New life in quadratic theories of gravity

Raquel Santos-Garcia

KAVLI IPMU

18th of July of 2018

Work in collaboration with
Enrique Alvarez, Jesus Anero and Sergio Gonzalez-Martin

arXiv:1703.07993, arXiv:1802.05922

Instituto de Física Teórica UAM-CSIC

UNIVERSIDAD AUTONOMA DE MADRID

elusives

invisiblesPlus

Motivation

General Relativity successful low energy theory

$$S_{EH} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\ R \qquad\qquad \kappa = \sqrt{8\pi G}$$

Motivation

General Relativity successful low energy theory

$$S_{EH} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\ R \qquad\qquad \kappa = \sqrt{8\pi G}$$

When we try to make a quantum formulation of the theory several problems appear

Motivation

In usual QFT, causality is defined as

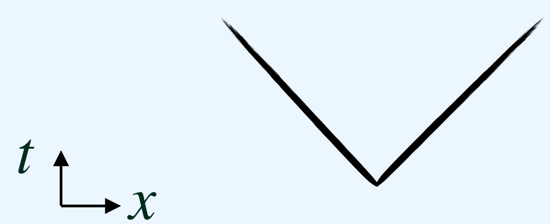
$$[\phi(x), \phi(y)] = 0 \ , \qquad (x-y)^2 < 0$$

Spacelike separations

Motivation

In usual QFT, causality is defined as

$$[\phi(x), \phi(y)] = 0 \ , \qquad (x-y)^2 < 0$$



Light cone around flat spacetime

Motivation

In usual QFT, causality is defined as

$$[\phi(x), \phi(y)] = 0 \ , \qquad (x-y)^2 < 0$$

Quantum fluctuations

Light cone around flat spacetime

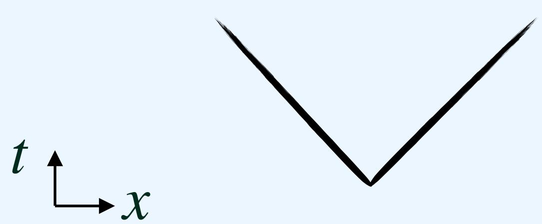
Causality?

Definition of time?

Motivation

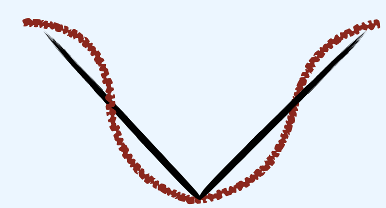
In usual QFT, causality is ensured if

$$[\phi(x), \phi(y)] = 0 \ , \qquad (x-y)^2 < 0$$



Light cone around flat spacetime

Quantum fluctuations



Causality?

Definition of time?

Does a fundamental QFT exist for the gravitational interaction?

Möller (1952)   Rosenfeld (1957)

Motivation

It could be that at high energies the metric is not the fundamental quantity in a quantum theory of gravity

Motivation

It could be that at high energies the metric is not the fundamental quantity in a quantum theory of gravity

$\longrightarrow$ At low energies, effective description in terms of spacetime variables

Motivation

It could be that at high energies the metric is not the fundamental quantity in a quantum theory of gravity

$\longrightarrow$ At low energies, effective description in terms of spacetime variables

Even if we do not know anything about the non-perturbative part, we can do a perturbative analysis

Motivation

It could be that at high energies the metric is not the fundamental quantity in a quantum theory of gravity

→ At low energies, effective description in terms of spacetime variables

Even if we do not know anything about the non-perturbative part, we can do a perturbative analysis

→ Background field method

B. De Witt (1967)

Metric perturbations around fixed background

’t Hooft and Veltman (1974)

Motivation

Around a fixed background spacetime we can use the usual QFT formalism and study the theory in the perturbation limit

Motivation

Perturbatively, GR $\longrightarrow$ Non renormalizable

Motivation

Perturbatively, GR $\longrightarrow$ Non renormalizable

Gravitational coupling is dimensionful $\quad [G] = -2$

Motivation

Perturbatively, GR $\longrightarrow$ Non renormalizable

Gravitational coupling is dimensionful $\quad [G] = -2$

$$\underbrace{R + R^2}_{\text{One-loop}} + \underbrace{R^3}_{\text{Two-loop}} + \ldots$$

Motivation

Perturbatively, GR $\longrightarrow$ Non renormalizable

Gravitational coupling is dimensionful $[G] = -2$

$$\underbrace{R + R^2}_{\text{One-loop}} + \underbrace{R^3}_{\text{Two-loop}} + \dots$$

Pure gravity non renormalizable at two loops   Goroff and Sagnotti (1985)

Motivation

What about quadratic theories of gravity? $S_Q \sim \int d^4x \sqrt{-g}\; R^2$

Motivation

What about quadratic theories of gravity? $S_Q \sim \int d^4x \sqrt{-g}\ R^2$

PROS

Dimensionless couplings $\longrightarrow$ Renormalizable

K. Stelle (1977)

Motivation

What about quadratic theories of gravity? $S_Q \sim \int d^4x\sqrt{-g}\; R^2$

PROS

Dimensionless couplings $\longrightarrow$ Renormalizable

K. Stelle (1977)

Closest analogy to a YM theory of gravity

$$R_{\mu\nu\rho\sigma} = \partial_\rho \Gamma^\mu{}_{\nu\sigma} - \partial_\sigma \Gamma^\mu{}_{\nu\rho} + \Gamma^\mu{}_{\lambda\rho}\Gamma^\lambda{}_{\nu\sigma} - \Gamma^\mu{}_{\lambda\sigma}\Gamma^\lambda{}_{\nu\rho}$$ Field strength

Motivation

What about quadratic theories of gravity? $S_Q \sim \int d^4x \sqrt{-g}\; R^2$

CONS

Propagators falling as $\sim \dfrac{1}{p^4}$

Motivation

What about quadratic theories of gravity? $S_Q \sim \int d^4x \sqrt{-g}\; R^2$

CONS

Propagators falling as $\sim \dfrac{1}{p^4}$

Källen-Lehmann spectral representation

$$\Delta(p) = \int_0^\infty d\mu^2 \rho(\mu^2) \frac{1}{p^2 - \mu^2 + i\epsilon} \qquad \rho(\mu^2) \geq 0$$

Motivation

What about quadratic theories of gravity? $S_Q \sim \int d^4x\sqrt{-g}\; R^2$

CONS

Propagators falling as $\sim \dfrac{1}{p^4}$

Källen-Lehmann spectral representation

$$\frac{1}{p^4 - m^4} = \frac{1}{p^2 - m^2} - \frac{1}{p^2 + m^2}$$

Ghost and/or tachyons

Motivation

What about quadratic theories of gravity? $S_Q \sim \int d^4x \sqrt{-g}\ R^2$

CONS

Propagators falling as $\sim \dfrac{1}{p^4}$

Källen-Lehmann spectral representation

$$\frac{1}{p^4 - m^4} = \frac{1}{p^2 - m^2} - \frac{1}{p^2 + m^2}$$

Unitarity is lost

Motivation

What about quadratic theories treated in first order formalism?

Motivation

What about quadratic theories treated in first order formalism?

First order $\longrightarrow$ metric and connection independents

Motivation

What about quadratic theories treated in first order formalism?

→ Renormalizability

Motivation

What about quadratic theories treated in first order formalism?

- Renormalizability
- Still room for unitarity

Motivation

What about quadratic theories treated in first order formalism?

- Renormalizability
- Still room for unitarity

$$R^{\mu}_{\nu\rho\sigma} \sim \nabla_{\nu}\Gamma^{\mu}_{\rho\sigma}$$

A single derivative in the curvature

Motivation

What about quadratic theories treated in first order formalism?

- Renormalizability
- Still room for unitarity

Possible UV completion of GR?

Motivation

What about quadratic theories treated in first order formalism?

- Renormalizability
- Still room for unitarity

Possible UV completion of GR?

A. Salvio and A. Strumia (2014)

M. B. Einhorn and T. Jones (2017)

J. F. Donoghue and G. Menezes (2018)

Motivation

Lee-Wick type of mechanisms

T. D. Lee and G. C. Wick (1969)

$\longrightarrow$ Able to fix unitarity diagram by diagram

Nevertheless, this mechanism cannot be implemented into the path integral formalism

D. G. Boulware and D. Gross (1969)

Outline

- First order formalism vs. Second order formalism
- First order quadratic gravity
- Physical content of the connection
- The coupling to matter in first order formalism
- Summary and Outlook

Outline

First order formalism vs. Second order formalism

- First order quadratic gravity
- Physical content of the connection
- The coupling to matter in first order formalism
- Summary and Outlook

First order vs Second order

Let us take the simple case of the EH action

First order vs Second order

Let us take the simple case of the EH action

$$g_{\mu\nu} \qquad\qquad \Gamma^{\mu}_{\nu\rho}$$

First order vs Second order

Let us take the simple case of the EH action

SO $\qquad g_{\mu\nu} \qquad \Gamma^{\mu}_{\nu\rho} \qquad$ Fixed relation

$$\Gamma^{\mu}_{\nu\rho} = \frac{1}{2} g^{\mu\lambda} \left(\partial_\nu g_{\lambda\rho} + \partial_\rho g_{\lambda\nu} - \partial_\lambda g_{\nu\rho} \right)$$

First order vs Second order

Let us take the simple case of the EH action

SO $\qquad g_{\mu\nu} \qquad \Gamma^{\mu}_{\nu\rho}$ Fixed relation

$$\Gamma^{\mu}_{\nu\rho} = \frac{1}{2} g^{\mu\lambda} \left(\partial_\nu g_{\lambda\rho} + \partial_\rho g_{\lambda\nu} - \partial_\lambda g_{\nu\rho} \right)$$

$$\delta S_{SO} = \int d^4x \sqrt{-g} \left(-R^{\mu\nu} + \frac{1}{2} R g^{\mu\nu} \right) \delta g_{\mu\nu}$$

