New multiplets in four dimensional N=2 conformal supergravity

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Introduction

- $\mathcal{N}$ extended conformal supergravity in four dimensions is a theory of gravity which is a representation of the $\mathcal{N}$-fold extended superconformal algebra $su(2,2|\mathcal{N})$.
- Due to the conformal symmetry, it contains off-shell representations with smaller number of components than the Poincare supergravity theory.
- On gauge fixing some of the conformal symmetries using compensator fields, one can obtain the corresponding Poincare supergravity theory.

Conformal gravity as a gauge theory

- SU(2,2) conformal algebra contains the generators P_a, M_{ab}, D, K_a.
- $[D, P_a] = P_a, [D, K_a] = -K_a$, $[P_a, K_b] = \eta_{ab}D - 2M_{ab}$,
  $[P_a, M_{bc}] = 2P_{[b}\eta_{c]a}$, $[K_a, M_{bc}] = 2K_{[b}\eta_{c]a}$,
  $[M_{ab}, M_{cd}] = 2\eta_{[a[c}M_{b]d]}$
- Introduce a gauge field corresponding to each generator: e^a_μ, ω^{ab}_μ, b_μ, f^a_μ.

- Transformation rule for the above fields is obtained by using the structure constants of the conformal algebra.

$$\delta h_\mu^A = \partial_\mu \varepsilon^A + \varepsilon^C h_\mu^B f_{BC}{}^A$$

For eg, $f_{d[bc]}{}^a = 2\eta_{a[b}\delta_{c]}^a$. Therefore $\delta_M e_\nu^a = \frac{1}{2}\lambda^{bc} e_\nu^d f_{d[bc]}{}^a$. Thus, $\delta_M e_\nu^a = -\lambda^{ab} e_{\nu b}$.

- Conformal curvatures:

$$R_{\mu\nu}{}^A = 2\partial_{[\mu} h_{\nu]}^A + h_\nu^C h_\mu^B f_{BC}{}^A$$

For eg: $R_{\mu\nu} = 2\partial_{[\mu} e_{\nu]}^a + 2b_{[\mu} e_{\nu]}^a + 2\omega_{[\mu}^{ab} e_{\nu]b}$

- Demand translations to act as general coordinate transformations.

$$\delta_P e_\mu^a = \delta_{\mathsf{cov}} e_\mu^a + \xi^\nu R_{\mu\nu}(P)^a$$

- Conventional constraints: $R_{\mu\nu}(P)^a = 0$, $R(M)_{\mu\nu}{}^{ab} e_b^\nu = 0$.
- ω_μ^{ab} and f_μ^a: Dependent gauge fields.

$$\begin{aligned} \omega(e,b)_\mu^{ab} &= \omega(e)_\mu^{ab} - 2 e_\mu^{[a} e^{b]\nu} b_\nu \\ f_\mu^a &= \frac{1}{2} R(e,b)_\mu^a - \frac{1}{12} R(e,b) e_\mu^a \end{aligned}$$

- Number of independent field components : $e_\mu^a(16), b_\mu(4)$
  Number of gauge transformation parameters for $su(2,2)$: 15.
  Therefore off-shell d.o.f $= 20 - 15 = 5$. But for Poincare gravity, we need 6 off-shell d.o.f!

- Consider

$$\mathcal{L} = -e\phi D^\mu D_\mu \phi$$

- ϕ has Weyl weight $+1$.
- $D_\mu \phi = \partial_\mu \phi - b_\mu \phi$,
  $D_\mu D^a \phi = (\partial_\mu - 2b_\mu) D^a \phi - \omega_\mu^{ab} D_b \phi + f_\mu^a \phi$
- Gauge fixing: $b_\mu = 0$, $\phi = \sqrt{6}/(\sqrt{2}\kappa)$
-

$$\mathcal{L} = -e\frac{1}{2\kappa^2}R$$

Outline

1. Introduction
   - Conformal gravity and gauge equivalence
   - N extended conformal supergravity
   - Matter multiplets and multiplet calculus
2. Real Scalar multiplet
   - 24+24 matter multiplet
   - Restricted real scalar multiplet = tensor multiplet
3. Construction of the action for the real scalar multiplet
4. Construction of the real scalar multiplet
   - dilaton Weyl multiplet in five dimensions
   - Supercurrent multiplet for the tensor multiplet and linearized tranformation rules
5. Dilaton Weyl multiplet in four dimensions
6. Conclusions and future work

N extended superconformal algebra in four dimensions

- Contains Q and S supercharges.
  $[K_a, Q^i] = \gamma_a S^i$, $[P_a, S^i] = \frac{1}{2}\gamma_a Q^i$,
  $\{Q^i, \bar{Q}_j\} = -(I - \gamma_5)\gamma^a P_a \delta^i{}_j$
- It contains an R symmetry algebra $SU(N)_R \times U(1)_R$.
- $\{Q^i, \bar{S}_j\} = \frac{1}{2}(I - \gamma_5)(2\sigma^{ab} M_{ab} + D - iA - 2V^i{}_j)$
- Fermionic charges are Majorana Fermions. Hence, the R symmetry algebra is chiral.
- $[D, Q^i] = \frac{1}{2}Q^i$, $[D, S^i] = \frac{1}{2}S^i$,...

