Continuum limit of fishnet graphs and AdS sigma model

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based on work with De-liang Zhong

Plan

Part I

Brief introduction to integrability in N=4 SYM

Brief overview of recent progress at computing correlation functions

Part II

Fishnet theory as a baby version of N=4 SYM

Fishnet theory as an integrable lattice regularization of the AdS5 sigma model

Part I

Motivation I

Understand dynamics of planar graphs and its relation to sigma models

['t Hooft]
[Polyakov]

Best possible starting point: N=4 SYM

[Maldacena'97]

1. String dual is (believed to be) known
2. Theory is (believed to be) integrable
   = there are methods for re-summing planar graphs

How does integrability works?

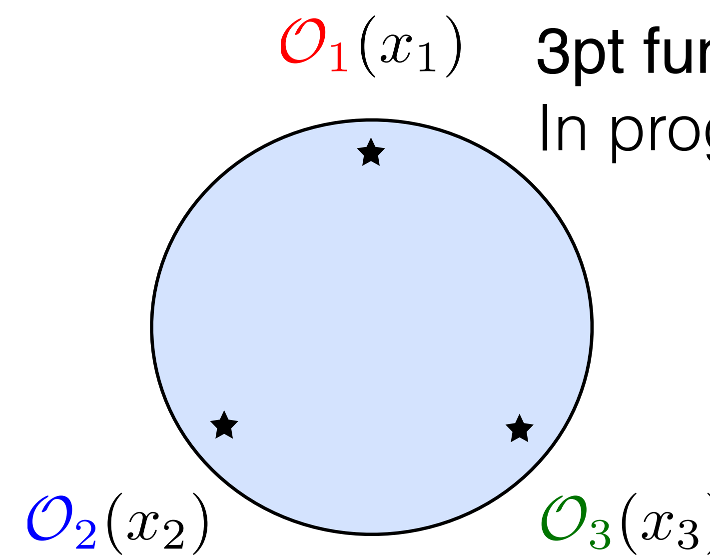
Elementary bubbles

Elementary planar correlators

2pt functions

Complete set of equations (planar)

$$\langle \mathcal{O}(x)^\dagger \mathcal{O}(0) \rangle = \frac{1}{x^{2\Delta}}$$



3pt functions

In progress

$$\langle \mathcal{O}_1(x_1) \mathcal{O}_2(x_2) \mathcal{O}_3(x_3) \rangle = \frac{C_{123}}{x_{12}^{\Delta_{12}} x_{23}^{\Delta_{23}} x_{13}^{\Delta_{13}}}$$

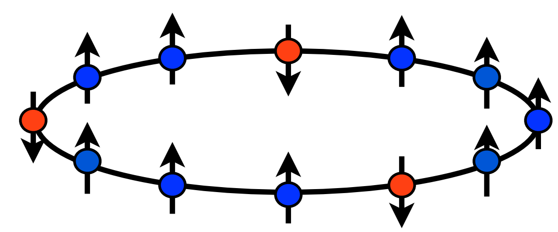
Should contain the seeds for computing all the correlators of the theory

Puncture = spin chain state

Local (single trace) operators

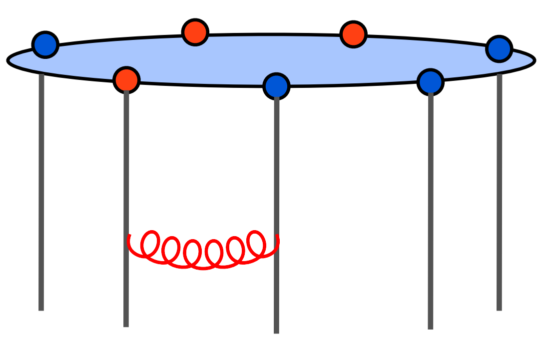
$$\mathcal{O} \sim \text{tr} \ldots \phi_1 \ldots \phi_2 \ldots \phi_1$$

Spin chain state



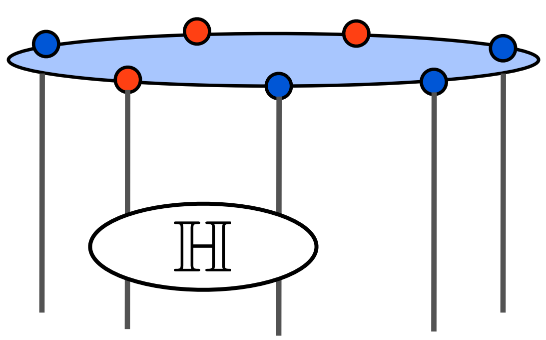
Composite operators renormalize non-trivially

quantum corrections
induce mixing of operators



$\sim$

equivalent to action of the
spin chain Hamiltonian



H = spin chain Hamiltonian = dilatation operator

$$\Delta = \Delta_0 + H_{\text{spin-chain}}(g^2)$$

Integrability

One-loop dilatation operator

$$H = 2g^2 \sum_{i=1}^{L} (I - P_{i,i+1}) + O(g^4)$$

[Faddeev,Korchemsky'95]
[Lipatov'95'97],
[Braun,Korchemsky,Derkachov,Manashov'98'99],[Belitsky'99]
[Minahan,Zarembo'02]
[Beisert,Staudacher'03]

is identical to Hamiltonian of the Heisenberg spin chain

Hans Bethe

Heisenberg spin chain is **integrable** :

- As many commuting conserved charges as degrees of freedom (L for SU(2) spin chain)

- S-matrix for fundamental excitations (magnons) above the ferromagnetic vacuum is totally factorized

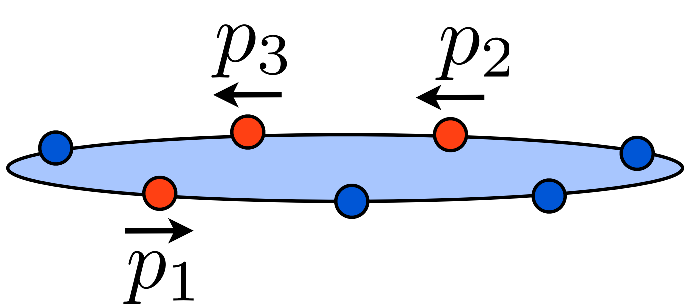
$$\mathbb{S}_{123} = \mathbb{S}_{23}\mathbb{S}_{13}\mathbb{S}_{12}$$

Werner Heisenberg

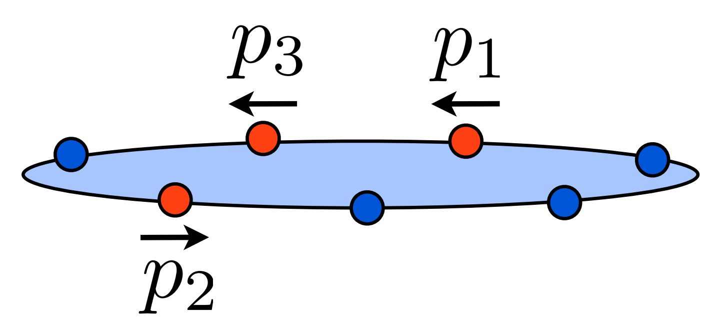
Puncture = Bethe state

See spins ↓ in spins ↑ background (BMN or ferromagnetic vacuum)
as **magnons** (spin waves) carrying energy and momentum

Write Bethe wave function (controlled by 2-by-2 elastic S matrix)



$+S(p_1, p_2)$



$+\ldots$

Impose periodicity conditions
= Bethe ansatz equations

$$e^{ip_i L} \prod_{j \neq i} S(p_i, p_j) = 1$$

Get energy spectrum

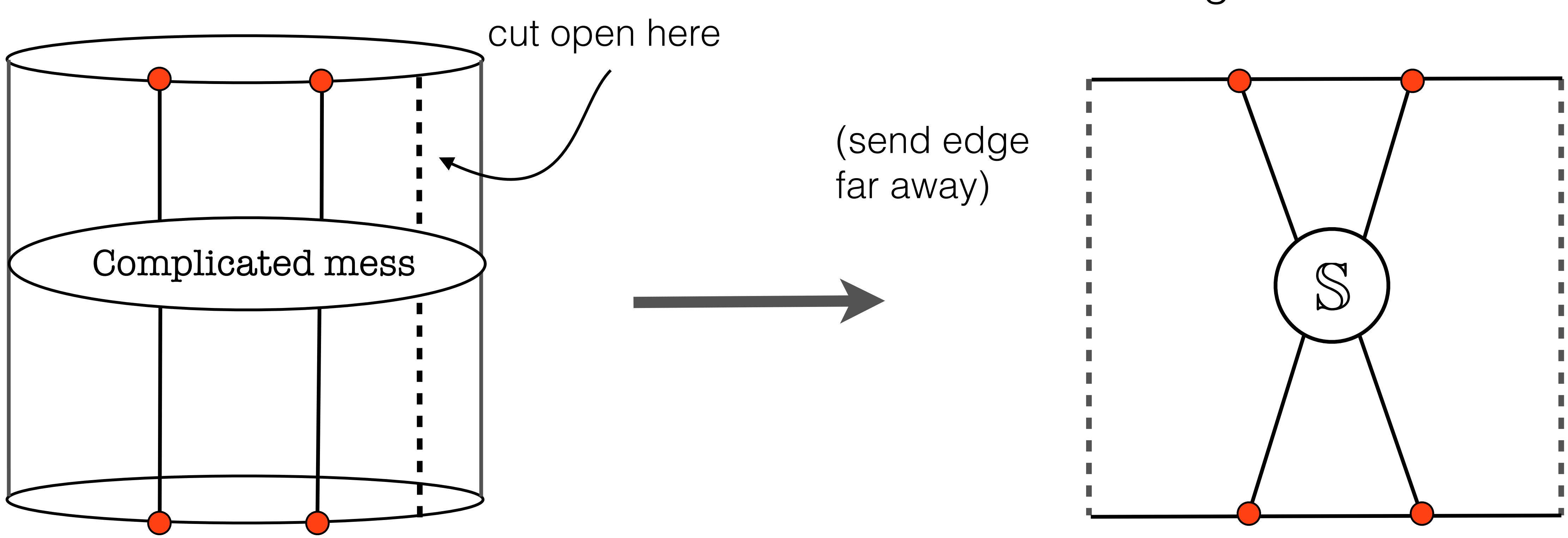
$$E = \sum_i E(p_i)$$

Higher loops

Long range Hamiltonian (messy and complicated) depends explicitly on length when range is greater than spin chain length

However one can delay the problem for asymptotically large length ~ decompactification

Geodesics to solution : Magnon **S-matrix**



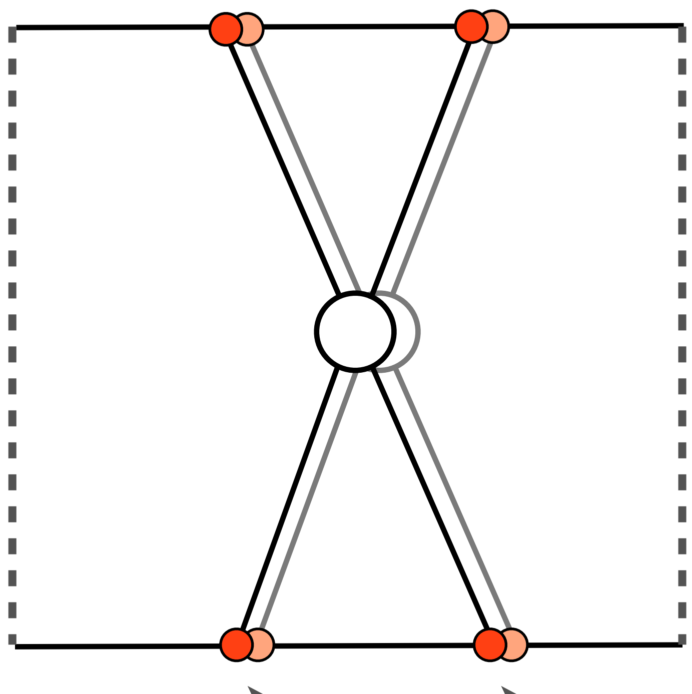
Akin to dilute gas approximation
Zoo of interactions reduces to 2-by-2 elastic scattering events