Einstein's field equation

First order vs Second order

Let us take the simple case of the EH action

FO $\quad g_{\mu\nu} \quad \Gamma^{\mu}_{\nu\rho}$ Independent fields

First order vs Second order

Let us take the simple case of the EH action

FO $\qquad g_{\mu\nu} \qquad \Gamma^{\mu}_{\nu\rho}$ $\qquad$ Independent fields

$$\Gamma^{\mu}_{\nu\rho} = \underbrace{\left\{ \begin{matrix} \mu \\ \nu\ \rho \end{matrix} \right\}}_{\text{Levi-Civita connection}} + K^{\mu}_{\nu\rho} + L^{\mu}_{\nu\rho}$$

First order vs Second order

Let us take the simple case of the EH action

FO $\qquad g_{\mu\nu} \qquad \Gamma^{\mu}_{\nu\rho}$ $\qquad$ Independent fields

$$\Gamma^{\mu}_{\nu\rho} = \left\{ \begin{matrix} \mu \\ \nu \ \ \rho \end{matrix} \right\} + K^{\mu}_{\nu\rho} + L^{\mu}_{\nu\rho}$$

Levi-Civita connection

Contorsion tensor

$$\frac{1}{2} g^{\rho\sigma} \left(T_{\mu\sigma\nu} + T_{\nu\sigma\mu} - T_{\mu\nu\sigma} \right)$$

First order vs Second order

Let us take the simple case of the EH action

FO $\quad g_{\mu\nu} \quad \Gamma^{\mu}_{\nu\rho}$ Independent fields

$$\Gamma^{\mu}_{\nu\rho} = \left\{ \begin{matrix} \mu \\ \nu\ \rho \end{matrix} \right\} + K^{\mu}_{\nu\rho} + L^{\mu}_{\nu\rho}$$

Levi-Civita connection

Contorsion tensor

$$\frac{1}{2} g^{\rho\sigma} \left(T_{\mu\sigma\nu} + T_{\nu\sigma\mu} - T_{\mu\nu\sigma} \right)$$

Related to non-metricity

$$Q_{\mu\nu\rho} = \nabla_{\mu} g_{\nu\rho}$$

First order vs Second order

Let us take the simple case of the EH action

FO $\qquad g_{\mu\nu} \qquad \Gamma^{\mu}_{\nu\rho} \qquad$ Independent fields

$$\Gamma^{\mu}_{\nu\rho} = \left\{ {\mu \atop \nu\ \rho} \right\} + \cancel{K^{\mu}_{\nu\rho}} + L^{\mu}_{\nu\rho}$$

Levi-Civita connection

Contorsion tensor

$$\frac{1}{2} g^{\rho\sigma} \left(T_{\mu\sigma\nu} + T_{\nu\sigma\mu} - T_{\mu\nu\sigma} \right)$$

Related to non-metricity

$$Q_{\mu\nu\rho} = \nabla_{\mu} g_{\nu\rho}$$

First order vs Second order

Let us take the simple case of the EH action

FO $\qquad g_{\mu\nu} \qquad \Gamma^{\mu}_{\nu\rho}$ $\qquad$ Independent fields

$$\delta S_{FO} = \int d^4x \sqrt{-g} \left\{ \left(-R^{\mu\nu} + \frac{1}{2} R g^{\mu\nu} \right) \delta g_{\mu\nu} + g^{\mu\nu} \delta^{\alpha}_{\mu} \left(\delta^{\beta}_{\nu} \nabla_{\lambda} - \delta^{\beta}_{\lambda} \nabla_{\nu} \right) \delta \Gamma^{\lambda}_{\alpha\beta} \right\}$$

First order vs Second order

Let us take the simple case of the EH action

FO $\qquad g_{\mu\nu} \qquad \Gamma^{\mu}_{\nu\rho}$ Independent fields

$$\delta S_{FO} = \int d^4x\sqrt{-g}\left\{\underbrace{\left(-R^{\mu\nu}+\frac{1}{2}Rg^{\mu\nu}\right)}_{}\delta g_{\mu\nu} + g^{\mu\nu}\delta^{\alpha}_{\mu}\left(\delta^{\beta}_{\nu}\nabla_{\lambda} - \delta^{\beta}_{\lambda}\nabla_{\nu}\right)\delta\Gamma^{\lambda}_{\alpha\beta}\right\}$$

Einstein's field equation

First order vs Second order

Let us take the simple case of the EH action

FO $\quad g_{\mu\nu} \quad \Gamma^{\mu}_{\nu\rho}$ Independent fields

$$\delta S_{FO} = \int d^4x\sqrt{-g}\left\{\underbrace{\left(-R^{\mu\nu} + \frac{1}{2}Rg^{\mu\nu}\right)}_{\text{Einstein's field equation}}\delta g_{\mu\nu} + \underbrace{g^{\mu\nu}\delta^{\alpha}_{\mu}\left(\delta^{\beta}_{\nu}\nabla_{\lambda} - \delta^{\beta}_{\lambda}\nabla_{\nu}\right)}_{\nabla_{\lambda}g^{\alpha\beta}=0}\delta\Gamma^{\lambda}_{\alpha\beta}\right\}$$

Einstein's field equation

$$\nabla_{\lambda}g^{\alpha\beta} = 0$$

First order vs Second order

Let us take the simple case of the EH action

FO $\qquad g_{\mu\nu} \qquad \Gamma^{\mu}_{\nu\rho}$ $\qquad$ Independent fields

$$\delta S_{FO} = \int d^4x\sqrt{-g}\left\{\left(-R^{\mu\nu} + \frac{1}{2}Rg^{\mu\nu}\right)\delta g_{\mu\nu} + g^{\mu\nu}\delta^{\alpha}_{\mu}\left(\delta^{\beta}_{\nu}\nabla_{\lambda} - \delta^{\beta}_{\lambda}\nabla_{\nu}\right)\delta\Gamma^{\lambda}_{\alpha\beta}\right\}$$

Einstein's field equation $\qquad$ $\nabla_{\lambda}g^{\alpha\beta} = 0$

First order vs Second order

Let us take the simple case of the EH action

FO $\quad g_{\mu\nu} \quad \Gamma^{\mu}_{\nu\rho}$ Independent fields

$$\Gamma^{\mu}_{\nu\rho} = \left\{ {\mu \atop \nu\ \rho} \right\} + \cancel{K^{\mu}_{\nu\rho}} + \cancel{L^{\mu}_{\nu\rho}}$$

Levi-Civita connection

Contorsion tensor

$$\frac{1}{2} g^{\rho\sigma} \left(T_{\mu\sigma\nu} + T_{\nu\sigma\mu} - T_{\mu\nu\sigma} \right)$$

Related to non-metricity

$$Q_{\mu\nu\rho} = \nabla_{\mu} g_{\nu\rho}$$

First order vs Second order

Let us take the simple case of the EH action

$$g_{\mu\nu} \qquad \Gamma^{\mu}_{\nu\rho}$$

For the EH action SO and FO classically equivalent

Palatini

First order vs Second order

Let us take the simple case of the EH action

$$g_{\mu\nu} \qquad\qquad \Gamma^{\mu}_{\nu\rho}$$

For the EH action SO and FO classically equivalent

Palatini

The equivalence also holds at one loop order J. Anero and RS (2017)

Outline

- First order formalism vs. Second order formalism

First order quadratic gravity

- Physical content of the connection
- The coupling to matter in first order formalism
- Summary and Outlook

First order quadratic gravity

Let us focus on the features of FO quadratic theories

First order quadratic gravity

Let us focus on the features of FO quadratic theories

The Riemann tensor does not enjoy the usual symmetries

LC $\qquad R_{[\mu\nu]\rho\sigma} \qquad R_{\mu\nu[\rho\sigma]} \qquad R_{\mu\nu\rho\sigma} = R_{\rho\sigma\mu\nu}$

First order quadratic gravity

Let us focus on the features of FO quadratic theories

The **Riemann tensor** does not enjoy the **usual symmetries**

Two different traces of the Riemann tensor

$$R^{+}[\Gamma]_{\nu\sigma} = g^{\mu\rho} R[\Gamma]_{\mu\nu\rho\sigma}$$
$$R^{-}[\Gamma]_{\mu\sigma} = g^{\nu\rho} R[\Gamma]_{\mu\nu\rho\sigma}$$
$$\mathscr{R}[\Gamma]_{\rho\sigma} = g^{\mu\nu} R[\Gamma]_{\mu\nu\rho\sigma}$$

$$\mathscr{R}_{\mu\nu} = R^{+}_{\mu\nu} - R^{-}_{\nu\mu}$$
$$R^{+} = g^{\mu\nu} R^{+}_{\mu\nu} = - g^{\mu\nu} R^{-}_{\mu\nu} = - R^{-}$$

