality of the fibration
$\rightarrow$ connection on $\mathcal{J}$ is $U(1)_D$ connection

Anomalies of $\mathcal{N} = 2$ Theories of Class F

The C_g fiber now has non-trivial curvature
$\rightarrow$ partial topological twist along the fiber

$$n_1 = 2r - (c_1(M_6) - c_1(M_4) + (e - a - b)c_1(\mathcal{J}))\,, \quad n_2 = 2\alpha$$

$\rightarrow$ notice: if fibration is trivial, twist is just with $c_1(C_g) \Rightarrow$ class S

After twisting one can pushforward

$$I_6 = \pi_* I_8 = \kappa_1 c_2(R) c_1(R) + \kappa_2 c_1(R)^3 + \kappa_3 c_1(R) p_1(M_4) + c_1(\mathcal{J})(\cdots)$$

Question: intrinsic 4d description of such anomaly polynomials?

Part II: Compactifications to 2d

Compactifications of $T[T^2, \mathcal{F}]$

Specialise: $\mathcal{N} = 4$ SYM where τ varies over 2d subspace of M_4, Σ
$\Rightarrow$ 2d chiral SCFTs
$\rightarrow$ corresponding to D3-branes wrapping Σ in F-theory

Two alternate approaches to compare with:

1. abelian $\mathcal{N} = 4$ compactified on Σ with topological duality twist
   [Haghighat, Murthy, Vafa, Vandoren; CL, Schäfer-Nameki, Weigand]
   Topological duality twist (TDT): To preserve SUSY compensate non-trivial transformation of supercharges under holonomy of C and $U(1)_D$ by R-symmetry transformation. [Martucci]
   $\rightarrow$ explicit reduction of the $\mathcal{N} = 4$ vector multiplet
2. AdS_3 solutions of F-theory
   $\rightarrow$ Type IIB solutions where 10d axio-dilaton varies over spacetime, with monodromies in the $SL(2, \mathbb{Z})$ duality group
   [Couzens, CL, Martelli, Schäfer-Nameki, Wong; Couzens, Martelli, Schäfer-Nameki]

AdS$_3$/CFT$_2$ Solutions of F-theory

F-theory Overview

F-theory geometrizes the axio-dilaton ($\tau = C_0 + ie^{-\phi}$) in Type IIB

$\rightarrow$ Type IIB $SL(2,\mathbb{Z})$ S-duality acts on τ [Vafa], [Morrison, Vafa]

$\rightarrow$ auxilliary elliptic fibration over compactification space

$\rightarrow$ complex structure τ of torus above $b \in B$ is axio-dilaton b

$$\begin{array}{ccccc} \mathbb{E}_\tau & \hookrightarrow & Y & & \\ & & \downarrow & & \\ & & B & \supset & \Sigma \end{array}$$

$\rightarrow$ singular fibers $\Rightarrow$ 7-branes sourcing τ

$\rightarrow$ elliptic fibration $\Rightarrow$ consistent configuration of (p,q) 7-branes

D3-branes in F-theory

$SL(2,\mathbb{Z})$ of axio-dilaton $\Rightarrow$ $SL(2,\mathbb{Z})$ Montonen–Olive duality of $\mathcal{N}=4$

$\rightarrow$ natural home of $\mathcal{N}=4$ SYM with varying coupling

Strings of F-theory

2d SCFTs arise in "string sector" of F-theory
$\rightarrow$ D3-branes on $\Sigma \subset B$

Strings of 6d $\mathcal{N} = (1, 0)$ SCFTs [del Zotto, Lockhart]

- tensionless strings are hallmark of superconformal symmetry in 6d
- instanton part of 6d Nekrasov PF $\leftrightarrow$ elliptic genera of strings

Strings of 6d $\mathcal{N} = (1, 0)$ Supergravities [Haghighat, Murthy, Vafa, Vandoren]

- 5d BPS black holes arise from 6d BPS strings on S^1
- microstate counting of strings in 6d $\rightarrow$ macroscopic entropy

General Solutions for IIB with AdS$_3$ Factor and $(0,2)$ SUSY

Consider general Type IIB solutions with an AdS$_p$ factor preserving some supersymmetry

→ no previously known solutions with full $SL(2,\mathbb{Z})$ monodromy

→ for poles in τ^{IIB} see [Couzens], [D'Hoker, Gutperle, Uhlemann]