The asymptotic description is valid up to corrections that are exponentially small in the volume (so called **wrapping or mirror**) corrections

[Ambjorn,Janik,Kristjansen'05]
[Bajnok,Janik'08]

Solving the problem

Method : Let the symmetries do the job



Residual symmetry group of BMN vacuum :

[Beisert'05]

$$\underset{\text{Left}}{PSU(2|2)} \times \underset{\text{Right}}{PSU(2|2)} \ltimes \mathbb{R}^3$$

Central extensions : contain energy (and coupling constant)

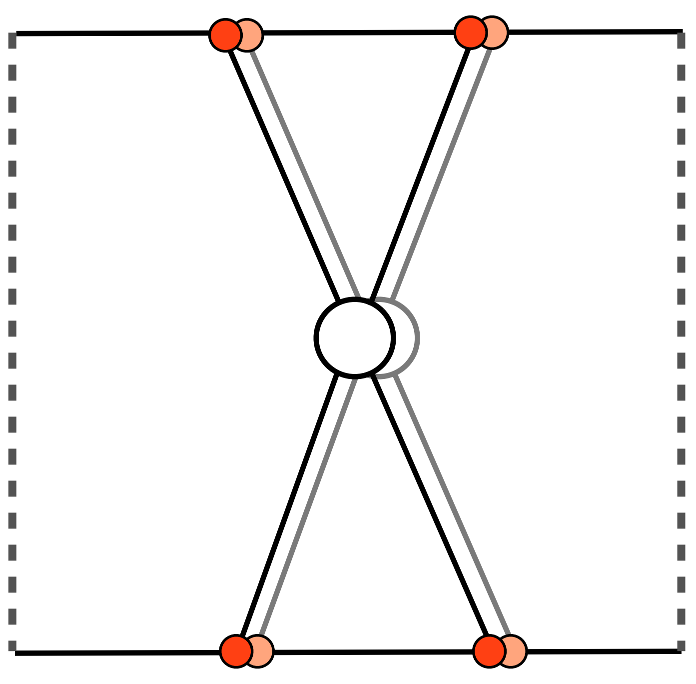
Magnon transforms in the bi-fundamental irrep

$$\underset{\text{Left}}{\mathbf{2}|\mathbf{2}} \otimes \underset{\text{Right}}{\mathbf{2}|\mathbf{2}}$$

(Dimension = 16
= 8 bosons + 8 fermions)

Solving the problem

Method : Let the symmetries do the job



Residual symmetry group of BMN vacuum :

[Beisert'05]

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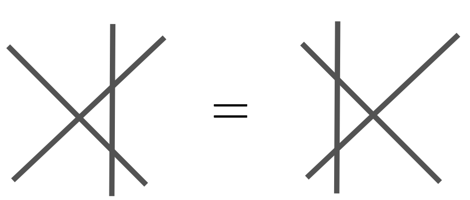
1) **Symmetries** fix dispersion relation

$$E = \sqrt{1 + 16\, g^2 \sin^2\left(\frac{p}{2}\right)}$$

2) **Symmetries** fix S-matrix (up to overall scalar factor)

$$\mathbb{S}_{12} \sim S^0_{12}\, \mathcal{S}_{12} \times \dot{\mathcal{S}}_{12}$$



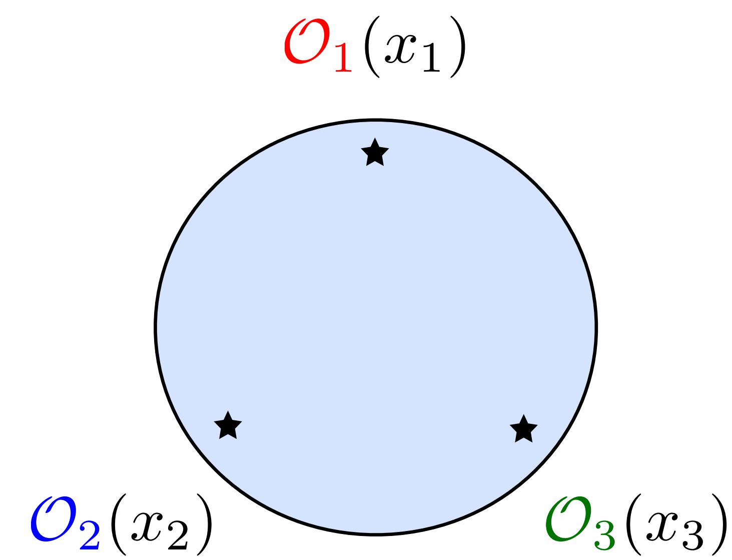
✓ Fulfills **Yang-Baxter** equation
✓ Scalar factor constrained by crossing symmetry

[Janik'05]

3pt functions

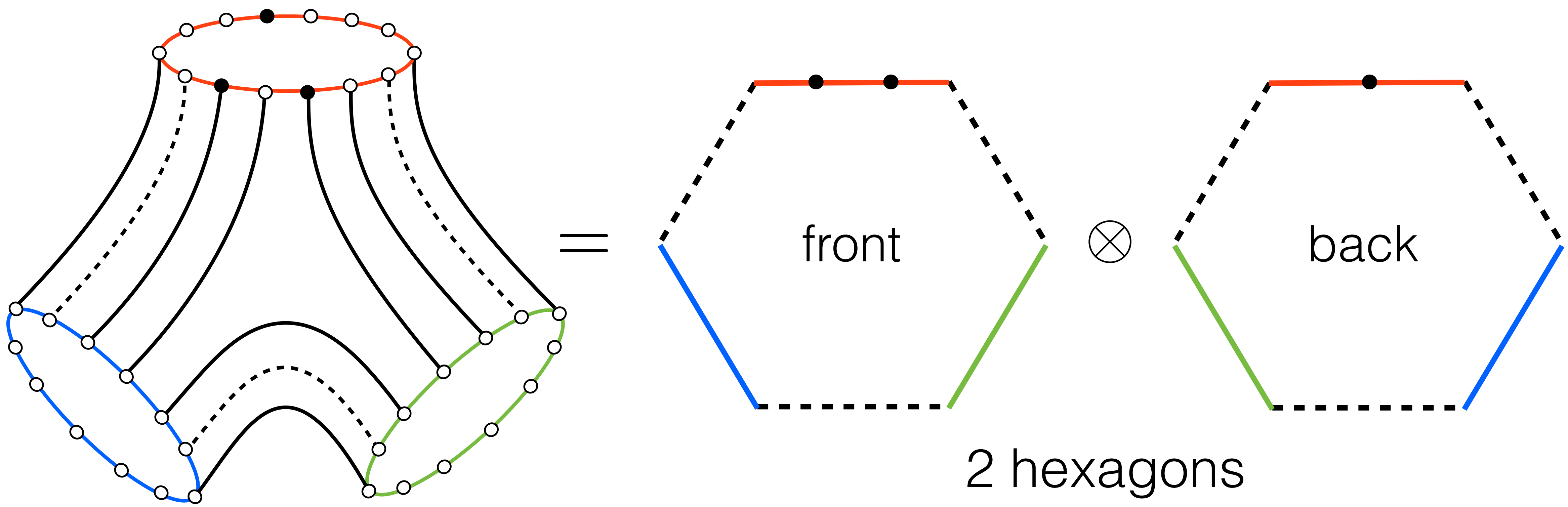
Structure constants:

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle = \frac{C_{123}}{x_{12}^{\Delta_{12}}x_{23}^{\Delta_{23}}x_{13}^{\Delta_{13}}}$$



Cutting strategy:
cut open into 2 open string like patches

1 pair of pants



2 hexagons

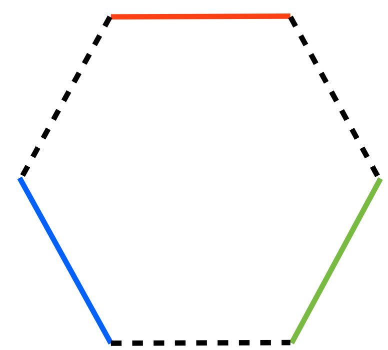
Use integrable bootstrap to find the hexagons

[BB,Komatsu,Vieira'15]

Hexagon form factors

6 edges:

3 spin chain edges
+
3 “mirror” edges for the cuts

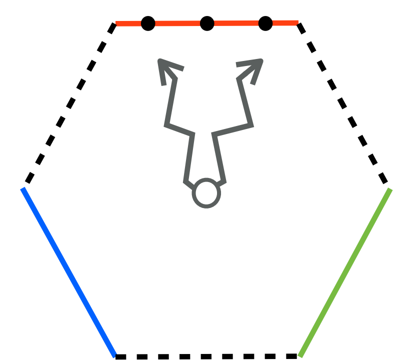


$$\sim \frac{1}{\sqrt{N}}$$

subsume the infinite class of planar graphs that fits inside its boarders

World-sheet picture:

2d spacetime with
a conical excess



Hexagon form factors:

Amplitudes for creation / annihilation of magnons on the edges of an hexagon

Use integrable bootstrap to find the hexagons

Main inputs : super-symmetries, analytical properties, guess work

Hexagon form factors

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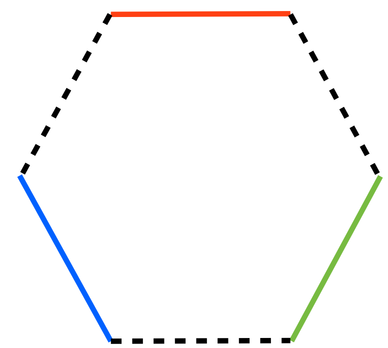