First order quadratic gravity

Let us focus on the features of FO quadratic theories

The Riemann tensor does not enjoy the usual symmetries

Two different traces of the Riemann tensor

$$R^+[\Gamma]_{\nu\sigma} = g^{\mu\rho} R[\Gamma]_{\mu\nu\rho\sigma}$$
$$R^-[\Gamma]_{\mu\sigma} = g^{\nu\rho} R[\Gamma]_{\mu\nu\rho\sigma}$$
$$\mathscr{R}[\Gamma]_{\rho\sigma} = g^{\mu\nu} R[\Gamma]_{\mu\nu\rho\sigma}$$

$$\mathscr{R}_{\mu\nu} = R^+_{\mu\nu} - R^-_{\nu\mu}$$
$$R^+ = g^{\mu\nu} R^+_{\mu\nu} = - g^{\mu\nu} R^-_{\mu\nu} = - R^-$$

$$R_{\mu\nu\rho\sigma}\,, \quad R^+_{\mu\nu}\,, \quad R^-_{\mu\nu}\,, \quad R$$

First order quadratic gravity

The most general first order quadratic action reads

$$S_{FOQ} = \int d^4x \sqrt{-g} \sum_{I=1}^{I=12} g_I O^I$$

$$O_I = R^{\mu}_{\nu\rho\sigma} (D_I)^{\nu\rho\sigma\nu'\rho'\sigma'}_{\mu\mu'} R^{\mu'}_{\nu'\rho'\sigma'}$$

D_I : Function of metrics and deltas

First order quadratic gravity

The most general first order quadratic action reads

$$S_{FOQ} = \int d^4x \sqrt{-g} \sum_{I=1}^{I=12} g_I O^I$$

$$O_I = R^{\mu}_{\nu\rho\sigma} (D_I)^{\nu\rho\sigma\nu'\rho'\sigma'}_{\mu\mu'} R^{\mu'}_{\nu'\rho'\sigma'}$$

D_I : Function of metrics and deltas

The theory is Weyl invariant

$$g_{\mu\nu} \longrightarrow \Omega^2(x)\; g_{\mu\nu} \qquad O_I \longrightarrow \Omega^{-4} O_I$$

$$\Gamma^{\lambda}_{\mu\nu} \longrightarrow \Gamma^{\lambda}_{\mu\nu} \qquad \sqrt{-g} \longrightarrow \Omega^4 \sqrt{-g}$$

First order quadratic gravity

The theory is up to now in the conformal phase so the symmetry has to be spontaneously broken

First order quadratic gravity

The theory is up to now in the conformal phase so the symmetry has to be spontaneously broken

$$L_s = \sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right)$$

First order quadratic gravity

The theory is up to now in the conformal phase so the symmetry has to be spontaneously broken

$$L_s = \sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right)$$

Renormalizing this sector we get $\Delta L_s = C_\epsilon R\phi^2$

First order quadratic gravity

The theory is up to now in the conformal phase so the symmetry has to be spontaneously broken

$$L_s = \sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right)$$

Renormalizing this sector we get $\Delta L_s = C_\epsilon R\phi^2$

$$\Big\downarrow \; <\phi> = \mathrm{v}$$

The spontaneous breaking of the symmetry generates an EH term $L_{EH} = M^2\sqrt{-g}\; R$

First order quadratic gravity

The theory is up to now in the conformal phase so the symmetry has to be spontaneously broken

$$L_s = \sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right)$$

Renormalizing this sector we get $\Delta L_s = C_\epsilon R\phi^2$

$$\Big\downarrow\; <\phi> = \mathrm{v}$$

The spontaneous breaking of the symmetry generates an EH term

$$L_{EH} = M^2\sqrt{-g}\; R$$

Dominates in the IR

I. Shapiro and S. D. Odintsov (1986)
S. L. Adler (1988)
P. G. Ferreira , T. Hill and G. Ross (2018)

First order quadratic gravity

More general connections are allowed

First order quadratic gravity

More general connections are allowed

$$\Delta H_{\mu\nu} = H^{SO}_{\mu\nu} - H^{FO}_{\mu\nu} = -\frac{1}{2}\nabla_\lambda K^\lambda_{(\mu\nu)} + \frac{1}{4} g_{\lambda\mu}\nabla^\rho K^\lambda_{(\rho\nu)} + \frac{1}{4} g_{\lambda\nu}\nabla^\rho K^\lambda_{(\rho\mu)}$$

$$H_{\mu\nu} = \frac{1}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} \qquad K^\lambda_{\mu\nu} = \frac{1}{\sqrt{-g}}\frac{\delta S}{\delta \Gamma^{\mu\nu}_\lambda}$$

M. Borunda, B. Jansen and M. Bastero-Gil (2008)

First order quadratic gravity

More general connections are allowed

$$\Delta H_{\mu\nu} = H^{SO}_{\mu\nu} - H^{FO}_{\mu\nu} = -\frac{1}{2}\nabla_\lambda K^\lambda_{(\mu\nu)} + \frac{1}{4} g_{\lambda\mu} \nabla^\rho K^\lambda_{(\rho\nu)} + \frac{1}{4} g_{\lambda\nu} \nabla^\rho K^\lambda_{(\rho\mu)}$$

$$H_{\mu\nu} = \frac{1}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} \qquad K^\lambda_{\mu\nu} = \cancel{\frac{1}{\sqrt{-g}}\frac{\delta S}{\delta \Gamma^{\mu\nu}_\lambda}} \quad \text{LC}$$

M. Borunda, B. Jansen and M. Bastero-Gil (2008)

First order quadratic gravity

More general connections are allowed

$$\Delta H_{\mu\nu} = H^{SO}_{\mu\nu} - H^{FO}_{\mu\nu} = -\frac{1}{2}\nabla_\lambda K^\lambda_{(\mu\nu)} + \frac{1}{4} g_{\lambda\mu}\nabla^\rho K^\lambda_{(\rho\nu)} + \frac{1}{4} g_{\lambda\nu}\nabla^\rho K^\lambda_{(\rho\mu)}$$

$$H_{\mu\nu} = \frac{1}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} \qquad K^\lambda_{\mu\nu} = \frac{1}{\sqrt{-g}}\frac{\delta S}{\delta \Gamma^{\mu\nu}_\lambda}$$

M. Borunda, B. Jansen and M. Bastero-Gil (2008)

Bigger solution space, where does gravitation live?

Outline

- First order formalism vs. Second order formalism
- First order quadratic gravity

Physical content of the connection

- The coupling to matter in first order formalism
- Summary and Outlook

Physical content of $A_{\mu\nu\lambda}$

The connection being an independent field, can introduce new degrees of freedom.

Physical content of $A_{\mu\nu\lambda}$

The connection being an independent field, can introduce new degrees of freedom.

Our aim is to find a complete basis of spin projectors so that we can decompose the three index tensor $A_{\mu\nu\lambda}$ in its propagating spin pieces.

Physical content of $A_{\mu\nu\lambda}$

The connection being an independent field, can introduce new degrees of freedom.

Our aim is to find a complete basis of spin projectors so that we can decompose the three index tensor $A_{\mu\nu\lambda}$ in its propagating spin pieces.

As we will see, a proliferation of spins occurs. It is crucial to check that we do not have ghosts encoded in those propagating spin components.

Physical content of $A_{\mu\nu\lambda}$

We take the EH action and expand the metric around flat space

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

Physical content of $A_{\mu\nu\lambda}$

We take the EH action and expand the metric around flat space

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

To quadratic order in the perturbation we get

$$S = \frac{1}{2}\int d^4x \; h^{\mu\nu} \; K^{EH}_{\mu\nu\rho\sigma} \; h^{\rho\sigma}$$

Physical content of $A_{\mu\nu\lambda}$

We take the EH action and expand the metric around flat space

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

To quadratic order in the perturbation we get

$$S = \frac{1}{2}\int d^4x \; h^{\mu\nu} K^{EH}_{\mu\nu\rho\sigma} h^{\rho\sigma}$$

Interaction between two index tensors

Physical content of $A_{\mu\nu\lambda}$

We take the EH action and expand the metric around flat space

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

To quadratic order in the perturbation we get

$$S = \frac{1}{2}\int d^4x \; h^{\mu\nu} \; K^{EH}_{\mu\nu\rho\sigma} \; h^{\rho\sigma}$$

We want to decompose a two index symmetric tensor in its spin components

Spin projectors $\longrightarrow$ Four index operators that project onto a certain spin

Physical content of $A_{\mu\nu\lambda}$

To project into the different components we have

$$k^\mu = \delta^\mu_0$$

In the rest frame

$$\theta_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$$

$$\omega_{\mu\nu} = \frac{k_\mu k_\nu}{k^2}$$

Physical content of $A_{\mu\nu\lambda}$

To project into the different components we have

$$k^\mu = \delta^\mu_0$$

$$\theta_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \longrightarrow$$ Projects onto spatial indices

$$\omega_{\mu\nu} = \frac{k_\mu k_\nu}{k^2} \longrightarrow$$ Projects onto time indices

Physical content of $A_{\mu\nu\lambda}$

To project into the different components we have

$$k^\mu = \delta^\mu_0$$

$$\theta_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \longrightarrow \text{Projects onto spatial indices}$$

$$\omega_{\mu\nu} = \frac{k_\mu k_\nu}{k^2} \longrightarrow \text{Projects onto time indices}$$

Barnes-Rivers Projectors

Barnes (1963) Rivers (1964)
P. Van Niewuheinzen (1973)

$$s = 2: \quad h^T_{ij} \equiv h_{ij} - \frac{1}{3} h \delta_{ij}$$

$$s = 1: \quad h_{0i}$$

$$s = 0: \quad h_{00}$$

$$s = 0: \quad h \equiv \delta^{ij} h_{ij}$$

Different spin representations SO(3)

Physical content of $A_{\mu\nu\lambda}$

To project into the different components we have

$$k^\mu = \delta^\mu_0$$

$$\theta_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \longrightarrow \text{Projects onto spatial indices}$$

$$\omega_{\mu\nu} = \frac{k_\mu k_\nu}{k^2} \longrightarrow \text{Projects onto time indices}$$