Dual CFTs can be difficult to understand

(p,q) 7-branes ⇒ genuinely non-perturbative effects

General starting point:

$$ds^2 = e^{2A} ds^2(\text{AdS}_3) + ds^2(M_7)$$

$$F_5 = (1 + *)\text{vol}(\text{AdS}_3) \wedge F^{(2)}$$

To preserve $(0,2)$ SUSY solve Killing spinor equation

$$\nabla_M \epsilon + \frac{i}{192} \Gamma^{P_1 P_2 P_3 P_4} F_{M P_1 P_2 P_3 P_4} \epsilon = 0$$

General Solutions for IIB with AdS_3 Factor and $(0,2)$ SUSY

General solution

[Couzens, CL, Martelli, Schäfer-Nameki, Wong]
[Couzens, Martelli, Schäfer-Nameki]

$$\begin{array}{ccc} S^1 & \hookrightarrow & M_7 \\ & & \downarrow \\ & & M_6 \end{array}$$

S^1 fibration provides $U(1)_r$ R-symmetry of $(0,2)$

τ variation combines into an auxilliary Kähler elliptic fibration M_8 over M_6 with non-trivial constraint

$$\Box_8 R_8 - \frac{1}{2} R_8^2 + R_{8ij} R_8^{ij} = 0$$

First consider more SUSY
$\to (0,4)$ SUSY $\Rightarrow$ dual to strings in 6d, F-theory on CY_3

Preserving $(0,4)$ SUSY

Requiring $(0,4)$ is highly constrained $A = \text{const}$ and

$$\begin{array}{ccccccc} S^1 & \hookrightarrow & S^3/\Gamma & & Y_3 & \hookleftarrow & \mathbb{E}_\tau \\ & & \downarrow & & \downarrow & & \\ M_6 = & & S^2 & \times & B_2 & & \end{array}$$

Killing spinors transform in $(\mathbf{2},\mathbf{1})$ of S^3 isometry

$$SO(4) = SU(2)_r \times SU(2)_L$$

$$SU(2)_r \to \text{superconformal R-symmetry}$$
$$SU(2)_L \to \text{additional flavour symmetry when } \Gamma = 1$$

We preserve the same SUSY for $\Gamma \subset SU(2)_L$ finite subgroup

$(0, 4)$ Solution

General F-theory solution of Type IIB SUGRA dual to 2d $(0, 4)$ is

$$\begin{array}{ccc} \mathbb{E}_\tau & \hookrightarrow & Y_3 \\ & & \downarrow \\ & & \mathrm{AdS}_3 \times S^3/\Gamma \times B_2 \end{array}$$

with F_5 flux

$$F_5 = (1 + *)\mathrm{vol}(\mathrm{AdS}_3) \wedge J_B$$

J_B is Kähler form on B Poincaré dual to a curve Σ

$\Rightarrow$ Σ, wrapped by D3-brane, ample in B

$$\boxed{\text{Central charge: } c_R = 3N^2 \Sigma \cdot \Sigma + 3N c_1(B) \cdot \Sigma}$$

Generalisation of previously known solutions:

$$\mathrm{AdS}_3 \times S^3 \times T^4 \quad \text{and } \mathrm{AdS}_3 \times S^3 \times K3$$

$T[T^2, \mathcal{F}]$ on Σ

$T[T^2, \mathcal{F}]$ on Σ

Consider $T[T^2, \mathcal{F}]$ on Σ with $\mathcal{L}_D$ supported on Σ
$\rightarrow$ $I_4 = \int_\Sigma I_6$ only has contributions from universal part of I_6

$\Sigma \subset B_2 \subset CY_3 \Rightarrow$ R-symmetry bundle decomposes

$$S_6^+ \rightarrow N^+ \otimes U \oplus N^- \otimes \overline{U}$$

$\rightarrow$ $N^\pm$: $SU(2)_\pm$ bundles – transverse rotations to CY
$\rightarrow$ U: $U(1)$ bundle – normal to $\Sigma \subset B_2$

Integrating I_6:

$$\begin{aligned} I_4 = d_G \left(c_2(N^-) - c_2(N^+) \right) \int_\Sigma c_1(U) \\ - \frac{1}{2} r_G \left(c_2(N^+) + c_2(N^-) - \frac{1}{2} p_1(M_2) \right) \int_\Sigma c_1(\mathcal{L}_D) \end{aligned}$$