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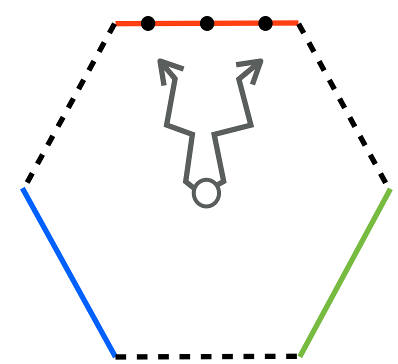


$$= \prod_{i<j} h(p_i, p_j)$$

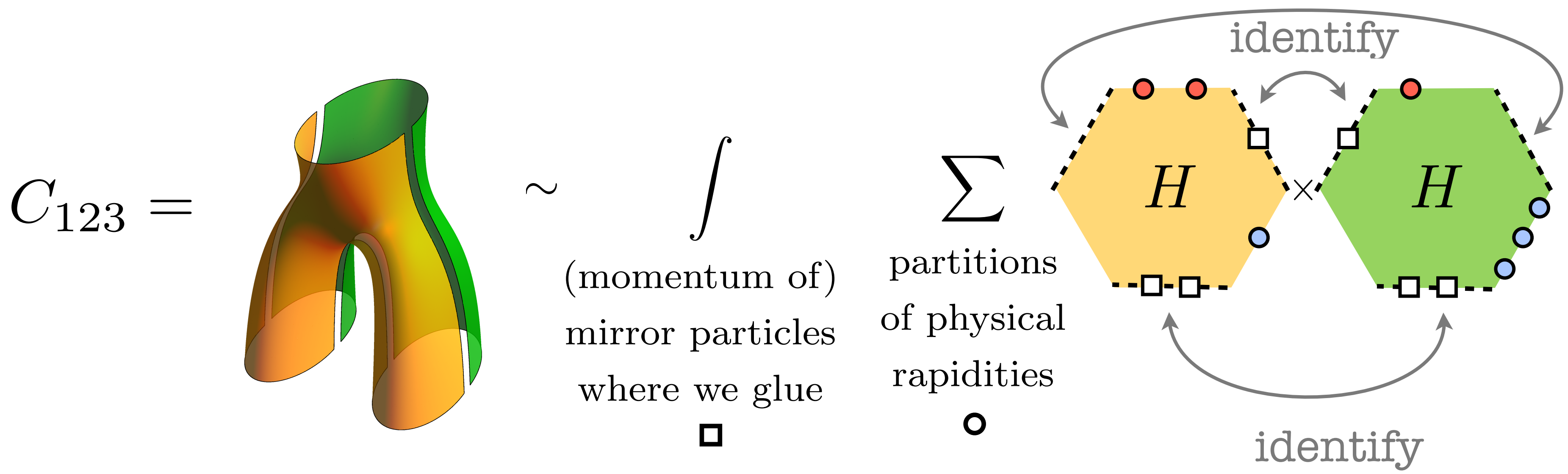
Hexagon form factors:

Amplitudes for creation / annihilation of magnons on the edges of an hexagon

Use integrable bootstrap to find the hexagons

Main inputs : super-symmetries, analytical properties, guess work

Final recipe

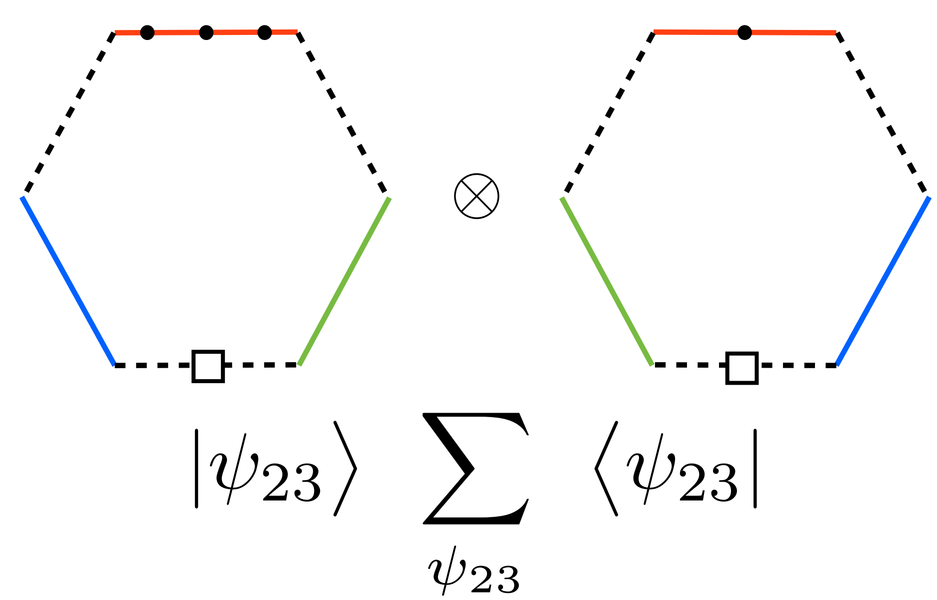


Hexagons are simple

Full result is more complicated. Gluing two hexagons back together entails

1) Summing over ways of distributing magnons on two hexagons

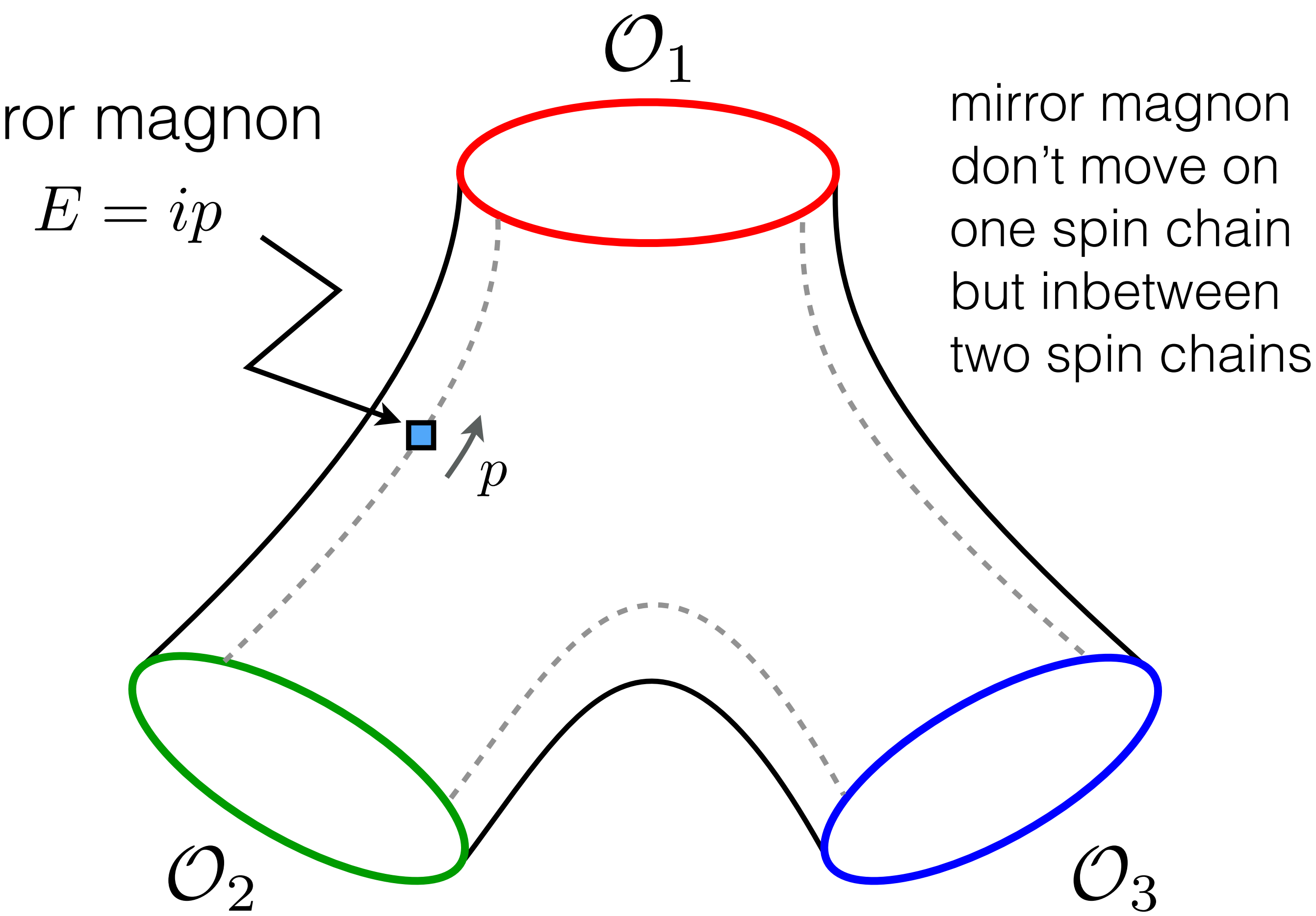
2) Inserting a resolution of the identity along each cut = sum over complete basis of so-called mirror magnons (that is, magnons carrying imaginary energies)



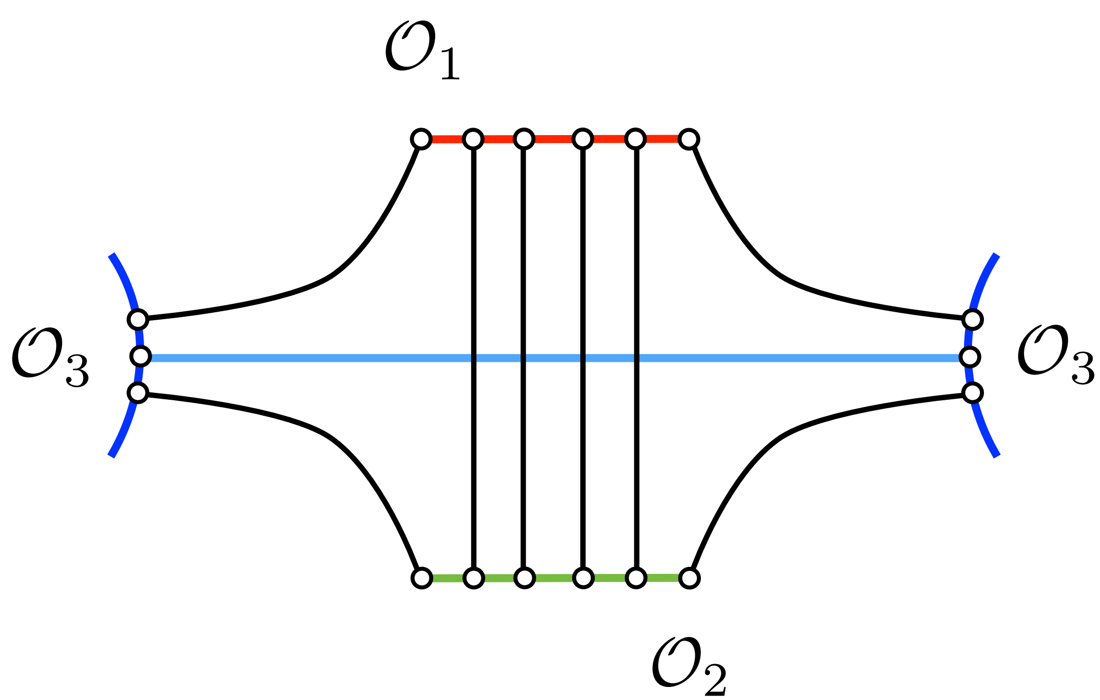
Mirror (virtual) corrections

Example of a mirror process:

Exchange of a mirror magnon
between the two hexagons in channel 12



Field theory counterpart
(graphs that don't fit inside a single hexagon)



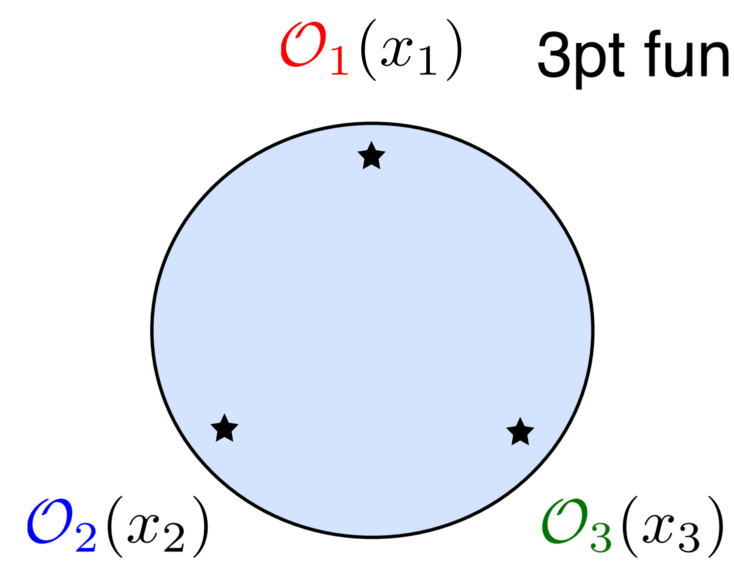
The insertion of a mirror magnon mimics the effect of a scalar propagator stretching across the mirror channel 12
(the thicker the bridge the smaller the effect)