Barnes-Rivers Projectors

$$s = 2: \quad h^T_{ij} \equiv h_{ij} - \frac{1}{3} h\delta_{ij} \quad \rightarrow \quad \left(P_2\right)^{\rho\sigma}_{\mu\nu} \equiv \frac{1}{2}\left(\theta^\rho_\mu \theta^\sigma_\nu + \theta^\sigma_\mu \theta^\rho_\nu\right) - \frac{1}{3}\theta_{\mu\nu}\theta^{\rho\sigma}$$

$$s = 1: \quad h_{0i} \quad \rightarrow \quad \left(P_1\right)^{\rho\sigma}_{\mu\nu} \equiv \frac{1}{2}\left(\theta^\rho_\mu \omega^\sigma_\nu + \theta^\sigma_\mu \omega^\rho_\nu + \theta^\rho_\nu \omega^\sigma_\mu + \theta^\sigma_\nu \omega^\rho_\mu\right)$$

$$s = 0: \quad h_{00} \quad \rightarrow \quad \left(P^w_0\right)^{\rho\sigma}_{\mu\nu} \equiv \omega_{\mu\nu}\omega^{\rho\sigma}$$

$$s = 0: \quad h \equiv \delta^{ij} h_{ij} \quad \rightarrow \quad \left(P^s_0\right)^{\rho\sigma}_{\mu\nu} \equiv \frac{1}{3}\theta_{\mu\nu}\theta^{\rho\sigma}$$

Physical content of $A_{\mu\nu\lambda}$

These projectors add up to the identity

$$\left(P_2\right)_{\mu\nu}^{\rho\sigma} + \left(P_1\right)_{\mu\nu}^{\rho\sigma} + \left(P_0^w\right)_{\mu\nu}^{\rho\sigma} + \left(P_0^s\right)_{\mu\nu}^{\rho\sigma} = I_{\mu\nu}^{\rho\sigma}$$

Physical content of $A_{\mu\nu\lambda}$

These projectors add up to the identity

$$\left(P_2\right)_{\mu\nu}^{\rho\sigma} + \left(P_1\right)_{\mu\nu}^{\rho\sigma} + \left(P_0^w\right)_{\mu\nu}^{\rho\sigma} + \left(P_0^s\right)_{\mu\nu}^{\rho\sigma} = I_{\mu\nu}^{\rho\sigma}$$

We are interested in forming a basis of four index projectors

$\longrightarrow$ 5 independent monomials with this symmetry

$$M_1 \equiv k_\mu k_\nu k_\rho k_\sigma$$

$$M_2 \equiv k_\mu k_\nu \eta_{\rho\sigma}$$

$$M_3 \equiv k_\mu k_\sigma \eta_{\nu\rho}$$

$$M_4 \equiv \eta_{\mu\nu}\eta_{\rho\sigma}$$

$$M_5 \equiv \eta_{\mu\rho}\eta_{\nu\sigma}$$

Physical content of $A_{\mu\nu\lambda}$

These projectors add up to the identity

$$\left(P_2\right)_{\mu\nu}^{\rho\sigma} + \left(P_1\right)_{\mu\nu}^{\rho\sigma} + \left(P_0^w\right)_{\mu\nu}^{\rho\sigma} + \left(P_0^s\right)_{\mu\nu}^{\rho\sigma} = I_{\mu\nu}^{\rho\sigma}$$

We are interested in forming a basis of four index projectors

$\longrightarrow$ 5 independent monomials with this symmetry

$$M_1 \equiv k_\mu k_\nu k_\rho k_\sigma$$
$$M_2 \equiv k_\mu k_\nu \eta_{\rho\sigma}$$
$$M_3 \equiv k_\mu k_\sigma \eta_{\nu\rho}$$
$$M_4 \equiv \eta_{\mu\nu}\eta_{\rho\sigma}$$
$$M_5 \equiv \eta_{\mu\rho}\eta_{\nu\sigma}$$

One extra operator
in the basis

$$\left(P_0^\times\right)_{\mu\nu}^{\rho\sigma} = \frac{1}{\sqrt{3}}\left(\omega_{\mu\nu}\theta^{\rho\sigma} + \theta_{\mu\nu}\omega^{\rho\sigma}\right)$$

Physical content of $A_{\mu\nu\lambda}$

These projectors add up to the identity

$$\left(P_2\right)_{\mu\nu}^{\rho\sigma} + \left(P_1\right)_{\mu\nu}^{\rho\sigma} + \left(P_0^w\right)_{\mu\nu}^{\rho\sigma} + \left(P_0^s\right)_{\mu\nu}^{\rho\sigma} = I_{\mu\nu}^{\rho\sigma}$$

Strategy

Take $\quad K_{\mu\nu\rho\sigma} = \sum_i c_i \, P_{i_{\mu\nu\rho\sigma}}$

Physical content of $A_{\mu\nu\lambda}$

These projectors add up to the identity

$$\left(P_2\right)_{\mu\nu}^{\rho\sigma} + \left(P_1\right)_{\mu\nu}^{\rho\sigma} + \left(P_0^w\right)_{\mu\nu}^{\rho\sigma} + \left(P_0^s\right)_{\mu\nu}^{\rho\sigma} = I_{\mu\nu}^{\rho\sigma}$$

Strategy

Take $$K_{\mu\nu\rho\sigma} = \sum_i c_i \, P_{i_{\mu\nu\rho\sigma}}$$

Divide the quadratic piece in the different spin components

$$h^{\mu\nu} \left(\sum_i c_i \, P_{i_{\mu\nu\rho\sigma}} \right) h^{\rho\sigma} = \sum_i \, h_i^{\mu\nu} \, \Box \, h_{\mu\nu}^i$$

Physical content of $A_{\mu\nu\lambda}$

So now we can decompose any four index operator into the spin projectors

$$S_{EH+gf} = \frac{1}{2}\int d^4x \; -\frac{1}{4}h^{\mu\nu}\left(P_2 + P_1 - \frac{1}{2}P_0^s + \frac{1}{2}P_0^w - \frac{\sqrt{3}}{2}P^\times\right)_{\mu\nu\rho\sigma} \Box \, h^{\rho\sigma}$$

Physical content of $A_{\mu\nu\lambda}$

So now we can decompose any four index operator into the spin projectors

$$S_{EH+gf} = \frac{1}{2}\int d^4x \ -\frac{1}{4}h^{\mu\nu}\left(P_2 + P_1 - \frac{1}{2}P_0^s + \frac{1}{2}P_0^w - \frac{\sqrt{3}}{2}P^\times\right)_{\mu\nu\rho\sigma} \Box \ h^{\rho\sigma}$$

Inverting the operator we get the propagator and the free energy

D. Dicus and S. Willenbrock (1969)

$$W\left[T_{(1)}, T_{(2)}\right] = \int d^4x T^{\mu\nu}_{(1)}\Delta_{\mu\nu\rho\sigma}T^{\rho\sigma}_{(2)} = \int d^4x \left(T^{\mu\nu}_{(1)} \ (P_2 - \frac{1}{2} \ P_0^s)_{\mu\nu\rho\sigma} \ T^{\rho\sigma}_{(2)}\right)$$

Interaction between external sources

Physical content of $A_{\mu\nu\lambda}$

So now we can decompose any four index operator into the spin projectors

$$S_{EH+gf} = \frac{1}{2}\int d^4x \ -\frac{1}{4}h^{\mu\nu}\left(P_2 + P_1 - \frac{1}{2}P_0^s + \frac{1}{2}P_0^w - \frac{\sqrt{3}}{2}P^\times\right)_{\mu\nu\rho\sigma} \Box \, h^{\rho\sigma}$$

Inverting the operator we get the propagator and the free energy

D. Dicus and S. Willenbrock (2004)

$$W\left[T_{(1)}, T_{(2)}\right] = \int d^4x T^{\mu\nu}_{(1)} \Delta_{\mu\nu\rho\sigma} T^{\rho\sigma}_{(2)} = \int d^4x \left(T^{\mu\nu}_{(1)} \ (P_2 - \frac{1}{2} \ P^s_0)_{\mu\nu\rho\sigma} \ T^{\rho\sigma}_{(2)} \right)$$

Positive definite

Physical content of $A_{\mu\nu\lambda}$

So now we can decompose any four index operator into the spin projectors

$$S_{EH+gf} = \frac{1}{2}\int d^4x \; -\frac{1}{4}h^{\mu\nu}\left(P_2 + P_1 - \frac{1}{2}P_0^s + \frac{1}{2}P_0^w - \frac{\sqrt{3}}{2}P^{\times}\right)_{\mu\nu\rho\sigma} \Box \; h^{\rho\sigma}$$

Inverting the operator we get the propagator and the free energy

$$W\left[T_{(1)}, T_{(2)}\right] = \int d^4x T^{\mu\nu}_{(1)}\Delta_{\mu\nu\rho\sigma}T^{\rho\sigma}_{(2)} = \int d^4x \left(T^{\mu\nu}_{(1)} \; (P_2 - \frac{1}{2}\; P^s_0)_{\mu\nu\rho\sigma} \; T^{\rho\sigma}_{(2)}\right)$$

On-shell asymptotic states $\longrightarrow$ A single spin 2 field propagates $h^{TT}_{\mu\nu}$

Graviton

Physical content of $A_{\mu\nu\lambda}$

So now we can decompose any four index operator into the spin projectors

$$S_{EH+gf} = \frac{1}{2}\int d^4x \ \ -\frac{1}{4}h^{\mu\nu} \left(P_2 + P_1 - \frac{1}{2}P_0^s + \frac{1}{2}P_0^w - \frac{\sqrt{3}}{2}P^{\times} \right)_{\mu\nu\rho\sigma} \Box \, h^{\rho\sigma}$$

Summary

Spin-2 and Spin-0 components mediating the interaction between external sources (off-shell)

Spin-2 massless unique asymptotic state
Graviton

Physical content of $A_{\mu\nu\lambda}$

We are working with symmetric connections

$$A_{\mu\nu\lambda} \in \mathscr{A} \equiv T_x \otimes Sym\left(T_x \otimes T_x\right)$$

