Susy Bolt solutions and their free energy via localization

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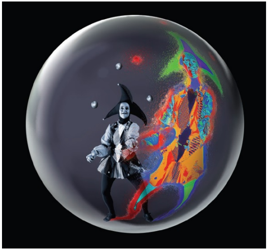
based on 1712.08861 with B. Willett and work in progress

Intro and motivation

Black holes seen as thermodynamic ensembles: they emit radiation and possess entropy. Microstate counting and entropy matching by [Strominger,Vafa '96] etc..

Black hole microstates via AdS/CFT:

Recently, entropy of AdS_4 black hole entropy was matched with computation in 3d dual field theory. [Benini, Hristov, Zaffaroni '15]



Intro and motivation

AdS_4 black hole solutions which admit embedding in M theory with field theory duals in the class of ABJM [Aharony, Bergman, Jafferis, Maldacena, '08] .

$1/4$ BPS static black hole solutions exist in 4d $\mathcal{N} = 2$ *gauged supergravity*: scalar potential allows for susy AdS_4 vacua.

- extremal black holes are flows from AdS_4 to $AdS_2 \times \Sigma_g$ near horizon geometry
- magnetic configurations: they realize the topological twist

Intro and motivation

Constant scalars require the horizon to be a Riemann surface Σ_g with genus $g > 1$ [Romans '92], [Sabra '99], [Caldarelli,Klemm '99]

Nontrivial scalar profiles and reduced amount of susy preserved = genuine static BPS black holes with *spherical* event horizon

1/4 BPS black holes [Cacciatori and Klemm, '09]

Previous static 1/2 BPS solutions [Duff and Liu,'99] contain naked singularities.

Intro and motivation

ABJM partition function on $S^1 \times \Sigma_g$ with magnetic fluxes s_i on Σ_g computed via susy localization, in the large N limit, [Benini, Hristov, Zaffaroni '15], [Benini, Zaffaroni '16]

$$\log \mathcal{Z}_{S^1 \times \Sigma_g} \approx -\frac{2\pi N^{3/2}}{3} \sqrt{2 m_1 m_2 m_3 m_4} \sum_{i=1}^{4} \frac{s_i}{m_i}$$

reproduces BPS black hole entropy upon extremization on m_i:

$$S_{BH} = \frac{A_{BH}}{4 G_N} = \log \mathcal{Z}|_{crit}(s^i)$$

Extremization corresponds to the attractor mechanism in supergravity.

Intro and motivation

Extend these considerations to more general case of $\mathcal{M}_{g,p} = S^1$ p-bundle over 2d Riemann surfaces Σ_g [Closset, Kim, Willett '17]

$$ds^2 = (d\psi + a)^2 + d\Omega_\kappa^2 \qquad -\frac{1}{4\pi}\int_{\Sigma_g} da = p$$

- case of $\mathcal{M}_{g,0} \simeq S^1 \times \Sigma_g$ reduces to the [Benini,Zaffaroni '15] and makes contact with black hole physics
- $\mathcal{M}_{0,1} \simeq S^3$ free energy (extensively studied, F-theorem, etc)
- holographic check with bulk dual solutions

Outline

Outline of the talk:

- BPS solutions: NUTs and Bolts
- Computation of the gravity on-shell action
- ABJM partition function on $\mathcal{M}_{g,p}$
- Conclusions and outlook

Minimal gauged $\mathcal{N} = 2$ supergravity

Euclidean minimal 4d gauged supergravity contains only the gravity multiplet (no vector multiplets or hypermultiplets). Bosonic action is Einstein-Maxwell-Λ:

$$S = \int \mathrm{d}^4 x \sqrt{g} \left[R - F_{\mu\nu} F^{\mu\nu} + \frac{6}{l^2} \right]$$

For a BPS solution the gravitino susy variation is zero

$$\delta_\epsilon \psi = \left(\partial_\mu + \frac{1}{4} \omega_{ab} \gamma^{ab} + \frac{i}{2l} \gamma_\mu + \frac{i}{l} A_\mu + \frac{i}{4} F_{\nu\rho} \gamma^{\nu\rho} \gamma_\mu \right) \epsilon = 0$$

where $\gamma_\mu \in Cliff(4,0)$ and $\{\gamma_\mu, \gamma_\nu\} = 2 g_{\mu\nu}$. Any solution M_4 of this theory uplifts locally to a solution of $M_4 \times Y^7$ of 11d supergravity.

