BLACK HOLE ENTROPY AND (0,4) SCFTs FROM F-THEORY

MS seminar, Kavli IPMU Tokyo

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Kilian Mayer

Utrecht University

based on arXiv 1808.05228 [T.W. Grimm, H. het Lam, KM, S. Vandoren]

work in progress: [C. Couzens, T.W. Grimm, H. het Lam, KM, S. Vandoren]

related work: 1509.00455 [Haghighat, Murthy, Vafa, Vandoren]

1705.04679 [Couzens, Lawrie, Martelli, Schäfer-Nameki, Wong]

OUTLINE

Part I

A. What and Why?

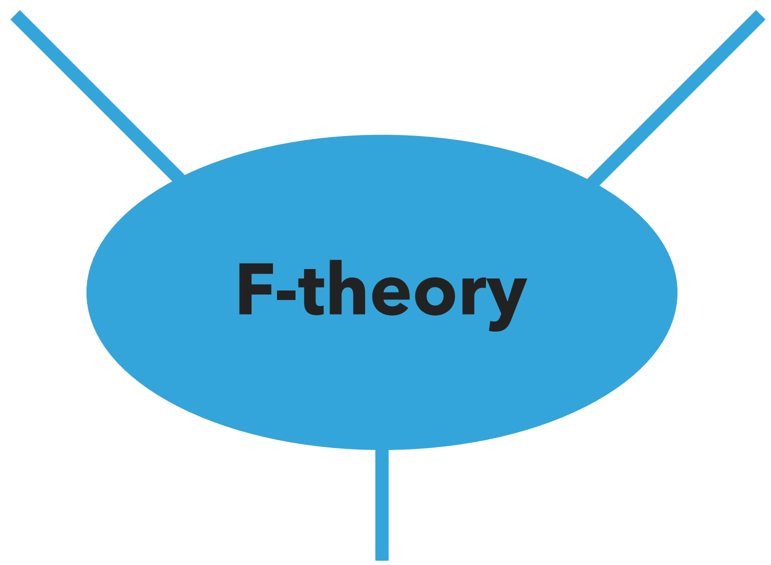
A. WHAT AND WHY?

study 4d and 5d SUSY black holes within the framework of F-theory

Construct SCFTs:
6d (1,0), 4d $\mathcal{N}$=3,

2d SCFTs, …

phenomenology,
GUT model building



QG in F-theory:
black holes, AdS/CFT, QG conjectures

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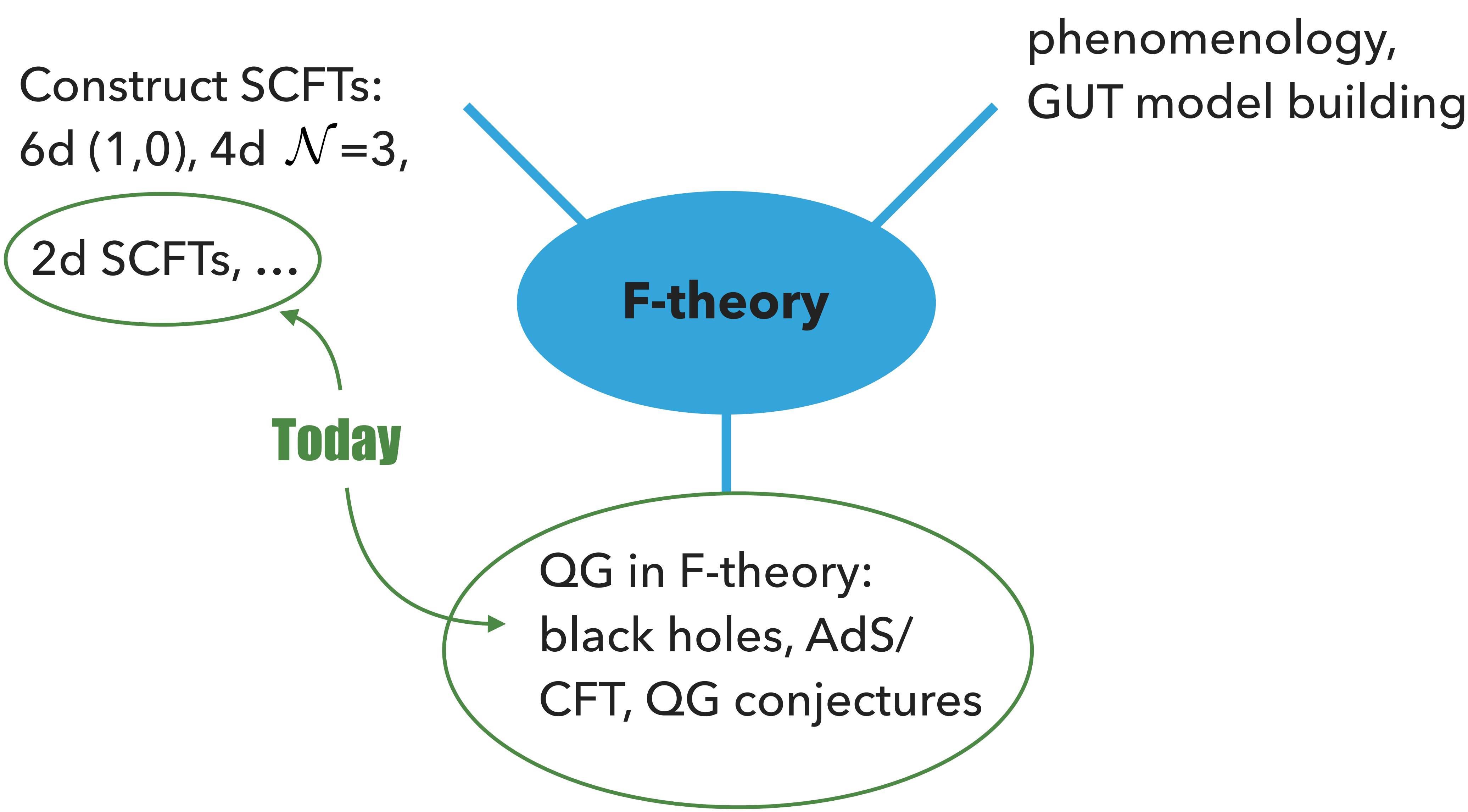

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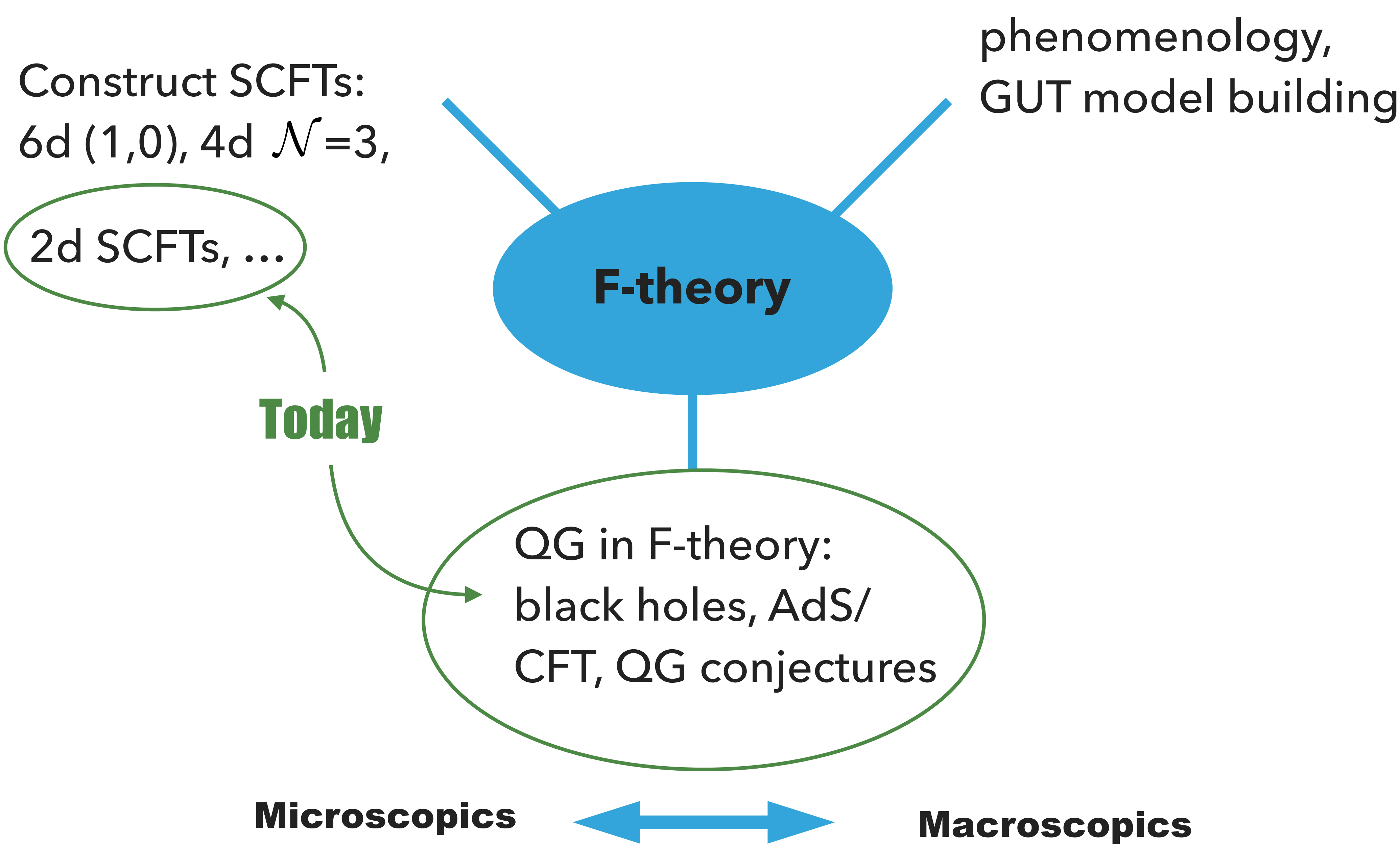

B. F-theory preliminaries

B. F-THEORY PRELIMINARIES

Why F-theory?