Physical content of $A_{\mu(\nu\lambda)}$

We are working with symmetric connections

$$A_{\mu\nu\lambda} \in \mathscr{A} \equiv T_x \otimes Sym\left(T_x \otimes T_x\right)$$

40 independent components

$$\square\square \otimes \square = \begin{array}{l}\square\square\\ \square\end{array} \oplus \square\square\square$$

Hook part

Totally Symmetric part

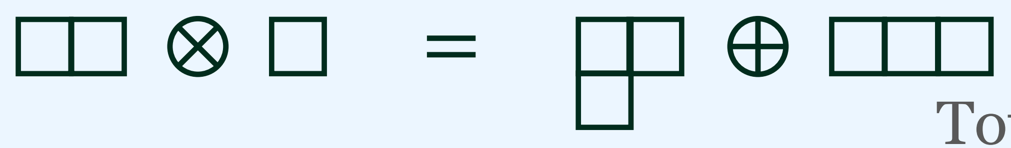
$$\{2,0\} \otimes \{1\} = \{3,0\} \oplus \{2,1\}$$

Physical content of $A_{\mu(\nu\lambda)}$

We are working with symmetric connections

$$A_{\mu\nu\lambda} \in \mathscr{A} \equiv T_x \otimes Sym\left(T_x \otimes T_x\right)$$

40 independent components



Hook part

Totally Symmetric part

$$\underline{20}_H = 2\,(\underline{2}) \oplus 3\;(\underline{1}) \oplus\;(\underline{0}) \qquad \underline{20}_S = (\underline{3}) \oplus (\underline{2}) \oplus 2\;(\underline{1}) \oplus 2\;(\underline{0})$$

$$P_H + P_S = I$$

Physical content of $A_{\mu(\nu\lambda)}$

We are working with symmetric connections

$$A_{\mu\nu\lambda} \in \mathscr{A} \equiv T_x \otimes Sym\left(T_x \otimes T_x\right)$$

$$\yng(2) \otimes \yng(1) = \yng(2,1) \oplus \yng(3)$$

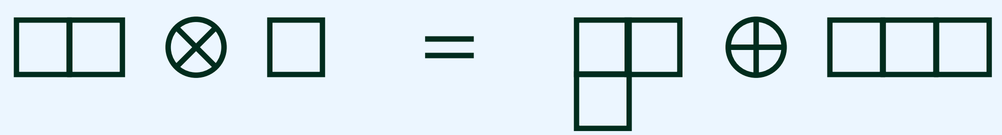
We have 22 independent monomials with this symmetry

$\longrightarrow$ 22 projectors in the basis $\quad (P_i)^{\mu\nu\lambda}_{\alpha\beta\gamma}$

Physical content of $A_{\mu(\nu\lambda)}$

We are working with symmetric connections

$$A_{\mu\nu\lambda} \in \mathscr{A} \equiv T_x \otimes Sym\left(T_x \otimes T_x\right)$$



We have 22 independent monomials with this symmetry

$\longrightarrow$ 22 projectors in the basis $(P_i)^{\mu\nu\lambda}_{\alpha\beta\gamma}$

We need 10 extra spin operators with mixed symmetry

Physical content of $A_{\mu(\nu\lambda)}$

We are working with symmetric connections

$$A_{\mu\nu\lambda} \in \mathscr{A} \equiv T_x \otimes Sym\left(T_x \otimes T_x\right)$$

$$\ydiagram{2} \otimes \ydiagram{1} = \ydiagram{2,1} \oplus \ydiagram{3}$$

We have 22 independent monomials with this symmetry

$\longrightarrow$ 22 projectors in the basis $(P_i)^{\mu\nu\lambda}_{\alpha\beta\gamma}$

We need 10 extra spin operators with mixed symmetry

One spin-3, four spin-2, eleven spin-1 and six spin-0

Physical content of $A_{\mu(\nu\lambda)}$

In FOQG around flat space

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

$$\Gamma^{\lambda}_{\mu\nu} = \bar{\Gamma}^{\lambda}_{\mu\nu} + A^{\lambda}_{\mu\nu}$$

$$A_{\alpha\beta\gamma}\, K^{\alpha\beta\gamma}_{\mu\nu\lambda}\, A^{\mu\nu\lambda} + h^{\mu\nu} M_{\mu\nu\rho\sigma} h^{\rho\sigma} + h^{\mu\nu} N_{\mu\nu}{}^{\rho\sigma}{}_{\lambda} A^{\lambda}_{\rho\sigma}$$

Physical content of $A_{\mu(\nu\lambda)}$

In FOQG around flat space $A_{\alpha\beta\gamma}\ K^{\alpha\beta\gamma}_{\mu\nu\lambda}\ A^{\mu\nu\lambda}$

All the dynamics encoded in the gauge field

Physical content of $A_{\mu(\nu\lambda)}$

In FOQG around flat space $A_{\alpha\beta\gamma} \, K^{\alpha\beta\gamma}_{\mu\nu\lambda} \, A^{\mu\nu\lambda}$

All the dynamics encoded in the gauge field

Taking for instance the simpler action

$$S_{FOQ} \equiv \int d^n x \sqrt{-g} \left(\alpha \; R[\Gamma]^2 + \beta \; R[\Gamma]_{\mu\nu} R[\Gamma]^{\mu\nu} + \gamma \; R[\Gamma]_{\mu\nu\rho\sigma} R[\Gamma]^{\mu\nu\rho\sigma} \right)$$

Physical content of $A_{\mu(\nu\lambda)}$

Taking for instance the simpler action

$$S_{FOQ} \equiv \int d^n x \sqrt{-g} \left(\alpha\, R[\Gamma]^2 + \beta\, R[\Gamma]_{\mu\nu} R[\Gamma]^{\mu\nu} + \gamma\, R[\Gamma]_{\mu\nu\rho\sigma} R[\Gamma]^{\mu\nu\rho\sigma} \right)$$

$\mathcal{P}$ Hook $\quad$ P Symmetric $\quad$ $\mathfrak{P}$ Mixed

$$\begin{aligned}(K_{\text{FOQ}})^{\mu\nu}_{\tau}{}_{\lambda}{}^{\rho\sigma} = \Big(& -2(2\gamma+\beta)\ \mathrm{P}_0^s - (4\gamma+9\alpha+2\beta)\ \mathcal{P}_0^s + (2\gamma-\beta)\ \mathfrak{P}_0^x - \frac{4}{3}(3\gamma+5\beta)\ \mathrm{P}_1^s \\ & - 2\gamma\ \mathcal{P}_1^s - \frac{4}{3}(3\gamma+\beta)\ \mathcal{P}_1^t - (2\gamma+\beta)\ \mathfrak{P}_1^{wx} + 4\beta\ \mathfrak{P}_1^{ss} - 2(2\gamma+\beta)\ (\mathrm{P}_2 + \mathcal{P}_2) \\ & - 4\gamma\ \mathcal{P}_2^s + 2(\beta+\gamma)\ \mathfrak{P}_2^x - 4\gamma \mathrm{P}_3 \Big)^{\mu\nu}_{\tau}{}_{\lambda}{}^{\rho\sigma} \Box\end{aligned}$$

Physical content of $A_{\mu(\nu\lambda)}$

Taking for instance the simpler action

$$S_{FOQ} \equiv \int d^n x \sqrt{-g}\left(\alpha\, R[\Gamma]^2 + \beta\, R[\Gamma]_{\mu\nu} R[\Gamma]^{\mu\nu} + \underline{\gamma}\, R[\Gamma]_{\mu\nu\rho\sigma} R[\Gamma]^{\mu\nu\rho\sigma}\right)$$

$\mathcal{P}$ Hook $\qquad$ P Symmetric $\qquad$ $\mathscr{P}$ Mixed

$$\begin{aligned}(K_{\mathrm{FOQ}})_{\tau\ \lambda}^{\mu\nu\ \rho\sigma} = \Big(& -2(2\gamma+\beta)\ \mathrm{P}_0^s - (4\gamma + 9\alpha + 2\beta)\ \mathcal{P}_0^s + (2\gamma-\beta)\ \mathscr{P}_0^x - \frac{4}{3}(3\gamma+5\beta)\ \mathrm{P}_1^s \\ & - 2\gamma\ \mathcal{P}_1^s - \frac{4}{3}(3\gamma+\beta)\ \mathcal{P}_1^t - (2\gamma+\beta)\ \mathscr{P}_1^{wx} + 4\beta\ \mathscr{P}_1^{ss} - 2(2\gamma+\beta)\ (\mathrm{P}_2 + \mathcal{P}_2) \\ & - 4\gamma\ \mathcal{P}_2^s + 2(\beta+\gamma)\ \mathscr{P}_2^x - \underline{4\gamma \mathrm{P}_3}\Big)_{\tau\ \lambda}^{\mu\nu\ \rho\sigma} \square \end{aligned}$$

Spin-3 is present and comes from the Riemann squared terms

Physical content of $A_{\mu(\nu\lambda)}$

Taking for instance the simpler action

$$S_{FOQ} \equiv \int d^n x \sqrt{-g} \left(\alpha\, R[\Gamma]^2 + \beta\, R[\Gamma]_{\mu\nu} R[\Gamma]^{\mu\nu} + \gamma\, R[\Gamma]_{\mu\nu\rho\sigma} R[\Gamma]^{\mu\nu\rho\sigma} \right)$$