Higher bubbles

Elementary planar correlators

2pt functions

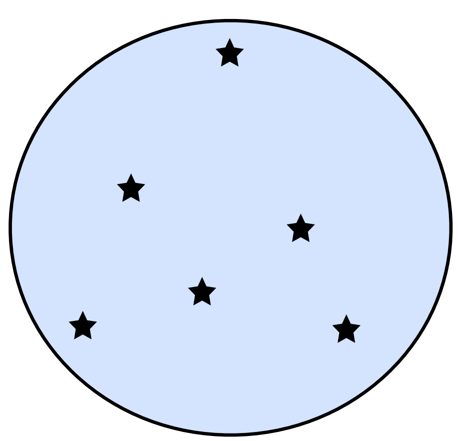
3pt functions

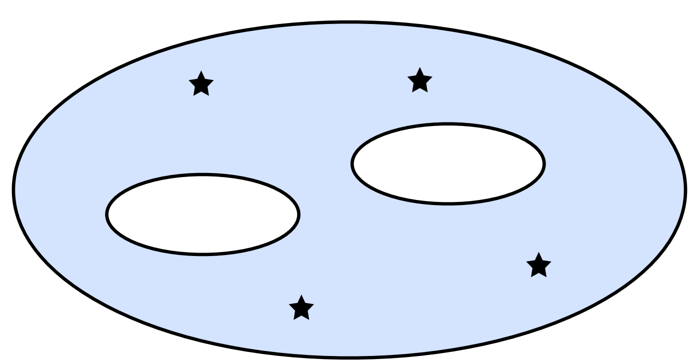


The same **planar** material allows us to attack :

[Fleury,Komatsu'16]
[Eden,Sfondrini'16]

Higher point functions &
String loops





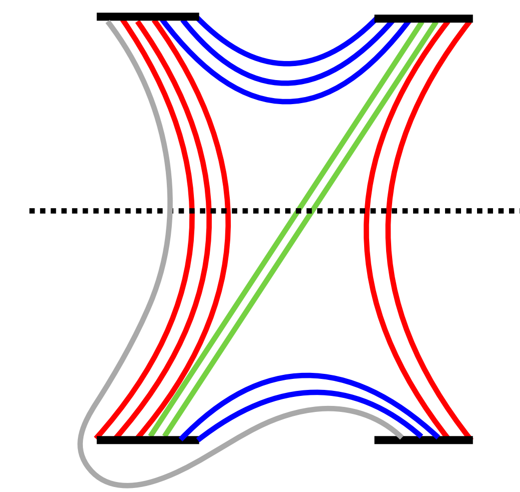
[Bargheer,Caetano,Fleury,Komatsu,Vieira'17'18]
[Eden,Jiang,Le Plat,Sfondrini'17]

Progess on amplitudes side as well [Ben-Israel,Tumanov,Sever'18]

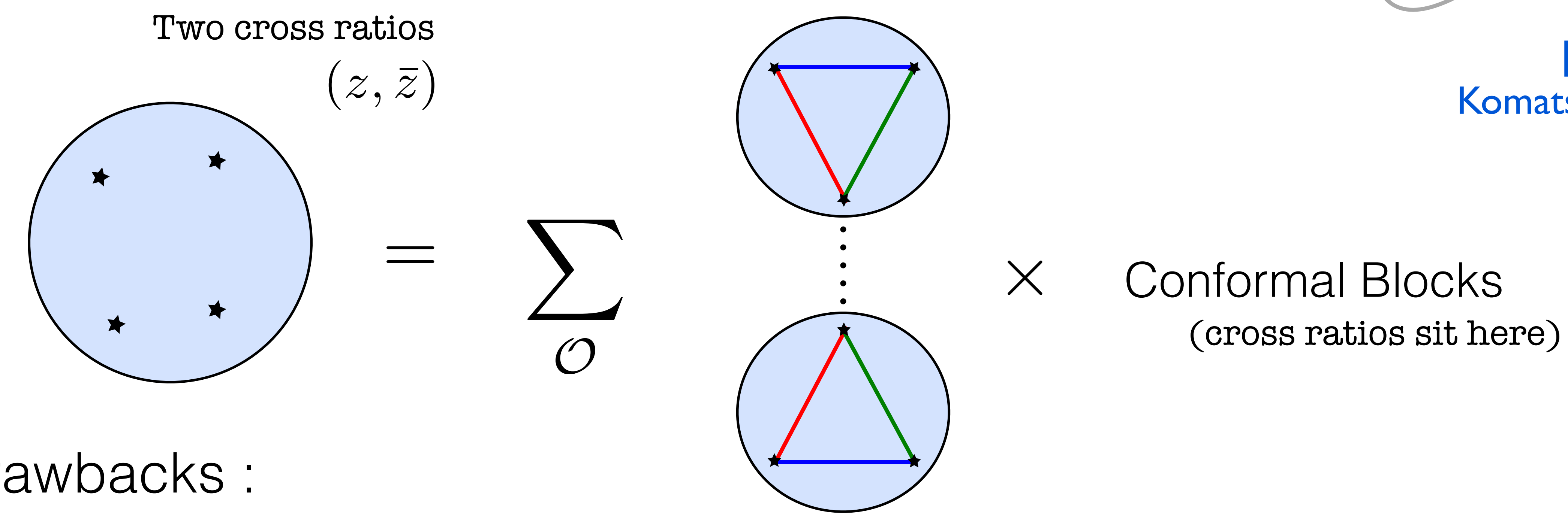
From 3 to 4

Using a good old technique

OPE cut



[BB,Coronado,Lam,
Komatsu,Vieira,Zhong'17]
[Bargheer'17]



Drawbacks :

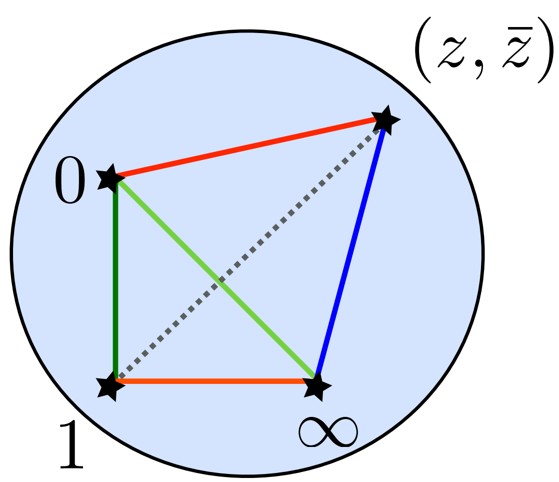
1) sum over a complete basis of super-conformal primaries
(not yet clear how to efficiently perform that step using integrability)

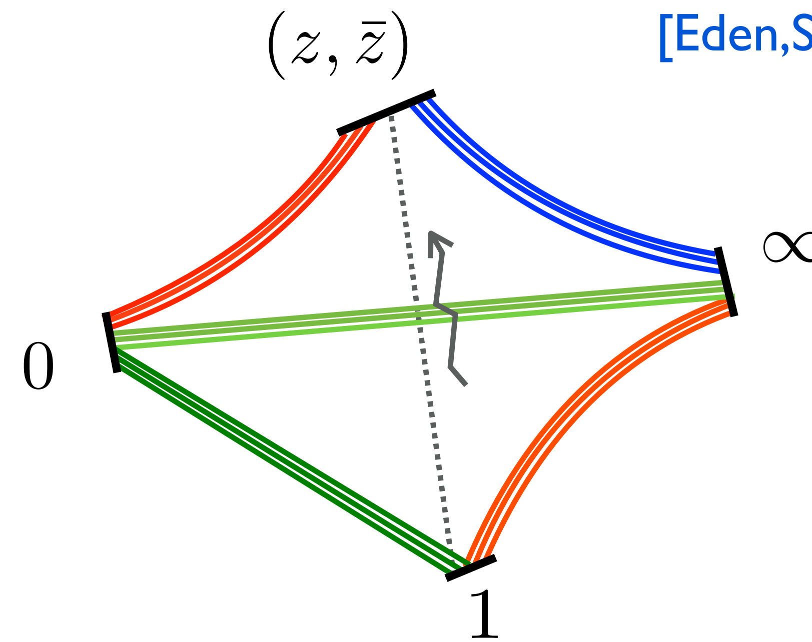
2) double trace operators must also be taken into account

Hexagonalization

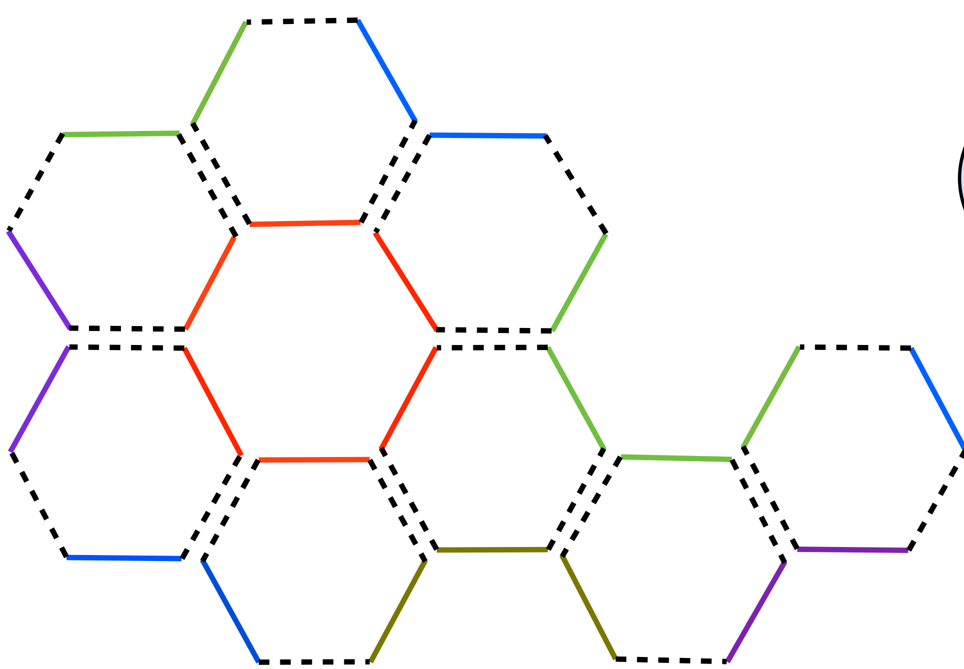
Any punctured sphere can be covered with hexagons

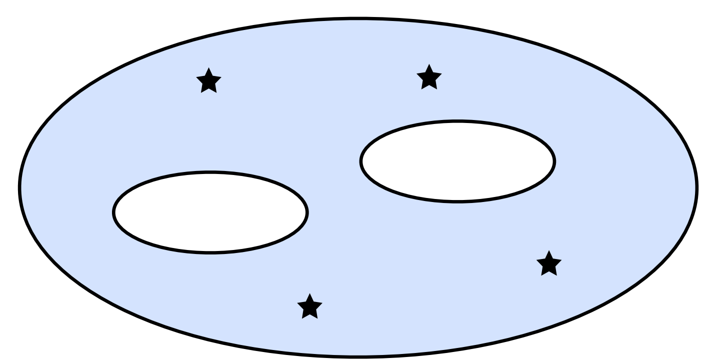
[Fleury,Komatsu'16]
[Eden,Sfondrini'16]





Should work at non-planar level





[Bargheer,Caetano,Fleury, Komatsu,Vieira'17]
[Eden,Jiang,Le Plat,Sfondrini'17]

Non planar gluing of hexagons

Summary I

Integrability helps formulating conjectures for higher correlators in N=4 SYM (reasonably well tested, although not everything is under control)

Are there **simpler** setups where similar ideas apply?
(setups where one could hope to prove these conjectures)

Possible direction: deform the theory, make it simpler, but maintain as many important properties as possible (conformal symmetry, integrability, duality, etc.)