Type IIB model building: Type IIB orientifolds with D3/D7 and O3/O7 planes

(i) perturbative string theory $\Rightarrow g_s = \mathrm{e}^{\phi} \ll 1$

(ii) probe approximation: no back-reaction of branes

Is this a good (enough) starting point to construct string compactifications?

→ Yes, at least under certain limiting assumptions!

(asymptotically back-reaction negligible, large volume, D7/O7 on top)

→ Way out: **F-theory**

B. F-THEORY PRELIMINARIES

A problem with D7-brane back-reaction:

BPS solution of Type IIB supergravity for p-branes (p < 7)

$$\mathrm{d}s^2 = H_p^{-\frac{1}{2}} \eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + H_p^{\frac{1}{2}} \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j$$

$$\mathrm{e}^{2\phi} = \mathrm{e}^{2\phi_0} H_p^{\frac{3-p}{2}} , \qquad H_p = 1 + \left(\frac{r_p}{r}\right)^{7-p}$$

For p = 7 (i.e. codimension 2) separate analysis needed

[Greene, Shapere, Vafa, Yau `90]

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back-reaction!!

For p = 7 (i.e. codimension 2) separate analysis needed

[Greene, Shapere, Vafa, Yau `90]

- D7-brane in the background $\underline{\mathbb{R}^{1,7}} \times \mathbb{C}$
- D7 bane couples to RR 0-form C_0 magnetically

Constraints from SUSY and equations of motions:

$$\bar{\partial}\tau = 0\,, \qquad \tau = C_0 + i\,\mathrm{e}^{-\phi}$$

$$\mathrm{d}\star F_9 = \delta_{\mathrm{D7}}\,, \qquad \star F_9 = \mathrm{d}C_0$$

Analysis in [Greene, Shapere, Vafa, Yau `90] shows: back-reaction of D7-branes not negligible, $\tau(z)$ generically strongly varying

Integrate around D7: $$\int_{\mathbb{C}} \mathrm{d} \star F_9 = \int_{S^1_{\mathrm{D7}}} \star F_9 = \int_{S^1_{\mathrm{D7}}} \mathrm{d}C_0 = 1$$

Solution close to the D7: $$\tau(z) = \tau_0 + \frac{1}{2\pi i} \log z + \dots$$

$$\Rightarrow \tau \to \tau + 1$$ **Monodromy**

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Type IIB invariant under $SL(2,\mathbb{R}) \xrightarrow{\mathrm{D}(-1)} SL(2,\mathbb{Z})$

$$SL(2,\mathbb{Z}): \quad \tau \to \frac{a\tau + b}{c\tau + d}, \qquad \begin{pmatrix} C_2 \\ B_2 \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} C_2 \\ B_2 \end{pmatrix}$$

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Type IIB invariant under $SL(2,\mathbb{R}) \xrightarrow{\mathrm{D}(-1)} SL(2,\mathbb{Z})$

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

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Monodromy by encircling D7 symmetry of the theory!

Need to consider [p,q] 7-branes in Type IIB/F-theory

[p, q] 7-brane induces a monodromy transformation

$$M_{p,q} = \begin{pmatrix} 1-pq & p^2 \\ -q^2 & 1+pq \end{pmatrix} \in SL(2,\mathbb{Z})$$

worldvolume action of [p,q] 7-branes unknown

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Insight in [Vafa `96] **:**

Type IIB axio-dilaton $\tau_{\rm IIB}$ ⟷ complex structure of T^2

$SL(2, \mathbb{Z})$ in Type IIB ⟷ modular group of T^2

- Interpret axio-dilaton $\tau_{\rm IIB}$ as complex structure of an additional *auxiliary* T^2
- back-reaction of 7-branes induces profile for $\tau_{\rm IIB}$

  ➡ non-triviality of the T^2 – fibration
- however: no 12d origin/interpretation of these T^2 dimensions (known)

  (no suitable 12d supergravity exisits, $\mathrm{vol}(T^2)$ not in the 10d effective action, ...)

F-THEORY / M-THEORY DUALITY

Various definitions of F-theory:

- Type IIB with varying τ and 7-branes
- sometimes via F-theory/heterotic duality
- F-theory via M-theory

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M-theory on $\mathbb{R}^{1,8} \times T^2$ $\quad\quad (T^2 = S^1_A \times S^1_B)$

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F-theory via M-theory

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$\downarrow$ reduce along S^1_A, $R_A \to 0$

weakly coupled Type IIA on $\mathbb{R}^{1,8} \times S^1_B$ with $g_s^{(\mathrm{IIA})} = \dfrac{R_A}{\ell_s}$

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↓ T-duality along S^1_B

Type IIB on $\mathbb{R}^{1,8} \times \tilde{S}^1_B$ with $\tilde{R}_B = \dfrac{\ell_s^2}{R_B}$ and $\mathrm{Im}\,\tau = \dfrac{R_A}{R_B}$

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Type IIB on $\mathbb{R}^{1,9}$

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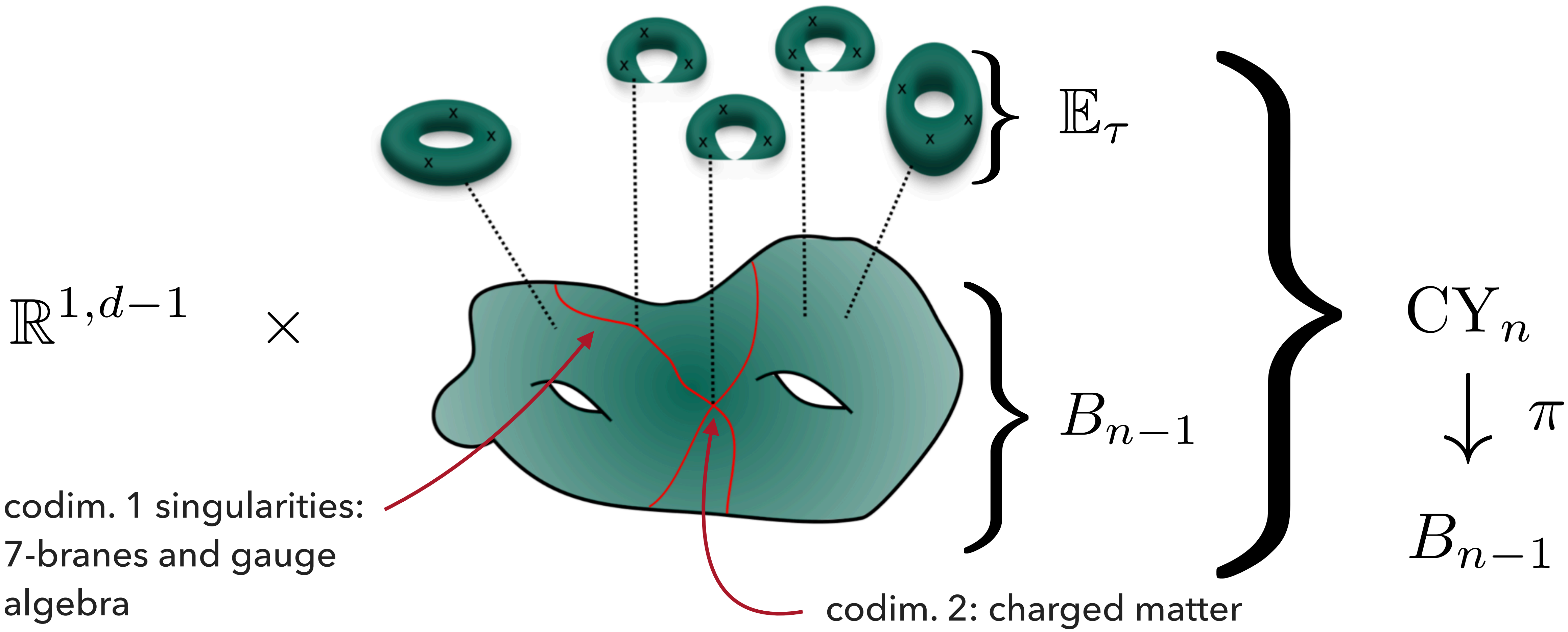
Type IIB on $\mathbb{R}^{1,9}$

For non-trivial fibration apply this duality fiber-wise!