$\mathcal{P}$ Hook $\qquad$ P Symmetric $\qquad$ $\mathscr{P}$ Mixed

$$\begin{aligned}(K_{\mathrm{FOQ}})^{\mu\nu}{}_{\tau}{}^{\rho\sigma}{}_{\lambda} = \Big(& -2(2\gamma+\beta)\ \mathrm{P}_0^s - (4\gamma + 9\alpha + 2\beta)\ \mathcal{P}_0^s + (2\gamma-\beta)\ \mathscr{P}_0^x - \frac{4}{3}(3\gamma+5\beta)\ \mathrm{P}_1^s \\ & - 2\gamma\ \mathcal{P}_1^s - \frac{4}{3}(3\gamma+\beta)\ \mathcal{P}_1^t - (2\gamma+\beta)\ \mathscr{P}_1^{wx} + 4\beta\ \mathscr{P}_1^{ss} - 2(2\gamma+\beta)\ \underline{(\mathrm{P}_2 + \mathcal{P}_2)} \\ & - 4\underline{\gamma\ \mathcal{P}_2^s} + 2(\beta+\gamma\underline{)\ \mathscr{P}_2^x} - 4\gamma \mathrm{P}_3 \Big)^{\mu\nu\ \ \rho\sigma}_{\ \ \ \tau\ \ \lambda} \Box\end{aligned}$$

We have in principle three Spin-2 components

Physical content of $A_{\mu(\nu\lambda)}$

Taking for instance the simpler action

$$S_{FOQ} \equiv \int d^n x \sqrt{-g} \left(\alpha\, R[\Gamma]^2 + \beta\, R[\Gamma]_{\mu\nu} R[\Gamma]^{\mu\nu} + \gamma\, R[\Gamma]_{\mu\nu\rho\sigma} R[\Gamma]^{\mu\nu\rho\sigma} \right)$$

$\mathcal{P}$ Hook P Symmetric $\mathcal{P}$ Mixed

$$\begin{aligned}(K_{\mathrm{FOQ}})^{\mu\nu}_{\tau}{}_{\lambda}^{\rho\sigma} = \Big(&- 2(2\gamma+\beta)\,\mathrm{P}^s_0 - (4\gamma + 9\alpha + 2\beta)\,\mathcal{P}^s_0 + (2\gamma-\beta)\,\mathcal{P}^x_0 - \frac{4}{3}(3\gamma+5\beta)\,\mathrm{P}^s_1 \\ &- 2\gamma\,\mathcal{P}^s_1 - \frac{4}{3}(3\gamma+\beta)\,\mathcal{P}^t_1 - (2\gamma+\beta)\,\mathcal{P}^{wx}_1 + 4\beta\,\mathcal{P}^{ss}_1 - 2(2\gamma+\beta)\,(\mathrm{P}_2 + \mathcal{P}_2) \\ &- 4\gamma\,\mathcal{P}^s_2 + 2(\beta+\gamma)\,\mathcal{P}^x_2 - 4\gamma \mathrm{P}_3 \Big)^{\mu\nu\ \ \rho\sigma}_{\ \ \ \tau\ \ \lambda} \Box\end{aligned}$$

We have in principle three Spin-0
and four Spin-1 components

Physical content of $A_{\mu(\nu\lambda)}$

Taking for instance the simpler action

$$S_{FOQ} \equiv \int d^n x \sqrt{-g} \left(\alpha \, R[\Gamma]^2 + \beta \, R[\Gamma]_{\mu\nu} R[\Gamma]^{\mu\nu} + \gamma \, R[\Gamma]_{\mu\nu\rho\sigma} R[\Gamma]^{\mu\nu\rho\sigma} \right)$$

$\mathcal{P}$ Hook $\qquad$ P Symmetric $\qquad$ $\mathfrak{P}$ Mixed

$$\begin{aligned}(K_{\mathrm{FOQ}})^{\mu\nu}_{\tau}{}_{\lambda}{}^{\rho\sigma} = \Big(& - 2(2\gamma+\beta)\ \mathrm{P}^s_0 - (4\gamma + 9\alpha + 2\beta)\ \mathcal{P}^s_0 + (2\gamma - \beta)\ \mathfrak{P}^x_0 - \frac{4}{3}(3\gamma+5\beta)\ \mathrm{P}^s_1 \\ & - 2\gamma\ \mathcal{P}^s_1 - \frac{4}{3}(3\gamma+\beta)\ \mathcal{P}^t_1 - (2\gamma+\beta)\ \mathfrak{P}^{wx}_1 + 4\beta\ \mathfrak{P}^{ss}_1 - 2(2\gamma+\beta)\ (\mathrm{P}_2 + \mathcal{P}_2) \\ & - 4\gamma\ \mathcal{P}^s_2 + 2(\beta+\gamma)\ \mathfrak{P}^x_2 - 4\gamma \mathrm{P}_3 \Big)^{\mu\nu\ \ \rho\sigma}_{\ \ \ \tau\ \ \lambda} \ \Box\end{aligned}$$

Which ones appear in the propagator? Which ones survive on-shell?

Physical content of $A_{\mu(\nu\lambda)}$

We need a gauge fixing to invert the operator

$$S_{gf} = \frac{1}{\chi} \int d^4x \; \eta^{\mu\nu} \; \eta^{\rho\sigma} \; \eta_{\tau\lambda} \; A^{\tau}_{\mu\nu} \; \Box \; A^{\lambda}_{\rho\sigma}$$

Physical content of $A_{\mu(\nu\lambda)}$

We need a gauge fixing to invert the operator

$$S_{gf} = \frac{1}{\chi}\int d^4x\ \eta^{\mu\nu}\ \eta^{\rho\sigma}\ \eta_{\tau\lambda}\ A^{\tau}_{\mu\nu}\ \Box\ A^{\lambda}_{\rho\sigma}$$

$\mathcal{P}$ Hook $\qquad$ P Symmetric $\qquad$ $\mathscr{P}$ Mixed

$$(K_{\mathrm{gf}})_{\tau\ \ \lambda}^{\mu\nu\ \ \rho\sigma} = \frac{1}{\chi}\Big(\mathrm{P}_0^w + 3\ \mathrm{P}_0^s + 3\ \mathcal{P}_0^s - 3\ \mathscr{P}_0^x + \mathscr{P}_0^{sw} + \mathscr{P}_0^{ws} + \mathrm{P}_1 - \frac{5}{3}\ \mathrm{P}_1^s + \mathcal{P}_1^w$$
$$+ \frac{2}{3}\ \mathcal{P}_1^t - \mathscr{P}_1^{wx} + \mathscr{P}_1^{ws} + \mathscr{P}_1^{sw} + \mathscr{P}_1^{sx} + 4\ \mathscr{P}_1^{ss}\Big)_{\tau\ \ \lambda}^{\mu\nu\ \ \rho\sigma}\ \Box$$

Physical content of $A_{\mu(\nu\lambda)}$

We need a gauge fixing to invert the operator

$$S_{gf} = \frac{1}{\chi}\int d^4x\ \eta^{\mu\nu}\ \eta^{\rho\sigma}\ \eta_{\tau\lambda}\ A^{\tau}_{\mu\nu}\ \Box\ A^{\lambda}_{\rho\sigma}$$

$\mathcal{P}$ Hook $\quad$ P Symmetric $\quad$ $\mathscr{P}$ Mixed

$$(K_{\rm gf})^{\mu\nu}_{\tau}{}^{\rho\sigma}_{\lambda} = \frac{1}{\chi}\Big(\mathrm{P}^w_0 + 3\ \mathrm{P}^s_0 + 3\ \mathcal{P}^s_0 - 3\ \mathscr{P}^x_0 + \mathscr{P}^{sw}_0 + \mathscr{P}^{ws}_0 + \mathrm{P}_1 - \frac{5}{3}\ \mathrm{P}^s_1 + \mathcal{P}^w_1 + \frac{2}{3}\ \mathcal{P}^t_1 - \mathscr{P}^{wx}_1 + \mathscr{P}^{ws}_1 + \mathscr{P}^{sw}_1 + \mathscr{P}^{sx}_1 + 4\ \mathscr{P}^{ss}_1\Big)^{\mu\nu\ \ \rho\sigma}_{\tau\ \ \lambda}\ \Box$$

Important to note: no spin-2 or spin-3 in the gauge fixing!

Physical content of $A_{\mu(\nu\lambda)}$

If we make $\beta = \gamma = 0$

$$(K_{R^2+\mathrm{gf}})^{\mu\nu}{}_{\tau}{}^{\rho\sigma}{}_{\lambda} = \frac{1}{\chi}\Big(\mathrm{P}_0^w + 3\,\mathrm{P}_0^s + (3-9\chi)\,\mathcal{P}_0^s - 3\,\mathscr{P}_0^x + \mathscr{P}_0^{sw} + \mathscr{P}_0^{ws} + \mathrm{P}_1^w - \frac{5}{3}\,\mathrm{P}_1^s$$
$$+ \mathcal{P}_1^w + \frac{2}{3}\,\mathcal{P}_1^t - \mathscr{P}_1^{wx} + \mathscr{P}_1^{ws} + \mathscr{P}_1^{sw} + \mathscr{P}_1^{sx} + 4\,\mathscr{P}_1^{ss}\Big)^{\mu\nu}{}_{\tau}{}^{\rho\sigma}{}_{\lambda}\,\Box$$

Physical content of $A_{\mu(\nu\lambda)}$

If we make $\beta = \gamma = 0$

$$(K_{R^2+\mathrm{gf}})^{\mu\nu}{}_{\tau}{}^{\rho\sigma}{}_{\lambda} = \frac{1}{\chi}\Big(\mathrm{P}_0^w + 3\ \mathrm{P}_0^s + (3-9\chi)\ \mathcal{P}_0^s - 3\ \mathscr{P}_0^x + \mathscr{P}_0^{sw} + \mathscr{P}_0^{ws} + \mathrm{P}_1^w - \frac{5}{3}\ \mathrm{P}_1^s$$

