What's new in G_2 ?

IPMU Tokyo, 30th October 2018

Andreas Braun

UNIVERSITY OF OXFORD

[1602.03521]
[1708.07215] + [1803.10755] with Sakura Schäfer-Nameki
[1701.05202] + [1712.06571] with Michele del Zotto
[1803.02343] with Michele Del Zotto, James Halverson, Magdalena Larfors, David R. Morrison, Sakura Schäfer-Nameki
+ upcoming ...

Broad challenge to string theorists:
what is the 'landscape' of 4D $\mathcal{N} = 1$ string vacua ??

- what can we build from string theory ?
- what can we not build (with ot without gravity ...swampland...) ?
- what can we learn ? Dualities ?
- $\mathcal{N} = 0$?!

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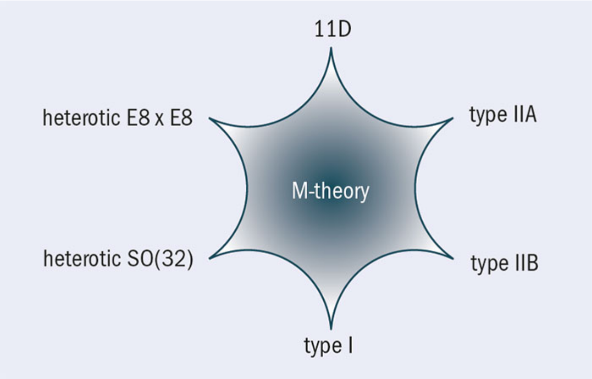
Easier in 'clean' setups with fewer ingredients, e.g. :

- orbifolds
- F-Theory

starting points:



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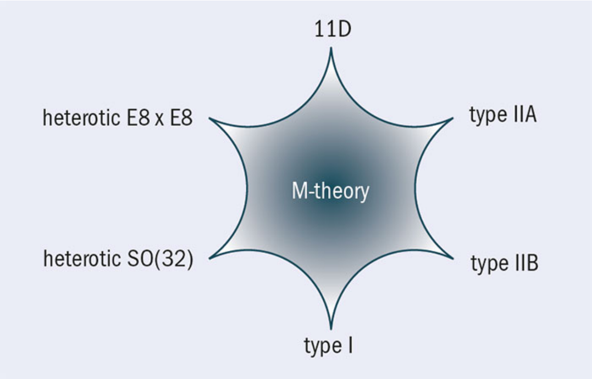

from the central node of M-theory/11D SUGRA and without sources for $T_{\mu\nu}$, obtain 4D $\mathcal{N} = 1$ by compactifying on a 7D manifold with

- a Ricci-flat metric
- a single covariantly constant spinor

G_2 manifolds

A G_2 manifold M is a 7D Riemannian manifold which allows a metric $g_{\mu\nu}$ with holonomy group G_2

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btw: manifolds of reduced holonomy (..simply connected..) which allow Ricci-flat metrics and cov. const. spinors:

	n	hol(g_M)	# cov.const spinors
Calabi-Yau	$2m$	$SU(m)$	2
Hyper-Kähler	$4m$	$Sp(m)$	$m+1$
$\mathbf{G_2}$	$\mathbf{7}$	$\mathbf{G_2}$	$\mathbf{1}$
Spin(7)	$\mathbf{8}$	**Spin(7)**	$\mathbf{1}$

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Moduli space of Ricci-flat metrics has real dimension $b^3(X)$ ... think of it as '$\delta\Phi_3$'.

M-Theory on G_2 manifolds

- everything is geometry !
- Compactifications of M-Theory: $b^3(X)$ 4D $\mathcal{N}=1$ chiral multiplets:

$$z_i = \int_{\Sigma_i} \Phi_3 + iC_3$$

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- Gauge Theory data: singularities

	codimension
ADE gauge group	4
non-chiral charged matter	6
chiral charged matter	7

- superpotential W from M2-brane instantons on associative three-cycles $\sim$ homology 3-spheres [Harvey, Moore '99]

we want detailed examples !

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Summary:

1. TCS: making smooth compact G_2 manifolds
2. heterotic duals of M-Theory on G_2
3. type II theories on G_2: mirror symmetry
4. bonus: Spin(7)

Making compact Calabi-Yau manifolds

A Calabi-Yau manifold X is a complex Kähler manifold with holomy group $SU(n)$ for $\dim_{\mathbb{C}} X = n$.

- there exists a Ricci-flat metric
- there are two covariantly constant spinors
- there are two independent calibrating forms ω and $\Omega^{n,0}$
- Yau's proof of the Calabi conjecture: X Kähler is Calabi-Yau iff $c_1(X) = 0$

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It is easy to make examples using algebraic geometry; use adjunction formula

$$c_1(X) = c_1(A) - c_1(L)$$

for hypersurfaces to compute $c_1(X)$.