F-THEORY ON CALABI-YAU MANIFOLDS



fibration	SUSY F-theory	SUSY M-theory
CY_2=K3	8d $\mathcal{N}=1$ (16)	7d $\mathcal{N}=2$ (16)
CY_3	6d $\mathcal{N}=(1,0)$ (8)	5d $\mathcal{N}=2$ (8)
CY_4	4d $\mathcal{N}=1$ (4)	3d $\mathcal{N}=2$ (4)
CY_5	2d $\mathcal{N}=(2,0)$ (2)	1d $\mathcal{N}=2$ SQM (2)

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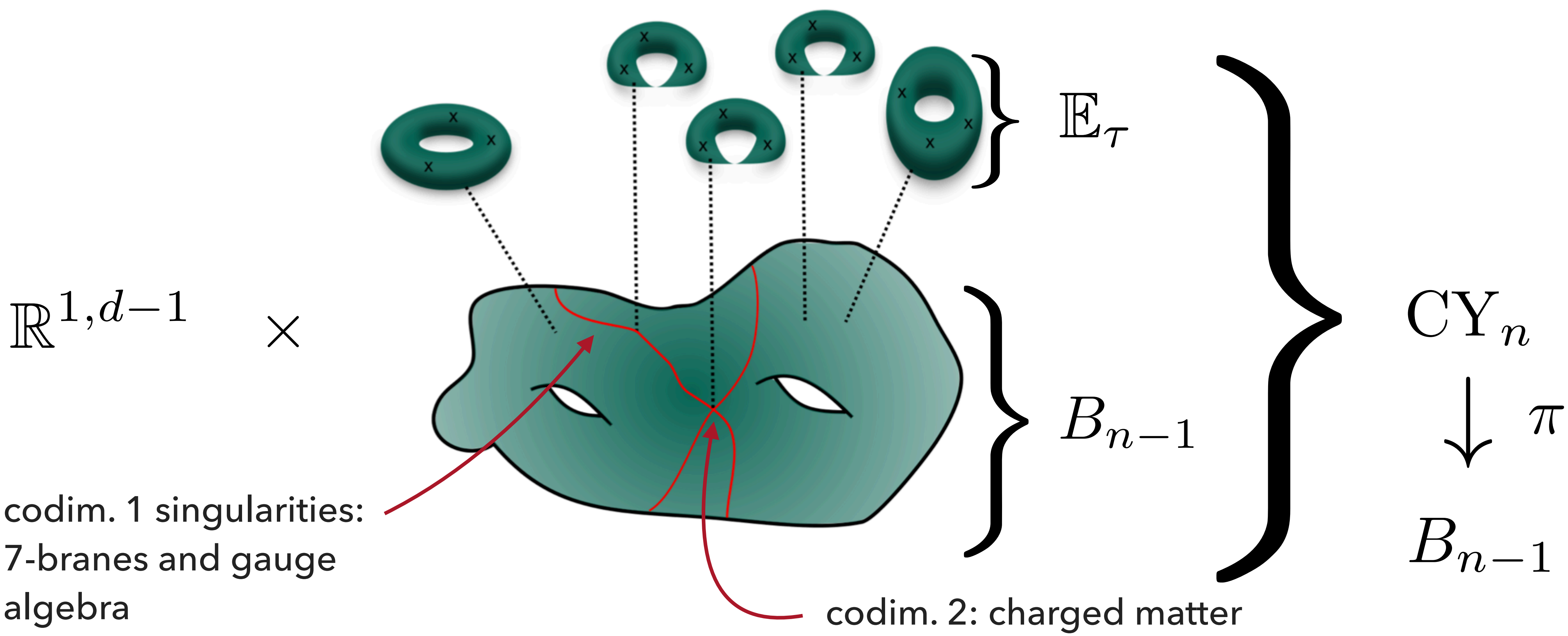

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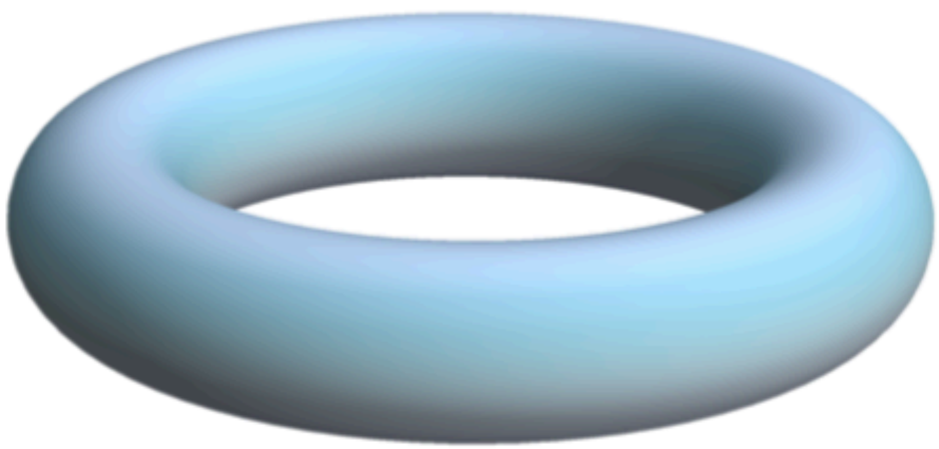
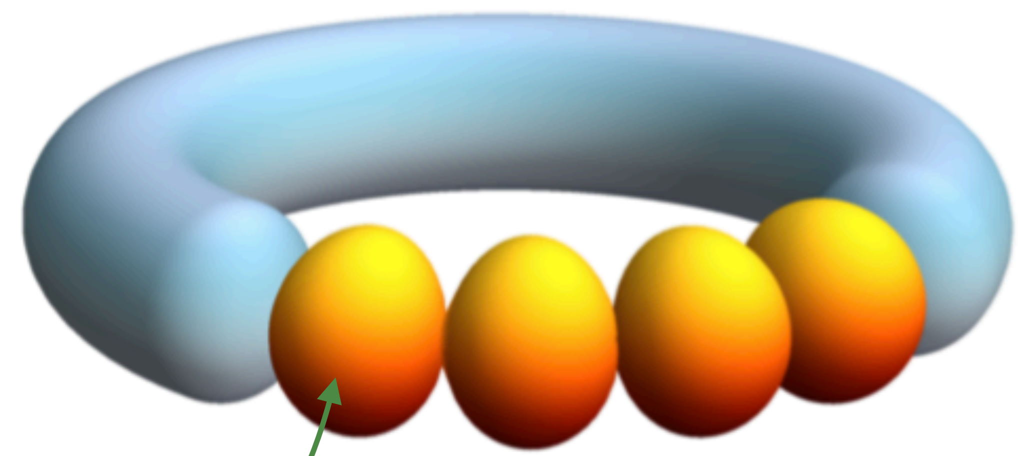


smooth fiber



resolution $\mathbb{P}^1$`s

resolved fiber

$\mathbb{P}^1$ `s have an intersection pattern as the nodes of affine Dynkin diagrams

→ intersection pattern dictates gauge algebra

F-THEORY ON CALABI-YAU MANIFOLDS

F-theory on X_{n+1}

reduce on S^1
and push to the
Coulomb branch

integrate out
massive KK modes
and W-bosons

M-theory on resolved X_{n+1}
at large volumes

d-dim. eff. action for
CY-zeromodes of $\hat{g}$, $\hat{C}_3$

F-theory on $X_{n+1} \times S^1$

d-dim. eff. action for
S^1-zeromodes and Cartan $U(1)$s

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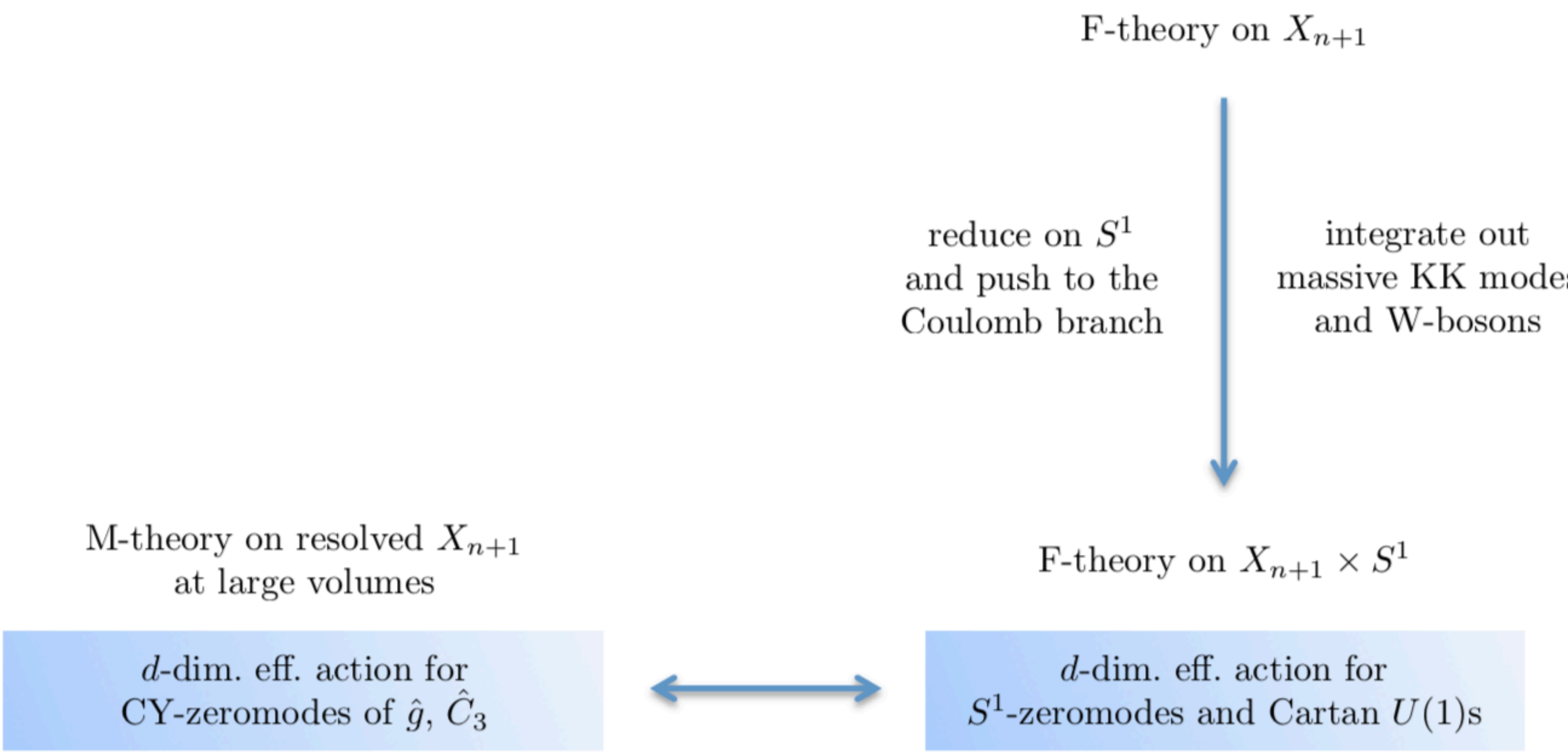