Superconformal gauge theory

- Begin with a superconformal gauge theory: Associate a gauge field to each charge from the superconformal algebra: $e_\mu{}^a$, $\omega_\mu{}^{ab}$, b_μ, $f_\mu{}^a$, $\mathcal{V}_\mu{}^i_j$, A_μ, $\psi_\mu{}^i$, $\phi_\mu{}^i$ with P^a, M^{ab}, D, K^a, V^i_j, A, Q^i, S^i respectively.
- Transformation rule for the above fields is obtained by using the structure constants of the superconformal algebra.
  $\delta h_\mu^A = \partial_\mu \varepsilon^A + \varepsilon^C h_\mu^B f_{BC}{}^A$
- Conformal curvatures: $R_{\mu\nu}{}^A = 2\partial_{[\mu} h_{\nu]}^A + h_\nu^C h_\mu^B f_{BC}{}^A$
- In this theory, the superconformal transformations act as internal symmetries.

- To realize this gauge theory as a theory of supergravity, impose constraints on conformal curvatures. Add matter fields (D, T_{ab}, χ^i) such that the bosonic and fermionic degrees of freedom match.
- Conventional constraints:

$$
\begin{aligned}
R_{\mu\nu}(P)^a &= 0, \\
\gamma^\mu(\hat{R}_{\mu\nu}(Q)^i + \frac{1}{2}\gamma_{\mu\nu}\chi^i) &= 0, \\
e_b^\nu \hat{R}_{\mu\nu}(M)_a{}^b - i\tilde{\hat{R}}_{\mu a}(A) + \frac{1}{4}T^+_{ab}T^{-b}_\mu - \frac{3}{2}De_{\mu a} &= 0
\end{aligned}
$$

- These constraints make some of the gauge fields to be dependent fields.

Weyl Multiplet

- Multiplet of fields obtained in this manner is known as the Weyl multiplet. This is the minimal multiplet containing the gauge fields of the superconformal algebra.
- Independent Bosonic fields:
  $e_\mu^a(16), b_\mu(4), A_\mu(4), \mathcal{V}_\mu{}^i{}_j(12), T_{ab}^{ij}(6), D(1)$. Number of bosonic gauge parameters: $M_{ab}(6), P^a(4), K^a(4), D(1), V(3), A(1)$.
  Off-shell bosonic d.o.f$=24$.
- Independent Fermionic fields: $\psi_\mu^i(32), \chi^i(8)$
  Number of fermionic gauge parameters: $Q^i(8), S^i(8)$
  Off-shell fermionic degrees of freedom $=$ 24.

- Algebra on the fields:

$$
\begin{aligned}
[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] &= \delta^{(cov)}(\xi) + \delta_M(\varepsilon) + \delta_K(\Lambda_K) + \delta_S(\eta) + \delta_{gauge} \\
[\delta_S(\eta), \delta_Q(\epsilon)] &= \delta_M(2\bar{\eta}^i \sigma^{ab} \epsilon_i + \text{h.c}) + \delta_D(\bar{\eta}_i \epsilon^i + \text{h.c}) \\
&\quad + \delta_A(i\bar{\eta}_i \epsilon^i + \text{h.c}) + \delta_V(-2\bar{\eta}^i \epsilon_j - (\text{h.c;traceless})) \\
[\delta_S(\eta_1), \delta_S(\eta_2)] &= \delta_K(\bar{\eta}_{2i} \gamma^a \eta_1^i + \text{h.c})
\end{aligned}
$$

where $\delta^{(cov)}(\xi) = \delta_{gct}(\xi) + \sum_T \delta_T(-\xi^\mu h_\mu(T))$.