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$$x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 + \cdots = 0 \qquad \subset \mathbb{CP}^4[x_1 : x_2 : x_3 : x_4 : x_5]$$

is a Calabi-Yau threefold.

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More fancy machinery: use reflexive polytopes to construct Calabi-Yau hypersurfaces (or complete intersections) in toric varieties.

$$\langle \Delta, \Delta^\circ \rangle \geq -1$$

Making compact G_2 manifolds

is harder than just doing complex algebraic geometry ...

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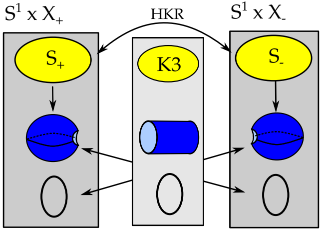
... classic Method: resolutions of orbifolds T^7/Γ [Joyce '96]

Making compact G_2 manifolds

New method: twisted connected sums (TCS): [Kovalev'03, Corti, Haskins, Nordström, Pacini '13]

$$M = \left[X_+ \times \mathbb{S}^1_+\right] \;\#\; \left[X_- \times \mathbb{S}^1_-\right]$$

for a pair of asymptotically cylindrical Calabi-Yau threefolds $S_\pm \to X_\pm \to_{\pi_\pm} \mathbb{C}$ with $\left[X_+ \times \mathbb{S}^1_+\right] \cap \left[X_- \times \mathbb{S}^1_-\right] = K3 \times \mathbb{S}^1 \times \mathbb{S}^1 \times I$ and $\phi : S_+ \leftrightarrow S_-$



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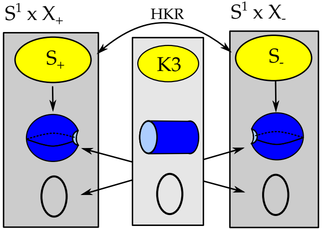

- $\exists$ millions of examples [Corti et al'13; AB'16], easy to make and work out $H^k(M, \mathbb{Z})$
- For M-Theory on M, there are $\mathcal{N} = 2$ and $\mathcal{N} = 4$ subsectors in the stretching limit [C. da Guio, Jockers, Klemm, Yeh '17; AB, del Zotto '17]

building blocks and cohomology

The acyl Calabi-Yau threefolds X can be constructed from compact ‘building blocks’ Z:

$$\begin{aligned} S \to Z \to \mathbb{P}^1 \\ c_1(Z) = [S] \end{aligned} \qquad \text{as} \qquad X = Z \setminus S_0$$

There is a natural restriction map

$$\rho : H^{1,1}(Z) \to H^{1,1}(S) \qquad \begin{aligned} N &\equiv \mathrm{im}(\rho) \\ K &\equiv \ker(\rho)/[S] \end{aligned}$$

then

$$\begin{aligned} H^2(M,\mathbb{Z}) &= N_+ \cap N_- \oplus K(Z_+) \oplus K(Z_-) \\ H^3(M,\mathbb{Z}) &= \mathbb{Z}[S] \oplus \Gamma^{3,19}/(N_+ + N_-) \oplus (N_- \cap T_+) \oplus (N_+ \cap T_-) \\ &\quad \oplus H^3(Z_+) \oplus H^3(Z_-) \oplus K(Z_+) \oplus K(Z_-) \end{aligned}$$

$$b^2(M) + b^3(M) = 23 + 2\left[|K_+| + |K_-| + h^{2,1}(Z_+) + h^{2,1}(Z_-)\right]$$

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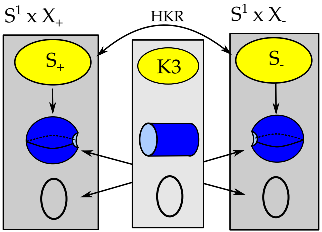

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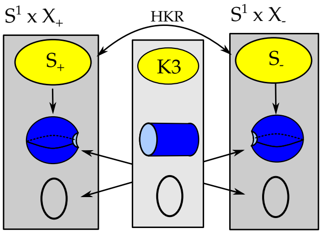

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$\rightarrow$ **use power of string dualities !** $\leftarrow$

Duality to heterotic strings

If G_2 manifolds are equipped with a (calibrated) $K3$ fibration, can apply fibrewise version of 7D duality between M-Theory and heterotic strings [Duff, Nilsson, Pope '83 '86, Witten '95]

$$\begin{array}{rcr} \text{heterotic} & \leftrightarrow & \text{M-Theory} \\ T^3 & \leftrightarrow & K3 \end{array}$$

to find SYZ fibration of a Calabi-Yau threefold + holomorphic vector bundles [Papadopoulos, Townsend '95; Harvey,Lowe, Strominger '95, Acharya '96; Harvey, Moore '99, Acharya, Witten '01, Gukov, Yau, Zaslow '02].