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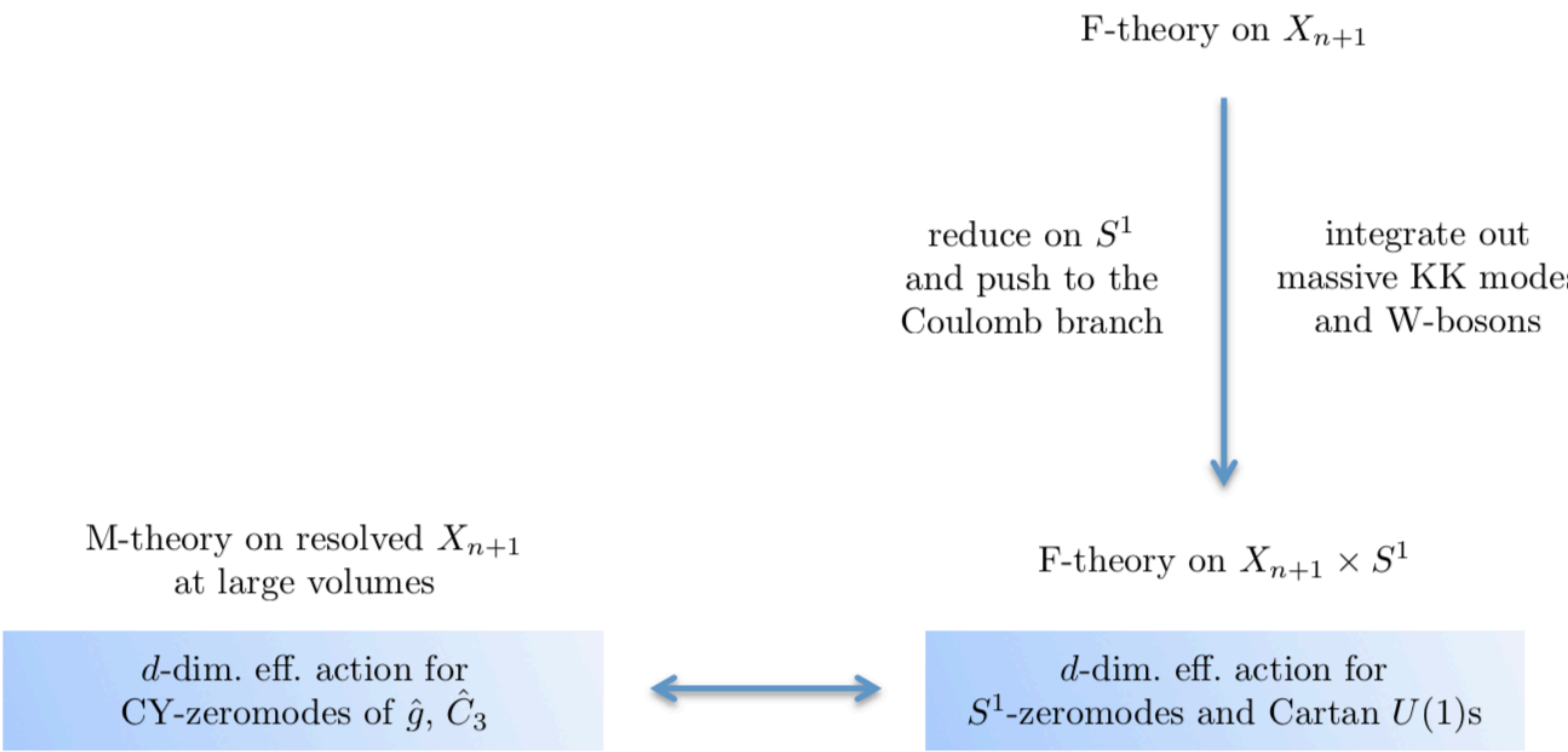

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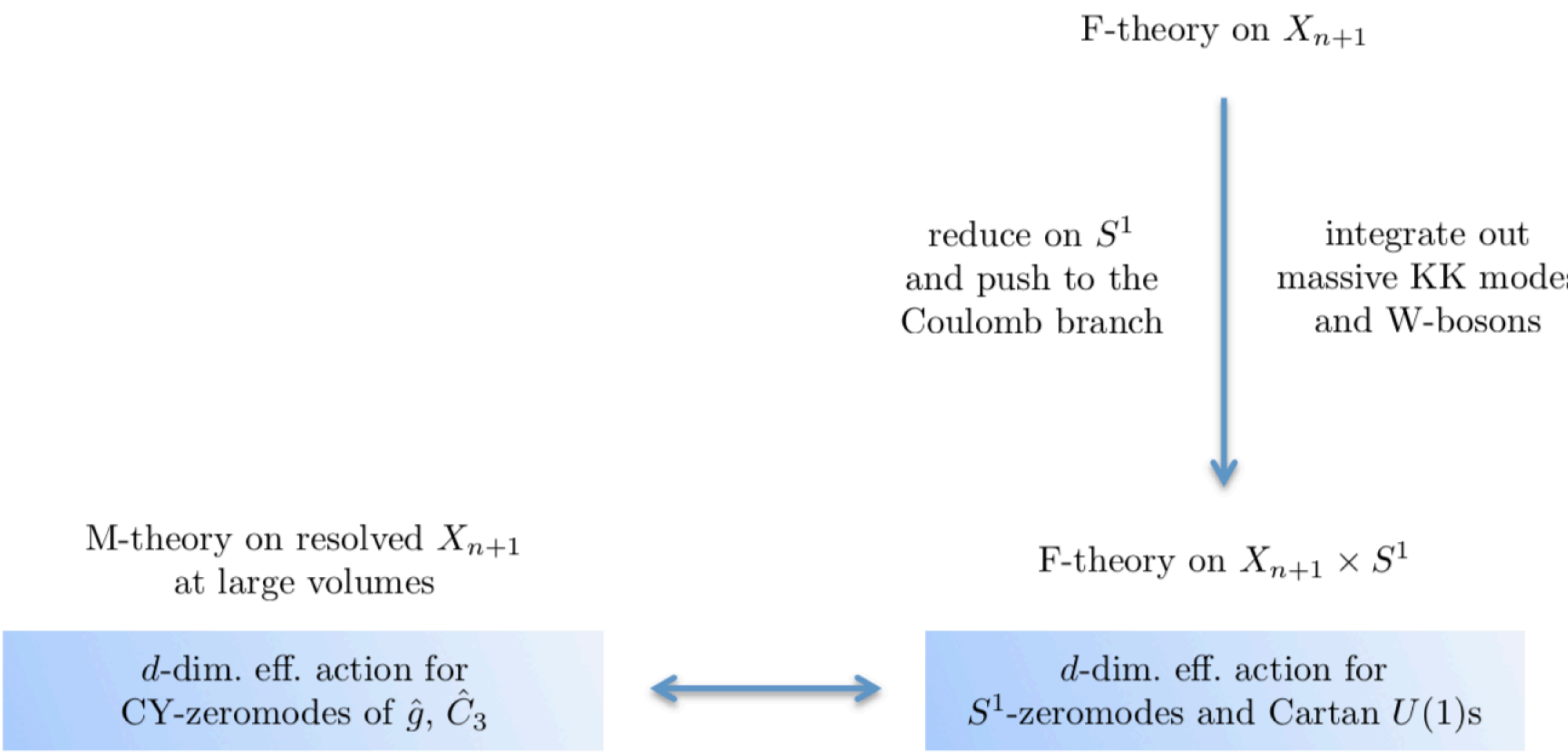

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- M2-branes wrapping $\mathbb{E}_\tau$: Kaluza-Klein modes

F-THEORY IN A NUTSHELL

mathematics of elliptic fibrations —map to→ physics of F-theory

$\simeq$ Type IIB string theory with varying axio-dilaton

Holy grail: establish *dictionary* between physics and mathematics

Physics of effective theory in $\mathbb{R}^{1,9-2n}$	Geometry of elliptic fibration Y_{n+1}
non-abelian gauge algebra	codim.-one singular fibers
localised charged matter representation localised uncharged matter	codim.-two singular fibers $\mathbb{Q}$-factorial terminal singularities in codim. two
triple Yukawa interactions (4d/2d) quartic Yukawa interactions (2d)	codim.-three singular fibers codim.-four singular fibers
abelian gauge algebra global structure of gauge group	free part of Mordell-Weil group torsional part of Mordell-Weil group

[from TASI lectures on F-theory, Weigand `18]

B. 5D BLACK HOLES FROM D3-BRANES

- F-theory on $\mathbb{R}^{1,5} \times \mathrm{CY}_3$: 6d supergravity with $\mathcal{N} = (1,0)$ $\quad \mathrm{CY}_3 \xrightarrow{\pi} B_2$

An interesting sub-sector of the theory:

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An interesting sub-sector of the theory: D3-branes wrapped on $C \subset B_2$

- strings of 6d $(1,0)$ SCFTs [e.g. del Zotto, Lockhart, Vafa, Haghighat, Tachikawa, Shimizu ...]
- excitations of 6d strings satisfy WGC and SDC in highly non-trivial way [Lee, Lerche, Weigand `18]
- 5d spinning black holes $\mathrm{AdS}_3/\mathrm{CFT}_{(0,4)}$

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$\mathrm{SO}(4)_\perp$

background: $\quad \mathbb{R}_t \times S^1 \times \mathbb{R}^4_\perp \times B_2$

	X^0	X^1	$X^2 \dots X^5$	X^6	X^7	X^8	X^9
D3	—	—	$\cdot$	—	—	$\cdot$	$\cdot$

MICROSCOPICS VS. MACROSCOPICS

- in IR: 2d $\mathcal{N} = (0,4)$ SCFT

  → $c_R = 6k_R$ $\mathrm{SU}(2)_R$ current algebra

- novel feature: identify $\mathrm{SU}(2)_L$ current algebra $\mathrm{SO}(4)_\perp = \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$
- entropy determined by Cardy's formula $S = 2\pi\sqrt{\frac{c_L}{6}\left(n - \frac{J^2}{k_L}\right)}$

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microscopics

- D3 brane on $C \subset B_2$ + D7's ➡ $\tau(z)$ varying on B_2
- worldvolume theory: $\mathcal{N} = 4$ SYM with varying coupling + monodromies
- supercharges transform under $SL(2,\mathbb{Z})$ ➡ *need generalization of top. twist*

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Topological duality twist [Martucci `14]

However: procedure misses D3-D7 modes

microscopics

dimensional reduction of D3-brane action and counting of left/right movers

[Haghighat, Murthy, Vafa, Vandoren `15]

[Lawrie, Schäfer-Nameki, Weigand `16]

$$c_R^{\mathrm{D3}} = 6k_R = 3C \cdot C + 3c_1(B_2) \cdot C$$
$$c_L^{\mathrm{D3}} = 3C \cdot C + c_1(B_2) \cdot C + \Delta c_L^{\mathrm{D3-D7}}$$
$$k_L = \frac{1}{2} C \cdot C - \frac{1}{2} c_1(B_2) \cdot C$$

notation: $C = q^\alpha \omega_\alpha \,, \qquad c_1(B_2) = c^\alpha \omega_\alpha \,, \qquad \omega_\alpha \in H^{1,1}(B_2, \mathbb{Z})$

$$A \cdot B = A^\alpha \eta_{\alpha\beta} B^\beta = \int_{B_2} A \wedge B$$

$$\eta_{\alpha\beta} = \int_{B_2} \omega_\alpha \wedge \omega_\beta$$

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dualize to M-theory!