- The field dependent transformation parameters are given by

$$
\begin{aligned}
\xi^\mu &= 2\bar{\epsilon}_2{}^i \gamma^\mu \epsilon_{1i} + \text{h.c} \\
\varepsilon^{ab} &= \bar{\epsilon}_1^i \epsilon_2^j T_{ij}^{ab} + \text{h.c} \\
\Lambda_K^a &= \bar{\epsilon}_1^i \epsilon_2^j D_b T_{ij}^{ba} - \frac{3}{2} \bar{\epsilon}_2^i \gamma^a \epsilon_{1i} D + \text{h.c} \\
\eta^i &= 6 \bar{\epsilon}_{[1}^i \epsilon_{2]}^j \chi_j
\end{aligned}
$$

Matter multiplets

- There is also an 8+8 tensor multiplet, on which the above algebra is realized, with field content G(A complex scalar), ϕ^i($SU(2)$ doublet of chiral fermions), $E_{\mu\nu}$(A two form gauge field) and L_{ij}($SU(2)$ triplet of scalars with 'reality' condition $L_{ij} = \epsilon_{ik}\epsilon_{jl}L^{kl}$, $(L^{ij})^* = L_{ij}$).
- There are other 8+8 multiplets in $\mathcal{N} = 2$ conformal supergravity, such as the vector multiplet, non-linear multiplet etc.
- The above matter multiplets can be used as compensator multiplets to obtain the physical Poincare supergravity.

Story of the chiral multiplet: multiplet calculus

- There is a 16+16 components chiral multiplet which reduces to the 8+8 restricted chiral multiplet when a consistent set of 8+8 constraints are imposed.
- There is a chiral weight 0 complex triplet of scalars B_{ij}, which is constrained to satisfy a 'reality' condition. $B_{ij} = \varepsilon_{ik}\varepsilon_{jl}B^{kl}$. Other contraints can be obtained by supersymmetric variation of this constraint.
- The restricted chiral multiplet is equivalent to tensor (vector) multiplet. i.e. gauge invariant quantities of these multiplets can be embedded in the restricted chiral multiplet.

- This was used to write a superconformal action for the (improved) tensor multiplet, vector multiplet and the Weyl multiplet.
- Obtaining an action for one multiplet through an action for another multiplet is known as multiplet calculus.
- This allowed for constructions of minimal Poincare supergravity theories as well as construction of supersymmetric higher derivative actions.
- The study of all the off-shell representations of the superconformal algebra and the corresponding actions is interesting, in this context.

Table of Contents

24+24 matter multiplet: Real scalar multiplet

- Field content of the multiplet is given by

Table: Field content of the 24+24 multiplet

Field	SU(2) Irreps	Weyl weight (w)	Chiral weight (c)	Chirality
ϕ	$\mathbf{1}$	1	0	–
$S_a{}^i{}_j$	$\mathbf{3}$	1	0	–
E_{ij}	$\mathbf{3}$	1	-1	–
C_{ijkl}	$\mathbf{5}$	2	0	–
Λ_i	$\mathbf{2}$	1/2	+1/2	+1
Ξ_{ijk}	$\mathbf{4}$	3/2	-1/2	+1

- It is the generalization of the flat space 24+24 multiplet constructed by Howe et al[1] to include coupling to conformal supergravity.

[1] P S Howe, K S Stelle, P K Townsend, Nucl. Phys. B 1983

Field redefinitions to simplify the transformation rule

- The Q-transformation of the above multiplet is highly non-linear in the fields, although the field components are S-invariant except Λ^i which transforms as $\delta_S \Lambda_i = -2\eta_i$.
- Q transformations are simplified by redefinition of the fields.
- Field content of the redefined multiplet is as follows.

Table: Field content of the redefined 24+24 multiplet

Field	SU(2) Irreps	Weyl weight (w)	Chiral weight (c)	Chirality
V	$\mathbf{1}$	-2	0	–
$A_a{}^i{}_j$	$\mathbf{3}$	-1	0	–
K_{ij}	$\mathbf{3}$	-1	-1	–
$\mathcal{C}_{ijkl}$	$\mathbf{5}$	0	0	–
ψ_i	$\mathbf{2}$	-3/2	+1/2	+1
ξ_{ijk}	$\mathbf{4}$	3/2	-1/2	+1

$$
\begin{aligned}
\delta V &= \bar{\epsilon}_k \psi^k + h.c.\ , \\
\delta \psi^i &= \not{D} V \epsilon^i - \not{A}^i{}_j \epsilon^j - 2K^{ij}\epsilon_j - 2V\eta^i\ , \\
\delta K^{ij} &= 2V\bar{\epsilon}^{(i}\chi^{j)} - \frac{2}{3}\bar{\epsilon}^{(i}\not{D}\psi^{j)} + \frac{1}{3}\bar{\epsilon}^k \xi_{lmk}\varepsilon^{il}\varepsilon^{jm} + \frac{1}{12}\bar{\epsilon}^{(i}\gamma\cdot T^- \psi_l \varepsilon^{j)l} \\
&\quad - 2\bar{\eta}^{(i}\psi^{j)}\ , \\
\delta A_a{}^i{}_j &= V\bar{\epsilon}_j\gamma_a\chi^i + \frac{2}{3}\bar{\epsilon}_j\gamma_a\not{D}\psi^i - 2\bar{\epsilon}_j D_a\psi^i - \frac{1}{3}\varepsilon^{li}\varepsilon^{nk}\bar{\epsilon}_n\gamma_a\xi_{ljk} \\
&\quad + \frac{1}{24}\bar{\epsilon}_j\gamma_a\gamma\cdot T^-\psi_k\varepsilon^{ik} - \bar{\eta}_j\gamma_a\psi^i - (h.c; traceless)\ ,
\end{aligned}
$$