No spin-2
or spin-3

$$+ \mathcal{P}_1^w + \frac{2}{3}\ \mathcal{P}_1^t - \mathscr{P}_1^{wx} + \mathscr{P}_1^{ws} + \mathscr{P}_1^{sw} + \mathscr{P}_1^{sx} + 4\ \mathscr{P}_1^{ss}\Big)^{\mu\nu}{}_{\tau}{}^{\rho\sigma}{}_{\lambda}\ \Box$$

Physical content of $A_{\mu(\nu\lambda)}$

If we make $\beta = \gamma = 0$

$$(K_{R^2+\mathrm{gf}})^{\mu\nu}{}_{\tau}{}^{\rho\sigma}{}_{\lambda} = \frac{1}{\chi}\Big(\mathrm{P}_0^w + 3\,\mathrm{P}_0^s + (3-9\chi)\,\mathcal{P}_0^s - 3\,\mathscr{P}_0^x + \mathscr{P}_0^{sw} + \mathscr{P}_0^{ws} + \mathrm{P}_1^w - \frac{5}{3}\,\mathrm{P}_1^s$$

No spin-2
or spin-3

$$+\mathcal{P}_1^w + \frac{2}{3}\,\mathcal{P}_1^t - \mathscr{P}_1^{wx} + \mathscr{P}_1^{ws} + \mathscr{P}_1^{sw} + \mathscr{P}_1^{sx} + 4\,\mathscr{P}_1^{ss}\Big)^{\mu\nu}{}_{\tau}{}^{\rho\sigma}{}_{\lambda}\,\Box$$

The projection onto spin-2 and spin-3 are zero modes

→ We would need to gauge fix them

Physical content of $A_{\mu(\nu\lambda)}$

If we make $\beta = \gamma = 0$

$$(K_{R^2+\mathrm{gf}})^{\mu\nu}{}_{\tau}{}^{\rho\sigma}{}_{\lambda} = \frac{1}{\chi}\Big(\mathrm{P}_0^w + 3\,\mathrm{P}_0^s + (3-9\chi)\,\mathcal{P}_0^s - 3\,\mathscr{P}_0^x + \mathscr{P}_0^{sw} + \mathscr{P}_0^{ws} + \mathrm{P}_1^w - \frac{5}{3}\,\mathrm{P}_1^s$$

No spin-2
or spin-3

$$+\,\mathcal{P}_1^w + \frac{2}{3}\,\mathcal{P}_1^t - \mathscr{P}_1^{wx} + \mathscr{P}_1^{ws} + \mathscr{P}_1^{sw} + \mathscr{P}_1^{sx} + 4\,\mathscr{P}_1^{ss}\Big)^{\mu\nu}{}_{\tau}{}^{\rho\sigma}{}_{\lambda}\,\Box$$

The projection onto spin-2 and spin-3 are zero modes

$\longrightarrow$ We would need to gauge fix them

In fact, for R^2 there are 13 zero modes

Physical content of $A_{\mu(\nu\lambda)}$

For $\alpha \neq \beta \neq \gamma \neq 0$ we can invert the operator in order to get the propagator. In doing so, all the projectors of the basis appear.

Physical content of $A_{\mu(\nu\lambda)}$

For $\alpha \neq \beta \neq \gamma \neq 0$ we can invert the operator in order to get the propagator. In doing so, all the projectors of the basis appear.

The next step is to couple external sources for the connection and see the spin components that survive.

Physical content of $A_{\mu(\nu\lambda)}$

For $\alpha \neq \beta \neq \gamma \neq 0$ we can invert the operator in order to get the propagator. In doing so, all the projectors of the basis appear.

The next step is to couple external sources for the connection and see the spin components that survive.

To see the asymptotic states, we need to compute the equations of motion for the different spin pieces of the connection.

Physical content of $A_{\mu(\nu\lambda)}$

For $\alpha \neq \beta \neq \gamma \neq 0$ we can invert the operator in order to get the propagator. In doing so, all the projectors of the basis appear.

The next step is to couple external sources for the connection and see the spin components that survive.

To see the asymptotic states, we need to compute the equations of motion for the different spin pieces of the connection.

We have not found any obvious problem with the spin-3 piece so far. A full analysis of this piece is needed in order to see if inconsistencies appear.

Outline

- First order formalism vs. Second order formalism
- First order quadratic gravity
- Physical content of the connection

The coupling to matter in first order formalism

- Summary and Outlook

The coupling to matter in FO

The coupling of bosons in first order formalism does not give any new feature as they do not couple to the connection.

The coupling to matter in FO

The coupling of bosons in first order formalism does not give any new feature as they do not couple to the connection.

We are interested in the coupling of fermions to gravity in first order formalism, as they turn out to be a source of torsion.

The coupling to matter in FO

The coupling of bosons in first order formalism does not give any new feature as they do not couple to the connection.

We are interested in the coupling of fermions to gravity in first order formalism, as they turn out to be a source of torsion.

This constitutes a difference between first order linear gravity and second order linear gravity, not present in the case of pure gravity.

Coupling of fermions in FO

We take the minimal coupling of fermions to gravity. To do that, we need to change to the vielbein formalism

$\underline{e_a^{\ \mu} \quad \omega_\mu^{ab}}$ Independent

Coupling of fermions in FO

We take the minimal coupling of fermions to gravity. To do that, we need to change to the vielbein formalism

$e_a^{\ \mu} \quad \omega_\mu^{ab}$ Independent

$$S_{DEH} \equiv -\frac{1}{\kappa^2}\int d^4x\ e\ \frac{1}{2} e_a^{\ \mu} e_b^{\ \nu} R_{\mu\nu}^{ab}[\omega] + \frac{i}{2}\int d^4x\ e\left(\bar{\psi} e_a^{\ \mu}\gamma^a \nabla_\mu \psi - \nabla_\mu \bar{\psi} e_a^{\ \mu}\gamma^a \psi\right)$$

Coupling of fermions in FO

We take the minimal coupling of fermions to gravity. To do that, we need to change to the vielbein formalism

$e_a^{\ \mu} \quad \omega_\mu^{ab}$ Independent

$$S_{DEH} \equiv -\frac{1}{\kappa^2}\int d^4x\, e\, \frac{1}{2} e_a^{\ \mu} e_b^{\ \nu} R_{\mu\nu}^{ab}[\omega] + \frac{i}{2}\int d^4x\, e \left(\bar{\psi} e_a^{\ \mu}\gamma^a \nabla_\mu \psi - \nabla_\mu \bar{\psi} e_a^{\ \mu}\gamma^a \psi\right)$$

Variations with respect to the vierbein $\quad G^a_{\ \mu} + \kappa T^a_{\ \mu} = 0$

Not symmetric!

$$T^a_{\ \mu} = \frac{i}{2}\kappa\left(\bar{\psi}\gamma^a \nabla_\mu \psi - \nabla_\mu \bar{\psi}\gamma^a \psi\right) - \frac{i}{2}\kappa e^a_{\ \mu}\left(\bar{\psi}\gamma^\nu \nabla_\nu \psi - \nabla_\nu \bar{\psi}\gamma^\nu \psi\right)$$

Coupling of fermions in FO

We take the minimal coupling of fermions to gravity. To do that, we need to change to the vielbein formalism

$e_a^{\ \mu} \quad \omega_\mu^{ab}$ Independent

$$S_{DEH} \equiv -\frac{1}{\kappa^2}\int d^4x\ e\ \frac{1}{2}e_a^{\ \mu}e_b^{\ \nu}R_{\mu\nu}^{ab}[\omega] + \frac{i}{2}\int d^4x\ e\left(\bar{\psi}e_a^{\ \mu}\gamma^a\nabla_\mu\psi - \nabla_\mu\bar{\psi}e_a^{\ \mu}\gamma^a\psi\right)$$

Variations with respect to the vierbein $G_\mu^a + \kappa T_\mu^a = 0$

Not symmetric!

Fermions act like a source of torsion

Coupling of fermions in FO

We take the minimal coupling of fermions to gravity. To do that, we need to change to the vielbein formalism

$$S_{DEH} \equiv -\frac{1}{\kappa^2}\int d^4x\; e\;\frac{1}{2}e_a^{\;\mu}e_b^{\;\nu}R_{\mu\nu}^{ab}[\omega] + \frac{i}{2}\int d^4x\; e\left(\bar{\psi}e_a^{\;\mu}\gamma^a\nabla_\mu\psi - \nabla_\mu\bar{\psi}e_a^{\;\mu}\gamma^a\psi\right)$$

The antisymmetric part of the variations with respect to the spin connection

$$T_{\nu\lambda\rho} = \frac{\kappa^2}{2}\epsilon_{\nu\lambda\rho\sigma}j_5^\sigma$$

Totally antisymmetric torsion proportional to the axial current

Coupling of fermions in FO

We can reintroduce it in the action

Coupling of fermions in FO

We can reintroduce it in the action

$$\tilde{S}_{DEH} \;=\; S_{DEH} \;+\; \frac{3\kappa^2}{16}\int d^4x\ e\ \bar{\psi}\gamma^{\sigma}\gamma_5\psi\ \bar{\psi}\gamma_{\sigma}\gamma^5\psi$$

H. Weyl (1950)

Coupling of fermions in FO

We can reintroduce it in the action

$$\underbrace{\tilde{S}_{DEH}}_{\text{Dirac-EH action with torsion}} = S_{DEH} + \frac{3\kappa^2}{16}\int d^4x\, e\, \bar{\psi}\gamma^\sigma\gamma_5\psi\ \bar{\psi}\gamma_\sigma\gamma^5\psi$$