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dualize to M-theory!

dual M-theory picture: M5 brane wrapping $\hat{C} = \pi^{-1}(C) \in H_4(\mathrm{CY}_3)$

M5 brane wrapping 4-cycle in CY_3 described by *MSW CFT*

[Maldacena, Strominger, Witten `97]

$$\Rightarrow \quad \Delta c_L^{\mathrm{D3-D7}} = 8c_1(B) \cdot C$$

macroscopics $\mathrm{AdS}_3 \times S^3 \times B_2$ computation...more later

Short summary

Today:

- generalize the setup in [Haghighat, Murthy, Vafa, Vandoren `15]
- compute CFT data from macroscopic side and match with microscopics
- express CFT data in terms of *geometric data* of CY_3

How: CFT data related to certain Chern-Simons coefficients

→ *calculate them!*

→ ‚classical' supergravity analysis not sufficient

→ need to include one-loop generated Chern-Simons terms

‚classical' supergravity + one-loop Chern Simons terms = microscopics

Part II

C. 4d Black Holes from D3-branes

B. MAKING 4D BLACK HOLES WITH D3-BRANES

background: $\mathbb{R}_t \times S^1 \times \quad \mathbb{R}^4_\perp \quad \times \qquad B_2$

	X^0	X^1	$X^2 \cdots X^5$	X^6	X^7	X^8	X^9
D3	$-$	$-$	$\cdot$	$-$	$-$	$\cdot$	$\cdot$

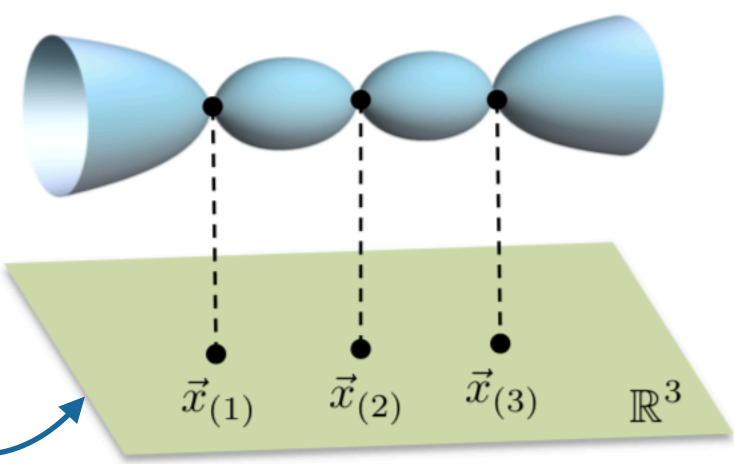
B. MAKING 4D BLACK HOLES WITH D3-BRANES

background: $\mathbb{R}_t \times S^1 \times \color{red}{\mathrm{TN}_m} \times B_2$

	X^0	X^1	$X^2 \dots X^5$	X^6	X^7	X^8	X^9
D3	—	—	·	—	—	·	·

- geometry of TN_m

$\mathrm{U}(1)_L \times \mathrm{SU}(2)_R$



centers on top: A_{m-1} singularity

interpolates between $\mathbb{C}^2/\mathbb{Z}_m$ & $\mathbb{R}^3 \times S^1$

carries ‚topological charge'

- near horizon geometry of D3-branes:

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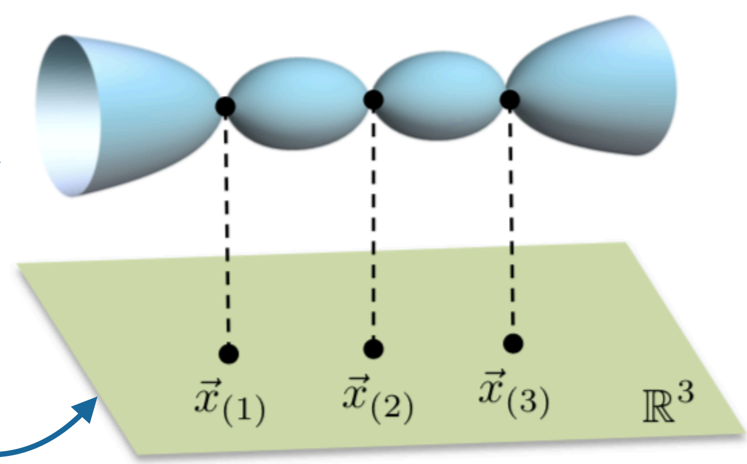
B. MAKING 4D BLACK HOLES WITH D3-BRANES

background: $\mathbb{R}_t \times S^1 \times \mathrm{TN}_m \times B_2$

	X^0	X^1	$X^2 \cdots X^5$	X^6	X^7	X^8	X^9
D3	—	—	·	—	—	·	·

- geometry of TN_m

$\mathrm{U}(1)_L \times \mathrm{SU}(2)_R$



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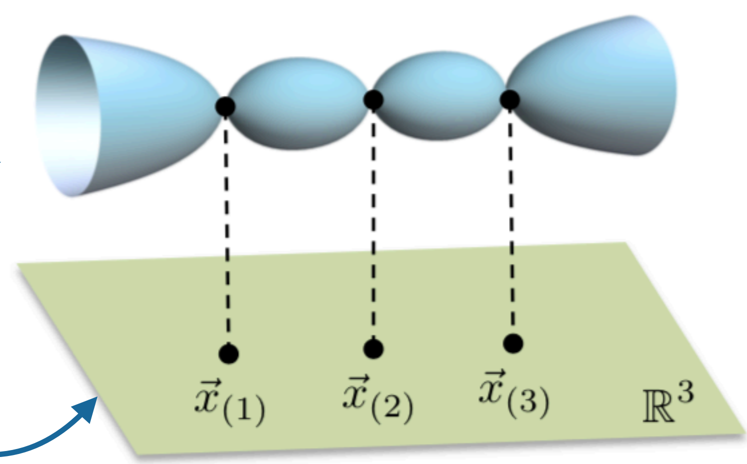

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MAKING 4D BLACK HOLES WITH D3-BRANES

Charges of the setup

q^α : characterize C

n : KK momentum around S^1

m : topological charge of TN_m $$p_1(\mathrm{TN}_m) = -\frac{1}{2}\int \mathrm{tr}\,\mathcal{R}\wedge\mathcal{R} = 2m$$

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Goals: *(i) compute* $c_{L,R}$, $k_{L,R}$ *in terms of the charges* q^α, m

(ii) compare macroscopics with microscopics

(iii) extend F-theory dictionary: physics ⟷ *geometry/topology*

MICROSCOPICS

- N D3-branes probing $\mathbb{Z}_m$ singularity [Kachru, Silverstein `98, Lawrence, Nekrasov, Vafa `98]

4d $\mathcal{N}=2$ SCFT: $\mathrm{SU}(N)^m$ quiver + m bifundamental hypers

$$\tau_i = \frac{\tau}{m}$$

type IIB axio-dilaton

- wrapped over $C \subset B_2$ → complexified gauge coupling τ varies over C

need for top. *duality* twist

Some issues with this approach:

→ ignoring D3-D7 string contributions

→ no (obvious) generalization of top. duality twist known…

microscopics → ***use suitable dual M-theory description***

TWO DUALITY CHAINS

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➡ ***microstate counting difficult task***

If $q^\alpha \gg mc^\alpha$

→ can eff. describe M5 system as

Single M5 on $\hat{C}_m = \hat{C} + mB_2$

→ ***MSW CFT techniques!***

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[Maldacena, Strominger, Witten `97]

- M-theory compactified on compact CY_3 ➔ 5d $\mathcal{N}=2$ supergravity

5d BPS string states: M5-brane wrapped on $[P] \in H_4(\mathrm{CY}_3)$

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compactify M5 brane theory on P → compute effective 2d spectrum

↳ counted by top. numbers

$$c_L = P \cdot P \cdot P + c_2(\mathrm{CY}_3) \cdot P$$

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→ ***apply this to our case of interest***

[Bena, Diaconescu, Florea `06]

MICROSCOPICS VIA MSW

Compute the following integrals for ell. fibered CY_3

$$P^3 = \int_{\mathrm{CY}_3} (q^\alpha \omega_\alpha + m\omega_0)^3 = 3mC \cdot C - 3m^2 c_1(B_2) \cdot C + m^3 c_1(B_2)^2$$

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to get k_L one has to work a bit harder...