$$
\begin{aligned}
\delta\xi_{ijk} = {}& \frac{3}{2} D_a A^{al}{}_{(i}\varepsilon_{j|m|}\varepsilon_{k)l}\epsilon^m - 3 D_a A_b{}^l{}_{(i}\varepsilon_{j|m|}\varepsilon_{k)l}\gamma^{ab}\epsilon^m \\
& - \frac{3}{2} V\gamma\cdot R(V)^l{}_{(i}\varepsilon_{j|m|}\varepsilon_{k)l}\epsilon^m + 6\not{D}K^{lm}\epsilon_{(i}\varepsilon_{j|l|}\varepsilon_{k)m} - \mathcal{C}_{ijkl}\epsilon^l \\
& - \frac{3}{4}\gamma\cdot T^-\epsilon^n K_{(ij}\varepsilon_{k)n} - \frac{3}{2}\bar{R}(Q)^l_{ab}\psi_{(i}\gamma^{ab}\epsilon^m\varepsilon_{j|l|}\varepsilon_{k)m} \\
& - \frac{3}{2}\bar{\chi}^l\psi_{(i}\epsilon^m\varepsilon_{j|l|}\varepsilon_{k)m} + \frac{3}{2}\bar{\chi}^l\gamma_{ab}\psi_{(i}\gamma^{ab}\epsilon^m\varepsilon_{j|l|}\varepsilon_{k)m} \\
& + \frac{3}{2}\bar{\chi}_{(i}\psi^l\epsilon^m\varepsilon_{j|l|}\varepsilon_{k)m} + 3\bar{\chi}^m\gamma_a\psi^l\gamma^a\epsilon_{(i}\varepsilon_{j|l|}\varepsilon_{k)m} \\
& - 6K^{nm}\eta_{(i}\varepsilon_{j|n|}\varepsilon_{k)m} - 6\not{A}^m{}_{(i}\eta^n\varepsilon_{j|m|}\varepsilon_{k)n} \ , \\
\delta\mathcal{C}_{ijkl} = {}& \bar{\epsilon}_{(i}\tilde{\Gamma}_{jkl)} + \varepsilon_{im}\varepsilon_{jn}\varepsilon_{kp}\varepsilon_{lq}\bar{\epsilon}^{(m}\tilde{\Gamma}^{npq)} - 4\bar{\eta}_{(i}\xi_{jkl)} \ ,
\end{aligned}
$$

where,

$$
\tilde{\Gamma}_{ijk} = -2\not{D}\xi_{ijk} + 12\chi_{(i}K^{lm}\varepsilon_{j|l|}\varepsilon_{k)m} \ .
$$

8+8 restricted real scalar multiplet

- The multiplet is a 24+24 multiplet rather than the more common 8+8. Can we restrict this multiplet to obtain an 8+8 multiplet?
- In the 24+24 real scalar multiplet, E_{ij}, has chiral weight -1. Therefore, we can not impose the 'reality' condition on E_{ij}.
- However, we can impose the constraint $E_{ij} = e^{-i\sigma/2}\mathcal{L}_{ij}$ where $\mathcal{L}_{ij}$ are 'real'.
- This can be rephrased in the form $\mathcal{R}_{ij} = \bar{E}_{ij} - e^{-i\sigma}E_{ij} = 0$ where $\bar{E}_{ij} = \varepsilon_{ik}\varepsilon_{jl}E^{kl}$.

- Supersymmetric variation of the above constraint gives us the full set of 16+16 constraints.
- We are left with a restricted real scalar multiplet with 8+8 degrees of freedom $(\mathcal{L}_{ij}, \mathcal{H}_{abc}, \phi, \sigma, \Lambda^i)$
- This multiplet is equivalent to the 8+8 tensor multiplet. i.e. it contains the gauge invariant objects of the tensor multiplet.