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TCS G_2 have K3 fibrations over S^3, dual heterotic geometry is always the 'Schoen' Calabi-Yau threefold $X_{19,19}$ with different bundles [AB, Schäfer-Nameki '17] !

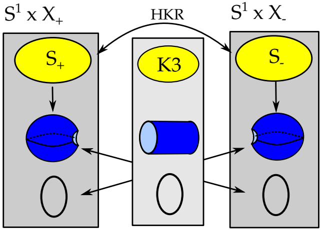
$$\left[\begin{array}{cc|c} 3 & 0 & \mathbb{P}^2 \\ 0 & 3 & \mathbb{P}^2 \\ 1 & 1 & \mathbb{P}^1 \end{array}\right]$$

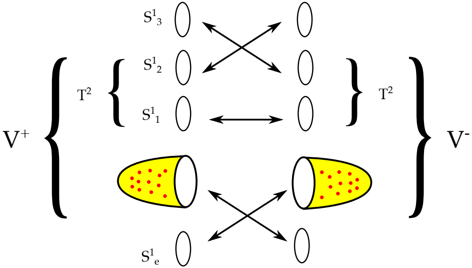
Many of these have an F-theory dual on X_4; **can prove equivalence of light fields !**

$$E \rightarrow X_4 \quad \rightarrow_{\pi_F} \quad \widetilde{dP_9 \times \mathbb{P}^1}$$

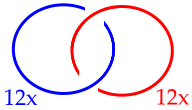
a few details

In hetetoric – M-Theory duality $\mathbb{S}^1_I$ of $T^3 \sim \omega_I$ of K3; $\omega_I \subset H^2(S) = [U^3 \oplus -E_8^{\oplus 2}] \otimes \mathbb{R}$
For each building block Z, only two out of the three forms ω_I vary non-trivially:





This gives a decompostion of $X_{19,19}$ as two copies of $[dP_9 \setminus T^2] \times T^2$ and shows the SYZ fibration; its discriminant is (also found in [Gross '04, Morrison, Plesser '15])



an example

Consider a generic Weierstrass elliptic fibration

$$E \to X_4 \quad \to_{\pi_F} \quad dP_9 \times \mathbb{P}^1 \,.$$

The topological data of X_4 is

$$h^{1,1}(X_4) = 12 \qquad h^{2,1}(X_4) = 112 \qquad h^{3,1}(X_4) = 140 \qquad \chi(X_4) = 288$$

so we need to include 12 space-filling D3-branes. Spectrum of F-Theory on X_4 is

$$n_v = 12 \qquad\qquad n_c = 11 + 112 + 140 + 3 \cdot 12 = 299$$

The dual M-Theory geometry is made from building blocks with

$$\begin{aligned} b^3(Z_+) &= 112 & \quad N(Z_+) &= U & \quad K(Z_+) &= 0 \\ b^3(Z_-) &= 20 & \quad N(Z_-) &= U \oplus E_8 \oplus E_8 & \quad |K(Z_-)| &= 12 \end{aligned}$$

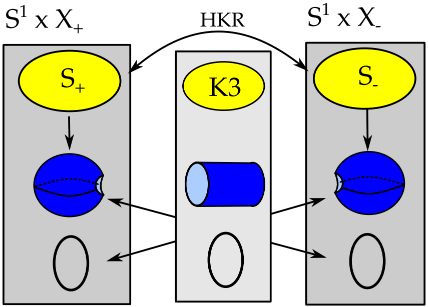
and

$$H^2(M,\mathbb{Z}) = N_+ \cap N_- \oplus K(Z_+) \oplus K(Z_-) = \mathbb{Z}^{12}$$

$$b^2(M) + b^3(M) = 23 + 2\left[|K_+| + |K_-| + h^{2,1}(Z_+) + h^{2,1}(Z_-)\right] = 23 + 2(12 + 112 + 20) = 311$$

Lessons:

- can systematically engineer codim $= 4$ and 6 singularities in TCS G_2 manifolds
- sometimes the gauge groups are (geometrically) non-Higgsable
- no chiral matter, i.e. no codim $= 7$ singularities ...



F-Theory on the Calabi-Yau fourfold

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has infinitely many corrections $\cong E_8$ to the superpotential from rigid divisors [Donagi, Grassi, Witten '96]. (I do not think they sum to an E_8 Θ-function though ... spoiled by D3s)

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aside: For heterotic on $X_{19,19}$, these are world-sheet instantons on rigid curves $\cong E_8 \oplus E_8$ [Curio, Lüst '97]; does evade the [Beasley, Witten '03] cancellation as $X_{19,19}$ is far from *favorable*; every appearing curve class has a unique holomorphic representative !