Dirac-EH action
with torsion

H. Weyl (1950)

Coupling of fermions in FO

We can reintroduce it in the action

$$\underbrace{\tilde{S}_{DEH}}_{\text{Dirac-EH action with torsion}} = \underbrace{S_{DEH}}_{\text{Dirac-EH action with no torsion}} + \underbrace{\frac{3\kappa^2}{16}\int d^4x\, e\, \bar{\psi}\gamma^\sigma\gamma_5\psi\ \bar{\psi}\gamma_\sigma\gamma^5\psi}_{\text{Extra quartic contact interaction between fermions}}$$

H. Weyl (1950)

Coupling of fermions in FO

We can reintroduce it in the action

$$\underbrace{\tilde{S}_{DEH}}_{\text{Dirac-EH action with torsion}} = \underbrace{S_{DEH}}_{\text{Dirac-EH action with no torsion}} + \underbrace{\frac{3\kappa^2}{16}\int d^4x\, e\, \bar{\psi}\gamma^\sigma\gamma_5\psi\ \bar{\psi}\gamma_\sigma\gamma^5\psi}_{\text{Extra quartic contact interaction between fermions}}$$

H. Weyl (1950)

This is precisely the difference between FO and SO formalisms

Coupling of fermions in FO

We can reintroduce it in the action

$$\underbrace{\tilde{S}_{DEH}}_{\text{Dirac-EH action with torsion}} = \underbrace{S_{DEH}}_{\text{Dirac-EH action with no torsion}} + \underbrace{\frac{3\kappa^2}{16}\int d^4x\, e\, \bar{\psi}\gamma^\sigma\gamma_5\psi\ \bar{\psi}\gamma_\sigma\gamma^5\psi}_{\text{Extra quartic contact interaction between fermions}}$$

H. Weyl (1950)

This is precisely the difference between FO and SO formalisms

Nevertheless, suppressed by the Planck mass

Coupling of fermions in FO

We can reintroduce it in the action

$$\underbrace{\tilde{S}_{DEH}}_{\text{Dirac-EH action with torsion}} = \underbrace{S_{DEH}}_{\text{Dirac-EH action with no torsion}} + \underbrace{\frac{3\kappa^2}{16}\int d^4x \; e \; \bar{\psi}\gamma^\sigma\gamma_5\psi \;\; \bar{\psi}\gamma_\sigma\gamma^5\psi}_{\text{Extra quartic contact interaction between fermions}}$$

H. Weyl (1950)

This is precisely the difference between FO and SO formalisms

What form does the torsion have when coupling fermions in FOQG?

Summary and Outlook

We have carried out a study of first order quadratic theories of gravity and found

Summary and Outlook

We have carried out a study of first order quadratic theories of gravity and found

- FOQG is a renormalizable gauge theory with room for unitarity

Summary and Outlook

We have carried out a study of first order quadratic theories of gravity and found

- FOQG is a renormalizable gauge theory with room for unitarity
- In principle, this theory is richer in its field content: we have spin-3, spin-2, spin-1 and spin-0 components

Summary and Outlook

We have carried out a study of first order quadratic theories of gravity and found

- FOQG is a renormalizable gauge theory with room for unitarity

- In principle, this theory is richer in its field content: we have spin-3, spin-2, spin-1 and spin-0 components

- The solution space its bigger than that of second order quadratic theories

Summary and Outlook

We have carried out a study of first order quadratic theories of gravity and found

- FOQG is a renormalizable gauge theory with room for unitarity
- In principle, this theory is richer in its field content: we have spin-3, spin-2, spin-1 and spin-0 components
- The solution space its bigger than that of second order quadratic theories
- The theory must undergo a spontaneous symmetry breaking so that EH dominates in the IR

Summary and Outlook

Future work is ongoing regarding some aspects of FOQG

Summary and Outlook

Future work is ongoing regarding some aspects of FOQG

- The coupling to external sources will shed light into the unitarity of the theory

Summary and Outlook

Future work is ongoing regarding some aspects of FOQG

- The coupling to external sources will shed light into the unitarity of the theory

  What are the sources of the connection?

  Is the free energy positive definite?

Summary and Outlook

Future work is ongoing regarding some aspects of FOQG

- The coupling to external sources will shed light into the unitarity of the theory

- The equations of motion of each component are needed to analyse the asymptotic states

Summary and Outlook

Future work is ongoing regarding some aspects of FOQG

- The coupling to external sources will shed light into the unitarity of the theory

- The equations of motion of each component are needed to analyse the asymptotic states

Are there ghostly degrees of freedom?

Summary and Outlook

Future work is ongoing regarding some aspects of FOQG

- The coupling to external sources will shed light into the unitarity of the theory
- The equations of motion of each component are needed to analyse the asymptotic states
- The coupling of fermions and the resulting torsion needs more study

Summary and Outlook

Future work is ongoing regarding some aspects of FOQG

- The coupling to external sources will shed light into the unitarity of the theory
- The equations of motion of each component are needed to analyse the asymptotic states
- The coupling of fermions and the resulting torsion needs more study

  Can we reintroduce it in the action?

  What kind of new interactions do we get?

Summary and Outlook

Future work is ongoing regarding some aspects of FOQG

- The coupling to external sources will shed light into the unitarity of the theory
- The equations of motion of each component are needed to analyse the asymptotic states
- The coupling of fermions and the resulting torsion needs more study
- The spin-3 part is worth of deeper study

C. Aragone and S. Deser (1979)

M. A. Vasiliev (1988)

Summary and Outlook

Future work is ongoing regarding some aspects of FOQG

- The coupling to external sources will shed light into the unitarity of the theory
- The equations of motion of each component are needed to analyse the asymptotic states
- The coupling of fermions and the resulting torsion needs more study
- The spin-3 part is worth of deeper study

Are there any inconsistencies in the interactions?

Thank you for your attention

Backup

Possible sources

$$J_{\alpha\beta\gamma} \equiv A k_\alpha\, \eta_{\beta\gamma} + B\left(k_\beta \eta_{\alpha\gamma} + k_\gamma \eta_{\alpha\beta}\right)$$

$$J_{\alpha\beta\gamma} = A j_\alpha T_{\beta\gamma} + B(j_\beta T_{\alpha\gamma} + j_\gamma T_{\alpha\beta})$$

Zero modes

$$Z_1 \equiv (\mathrm{P}_0^w + \mathrm{P}_0^s - \mathscr{P}_0^{ws})_{\lambda\mu\nu}^{\alpha\beta\gamma}\ \Omega_{\alpha\beta\gamma}$$

$$Z_2 \equiv \left(-\mathrm{P}_1^w + \mathrm{P}_1^s + 3\mathcal{P}_1^w - \tfrac{3}{8}\mathscr{P}_1^{sw} - \tfrac{3}{2}\mathscr{P}_1^{wst}\right)_{\lambda\mu\nu}^{\alpha\beta\gamma}\ \Omega_{\alpha\beta\gamma}$$

$$Z_3 \equiv \left(2\mathcal{P}_1^w + \mathcal{P}_1^t - \tfrac{3}{2}\mathscr{P}_1^{sw}\right)_{\lambda\mu\nu}^{\alpha\beta\gamma}\ \Omega_{\alpha\beta\gamma}$$

$$Z_4 \equiv \left(-2\mathrm{P}_1^w + \mathcal{P}_1^w + \mathscr{P}_1^{ws} - \tfrac{1}{8}\mathscr{P}_1^{sw} - \tfrac{1}{2}\mathscr{P}_1^{wst}\right)_{\lambda\mu\nu}^{\alpha\beta\gamma}\ \Omega_{\alpha\beta\gamma}$$

$$Z_5 \equiv \left(-2\mathrm{P}_1^w + \mathcal{P}_1^w - \tfrac{3}{4}\mathscr{P}_1^{sw} + \mathscr{P}_1^{sx} - \mathscr{P}_1^{wst}\right)_{\lambda\mu\nu}^{\alpha\beta\gamma}\ \Omega_{\alpha\beta\gamma}$$

$$Z_6 \equiv \left(-\tfrac{7}{6}\mathrm{P}_1^w + \tfrac{14}{3}\mathcal{P}_1^w - \tfrac{21}{16}\mathscr{P}_1^{ws} + \mathscr{P}_1^{ss} - \tfrac{7}{4}\mathscr{P}_1^{wst}\right)_{\lambda\mu\nu}^{\alpha\beta\gamma}\ \Omega_{\alpha\beta\gamma}$$

$$Z_7 \equiv (\mathcal{P}_1^s)_{\lambda\mu\nu}^{\alpha\beta\gamma}\ \Omega_{\alpha\beta\gamma}$$

$$Z_8 \equiv (\mathscr{P}_1^{wx})_{\lambda\mu\nu}^{\alpha\beta\gamma}\ \Omega_{\alpha\beta\gamma}$$

$$Z_9 \equiv (\mathrm{P}_2)_{\lambda\mu\nu}^{\alpha\beta\gamma}\ \Omega_{\alpha\beta\gamma}$$

$$Z_{10} \equiv (\mathcal{P}_2)_{\lambda\mu\nu}^{\alpha\beta\gamma}\ \Omega_{\alpha\beta\gamma}$$

$$Z_{11} \equiv (\mathcal{P}_2^s)_{\lambda\mu\nu}^{\alpha\beta\gamma}\ \Omega_{\alpha\beta\gamma}$$

$$Z_{12} \equiv (\mathscr{P}_2^{\mathrm{x}})_{\lambda\mu\nu}^{\alpha\beta\gamma}\ \Omega_{\alpha\beta\gamma}$$

$$Z_{13} \equiv (\mathrm{P}_3)_{\lambda\mu\nu}^{\alpha\beta\gamma}\ \Omega_{\alpha\beta\gamma}$$