General solution with NUT

$\mathcal{M}_{g,p}$ boundary: related to the presence of NUT charge s.

General solution to the equations of motion: [Chamblin, Emparan, Johnson,Myers '98]

$$ds^2 = \frac{\lambda(r)}{r^2 - s^2}(d\tau + 2s\, f(\theta)d\phi)^2 + \frac{dr^2(r^2 - s^2)}{\lambda(r)} + (r^2 - s^2)d\Omega_k^2$$

$$\lambda(r) = (r^2 - s^2)^2 + (\kappa - 4s^2)(r^2 + s^2) - 2Mr + P^2 - Q^2$$

and

$$f(\theta) = \begin{cases} \cos\theta \\ -\theta \\ -\cosh\theta \end{cases} \qquad d\Omega_\kappa^2 = \begin{cases} d\theta^2 + \sin^2\theta d\phi^2 & \text{for} \quad \kappa = 1 \\ d\theta^2 + d\phi^2 & \text{for} \quad \kappa = 0 \\ d\theta^2 + \sinh^2\theta d\phi^2 & \text{for} \quad \kappa = -1 \end{cases}$$

Compact Σ_g obtained by appropriately taking the quotient of $\mathbf{R}^2$ and $\mathbf{H}^2$

General solution

The gauge field is

$$A_\tau = \frac{-2sQr + P(r^2 + s^2)}{r^2 - s^2}$$

$$A_\phi = \begin{cases} \cos\theta \frac{P(r^2+s^2)-2s\,Q\,r}{r^2-s^2} & \text{for} \quad \kappa = 1 \\ -\theta \frac{P(r^2+s^2)-2s\,Q\,r}{r^2-s^2} & \text{for} \quad \kappa = 0 \\ -\cosh\theta \frac{P(r^2+s^2)-2s\,Q\,r}{r^2-s^2} & \text{for} \quad \kappa = -1 \end{cases}$$

Boundary ($r \to \infty$) is a circle bundle over a constant curvature Σ_g

$$ds^2 = \frac{dr^2}{r^2} + r^2 \left(4s^2(d\psi + f(\theta)d\phi)^2 + d\Omega_\kappa^2\right)$$

In particular for the choice $p = 1$, $g = 0$ and imposing periodicity $\Delta\psi = 4\pi$ the boundary is a biaxially squashed S^3.

NUTs and Bolts (spherical)

Impose regularity (non-singular solutions). Two classes depending on the dimension of the fixed point for the Killing vector ∂_τ.

- "NUT" (AdS-Taub-NUT): isolated fixed point
- "BOLT" (AdS-Taub-Bolt): 2d fixed point

(spherical) NUTs and Bolts

"NUT" ($\mathbf{R}^4$ topology)

- $\lambda(r)$ has double root at $r = s$
- Regularity condition gives $\Delta\psi = 4\pi$
  (if $\Delta\psi = 4\pi/p$ "mildly singular")
- Boundary is a squashed 3-sphere S^3

"Bolt" (topology $\mathcal{O}(-p) \to S^2$)

- $\lambda(r)$ has simple root at $r_b > s$
- regularity condition gives $\Delta\psi = 4\pi/p$
- boundary is squashed Lens space $S^3/\mathbb{Z}_p$

more general NUTs and Bolts

Spherical NUTs and Bolts can be generalized into configurations where Σ_g is a higher genus Riemann surface:

- toroidal and higher genus Bolt solutions, with topology $\mathcal{O}(-p) \to \Sigma_g$ and $\Delta\psi = \frac{4\pi(g-1)}{p}$ for $g > 1$, $\Delta\psi = \frac{4\pi}{p}$ (g=1).

Imposing regularity (+BPS) in general restricts the moduli space of solutions. i.e. constraints between the parameters $Q(s), M(s), P(s)$ and solutions might exist for a certain squashing interval $s \in [0, s_0]$.

BPS solutions

BPS solutions are found for both NUTs and Bolts.

Lorentzian solutions (black holes) with NUT charge, along with their susy properties, studied in [Alonso-Alberca, Meessen, Ortin, '99] and more recently in [Nozawa, Klemm '13].
Euclidean spherical Taub-NUT-AdS and Taub-Bolt analyzed in [Martelli, Passias, Sparks '12].

They preserve $1/2$ or $1/4$ of supersymmetry.