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D. Computation of central charges and levels: Macro

6D N=(1,0) SUPERGRAVITY & F-THEORY

- *starting point:* 6d $\mathcal{N}=(1,0)$ supergravity describing F-theory on CY_3

[Ferrara, Minasian, Sagnotti `97; Bonetti, Grimm `11]

- 6d $\mathcal{N}=(1,0)$ supergravity has 4 types of multiplets

gravity	$g_{\mu\nu} \oplus \psi_L \oplus B^+_{\mu\nu}$	n_V *vector*	$A_\mu \oplus \lambda_L$
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tensor multiplets in supergravity

- ➔ denote collectively tensor and gravity multiplet two forms B^α, $\alpha = 1, \ldots, n_T + 1$
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obey quadratic constraint $\Omega_{\alpha\beta} j^\alpha j^\beta = \boldsymbol{j} \cdot \boldsymbol{j} = 1$

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more on tensor multiplets in supergravity

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$$J_{\mathrm{CY}_3} = v^0 \omega_0 + v^\alpha \pi^*(\omega_\alpha)$$

$$C_3 = A^\alpha \wedge \pi^*(\omega_\alpha) + \dots$$

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- number of multiplets determined by top. data $n_T = h^{1,1}(B_2) - 1$

hypermultiplets in supergravity

- hypermultiplet scalars q^U , $U = 1, \dots, 4n_H$

  parametrize quaternionic manifold

  play no further role in supergravity analysis, later more….

always assume in the following anomaly cancellation $n_H - n_V = 273 - 29n_T$

total 6d N=(1,0) action (two derivatives)

$$S_{6\mathrm{d}} = \frac{1}{(2\pi)^3}\int_{M_6} \frac{1}{2}R * 1 - \frac{1}{4}g_{\alpha\beta}G^\alpha \wedge *G^\beta - \frac{1}{2}g_{\alpha\beta}\mathrm{d}j^\alpha \wedge *\mathrm{d}j^\beta - \frac{1}{2}h_{UV}\mathrm{d}q^U \wedge *\mathrm{d}q^V$$

equations of motion have black string solution

$$\mathrm{AdS}_3 \times S^3/\mathbb{Z}_m \to \mathbb{R}^{1,1} \times \mathrm{TN}_m$$

near horizon ⟶ **asymptotics**

black string solution: some properties [het Lam, Vandoren `18]

$$\mathrm{d}s^2_{6\mathrm{d}} = 2H^{-1}\mathrm{d}u\big(\mathrm{d}v - \frac{1}{2}H_5\mathrm{d}u\big) + H\mathrm{d}s^2(\mathrm{TN}_m)$$

$$H = \sqrt{\boldsymbol{H_1}\cdot\boldsymbol{H_1}}$$

$$H_1^\alpha = \mu_\infty^\alpha + \frac{Q^\alpha}{4r}$$

$r \to 0$

near horizon (IR) limit

$$\mathrm{AdS}_3 \times S^3/\mathbb{Z}_m \quad \text{with} \quad R^2(S^3/\mathbb{Z}_m) = m\sqrt{\boldsymbol{Q}\cdot\boldsymbol{Q}}$$

→ in addition: non-trivial profile of j^α and G^α

6d tensor branch attractor flow

→ Q^α is macro charge of the string under B^α

$$\int_{\partial\mathrm{TN}_m} G^\alpha = (2\pi)^3 Q^\alpha$$

How to relate the macro charges Q^α to the micro data $(q^\alpha, m) \Leftrightarrow (C, m)$?

Connecting micro and macro data

- consider D3-brane wrapped around $C \subset B_2$, extended along Σ in 6d
- string couples to a two-form $S_{\text{string}} = -\mathcal{Q}\int_\Sigma B = -\mathcal{Q}\int_{M_6} B \wedge \delta(\Sigma)$
- total action $S_{\text{tot}} = S_{\text{6d}} + S_{\text{string}}$

reduce CS action of D3-branes on C

$$S_{\text{string}} = -\frac{N}{2\pi}\int_{\Sigma\times C} C_4^+ = -\frac{N}{2\pi}\int_{\Sigma\times C} B^\alpha \wedge \omega_\alpha = -\frac{N}{2\pi}\int_\Sigma B^\alpha \int_C \omega_\alpha = -\frac{N}{2\pi}\int_\Sigma q^\alpha \eta_{\alpha\beta} B^\beta$$

equation of motion of tensors

$$\mathrm{d}\big(g_{\alpha\beta} * G^\beta\big) = (2\pi)^2 \eta_{\alpha\beta} N q^\beta \delta(\Sigma)$$

integrate

$$\int_{\text{TN}_m} \mathrm{d}\big(g_{\alpha\beta} * G^\beta\big) = \eta_{\alpha\beta}\int_{\partial\text{TN}_m} G^\beta = (2\pi)^2\eta_{\alpha\beta}Q^\beta = (2\pi)^2 N \eta_{\alpha\beta} q^\beta$$

$$\boxed{Q^\alpha = N q^\alpha}$$

Subleading contributions: higher-derivative corrections

6d effective action obtains higher-derivative corrections

$$S_{\text{hd}} \sim \int_{M_6} \eta_{\alpha\beta} c^\alpha B^\beta \wedge \operatorname{tr} \mathcal{R} \wedge \mathcal{R}$$

→ relevant for gen. Green-Schwarz mechanism [Sagnotti `92; Sadov `96]

Subleading contributions: higher-derivative corrections

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Where does this come from in M-theory/Type IIB picture?

M-theory

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$$S_{\rm 11d} = \int_{M_{11}} C_3 \wedge \left[{\rm tr}\, \mathcal{R}^4 - \frac{1}{4} ({\rm tr}\, \mathcal{R}^2)^2 \right]$$

compactify on CY_3 ⟶ $$S_{\rm 5d} = \int_{M_5} c_{2\,\alpha} A^\alpha \wedge {\rm tr}\, \mathcal{R} \wedge \mathcal{R}$$

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F-theory uplift ⟶

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10d bulk effective action does **not** contain such a correction

- τ is varying: localized sources embedded in 10d space-time
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$$C_4^+ = B^\alpha \wedge \omega_\alpha$$

- $$S_{R^2} = \frac{1}{(2\pi)^3}\frac{1}{96}\int_{M_6} B^\alpha \wedge \mathrm{tr}\,\mathcal{R}\wedge\mathcal{R}\Big(\int_{\rm D7}\omega_\alpha + 2\int_{\rm O7}\omega_\alpha\Big)$$

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- $$\int_{\rm D7}\omega_\alpha + 2\int_{\rm O7}\omega_\alpha = \int_{B_2}\omega_\alpha \wedge \big([\mathrm{D7}] + 2[\mathrm{O7}]\big)$$
- elliptic fibration is Calabi-Yau → $[\Delta] = [\mathrm{D7}] + 2[\mathrm{O7}] = 12 c_1(B_2)$

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- $$\int_{\rm D7}\omega_\alpha + 2\int_{\rm O7}\omega_\alpha = \int_{B_2}\omega_\alpha\wedge([{\rm D7}]+2[{\rm O7}]) = 12\int_{B_2}\omega_\alpha \wedge c_1(B_2) = 12\eta_{\alpha\beta}c^\beta$$
- elliptic fibration is Calabi-Yau ➔ $[\Delta] = [{\rm D7}] + 2[{\rm O7}] = 12 c_1(B_2)$

(„D7 tadpole cancellation")

equation of motion of tensors…again

$$\mathrm{d}\big(g_{\alpha\beta} * G^{\beta}\big) = (2\pi)^2 \eta_{\alpha\beta} N q^{\beta} \delta(\Sigma) + \frac{1}{8} \eta_{\alpha\beta} c^{\beta} \mathrm{tr}\, \mathcal{R} \wedge \mathcal{R}$$

↓ integrate

- micro-macro charge relation $Q^{\alpha} = N q^{\alpha} + \frac{1}{8} \frac{1}{(2\pi)^2} c^{\alpha} \int_{\mathrm{TN}_m} \mathrm{tr}\, \mathcal{R} \wedge \mathcal{R}$

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$$\Rightarrow \boxed{Q^{\alpha} = N q^{\alpha} - \frac{m}{2} c^{\alpha}}$$

charge shift due to non-trivial topology of Taub-NUT space

- in the following: set $N = 1$

(i) „classical“ contribution

HOW TO COMPUTE LEVELS & CENTRAL CHARGES?

- central charges and levels are related to anomalies on the 2d CFT side

$c_L - c_R$ ⟷ *gravitational anomaly*

k_R ⟷ *`t Hooft anomaly of* $\mathrm{SU}(2)_R$ *– R-symmetry*

k_L ⟷ *`t Hooft anomaly of* $\mathrm{SU}(2)_L/\mathrm{U}(1)_L$

→ *bulk description known*

- compactify to 3d, e.g. $\mathrm{AdS}_3 \times S^3/\mathbb{Z}_m$
- gauge isometries of space you reduce on $\mathrm{U}(1)_L \times \mathrm{SU}(2)_R$

→ produces gauge fields in Kaluza-Klein ansatz A_L, A_R

- from compactified action read off $k_{L,R},\, c_L - c_R$

HOW TO COMPUTE LEVELS & CENTRAL CHARGES?