Example : **fishnet** theory (a laboratory for “integrabilists”)

Part II

Motivation II

Understand dynamics of planar graphs and its relation to sigma models

['t Hooft]
[Polyakov]

Best possible starting point: N=4 SYM

[Maldacena'97]

1. String dual is (believed to be) known
2. Theory is (believed to be) integrable
   = we have methods for re-summing planar graphs

Use solution to gain knowledge about other models by deforming / twisting the theory ⟶ partial re-summations
Reduce complexity, but maintain as many important properties as possible: conformal symmetry, integrability, etc

Fishnet theory

Baby version of N = 4 SYM

A theory for matrix scalar fields with quartic coupling

[Gurdogan,Kazakov'15]
[Caetano,Gurdogan,Kazakov'16]

$$\mathcal{L}_{\text{fishnet}} = N\text{tr}\left[\partial_\mu\phi_1\partial_\mu\phi_1^* + \partial_\mu\phi_2\partial_\mu\phi_2^* + (4\pi g)^2\phi_1\phi_2\phi_1^*\phi_2^*\right]$$

It can be obtained by 1) twisting N=4 SYM theory (γ deformation)
2) sending the deformation parameter to i-infinity
while taking YM coupling to zero

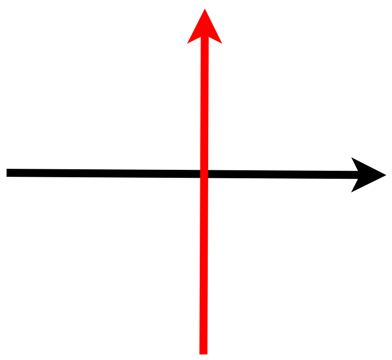
[Leigh,Strassler]
[Frolov'05]
[Lunin,Maldacena'05]

Gluons decouple

$$D_\mu\phi = (\partial_\mu\phi + ig_{YM}A_\mu\phi) \to \partial_\mu\phi$$

Of all quartic couplings only 1 remains

$$\text{tr}\,[\Phi^{ab},\Phi^{cd}][\Phi_{ab},\Phi_{cd}] \to \text{tr}\,\phi_1\phi_2\phi_1^*\phi_2^*$$



fishnet vertex

Fishnet theory

Baby version of N = 4 SYM

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[Leigh,Strassler]
[Frolov'05]
[Lunin,Maldacena'05]

1. Massive cut : gluons and gauginos are gone (no SUSY)
2. Gauge group becomes a flavour group
3. Conformal symmetry is preserved for any coupling
(at least in planar limit and for fine-tuned double-trace couplings)
4. Integrability is retained

[Grabner,Gromov,Kazakov,Korchemsky'17]
[Sieg,Wilhelm'16]

Fishnet theory

Baby version of N = 4 SYM

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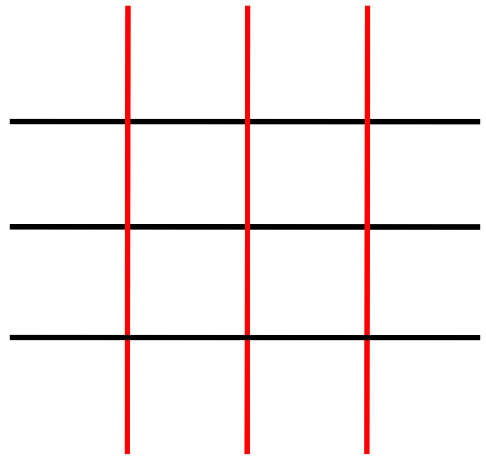
Planar graphs all look the same



bulk of any graph

Integrability much less mysterious here

Follow from properties of quartic vertex in d=4

[Zamolodchikov'80]
[Isaev'03]
[Gromov,Kazakov,Korchemsky,Negro,Sizov'17]
[Chicherin,Kazakov,Loebbert,Muller,Zhong'16]

Win: simplicity (very few graphs)

Lose: unitarity

Continuum limit & string?

What about duality to string in AdS?

Extremal twisting procedure forces the YM coupling to be small

⟶ string in highly curved AdS?

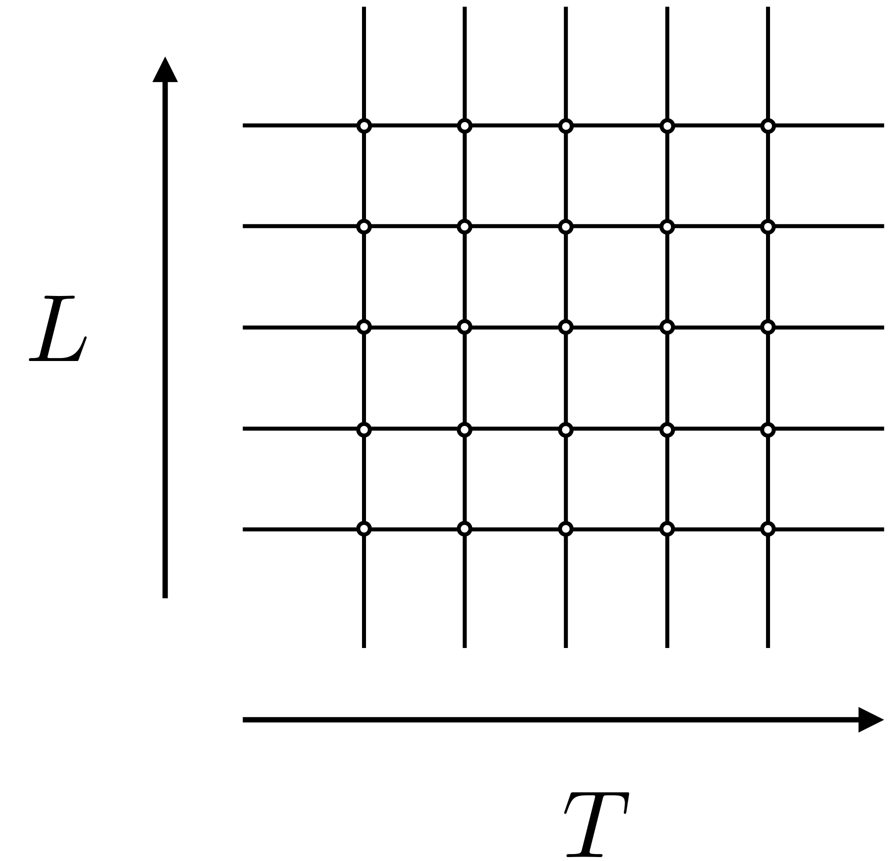
Related question: **continuum limit** of fishnet graphs?

Important observation concerning large order behaviour [Zamolodchikov'80]

Zamolodchikov's thermodynamical scaling $L, T \to \infty$

$$\log Z_{L,T} = -L \times T \log g_{cr}^2$$

$$g_{cr} = \frac{\Gamma(3/4)}{\sqrt{\pi}\Gamma(5/4)} = 0.7\ldots$$



Critical coupling:
Point at which graphs become "dense"

Exploring fishnet using N=4 SYM

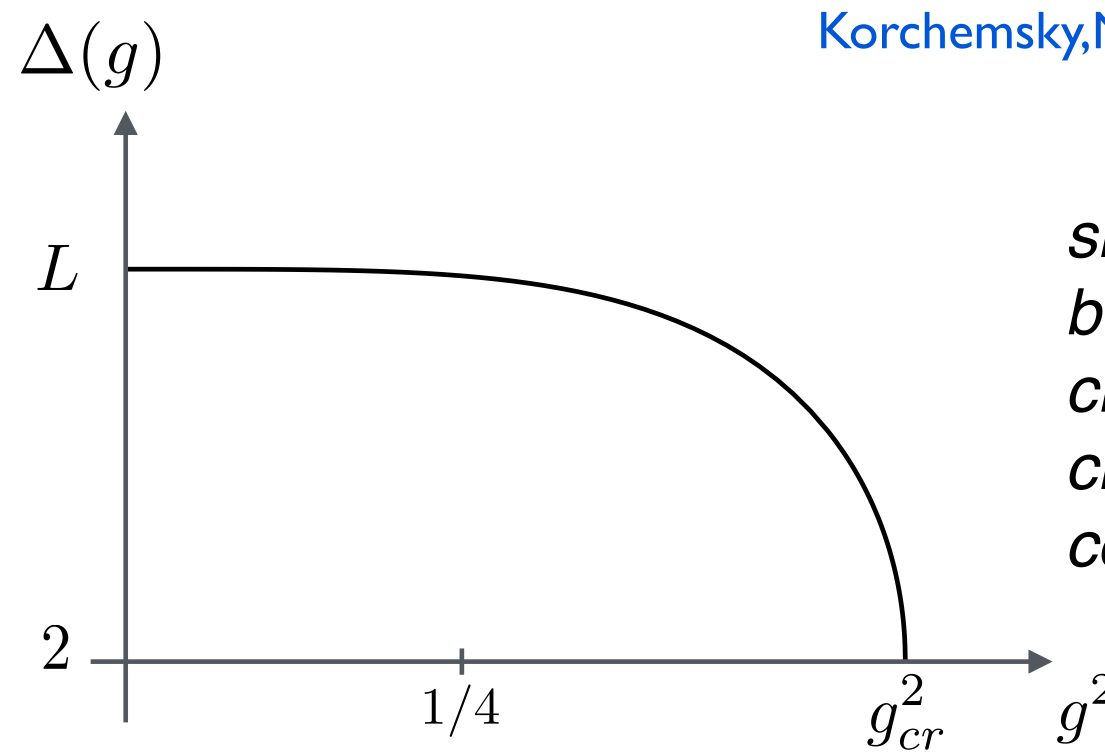
Goal: investigate continuum limit using N=4 SYM integrable techniques

Probe: scaling dimension Δ of BMN vacuum operator $\mathcal{O} = \mathrm{tr}\,\phi_1^L$

see also [Gromov,Kazakov, Korchemsky,Negro,Sizov'17]

Qualitative picture
in thermodynamical limit

$$\Delta \sim L f(g)$$
$$L \to \infty$$



singular behaviour close to critical coupling

Near critical regime:

fishnet graph = AdS5 sigma model

with dictionary

BMN operator = tachyon	$\mathrm{tr}\,\phi_1^L \leftrightarrow V_\Delta \sim e^{-i\Delta t}$
coupling = worldsheet energy	$\log g^{2L} = \log g_{cr}^{2L} + E_{2d}(\Delta, L)$