The 8+8 tensor multiplet

- Contains a complex scalar G, a triplet of 'real' scalars L_{ij} (reality condition $L_{ij} = \varepsilon_{ik}\varepsilon_{jl}L^{kl}$ where $(L_{ij})^* = L^{ij}$), a two form gauge field $E_{\mu\nu}$ and a doublet of Majorana fermions ϕ^i.
- The transformation rules are

$$
\begin{aligned}
\delta L_{ij} &= 2\bar{\epsilon}_{(i}\varphi_{j)} + 2\varepsilon_{ik}\varepsilon_{jl}\bar{\epsilon}^{(k}\varphi^{l)} \ , \\
\delta\varphi^i &= \not{D}L^{ij}\epsilon_j + \not{H}\varepsilon^{ij}\epsilon_j - G\epsilon^i + 2L^{ij}\eta_j \ , \\
\delta G &= -2\bar{\epsilon}_i\not{D}\varphi^i - 6\bar{\epsilon}_i\chi_j L^{ij} + \frac{1}{4}\varepsilon^{ij}\bar{\epsilon}_i\gamma\cdot T^+\varphi_j + 2\bar{\eta}_i\varphi^i \ , \\
\delta E_{\mu\nu} &= i\bar{\epsilon}^i\gamma_{\mu\nu}\varphi^j\varepsilon_{ij} + 2iL_{ij}\varepsilon^{jk}\bar{\epsilon}^i\gamma_{[\mu}\psi_{\nu]k} + \text{h.c} \ .
\end{aligned}
$$

Restricted real scalar multiplet=tensor multiplet

- Consider the following combinations of tensor multiplet fields,

$$
\begin{aligned}
\phi^4 &= L^2 \\
\Lambda^i &= -2L^{-2}L^{ij}\varphi_j \\
E_{ij} &= L^{-4}L_{ij}L^{kl}\bar{\varphi}_k\varphi_l - L^{-2}\bar{G}L_{ij} \\
S_a{}^i{}_j &= 2L^{-2}H_aL^{ik}\varepsilon_{kj} + 4L^{-4}L^{ik}L_{jm}\bar{\varphi}^m\gamma_a\varphi_k - L^{-2}\bar{\varphi}^i\gamma_a\varphi_j \\
&\quad - \frac{1}{2}L^{-2}\delta^i_j\bar{\varphi}^m\gamma_a\varphi_m + L^{-2}\left(L^{ik}D_aL_{jk} - L_{jk}D_aL^{ik}\right) \\
\Xi_{ijk} &= -24L^{-6}\varphi^lL^{mn}\bar{\varphi}_m\varphi_nL_{(ij}L_{k)l} + 6L^{-4}\varphi^l\bar{\varphi}_l\varphi_{(i}L_{jk)} \\
&\quad - 6L^{-4}L^{ln}\not{D}L_{l(i}L_{jk)}\varphi_n + 6L^{-4}L^{nm}\not{H}\varphi_nL_{(ij}\varepsilon_{k)m} \\
&\quad + 12L^{-4}\bar{G}L_{(ij}L_{k)l}\varphi^l + 6L^{-2}\not{D}\varphi_{(i}L_{jk)} + 18L^{-2}L_{(ij}L_{k)l}\chi^l \\
&\quad - \frac{3}{4}L^{-2}\gamma\cdot T^-\varphi^lL_{(ij}\varepsilon_{k)l} \\
C_{ijkl} &= 6L^{-4}G\bar{G}L_{(ij}L_{kl)} + \ldots.
\end{aligned}
$$

- These fields transform exactly like the real scalar multiplet fields, but with 8+8 off-shell degrees of freedom.
- From the above identification, one can read off the relation between the tensor multiplet fields and the restricted real scalar multiplet fields.

$$
\begin{aligned}
\phi^4 &= L^2 \ , \\
\Lambda^i &= -2L^{-2}L^{ij}\varphi_j \ , \\
e^{-i\sigma/2} &= \left(\frac{\mathcal{Z}}{\bar{\mathcal{Z}}}\right)^{1/2} \ , \\
\mathcal{L}_{ij} &= |\mathcal{Z}|\, L_{ij} \ , \\
\mathcal{H}_a &\sim H_a \ , \quad \text{Up to fermion bilinears} \ . \\
\mathcal{Z} &= L^{-4}L^{kl}\bar{\varphi}_k\varphi_l - L^{-2}\bar{G} \ .
\end{aligned}
$$

Table of Contents

Action for the 24+24 multiplet and multiplet calculus: Work in progress

- As the 24+24 multiplet admits a tensor multiplet embedding, this enables us to do multiplet calculus.
- Can we construct combinations of the real scalar multiplet fields to obtain a chiral multiplet?
- $\hat{A} = E_{ij} E_{kl} \varepsilon^{ik} \varepsilon^{jl}$ is a chiral field. Further variation gives the other chiral multiplet components.
- This gives us a higher derivative action for the tensor multiplet.
- This is the same action one would obtain for the tensor multiplet from the chiral density formula.
- Can we develop a density formula for the real scalar multiplet, independent of the chiral multiplet.
- The answer appears to be yes. (Work in progress).