BPS solutions

BPS solutions:

[Alonso-Alberca, Meessen, Ortin, '99]; [Martelli, Passias, Sparks '12].

1/2 BPS solution

$$P = -s\sqrt{4s^2 - 1} \qquad M = Q\sqrt{4s^2 - 1}$$

1/4 BPS solution

$$P = \frac{1}{2}(4s^2 - 1) \qquad M = 2sQ$$

Focus on the 1/4 BPS solutions, and impose regularity. End goal is to compute the on-shell action and compare with the ABJM free energy on $\mathcal{M}_{g,p}$ computed via localization.

BPS NUTs and Bolts

Generalize these BPS solutions to Σ_g, construct Killing spinor [Toldo, Willett]

1/4 BPS solution

$$P = \frac{1}{2}(4s^2 - \kappa) \qquad M = 2sQ$$

Killing spinor equation $\delta_\epsilon \psi = 0$ is solved by

$$\epsilon_\pm = \begin{pmatrix} f_1(r) \\ f_2(r) \end{pmatrix} \otimes \chi \qquad f_1(r) = \sqrt{\frac{(r-r_3)(r-r_4)}{r+s}} \qquad f_2(r) = i\sqrt{\frac{(r-r_1)(r-r_2)}{r-s}}$$

with $\chi = \begin{pmatrix} 0 \\ \chi_{(0)} \end{pmatrix}$, $\chi_{(0)}$ constant. In particular, Killing spinor has only radial dependence.

Regularity and parameter range

- NUT solutions are obtained by a double root at $r = s$, giving $Q = \frac{1}{2}(4s^2 - \kappa)$.
- For the Bolt we need to impose the regularity condition

$$\left.\frac{r^2 - s^2}{2s\lambda'(r)}\right|_{r_0} = \frac{2}{p}$$

which constrains the value of Q:

$$Q_+^\pm = \frac{p^2 \mp (16s^2 - p)\sqrt{f_+}}{128s^2} \qquad Q_-^\pm = -\frac{p^2 \mp (16s^2 + p)\sqrt{f_-}}{128s^2}$$

$$f_\pm = (16s^2 \pm p)^2 - 128\kappa s^2$$

obtaining up to four different branches, denoted with $\text{Bolt}_\pm$.

Regularity and flux

Regularity imposes constraints on the range of parameter s [Martelli, Passias, Sparks '12]. Bolt solutions exist for a finite range of s for $p = 1, 2$. For $p \geq 3$ Bolts are present for every $s > 0$.

Flux through the Bolt computed as

$$\mathcal{F}_{Bolt\pm} = \int_{\Sigma_g} \frac{F}{2\pi} = (g-1) \pm \frac{p}{2}$$

needs to be quantized for the uplift in 11d to be well defined:

$$\pm p + 2(g-1) = 0 \mod I(Y^7) \quad \text{for uplift on } S^7$$

In particular $I(S^7) = 4$, hence $\pm p + 2(g-1) = 0 \mod 4$.

Renormalized on-shell action

Evaluating bulk supergravity action

$$I = -\frac{1}{16\pi G_4} \int d^4x \sqrt{g}(R + 6 - F^2)$$

on a NUT/Bolt solution leads to divergencies: regularize with the introduction of cutoff r_{inf} and add the boundary term from holographic renormalization [Skenderis, '02]

$$I_{ct} = \frac{1}{8\pi G_4} \int_{\partial M} d^3x \sqrt{\gamma}\,(2 + \frac{1}{2} R(\gamma) - K)$$

Manifold closes off at $r_0 = s$ for NUT and $r_0 = r_b$ for Bolt.

Renormalized on-shell action

Evaluating bulk terms we obtain

$$I_{grav}^{bulk} = \frac{1}{8\pi G_4}\frac{16\pi^2}{p}[2sr_{inf}^3 - 6s^3 r_{inf} - 2sr_0 + 6s^3 r_0]$$

and

$$I_{F,NUT} = \frac{2\pi}{G_4}\frac{(\kappa - 4s^2)^2}{4}$$

$$I_{F,Bolt}^{bulk} = \frac{\pi s r_b \left(r_b^2\left(\left(\kappa - 4s^2\right)^2 + 4Q^2\right) + 8\left(4s^2 - \kappa\right) sQr_+\right)}{2G_4\left(s^2 - r_b^2\right)^2 \, p} + \\ + \frac{\pi s r_b \left(s^2\left(\left(\kappa - 4s^2\right)^2 + 4Q^2\right)\right)}{2G_4\left(s^2 - r_b^2\right)^2 \, p}$$