Wheels and magnons

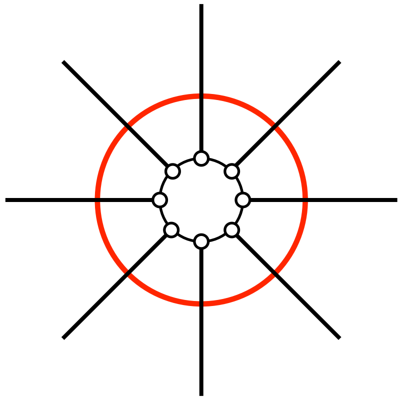
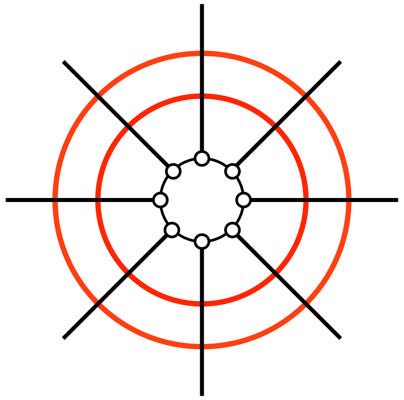
Computation of anomalous dimension

[Gurdogan,Kazakov'15]
[Caetano,Gurdogan,Kazakov'16]
[Gromov,Kazakov,
Korchemsky,Negro,Sizov'17]

BMN vacuum
(not protected)

$$\Delta = \Delta_L(g) = L + \gamma$$

Graphs: loop corrections come from wheel diagrams

$Z \;=\; 1 \;+\; g^{2L}$  $+\; g^{4L}$  $+\; \cdots$

wave-function
renormalization

1 wheel

2 wheels

It depends on UV cut off

$$R \sim \log \Lambda_{UV}$$

Anomalous dimension controls the logarithmic dependence on cut off

$$\log Z \sim -\gamma \times R$$

Wheels and magnons

Computation of anomalous dimension

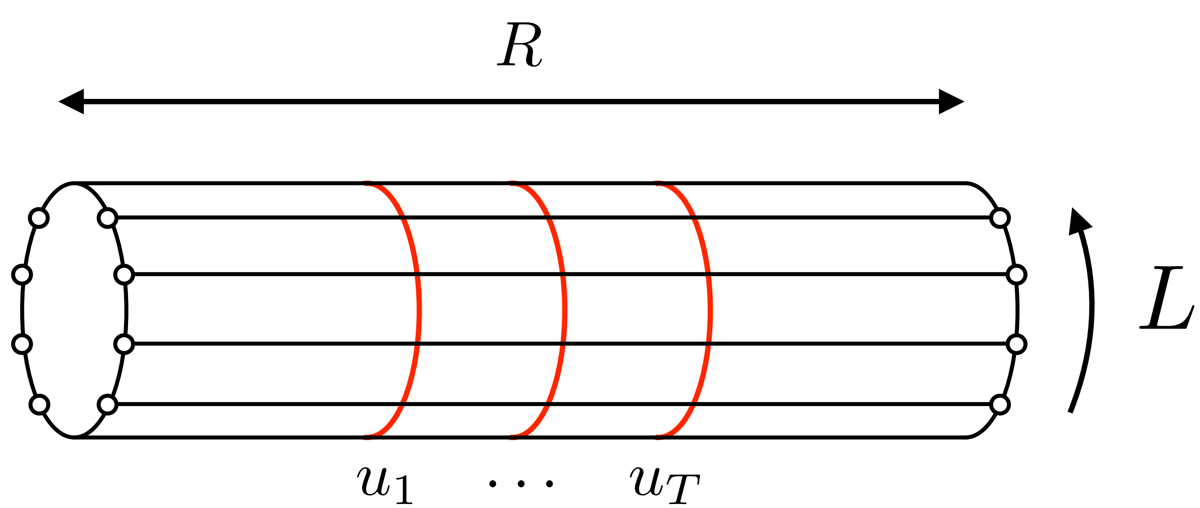
[Gurdogan,Kazakov'15]
[Caetano,Gurdogan,Kazakov'16]
[Gromov,Kazakov, Korchemsky,Negro,Sizov'17]

BMN vacuum (not protected) $\Delta = \Delta_L(g) = L + \gamma$

Integrability: quantum mechanical interpretation of the graphs

1+1d picture: partition function on $\mathbb{R} \times S_L$

$$\mathcal{Z}_{L,R} = \sum_{T \geqslant 0} g^{2LT} \times$$



d.o.f = magnon = wheel

magnon carries a rapidity "u", momentum along euclidean time direction, and a discrete label "a", for harmonics on 3-sphere

scaling dimension = free energy of a gas of magnon

$$\log \mathcal{Z}_{L,R} = -\Delta_L(g) R + O(R^0)$$

TBA equations

[Yang, Yang'60s]
[Zamolodchikov'90s]

Factorized scattering allows us to obtain free energy from TBA eps

$$\log Y_a(u) = Lh - L\epsilon_a(u) + \sum_b \mathcal{K}_{ab} * \log\left(1 + Y_b(u)\right) + \ldots$$

Coupling constant only enters as chemical potential $h = \log g^2$

Length **L** of operator acts as inverse temperature

dynamical input:

1) energy of magnon $\epsilon_a(u) = \log\left(u^2 + \frac{1}{4}a^2\right)$

2) scattering kernel $\mathcal{K}_{ab}(u,v) = -i\partial_u \log S_{ab}(u,v)$

Solution to TBA determines the free energy = scaling dimension

$$\Delta = L - 2\sum_a \int \frac{du}{2\pi} \log\left(1 + Y_a(u)\right)$$

Contain exact dependence of scaling dimension on the coupling

TBA equations

Iterative solution = small Y expansion

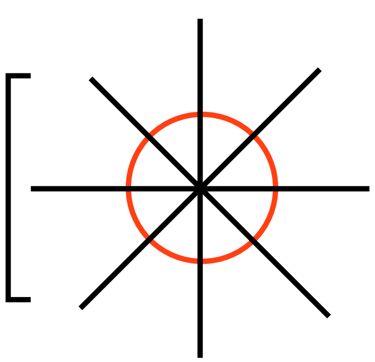
$$Y_a(u) \simeq a^2 e^{L(h-\epsilon_a)} \ll 1$$

(Boltzmann weight free magnon)

valid at weak coupling and large (enough) L

Reproduce perturbation theory $$\Delta = L - 2\sum_a \int \frac{du}{2\pi} Y_a(u) + \ldots$$

matches divergent part of 1-wheel graph

div $$\Bigg[$$  $$\Bigg] = -\sum_{a\geqslant 1} a^2 \int \frac{du}{\pi} \frac{g^{2L}}{(u^2+a^2/4)^L} \propto g^{2L}\zeta(2L-3)$$

[Broadhurst'85]

[Gurdogan,Kazakov'15]

Thermodynamic limit

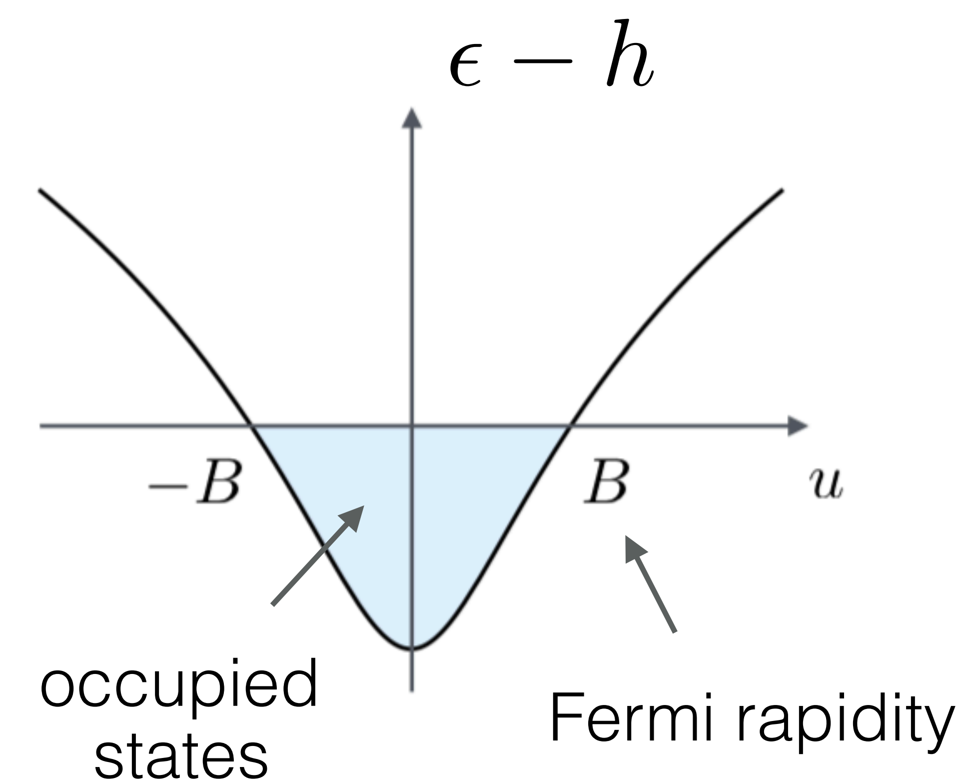
Thermodynamic limit $L \to \infty$

Interesting when chemical potential gets bigger than mass of lightest magnon

$$h = \epsilon(u = 0) = \log 1/4$$

that is for

$$g > 1/2$$



A **Fermi sea** forms

All states below the Fermi rapidity are filled

Increasing coupling amounts to increasing B

Comment: only the s-wave (lightest) magnons condense
(higher Lorentz harmonics decouple)

Linear integral equation

In the thermodynamic limit the TBA eq linearize

Single **linear integral equation** for the distribution of energy levels

$$\chi(u) = C - \epsilon(u) + \int_{-B}^{B} \frac{du}{2\pi} \mathcal{K}(u-v)\chi(v)$$

BC: $\chi(u = \pm B) = 0$

Chemical potential: $C = \log g^2 - \int_{-B}^{B} \frac{du}{2\pi} k(u)\chi(u)$

Kernel: $\mathcal{K}(u) = 2\psi(1+iu) + 2\psi(1-iu) + \frac{2}{1+u^2}$

Scaling dimension: $\Delta/L = 1 - \int_{-B}^{B} \frac{du}{\pi} \chi(u)$

Critical regime

Small B : dilute gas (small magnon density) free regime

$$j = -df/dh \sim 0 \qquad \varepsilon = f + hj \sim 1$$

Critical regime : dense gas (large magnon density) $\quad B \to \infty$

$$\varepsilon \sim j \log g_{cr}^2 \quad \longleftrightarrow \quad \log Z_{L,T} \sim -LT \log g_{cr}^2$$

(Zamolodchikov's micro-canonical scaling)