Table of Contents

Motivation: Dilaton Weyl multiplet in five dimensions

- In $d = 6$ and 5, it was found that there is not one, but two possible Weyl multiplets.
- The new Weyl multiplet constructed had a dilaton field with Weyl weight $+1$, and hence was called dilaton Weyl multiplet.
- Two routes to construction of dilaton Weyl multiplet in five dimensions[2]:
  a) Multiplet of supercurrents of a non conformal rigid supersymmetry multiplet - vector multiplet in five dimensions.
  b) Coupling the improved vector multiplet to conformal supergravity.
- In four dimensions, the second route gives a 24+24 dilaton Weyl multiplet[3].

[2] E Bergshoeff, S Cucu, M Derix, T de Wit, R Halbersma, A van Proyen, JHEP 2001

[3] D Butter, SH, I Lodato, B Sahoo, JHEP, 2018

Current multiplet method

- Compute the multiplet of supercurrents for a rigid supersymmetry multiplet.
- Couple the supercurrents to fields to obtain the linearized supersymmetry transformations.
- Complete the non-linear supersymmetry transformations using the superconformal algebra.
- A non-conformal multiplet has an energy momentum tensor with a non zero trace. This can be coupled to a scalar field which can be interpreted as the dilaton.
- But the action for rigid vector multiplet in four dimensions is conformal. However the rigid tensor multiplet action is non-conformal in four dimensions.

- Transformation rules for the rigid tensor multiplet on substitution of the equation of motion for the auxiliary field G.

$$\delta E_{\mu\nu} = i\bar{\epsilon}^i \gamma_{\mu\nu}\phi^j \epsilon_{ij} + \text{h.c.}\ ,$$
$$\delta\phi^i = \not{\partial} L^{ij}\epsilon_j + \epsilon^{ij}\not{H}\epsilon_j\ ,$$
$$\delta L^{ij} = 2\bar{\epsilon}^{(i}\phi^{j)} + 2\epsilon^{ik}\epsilon^{j\ell}\bar{\epsilon}_{(k}\phi_{\ell)}\ .$$

- Action for the tensor multiplet

$$S = \int d^4x \left[H_\mu H^\mu - \bar{\phi}^i \overleftrightarrow{\not{\partial}} \phi_i - \frac{1}{2}\partial_\mu L^{ij}\partial^\mu L^{ij} \right]\ ,$$

where H^μ is the Hodge dual of the three form field strength.

- The above action is not conformal.

- 48+48 component multiplet of supercurrents for the above action: $\theta_{\mu\nu}, \sigma, v_{\mu i}{}^{j}, t_{\mu i}{}^{j}, a_\mu, b^-_{\mu\nu}, \tilde{a}_\mu, e_{ij}, d, c_{ijkl}, J_{\mu i}, \lambda_i, \xi^i, \Sigma_{ijk}$ [D Butter, S Kuzenko, 2010]
- Couple the currents to fields via a first order action.

$$S = \int d^4x \left[\frac{1}{2}\theta^{\mu\nu}h_{\mu\nu} + \sigma\varphi + d\mathcal{D} + \frac{1}{24}c^{ijkl}C_{ijkl} + \frac{1}{4}b^-_{\mu\nu}T^{-\mu\nu} \right.$$
$$-2v_\mu{}^i{}_j V^{\mu j}{}_i + e^{ij}E_{ij} + 4\,a_\mu A^\mu + \left(t_\mu{}^i{}_j - v_\mu{}^i{}_j\right) S^{\mu j}{}_i + 2\bar{J}_{\mu i}\psi^{\mu i} + \bar{\lambda}_i\Lambda^i$$
$$\left. +\bar{\xi}_i\zeta^i + \frac{1}{3}\bar{\Sigma}_{ijk}\Xi^{ijk} + \tilde{a}_\mu\tilde{A}^\mu + \text{h.c.}\right] .$$

- Demand the invariance of the action to obtain linearized transformation of the fields.
- Conservation of a current $\implies$ gauge symmetry of a field.

- We obtain the linearized transformation rules for 48+48 component multiplet of fields which contains a real scalar of Weyl weight +1.
- Is this multiplet reducible? i.e. can we decouple this into two or more multiplets by use of field redefinitions.
- Assumption: Such a redefiniton should be apparent even at the linearised level.

- A 24+24 standard Weyl multiplet decouples and we are left with a 24+24 matter multiplet coupled to the standard Weyl multiplet.
- The matter multiplet contains the real scalar field with Weyl weight +1.
- Use the superconformal algebra to complete the supersymmetric variations of the 24+24 matter multiplet.