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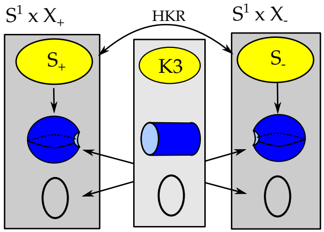

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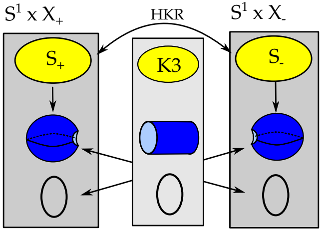

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Associatives are sections of coassociative fibration by T^4 !

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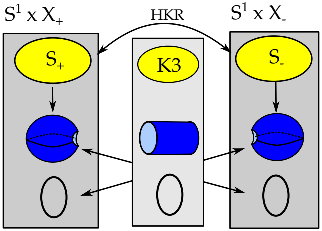

SYZ/elliptic fibrations on $X_\pm$ lift to T^3 and T^4 fibrations on G_2 manifold M and construction of multiple mirror duals [AB '16; AB, del Zotto, '17], can prove invariance of $b^2(M) + b^3(M)$ and equivalence with CFT results for orbifolds [Gaberdiel,Kaste '04]

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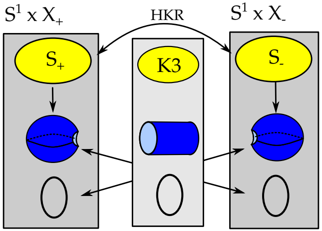

SYZ/elliptic fibrations on $X_\pm$ lift to T^3 and T^4 fibrations on G_2 manifold M and construction of multiple mirror duals [AB '16; AB, del Zotto, '17], can prove invariance of $b^2(M) + b^3(M)$ and equivalence with CFT results for orbifolds [Gaberdiel,Kaste '04]
cute aside: tor$H^3(M, \mathbb{Z}) =$ tor$H_2(M, \mathbb{Z})$ matches discrete torsion of CFT

dual tops

Calabi-Yau manifolds with $c_1(X) = 0$ can be constructed from reflexive polytopes

$$\langle \Delta, \Delta^\circ \rangle \geq -1 \, .$$

Swapping the roles of Δ, Δ° gives the mirror map !

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In complete analogy, building blocks Z with $c_1(Z) = [S]$ can be constructed from 'tops'

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which are 'halves' of reflexive polytopes [AB '16].Exchanging $\Diamond \leftrightarrow \Diamond^\circ$ gives the mirror $X^\vee = Z^\vee \setminus S^\vee$ of $X = Z \setminus S$, find:

$$K(X^\vee) = h^{2,1}(X)$$

so that

$$b^2(M) + b^3(M) = 23 + 2\left[K_+ + K_- + H^{2,1}(Z_+) + H^{2,1}(Z_-)\right]$$

is invariant [AB, del Zotto '17,'18].

lessons/future

- smooth G_2s can have singular mirrors/works similar to K3 mirror symmetry: B-field makes states massive !
- can actually show that $H^\bullet(M, \mathbb{Z})$ is preserved by mirror map of [AB, del Zotto '17,'18]
- future: use mirrors to count (co)associatives, spectra of D-branes;
- future: relation to 3D $\mathcal{N} = 2$ field theories !?

Spin(7)

engineer 3D $\mathcal{N} = 1$ theories via M-Theory.

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of an acyl Calabi-Yau fourfold $CY_3 \to X_4 \to \mathbb{C}$ and an acyl G_2 manifold M with $X_4 \cap M = CY_3 \times \mathbb{S}^1 \times I$.

- These are duals of heterotic models on TCS G_2 manifolds; recover T^3 fibration appearing in mirror maps !
- We checked the equivalence of light fields in examples !

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- For many of examples, these are simply resolution of quotients of compact CY_4.
- Learn how to construct vector bundles on G_2 manifolds and singular Spin(7) manifolds
- Decomposition shows existence of subsectors with enhanced SUSY: $\mathcal{N} = 2$ and $\mathcal{N} = 4$ in 3D
  $\to$ F-Theory on Spin(7) ? [Vafa'96; Grimm, Bonetti '13] Relation to [Witten '94,'95] ?

Summary

New examples, results and conjectures on compact manifolds of exceptional holonomy $=$ M-theory compactifications to 3D and 4D with minimal SUSY.

Geometries made by gluing ‘simpler’ pieces.

... different theories/dualities teach different lessons which nicely tie into each other ...

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Thank you !