All energy levels are filled, distribution covers real axis

$$\chi_{cr}(u) = C_{cr} - \epsilon(u) + \int_{-\infty}^{\infty} \frac{dv}{2\pi} \mathcal{K}(u-v)\chi_{cr}(v) \quad \Rightarrow \quad \chi_{cr} = \log \frac{\sqrt{2}\cosh\theta + 1}{\sqrt{2}\cosh\theta - 1}$$

with $\theta = \pi u/2$

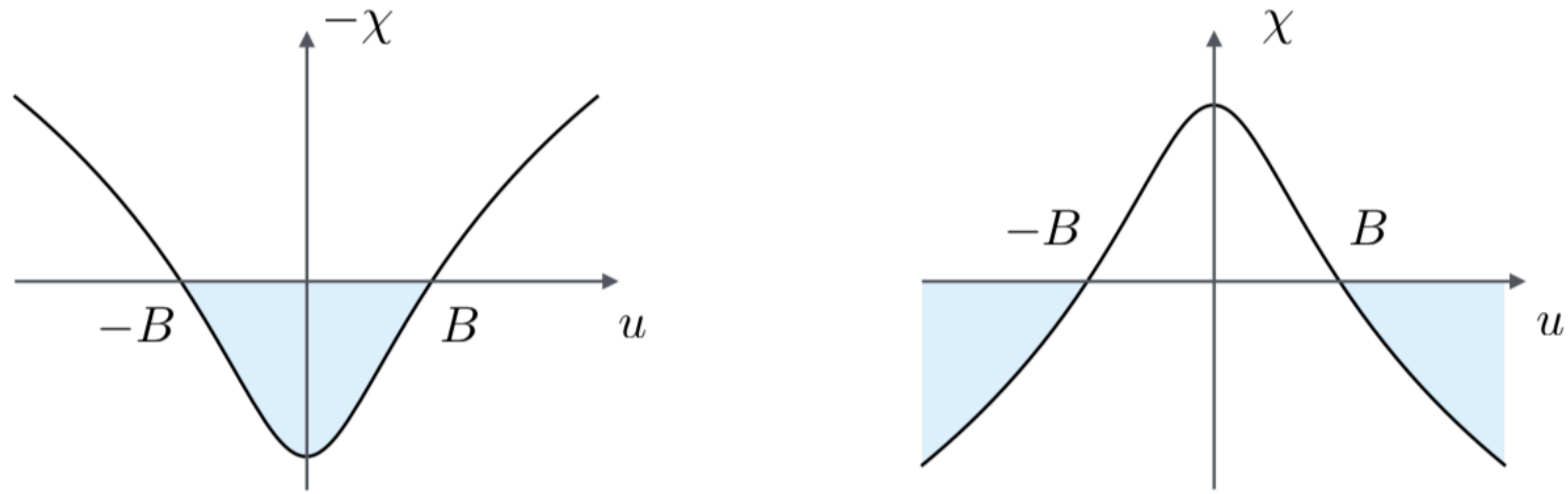
(i) Scaling function vanishes $\quad f = \Delta/L \to 0$

(ii) Chemical potential (coupling) approaches predicted value

$$\chi_{cr} \sim e^{-|\theta|} \quad \Rightarrow \quad C_{cr} = 0 \quad \Rightarrow \quad g_{cr} = \Gamma(3/4)/\sqrt{\pi}\Gamma(5/4)$$

Near-critical regime

Particle-hole transformation (similar to map between ferro and anti-ferro)



Fermi sea of magnons | dual Fermi sea

Dual equation for dual excitations obtained mathematically after

1) dualizing kernel

$$K = -\frac{\mathcal{K}}{1-\mathcal{K}*} = -\mathcal{K} - \mathcal{K}*\mathcal{K} - \ldots$$

2) acting on both sides of the equation with $1 - K*$

Dual equations

Dual equation: $$\chi(\theta) = E(\theta) + \int_{\theta^2 > B^2} \frac{d\theta'}{2\pi} K(\theta - \theta')\chi(\theta')$$

Dual energy formula: $$\log g^2 = \log g_{cr}^2 + \int_{\theta^2 > B^2} \frac{d\theta}{2\pi} P'(\theta)\chi(\theta)$$

No chemical potential
but extra BC (at ∞) $\chi(\theta) \sim -2\rho \log\theta$ $\rho = \Delta/L =$ charge density

1) *Dual kernel*: $$K(\theta) = \frac{\partial}{i\partial\theta} \log \frac{\Gamma(1+\frac{i\theta}{2\pi})\Gamma(\frac{1}{2}-\frac{i\theta}{2\pi})\Gamma(\frac{3}{4}+\frac{i\theta}{2\pi})\Gamma(\frac{1}{4}-\frac{i\theta}{2\pi})}{\Gamma(1-\frac{i\theta}{2\pi})\Gamma(\frac{1}{2}+\frac{i\theta}{2\pi})\Gamma(\frac{3}{4}-\frac{i\theta}{2\pi})\Gamma(\frac{1}{4}+\frac{i\theta}{2\pi})}$$

2) *Dual dispersion relation*:

$$E(\theta) = \chi_{cr}(\theta) = \log\left[\frac{\sqrt{2}\cosh\theta + 1}{\sqrt{2}\cosh\theta - 1}\right] \sim \frac{m}{2} e^{-|\theta|}$$

$$P(\theta) = -iE(\theta + i\frac{\pi}{2}) = i\log\left[\frac{\sqrt{2}\sinh\theta + i}{\sqrt{2}\sinh\theta - i}\right] \sim \mp\frac{m}{2} e^{-|\theta|}$$

here $m = 4\sqrt{2}$ acts like a mass scale

Interpretation

What is the dual system describing?

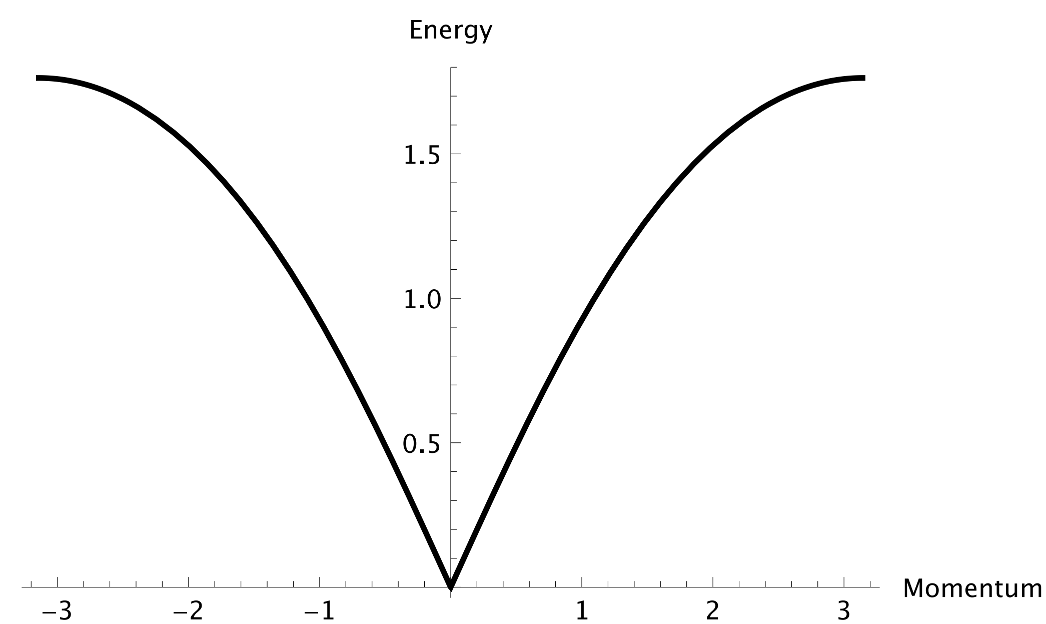
1) **Kernel**: $$K = -i\partial_\theta \log S_{O(6)}$$

Particles scatter as in 2d O(6) non-linear sigma model!

[Zamolodchikov&Zamolodchikov'78]

2) **Dispersion relation**:

$$\sinh^2 (\frac{1}{2}E) = \sin^2 (\frac{1}{2}P)$$



Gapless excitations (unlike O(6) model)

E decreases when θ increases
Support is non-compact, density is not normalizable (cannot count excitations)

No mass gap + continuous spectrum
Suggest: sigma model with **non-compact** target space
Proposal: integrable lattice completion of AdS_5 sigma model

Dual theory: hyperbolic sigma model

2d sigma model

$$\mathcal{L} = -\frac{1}{2e^2} G^{AB} \partial_a Y_A \partial^a Y_B$$

Weak coupling (large radius of curvature) : $e^2 \ll 1$

Beta function related to Ricci scalar

$$AdS_{d+1} \ : \ -Y_0^2 + Y_\perp^2 - Y_{d+1}^2 = -1$$

Negative curvature ⟶ a **positive** beta function $\mu \frac{\partial}{\partial \mu} e^2(\mu) = \frac{d}{2\pi} e^4(\mu) + \dots$

Alternatively, the coupling **grows** with the energy $\frac{1}{e^2(\mu)} = \frac{d}{2\pi} \log(\Lambda/\mu)$

1. Theory is weakly coupled in IR
2. There is no mass gap
3. Integrable but gapless and no good particle picture

akin to massless factorized scattering theories

[Zamolodchikov&Zamoldchikov'92]
[Fendley,Saleur,Zamolodchikov'93]
[Fateev,Onofri,Zamolodchikov'93]

Dual state: tachyon

Consider sigma model on cylinder of **radius L**

Interested in 2d "ground state" energy: **tachyon**

(best candidate for extremum of energy at given charge = global time energy)

$$V_\Delta \sim e^{-i\Delta t}$$

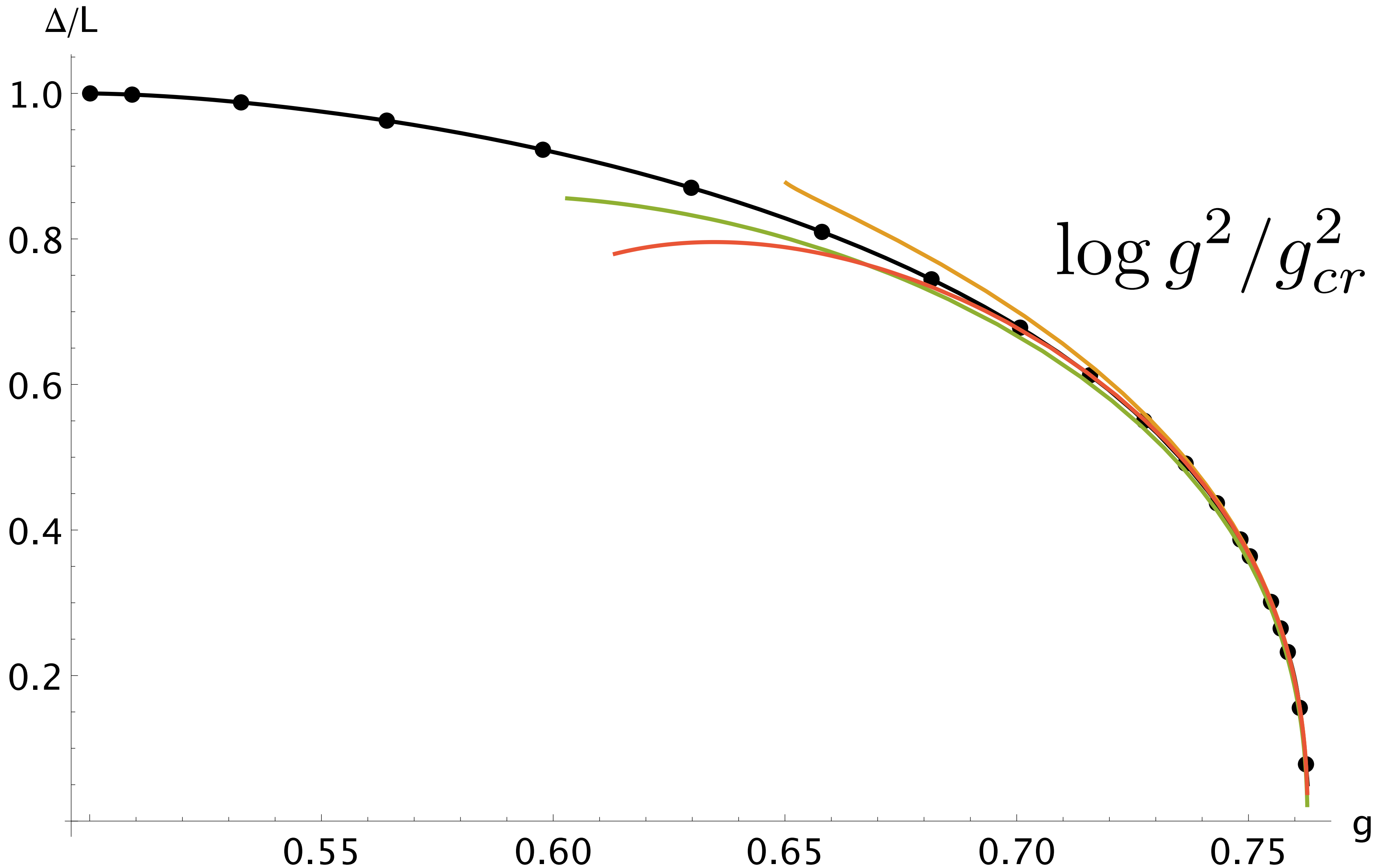
Classically it corresponds to solution $$Y^0 \pm iY^{d+1} = e^{\pm iH\tau}$$ $$Y_\perp = 0$$