Renormalized on-shell action

Boundary counterterms give

$$I_{ct} = \frac{1}{8\pi G_4}\frac{16\pi^2}{p}[4Qs^2 - 2sr_{inf}^3 + 6s^3 r_{inf} + O(r_{inf}^{-1})]$$

Putting all together, the renormalized on-shell action has a simple form, independent of the squashing parameter s:

$$I_{ren,NUT} = \frac{\pi(1-g)}{2G_4},$$

$$I_{ren,Bolt_\pm} = I_{bulk} + I_{ct} = \frac{\pi\left(4(1-g) \mp p\right)}{8G_4}$$

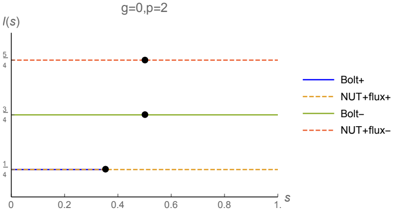
(g=0 case already found in [Martelli,Passias,Sparks, '12])

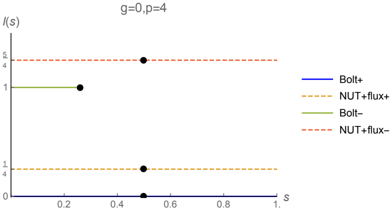
Renormalized on shell action

Also consider AdS-TN/$\mathbb{Z}_p$ with $\pm\frac{p}{2} - 1$ units of magnetic flux, with same boundary data. On shell action is higher

$$I_{NUT+flux} = \frac{\pi}{2G_4}\left(\frac{(g-1)}{p} + \frac{1}{p}(\pm\frac{p}{2} - \kappa)^2\right)$$

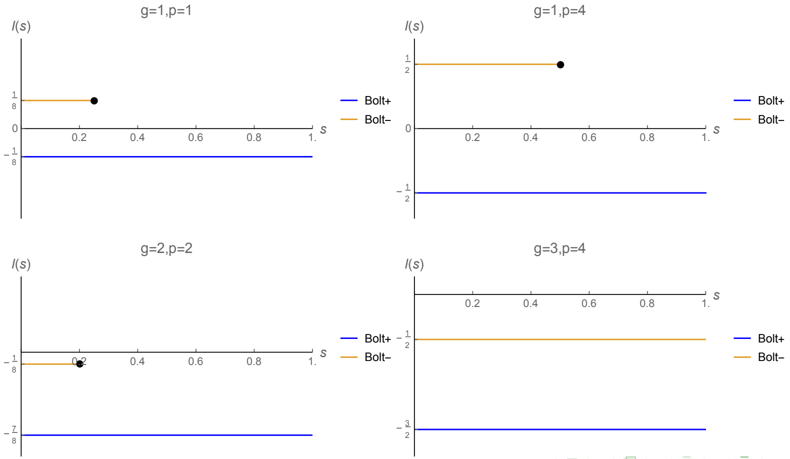
Compare vales of I_{NUT} and $I_{Bolt\pm}$ and range of existence:





Renormalized on shell action

More on moduli space..



Recap

Free energy of SUSY configurations which uplift to 11d on S^7.

- Bolt solutions with boundary $\mathcal{M}_{g,p}$

$$I_{Bolt_{\pm}} = \frac{\pi\,(4(1-g)\mp p)}{8G_4} = \frac{\pi N^{3/2}}{6\sqrt{2}}\,(4(1-g)\mp p)$$

which exist for $\pm p + 2(g-1) = 0 \mod 4$.

- For $p = 1, g = 0$ boundary is squashed S^3 we have the NUT

$$I_{NUT} = \frac{\sqrt{2}\pi N^{3/2}}{3}$$

We used $1/G_4 = 2\sqrt{2}N^{3/2}/3$.

Free energy result for S^3

$\mathcal{N} = 2$ CS theories on the squashed S^3 studied in [Hama, Hosomichi, Lee '11], and found to be independent of the squashing parameter. Computation of the ABJM partition function on S^3 gives [Jafferis, Klebanov, Pufu, Safdi, '11]

$$\log Z_{S^3} = -\frac{4\pi N^{3/2}}{3}\sqrt{2[m_1][m_2][m_3][m_4]}$$

At the conformal point, we have

$$F_{S^3} = -Log(Z_{S^3}) = \frac{\sqrt{2}\pi N^{3/2}}{3} = I_{NUT}$$

Indeed the NUT free energy does not depend on the squashing, already noticed in [Martelli, Passias, Sparks '12].