Concretely: look for Chern-Simons terms in the 3d action and read off

$$S_{\rm CS} = \frac{k_L}{8\pi}\int A_L \wedge {\rm d}A_L + \frac{k_R}{4\pi}\int {\rm tr}\left(A_R\wedge {\rm d}A_R + \tfrac{2}{3}A_R^3\right) + \frac{c_L - c_R}{96\pi}\int {\rm tr}\left(\omega\wedge{\rm d}\omega + \tfrac{2}{3}\omega^3\right)$$

$\mathrm{U}(1)_L$ CS-term — $\mathrm{SU}(2)_R$ CS-term — grav. CS-term

[Witten `98; Kraus, Larsen `05, Hansen, Kraus `06; …]

- read off k_R $\xrightarrow{\text{use } \mathcal{N}=(0,4)}$ $c_R = 6k_R$ $\xrightarrow{\text{use grav. CS}}$ $c_L = c_R + (c_L - c_R)$
- read off k_L

→ ***central charges and levels of the N=(0,4) SCFTs fully encoded in Chern-Simons terms***

‚CLASSICAL' LEVELS & CENTRAL CHARGES

- reduce two-derivative supergravity action & collect Chern–Simons terms

$$S_{2-\text{der}} = \frac{1}{2} m \eta_{\alpha\beta} Q^\alpha Q^\beta \left[\frac{1}{8\pi} \int \omega_{\text{CS}}(A_L) + \frac{1}{4\pi} \int \omega_{\text{CS}}(A_R) \right]$$

$$k_L^{\text{2-der}} = k_R^{\text{2-der}} = \frac{1}{6} c_R^{\text{2-der}} = \frac{1}{2} m \eta_{\alpha\beta} Q^\alpha Q^\beta = \frac{1}{2} m \left(q^\alpha - \frac{m}{2} c^\alpha \right) \left(q^\beta - \frac{m}{2} c^\beta \right)$$

$$(c_L - c_R)^{\text{2-der}} = 0$$

- **subleading contributions:** higher-derivative corrections

→ want to produce CS-terms for the KK-gauge fields $\sim \int \eta_{\alpha\beta} c^\alpha B^\beta \wedge \text{tr}\, \mathcal{R} \wedge \mathcal{R}$

contains $\omega_{\text{CS}}^{\text{grav}}$ → $c_L - c_R \neq 0$

‚CLASSICAL' LEVELS & CENTRAL CHARGES

- reduce **four**-derivative supergravity action & collect Chern–Simons terms

$$S_{\text{4-der}} = \frac{1}{16\pi}\eta_{\alpha\beta}c^\alpha\left(q^\beta - \frac{1}{2}mc^\beta\right)\left[\int \omega_{\text{CS}}^{\text{grav}} + 4\omega_{\text{CS}}(A_R)\right]$$

$$\Rightarrow \quad k_L^{\text{4-der}} = 0$$

$$k_R^{\text{4-der}} = \eta_{\alpha\beta}c^\alpha\left(q^\beta - \frac{m}{2}c^\beta\right)$$

$$(c_L - c_R)^{\text{4-der}} = 6\eta_{\alpha\beta}c^\alpha\left(q^\beta - \frac{m}{2}c^\beta\right)$$

- adding up two– and four–derivative contributions

$$c_L^{\text{class}} = 3mC^2 - 3m^2c_1(B)\cdot C + \frac{3}{4}m^3c_1(B)^2 + 12c_1(B)\cdot C - 6mc_1(B)^2,$$

$$c_R^{\text{class}} = 6k_R^{\text{class}} = 3mC^2 - 3m^2c_1(B)\cdot C + \frac{3}{4}m^3c_1(B)^2 + 6c_1(B)\cdot C - 3mc_1(B)^2,$$

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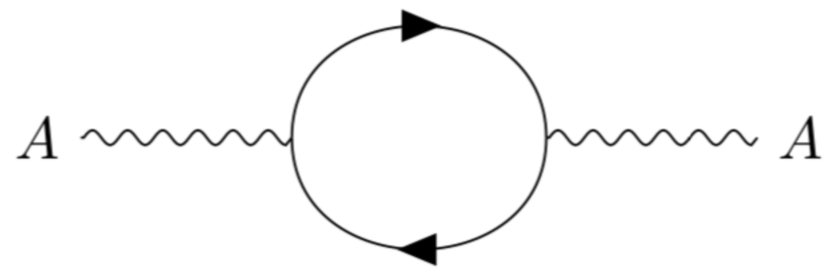
Do we recover the micro result? ***Not even close… what did we miss?***

(ii) „quantum“ contribution

One-loop Chern-Simons terms

- ***fact:*** Chern-Simons terms can be loop induced



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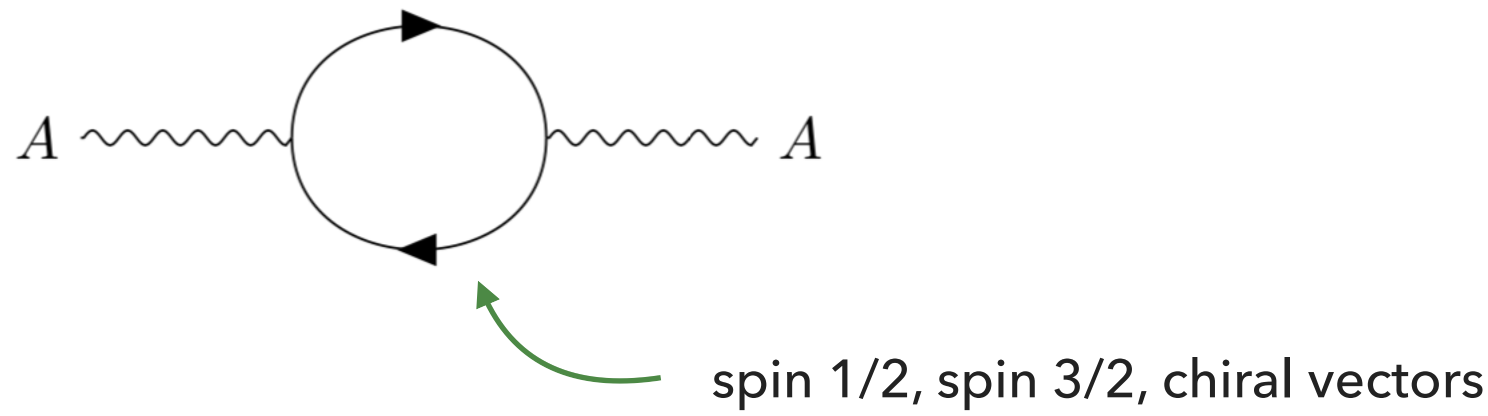

- loop induced Chern-Simons terms give corrections to $k_{L,R},\, c_L - c_R$

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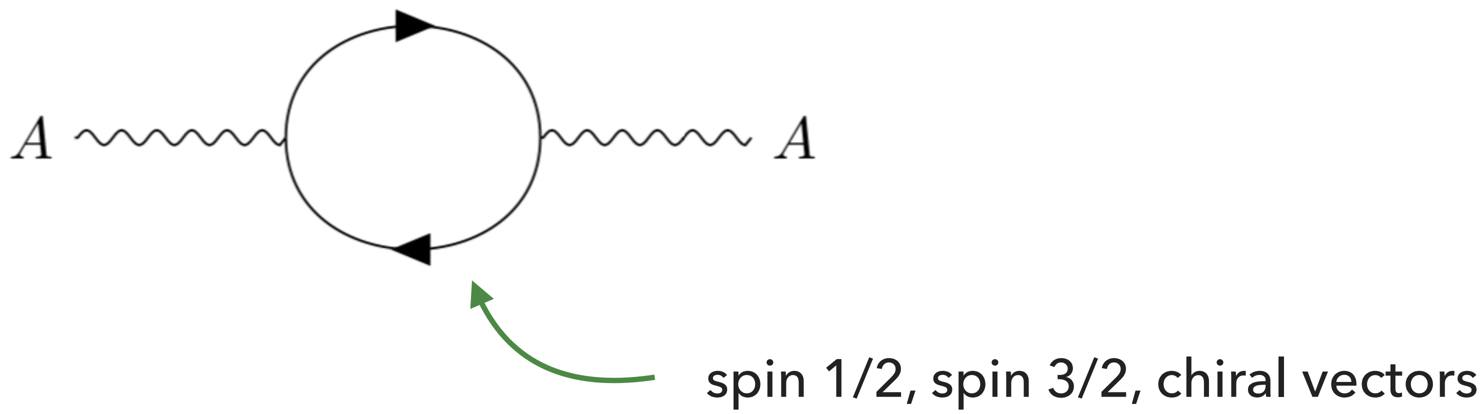

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Three-step plan to success

(1) determine (relevant) Kaluza-Klein spectrum on $S^3/\mathbb{Z}_m$

→ use known spectrum of $\mathcal{N}=(2,0)$ on S^3 ↷ charged under $\mathrm{SO}(4)=\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$

→ truncate to $\mathcal{N}=(1,0)$ spectrum

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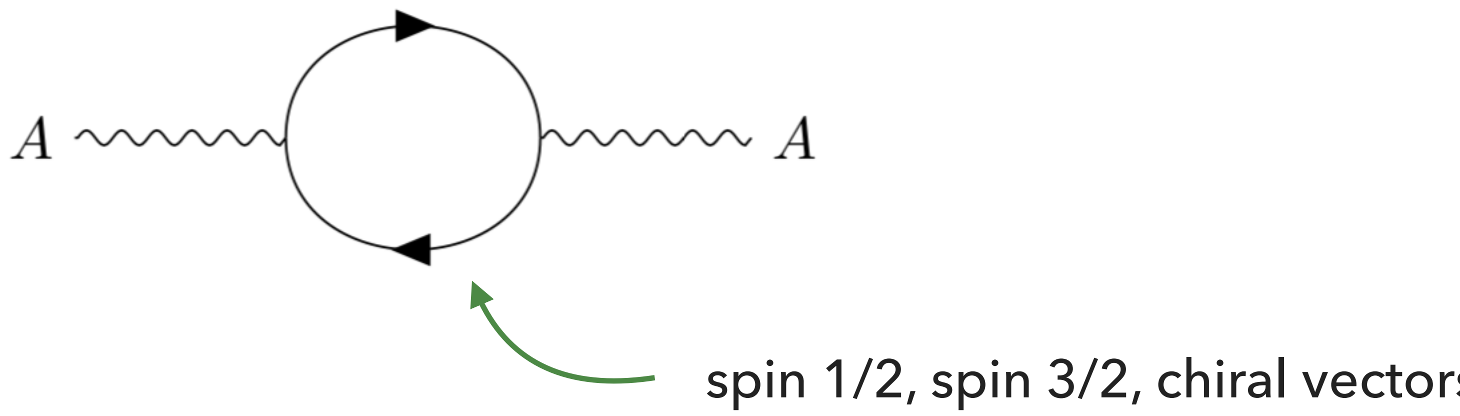