Classical (c-o-m) energy $$E = -\frac{e^2\Delta^2}{2L}$$

Same as in O(d+2) model if not for the sign of the coupling $e^2 \leftrightarrow -e^2$

Numerics

"Quadratic Casimir" scaling near critical point fits numerical sol. of linear eq.



$$\log g^2/g_{cr}^2 = -\frac{e^2\Delta^2}{2L^2}$$

WS PT
at 1st, 2nd
and 3rd
order

$g_{\rm thr} = 0.5$

$g_{\rm cr} = 0.76276$

thermodynamical window

All order perturbation theory

Argument that dual integral equation describes the tachyon of the AdS model to *all* orders in perturbation theory

Solutions to eqs for sphere and hyperboloid are the same to any order in $1/B$ if we formally flip the sign of the Fermi rapidity $B \to -B$ [Volin'09]

Since the Fermi rapidity plays role of the inverse running coupling

$$B \sim 1/e^2 \sim \log\left(L/\Delta\right) \gg 1$$

at energy scale $\sim \rho = \Delta/L$

flipping its sign has the same effect as flipping the curvature of the space

Two descriptions co-exist

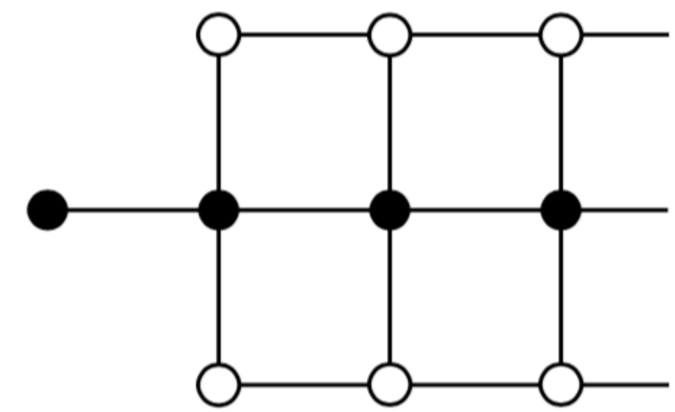
Original magnon TBA (massive + chemical potential)

$$\log Y_1 = L \log g^2 - L\epsilon + \mathcal{K} * \log(1+Y_1) + \ldots$$

$$\Delta = L - \sum_{a=1}^{\infty} \int \frac{du}{\pi} \log(1+Y_a)$$

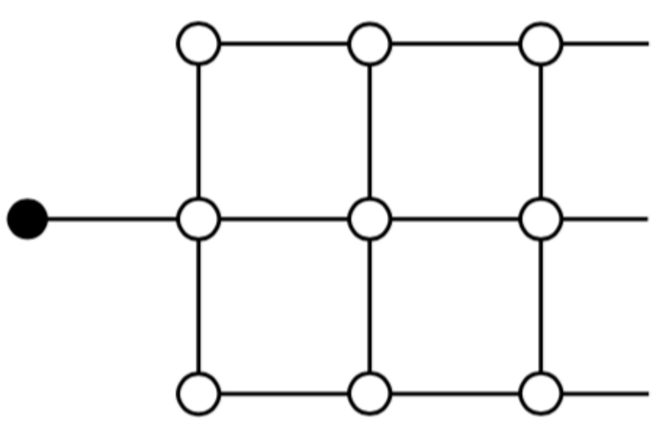
input : coupling $\log g^2$

output : scaling dimension Δ



massive TBA

(black nodes = energy carriers)



massless TBA

(only 1 momentum carrier)

Dual TBA (massless + no chemical potential)

$$\log Y_1 = LE - K_{O(6)} * \log(1+1/Y_1) + \ldots$$

$$\log g^{2L}/g_{cr}^{2L} = -\int \frac{d\theta}{2\pi} P'(\theta) \log(1+1/Y_1)$$

Δ = input (label tachyon rep)

$\log g^2$ = output (sigma model energy)

Finite size effects : central charge

TBA analysis in CFT limit (1/L effect = Casimir energy)

[Zamolodchikov'90s]
[Klassen-Melzer'90s]

$$L \gg 1 \qquad \Delta = O(1)$$

Problem split into left and right movers = scale-invariant (kink) solution
Kink is characterized by its asymptotic values on far its left and far right

Standard di-log analysis yields TBA central charge $c = c_0 - c_\infty$

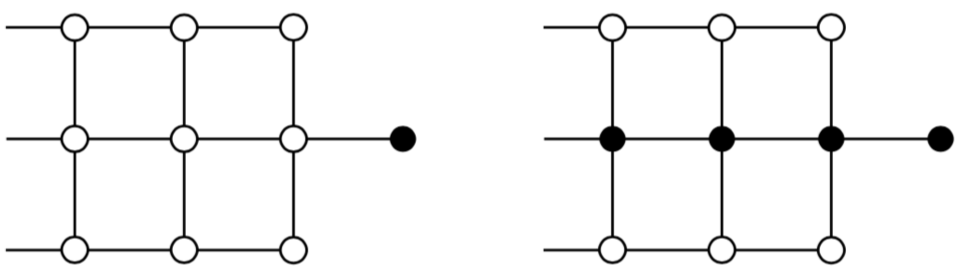
where $$c_\star = \sum_i \mathcal{L}\left(\frac{Y_i^\star}{1+Y_i^\star}\right)$$

with Rogers dilogarithm $$\mathcal{L}(x) = \frac{6}{\pi^2}\left(\mathrm{Li}_2(x) + \frac{1}{2}\log x \log(1-x)\right)$$

Stationary solutions to TBA are known

see e.g.
[Balog,Hegedus'04]

symmetric phase
O(6)
$c_0 = 7$



broken phase
O(4)
$c_\infty = 2$

TBA central charge: $c = 5$

Finite size effects : central charge

CFT analysis : close to IR fixed point, i.e. large L, the 2d CFT gives information about the behaviour of the energy levels

Operator-state correspondence: energy maps to 2d anomalous dimension of vertex operator (here tachyon)

$$V_\Delta \sim e^{-i\Delta t} \qquad\qquad E_{2d} = -\frac{\pi c_{eff}(L)}{6L} - \frac{e^2\Delta(\Delta-d)}{2L} + O(e^4)$$

running coupling at distance L: $e^2 \sim \dfrac{2\pi}{d\log L} \ll 1$

effective central charge at distance L: $c_{eff}(L) = d+1+O(e^2)$

(count the number of Goldstone bosons = dimensions of AdS_{d+1})

Agreement with TBA (for d=4 ie AdS5)

Spinning the wheels

Operators with spin $\mathrm{tr}\,\partial^M \phi_1^L$

Scaling dimension at weak coupling $\Delta = L + M + O(g^{2L})$

Conformal primaries map to solutions of Bethe equations for non-compact spin chain

$$1 = \left(\frac{v_k - i/2}{v_k + i/2}\right)^L \prod_{j\neq k}^{M} \frac{v_k - v_j - i}{v_k - v_j + i} \times e^{i\Phi_k}$$

dressing factor = long-range corrections from wheels

Anomalous dimension is obtained as before

(with the Y's solving TBA eqs with extra source terms from the v's)

$$\gamma = -\sum_{a\geqslant 1} \int \frac{du}{\pi} \log\left(1 + Y_a(u)\right)$$

We can repeat the same game as before and dualize the equations

Spinning the wheels

Dual energy formula

$$L \log g^2/g_{cr}^2 = \sum_{i=1}^{M} E(\theta_i) - \int \frac{d\theta}{2\pi} P'(\theta) \log\left(1 + 1/Y_1(\theta)\right)$$

mechanical energy of transverse excitations $+$ "vacuum" or "center-of-mass" energy

Transverse energy is positive, while vacuum energy is negative, in agreement with signature of (Minkowskian) AdS space

Dual Bethe equations (neglecting effects triggered by tachyon background)

$$e^{iP(\theta_k)L} \prod_{j \neq k}^{M} S_{O(6)}(\theta_k - \theta_j) = 1$$

Same equations as for O(6) if not for the momentum $P(\theta) = \mp \frac{m}{2} e^{-|\theta|}$

(low momentum = large rapidity)

Spinning the wheels

Here again we find relation to the compact sigma model. The only difference is that the momentum decreases at large rapidity in our case. This again has the effect of flipping the sign of the coupling.

Hence, to any order in PT we expect agreement with sigma model analysis, at least as long as BAEs are applicable

In the sigma mode these states should correspond to vertex operators of the type

$$V_{\Delta,M,N} \sim \partial^N (Y_1 + iY_2)^M \times e^{-i\Delta t}$$

with number of 2d light-cone derivatives mapping to the level N

Summary II

Conformal fishnet theory: laboratory for applying integrability to correlators, etc.

Duality: fishnet graphs define an integrable lattice regularization of the 2d AdS5 sigma model

Dual sigma model description is weakly coupled when fishnet lengths are large

i.e. large L and "small" quantum numbers = low worldsheet energy

String?

String worldsheet?

Marginality condition of sort $0 = L\mu + E_{2d}(L)$

with cosmological constant $\mu = \log g_{cr}^2 / g^2$

On-shell condition comes from the geometric sum over the wheels

$$\sum_{T \geqslant 0} (g/g_{cr})^{2LT} e^{-TE_{2d}(L,\Delta)} = \frac{1}{1 - (g/g_{cr})^{2L} e^{-E_{2d}(L,\Delta)}}$$

with T acting as a discrete proper time (Schwinger parameter)

Non-critical string with tunable intercept exists in flat space,
at least classically

Could the conformal fishnet be an AdS version of it?

THANK YOU!