Table of Contents

Dilaton Weyl multiplet in four dimensions

- A 24+24 Weyl multiplet which contains a dilaton, hence named Dilaton Weyl multiplet. Independent fields are $e_\mu{}^a$, $\psi_\mu{}^i$, b_μ, A_μ, $\mathcal{V}_\mu{}^i{}_j$, X, W_μ, $\tilde{W}_\mu$, Ω_i.
- The transformation rule is given by

$$\begin{aligned}
\delta e_\mu{}^a &= \bar{\epsilon}^i \gamma^a \psi_{\mu i} + \text{h.c} \\
\delta \psi_\mu{}^i &= 2\mathcal{D}_\mu \epsilon^i - \frac{1}{8}\varepsilon^{ij}\bar{X}^{-1}\gamma.\left(\mathcal{F}^- + i\mathcal{G}^-\right)\gamma_\mu \epsilon_j - \gamma_\mu \eta^i \\
\delta b_\mu &= \frac{1}{2}\bar{\epsilon}^i \phi_{\mu i} - \frac{1}{4}X^{-1}\bar{\epsilon}^i \gamma_\mu \not{D}\Omega_i - \frac{1}{2}\bar{\eta}^i \psi_{\mu i} + \text{h.c} + \Lambda_K^a e_{\mu a} \\
\delta A_\mu &= \frac{i}{2}\bar{\epsilon}^i \phi_{\mu i} + \frac{i}{2}X^{-1}\bar{\epsilon}^i \gamma_\mu \not{D}\Omega_i + \frac{i}{2}\bar{\eta}^i \psi_{\mu i} + \text{h.c} \\
\delta V_\mu{}^i{}_j &= 2\bar{\epsilon}_j \phi_\mu^i - \bar{X}^{-1}\bar{\epsilon}_j \gamma_\mu \not{D}\Omega^i + 2\bar{\eta}_j \psi_\mu^i - (\text{h.c; traceless})
\end{aligned}$$

- $$\delta X = \bar{\epsilon}^i \Omega_i$$
  $$\delta \Omega_i = 2 \not{D} X \epsilon_i + \frac{1}{4} \epsilon_{ij} \gamma . \mathcal{F} \epsilon^j - \frac{i}{4} \epsilon_{ij} \gamma . \mathcal{G}^- \epsilon^j + 2 X \eta_i$$
  $$\delta W_\mu = \varepsilon^{ij} \bar{\epsilon}_i \gamma_\mu \Omega_j + 2 \varepsilon_{ij} \bar{X} \bar{\epsilon}^i \psi_\mu^j + \text{h.c}$$
  $$\delta \tilde{W}_\mu = i \varepsilon^{ij} \bar{\epsilon}_i \gamma_\mu \Omega_j - 2 i \varepsilon_{ij} \bar{X} \bar{\epsilon}^i \psi_\mu^j + \text{h.c}$$

- Fields should satisfy the constraint
  $$X D^2 \bar{X} + \frac{1}{2} \bar{\Omega}^k \not{D} \Omega_k + \frac{1}{4} \mathcal{F} \cdot \mathcal{F}^+ + \frac{1}{4} \mathcal{G} \cdot \mathcal{G}^+ - \text{h.c} = 0 \qquad (1)$$
  We can solve this constraint for a three-form gauge field where the constraint appears as the Bianchi identity.
- i.e. define
  $$X \bar{X} D_a \log \frac{X}{\bar{X}} - \frac{1}{2} \bar{\Omega}^k \gamma_a \Omega_k = \frac{1}{3!} \varepsilon_{abcd} H^{bcd} \ .$$