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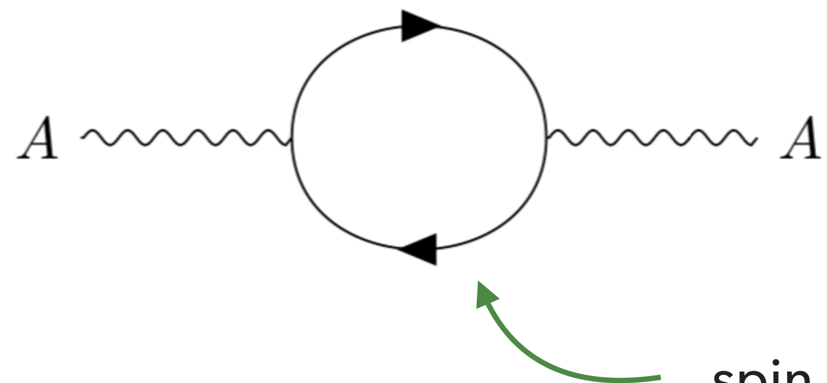

- loop induced Chern-Simons terms give corrections to $k_{L,R}$, $c_L - c_R$

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(3) sum over Kaluza-Klein tower projected on $\mathbb{Z}_m$– invariant states (ζ- function reg.)

(1) Kaluza-Klein spectrum

- Kaluza-Klein spectrum of (2,0) theory worked out by [Deger, Kaya, Sezgin, Sundell `98; de Boer `98]
- Kaluza-Klein tower transforms under $\mathrm{SO}(4) = \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$

  $j_L \qquad\qquad j_R$

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- additional information: sign of the mass matrix eigenvalue $\mathrm{sgn}(M)$

notation $(j_L, j_R)^{\mathrm{sgn}(M)}$

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spin 3/2 $$2 \bigoplus_{j_L = \frac{1}{2}, \frac{3}{2}, \ldots}^{\infty} (j_L, j_L \pm \tfrac{1}{2})^{\mp}$$

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spin 1/2

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(2) One-loop Chern-Simons terms

- need to consider three species of fields spin 1/2 spin 3/2 chiral vectors

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	spin 1/2	spin 3/2	vectors
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$c_L - c_R$			

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(3) Sum over invariant states

- **main question:** how does the $\mathbb{Z}_m$ quotient act on the Kaluza-Klein states?

  → check how $\mathbb{Z}_m$ acts on spherical harmonics on S^3

  → group theory exercise…

- quotient is generated by $\mathbb{Z}_m \subset \mathrm{U}(1)_L \subset \mathrm{SU}(2)_L$

$(2j_L + 1)$ – dimensional irrep of $\mathrm{SU}(2)_L$ → $2j_L$ – fold sym. tensor product of $\mathbf{2}$

- $\mathbb{Z}_m$ acts on $\mathbf{2}$ as $\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \to \begin{pmatrix} \mathrm{e}^{\frac{2\pi i}{m}} & 0 \\ 0 & \mathrm{e}^{-\frac{2\pi i}{m}} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$

  → ***induces action on symmetric tensor representation***

- projection condition can be easily calculated

$$j_L^{(3)} = \frac{1}{2} m k \,, \qquad k \in \mathbb{Z}$$

Summary

Kaluza-Klein spectrum

spin 3/2

$$2 \bigoplus_{j_L=\frac{1}{2},\frac{3}{2},\dots}^{\infty} \left(j_L, j_L \pm \tfrac{1}{2}\right)^{\mp}$$

spin 1/2

$$2 \bigoplus_{j_L=\frac{3}{2},\frac{5}{2},\dots}^{\infty} \left(j_L, j_L \pm \tfrac{3}{2}\right)^{\mp} \oplus 2 \bigoplus_{j_L=1,2,\dots}^{\infty} \left(j_L, j_L \pm \tfrac{1}{2}\right)^{\pm} \oplus 2(n_T + n_H) \bigoplus_{j_L=\frac{1}{2},\frac{3}{2},\dots}^{\infty} \left(j_L, j_L \pm \tfrac{1}{2}\right)^{\pm}$$

vectors

$$\bigoplus_{j_L=1,2,\dots}^{\infty} \left(j_L, j_L \pm 1\right)^{\mp} \oplus n_T \bigoplus_{j_L=1,2,\dots}^{\infty} \left(j_L, j_L \pm 1\right)^{\pm}$$

Loop corrections

	spin 1/2	spin 3/2	vectors
k_L	$\mathrm{sgn}(M)(j_L^{(3)})^2(2j_R+1)$	$3\mathrm{sgn}(M)(j_L^{(3)})^2(2j_R+1)$	$-2(j_L^{(3)})^2(2j_R+1)$
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Projection condition

$$j_L^{(3)} = \frac{1}{2}mk \qquad k \in \mathbb{Z}$$

left level k_L

$$k_L^{\text{sugra}} = \frac{1}{2}mC\cdot C - \frac{1}{2}m^2 c_1(B_2)\cdot C + \frac{1}{8}m^3 c_1(B_2)\cdot c_1(B_2)$$

$$k_L^{\text{quantum}} = -\frac{1}{8}m^3 c_1(B_2)\cdot c_1(B_2)$$

→ $$k_L^{\text{total}} = \frac{1}{2}mC\cdot C - \frac{1}{2}m^2 c_1(B_2)\cdot C = k_L^{\text{micro}}$$

right level k_R

$$k_R^{\text{sugra}} = \frac{1}{2}mC\cdot C - \frac{1}{2}m^2 c_1(B_2)\cdot C + \frac{1}{8}m^3 c_1(B_2)^2 + c_1(B_2)\cdot C - \frac{1}{2}mc_1(B_2)^2$$

$$k_R^{\text{quantum}} = \frac{m^3}{24}c_1(B_2)^2 + \frac{m}{3}c_1(B_2)^2 + m$$

→ $$c_R^{\text{total}} = 6k_R^{\text{total}} = 3mC\cdot C - 3m^2 c_1(B_2)\cdot C + m^3 c_1(B_2)^2 + 6c_1(B_2)\cdot C + 6m - mc_1(B_2)^2$$

gravitational Chern-Simons level $c_L - c_R$

$$(c_L - c_R)^{\text{sugra}} = 6c_1(B_2)\cdot C - 3mc_1(B_2)\cdot c_1(B_2)$$

$$(c_L - c_R)^{\text{quantum}} = 6m + 2mc_1(B_2)\cdot c_1(B_2)$$

→ $$c_L^{\text{total}} = c_R^{\text{total}} + (c_L - c_R)^{\text{total}}$$
$$= 3mC\cdot C - 3m^2c_1(B_2)\cdot C + m^3c_1(B_2)^2 + 12c_1(B_2)\cdot C + 12m - 2mc_1(B_2)^2$$

final result

$$c_L = 3mC\cdot C - 3m^2c_1(B_2)\cdot C + m^3c_1(B_2)^2 + 12c_1(B_2)\cdot C + 12m - 2mc_1(B_2)^2$$
$$c_R = 6k_R = 3mC\cdot C - 3m^2c_1(B_2)\cdot C + m^3c_1(B_2)^2 + 6c_1(B_2)\cdot C + 6m - mc_1(B_2)^2$$
$$k_L = \frac{1}{2}mC\cdot C - \frac{1}{2}m^2c_1(B_2)\cdot C$$

→ ***matches microscopic prediction!***

E. Other families: ADE Black Holes

D. Other families: ADE Black holes

- up to now restricted to Taub-NUT space ➞ $\mathbb{C}^2/\mathbb{Z}_m$ singularity close to center
- ***recall:*** F-theory AdS_3 solutions preserving $\mathcal{N}=(0,4)$ SUSY have ADE classification

$$\mathrm{AdS}_3 \times S^3/\Gamma_{\mathrm{ADE}} \times (\mathrm{CY}_3 \to B_2)$$

$\Gamma_{\mathrm{A}} = \mathbb{Z}_m \subset \mathrm{SU}(2)$ cyclic group

$\Gamma_D = \mathbb{D}^*_m \subset \mathrm{SU}(2)$ bin. dihedral group

$\Gamma_E = \mathbb{T}^*, \mathbb{O}^*, \mathbb{I}^* \subset \mathrm{SU}(2)$

- microscopic interpretation: D3-brane with transverse *ALF space*

hyperkähler manifold with $\mathbb{C}^2/\Gamma_{\mathrm{ADE}}$ singularity

- no (straightforward) dual M5-brane picture in M-theory

➞ MSW techniques for micro don't apply

However: macroscopic computation can be done! ✔

F. Summary

→ *studied IR CFT of wrapped D3-branes in F-theory*

→ *computed central charges and levels of 2d N=(0,4) IR SCFT*

↳ determine entropy

→ $c_{L,R}$ and $k_{L,R}$ *are determined by geometric/topological data in the setup*

- first Pontryagin number of Taub-NUT space m
- first Chern class of the base of the fibration $c_1(B_2)$
- (co-)homology class of the curve wrapped by the D3 C

→ *obtained matching of micro and macro result (for a fairly involved expression)*

→ *one–loop Chern–Simons terms crucial for the matching*

F. Outlook

→ *study the remaining possible quotients in the ADE classification*

macro computation done ✓

micro difficult

generalization of top. duality twist?

→ *wrapped D3-branes probing singularities in F-theory*

→ *look at D3-branes in Type IIB on K3 x T^2 with ADE quotients*

*does **not** follow by $c_1(B_2 = K3) = 0$*

→ *both macroscopics and microscopics under good control, yet non-trivial*