ABJM free energy on $\mathcal{M}_{g,p}$

We want to reproduce the Bolt free energy.

Recently the partition function of superconformal $\mathcal{N} = 2$ theories on $\mathcal{M}_{g,p}$ computed in [Closset, Kim, Willett '17]. It has the form of sum over vacua

$$Z_{\mathcal{M}_{g,p}} = \sum_{u_a \in S_{BE}} \mathcal{F}_I(u_a, m_i)^p \, \mathcal{H}_I^{g-1}(u_a, m_i) \, \Pi_I^i(u_a, m_i)^{s_i}$$

where

- $\mathcal{F}_I(u_a, m_i) = exp\left(2\pi i \left(\mathcal{W}^I - \sum_i m_i \frac{\partial \mathcal{W}^I}{\partial m_i}\right)\right)$ "fibering operator"
- $\mathcal{H}_I = exp\left(2i\pi\Omega^I\right)$ "handle gluing operator"
- $\Pi_I^i = exp\left(2\pi i \frac{\partial \mathcal{W}}{\partial m_i}\right)$ "flux operator"

Large N ABJM free energy on $\mathcal{M}_{g,p}$

Large N limit of the partition function is

$$\log Z_{\mathcal{M}_{g,p}}(m_i, n_i) = pW_{ext} - \sum_i (pm_i - s_i - (g-1)r_i)\partial_i W_{ext}$$

and inserting the superpotential W_{ext}

$$\log Z_{\mathcal{M}_{g,p}} = \frac{2\pi N^{3/2}}{3}\sqrt{2[m_1][m_2][m_3][m_4]}\left(2p - \sum_i \frac{-pn_i + s_i + (g-1)r_i}{[m_i]}\right)$$

with the following constraints

$$\sum_{j=0}^{3} m_j = \sum_j s_j = 0\,, \qquad \sum_j [m_j] = 1\,, \qquad \sum_j (r_j - 1) + 2 = 0$$

Matching for minimal solutions

Minimal sugra: condition of constant scalars and equal fluxes gives

$$[m_i] \text{ and } -pn_i + s_i + (g-1)r_i \text{ independent of } i$$

hence $p + 2(g-1) = 0 \;\; mod \; 4$ and $[m_i] = [m] = 1/4$ which gives

$$-\log Z^{ABJM}_{\mathcal{M}_{g,p}} = \frac{\pi N^{3/2}}{6\sqrt{2}} (4(1-g) - p) = I_{Bolt+}$$

which matches the supergravity result!

For $p = 0$ reduces to [Azzurli, Bobev, Crichigno, Min, Zaffaroni '17] which gives the entropy of minimal sugra black holes with Σ_g horizons.

Recap

- For $k = 1$ ABJM theory Bolt free energy in sugra matches [CT, Willett '17] the large N localization result for $\mathcal{M}_{gp}$ with

$$p \pm 2(g-1) = 0 \mod 4$$

- S^3 boundary treated separately, and was computed before [Jafferis, Klebanov, Pufu, Safdi, '11]: $I_{NUT} = I_{AdS4} = -Z_{ABJM}(S^3)$ (Also S^3/Z_p (no flux) in [Alday,Fluder,Sparks])
- notice that for $p = 2$ NUT and Bolt free energy coincide (solutions are continuously connected)

... How about the $p = 1$ Bolt?

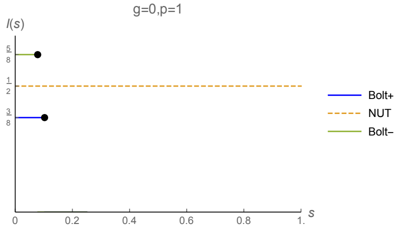
Curious case of $g = 0, p = 1$ Bolt

There might be cases in which the S^3 boundary $(g = 0, p = 1)$ admits $p = 1$ Bolt fillings for certain parameter ranges $s \in [s_-, s_+]$:

Bolt free energy is

$$I_{Bolt+} = \pi \frac{4-p}{8G_N} = \frac{3\pi}{8G_4}$$

$$I_{Bolt-} = \pi \frac{4+p}{8G_N} = \frac{5\pi}{8G_4}$$



Notice that

$$I_{Bolt_+} < \frac{\pi}{2G_4} = I_{NUT} < I_{Bolt-}$$

Curious case of $g = 0, p = 1$ Bolt

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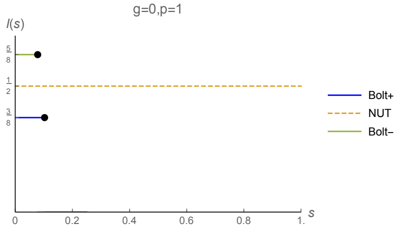

Notice that

$$I_{Bolt_+} < \frac{\pi}{2G_4} = I_{H_4} < I_{Bolt-}$$

$V^{5,2}$ and $g = 0, p = 1$ Bolt

Free energy for theory dual to $V^{5,2}$ on S^3 computed by [Martelli, Sparks '11] coincides with I_{NUT}.

Condition for uplift on M-theory on $Y^7 = V^{5,2}$ is $(I(V^{5,2}) = 3)$

$$p \pm 2(g-1) = 0 \mod 3$$

hence $p = 1$ Bolt- uplifts! Its free energy is $I_{Bolt-} = \frac{5}{4} I_{NUT}$.

Its free energy is reproduced by the partition function on $\mathcal{M}_{g,p}$. For $p = 1$ our procedure yields a subleading saddle point: $I_{Bolt-} > I_{NUT}$.

Summary and Outlook

Free energy of Bolts with $\mathcal{M}_{g,p}$ boundary, when uplift is possible, is reproduced by ABJM partition function via susy localization.

Goals:

- subtleties when different $p = 1$ Bolt fillings are allowed, i.e. for the $V^{5,2}$ theory
  - compare S^3 free energy computed with previous methods
- NUT/Bolt solutions in gauged supergravity with matter multiplets [Colleoni, Klemm '11; Erbin, Halmagyi '15]
  - subtleties with holographic renormalization due to scalars boundary conditions in progress

Summary and Outlook

Goals (continued):

We constructed rotating magnetically charged 1/4 BPS AdS_4 black holes with compact horizon [Hristov, Katmadas, Toldo, to appear] supported by a θ, r dependent complex scalar field.

- counting microstates in dual ABJM theory?
- NH geometry contains S^1 fibration of AdS_2: same universality class of fast spinning black holes in our universe

Thanks for attention!

Boundary metric is

$$ds^2 = \frac{q^2 \Delta_\theta}{\Xi} \left[-dt^2 + \frac{\Xi d\theta^2}{\Delta_\theta^2} - \frac{\sin^2\theta}{\Delta_\theta} \left(d\phi + \frac{j}{l^2} dt \right)^2 \right] .$$

where $\Xi = 1 - \frac{j^2}{l^2}$, $\Delta_\theta = 1 - \frac{j^2}{l^2} \cos^2\theta$

The 11d uplift ansatz for the metric ($k = 1$) is

$$ds^2_{11} = ds^2_4 + ds^2_{B_6} + (d\chi + \frac{1}{2}A)^2$$

To have a well-defined S^1 bundle over $M_4 \times B_6$, the coordinate χ needs to have period $2\pi I/4$, where I is the Fano index of B_6. The connection $\frac{1}{2}A$ should satisfy the requirement

$$\frac{4}{2\pi I}\int_{S^2}\frac{1}{2}F = m \in \mathbb{Z}$$

- For S^7 have $I(S^7) = 4$ and the connection $\frac{1}{2}A$ should satisfy the requirement

$$\frac{1}{2\pi}\int_{S^2}\frac{1}{2}F = m \in \mathbb{Z} \qquad \rightarrow \qquad \frac{1}{2\pi}\int_{S^2}\frac{1}{2}F = \pm\frac{p}{2} - 1 \notin \mathbb{Z}$$

The $p = 1$ Bolt is not allowed.

- For $V^{5,2} = SO(5)/SO(3)$ instead $I(V^{5,2}) = 3$, so

$$\frac{1}{2\pi}\int_{S^2}\frac{1}{2}F = \frac{2}{3}\left(\pm\frac{p}{2} - 1\right) \in \mathbb{Z}$$

Bolt-, p=1 is allowed by the quantization condition.