- The constraint now becomes a Bianchi identity and reads,

$$D_{[a}H_{bcd]} = \frac{3}{8}F_{[ab}F_{cd]} + \frac{3}{8}G_{[ab}G_{cd]}$$

- We get

$$\begin{aligned}
\delta e_\mu{}^a &= \bar{\epsilon}^i\gamma^a\psi_{\mu i} + \text{h.c.} - \Lambda_D e_\mu{}^a + \Lambda_M^{ab}e_{\mu b} \\
\delta\psi_\mu{}^i &= 2\mathcal{D}_\mu\epsilon^i - \frac{1}{16}\varepsilon^{ij}\bar{X}^{-1}\gamma\cdot\left(\mathcal{F}^- + i\mathcal{G}^-\right)\gamma_\mu\epsilon_j - \gamma_\mu\eta^i - \frac{1}{2}\Lambda_D\psi_\mu{}^i \\
&\quad - \frac{i}{2}\Lambda_A\psi_\mu{}^i + \Lambda^i{}_j\psi_\mu{}^j + \frac{1}{4}\Lambda_M^{ab}\gamma_{ab}\psi_\mu{}^i \\
\delta b_\mu &= \frac{1}{2}\bar{\epsilon}^i\phi_{\mu i} - \frac{1}{4}X^{-1}\bar{\epsilon}^i\gamma_\mu\,\not{D}\Omega_i - \frac{1}{2}\bar{\eta}^i\psi_{\mu i} + \text{h.c.} + \Lambda_K^a e_{\mu a} + \partial_\mu\Lambda_D \\
\delta\mathcal{V}_\mu{}^i{}_j &= 2\bar{\epsilon}_j\phi_\mu^i - \bar{X}^{-1}\bar{\epsilon}_j\gamma_\mu\,\not{D}\Omega^i + 2\bar{\eta}_j\psi_\mu^i - (\text{h.c.; traceless}) - 2\partial_\mu\Lambda^i{}_j \\
&\quad + \Lambda^i{}_k\mathcal{V}_\mu{}^k{}_j - \Lambda^k{}_j\mathcal{V}_\mu{}^i{}_k \\
\delta X &= \bar{\epsilon}^i\Omega_i + \left(\Lambda_D - i\Lambda_A\right)X
\end{aligned}$$

$$
\begin{aligned}
\delta\Omega_i &= 2\,\not{D} X\epsilon_i + \frac{1}{4}\epsilon_{ij}\gamma\cdot\mathcal{F}\epsilon^j - \frac{i}{4}\epsilon_{ij}\gamma\cdot\mathcal{G}^-\epsilon^j + 2X\eta_i \\
&\quad + \left(\frac{3}{2}\Lambda_D - \frac{i}{2}\Lambda_A\right)\Omega_i - \Lambda^j{}_i\Omega_j + \frac{1}{4}\Lambda_M^{ab}\gamma_{ab}\Omega_i \\
\delta W_\mu &= \varepsilon^{ij}\bar{\epsilon}_i\gamma_\mu\Omega_j + 2\varepsilon_{ij}\bar{X}\bar{\epsilon}^i\psi_\mu^j + \text{h.c.} + \partial_\mu\lambda \\
\delta\tilde{W}_\mu &= i\varepsilon^{ij}\bar{\epsilon}_i\gamma_\mu\Omega_j - 2i\varepsilon_{ij}\bar{X}\bar{\epsilon}^i\psi_\mu^j + \text{h.c.} + \partial_\mu\tilde{\lambda} \\
\delta B_{\mu\nu} &= \tfrac{1}{2}W_{[\mu}\delta_Q W_{\nu]} + \tfrac{1}{2}\tilde{W}_{[\mu}\delta_Q\tilde{W}_{\nu]} + \bar{X}\bar{\epsilon}^i\gamma_{\mu\nu}\Omega_i + X\bar{\epsilon}_i\gamma_{\mu\nu}\Omega^i \\
&\quad + 2\,X\bar{X}\bar{\epsilon}^i\gamma_{[\mu}\psi_{\nu]\,i} + 2\,X\bar{X}\bar{\epsilon}_i\gamma_{[\mu}\psi_{\nu]}^i + 2\partial_{[\mu}\Lambda_{\nu]} - \frac{\lambda}{4}F_{\mu\nu} - \frac{\tilde{\lambda}}{4}G_{\mu\nu}
\end{aligned}
$$

Table of Contents

Conclusions

- We have obtained a 24+24 matter multiplet in $\mathcal{N} = 2$ conformal supergravity in four dimensions by following the current multiplet procedure for the rigid on-shell tensor multiplet.
- This is the generalization of Howe, Stelle, Townsend's flat space multiplet to include coupling to conformal supergravity.
- We can impose 16+16 constraints on the real scalar multiplet to obtain a 8+8 restricted multiplet.
- This restricted multiplet is equivalent to the tensor multiplet.
- Thus the real scalar multiplet is similar to the chiral multiplet, which allowed for a formulation of superconformal tensor calculus.
- We have obtained the 24+24 dilaton Weyl multiplet in four dimensional $\mathcal{N} = 2$ conformal supegravity.

Future directions

- Compute the action for the 24+24 real scalar multiplet.
- This would allow us to write new superconformal invariant action for the tensor multiplet.
- This could potentially lead to new higher derivative invariants in Poincare supergravity.
- Can vector multiplet and Weyl multiplet be embedded in the real scalar multiplet?
- Off-shell dimensional reduction of the dilaton Weyl multiplet from five to four dimensions
- This will allow to write all curvature squared invariants in four dimensions in terms of the dilaton Weyl multiplet.