$T[T^2, \mathcal{F}]$ on Σ

Topological duality twist relates

$$c_1(U) = -\frac{1}{2}c_1(T_\Sigma) + \frac{1}{2}c_1(\mathcal{L}_D)$$

and for 6d supersymmetry [Bianchi, Collinucci, Martucci]

$$\mathcal{L}_D = \mathcal{O}(-K_B)$$

Thus

$$\begin{aligned} I_4 = {} & c_2(N^-)\left(\frac{1}{2}d_G\Sigma\cdot\Sigma - \frac{1}{2}r_G c_1(B)\cdot\Sigma\right) \\ & + c_2(N^+)\left(-\frac{1}{2}d_G\Sigma\cdot\Sigma - \frac{1}{2}r_G c_1(B)\cdot\Sigma\right) \\ & + \frac{1}{4}r_G c_1(B)\cdot\Sigma p_1(TW_2)\,. \end{aligned}$$

Matches AP for strings in 6d [Berman, Harvery; Shimizu, Tachikawa]

$T[T^2, \mathcal{F}]$ on Σ

Central charge:

$$c_R = 6k_+ = 3d_G \Sigma \cdot \Sigma + 3r_G c_1(B) \cdot \Sigma$$

Matches AdS_3 supergravity dual

Similar comparisons

→ APs for abelian $\mathcal{N} = 4$ SYM on $\Sigma \subset B \subset K3, CY_3, CY_4, CY_5$

→ directly from zero-mode spectrum [CL, Schäfer-Nameki, Weigand]

Recall: $\mathcal{N} = 4$ bulk AP differed from AP from integrating I_8 by

$$8r_G\, c_1(\mathcal{L}_D) p_1(M_4)$$

In 2d this is

$$8r_G\, c_1(B) \cdot \Sigma\, p_1(M_4)$$

→ exact gravitational anomaly contribution from defects/3–7 strings

[CL, Schäfer-Nameki, Weigand]

Part III: Extensions

6d Conformal Matter Theories on T^2 Fibration

Recently a large class of 6d $(1,0)$ SCFTs has been found

[Heckman, Morrison, (Rudelius,) Vafa; Bhardwaj]

$\rightarrow$ can equally consider compactifications on fiber of C_g fibration

In [Ohmori, Shimizu, Tachikawa, Yonekura] shown that compactification of 6d $(2,0)$ of type G on punctured S^2 is the same as (G,G) minimal conformal matter [Del Zotto, Heckman, Tomasiello, Vafa] on T^2

$\rightarrow$ the reduction on a T^2 fibration should be in class F

$\rightarrow$ generalised bifundamental with varying coupling

$I_8^{(G,G)}$ known from [Ohmori, Shimizu, Tachikawa, Yonekura] and

$$\pi_* I_8^{(G,G)} = \left[\frac{N}{2}\left(|\Gamma|(r_G+1)-2\right) - \frac{d_G}{2}\right] c_2(R) c_1(\mathbb{L}) + \left[\frac{N}{4} - \frac{d_G}{24}\right] c_1(\mathbb{L}) p_1(B) - \left[\frac{1}{4}\left(\mathrm{tr}_{\mathrm{adj}} F_0^2 + \mathrm{tr}_{\mathrm{adj}} F_N^2\right)\right] c_1(\mathbb{L}) + \cdots$$

Duality twisted compactification to 2d on Σ agrees with

[Apruzzi, Hassler, Heckman, Melnikov]

Conclusions/Future Directions

1. Conclusions

   → Duality defects ⇒ new terms in anomaly polynomial of 4d SUSY theories

$$I_6 = \frac{1}{2} d_G c_3(S_6^+) - \frac{1}{2} r_G c_2(\mathcal{S}_6^+) c_1(\mathcal{L}_D) - \frac{1}{24}(-6 r_G) c_1(\mathcal{L}_D) p_1(T_4) - \frac{61}{4} r_G c_1(\mathcal{L}_D)^3$$

   → Seen by reducing 6d $(2,0)$ AP along fiber of C_g fibration
   → for T^2 fibration, interpretation as $\mathcal{N}=4$ SYM with varying coupling
   → 4d theories compactified on Σ match known AdS_3 duals

2. Future Directions

   → Punctures – anomaly analysis as in [Bah, Nardoni]
   → Realisations of hyperelliptic fibrations – general $\mathcal{N}=2$ theories
   → Duality defects
   → Fibral dimensional reductions of different theories – 6d $\mathcal{N}=(1,0)$, . . .